Shocking Stuff: Technology, Hours, and Factor Substitution*

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Abstract

The reaction of hours worked to technology shocks represents one of the key controversies between Real Business Cycle and New Keynesian explanations of the business cycle. It sparked a large empirical literature with contrasting results. However, we demonstrate that, with a more general and data coherent supply and production framework ("normalized" factor-augmenting CES technology), both models can plausibly generate impacts of either sign. We further develop an analytical rule which establishes the threshold between positive and negative contemporaneous correlations for both models. The key margin determining its sign lies in the wedge between the substitution elasticity and the capital income share, and the factor bias of technology shocks. In the New Keynesian model this rule is also dependent on the response of marginal costs. Our findings are supplemented by an extensive robustness analysis. We conclude that the impact of technology on hours can hardly be taken as evidence in support of any particular business-cycle model. Our results, however, may help interpret the possible time-variation in technology and hours correlations over time.

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1 Introduction

The reaction of hours worked to a technology shock has been a key controversy in macroeconomics over the last decade. According to the standard real business cycle (RBC) model, hours worked should rise after a (positive) productivity shock. However, in an influential paper, Galí (1999), using a structural VAR (SVAR) with long-run restrictions, found the impact to be negative. This evidence has since been interpreted as favoring the New-Keynesian (NK) sticky-price model of business-cycle fluctuations.[1]

Subsequent literature was mostly supportive of a negative correlation between technology and hours (e.g., Francis, Owyang and Theodorou (2003), Francis and Ramey (2005)). Christiano, Eichenbaum and Vigfusson (2003), though, challenged these results arguing that they were driven by the way researchers treat hours worked; using hours on a per capita basis, they found a positive hours-technology short run correlation.

Econometric identification, hence, took center stage in the debate. Fernald (2007) emphasized the importance of low-frequency trend-breaks in productivity and found a negative impact of technology on hours. Dedola and Neri (2007) use sign restrictions for VAR identification and found that hours worked are likely to increase. Uhlig (2004), using a medium-run identification scheme, also finds support for a mildly positive impact. Pesavento and Rossi (2005), use an agnostic method that does not require choosing between a specification in levels or in first differences, found hours worked fall after a technology shock, but that the effect is short lived. Chari, Kehoe and McGrattan (2008), however, maintain that SVAR models are incapable of distinguishing between different explanations of the business cycle (i.e. RBC .v. sticky prices). They argue that, if non-technology shocks account for an important part of business-cycle fluctuations, SVAR models can erroneously favor a NK model when the data has been generated using an RBC model. Basu, Fernald and Kimball (2006) construct a measure of aggregate technology change from sectoral-level data. This method is free from the shortcomings of the SVAR identification restrictions. They found technological change and factor inputs to be negatively correlated. They conclude that technology improvements are contractionary on impact, which would constitute an important

[1] It is well known that a standard RBC model could also generate a negative technology-hours response if the coefficient of relative risk aversion is sufficiently high (above unity). Rotemberg (2003) also showed that an RBC model with protracted technical diffusion could generate a negative technology-hours correlation. In the NK case, the presence of nominal rigidities (i.e., that some fraction of firms cannot reset current prices) implies that aggregate demand grows by less than the growth in technology, generally entailing a initial reduction in employment. See Galí (2008) and Galí and Rabanal (2004) for thorough discussions. Fernández-Villaverde (2009) provides an effective review and surveys of modern DSGE models.

stylized fact. Finally, recent work is suggestive of a time-varying correlation, e.g., Galí and Gambetti (2009).[2]

The effect of technical change on employment is, in fact, a long-standing debate in economics - see, for example Wicksell (1911)'s discussion of the historical "machinery question". The traditional Ricardian effect - defended by Hicks (1969) - supported the idea that technological advancement reduces employment in the short run, but increases it in the long run.[3] The kind of mechanism envisaged, however, did not rest on the introduction of nominal rigidities that characterizes much of modern macroeconomics, but on aspects of the production process: such as the degree to which different factors substitute or complement one another and the extent to which technical change is non neutral.

Modern business cycle models, though, have generally abstracted from these aspects.[4] They tend to impose aggregate (unitary elasticity) Cobb-Douglas production which is both highly restrictive and uninformative regarding biases in technical change. The choice may be considered startling given the avowed interest of the literature in promoting (or testing) the cyclical importance of "technology" shocks. In that light, our work may be judged as synthesizing developments in production and growth theory - where non-unitary substitution elasticities and factor-augmenting technology shocks are relied upon to describe various economic phenomena (e.g., Acemoglu (2009)) - with developments in dynamic stochastic general equilibrium models used to explain data developments and inform policy.

Indeed, there is now mounting evidence that aggregate production may be better characterized by a *non-unitary* substitution elasticity and especially so at business-cycle frequencies; Chirinko (2008) suggests 0.4-0.6 as a benchmark range for the US substitution elasticity.[5] Moreover, as Acemoglu (2009) (Ch. 15) com-

[2]Alexopoulos (2010) pursues a novel approach: using an index of information technology publications, she finds output and (albeit to a small extent) employment rise following a technology shock in line with the RBC interpretation and also a NK model with accommodative policy.

[3]This is in contrast to the Marxian theories that supported the existence of permanent (negative) effects on employment (see Beach (1971) for a discussion).

[4]Francis and Ramey (2005) examined the hours correlation in the (non-normalized) Leontief case. Although at business-cycle frequencies low substitution elasticities might be expected, *zero* factor substitution is a very strong assumption with the counter-factual implication of common business-cycle volatility for capital and labor. Furthermore, in the technology and growth literature, the Leontief form is usually ruled out given its dis-equilibrium implications for growth and optimal savings, e.g., Barro and Sala-i-Martin (2004).

[5]Jones (2003) and Jones (2005) argued that capital shares exhibit such protracted swings and trends in many countries as to be inconsistent with Cobb-Douglas (see also Blanchard (1997), McAdam and Willman (2008)). Building on Houthakker (1955)'s idea that production combinations reflect the (Pareto) distribution of innovation activities, he proposes a "nested" production function. Given such parametric innovation activities, this function will exhibit a

ments, there is little reason to suppose that over business-cycle frequencies, technical change will be neutral or embody balanced growth characteristics: technical change typically benefits some factors and some agents more than others. Furthermore, the acclaimed acceleration in labor productivity during the second half of the 1990s (see Basu et al. (2003), Fernald and Ramnath (2004) and Jorgenson (2001)) underpins the need for a careful incorporation of technical improvements.[6]

When investigating the ramifications of a non-unitary substitution elasticity and factor-augmenting technology shocks in dynamic macro-models one necessarily faces the issue of *normalization*, even though the topic is not yet widely known (following the seminal contributions of La Grandville (1989*b*) and Klump and de La Grandville (2000)).[7] Normalization essentially implies representing the supply side of the model (i.e., production function and factor returns) in consistent indexed number form. In our context, normalization turns out to be absolutely crucial to ensure the validity of comparative statics, and for meaningful and consistent calibration of the deep parameters of the supply side of the model.

Accordingly, while remaining agnostic about empirical identification methods, we generalize the supply side of both standard RBC and NK models. In doing so, we demonstrate that both models can yield positive or negative responses of hours worked to technology shocks. We further derive a threshold rule for the sign of this response. The key margin involved is the whether the elasticity of substitution exceeds the capital income share and whether technology shocks are capital or labor saving (in the NK model policy reactions also matter). We explain the intuition behind this rule and carry out a comprehensive robustness analysis. This threshold rule may further help interpret the time variation in technology-hours correlations that some researchers report.

The paper is organized as follows. The next section discusses the importance of normalization alongside biased technical change in the more general Constant Elasticity of Substitution (CES) production function. Sections 3 and 4 present the RBC and NK models, which are used in simulation analysis. Section 5 discusses calibration. Section 6 presents and discusses the results and section **??** provides some sensitivity analysis. Section 7 derives a threshold rule determining the sign of the response of hours to technology shocks. Following that we touch

(far) less than unitary substitution elasticity over business-cycle frequencies but asymptote to Cobb-Douglas.

[6]On could add a great many other topics to this list of the importance of technical change and non-unitary substitution: the impact of technical change on the welfare consequences of new technologies (Marquetti (2003)); labor-market inequality and skills premia (Acemoglu (2002*b*)); the evolution of factor income shares and non-balanced growth (Acemoglu (2002*a*), McAdam and Willman (2008)), the efficacy of stabilization policy (e.g., Chirinko (2008)) etc.

[7]As far as we know, ours is the first attempt to incorporate and consistently implement normalized supply sides into fully-fledged micro-founded macro–economics models.

specifically on issues related to non-separability in the utility function. Section 8 discusses possible time variation in the technology-hours impact. Finally, section 9 concludes.

2 The Normalized CES Production Function

The technology assumption adopted by modern business cycle models has almost exclusively been Cobb Douglas. This functional form constrains the substitution elasticity between factors of production to unity and, therefore, is unable to separately identify capital and labor augmenting technology shocks. The more general CES production function, by contrast, nests Cobb-Douglas as a special case and admits the possibility of neutral and non-neutral technical change.

2.1 Normalization

At a simple level, one can think of normalization as removing the problem that arises from the fact that labor and capital are measured in different units - although its importance goes well beyond that. Under Cobb-Douglas, normalization plays no role since, due to its multiplicative form, differences in units are absorbed by the scaling constant. The CES function, by contrast, is highly non-linear, and so, unless correctly normalized, out of its three key parameters - the efficiency parameter, the distribution parameter, the substitution elasticity - only the latter is deep. The other two parameters turn out to be affected by the size of the substitution elasticity and factor income shares. Accordingly: i) if one is interested in model sensitivity with respect to production parameters (as here), normalization in indispensable to have interpretable comparisons; and ii) without a proper normalization, nothing ensures that factor shares equal the distribution parameter, hence invalidating inference based on impulse-response functions (IRFs).

Let us start with the general definition of a linear homogenous production function:

$$Y_t = F\left(\Gamma_t^k K_t, \Gamma_t^h H_t\right) = \Gamma_t^h H_t f\left(\kappa_t\right) \tag{1}$$

where Y_t is output, K_t capital and H_t the labor input. The terms Γ_t^k and Γ_t^h capture capital and labor-augmenting technical progress, respectively. To circumvent problems related to the "Diamond-McFadden impossibility theorem"[8], researchers usually assume specific functional forms for these functions, e.g., $\Gamma_t^k = \Gamma_0^k e^{z_t^k}$ and $\Gamma_t^h = \Gamma_0^h e^{z_t^h}$ where z_t^i can be a stochastic or deterministic technical progress function associated to factor i. The case where $z_t^k = z_t^h > 0$ denotes Hicks-Neutral technology; $z_t^k > 0$, $z_t^h = 0$ yields Solow-Neutrality; $z_t^k = 0$, $z_t^h > 0$ represents

[8]See Diamond and McFadden (1965), Diamond, McFadden and Rodriguez (1978).

Harrod-Neutrality; and $z_t^k > 0 \neq z_t^h > 0$ indicates general factor-augmentation. The term $\kappa_t = \left(\Gamma_t^k K_t\right) / \left(\Gamma_t^h H_t\right)$ is the capital-labor ratio in efficiency units. Likewise define $\varphi_t = y_t / \left(\Gamma_t^h H_t\right)$ as per-capita production in efficiency units.

The elasticity of substitution can then be expressed as:

$$\sigma = -\frac{f'(\kappa)\left[f(\kappa) - \kappa f'(\kappa)\right]}{\kappa f''(\kappa)\, f(\kappa)}. \tag{2}$$

This definition can be viewed as a second-order differential equation in κ having the following general CES production function as its solution:

$$\varphi_t = a\left[\kappa_t^{\frac{\sigma-1}{\sigma}} + b\right]^{\frac{\sigma}{\sigma-1}} \Rightarrow Y_t = a\left[\left(\Gamma_t^k K_t\right)^{\frac{\sigma-1}{\sigma}} + b\left(\Gamma_t^h H_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{3}$$

where a and b are two arbitrary constants of integration with the following correspondence with the original Arrow et al. (1961) non-normalized form, which, after some rearrangements can be presented in conventional form:

$$Y_t = \mathbb{C}\left[\alpha\left(\Gamma_t^k K_t\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(\Gamma_t^h H_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{4}$$

where efficiency parameter $\mathbb{C} = a(1+b)^{\frac{\sigma}{\sigma-1}}$ and distribution parameter $\alpha = 1/(1+b)$.

An economically meaningful identification of these integration constants a and b (and further by $\mathbb{C}$ and α) is given by the fact that σ is a point elasticity relying on three baseline (or "normalized", $t = 0$) values: a given capital intensity $\kappa_0 = \Gamma_0^k K_0 / \left(\Gamma_0^h H_0\right)$, a given marginal rate of substitution, $\frac{\partial(Y_0/H_0)}{\partial(Y_0/K_0)}$, and a given level of per-capita production $\varphi_0 = Y_0 / \left(\Gamma_0^h H_0\right)$. For simplicity, and without loss of generality, we scale the components of technical progress such that $\Gamma_0^k = \Gamma_0^h = 1$. Accordingly, we can transform (4) into the key normalized form,

$$Y_t = Y_0\left[\alpha_0\left(\frac{\Gamma_t^k K_t}{K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0)\left(\frac{\Gamma_t^h H_t}{H_0}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{5}$$

where distribution parameter $\alpha_0 = r_0 K_0 / (r_0 K_0 + w_0 H_0)$ has a clear economic interpretation: the capital income share evaluated at the point of normalization. We see that all parameters of (5) are deep, demonstrated by the fact the at the point of normalization, the left-hand-side equals the right-hand side for all values of σ, α_0 and the parameterization of Γ_t^k and Γ_t^h.

By contrast, comparing (4) with (5), the parameters of the non-normalized function depend on the normalized value of the factors and the factor returns as well as on the σ value itself:

$$\mathbb{C}(\sigma, \cdot) = Y_0 \left[\frac{r_0 K_0^{1/\sigma} + w_0 H_0^{1/\sigma}}{r_0 K_0 + w_0 H_0} \right]^{\frac{\sigma}{\sigma-1}} \tag{6}$$

$$\alpha(\sigma, \cdot) = \frac{r_0 K_0^{1/\sigma}}{r_0 K_0^{1/\sigma} + w_0 H_0^{1/\sigma}}. \tag{7}$$

Accordingly, in the non-normalized formulation, parameters $\mathbb{C}$ and α have no theoretical or empirical meaning. Hence, varying σ, whilst holding $\mathbb{C}$ and α constant, is inconsistent for comparative-static purposes. Each of the resulting CES functions goes through different fixed points and we can say that each resulting CES function belongs to "different families", La Grandville (2009).

Since, in the non-normalized case, parameter α depends on the point of normalization as well as on σ, it is obvious that the dynamic responses to shocks can change as we vary σ, since the elasticity of output w.r.t capital and labor will change. That is, by changing σ we would also be changing the capital and labor shares. Also, parameter α in a non-normalized CES will not match the capital share. Hence, in a calibrated dynamic simulation we would not be controlling for this important parameter, making IRFs invalid. In our dynamic general equilibrium setting, we are interested in the dynamic responses of variables in a stationary model. Hence, we need to ensure that factor shares in steady state (the initial and end point of our simulations) are constant and equal to α_0 and $1 - \alpha_0$. Also, output, capital, labor, consumption and factor payments are common at this point for different σ's. We hence choose to make the steady state our normalization point.

A logical way to proceed is then to choose a steady state and then calibrate the model using this as the normalization point. We can, for instance, set Y_0 and H_0 to 1. Since the real interest rate is determined by preferences and depreciation, we can then, given the income/factor income identity,

$$\underbrace{Y_0}_{=1} \equiv \underbrace{r_0 K_0}_{\alpha_0} + \underbrace{w_0 H_0}_{1-\alpha_0} \tag{8}$$

define the steady-state capital stock as $K^* = \alpha_0 / r_0$, where α_0 and r_0 are the capital income share and real interest rate at the chosen steady-state. The real normalized/steady-state wage is solved as $w^* = 1 - \alpha_0$. This ensures that the model is consistent, so factor shares sum to one (as does the CES production function) and consumption plus investment equals output.

Hence, we calibrate the model's parameters and enter all the supply side (CES and first order conditions for capital and labor) in normalized form. Crucially, this implies also that changing σ does not change our steady state or factor shares, IRFs are directly comparable, and parameter values are consistent with their economic interpretation.

2.2 CES Production and Factor-Augmenting Technology

The CES production function, (4) or (5), nests Cobb-Douglas when $\sigma = 1$; the Leontief function (i.e., fixed factor proportions) when $\sigma = 0$; and a linear production function (i.e., perfect factor substitutes) when $\sigma \to \infty$. Although factors are always substitutes, the higher is σ the greater the similarity between capital and labor. Thus, when $\sigma < 1$, we say that factors are gross complements in production and gross substitutes when $\sigma > 1$ (La Grandville (1989*a*), Acemoglu (2002*a*)).

In business-cycle models - RBC or NK - factor substitutability and non-neutral technical change will matter in so far as they influence developments in output, relative prices, factor intensities, income shares and cost pressures. Movements in these variables affect the inter-temporal decisions of consumers and firms. Some indications of the key role played by factor substitution can be gauged from the following.

Assuming competitive markets and profit maximization, relative factor income shares and relative marginal products are (dropping time subscripts):

$$\Theta = \frac{rK}{wH} = \frac{\alpha}{1-\alpha}\left(\frac{\Gamma^K K}{\Gamma^N H}\right)^{\frac{\sigma-1}{\sigma}} \tag{9}$$

$$\iota = \frac{F_K}{F_H} = \frac{r}{w} = \frac{\alpha}{1-\alpha}\left[\left(\frac{K}{H}\right)^{-\frac{1}{\sigma}}\left(\frac{\Gamma^k}{\Gamma^h}\right)^{\frac{\sigma-1}{\sigma}}\right] \tag{10}$$

It is straightforward to show that the effect of technical bias and capital deepening on factor income shares and factor prices is related to whether factors are gross complements or gross substitutes:

$$sign\left\{\frac{\partial \iota}{\partial\left(\Gamma^K/\Gamma^H\right)}\right\}, sign\left\{\frac{\partial \Theta}{\partial\left(K/H\right)}\right\}, sign\left\{\frac{\partial \Theta}{\partial\left(\Gamma^K/\Gamma^H\right)}\right\} = sign\left\{\sigma - 1\right\} \tag{11}$$

Consider also the following:

Definition
An increase in factor-J augmenting (J=K, H) technical change"favors" factor J (i.e., implying $F_j > F_i$ $(j \neq i)$ and raising J's income share, $p_j J$, for given factor proportions) if factors are gross substitutes $(\sigma > 1)$. *The effects reverse if factors are gross complements* $(\sigma < 1)$.

To illustrate, an increase in capital-augmenting technical change assuming gross complements decreases the relative marginal product and factor share of capital (and the opposite for gross substitutes). Hence, it is only in the gross-substitutes case that, for instance, that a factor J augmenting change in technology is J-biased (i.e., raises factor J's relative marginal product and factor share for given factor proportions).[9] Naturally, the relations between the substitution elasticity, technical bias and factor shares evaporate under Cobb-Douglas: factor income shares are time-invariant and relative factor prices are purely determined by capital deepening.

Equations (9)-(11) illustrates the importance of factor substitution and technical bias. For instance, the impact of technology shocks on factor payments depends on the substitution elasticity and the factor bias of the shock. This influences the dynamic response of interest, wages (and hence hours) to technology shocks.

3 The Real Business Cycle model

The standard RBC model is a variant of the representative agent neoclassical model, where business cycles are economic fluctuations due to non-monetary sources (primarily, changes in technology). The model is well known and can be introduced compactly.

The standard model with CES production technology in the supply side is given by (for expositional simplicity we omit the expectations operator):

$$C_t^{-\sigma_c} = \beta C_{t+1}^{-\sigma_c}[1 + r_{t+1} - \delta] \tag{12}$$

$$w_t = \upsilon H_t^{\gamma} C_t^{\sigma_c} \Rightarrow H_t = \left[\frac{w_t}{\upsilon C_t^{\sigma_c}}\right]^{\frac{1}{\gamma}} \tag{13}$$

$$Y_t = CES_t = Y_0 e^{z_t^{\mathbb{H}}} \left[\alpha_0 (\frac{e^{z_t^k} K_{t-1}}{K_0})^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0)(\frac{e^{z_t^h} H_t}{H_0})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{14}$$

$$w_t = (1-\alpha_0)\left(\frac{Y_0}{H_0} e^{z_t^{\mathbb{H}}} e^{z_t^h}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{H_t}\right)^{\frac{1}{\sigma}} \tag{15}$$

$$r_t = \alpha_0 \left(\frac{Y_0}{K_0} e^{z_t^{\mathbb{H}}} e^{z_t^k}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{K_{t-1}}\right)^{\frac{1}{\sigma}} \tag{16}$$

$$C_t + K_t - (1-\delta)K_{t-1} \leq Y_t \tag{17}$$

$$\log(z_t^j) = \rho \log(z_{t-1}^j) + \varepsilon_t^j \tag{18}$$

where C_t, w_t and r_t are, respectively, real consumption, real wages and the real interest rate; H_t is hours worked; and Y_t is output. Parameters β, δ and υ represent,

[9] The effect of capital deepening on factor prices however is independent on the elasticity of substitution, $\frac{\partial \iota}{\partial (K/N)} < 0 \; \forall \sigma$.

respectively, the discount factor, the capital depreciation rate and a scaling constant. z_t^j are technology shocks for $j = k, h, \mathbb{H}$ (i.e., capital-augmenting, labor-augmenting, and Hicks-neutral shocks respectively). Equations (12) and (13) represent the household's optimal consumption and labor supply choices given the following utility function (e.g., Galí (2008), chap. 2):[10]

$$U_t = \frac{C_t^{1-\sigma_c}}{1-\sigma_c} - \upsilon \frac{H_t^{1+\gamma}}{1+\gamma} \tag{19}$$

where σ_c is the coefficient of relative risk aversion and γ is the inverse of the Frisch elasticity. Solving equation (13) (or below, (A.3)) for hours demonstrates that, after a shock, hours rise if real wages grow faster than consumption (i.e., the substitution effect on labor supply of higher real wages dominates the negative effect of smaller marginal utility of consumption). Equations (14) to (16) are the CES production function and its factor derivatives in normalized form. Equation (17) is the resource constraint. The technology shocks follow an AR(1) process.

The model could straightforwardly be expanded to contain monopolistic competition and investment adjustment costs as in the NK model discussed below. In the limiting case of fully-flexible prices the NK model would encompass that more generalized RBC version as special case. Where discussing the threshold rule for the sign of the technology-hours response (Section 7), for the ease of comparison, we treat the RBC model as a special case of the flex-price NK model.

4 The New Keynesian Model

The NK model builds on the RBC framework with the addition of monopolistic competition, nominal rigidities, investment adjustment costs and a monetary policy rule.

4.1 Households

As before, the representative household maximizes utility function, (19) supplemented by (separable) real money balances m_t,

$$U_t = \frac{C_t^{1-\sigma_c}}{1-\sigma_c} - \upsilon \frac{H_t^{1+\gamma}}{1+\gamma} + \digamma(m_t) \tag{20}$$

with $\digamma' > 0, \digamma'' < 0$. The consumption good is assumed to be a composite good produced with a continuum of differentiated goods, $C_{i,t}$, $i \in [0, 1]$, via the

[10]We employ here a separable utility function. This makes our analysis more general. Results for RBC and NK models with non-separable preferences are shown in Appendices ?? and ??.

aggregator function:

$$C_t = \left[\int_0^1 C_{i,t}^{1-\frac{1}{\eta}} di\right]^{\frac{1}{1-\frac{1}{\eta}}} \quad (21)$$

where η represents the intra-temporal elasticity of substitution across different varieties of consumption goods. To find total consumption demand for each variety i, one can solve the standard problem of minimizing total cost subject to (21), which yields the downward sloped demand function:

$$C_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} C_t \quad (22)$$

where P_t is the nominal price index given by:

$$P_t = \left[\int_0^1 P_{it}^{1-\eta} di\right]^{\frac{1}{1-\eta}} \quad (23)$$

Households are assumed to have access to a complete set of nominal state-contingent assets. The household maximizes (19) subject to a sequence of flow budget constraints given by,

$$Q_t b_t^h + m_t^h + C_t + I_t = \frac{b_{t-1}^h + m_{t-1}^h}{\pi_t} + w_t H_t + r_t^k K_{t-1} + \Pi_t \quad (24)$$

where b_t represents the quantity of one-period nominally riskless discount bonds (or any one-period claim) purchased (or issued) in period t and maturing in period $t+1$. Each bond pays one unit of money at maturity and its price is Q_t. Because $1 - Q_t$ corresponds to the nominal loss of purchasing one unit of money instead of purchasing bonds the equality $Q_t = R_t^{-1}$ must hold, where R_t is the gross nominal interest rate. Variable I_t is investment, K_t capital stock, r_t^k the rental price of the capital stock, w_t the real wage rate, $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate and Π_t denotes profits received from the ownership of firms.

The resource constraint is,

$$C_t + I_t \leq Y_t \quad (25)$$

and capital accumulation is given by,

$$K_t = (1-\delta)K_{t-1} + I_t\left[1 - \frac{\psi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] \quad (26)$$

Changes in the capital stock are thus assumed to be subject to a convex adjustment cost with $\psi > 0$.

We assume the investment good I_t to be a composite made of the aggregator function type (21). Hence, investment demand for each variety has the same form as the consumption function: $I_{it} = (P_{it}/P_t)^{-\eta} I_t$.

The household chooses C_t, b_t, H_t, K_t, I_t, and m_t to maximize utility (19) subject to (24), (26) and the no-Ponzi-game constraint. Letting λ_t and $\lambda_t q_t$ denote the Lagrange multipliers associated with constraints (24) and (26), respectively, yields,

$$\sum_{t=0}^{\infty}\beta^t \left\{ \begin{array}{c} \left[\frac{C_t^{1-\sigma_c}}{1-\sigma_c} - \upsilon\frac{H_t^{1+\gamma}}{1+\gamma} + F\left(m_t\right)\right] \\ +\lambda_t\left[w_t H_t + r_t^k K_{t-1} - C_t - I_t - R_t^{-1} b_t^h - m_t^h + \frac{b_{t-1}^h + m_{t-1}^h}{\pi_t} + \Pi_t\right] \\ +\lambda_t q_t\left[(1-\delta)K_{t-1} + I_t\left(1 - \frac{\psi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) - K_t\right] \end{array} \right\} \tag{27}$$

The first-order conditions are:

$$C_t^{-\sigma_c} = \lambda_t \tag{28}$$

$$\lambda_t = \beta R_t \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{29}$$

$$w_t = \upsilon\frac{H_t^{\gamma}}{\lambda_t} \Rightarrow H_t = \left[\frac{1}{\upsilon} w_t \lambda_t\right]^{\frac{1}{\gamma}} \tag{30}$$

$$\lambda_t q_t = \beta\lambda_{t+1}\left[r_{t+1}^k + (1-\delta)\, q_{t+1}\right] \tag{31}$$

$$\begin{aligned} \lambda_t = \lambda_t q_t &\left[1 - \frac{\psi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \psi\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] \\ &+ \beta\psi\lambda_{t+1} q_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - 1\right) \end{aligned} \tag{32}$$

$$F' = \lambda_t - \beta\frac{\lambda_{t+1}}{\pi_{t+1}} \tag{33}$$

The last condition determines the demand for money function. However, in the current framework, with a Taylor-rule based monetary policy (described below), money demand is purely recursive.

4.2 Firms

We assume that single firms operating in a monopolistically competitive environment produce one good variety i. The firm does so by using capital and labor following a production technology:

$$e^{z_t^{\mathbb{H}}} F(e^{z_t^k} K_{it}, e^{z_t^h} H_{it}) - \chi \tag{34}$$

where the F is the same normalized CES production function presented before and χ represents fixed costs in production.[11] Given the consumption and investment demand functions, aggregate demand for good i will then be given by:

$$Y_{it} = \frac{P_{it}}{P_t}^{-\eta} Y_t \tag{35}$$

Real profits for firm i expressed in terms of the composite good are,

$$\Pi_{it} = \frac{P_{it}}{P_t} Y_{it} - r_t^k K_{it-1} - w_t H_{it} \tag{36}$$

We assume, as usual in this literature, that firms rent labor and capital services from a centralized market and that capital input can be readily reallocated across industries. The objective of the firm is to choose the plan H_{it}, K_{it-1} and p_{it}, so as to maximize the present discounted value of the profit stream assuming that the firm satisfies demand at the posted price subject to the production function constraint. Hence,

$$\sum_{s=t}^{\infty} R_{ts}^{-1} P_s \left\{ \Pi_{is} + mc_{is} \left[e^{z_t^{\mathbb{H}}} F(e^{z_t^k} K_{it-1}, e^{z_t^h} H_{it}) - \chi - Y_{is} \right] \right\} \tag{37}$$

where $R_{ts} = \prod_{\tau=t}^{s} R_\tau$ with $R_{tt} = 1$ and $R_{tt+1} = R_t$.Variable mc_{it} is the Lagrange multiplier related to the production function constraint. We derive the following cost-minimization conditions with respect to labor and capital:

$$mc_{it} e^{z_t^{\mathbb{H}}} F_h(e^{z_t^k} K_{it-1}, e^{z_t^h} H_{it}) = w_t \tag{38}$$

$$mc_{it} e^{z_t^{\mathbb{H}}} F_k(e^{z_t^k} K_{it-1}, e^{z_t^h} H_{it}) = r_t^k \tag{39}$$

We see that the Lagrange multiplier mc_{it} has a clear economic interpretation, i.e. the real marginal cost of the firm. It is also straightforward to show that mc_{it} is the same for all firms i. The ratio of (38) and (39) imply that $\frac{F_k(K_{it-1}, H_{i,t})}{F_h(K_{i,t-1}, H_{i,t})} = \frac{r_t^k}{w_t}$, the right-hand-side of which is common to all firms. We know that the marginal productivities of labor and capital for any linearly homogeneous production function can be presented in terms of capital intensity $\frac{K_{it-1}}{H_t}$ and common technical progress. Now common input prices imply that capital intensities and hence also marginal productivities and marginal costs must be equal across firms, i.e. $mc_{it} = mc_t$.

[11] These are chosen to ensure zero profits in steady state. This in turn guarantees that there is no incentive for other firms to enter the market in the long run (for example, Coenen, McAdam and Straub (2008)).

The second part of the firm's optimization problem is to set the optimal price level subject to Calvo pricing, where in each period the probability of re-optimizing the price level is $1-\theta$ (for simplicity, we do not assume price indexation). Therefore, utilizing the demand function (35) the Lagrangian (37) covering only those firms which are allowed to re-optimize their price level on period t, can be transformed into,

$$\sum_{s=t}^{\infty} R_{ts}^{-1} P_s \theta^{s-t} \left\{ \begin{array}{c} \left(\frac{\tilde{P}_{it}}{P_s}\right)^{1-\eta} Y_s - r_t^k K_{it-1} - w_t H_{it} \\ +mc_s \left[e^{z_t^{\mathbb{H}}} F(e^{z_t^k} K_{it-1}, e^{z_t^h} H_{it}) - \chi - \left(\frac{\tilde{P}_{it}}{P_s}\right)^{-\eta} Y_s \right] \end{array} \right\} \tag{40}$$

where we denote the re-optimized price level by $\tilde{P}_{it}$. Maximizing (40) with respect to $\tilde{P}_{it}$ yields,

$$\sum_{s=t}^{\infty} R_{ts}^{-1} \theta^{s-t} \left(\frac{\tilde{P}_{it}}{P_s}\right)^{-1-\eta} Y_s \left(mc_s - \frac{\eta-1}{\eta} \frac{\tilde{P}_{it}}{P_s} \right) = 0 \tag{41}$$

This expression tells us that the optimal price equals a weighed sum of future expected mark-ups and that the re-optimized price level is same for all firms, i.e. $\widetilde{P}_{it} = \widetilde{P}_t$. Utilizing (29), this condition can be expressed recursively as:

$$x1_t = \widetilde{p}_t^{-1-\eta} Y_t mc_t + \theta\beta \left[\frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{\eta} \left(\frac{\widetilde{p}_t}{\widetilde{p}_{t+1}} \right)^{-1-\eta} x1_{t+1} \right] \tag{42}$$

$$x2_t = \widetilde{p}_t^{-\eta} Y_t + \theta\beta \left[\frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{\eta-1} \left(\frac{\widetilde{p}_t}{\widetilde{p}_{t+1}} \right)^{-\eta} x2_{t+1} \right] \tag{43}$$

$$x2_t = \frac{\eta}{\eta-1} x1_t \tag{44}$$

where $\widetilde{p}_t = \frac{\widetilde{P}_t}{P_t}$.

4.3 Monetary authority

The monetary authority follows a Taylor-type rule:

$$\log(R_t/R) = \alpha_r \log(R_{t-1}/R) + \alpha_\pi \log(\pi_t/\pi) + \alpha_y \log(Y_t/Y_t^f) \tag{45}$$

where r_t denotes the nominal interest rate. Consistent with the DSGE model, potential output, Y^f, is defined as the level of output that would prevail under flexible prices and wages, Smets and Wouters (2007).

Aside from this simple rule, we also consider the responses of the economy under optimal (Ramsey) policy according to which the monetary authority sets the optimal path of all variables in the economy by maximizing agents' welfare subject to the relations describing the competitive economy, e.g., Levine, McAdam and Pearlman (2008).

4.4 Aggregation and Equilibrium

Looking at expression (41) we see that all firms that are able to change their price in a given period chose the same price so we can drop subscript i from the equilibrium conditions. By taking into account relative price dispersion across varieties, the resource constraint in this model is given by the following three expressions:

$$Y_t = \frac{1}{S_t}[e^{z_t^{\mathbb{H}}} F(\cdot, t) - \chi] \tag{46}$$

$$Y_t = C_t + I_t \tag{47}$$

$$S_t = (1-\theta)\tilde{p}_t^{-\eta} + \theta \pi_t^{\eta} S_{t-1} \tag{48}$$

where S is a state variable that measures the resource costs induced by the inefficient price dispersion present in the Calvo problem in equilibrium[12].

5 Calibration

The calibration reflects common practice (see **Table 1**). We set the discount factor to represent a discount rate of around 4% per year. Utility function parameters are set consistent with balanced growth. The normalized capital share is set to 0.4. For the investment adjustment cost parameter we chose 2.5 which is a common benchmark value (e.g., Christiano, Eichenbaum and Evans (2005)), although in our sensitivity exercises below we vary that around a wide support. The price elasticity of demand, η, ensures a steady-state price mark-up of 20% over marginal costs. The depreciation rate of capital is 10% per year. The Calvo parameter implies a fixed price duration of 4 quarters. The substitution elasticity is set to a range from 0.4 above Leontief (i.e., at the lower end of the "Chirinko interval"), at Cobb Douglas, and at 0.4 above Cobb-Douglas, thus traversing gross complements and substitutes. The auto-regressive parameter of technology shocks is set to 0.95. For

[12] Although this variable is redundant when there is zero steady-state inflation (see Schmitt-Grohé and Uribe (2007)), we introduce it because we also analyzed the results with positive inflation for robustness. The results regarding the reaction of hours did not change and are available on request.

simplicity, in our core calibration, we assume monetary policy only responds to deviations of inflation from the steady state with a coefficient just respecting the Taylor principle. Moreover, both models are normalized around the same steady state point. Parameter υ is set to equate the real wage expressions in (13) and (15) - see Section 7 for further details.

Households		
β	Discount Factor	$1.04^{-1/4}$
γ	Inverse Frisch Elasticity	1
σ_c	Coefficient of Relative Risk Aversion	1
υ	Leisure Scaling Constant	0.84
Firms		
η	Price Elasticity of Demand	6
θ	Calvo Parameter	0.75
δ	Depreciation Rate	$1.1^{1/4} - 1$
ψ	Investment Adjustment Costs	2.5
α_0	Normalized Capital Income Share	0.4
σ	Elasticity of Substitution	$[0.4, 1, 1.4]$
ρ	Shock Auto-Regressive Parameter	0.95
Monetary policy		
π	Steady State Inflation	1
α_π	Taylor Coefficient on Inflation	1.1
α_y	Taylor Coefficient on Output Gap	0.0
α_r	Taylor Coefficient on Lagged Interest Rate	0.0

Table 1: Parameter Calibration

6 Simulations Results

Figures 1 to **4** depict the dynamic responses of selected variables to a persistent one percentage point increase in ε_t^k and ε_t^h in the standard RBC and NK models, respectively.[13, 14] For the NK model, we additionally show the responses of inflation

[13]We also computed dynamic responses for $\varepsilon_t^{\mathbb{H}}$ (see Appendix **??**).

[14]Both models were solved and simulated using first-order approximation methods around their non-stochastic steady state using Dynare, Juillard (2009).

and real marginal costs. The shocks are conducted against the parameters values shown in Table 1 with the substitution variations $\sigma \in (0.4, 1, 1.4)$. Variations in the latter, recall, are admissible in our framework since we expressed the supply side in normalized form.

Looking over all results we see that the effect of (any) positive technology shock is to stimulate output, consumption and investment. Movements in factor income shares (excluding Cobb Douglas, where factor shares are constant) are symmetrical and, following the analysis in Section 2.2, favor either factor depending on the source of the technology improvement and whether factors are gross complements or not.

Where qualitative differences may arise lies in the hours response. Importantly, these differences may arise not only across the models but also *within* them. Capital-augmenting technology-hours impacts are positive in the standard RBC model, whilst labor-augmenting technology-hours impacts are negative in the NK one (see summary **Table 2)**. However, the off-diagonal elements reveal that the models are capable of generating technology-hours impacts *of either sign*.[15]

	RBC	NK
$\frac{dh_t}{dz_t^k}$	> 0 $(a.s.)$	$\lesseqgtr 0$
$\frac{dh_t}{dz_t^h}$	$\lesseqgtr 0$	< 0 $(a.s.)$

Table 2: Sign Responses

To understand why we refer back to Section 2.2. A capital-augmenting (labor-augmenting) shock under gross complementarity raises (lowers) the labor income share and the w/r ratio (equivalently, the marginal product of labor to capital). Under gross substitutability the signs of these effects switch. These results were derived in the frictionless and fully competitive markets and, hence, in this respect compatible with standard RBC model. On the other hand, somewhat problematic may be that they are the comparative static results, the discrete time counterparts of which are permanent technology shocks. However, to retain the steady state well defined our shocks are not permanent. The imposed high auto-correlation of the shocks ($\rho = 0.95$) improves, we think, essentially the comparability of our results with the aforementioned comparative static results. In the context of NK model simulations price stickiness and the adjustment costs of investment introduce their own ingredients that complicates the straightforward interpretations.

However, Figures 1-4 verify the sign dependency of shock impacts on $sign\,\{\sigma - 1\}$

[15]The negative hours response for the RBC model with labor-augmenting shocks and the positive one for capital-augmenting shocks in the NK model are more pronounced, the lower is the value of the substitution elasticity.

even in the NKM simulations. Factor shares and relative prices (marginal productivities) behave as anticipated. Differences in consumption and investment responses corresponding to alternative σ values are, in turn, explained by investment reactions to changes in relative marginal productivities of capital and labor. When capital and labor are gross complement (substitutes), in Figures 1 and 3, then a capital augmenting shock decreases (increases) the relative price and the marginal productivity of capital with a disincentive (an incentive) investment effect and, hence, the split of the increased in total income favors (**discourages**) consumption over investment.

Difference in the consumption-investment split resulting from the labor augmenting shock (Figures 3 and 4) can be explained analogously. Hence, we conclude that due to investment reactions to relative factor prices (marginal factor productivities) the impact response of consumption (what is otherwise called the *marginal propensity to consume*) tends to decrease (increase) with rising substitution elasticity for capital (labor) augmenting shocks.

We shall revisit this property in Section 7 where we give a more analytic and generalized interpretation of the technology-hours response in the models. It transpires that in most cases, the rule has threshold characteristics such that over some ranges of factor substitution, the technology-hours response may change sign. Although necessarily more involved, that rule embodies the same intuition and logic as discussed here: for a given shock and substitution elasticity, we know how consumption and savings behave at the margin.

To anticipate some of those interactions, we perform some additional analysis. **Figures 5a to 5d** analyze the impact response of hours along a $\sigma \in (0, 2]$ support, with a steady-state capital income share of $\alpha_0 = 0.4$ (the baseline) and $\alpha_0 = 0.7$ (an admittedly counter-factual value). For instance, for the standard RBC labor augmenting shock the figures indeed indicate a selective threshold relationship such that when $\sigma > \alpha_0 \varpi$ the impact effect of technology shocks changes sign (where ϖ is some, as yet undefined, wedge). Thus for all σ values sufficiently above α_0, the RBC labor-augmenting/hours sign flips from negative to positive. For the NK model, it is the capital-augmenting shocks which switch sign but now other factors as the capital income share play more crucial role in the switching-rule.

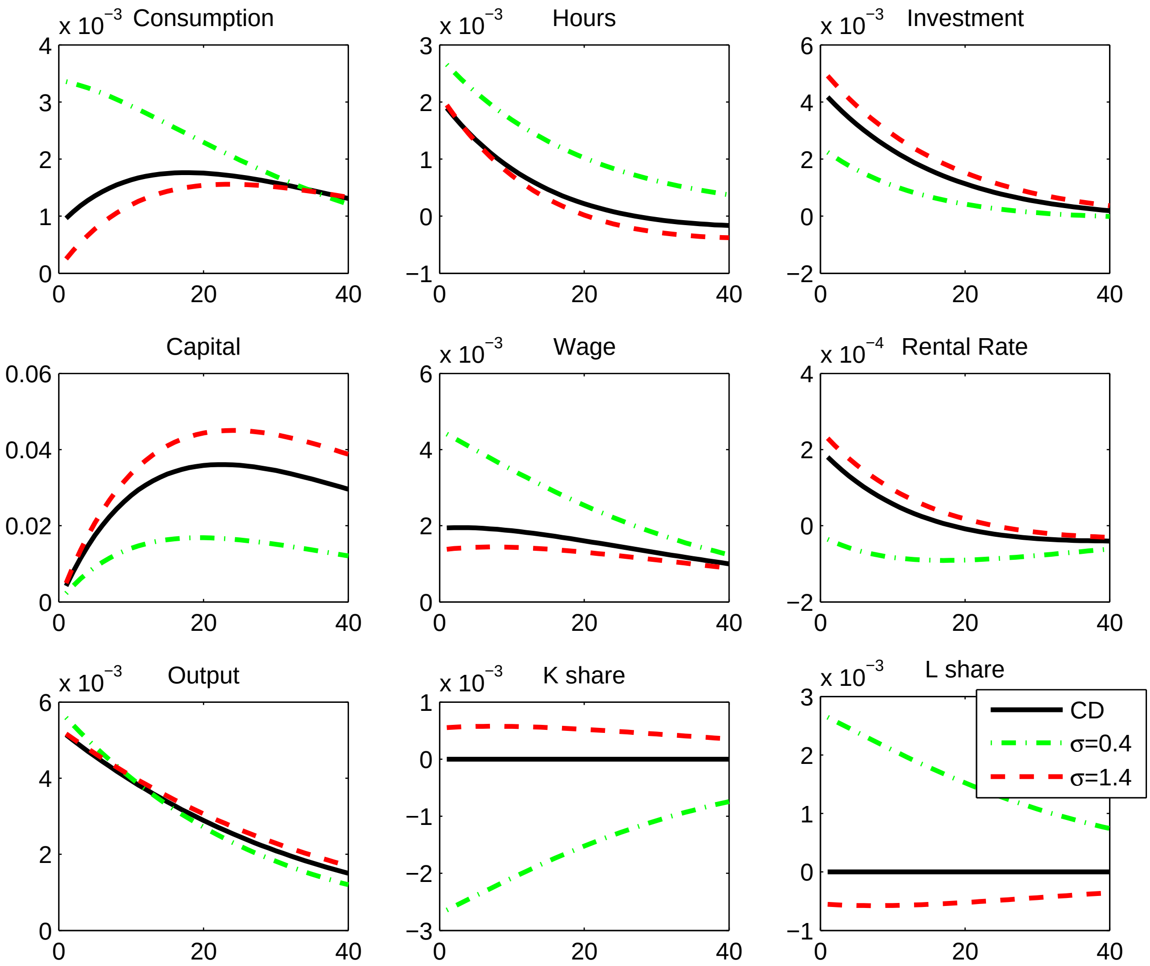
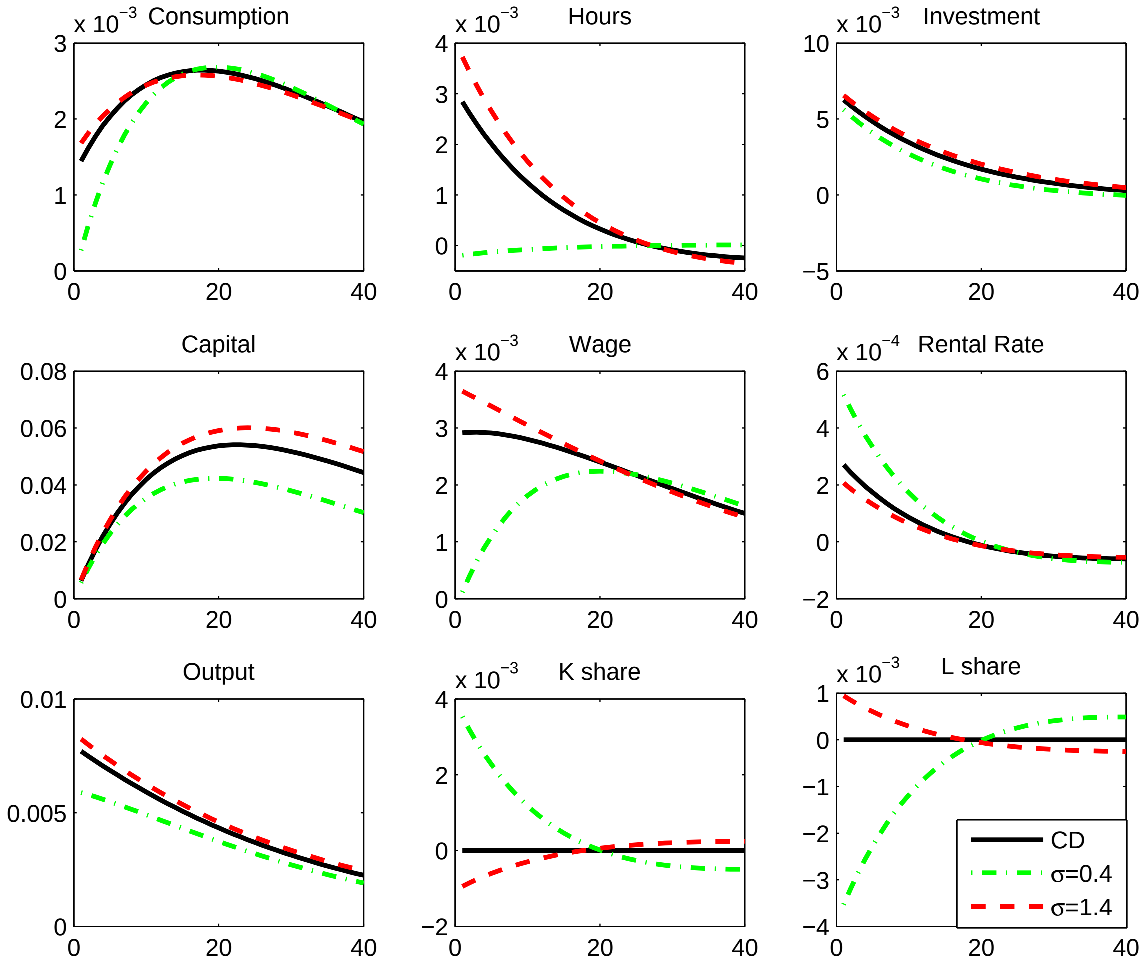
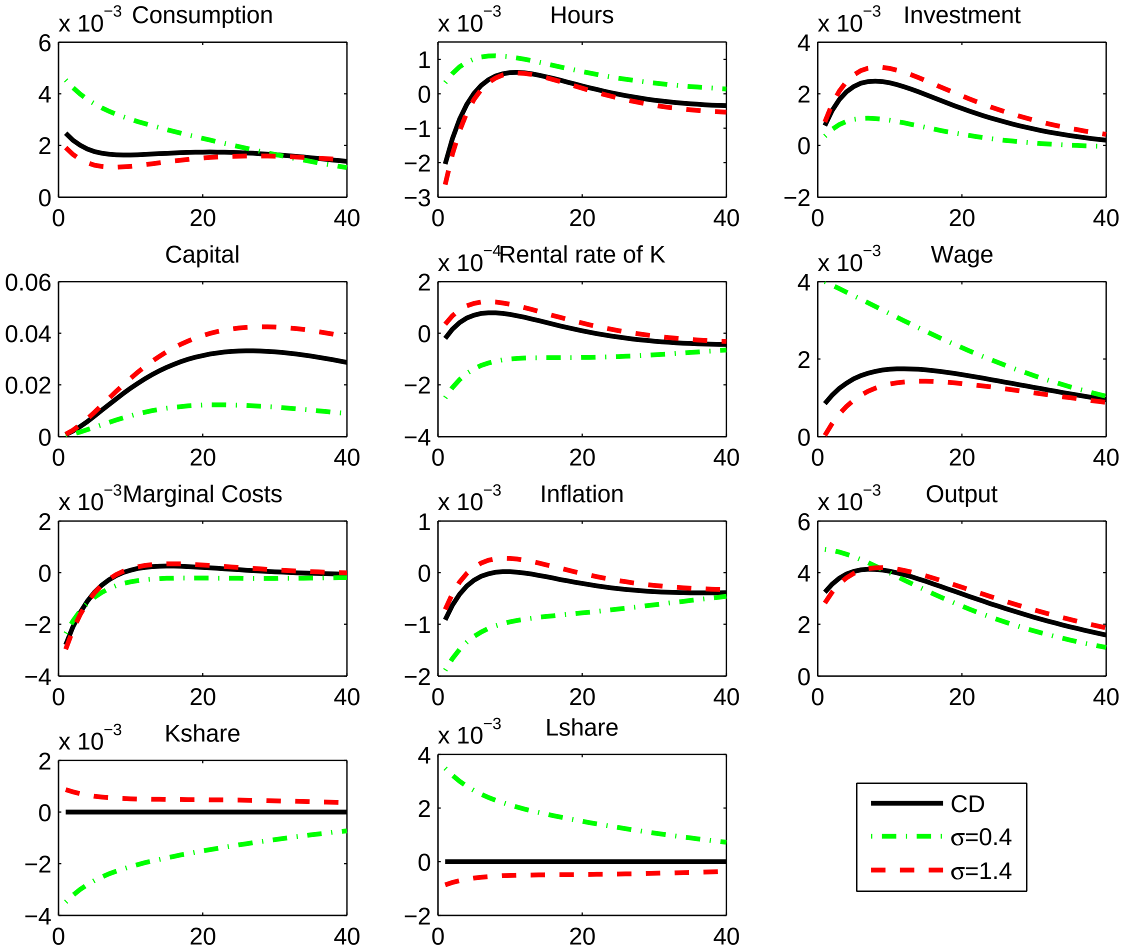


Figure 1: RBC model - Capital Augmenting Shock



Note: Hours Impact when $\sigma = 0.4$ is negative (at -0.0189)

Figure 2: RBC model - Capital Augmenting Shock



Note: Hours Impact when $\sigma = 0.4$ is positive (at 0.03)

Figure 3: RBC model - Capital Augmenting Shock

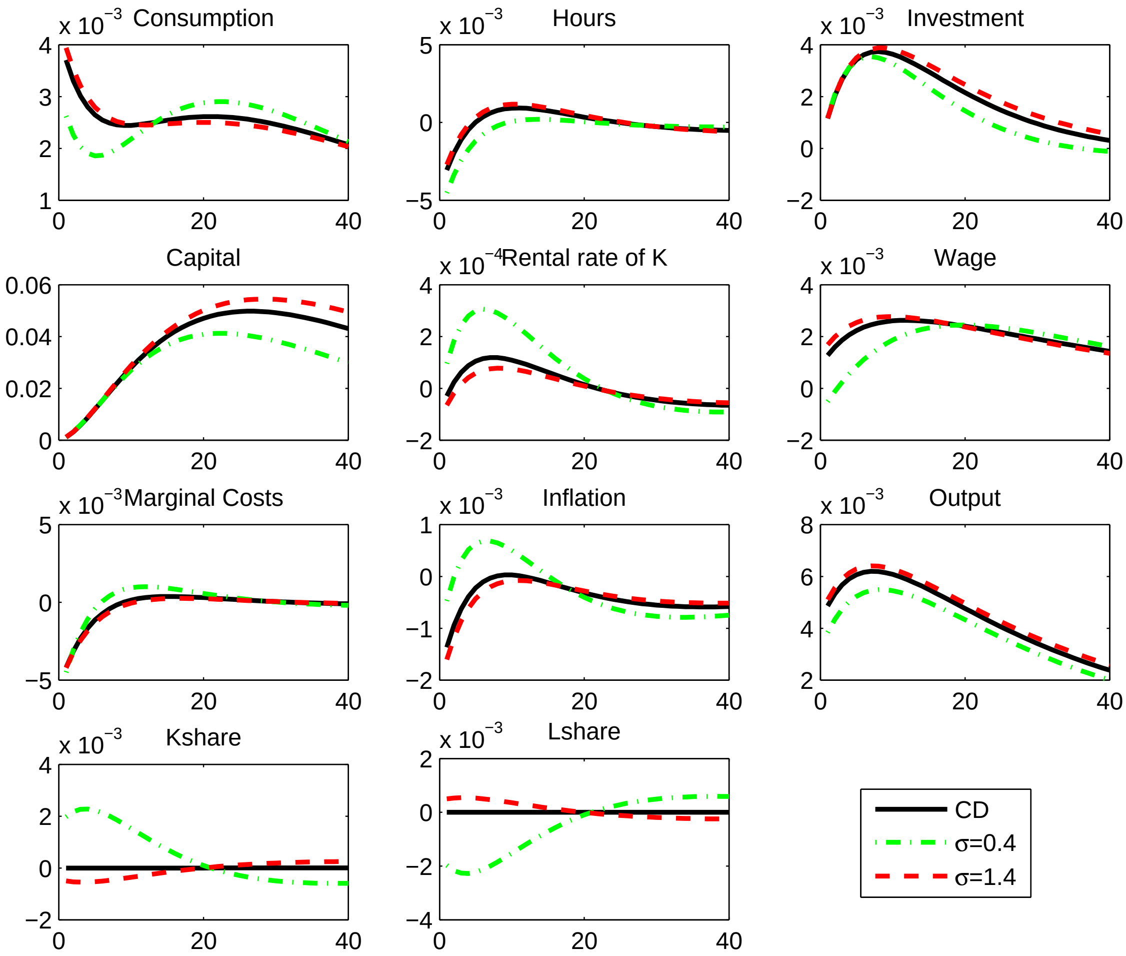
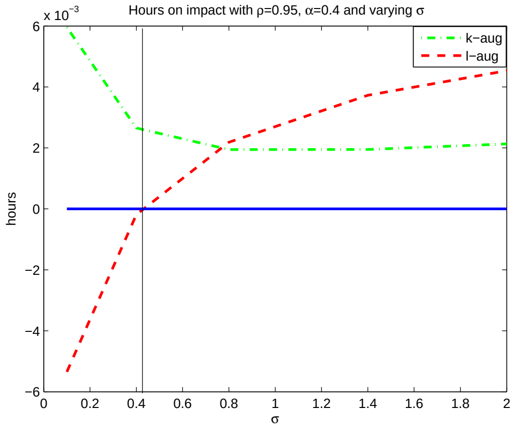
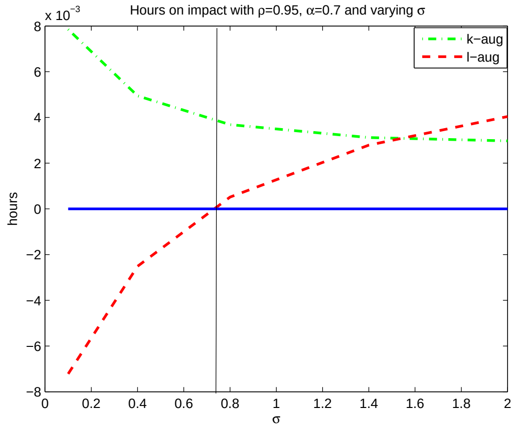


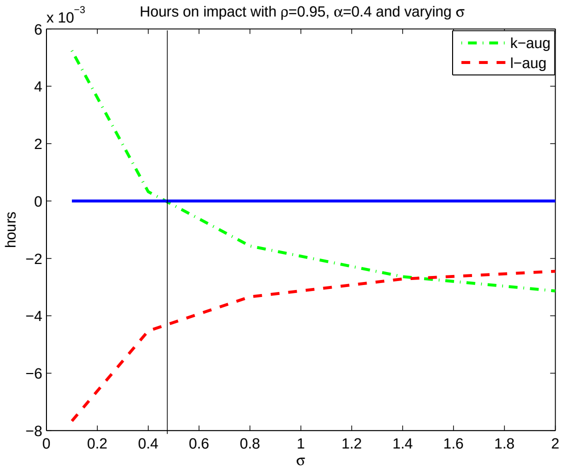
Figure 4: NK model - Labor Augmenting Shock



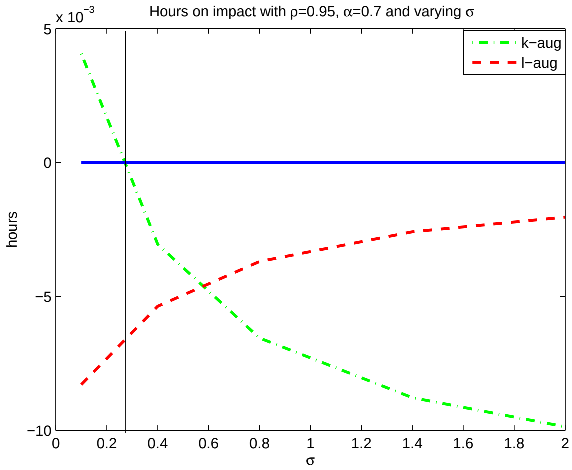
(a) Sensitivity for σ with $\alpha = 0.4$, RBC Model



(b) Sensitivity for σ with $\alpha = 0.7$, RBC Model



(c) Sensitivity for σ with $\alpha = 0.4$, NK Model



(d) Sensitivity for σ with $\alpha = 0.7$, NK Model

Figure 5: Changes in σ and α

7 Technology and Hours: A Threshold Rule

We now derive a rule determining the sign of the technology-hours impact: i.e., a threshold value of the substitution elasticity, $\widetilde{\sigma}$, whereby the technology-hours shock switches sign. Initially, we define the rule for an encompassing RBC model - i.e., allowing for imperfect competition and investment adjustment costs - and thereafter for the NK model where nominal rigidities and a policy role are present. The rules highlights that the crux of the technology-hours correlation hinges on the marginal response of saving and, in the NKM, also marginal costs to shocks. Hence, the rules do not give closed form solutions to threshold σ, except in the special case of infinite investment adjustment costs in the RBC model, when income changes are fully transmitted to consumption and the marginal propensity to consume (*mpc*) is one. In other cases, however, the rules open a way for a general geometrical analysis of the sign-dependency of technology-hours impact on the substitution elasticity σ. The important advantage of rules containing endogenous *mpc* and *mc* is that via their reactions all inter-temporal effects reflecting also the frictional elements of the models are transmitted to the threshold rule and in our numerical simulations we can concentrate our attention to the behavior of these two variables.

7.1 The Encompassing RBC Case

To improve comparability between the RBC and the NK model we supplement our standard RBC model to encompass imperfect competition and fixed costs with zero profits in the steady state, i.e. $\chi = \frac{Y_0}{\eta-1}$. Without loss of generality, we normalize the CES function around the following steady-state: $Y_0 = H_0 = 1 \Rightarrow K_0 = \frac{\alpha_0}{r_0}$, $w_0 = 1-\alpha_0$ and $C_0 = Y_0 - \delta K_0 = \frac{r_0 - \delta\alpha_0}{r_0}$. and, hence,

$$Y_t = \left(\frac{\eta}{\eta-1}\right)\left[\alpha_0\left(e^{z_t^k}\frac{r_0}{\alpha_0}K_{t-1}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0)\left(e^{z_t^h}H_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - \frac{1}{\eta-1} \tag{49}$$

Using the fact that, around our baseline, $dY_t = d\log Y_t$, differentiating (49) around the steady state yields:

$$d\log Y_t = \left(\frac{\eta}{\eta-1}\right)\left[\alpha_0 dz_t^k + (1-\alpha_0)\left(dz_t^h + d\log H_t\right)\right] \tag{50}$$

Equation (50) measures the first-period impact effect of technology shocks on production. We see that the production effect is strengthened by the increasing returns to scale term $\left(\frac{\eta}{\eta-1}\right)$. Although (50) is seemingly independent of the elasticity of substitution that is not actually the case, because the effects transmitted via the reactions of hours to shocks depend strongly on the size of substitution elasticity. To solve those effects we need the FOCs of the profit maximizing firm, labor supply condition in the RBC model and the goods market equilibrium condition:[16]

$$\log r_t = \log(r_0) + \frac{\sigma-1}{\sigma}\log e^{z_t^k} + \frac{1}{\sigma}(\log Y_t - \log K_t) \tag{51}$$

$$\log w_t = \log(1-\alpha_0) + \frac{\sigma-1}{\sigma}\log e^{z_t^h} + \frac{1}{\sigma}(\log Y_t - \log H_t) \tag{52}$$

$$\log w_t = \log v + \gamma \log H_t + \sigma_c \log C_t \tag{53}$$

$$C_t + K_t - (1-\delta)K_{t-1} = Y_t \tag{54}$$

Normalization around the steady state implies that, in (53), the scaling constant becomes $v = \frac{(1-\alpha_0)r_0^{\sigma_c}}{(r_0-\delta\alpha_0)^{\sigma_c}}$.

As in a closed economy investment $I_t = K_t - (1-\delta)K_{t-1}$ must equal saving, equation (54) also measures the split of income into consumption and saving. In the steady state (or point of normalization) the average propensity to consume $apc = C_0 = 1 - \frac{\delta}{r_0}\alpha_0$, where $r_0 = \frac{1}{\beta} - 1 + \delta \Rightarrow apc \in [1-\alpha_0, 1]$ given $\frac{\delta}{r_0} \in [0, \beta]$. Now define the marginal propensity to consume $mpc_t = \frac{dC_t}{dY_t} = apc\frac{d\log C_t}{d\log Y_t}$ and apply it in (50) to obtain:

$$d\log C_t = \frac{mpc_t}{apc}\left(\frac{\eta}{\eta-1}\right)\left[\alpha_0 dz_t^k + (1-\alpha_0)\left(dz_t^h + d\log H_t\right)\right] \tag{55}$$

Next differentiate the labor supply condition (53) and use (55) to obtain:

$$\begin{aligned} d\log w_t = &\left[\gamma + \sigma_c\frac{mpc_t}{apc}\left(\frac{\eta}{\eta-1}\right)(1-\alpha_0)\right]d\log H_t + \sigma_c\gamma \\ &+ \sigma_c\frac{mpc_t}{apc}\left(\frac{\eta}{\eta-1}\right)\left[\alpha_0 dz_t^k + (1-\alpha_0)\,dz_t^h\right] \end{aligned} \tag{56}$$

The FOC for labor (52), in turn, implies,

$$d\log w_t = \frac{\sigma-\alpha_0}{\sigma}dz_t^h + \frac{\alpha_0}{\sigma}\left(dz_t^k - d\log H_t\right) \tag{57}$$

and conditions (56) and (57) imply:[17]

$$d\log H_t = \frac{1}{\Upsilon}\left[\Upsilon_k^h dz_t^k + \Upsilon_h^h dz_t^h\right] \tag{58}$$

[16] In spite of imperfect competition equations (51) and (52) do not contain the mark-up term. It is canceled out by the assumed link between fixed costs and imperfect competition.

[17] The threshold rule for the non-separable case is described in Appendix **??**.

where,

$$\Upsilon = \gamma + \sigma_c \frac{mpc_t}{apc}\left(\frac{\eta}{\eta-1}\right)(1-\alpha_0) + \frac{\alpha_0}{\sigma}$$
$$\Upsilon_k^h = \frac{\alpha_0}{\sigma}\left[1 - \sigma\sigma_c \frac{mpc_t}{apc}\left(\frac{\eta}{\eta-1}\right)\right]$$
$$\Upsilon_h^h = \left[1 - \sigma_c \frac{mpc_t}{apc}\left(\frac{\eta}{\eta-1}\right)(1-\alpha_0)\right] - \frac{\alpha_0}{\sigma}$$

Equation (58) is our key relationship.

Given that $\Upsilon > 0$ $(a.s)$[18] the sign of the technology-hours correlation depends on Υ_k^h and Υ_h^h:

$$\frac{dH_t}{dz_t^k} > 0 \quad \text{if} \quad mpc_t < \tfrac{1}{\sigma}\underbrace{\frac{apc}{\sigma_C}\left(\frac{\eta-1}{\eta}\right)}_{\phi} \tag{59}$$

$$\frac{dH_t}{dz_t^h} > 0 \quad \text{if} \quad mpc_t < \frac{\left(1-\frac{\alpha_0}{\sigma}\right)}{(1-\alpha_0)}\phi \tag{60}$$

The right-hand-side of (59) is a hyperbola: $\frac{\phi}{\sigma} \in [\infty, 0]$ as $\sigma \to \infty$. It moves towards the origin the larger the risk aversion, capital share and $\frac{\delta}{r_0}$ ratio and the smaller is the price elasticity but is independent of investment adjustment costs. **By contrast, within the interval $1-\alpha_0 \leq apc \leq 1$ it moves away from the origin, the closer to unity is the *apc*.** Although we forthwith use simulation approach to analyze the behavior of the *mpc*-schedule allowing deviations from the assumptions of the standard RBC model (e.g. costly investment adjustment).[19] Section 6 already gave a flavor of how these rules worked.

If the *mpc*-schedule and $\frac{\phi}{\sigma}$ do not intersect the technology-hours impact never switches sign. Further, if investment adjustment costs are infinite, $\psi \to \infty$, no investment (or saving) response to technology shock arises, implying $dc_t = dy_t \Rightarrow mpc_t = 1$ independently from the size of all other parameters.

Similar reasoning pertains to rule (60).

7.1.1 RBC: Capital Augmenting Shocks (Figs 1. and 2.)

Figures 1 and **2** illustrate the workings of rule (60) in terms of the substitution elasticity, for temporary and persistent shocks. As earlier discussed, the *mpc*-

[18] Although we cannot exclude the possibility that $\Upsilon < 0$, our numerical analysis judged it highly improbable. We see that $\Upsilon > 0$ if $mpc_t > -\left(\gamma + \frac{\alpha_0}{\sigma}\right)\frac{apc}{\sigma_c(1-\alpha_0)}\left(\frac{\eta-1}{\eta}\right)$ the right hand side of which is always negative. Hence, the sign condition may hold also with negative mpc_t. Therefore, in the following our default assumption is that $\Upsilon > 0$.

[19] The full set of results on which the below sections are based are collected in Appendix **??**

behavior is anticipated the better on the basis of the comparative static results implied by *Definition*, the closer the simulated model is to the standard RBC model and the more persistent is the shock, i.e. $\rho \to 1$. The *mpc* schedules of Figures 1 and 2 correspond to zero, finite and infinite investment adjustment costs.

Zero Adjustment Costs We already know (Section 6) that for capital augmenting shocks and finite adjustment costs that the *mpc*-schedule is decreasing in the substitution elasticity.[20] The main difference - compared to the persistent shock - is the lower size of the *mpc* (the intuition being a transitory income increase is allocated to consumption smoothly over the entire optimization horizon). Hence, following a transitory shock the *mpc* is small for all σ and below $\frac{\phi}{\sigma}$(see **Fig 1.**). Although increasing risk aversion moves $\frac{\phi}{\sigma}$ closer to the origin, the *mpc*-schedule also shifts downwards. Hence the standard RBC model generates a positive hours response to a transitory capital-augmenting shock (confirming Table 2).

Persistent shocks develop differently (see **Fig 2.**). For small σ, the *mpc* can exceed unity (the more so the higher is risk aversion). With persistence, the *mpc*-schedule steepens and intersects the horizontal axis at a finite σ (beyond which $mpc < 0$). With small risk aversion (e.g., $mpc(\sigma_C^1 < 1)$), the *mpc*-schedule remains below $\frac{\phi}{\sigma}$ and the hours reaction is, as before, positive. However, increasing risk aversion shifts up the *mpc*-schedule, $(mpc(\sigma_C^2))$, especially for small σ, but still intersects the horizontal axis with high σ. Hence, the negative slope of the *mpc*-schedule steepens. The hyperbola $\frac{\phi}{\sigma}$, in turn, moves closer to origin. Therefore, with sufficiently high σ_c there is a interval $(\widetilde{\sigma}_L;\ \widetilde{\sigma}_H)$ over which $mpc_t \geq \frac{\phi}{\sigma}$ and the hours reaction is negative (and that interval widens in σ_c).[21]

Finite Adjustment Costs For a temporary shock the *mpc*-schedule moves upwards as ψ increases (as in Fig.1, $mpc\,(\psi = 0) \to mpc\,(\psi > 0)$). In fact, only very small adjustment costs are needed to ensure that $mpc > \frac{\phi}{\sigma}$ and that intersection point moves leftward along $\frac{\phi}{\sigma}$ until $\widetilde{\sigma} = \phi$, where $mpc_t = 1$ and $\psi \to \infty$.

For a persistent shock, increasing adjustment costs flattens the downward sloping *mpc*-schedule (as in the dashed $mpc\,(\sigma_C^2,\ \psi > 0)$, Fig 2.) so that the points above the horizontal unit line decrease and the points below the unit line increase. Accordingly, the point where $mpc > \frac{\phi}{\sigma}$(i.e., technology-hours shock switches from positive to negative), shifts to the right.

Likewise the second intersection point shifts to the right and, due to the decreasing negative slope of the hyperbola $\frac{\phi}{\sigma}$, this shift is larger than the shift of the

[20] This was earlier shown for persistent shocks, but our simulations (Table 1, Appendix B) show it also holds also for transitory shocks.

[21] In Table 3 the size of risk aversion that is needed for the *mpc*-schedule to start having values above the hyperbola is between 1 and 2.

first intersection point and, hence, the interval σ over which the response of hours worked to a capital augmenting shock is negative widens towards infinity along with σ.

7.1.2 RBC: Labor Augmenting Shocks (Fig 4.)

Our earlier comparative statics suggested an upward sloping *mpc*-schedule (which is confirmed numerically, Table 5, Appendix B).[22] In addition, unless coupled with high risk aversion, the *mpc*-schedule intersects the horizontal σ-axis from below. However, a decrease in shock persistency rotates the schedule and, especially with high σ, lowers *mpc*-values. Interestingly, for a transitory shock the labor augmenting *mpc*-schedule converges exactly to the capital augmenting schedule. This situation is shown in Fig 4 by the curve $mpc\,(\psi = 0)$.

The rhs of (60), $\frac{\phi}{1-\alpha_0}\left(1 - \frac{\alpha_0}{\sigma}\right)$, defines a hyperbola going from $-\infty \rightarrow \frac{\phi}{1-\alpha_0}$(its horizontal upper asymptote) as $\sigma \rightarrow \infty$.[23] The hyperbola always intersects the σ-axis at $\sigma = \alpha_0$; this intersection point is independent from other model parameter values. Changes in the arguments of $\phi\,(apc, \sigma_c, \eta)$, which increase (decrease) ϕ moving the asymptote upwards, move all points of the hyperbola - except the intersection $\sigma = \alpha_0$ - closer to (further off) the σ-axis. Now, the *mpc*-schedule corresponding to a temporary shock and locating above but close to the horizontal σ-axis intersects the upward sloping hyperbola only slightly to the right of the point $\sigma = \alpha_0$ and, hence, the wedge $\widetilde{\sigma} - \alpha_0$ is positive.

An increase in investment adjustment costs raises the *mpc*-schedule without any effect on the hyperbola, hence wedge $\widetilde{\sigma} - \alpha_0$ widens. As the hyperbola's upper asymptote, $\frac{\phi}{1-\alpha_0}$, approaches zero, when e.g. $\sigma_c \rightarrow \infty$, it is possible that the mpc-schedule and the hyperbola do not intersect and $\frac{dH_t}{dz_t^h} < 0 \ \forall\sigma$.

For a persistent shock, the *mpc*-schedule is upward sloping, at least initially. When both σ and σ_c are small the *mpc*-impact can be negative and intersection can occur below the σ-axis (as in **Fig 5**., $\widetilde{\sigma}_1 < \alpha_0$).[24] However, a rise in risk-aversion shifts the *mpc*-schedule upwards, especially, the part of the curve corresponding to small σ. Although also the points of the hyperbola to the left (to the right) from the point $\sigma = \alpha_0$ move upwards (downwards), results indicate that the points of the mpc-schedule move more and the negative wedge between $\widetilde{\sigma}$ and α_0 becomes smaller or may turn positive.

[22] Table 5 of Appendix B shows that with $\rho = 0.99$ and $\sigma_c = 8$ the mpc-schedule is, in fact, downward sloping. However, when we repeated the calculations corresponding $\rho = 0.999$, then the mpc values corresponding to $\sigma = (0.3,\ 0.9,\ 1.4.\ \ 2,\ 10)$ were (0.494, 0.593, 0.635, 0.644) indicating the upward sloping schedule.

[23] The value of that asymptote may vary from 0 to ∞ although values much above unity appear unlikely.

[24] The case in the top panel of Table 7 is an example ($\widetilde{\sigma} = 0.33$, $\alpha_0 = 0.4$ and $\sigma_c = 0.5$).

7.2 The NK Case

Price-setting frictions introduce another channel whereby technology shocks affect hours worked. While in the RBC case frictionless nominal adjustment insulates the effects of technology shocks on real marginal costs, that is not the case here with implications for labor demand and supply decisions.

The following analysis is based on the observation that the *rhs* of (52) is the normalized CES condition for F^H and, hence, on the basis of NK equation (38) we can write,

$$d \log w_t = \frac{\sigma - \alpha_0}{\sigma} dz_t^h + \frac{\alpha_0}{\sigma} \left(dz_t^k - d \log H_t \right) + d \log mc_t \tag{61}$$

This is otherwise the same as the RBC condition (57) except for the additional (endogenous) marginal cost term. Coupled with labor supply equation (57) we end up with,

$$d \log H_t = \frac{1}{\Upsilon} \left[\Upsilon_h^k dz_t^k + \Upsilon_h^h dz_t^h + \frac{\partial \log mc_t}{\partial z_t^k} dz_t^k + \frac{\partial \log mc_t}{\partial z_t^h} dz_t^h \right] \tag{62}$$

where parameters Υ, Υ_h^k and Υ_h^h are as in (58). Thus,

$$\frac{dH_t}{dz_t^k} > 0 \quad \text{if} \quad mpc_t - \tfrac{\phi}{\alpha_0} \tfrac{\partial \log mc_t}{\partial z_t^k} \quad < \tfrac{\phi}{\sigma} \tag{63}$$

$$\frac{dH_t}{dz_t^h} > 0 \quad \text{if} \quad mpc_t - \tfrac{\phi}{1-\alpha_0} \tfrac{\partial \log mc_t}{\partial z_t^h} < \tfrac{\left(1-\frac{\alpha_0}{\sigma}\right)}{(1-\alpha_0)} \phi \tag{64}$$

The rhs of conditions (63) and (64) are exactly the same hyperbolas as in the sign conditions (59) and (60). The lhs, however, differs because relevant sign criteria are the composite functions of the responses of the marginal propensity to consume and real marginal costs to the technology shocks.

7.2.1 NK: Capital Augmenting Shocks (Figs 3.)

Compared to the extended RBC model, in the NK case price stickiness and the monetary rule play a role.

In the case of a temporary capital-augmenting shock (not shown) an increase in θ moves the composite *mpc&mc* schedule (i.e.,$mpc_t - \frac{\phi}{1-\alpha_0} \frac{\partial \log mc_t}{\partial z_t^h}$) term only marginally: the effect is so small that, at least in the interval $\sigma \in (0, 10)$, the values of $\frac{\phi}{\sigma}$ exceed the values of the composite function and the hours impact is always positive (as in the RBC model, in Fig 1.).

Under a persistent shock (**Fig 3.**) marginal cost reactions matter more. Marginal costs decrease in response to a persistent capital-augmenting shock when σ-values are small - i.e., see region $mpc\&mc\ (\theta_0) > mpc$ - and they increase otherwise. Thus the downward slope of $mpc\&mc$ steepens as price stickiness increases. Hence, an increase of θ shifts the interval of σ over which the technology-hours impact is negative leftward, however, without implying any essential differences in the sign of technology-hours impact in the RBC, on one hand, and the NK model, on the other hand.

7.2.2 NK: Labor Augmenting shocks (Figs 4. and 5.)

Consider first a temporary shock (**Fig 4.**). For an increase in θ the composite $mpc\&mc\,(\theta > 0, \psi = 0)$ shifts above $mpc\,(\psi = 0)$ term increases in and hence, compared to the RBC model, the $\widetilde{\sigma}\ -\alpha_0$ wedge is wider.

By contrast, if the shock is persistent, **Fig 5.**, an increase in price stickiness increases marginal costs shifting the composite function, $mpc\&mc\,(\sigma_C^1, \psi = 0)$, below $mpc\,(\sigma_C^1, \psi = 0)$ schedule (with the possibility of high negative values). This shifts the intersection point of the composite function and the hyperbola and, accordingly, shifts $\widetilde{\sigma}$ towards zero. Therefore, the technology-hours impact can be positive in the NK model with practically all σ, when coupled with a persistent shock.

An increase in adjustment cost raises, as in the RBC model, the *mpc*-schedule and decreases marginal costs (now independently from whether the shock is temporary or persistent). Hence the composite function rises markedly above the *mpc*-schedule (at $mpc\&mc\,(\sigma_C^1, \psi_1 > 0)$) and the possibility exists that it does not intersect the hyperbola. Hence, when coupled with a persistent shock, it is possible that the technology-hours impact can be negative in the NK model with practically all σ.

8 A Time-Varying Correlation?

Potentially, our study can help interpret some recent evidence that has suggested that technology-hours responses are time-varying. Fisher (2006), Fernald (2007), and Gambetti (2006), amongst others, report changes in the impact of technology shocks since the mid 1960s. Technology shocks appear to have a strong negative hours effect before the 1980s which then become almost insignificant (with another possible change in the mid-1990s).[25] At the same time, the dynamics of the labor share in the US show a sharp decrease after the early 1980s and then a steady but slow increase until the mid 1990s. It is then likely that shocks that reduce

[25]Results differ by study but, in general, changes on the impact response can be observed around 1973, 1982 and also the first half of the 1990s

the labor share were more prominent in the 1980s as suggested by theories of directed technical change (Acemoglu (2002*a*)) since firms introduce technologies that reduce the use of the factors with a larger cost share.[26]

Thus, for an observed evolution of factor volumes, prices and income shares - and preferred σ - we may back out the sequence of labor- and capital-augmenting technology shocks consistent with that evolution. Accordingly, it is entirely plausible that the response of hours to technology shocks would be time-varying if the technology shocks are themselves time varying in their relative intensity. The value of the substitution elasticity will matter for the transmission of those shocks, and indeed may itself change over time, or, at the least, will differ across countries.[27]

In the same vein, if the introduction of certain technology (e.g., IT technologies) changes the depreciation rate of capital, this can have an impact on the response of hours as can easily be seen in the derivation of the rules for the sign of the impact. Such changes modify the proportion of output devoted to saving, hence affecting the response of hours for given parameter values.[28] Likewise, variations in risk aversion (from say the "Great Inflation" to the "Great Moderation") may be time varying[29]. Thus, our analysis may help interpret some of the time-varying evidence reported and provide a model-based foundation to such approaches.[30]

9 Conclusions

We re-examined the impact of technology shocks on hours worked in business cycle models. The usual interpretation being that, in an RBC model, hours increase after a positive technology shock but initially fall in a NK model. This difference has been taken as a means of empirically discriminating between different theories of business-cycle fluctuations and remains a key controversy in macroeconomics.

Our contributions to this key debate are the following:

[26]McAdam and Willman (2008) develop a similar argument for the euro area.

[27]Muredduy and Strauchz (2010) reports that time-variations in sectoral substitution elasticities for the US and euro area map to those in the share of ICT investment.We may also recall the Jones short-run long run debate on the substitution elasticity. Similarly, Yuhn (1991), following the La Grandville (1989*b*) conjecture, examined whether the high growth rate of east Asian countries was due not to higher technical progress, but to a higher elasticity of substitution.

[28]Depreciation rates have trended upwards in recent years - see Evans (2000). This is compatible with the commonly-held view that the share of equipment in capital has increased while the share of structures has decreased and hence investment is characterized by shorter mean lives.

[29]For instance, Canova (2009) notes a considerable drop in risk aversion from the 1980s onwards for the US, albeit in the context of explaining the "Great Moderation".

[30]This is important, since much of the relevant literature is motivated by the concerns of statistical fit rather than underlying economic stories.

First, given the evidence, we argued that it is no longer defensible for business-cycle models to ignore non-unitary factor substitution and, by implication, factor-biased technical change; Cobb-Douglas is typically rejected by the data, and factor shares display important short- and medium-run fluctuations.

Second, given our interest in supply-side sensitivities, we demonstrated that introducing a Constant Elasticity of Substitution production function with differing substitution possibility requires "normalization" for the calibration of dynamic general equilibrium models. By using normalized CES production functions, we re-stated that technical change can be either capital or labor saving. In the standard RBC model, capital-augmenting shocks yield positive hours responses, whilst labor-augmenting shocks can lead to either response sign. In the NK model, however, labor-augmenting shocks yield negative responses and capital-augmenting shocks, positive or negative responses.

Third, we derived a threshold rule for the determination of the technology-hours sign, the key margin of which lies in the difference between the substitution elasticity and the capital income share, as well as the factor bias of technology shocks.

We conclude that the impact of technology shocks on hours worked can hardly be taken as evidence in support of any particular business-cycle model. This is not to say that empirical evidence *cannot* discriminate between models only that concentrating on the hours response may lead to ambiguous or weak evidence. Consequently, researchers may consider a wider class of discriminatory metrics and try to understand better the entire response space of their model (i.e., under what circumstances certain impacts are or are not generated).

Finally, our threshold rule may help shed light on possible time-variation in the hours-technology correlation.

Our analysis opens important new avenues for research. For instance, if it is not satisfactory for business-cycle models to assume unitary factor substitution, the practice implemented here of appropriately normalizing the supply side and analyzing the consequent sensitivities should become standard. Moreover, although theoretical, our results *can* have empirical implications. We might, for instance, be able to exploit changes in factor income shares to identify different sorts of technology shocks in modeling and SVAR analysis. The models presented here can also serve as a benchmark to study the business cycle properties of movements in factor shares.

References

Acemoglu, Daron. 2002*a*. "Directed technical change." *Review of Economic Studies*, 69: 781–809.

Acemoglu, Daron. 2002*b*. "Technical Change, Inequality and the Labor Market." *Journal of Economic Literature*, 40(1): 7–72.

Acemoglu, Daron. 2009. *Introduction to Modern Economic Growth.* MIT Press.

Alexopoulos, Michelle. 2010. "Read all about it! What happens following a technology shock?" *American Economic Review*, forthcoming.

Arrow, Kenneth J., Hollis B. Chenery, Bagicha S. Minhas, and Robert M. Solow. 1961. "Capital-labor substitution and economic efficiency." *Review of Economics and Statistics*, 43(3): 225–250.

Barro, Robert J., and Xavier Sala-i-Martin. 2004. *Economic Growth.* Cambridge, MA.:2nd Edition, MIT Press.

Basu, Susanto, John G. Fernald, and Miles S. Kimball. 2006. "Are technology improvements contractionary?" *American Economic Review*, 96: 1418–1448.

Basu, Susanto, John G. Fernald, Nicholas Oulton, and Sylaja Srinivasan. 2003. "The case of the missing productivity growth, or does information technology explain why productivity accelerated in the U.S. and not in the U.K.?" *NBER Macroeconomics Annual*, 18: 9–63.

Beach, Earl F. 1971. "Hicks on Ricardo on Machinery." *Economic Journal*, 81: 916–922.

Blanchard, Olivier J. 1997. "The Medium Run." *Brookings Papers on Economic Activity*, 2: 89–158.

Canova, Fabio. 2009. "What Explains The Great Moderation in the U.S.? A Structural Analysis." *Journal of the European Economic Association*, 7(4): 697–721.

Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan. 2008. "New Keynesian Models: Not Yet Useful for Policy Analysis." *American Economic Journal: Macroeconomics*, 1(1): 24–66.

Chirinko, Robert S. 2008. "Sigma: The Long and Short of It." *Journal of Macroeconomics*, 30(2): 671–686.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy*, 113(1): 1–45.

Christiano, Lawrence J., Martin Eichenbaum, and Robert Vigfusson. 2003. "What Happens After a Technology Shock?" National Bureau of Economic Research NBER Working Papers 9819.

Coenen, Günter, Peter McAdam, and Roland Straub. 2008. "Tax reform and labour-market performance in the euro area: a simulation-based analysis using the New Area-Wide Model." *Journal of Economic Dynamics and Control*, 32(8): 2543–2583.

Dedola, Luca, and Stefano Neri. 2007. "What does a technology shock do? A VAR analysis with model-based sign restrictions." *Journal of Monetary Economics*, 54(2): 512–549.

Diamond, Peter A., and Daniel McFadden. 1965. "Identification of the Elasticity of Substitution and the bias of Technical Change: An Impossibility Theorem." Working Paper No. 62, University of California Berkeley.

Diamond, Peter A., Daniel McFadden, and Miguel Rodriguez. 1978. "Measurement of the elasticity of substitution and bias of technical change." In *Production Economics, Vol. 2.* , ed. Melvyn Fuss and Daniel McFadden, 125–147. Amsterdam and North Holland.

Evans, Paul. 2000. "Stylized Facts and their implications for Growth Theory." Mimeo, Ohio State University.

Fernald, John G. 2007. "Trend breaks, long-run restrictions, and contractionary technology improvements." *Journal of Monetary Economics*, 54(8): 2467–2485.

Fernald, John G., and Shanthi Ramnath. 2004. "The Acceleration in U.S. Total Factor Productivity after 1995: The Role of Information Technology." *Economics Perspectives, Federal Reserve Bank of Chicago*, 1Q: 52–67.

Fernández-Villaverde, Jesús. 2009. "The Econometrics of DSGE Models." National Bureau of Economic Research NBER Working Papers 14677.

Fisher, Jonas D. M. 2006. "The Dynamic Effects of Neutral and Investment-Specific Technology Shocks." *Journal of Political Economy*, 114(3): 413–451.

Francis, Neville, and Valerie A. Ramey. 2005. "Is the technology-driven real business cycle hypothesis dead? Shocks and aggregate fluctuations revisited." *Journal of Monetary Economics*, 52(8): 1379–1399.

Francis, Neville, Michael T. Owyang, and Athena T. Theodorou. 2003. "The use of long-run restrictions for the identification of technology shocks." *Federal Reserve Bank of St. Louis Review*, 85(6): 53–66.

Galí, Jordi. 1999. "Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?" *American Economic Review*, 89(1): 249–271.

Galí, Jordi. 2008. *Monetary Policy, Inflation and the Business Cycle.* Princeton University Press.

Galí, Jordi, and Luca Gambetti. 2009. "On the Sources of the Great Moderation." *American Economic Journal: Macroeconomics*, 1(1): 26–57.

Galí, Jordi, and Pau Rabanal. 2004. "Technology Shocks and Aggregate Fluctuations: How Well Does the RBC Model Fit Postwar U.S. Data?" *NBER Macroeconomics Annual*, 225–288.

Gambetti, Luca. 2006. "Time-Varying Dynamics Matter: Technology Shocks and the Response of Hours Worked." Mimeo.

Hicks, John R. 1969. *A Theory of Economic History.* Oxford University Press.

Houthakker, Hendrik S. 1955. "The Pareto Distribution and the Cobb-Douglas Production Function in Activity Analysis." *Review of Economic Studies*, 23: 27–31.

Jones, Charles I. 2003. "Growth, Capital Shares, and a New Perspective on Production Functions." *mimeo*, Stanford University.

Jones, Charles I. 2005. "The Shape of Production Functions and the Direction of Technical Change." *Quarterly Journal of Economics*, 120(2): 517–549.

Jorgenson, Dale W. 2001. "Information technology and the U.S. economy." *American Economic Review*, 91: 1–32.

Juillard, Michel. 2009. "Dynare Manual." Available at `http://www.dynare.org`.

Klump, Rainer, and Olivier de La Grandville. 2000. "Economic growth and the elasticity of substitution: Two theorems and some suggestions." *American Economic Review*, 90: 282–291.

La Grandville, Olivier de. 1989*a*. "Curvature and the elasticity of substitution: Straightening it out." *Journal of Economics*, 66(1): 23–34.

La Grandville, Olivier de. 1989*b*. "In Quest of the Slutzky Diamond." *American Economic Review*, 79: 468–481.

La Grandville, Olivier de. 2009. *Economic Growth: A Unified Approach.* Cambridge University Press.

Levine, Paul, Peter McAdam, and Joseph G. Pearlman. 2008. "Quantifying and Sustaining Welfare Gains from Monetary Commitment." *Journal of Monetary Economics*, 55(2): 1253–78.

Marquetti, Adalmir A. 2003. "Analyzing historical and regional patterns of technical change from a classical-Marxian perspective." *Journal of Economic Behavior and Organization*, 52: 191–200.

McAdam, Peter, and Alpo Willman. 2008. "Medium Run Redux." *Working Paper No. 915*, European Central Bank.

Muredduy, Francesco, and Rolf Strauchz. 2010. "Accounting for the Productivity Slowdown." Mimeo, European Central Bank.

Pesavento, Elena, and Barbara Rossi. 2005. "Do Technology Shocks Drive Hours Up Or Down? A Little Evidence From An Agnostic Procedure." *Macroeconomic Dynamics*, 9(04): 478–488.

Prescott, Edward C. 1986. "Theory Ahead of Business Cycle Measurement." *Quarterly Review, Federal Reserve Bank of Minneapolis*, Fall: 9–22.

Rotemberg, Julio J. 2003. "Stochastic Technical Progress, Smooth Trends, and nearly distinct Business Cycles." *American Economic Review*, 93(5): 1543–1559.

Schmitt-Grohé, Stephanie, and Martin Uribe. 2007. "Optimal simple and implementable monetary and fiscal rules." *Journal of Monetary Economics*, 54(6): 1702–1725.

Smets, Frank, and Rafael Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97(3): 586–606.

Uhlig, Harald. 2004. "Do Technology Shocks Lead to a Fall in Total Hours Worked?" *Journal of the European Economic Association*, 2(2-3): 361–371.

Wicksell, Knut. 1911. *Lectures on Political Economy.* London: G. Routledge. 2nd edn, trans. E. Classen, 2 vols, 1934-5.

Yuhn, Ky-hyang. 1991. "Economic Growth, Technical Change Biases, and the Elasticity of Substitution: A Test of the De La Grandville Hypothesis." *Review of Economics and Statistics*, 73(2): 340–346.

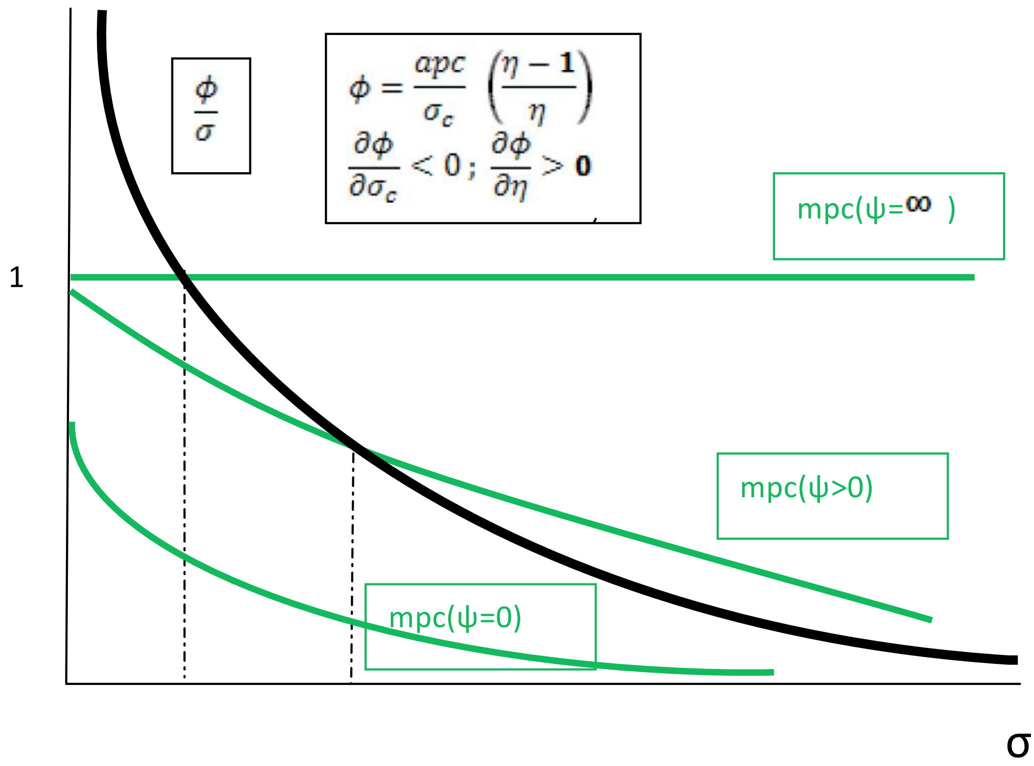


Figure 6: Transitory Capital-Augmenting Shock (RBC)

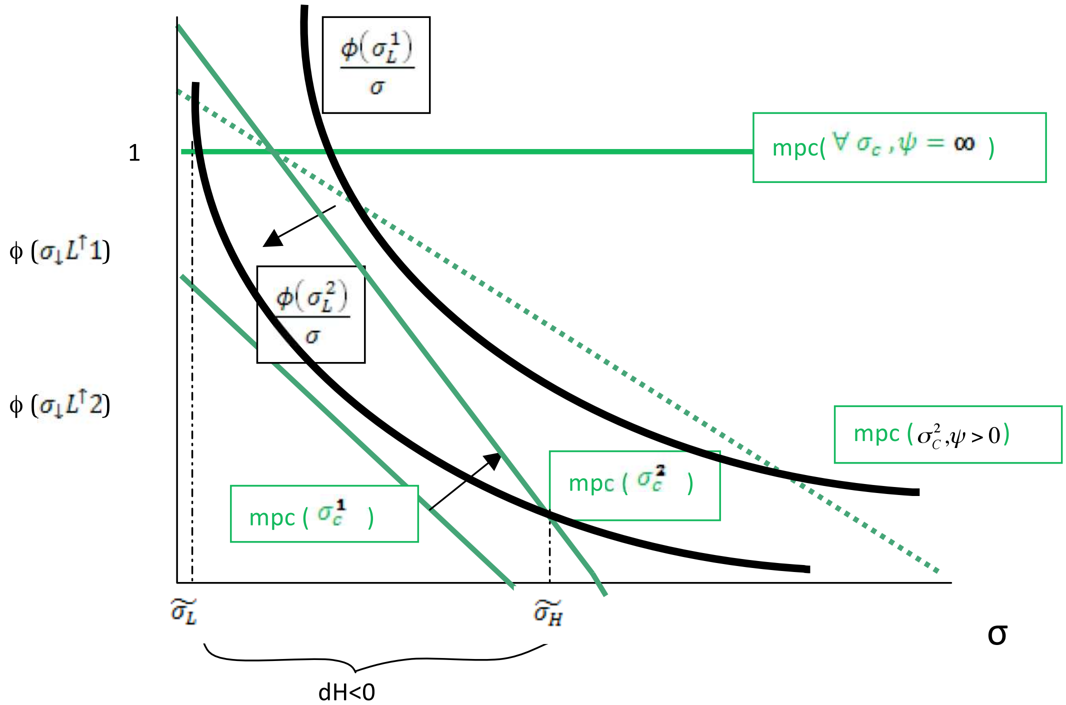


Figure 7: Persistent/Permanent Capital-Augmenting Shock (RBC)

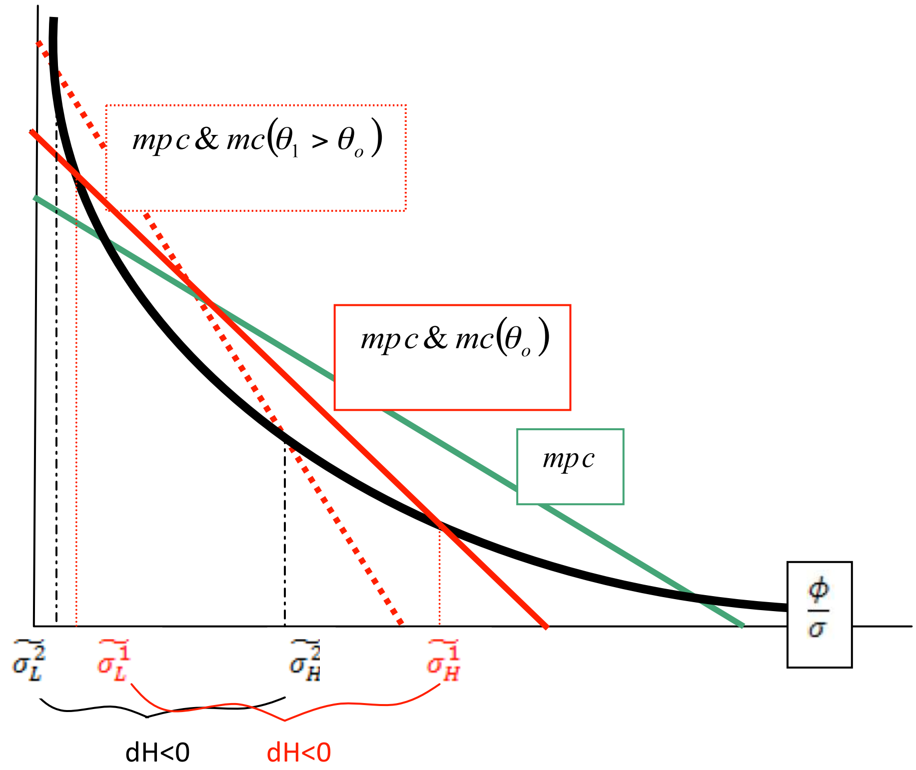


Figure 8: Persistent/Permanent Capital-Augmenting Shocks (NK)

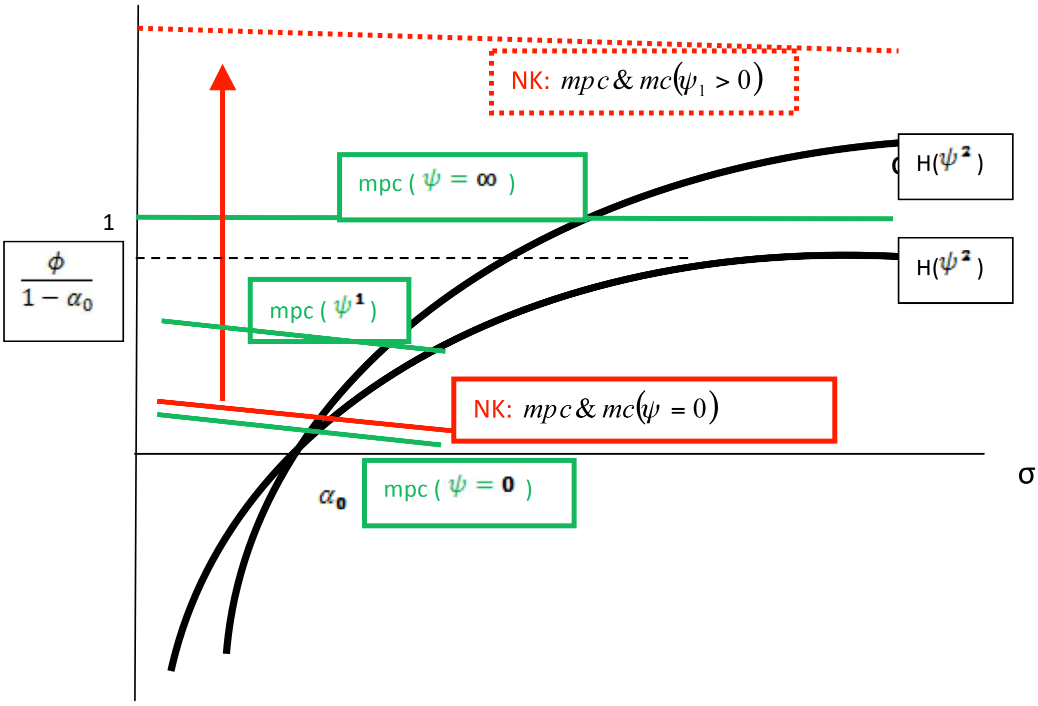


Figure 9: Temporary Labor-Augmenting Shock (RBC and NK)

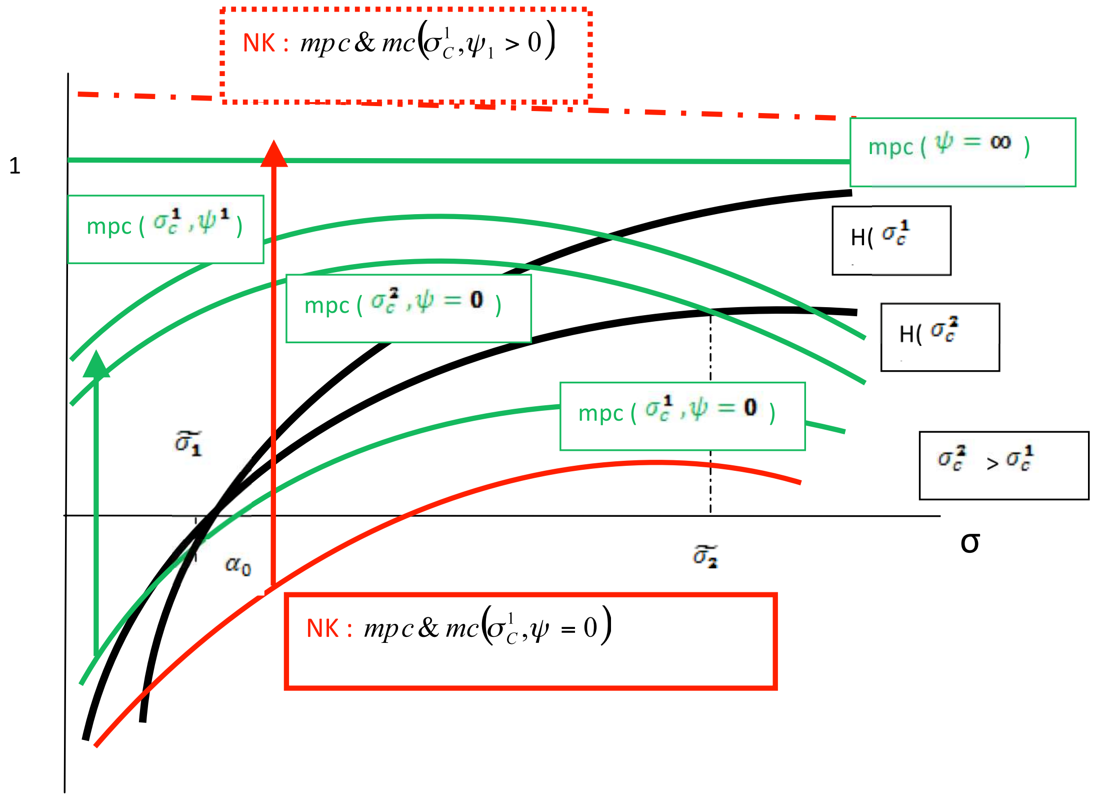


Figure 10: Persistent Labor-Augmenting Shock (RBC and NK)