Gross and net loan flows under search and matching frictions in labour and credit markets

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Abstract

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The interplay of imperfections in several markets is the new frontier of New Keynesian DSGE model research. Following a suggestion by Wasmer and Weil (2004) in a partial equilibrium environment, we introduce search and matching frictions in both labour and financial markets into a cash in advance New Keynesian DSGE model. In this economy output depends on both the number of employed workers and the number of hours worked (intensive and extensive margins). We compare the offspring of this model with some of the empirical findings documented by the recent literature on gross credit flows dynamics over the business cycle and show that our results are in line with several of these findings.

Keywords: Credit frictions; search and matching; business cycle

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1. Introduction

The interplay of imperfections in several markets is the new frontier of New Keynesian DSGE model research. Yet, with the exception of limited contributions (e.g., Wasmer and Weil, 2004; Belke and Fehn, 2001; Nicoletti and Pierrard, 2006), the joint effect of labour and financial markets imperfections remains a mostly unexplored field of research,.

Following a suggestion put forward by Wasmer and Weil (2004) in a different theoretical environment, we introduce search and matching frictions in both the labour and the financial markets into a cash in advance New Keynesian DSGE theoretical model with flexible prices. The aim is to construct a dynamic framework in which, before production begins, wholesale competitive firms search for credit by financiers (banks) posting loan offers; the firms that have these loans granted post vacancies in the labour market, where unemployed workers are searching for jobs. The firms matching with workers obtain bank advances to pay for the wage bill. At the end of the period wholesale firm production is sold to retail firms transforming the homogeneous good into differentiated goods bought by households. Loans are then repaid and households receive profit income from financial intermediaries and firms, and the principal plus interest on deposits. A fraction of the wholesale firms producing in a given period − determined on the basis of a separation rate (or a survival rate) specifying the fraction of labour matches which is destructed at the end of the production period − obtains loans also in the next period, influencing the dynamics of bank advances.

In this economy there emerges a *default* problem, as some of the firms matching with banks and obtaining the loans necessary to post vacancies in the labor market may not be able to match with workers and so to start production and repay their debt with the bank. Our model hence allows us to understand the way in which banks translate their risk of default into the loan rate they charge to firms and the effects this produces on the other economic variables.

Our set set-up also allows to properly highlight the general equilibrium mechanism influencing inflation and unemployment: credit market frictions influence loan availability; this influences the difficulties for firms to finance production and so their willingness to search, which has an effect on labour market results, and so on, giving rise to a cumulative chain of reactions.

Our contribution is in the same spirit of Nicoletti and Pierrard (2006), who compare the offspring of their model with some of the empirical findings documented by Dell'Ariccia and Garibaldi (2005). Gross credit flows (the sum of creation and destruction, respectively equal to the sum of the change in bank loans at all banks that increased loans since last quarter and to the absolute value of the change in loans at all banks that decreased loans) have a much more volatile behavior than net credit flows (the difference between creation and destruction).[1] Along each cycle it occurs a massive reallocation of credit, as measured by the difference between the sum of gross flows (contraction and expansion) and their difference. To this aim, Nicoletti and Pierrard (2006) model gross credit expansion flows as new matches on the credit market and credit contraction flows as destruction of existing matches.

Our paper improves on Nicoletti and Pierrard's (2006) results on several grounds. First, our model provides a clearer and more realistic interpretation of the economic process, as: (i) the cash-in-advance setting requires that bank advance the funds necessary to pay for the cost of the job vacancies and the wage bill (which are instead paid by firms in NP); (ii) firms demand a variable quantity of loans (rather than looking for a match with only one bank) and produce a quantity of output depending on employment (rather than producing a single unit of output with one unit of capital provided by banks[2] and one worker); (iii) the wage and the interest rate on loans are

[1] Net flows hence measure aggregate credit change, whereas gross flows measure of how much credit is expanding and contracting, or the reallocation of lending across borrowers.

[2] The need for this unit of capital is rather artificial in Nicoletti and Pierrard's (2006) model, as it is assumed to be necessary to look for a worker, even though it does finances firms' job vacancy posting not and does not enter the production function.

determined according to a sequential Nash bargaining procedure, as in Wasmer and Weil (2004). Second, our approach allows to directly calculate both gross credit expansion and credit contraction flows, instead of approximating them as new matches and destruction of existing matches on the credit market. Third, we extend the comparison of the model results to empirical findings disregarded by Nicoletti and Pierrard (2006). Fourth, on top of Dell'Ariccia and Garibaldi (2005), we also consider the more recent empirical findings on gross and net loan flows produced by Craig and Haubrich (2006) and Contessi and Francis (2009). We show that the model results are in line with several of these findings.

The paper is structured as follows. In the next section we describe the model economy. In section 3. we discuss our calibration strategy. In section 4. we present the dynamic properties of the model and our results on credit flows. Section 5. concludes.

2. The model economy

The economy is composed by four sets of agents: households, firms, banks and a monetary authority. Following a suggestion by Wasmer and Weil (2004) in a different theoretical environment, we introduce search and matching in both the labour and the financial markets into a cash in advance New Keynesian DSGE model with flexible prices.

The *timing* of the model follows from the hypothesis that firms, which do not possess their own capital, must obtain advances from banks in order to pay for the cost of job vacancy posting. Before production begins, wholesale competitive firms must hence search for lines of credit posted by banks (of total number V_t^B) which also collect deposits (*D)* from households. Each realized match provides firms with one line of credit of nominal value k_n^F that corresponds to the cost the firm must sustain in order to post one vacancy in the labour market, where unemployed workers are searching for jobs. The firm that finds a match in this market obtains from the bank a new line of credit which allows it to pay the wage (*w*) to the matched worker; after wages are paid production occurs.

It follows from these assumptions that, in each period *t*, the total number of matched credit lines which finance job vacancies, L_t^v, is equal to the total number of job vacancies posted by firms, V_t^F, and that the total number of matched credit lines financing wages, L_t^N, is equal to the total number of employed workers, N_t. It also follows that the nominal value of the credit lines financing job vacancies is equal to $k_n^F V_t^F$ and that the nominal value of the credit lines financing wages is equal to the wage bill, $W_t h_t N_t$.

A job vacancy which is not filled produces a default on the corresponding line of credit. We assume that this destroys that line of credit and that the cost of the default is borne by the bank. The presence of this risk of default is taken into account by banks and it hence influences the rate of interest banks charge on lines of credit (loans).

Monopolistically competitive retail firms transform wholesale homogeneous goods into differentiated retail goods which are sold to households. At the end of the period, banks receive from firms the principal plus interest on loans; households receive profit income from financial intermediaries and firms, and the principal plus interest on deposits. A fraction of the wholesale firms that produced in a given period, determined on the basis of a separation rate specifying the fraction of labour matches which is destructed at the end of the production period, obtains loans also in the next period, influencing the dynamics of bank advances. The monetary authority sets the rate of interest according to a rule to be specified below.

2.1 Matching

In the labor market, the search for workers is costly and the existence of search frictions prevents some workers from finding jobs and some posted vacancies from being filled. Similarly, search frictions in the credit market prevent some firms from obtaining lines of credit and some banks from filling their posting of lines of credit. Banks and wholesale firms choose the number of *vacancies* (regarding lines of credit for banks and jobs for firms) they want to post. Their aggregate numbers are denoted by V_t^B and s_t^F.

Denoting s_t^F and s_t^W the demand for lines of credit by firms and the fraction of workers searching for jobs, respectively, the number of new *matches* in the markets for labor and for lines of credit are determined by the *matching* functions $M_t = M\left(s_t^F, s_t^W\right)$ and $H_t = H\left(V_t^B, s_t^F\right)$, which are both homogeneous of degree one and increasing in their arguments. The matching function on the labor market arises by considering that there is a one to one correspondence between a credit line and a labor vacancy.

It follows that $p_t^B = H\left(V_t^B, s_t^F\right)/q_t^F s_t^F$ is the probability that a line of credit demanded by a firm matches with a line of credit vacancy (or, more simply, a credit vacancy) posted by a bank[3], $q_t^B = H\left(V_t^B, s_t^F\right)/q_t^F V_t^B$ is the probability for a bank of filling a posted credit vacancy, $q_t^F = M\left(s_t^F, s_t^W\right)/p_t^B s_t^F$ is the probability that a firm matches with a worker and $p_t^F = M\left(s_t^F, s_t^W\right)p_t^B s_t^W$ is the probability that a worker matches with a firm. Then, in each period it must be: $M\left(s_t^F, s_t^W\right) = q_t^F s_t^W p_t^F = q_t^F p_t^B s_t^F = q_t^F p_t^B s_t^F = q_t^F V_t^F$ and $H\left(V_t^B, s_t^F\right) = V_t^B q_t^B q_t^F = s_t^F p_t^B q_t^F$.

As in existing models with search and matching in the labor market, $q_t^F = q^F\left(\theta_t^L\right)$, with $q^{F'}\left(\theta_t^L\right) < 0$, is a function of the aggregate labor market tightness (from the viewpoint of the firm), $\theta_t^L \equiv s_t^F / s_t^W$. Similarly, $p_t^B = p^B\left(\theta_t^C\right)$, with $p^{B'}\left(\theta_t^C\right) < 0$ is a function of the aggregate tightness in the market for credit lines (from the viewpoint of the firm), $\theta_t^C \equiv s_t^F / V_t^B$, which for simplicity we label as credit market tightness henceforth. It follows that the matching probabilities p_t^F and q_t^B are also functions of tightness in the markets for labor and for credit (lines), respectively: $p_t^F = p^F\left(\theta_t^L\right)$, with $p^{F'}\left(\theta_t^L\right) > 0$, and $q_t^B = q^B\left(\theta_t^C\right)$ with $q^{B'}\left(\theta_t^C\right) > 0$.

To describe the matching process we employ the Cobb-Douglas functions for the expected matches in the labor market, $M_t / p_t^B = \eta\left(s_t^F\right)^{\xi}\left(s_t^W\right)^{1-\xi}$, and for the expected matches of the labor and credit market, $H_t / q_t^F = \upsilon\left(V_t^B\right)^{\zeta}\left(s_t^F\right)^{1-\zeta}$. The first one allow us to specify the labor market probabilities as functions of θ_t^L in a form which will be useful when analyzing steady states and log-linearizing the model: $q_t^F = \frac{\eta}{p_t^B}\left(1/\theta_t^L\right)^{1-\xi}$ and $p_t^F = \frac{\eta}{p_t^B}\left(\theta_t^L\right)^{\xi}$. It follows that: $q_t^F = p_t^F / \theta_t^L$. If the labor market tightness increases, the probability that a firm finds a worker, q_t^F, diminishes, whereas the probability that a worker finds a firm, p_t^F, increases.

Similarly, we use $\frac{H_t}{q_t^F} = \upsilon\left(V_t^B\right)^{\zeta}\left(s_t^F\right)^{1-\zeta}$ to obtain $p_t^B = \frac{\upsilon}{q_t^F}\left(1/\theta_t^C\right)^{\zeta}$ and $q_t^B = \frac{\upsilon}{q_t^F}\left(\theta_t^C\right)^{1-\zeta}$. It follows that: $p_t^B = q_t^B / \theta_t^C$. If credit market tightness increases (because the number of lines of credit

[3] The probability p_t^B is the ratio between the expected matches on the labor market that increase the stock of employment and the demand for credit lines by firms.

demanded by firms goes up, or because the number of credit vacancies posted by banks falls), the probability that a line of credit demanded by a firm matches with a line of credit posted by a bank, p_t^B, diminishes, whereas the probability that a credit vacancy is filled, q_t^B, increases. It is worth noticing that the four matching probabilities (the two markets) are interdependent.

2.2 Households

There exists a continuum of households of mass one maximizing the expected discounted value of their utility. The preferences of the representative household are defined over a composite consumption good, consisting of the differentiated goods produced by retail firms, and leisure. Household members can be either employed (N_t) in a labour match with wholesale firms, at the real wage $w_t = W_t / P_t$ bargained between workers and firms, or unemployed $(1 - N_t)$ and enjoy a fixed amount of benefits, W^u, paid by lump sum on banks' and retailers' profits. The employed worker must work and h_t hours (intensive margin), where h_t is determined by the efficient Nash bargained to be described below. We make the usual assumption that consumption risks are fully pooled within the household.

Households enter each period with a given amount of nominal cash holding ("banknotes") B_t and buy retail goods using their money endowments and their wage income $(W_t h_t N_t)$ from employment plus benefits from unemployment net of nominal deposits with banks (D_t^n). It follows that $B_t + W_t h_t N_t + (1 - N_t) W^u - D_t^n$ is spent to purchase consumption goods from retail firms $P_t C_t$, which refers to an aggregate of goods. As in Dixit and Stiglitz (1977), it is set as $\int_0^1 p_{it} c_{it} di = P_t C_t$, where $c_{it} = (p_{it} / P_t)^{-\varepsilon} C_t$ and $\varepsilon > 1$ is the parameter governing the elasticity of individual goods, and the cost of one unit of the consumption basket is given by the aggregation of the prices of the differentiated products, $P_t = \left[\int_0^1 p_{it}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$.

The purchase of consumption goods is subject to the CIA constraint: $P_t C_t \leq B_t + W_t h_t N_t + W^u (1 - N_t) - D_t^n$. At the end of the period households receive retail firms' and banks profits' net of lump-sum government taxes used to pay benefits to unemployed workers, denoted by Π_t^F e Π_t^B, and obtain the reimbursement of their deposits plus the interest on them: $(1 + r_t^D) D_t^n = R_t^D D_t^n$. It follows that the amount of money carried over to the following period is: $B_{t+1} = B_t + W_t h_t N_t - D_t^n - P_t C_t + W_t^u (1 - N_t) + \Pi_t^F + \Pi_t^B + R_t^D D_t^n$. Substituting the CIA constraint into this equation we get: $B_{t+1} = \Pi_t^F + \Pi_t^B + R_t^D D_t^n$. Calculating this equation a period backward and substituting the result into the CIA constraint gives: $P_t C_t = W_t h_t N_t + W_t^u (1 - N_t) + \Pi_{t-1}^F + \Pi_{t-1}^B - D_t^n + R_{t-1}^D D_{t-1}^n$. Defining $D_t = D_t^n / P_t$, this can be expressed in real terms as:

$$C_t = w_t h_t N_t + w^u (1 - N_t) + \frac{\Pi_{t-1}^F}{P_t} + \frac{\Pi_{t-1}^B}{P_t} - D_t + R_{t-1}^D D_{t-1} \tag{1}$$

Equation (1) states that consumption and savings are financed by real labor income $w_t h_t N_t$, by real benefits, $w^u(1-N_t)$, by the interest on deposits, $r_{t-1}^D D_{t-1}/P_t$, and by profits from banks and retailers, $\left(\Pi_{t-1}^F + \Pi_{t-1}^B\right)/P_t$.

Households hence solve the problem:

$$W_t^H = Max\left[U(C_t, h_t) + \beta E_t W_{t+1}^H\right]$$

$$s.t.\ (1)$$

where β is the household's subjective discount factor. A CRRA specification for $U(C_t, h_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \vartheta \frac{h_t^{1+\phi}}{1+\phi}$ provides the first order conditions which lead to the standard Euler equation:

$$\lambda_t = R_t^D \beta E_t \left(\frac{P_t}{P_{t+1}}\right) \lambda_{t+1} \tag{2}$$

where $R_t^D = \left(1 + r_t^D\right)$ is the nominal interest factor on deposits, $\lambda_t = \left(C_t - h_C C_{t-1}\right)^{-\sigma}$ is the marginal utility of consumption and h_C represents habits in consumption.

The dynamics of employment is described by:

$$N_t = (1-\rho)N_{t-1} + p_t^B p_t^F s_t^W \tag{3}$$

where $s_t^w = 1 - (1-\rho)N_{t-1}$ and $\rho = \left(\rho^F + \rho^B - \rho^F \rho^B\right)$. The first term on the right hand side of equation (3) represents the number of workers who have not separated from firms which were producing in the previous period and which maintain the lines of credit allowing them to finance the wages to be paid to those workers, so that $\rho^F \in [0,1]$ e $\rho^B \in [0,1]$ represent, respectively, the exogenous separation rates relative to the firm-worker[4] and firm-bank relations. We assume that separation in one market (labor or credit) implies also separation in the other one; if one separation occurs, the firm will have to go once again through all the matching phases, starting from the demand for a line of credit of nominal value k_n^F. The second term represents the new matches in the labor market, to be further analyzed below.[5] Unemployment is determined *ex post* as: $U_t = (1 - N_t)$.

2.3 Wholesale firms

There exists a continuum of wholesale firms in the unit interval producing homogenous goods in a competitive sector. The production function of the individual firm *i* is:

$$Y_{it} = A_t h_{it}^{\alpha} N_{it} \tag{4}$$

[4] Hall (2005) documented that the separation rate does not vary considerably along the business cycle, but this is not uncontroversial (see, e.g., Davis, Haltiwanger e Schush, 1996). Endogenous separations rate are considered, e.g., by Walsh (2003).

[5] The new matches are calculated in the same period *t* because we assume that the fraction of households searching jobs is formed at the beginning of each period.

where A_t is a productivity shock with unit mean $E_t(A_t)=1$. Aggregating (4) over firms we get: $Y_t = A_t h_t^{\alpha} N_t$.

The representative firm must determine the optimal number of job vacancies to post. We assume that job vacancy posting is "produced" at no cost by a specialized firm. Vacancies are costs for producing firms and proceeds for the specialized firm which enter aggregate profits that can be spent by households on the basis of the consumption demand for the individual good. This allows us to write $c_{it} = Y_{it}$ or,[6] in aggregate terms, $C_t = Y_t$. The demand for the differentiated good is given by: $Y_{it} = (p_{it} / P_t)^{-\varepsilon} Y_t$.

The number of job vacancies posted by each firm i at time t is equal to the realized matches in the market for lines of credit. We make the timing assumption that credit lines are transformed into job vacancies immediately (Gertler, Sala, Trigari, 2008; Blanchard and Galì, 2006).

$$V_{it}^F = p_t^B s_{it}^F \tag{5}$$

where $s_t^F = 1-(1-\rho)L_{t-1}^N$. The number of workers available for production in each firm i at time t is:

$$N_{it} = (1-\rho)N_{it-1} + q_t^F V_{it}^F \tag{6}$$

Similarly to equation (3), the first term on the right hand side of equation (6) represents the number of workers who have not separated from firm i (if it was active in the previous period and has not separated from the bank). The second term represents the new actual demand for labor, represented by the new vacancies posted by the firm (that has obtained the necessary credit lines), V_{it}^F, multiplied by the probability, q_t^F, that a vacancy is filled. Aggregating (6) over all firm we obtain: $N_t = (1-\rho)N_{t-1} + q_t^F V_t^F$.

The real value of the firm, F_{it}, is:

$$F_{it} = -f_t s_{it}^F + \frac{A_{it} h_{it}^{\alpha}}{\mu_t} N_{it} - R_t^L w_t h_{it} N_{it} - R_t^L k^F \left(q_t^F p_t^B s_{it}^F\right) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left(F_{t+1}\right) \tag{7}$$

where $f_{it} = f / \lambda_{it}$ denotes a utility unit cost born by the entrepreneur, reflecting the time it takes for a prospective line of credit to find a match, and $k^F = k_n^F / P_t$ is the real cost of filling a vacancy (being k_n^F the nominal cost). As wholesale firms sell goods at the competitive price P_t^w, the real value of a firm's output expressed in terms of consumption goods is $P_t^w Y_{it} / P_t = Y_{it} / \mu_t$, where $\mu_t = P_t / P_t^w$ is the mark up of the retail sector over the price of the wholesale good. Recalling that the representative wholesale firm borrows from the bank, at the nominal interest rate factor on loans $R_t^L = (1 + r_t^L)$, the funds necessary to post its vacancies and to hire workers, $\left(R_t^L k^F q_t^F V_{it}^F + R_t^L w_t h_t N_{it}\right)$ represents the firm's real repayment to the bank. Finally, $\beta E_t \frac{\lambda_{it+1}}{\lambda_{it}}$ is the firm's discount rate.

The firm chooses V_{it}^F by setting s_{it}^F; its decision on the credit lines for job vacancies to demand yields:

[6] We can hence take the vacancy cost in nominal terms and exclude it from the aggregate resource constraint.

$$\frac{f_t}{q_t^F p_t^B} + R_t^L k^F = \frac{A_{it} h_{it}^{\alpha}}{\mu_t} - R_t^L w_{it} h_{it} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial F_{it+1}}{\partial N_{it}} \tag{8}$$

By using the envelope theorem we obtain:

$$\frac{\partial F_{it}}{\partial N_{t-1}} = \left(\frac{A_{it} h_{it}^{\alpha}}{\mu_t} - R_t^L w_{it} h_{it} \right)(1-\rho) + \beta(1-\rho) E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial F_{t+1}}{\partial N_t} \tag{9}$$

Combining equations (8) and (9) we get the firm's first order condition:

$$\frac{f_{it}}{p_t^B q_t^F} + R_t^L k^F = \frac{A_{i1} h_{it}^{\alpha}}{\mu_t} - R_t^L w_t h_{it} + \beta(1-\rho) E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{f_{it+1}}{q_{t+1}^F p_{t+1}^B} + R_{t+1}^L k^F \right] \tag{10}$$

Equation (10) shows that the condition to demand a new line of credit depends on the firm's discounted stream of earnings and of savings on job vacancy posting.

2.4 Wage bargaining

The real wage, determined by an efficient Nash bargaining between the firm and the *k-th* worker, is obtained by maximizing $\left[\nabla_t^F\right]^{1-d}\left[\nabla_t^W\right]^d$, where d represents the bargaining power of workers and $(1-d)$ is that of firms, where ∇_t^F is the firm's surplus and ∇_t^W is the worker's surplus. ∇_t^F is equal to the difference between the firms' value if a match is obtained, $\nabla_t^J = \frac{\partial F_{it}}{\partial N_{it}^F} = \frac{A_{it} h_{it}^{\alpha}}{\mu_t} - R_t^L w_{it} h_{it} + \beta(1-\rho) E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{f_{it+1}}{q_{t+1}^F p_{t+1}^B} + R_{t+1}^L k^F \right)$, and the value if a match is not obtained, ∇_t^V. Equation (10) and the free entry condition $\nabla_t^V = 0$ [7] allow us to write:

$$\nabla_t^F = \nabla_t^J - \nabla_t^V = \nabla_t^J = \frac{f_{it}}{q_t^F p_t^B} + R_t^L k^F \tag{11}$$

The worker surplus, ∇_t^W, is equal to the value the worker enjoys when being matched, ∇_t^M, relative to not being matched, ∇_t^N. ∇_t^M is equal to the wage obtained in period t plus the expected values of the possible states of the worker entering the following period, ∇_{t+1}^E:

$$\nabla_t^M = w_t h_{it} - \frac{g(h_{it})}{\lambda_t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \nabla_{t+1}^E \tag{12}$$

where $g(h_{it}) = \vartheta \frac{h_{it}^{1+\phi}}{1+\phi}$ is the disutility of labor (hours) and ϑ is a given parameter.

The following period the worker can still be in a match with a firm which has not separated from a bank and enjoy the value ∇_{t+1}^M, or be in search because at least one separation occurred, which

[7] Recall that defaulting firms leaves the market at no cost.

implies a search processes in both markets (credit and labor). In the latter case, the worker obtains ∇_{t+1}^{M} if both matches are generated and ∇_{t+1}^{N} otherwise. It is hence:

$$\begin{aligned}\nabla_{t+1}^{E} &= (1-\rho)\nabla_{t+1}^{M} + [1-(1-\rho)]p_{t+1}^{B}p_{t+1}^{F}\nabla_{t+1}^{M} + [1-(1-\rho)](1-p_{t+1}^{B}p_{t+1}^{F})\nabla_{t+1}^{N} = \\ &= (1-\rho)\nabla_{t+1}^{M} + \rho(p_{t+1}^{B}p_{t+1}^{F})\nabla_{t+1}^{M} + \rho(1-p_{t+1}^{B}p_{t+1}^{F})\nabla_{t+1}^{N}\end{aligned} \tag{13}$$

∇_{t+1}^{N} is given by the sum of home production (expressed in terms of consumption goods) and the discounted value for a worker entering the following period without being employed in a match:

$$\nabla_{t}^{N} = w^{u} + \beta E_{t}\frac{\lambda_{t+1}}{\lambda_{t}}\nabla_{t+1}^{U} \tag{14}$$

where:

$$\nabla_{t+1}^{U} = p_{t+1}^{B}p_{t+1}^{F}\nabla_{t+1}^{M} + (1-p_{t+1}^{B}p_{t+1}^{F})\nabla_{t+1}^{N} \tag{15}$$

The worker surplus is given by the difference between equations (12) and (14):

$$\nabla_{t}^{W} = \nabla_{t}^{M} - \nabla_{t}^{N} = \left(w_{t}h_{it} - \frac{g(h_{it})}{\lambda_{t}} - w^{u}\right) + \beta E_{t}\frac{\lambda_{t+1}}{\lambda_{t}}\left(\nabla_{t+1}^{E} - \nabla_{t+1}^{U}\right) \tag{16}$$

Using (13), (15) and (16) we hence obtain:

$$\nabla_{t}^{W} = \left(w_{t}h_{it} - \frac{g(h_{it})}{\lambda_{t}} - w^{u}\right) + \beta E_{t}\frac{\lambda_{t+1}}{\lambda_{t}}(1-\rho)(1-p_{t+1}^{B}p_{t+1}^{F})\nabla_{t+1}^{W} \tag{17}$$

The optimality condition, is: $(1-d)\frac{\partial\nabla_{t}^{F}}{\partial w_{t}}\nabla_{t}^{W} + d\frac{\partial\nabla_{t}^{W}}{\partial w_{t}}\nabla_{t}^{F} = 0$. In order to provide straightforward comparison with the literature, and being $\frac{\partial\nabla_{t}^{W}}{\partial w_{t}} = h_{it}$ and $\frac{\partial\nabla_{t}^{F}}{\partial w_{t}} = -R_{t}^{L}h_{it}$, we write it here as:

$$\{\delta_{t}^{F}\}(1-d)\nabla_{t}^{W} = d\left[\nabla_{t}^{F}\right] \tag{18}$$

where $\delta_{t}^{F} = R_{t}^{L}$ By substituting (11) and (16) into (18), and using (17) and (10), we get the following wage equation:

$$\begin{aligned}w_{t} &= (1-\chi_{t})\left[\frac{mrs_{t}}{1+\phi} + \frac{w^{u}}{h_{t}}\right] + \chi_{t}\left[\frac{mpl_{t}}{\alpha\mu_{t}} - R_{t}^{w} + \beta(1-\rho)\frac{1}{h_{t}}E_{t}\frac{\lambda_{t+1}}{\lambda_{t}}\left[f_{t+1}\theta_{t+1}^{F} + R_{t+1}^{L}k^{F}p_{t+1}^{B}p_{t+1}^{F}\right]\right] + \\ &+ \chi_{t}(1-p_{t+1}^{B}p_{t+1}^{F})\left[\beta(1-\rho)\frac{1}{h_{t}}E_{t}\frac{\lambda_{t+1}}{\lambda_{t}}\left(\frac{f_{t+1}}{q_{t+1}^{F}p_{t+1}^{B}} + R_{t+1}^{L}k^{F}\right)\right]\left[1-\chi_{t}\frac{\chi_{t+1}(1-\chi_{t})}{(1-\chi_{t+1})\chi_{t}}\right]\end{aligned} \tag{19}$$

where: $\chi_t = \frac{d}{\left[\delta_t^F (1-d)+d\right]}$, $mrs_t = \vartheta \frac{h_t^{\phi}}{\lambda_t}$ is the worker's marginal rate of substitution, $mpl_t = \frac{\partial A_t h_t^{\alpha}}{\partial h_t} = \alpha A_t h_t^{\alpha-1}$ is the marginal product of labor and $R_t^w = r_t^L w_t$ is the interest paid by the firm on the real wage borrowed from a bank. We hence obtain a variation of the conventional sharing rule where the relative share $\chi_t = \frac{d}{\left[\delta_t^F (1-d)+d\right]}$ depends not only on the bargaining power but also on the effect of the wage on the firms's surplus (relative allocational effect).

When linearizing the model, it is however useful to use a different formulation of the bargained wage:

$$w_t = (1-d)\left(\frac{mrs_t}{1+\phi} + \frac{w^u}{h_t}\right) + \frac{d}{R_t^L}\left[\frac{mpl_t}{\alpha\mu_t} + (1-\rho)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{1}{h_t}\left(\frac{f}{q_{t+1}^F p_{t+1}^B} + k^F R_{t+1}^L\right)\left(1 - \frac{R_t^L\left(1-p_{t+1}^B p_{t+1}^F\right)}{R_{t+1}^L}\right)\right] \tag{20}$$

This equation depicts the wage as a weighted average of the worker's disutility from supplying hours of work, plus the foregone flow benefit from unemployment and the firm's revenues plus future expected net present value from employment.

Further, as it is necessary when calculating the bargained interest rate, it is useful to calculate here:

$$\frac{\partial w_t}{\partial R_t^L} = -\frac{d}{\left(R_t^L\right)^2}\left[\frac{mpl_t}{\alpha\mu_t} + (1-\rho)\beta E_t \frac{\lambda_{t+1}}{h_t\lambda_t}\left(\frac{f_{t+1}}{q_{t+1}^F p_{t+1}^B} + k^F R_{t+1}^L\right)\right] \tag{21}$$

The efficient bargaining determines also the number of hours. The optimality condition is $\frac{\partial \nabla_t^F}{\partial h_t}(1-d)\nabla_t^W = d\frac{\partial \nabla_t^W}{\partial h_t}\nabla_t^F$; being $\frac{\partial \nabla_t^F}{\partial h_t} = \left(\frac{mpl_t}{\mu_t} - w_t R_t^L\right)$ and $\frac{\partial \nabla_t^W}{\partial h_t} = (mrs_t - w_t)$, it can be written as: $(1-d)\left(\frac{mpl_t}{\mu_t} - w_t R_t^L\right)\nabla_t^W = d(mrs_t - w_t)\nabla_t^F$. From equation (18) we know that $\nabla_t^W = \frac{d}{R_t^L(1-d)}\nabla_t^F$, so optimal hours are obtained from the condition: $\frac{mpl_t}{R_t^L \mu_t} = mrs_t$, that is:

$$h_t = \left(\vartheta \frac{R_t^L \mu_t}{\alpha A \lambda_t}\right)^{1/(\alpha-1-\phi)} \tag{22}$$

As it is necessary when calculating the bargained interest rate, it is useful to calculate here:

$$\frac{\partial h_{it}}{\partial R_t^L} = \frac{1}{(\alpha-1-\phi)}\frac{h_{it}}{R_t^L} \tag{23}$$

2.5 Retail firms

Retail firms purchase the goods produced by the wholesale sector and transform them into the differentiated products purchased by households. Cost minimization provides the condition that the retail firm's nominal marginal cost (MC_t^n) be equal to the price charged by the wholesale firm for its product:

$$MC_t^n = P_t MC_t = P_t^w \tag{24}$$

where $MC_t = P_t^w / P_t = 1/\mu_t$ is the real marginal cost.

If in the retail sector prices are adjusted according to the Calvo (1983) rule, each period a firm can adjust its price with probability $1-\omega$. Given MC_t^n and setting $p_t^* = p_{it}$, in a symmetric equilibrium all firms set the price so as to maximize the expected lifetime profits (subject to the demand $Y_{it} = (p_{it}/P_t)^{-\varepsilon} Y_t$): $E_t \sum_{l=0}^{\infty} \omega^l \beta^l \frac{\lambda_{t+l}}{\lambda_l} \left[\left(\frac{p_t^*}{P_{t+l}} \right)^{1-\varepsilon} - MC_{t+l} \left(\frac{p_t^*}{P_{t+l}} \right)^{-\varepsilon} \right] Y_{t+l}$. This provides the price equation:

$$\frac{p_t^*}{P_{t+l}} = \Theta \frac{E_t \sum_{l=0}^{\infty} \omega^l \beta^l MC_{t+l} (P_{t+l}/P_t)^{-\varepsilon} C_{t+l}^{\ 1-\sigma}}{E_t \sum_{l=0}^{\infty} \omega^l \beta^l (P_{t+l}/P_t)^{\varepsilon-1} C_{t+l}^{\ 1-\sigma}} \tag{25}$$

where: $\Theta = \varepsilon/(\varepsilon-1)$. From (25) the usual (log-linearized) NEK Phillips curve is obtained:

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa} \hat{MC}_t \tag{26}$$

where $\tilde{\kappa} = \frac{(1-\beta\omega)(1-\omega)}{\omega}$ and the hat denotes percentage deviations from the steady state. Under flexible prices, equation (25) reduces to the standard Blanchard and Kiyotaki (1987) equation:

$$p_t^* / P_{t+j} = \Theta MC_{t+j} \tag{27}$$

2.6 Banks

Banks, operating in a competitive market, collect nominal deposits (D_t^n) from households at the interest rate on deposits r_t^D, post vacancies in the credit market (V_t^B) at the utility unit cost $b_{it} = b/\lambda_{it}$ and provide loans, upon which the interest rate on loans r_t^L is charged, to wholesale firms.

Each match in the credit market (the total number being equal to $V_t^B q_t^B$) provides firms with the funds necessary to post one vacancy in the labor market $(k^F V_{it}^F)$. As already clarifies, only a share of these funded vacancies do however match with workers. Each of these matches provides a line of

credit to pay for the wage bill $(w_t N_{it})$ to be anticipated to matched workers; the proceeds from sales allow firms to repay the loans and to pay the charged interest to the bank. In the following period, each of these firms will continue to have their wage bill financed by the banks, unless a separation occurs in at least one market. As already described, the exogenous separation rate in the credit market is denoted $\rho^B \in [0;1]$. The funded credit lines for vacancies that do not produce a labor match get destroyed and are not repaid by firms.

We make also in this case the timing assumption that matched credit lines are transformed into lines of credit financing vacancies immediately. Given these assumptions, and financing wages evolve, respectively, according to:

$$L_{jt}^V = q_t^B V_{jt}^B \tag{28}$$

$$L_{jt}^N = (1-\rho) L_{jt-1}^N + q_t^F q_t^B V_{jt}^B \tag{29}$$

Aggregating (30) over all banks we obtain: $L_t^N = (1-\rho) L_{t-1}^N + q_t^F q_t^B V_t^B$.

Recalling that the funded credit lines for vacancies that are not "transformed" into a labor match get destroyed and are not repaid by firms (the bank hence looses the anticipated funds and the interest on this amount), the real value of a bank, J_{jt}, can be written as:.

$$J_{jt} = \left(R_t^L - R_t^D\right) w_t h_{it} L_{jt}^N + \left(R_t^L q_t^F - R_t^D\right) k^F L_{jt}^V - b_t V_{jt}^B + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(J_{jt+1}\right) \tag{30}$$

where $b_t = \dfrac{b}{\lambda_t}$.The bank chooses L_{jt+1}^V by setting V_{jt}^B and its decision on the credit lines for job vacancies to post yields:

$$\frac{b_t}{q_t^F q_t^B} - \left(R_t^L q_t^F - R_t^D\right) \frac{k^F}{q_t^F} - \left(R_t^L - R_t^D\right) w_t h_{it} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial J_{t+1}}{\partial L_{jt}^N} \tag{31}$$

By using the envelope theorem we obtain:

$$\frac{\partial J_t}{\partial L_{jt-1}^N} = (1-\rho)\left(R_t^L - R_t^D\right) w_t h_{it} + \beta(1-\rho) E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial J_{t+1}}{\partial L_{jt}^N} \tag{32}$$

Combining equations (31) and (32) we get what we interpret as the "credit creating condition":

$$\frac{b_t}{q_t^F q_t^B} - \left(R_t^L q_t^F - R_t^D\right) \frac{k^F}{q_t^F} = \left(R_t^L - R_t^D\right) w_t h_{it} + \beta(1-\rho) E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{b_{t+1}}{q_{t+1}^F q_{t+1}^B} - \left(R_{t+1}^L q_{t+1}^F - R_{t+1}^D\right) \frac{k^F}{q_{t+1}^F} \right] \tag{33}$$

Similar to the firm's case, the condition to offer a new line of credit depends on the bank's discounted stream of earnings and of savings on credit vacancy posting and on defaults.

2.7 Loan rate bargaining

In line with Wasmer and Weil (2004), we assume a sequential bargaining framework: the rate of interest on loans is first negotiated by banks and firms; workers and firms then bargain over the wage and hours (efficient bargaining). We hence solve the problem backward, taking into account the effect of R_t^L on *w* and on h_t.

$\nabla^B = \nabla^C - \nabla_t^{VB}$ is the bank's surplus, which is equal to the value to the bank if a match with the firm is generated, $\nabla^C = \frac{\partial J_t}{\partial L_{jt}^N}$, less the value if a match is not generated, which is $\nabla_t^{VB} = 0$ for the free entry condition. We hence obtain: $\nabla_t^C = \left(R_t^L - R_t^D\right) w_t h_{it} + (1-\rho)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[\frac{b_{t+1}}{q_{t+1}^F p_{t+1}^B} - \left(R_{t+1}^L q_{t+1}^F - R_{t+1}^D\right)\frac{k^F}{q_{t+1}^F}\right]$. The optimality condition is:

$$(1-z)\gamma_t^B \nabla_t^J = z\gamma_t^F \nabla_t^C \tag{34}$$

where *z* represents the bargaining power of banks, $(1-z)$ is that of firms, $\gamma_t^B = w_t h_t + \left(R_t^L - R_t^D\right)\left(\frac{\partial w_t}{\partial R_t^L} h_t + \frac{\partial h_t}{\partial R_t^L} w_t\right)$ and $\gamma_t^F = \frac{mpl_t}{\mu_t}\frac{\partial h_t}{\partial R_t^L} + w_t h_t + R_t^L\left(\frac{\partial w_t}{\partial R_t^L} h_t + w_t \frac{\partial h_t}{\partial R_t^L}\right)$

Using (21) and (23) we obtain:

$$\begin{aligned} R_t^L = {} & \psi_t \frac{1}{w_t}\left[\frac{mpl_t}{\mu_t} + (1-\rho)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{1}{h_t}\left(\frac{f}{q_{t+1}^F p_{t+1}^B} + R_{t+1}^L k^F\right)\right] + \\ & + (1-\psi_t)\frac{1}{w_t}\left\{w_t R_t^D - (1-\rho)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{1}{h_t}\left[\frac{b}{q_{t+1}^F q_{t+1}^B} - \left(R_{t+1}^L q_{t+1}^F - R_{t+1}^D\right)\frac{k^F}{q_{t+1}^F}\right]\right\} \end{aligned} \tag{35}$$

where: $\psi_t = \frac{(1-z)\gamma_t^B}{\left[(1-z)\gamma_t^B + z\gamma_t^F\right]}$.

Similarly to what obtained as for the wage bargaining, the interest rate on loans turns out to be a weighted average of the firm's revenues plus future expected net present value from entering a credit relation, on the one side, and the rate of interest on deposits plus the banks' future expected net present value from entering a credit relations. The weights, once again, not only depend on the relative bargaining power *z*, but also on the wage relative allocational effect, encapsulated in γ_t^B and γ_t^F.

If $\gamma_t^B / \gamma_t^F = 1$ we obtain the determination of w_t and R_t^L in terms of two separate Nash bargaining rules:

$$\begin{aligned} R_t^L = {} & \frac{(1-z)}{w_t}\left\{R_t^D w_t - (1-\rho)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{1}{h_t}\left[\frac{b}{q_{t+1}^F q_{t+1}^B} - \left(R_{t+1}^L q_{t+1}^F - R_{t+1}^D\right)\frac{k^F}{q_{t+1}^F}\right]\right\} + \\ & + \frac{z}{w_t}\left\{\frac{mpl_t}{\mu_t} + (1-\rho)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{1}{h_t}\left(\frac{f}{q_{t+1}^F p_{t+1}^B} + R_{t+1}^L k^F\right)\right\} \end{aligned} \tag{35}$$

and

$$w_t = (1-\chi_t)\left[\frac{mrs_t}{1+\phi} + \frac{w^u}{h_t}\right] + \chi_t\left[\frac{mpl_t}{\alpha\mu_t} - r_t^L w_t + \beta(1-\rho)\frac{1}{h_t}E_t\frac{\lambda_{t+1}}{\lambda_t}\left[f\theta_{t+1}^F + R_{t+1}^L k^F p_{t+1}^B p_{t+1}^F\right]\right] + \\ + \chi_t\left(1-p_{t+1}^B p_{t+1}^F\right)\left[\beta(1-\rho)\frac{1}{h_t}E_t\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{f}{q_{t+1}^F p_{t+1}^B} + R_{t+1}^L k^F\right)\right]\left[1-\chi_t\frac{\chi_{t+1}(1-\chi_t)}{(1-\chi_{t+1})\chi_t}\right] \tag{35}$$

2.7 Monetary authorities

A central bank employs the following monetary rule to set the policy rate, which we assume for simplicity equal to the rate on deposits:

$$R_t^D = \left(R_{t-1}^D\right)^{\rho_R} \cdot \left(P_t / P_{t-1}\right)^{(1-\rho_R)\delta_\pi} \cdot \left(Y_t\right)^{(1-\rho_R)\delta_Y} \cdot e^{\upsilon_t} \tag{36}$$

where ρ_R is the degree of interest rate smoothing and δ_π, δ_Y and δ_U are the weights assigned to the targets of inflation $\left(P_t / P_{t-1}\right)$, output and unemployment, respectively, and υ_t is a white noise stochastic process.

2.8 The complete model

The complete set of equations composing the model, which will be linearized in order to simulate its dynamic properties, can be summarized as follows:

$$\lambda_t = R_t^D \beta E_t\left(\frac{P_t}{P_{t+1}}\right)\lambda_{t+1} \tag{37}$$

$$\lambda_t = \left(C_t - h_C C_{t-1}\right)^{-\sigma} \tag{38}$$

$$V_{it}^F = p_t^B s_{it}^F \tag{39}$$

$$N_t = (1-\rho)N_{t-1} + p_t^B p_t^F s_t^W \tag{40}$$

$$N_t = (1-\rho)N_{t-1} + q_t^F p_t^B s_t^F \tag{41}$$

$$s_t^W = 1-(1-\rho)N_{t-1} \tag{42}$$

$$U_t = 1 - N_t \tag{43}$$

$$Y_t = A_t h_t^\alpha N_t \tag{44}$$

$$\frac{p_t^*}{P_{t+l}} = \Theta\frac{E_t\sum_{l=0}^{\infty}\omega^l\beta^l MC_{t+l}\left(P_{t+l}/P_t\right)^{-\varepsilon}C_{t+l}^{\;1-\sigma}}{E_t\sum_{l=0}^{\infty}\omega^l\beta^l\left(P_{t+l}/P_t\right)^{\varepsilon-1}C_{t+l}^{\;1-\sigma}} \tag{45}$$

$$Y_t = C_t \tag{46}$$

$$\frac{f_t}{q_t^F p_t^B} = \frac{A_t h_t^\alpha}{\mu_t} - R_t^L\left(k^F + w_t h_t\right) + \beta(1-\rho)E_t \frac{\lambda_{t+1}}{\lambda_t}\left[\frac{f_{t+1}}{q_{t+1}^F p_{t+1}^B} + R_{t+1}^L k^F\right] \tag{47}$$

$$w_t = (1-d)\left(\frac{mrs_t}{1+\phi} + \frac{w^u}{h_t}\right) + \frac{d}{R_t^L}\left[\frac{mpl_t}{\alpha\mu_t} + (1-\rho)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{1}{h_t}\left(\frac{f_{t+1}}{q_{t+1}^F p_{t+1}^B} + k^F R_{t+1}^L\right)\left(1 - \frac{R_t^L\left(1 - p_{t+1}^B p_{t+1}^F\right)}{R_{t+1}^L}\right)\right] \tag{48}$$

$$h_t = \left(\vartheta \frac{R_t^L \mu_t}{\alpha A \lambda_t}\right)^{1/(\alpha-1-\phi)} \tag{49}$$

$$L_t^V = q_t^B V_t^B \tag{50}$$

$$s_t^F = 1 - (1-\rho)L_{t-1}^N \tag{51}$$

$$L_t^N = (1-\rho)L_{jt-1}^N + q_t^F q_t^B V_t^B \tag{52}$$

$$L_t^N = (1-\rho)L_{t-1}^N + q_t^F p_t^B s_t^F \tag{53}$$

$$\frac{b_t}{q_t^F q_t^B} - \left(R_t^L q_t^F - R_t^D\right)\frac{k^F}{q_t^F} - \left(R_t^L - R_t^D\right)w_t h_t = (1-\rho)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[\frac{b_{t+1}}{q_{t+1}^F q_{t+1}^B} - \left(R_{t+1}^L q_{t+1}^F - R_{t+1}^D\right)\frac{k^F}{q_{t+1}^F}\right] \tag{54}$$

$$\begin{aligned} R_t^L = {} & \psi_t \frac{1}{w_t}\left[\frac{mpl_t}{\mu_t} + (1-\rho)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{1}{h_t}\left(\frac{f_{t+1}}{q_{t+1}^F p_{t+1}^B} + R_{t+1}^L k^F\right)\right] + \\ & + (1-\psi_t)\frac{1}{w_t}\left\{w_t R_t^D - (1-\rho)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{1}{h_t}\left[\frac{b_{t+1}}{q_{t+1}^F q_{t+1}^B} - \left(R_{t+1}^L q_{t+1}^F - R_{t+1}^D\right)\frac{k^F}{q_{t+1}^F}\right]\right\} \end{aligned} \tag{55}$$

$$\gamma_t^B = w_t h_t + \left(R_t^L - R_t^D\right)\left(\frac{\partial w_t}{\partial R_t^L} h_t + \frac{\partial h_t}{\partial R_t^L} w_t\right) \tag{56}$$

$$\gamma_t^F = \frac{mpl_t}{\mu_t}\frac{\partial h_t}{\partial R_t^L} + w_t h_t + R_t^L\left(\frac{\partial w_t}{\partial R_t^L} h_t + w_t \frac{\partial h_t}{\partial R_t^L}\right) \tag{57}$$

$$\psi_t = \frac{(1-z)\gamma_t^B}{\left[(1-z)\gamma_t^B + z\gamma_t^F\right]} \tag{58}$$

$$\frac{\partial w_t}{\partial R_t^L} = -\frac{d}{\left(R_t^L\right)^2}\left[\frac{mpl_t}{\alpha\mu_t} + (1-\rho)\beta E_t \frac{\lambda_{t+1}}{h_t \lambda_t}\left(\frac{f_{t+1}}{q_{t+1}^F p_{t+1}^B} + k^F R_{t+1}^L\right)\right] \tag{59}$$

$$\frac{\partial h_t}{\partial R_t^L} = \frac{1}{(\alpha-1-\phi)}\frac{h_t}{R_t^L} \tag{60}$$

$$mpl_t = \alpha A_t h_t^{\alpha-1} \tag{61}$$

$$mrs_t = \vartheta \frac{h_t^\phi}{\lambda_t} \tag{62}$$

$$MC_t = 1/\mu_t \tag{63}$$

$$\theta_t^C = s_t^F / V_t^B \tag{64}$$

$$\theta_t^L = s_t^F / s_t^W \tag{65}$$

$$M_t = p_t^B \eta \left(s_t^F\right)^{\xi} \left(s_t^W\right)^{1-\xi} \tag{66}$$

$$H_t = q_t^F \upsilon \left(V_t^B\right)^{\zeta} \left(s_t^F\right)^{1-\zeta} \tag{67}$$

$$R_t^D = \left(R_{t-1}^D\right)^{\rho_R} \cdot \left(P_t / P_{t-1}\right)^{(1-\rho_R)\delta_\pi} \cdot \left(Y_t\right)^{(1-\rho_R)\delta_Y} \cdot e^{\upsilon_t} \tag{68}$$

The model is closed by adding the following equilibrium conditions:

$$s_t^F q_t^F = p_t^F s_t^W \tag{69}$$

$$p_t^B s_t^F = q_t^B V_t^B \tag{70}$$

$$U_t = s_t^W \left(1 - p_t^F p_t^B\right) \tag{71}$$

$$1 - L_t^N = \left(1 - p_t^B q_t^F\right) s_t^F \tag{72}$$

3. Calibration

In order to study the dynamic properties of the model, we log-linearize equations (37)-(72) around the steady state and assign to the model parameters the most recent values employed by the literature. The values of the calibrated parameters are summarized in the following table. The other coefficients of the log-linear model are obtained by steady state conditions summarized in the Appendix.

In the exercise we present here, we use a very simple version of the model. We hence disregard habit in consumption $(h_C = 0)$ and assume logarithmic utility from consumption $(\sigma = 1)$. Even though in this model the elasticity of output to hours does not correspond to the labor share (as it depends on the outcome of the bargaining process), we follow the convention of setting α = 2/3 (Gertler, Sala and Trigari, 2008). We also follow the conventional attitude of setting the quarterly discount factor so as to obtain a quarterly real steady state rate of interest on deposits of approximately 1 per cent $(\beta = 0.99)$.

The AR coefficient on technology shock is set at 0.8, indicating a fairly high persistency. The steady state TFP shock on the production function is set equal to its unitary mean (Ravenna and Walsh, 2008). The steady state employment rate N (and so L^N) is calibrated at 0.8, a value lower than in the data because we interpret the unmatched workers as being both unemployed and partly out of the labor force, in line with the abstraction we made from labor force participation decisions (Trigari, 2006). The steady state value for the mark-up is μ = 1.1, implying a 10 per cent retail mark-up on wholesale prices. The value of the steady state interest rate on loans is R^L = 1.015, which implies a spread between the (annualized) lending and deposit rates of approximately 2 per cent. We impose a steady state relation between the cost of a vacancy and output $k^F = tY$, with t = 0.1. The replacement rate is set at $\overline{\omega} = 0.6$, which is between the values of 0.4 proposed by Shimer (2005) and 0.8 used by Hall, the latter being based on a broader interpretation that permits utility from leisure (2008). Firm's probability of not adjusting prices is the usual value $\omega = 0.75$.

Variable	Description	Value
α	elasticity of output to hours	0.68
β	quarterly discount factor	0.99
h_c	coefficient of habit in consumption	0.0
σ	consumption curvature	1
ρ^F	firm-worker separation rate	0.1
ρ^B	bank-firm separation rate	0.1
δ^π	weight of inflation in the policy rule	1.5
δ^y	weight of output in the policy rule	0.5
d	worker bargaining power in real wage bargaining	0.15
z	firm bargaining power in lending rate bargaining	0.5
ξ	elasticity of matches to labor market searchers	0.5
ζ	credit matching elasticity	0.5
ρ^r	interest rate smoothing	0.9
ρ^A	AR coefficient on technology shock	0.8
A	SS TFP shock on production function	1
N	SS Employment	0.8
q^F	SS Job finding probability	0.7
q^B	SS credit finding probability	0.7
μ	SS retail mark-up	1.1
L^N	SS lines of credit	N
ν	coefficient in credit matching function	1
η	coefficient in labor matching function	1
R^L	SS loan interest rate	1.015
ϕ	inverse of the elasticity in the supply of hours	1
h	SS hours	0.33
t	vacancy cost relative to y	0.1
$\overline{\omega}$	replacement rate	0.6
ω	probability of not adjusting prices	0.75

The elasticity of intertemporal substitution in the supply of hours is equal to $1/\phi$. In the face of the existing controversy on this coefficient, we follow the standard business cycle literature and set ϕ equal to 1.[8] As for the policy rule, we use the usual estimates by Clarida, Galí and Gertler (2000) and set the interest rate smoothing coefficient ρ^r equal to 0.9 and the parameters attached to inflation δ^π and to output δ^y equal to 1.5 and to 0.5, respectively. Finally, we normalize the value of the time spent working in the steady state, h, to 0.33 (one third of the day) and obtain the value of ϑ (the coefficient of the CRRA equations for hours) which is coherent with the chosen normalization.

As for credit parameters for which we have limited evidence, we adopt the strategy of setting their values equal to their labor market counterparts. As a consequence, we calibrate the separation rates in both the labor and the credit market at 0.1, as in Ravenna and Walsh (2008). We set the

[8] Other contributions exploit instead the evidence provided by microeconomic estimates and set much lower values for this elasticity. Trigari (2006), for example, sets the value of ϕ equal to 10, which implies a labor supply elasticity of 0.1.

elasticity of matches to labor market searchers according to the evidence provided by Hagerdon and Manovskii (2008), which is also the midpoint of the evidence typically cited in the literature (Gertler, Sala and Trigari, 2008), and do the same for credit market matches, so as to calibrate $\xi = \zeta = 0.5$. We assume a unitary efficiency of matching in both markets and hence set the coefficients in the credit matching functions: $\eta = \nu = 1$. The steady state job finding probability is taken as a summary from wide evidence, and the same value is used also for its credit market counterpart: $q^F = q^B = 0.7$.

We do not however follow the same convention as far as the bargaining powers are concerned. Whereas the literature has usually set $d = 0.5$, the recent estimations by Hagerdon and Manovskii (2008) suggest a much lower value of the worker bargaining power, i.e., $d = 0.15$. In the absence of robust evidence, we prefer to set the firm power in the bargaining over the rate of interest equal to the midpoint of possible values and calibrate $z = 0.5$.

4. Results

4.1 Dynamic properties of the model

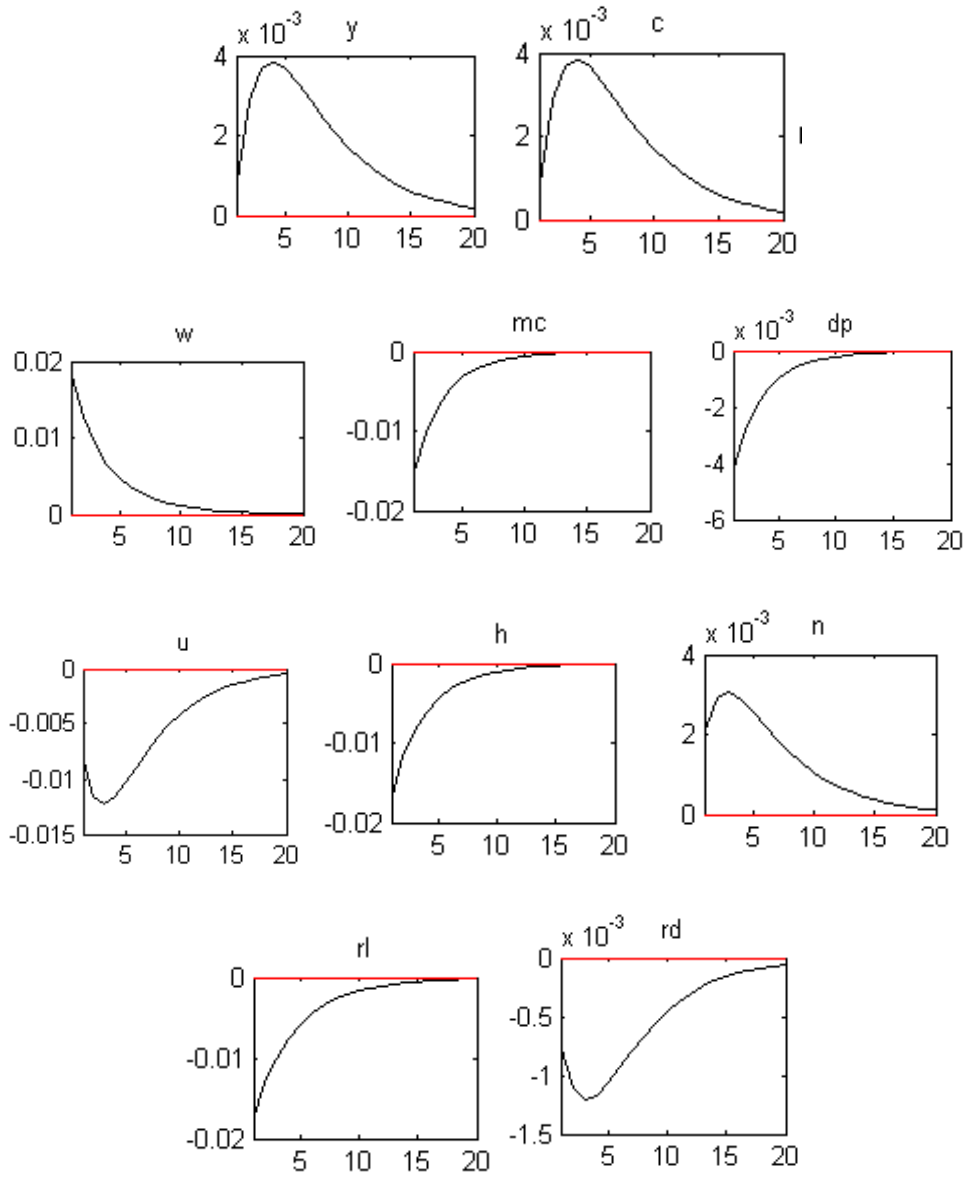
In order to focus on credit flows and to compare our results with the empirical literature, in this paper we limit our attention to the response of the model variables to a positive technological shock. Figure 1 shows the main impulse response functions (IRF) we obtain by employing the baseline model. The IRF show a dynamic behavior in the macroeconomic variables in line with the NEK DSGE models with search and matching in the credit market, but also display interesting and novel features.

As shown by the first row of diagrams of Figure 1, a technology shock increases income (*y*) and consumption (*c*). The second row of diagrams clarifies that wages (*w*) also go up, whereas marginal costs (*mc*) and inflation (dp) fall. This is no surprise.

More interesting is the evidence provided by the third row of diagrams, which highlights the presence of a novel phenomenon: whereas unemployment (*u*) falls and jobs (*n*) increase, the expansion of income produces a fall in the intensive margin (*h*). The reduction in the number of hours provides a new version of the well known "productivity-employment puzzle", i.e., the possibility that employment falls after a technological innovation expanding output. NK DSGE models with sticky prices are consistent with this finding, as long as monetary policy is not too accommodative and they deal only with the extensive labor margin (Gali, 1999; Gali and Rabanal, 2004; Basu, Fernald and Kimball, 2004).

This issue must be considered together with the other interesting feature of our model, that is, the relative behavior of the rates of interest on deposits (*rd*) and on loans (*rl*). Given the behavior of the policy rate (*rd*), driven by the behavior of the variables entering the monetary policy rule, the functioning of the credit market allows bank to magnify the behavior of the loan rate, with effects on the dynamics of credit market variables to be discussed in the next section. The sharp reduction in the interest rate on loans allows workers to obtain higher wages and to sustain "employment". Yet, as workers and firms bargain on both the wage and the intensive margin, our intuition is that they "exchange" higher wages for lower working hours, but since firm need to expand production, they are forced to increase the number of employed workers. A complete understanding of this behavior of the model lies however beyond the aims of the present paper and, also considering that the theoretical and empirical debate on the productivity-employment puzzle is still alive and that the issue is not at all controversial, we are forced to leave the deepening of this characteristic of our model to future research.

Figure 1. Macroeconomic variables and interest rates



4.2 Loan flows

Three of Dell'Ariccia and Garibaldi's (2005) results about gross credit flows in the U.S. banking system between 1979 and 1999 are especially important for the present paper:

a. gross flows are much larger than net flows (credit expansion and contraction coexist at any phase of the cycle);
b. cyclical fluctuations of gross flows are more than an order of magnitude larger than GDP fluctuations;

c. credit contraction is more volatile than credit expansion and excess credit reallocation (the sum of gross flows in excess of net changes) is negatively correlated with GDP fluctuations.[9]

By using a longer data series (1959-2004 versus 1979-1999), Craig and Haubrich (2006) confirm that changes in net lending hide the larger and more variable changes in gross lending flows, which average over six times the net changes.[10] They also find high levels of both loan creation and destruction in all periods over the business cycle and stress that changes in both creation and destruction contribute to reduced loan growth in recessions. Finally, loan growth slows, on average, by 1.2 percent (quarterly) between expansions and recessions, and this is apportioned between a 0.7 percent drop in creation and a 0.6 percent increase in destruction.

Contessi and Francis (2009) study gross credit loans and the reallocation in excess of the net credit change (the sum of credit expansion and credit contraction plus the absolute value of their difference) of U.S. commercial banks between the first quarter of 1999 and the fourth quarter of 2008. This extension is relevant, given the significant restructuring, the reduction in the number of banks and the increase in the size of the average bank which occurred in the U.S. banking system between the periods 1979-1999, considered the Dell'Ariccia and Garibaldi (2005), and 1999-2008. Their main results are:

(1) there are significant gross flows at any point of the cycle and they are much larger than net flows, implying that at any phase of the cycle significant credit contraction and credit expansion co-exist;
(2) credit expansion in total loans is pro-cyclical whereas contraction and excess reallocation of credit are counter-cyclical;
(3) credit contraction tends to sharply increase during recessions while credit expansion decreases; this generally leaves net flow growth positive although small;
(4) credit expansion of gross flows is more volatile than credit contraction; both volatilities are larger than that of GDP, but not nearly as volatile as they were in the previous 20 years;
(5) credit expansion in the past 10 years is larger in magnitude and more volatile than in the previous 20 years while credit contraction has similar volatility and average;
(6) in comparison with the evidence by Dell'Ariccia and Garibaldi (2005), credit expansion is more volatile while credit contraction is very similar and excess credit reallocation remains significant at around 3 percent of total quarterly loans.

In order to compare our results with this empirical literature, we define the following variables, which are added to the model summarized in section 2.8 above.

Gross credit expansion (*Fplus*) is equal to the value of the new matches in the credit market financing labor vacancy posting, plus the value of bank loans necessary to advance the wage bill to the new matches in the labor market, plus the bank loans necessary to advance the wage bill to the firms producing in the previous period that separate neither from workers nor from banks:

$$F_t^+ = q_t^B V_t^B k^F + L_t^N w_t h_t \tag{73}$$

[9] The other two key findings are that the cyclical behavior of the components of aggregate gross flows follows distinctive sectoral patterns and that the behavior of gross credit flows in the 1991 moderate recession features a high and persistent credit contraction.

[10] Craig and Haubrich (2006) also study bank entry and exit and the distribution of changes across banks, and compare gross loan flows with gross job flows.

Gross credit contraction (*Fminus*) is equal to the value of the wage bill that banks do not anticipate to firms that separate from workers and/or from banks plus the value of defaults on the credit lines financing labor vacancy posting:

$$F_t^- = \left(\rho L_{t-1}^N\right) w_t h_t + \left(1 - q_t^F\right) k^F L_t^V = \left(\rho L_{t-1}^N\right) w_t h_t + \left(1 - q_t^F\right) k^F q_t^B V_t^B \tag{74}$$

Gross credit flows (*Fgross*) are equal to the difference between gross credit expansion and the module of gross credit contraction (the extension of new loans and the cancellation of expired and non-performing loans):

$$F_t^G = F_t^+ + \left|F_t^-\right| \tag{75}$$

Net credit flows (*Fnet*) are equal to the difference between gross credit expansion and gross credit contraction:

$$F_t^N = F_t^+ - F_t^- \tag{76}$$

Excess credit reallocation (*Excess*) is equal to the difference between gross credit flows and the module of net credit flows (gross flows in excess of net changes):

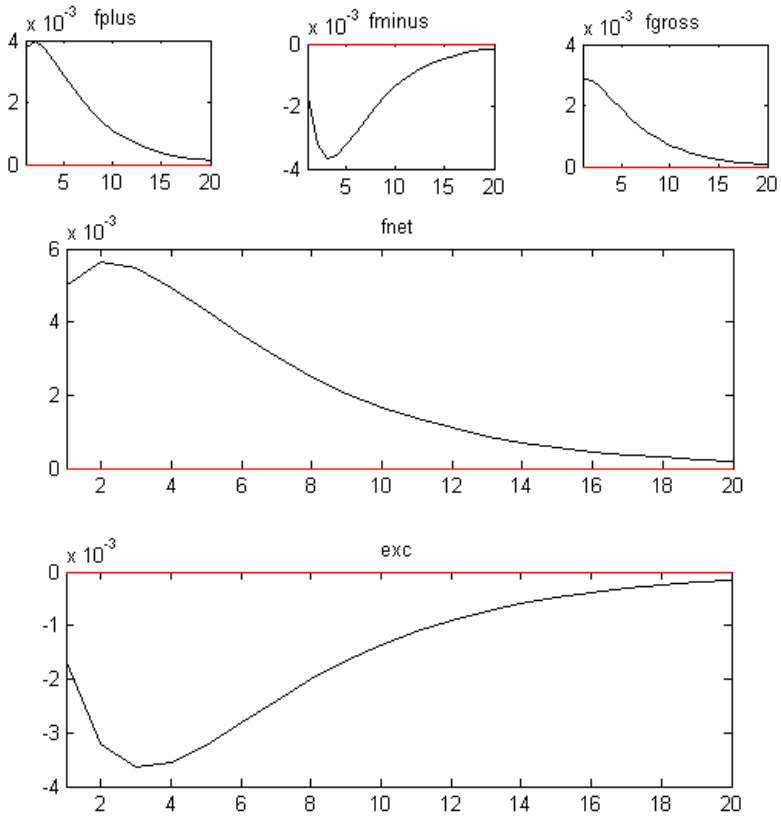
$$EXC = F_t^G - \left|F_t^N\right| \tag{77}$$

The dynamic behavior displayed in our model by the five variables is shown in Figure 2. It clearly shows that gross credit expansion, gross credit flows and net credit flows increase at impact after a technology innovation and return to steady state in around 20 quarters.

This dynamic behavior is in line with several results provided by the empirical literature summarized above:

1. gross flows are much larger than net flows: *Fgross* (0.9030) > *Fnet* (0.6131), but are less volatile than GDP (0.006243 < 0.009439);
2. gross credit expansion is pro-cyclical and has a high contemporaneous correlation with GDP (around 0.92);
3. gross credit contraction and excess credit reallocation are countercyclical;
4. both gross credit contraction and excess credit reallocation have a high contemporaneous correlation with GDP (both around -0.99);
5. in absolute value, the correlation between gross credit contraction and GDP is larger than the correlation between gross credit expansion and GDP (|-0.9896 | > | 0.9164 |).
6. gross credit expansion volatility (standard deviation) is larger than gross credit contraction (0.009032 > 0.008738).

Figure 2. Gross and net credit flows



5. Conclusions

In this paper we extended to a cash in advance New Keynesian DSGE theoretical model with sticky prices Wasmer and Weil's (2004) attempt to introduce search and matching frictions in both the labour and the financial markets. In this economy households are depicted in a standard fashion, and so are retail firms producing under monopolistic competition differentiated goods consumed by households. Before starting production, wholesale competitive firms, producing a homogeneous good, search for credit by banks posting loan offers; the firms obtaining loans post vacancies in the labour market, where unemployed workers are searching for jobs. The firms filling their vacancies obtain bank advances to pay for the wage bill. At the end of the period their production is sold to retail firms, loans are repaid and households receive profit income from financial intermediaries and firms, and the principal plus interest on deposits. The wage and the interest rate on loans are determined according to a sequential Nash bargaining procedure.

A fraction of the wholesale firms producing in a given period − determined on the basis of a separation rate specifying the fraction of labour matches which is destructed at the end of the production period − obtains loans also in the next period. The firms matching with banks and obtaining the loans necessary to post vacancies in the labor market may not be able to match with workers and so to start production. They hence cannot repay their debt with the bank and default on

the corresponding loans. Our approach allows to directly calculate both gross credit expansion and credit contraction flows, which can be compared with the existing empirical evidence.

The dynamic properties of our model are consistent with the main cyclical evidence reported in the NEK DSGE literature, but we are able to identify a "productivity-employment puzzle" with respect to hours worked and not to jobs. A technology shock increases income, consumption and wages, whereas marginal costs and inflation fall. Whereas unemployment falls and jobs increase, the expansion of income produces a decrease in hours worked. This occurs together with a peculiar behavior of the rates of interest on deposits and on loans. Given the behavior of the policy rate, the functioning of the credit market allows bank to magnify the fall in the loan rate, with positive effects on higher wages and unemployment. As workers and firms bargain on both wage and hours, our intuition is that they "exchange" higher wages for lower working hours, but firm need to expand production and they are hence forced to increase the number of employed workers.

The behavior of the model produces also results that are in line with several of the empirical findings on credit flows. More in particular, gross flows are much larger than net flows, but are less volatile than GDP; gross credit expansion is pro-cyclical and has a high contemporaneous correlation with GDP; gross credit contraction and excess credit reallocation are countercyclical; both gross credit contraction and excess credit reallocation have a high contemporaneous correlation with GDP; in absolute value, the correlation between gross credit contraction and GDP is larger than the correlation between gross credit expansion and GDP); gross credit expansion volatility (standard deviation) is larger than gross credit contraction.

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Appendix. Steady state calculations

- From the Euler equation (37) we obtain: $R^D = 1/\beta$.
- Given N, from employment dynamics (41), being $p_t^B s_t^F = V_t^F$, we get $V^F = \rho N / q^F$; from $p^B s^F = V^F$ it is also $p^B = \rho N / q^F s^F = V^F / s^F$.
- Similarly, given $L^N = N$, from the dynamics of the lines of credit financing wages (52) we have: $V^B = \rho L^N / q^F q^B$ and $p^B = \frac{\rho L^N}{q^F s^F} = \frac{q^B}{q^B}\frac{\rho L^N}{q^F s^F} = \frac{\rho L^N}{q^F q^B}\frac{1}{s^F} q^B = \frac{V^B}{s^F} q^B = \frac{q^B}{\theta^C}$.
- From the dynamics of the demand for lines of credit by firms (51) we obtain $s^F = 1 - (1-\rho)L^N$.
- Similarly, from the dynamics of workers searching for jobs (42), we obtain $s^W = 1 - (1-\rho)N$.
- The steady state values of tightness in the credit and in the labour markets are straightforwardly obtained from (64) and (65): $\theta^C = s^F / V^B$ and $\theta^L = s^F / s^W$.
- From employment dynamics (40) we have: $p^F = \frac{\rho N}{p^B s^W} = \frac{\rho N}{q^F}\frac{1}{p^B s^W} q^F = \frac{V^F}{p^B s^W} q^F \frac{p^B s^F}{p^B s^W} q^F = \frac{s^F}{s^W} q^F = \theta^L q^F$.
- The steady state values of matches in the credit and in the labour markets are straightforwardly obtained from (66) and (67): $H = q^F \upsilon (V^B)^{\zeta} (s^F)^{1-\zeta}$ and $M = p^B \eta (s^F)^{\xi} (s^W)^{1-\xi}$.
- Being N and h calibrated, and setting A = 1, from the production function (44) we get $Y = Ah^{\alpha} N$.
- From the aggregate resource constraint (46) we have $C = Y$.
- From (38), the value of the Lagrange multiplier is: $\lambda = (1-h_C)^{-\sigma} C^{-\sigma}$.
- Given RL, from (60) we obtain: $\partial h / \partial R^L = h / (\alpha - 1 - \phi) R^L$.
- From the condition for optimal hours (49) we get: $h = \left(\vartheta \frac{R^L \mu}{\alpha A \lambda} \right)^{1/\alpha - 1 - \phi}$, that is: $\vartheta = \frac{\alpha A \lambda}{R^L \mu (h)^{1+\phi-\alpha}}$.
- From (62) we obtain $mrs = \vartheta h^{\phi} / \lambda$.
- From (61) we obtain $mpl = \alpha A h^{\alpha - 1}$.
- We impose the relationship $k^F = t(Y)$, where t is a constant.
- We calibrate the replacement rate ϖ and write $w^U = \varpi w$.
- Inserting the firm FOC (47) into the wage equation (48) we obtain:

$$w = \left(\frac{h[1-\beta(1-\rho)]}{[h-(1-d)\overline{\omega}][1-\beta(1-\rho)] + hd\beta(1-\rho)p^B p^F} \right) \times$$

$$\times \left\{ \frac{(1-d)mrs}{1+\phi} + \frac{d}{R^L} \left[\frac{mpl}{\alpha\mu} + \frac{\beta(1-\rho)p^B p^F}{h} \left(\frac{1}{[1-\beta(1-\rho)]} \left[\frac{Ah^{\alpha}}{\mu} - R^L k^F [1-\beta(1-\rho)] \right] + k^F R^L \right) \right] \right\}$$

- We may now calculate: $w^U = \varpi w$
- We insert the value of w back into (47) and obtain:
  $$f = \frac{\lambda q^F p^B}{[1-\beta(1-\rho)]}\left[\frac{Ah^\alpha}{\mu} - R^L\{[1-\beta(1-\rho)]k^F\}\right] - \frac{R^L \lambda q^F p^B h}{[1-\beta(1-\rho)]} w$$
- We insert the value of w into (54) and obtain: $b = \lambda q^B \left(R^L q^F - R^D\right)k^F + \frac{\lambda q^F q^B \left(R^L - R^D\right)h}{[1-(1-\rho)\beta]} w$
- From (59) we obtain: $\frac{\partial w}{\partial R^L} = -\frac{d}{\left(R^L\right)^2}\left[\frac{mpl}{\alpha\mu} + (1-\rho)\beta\frac{1}{h}\left(\frac{f}{\lambda q^F p^B} + k^F R^L\right)\right]$.
- From (56) we obtain: $\gamma^B = wh + \left(R^L - R^D\right)\left(\frac{\partial w}{\partial R^L}h + \frac{\partial h}{\partial R^L}w\right)$.
- From (57) we obtain: $\gamma^F = \frac{mpl}{\mu}\frac{\partial h}{\partial R^L} + wh + R^L\left(\frac{\partial w}{\partial R^L}h + w\frac{\partial h}{\partial R^L}\right)$.
- From (58) we obtain: $\psi = \frac{(1-z)\gamma^B}{[(1-z)\gamma^B + z\gamma^F]}$.
- From (73)-(77) we obtain: $F^+ = q^B V^B k^F + L^N wh$, $F^- = \rho L^N wh + \left(1-q^F\right)k^F q^B V$, $F^G = F^+ + \left|F^-\right|$, $F^N = F^+ - F^-$ and $EXC = F^G - \left|F^N\right|$.