

# Financial Intermediaries, Leverage Ratios and Business Cycles

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## Abstract

I document the cyclical properties of aggregate financial variables (debt, equity, and the leverage ratio) of the U.S. financial sector. In particular, (i) debt is strongly procyclical while leverage ratio and equity are acyclical, and (ii) aggregate financial variables are more volatile than output. I then build a DSGE model augmented with profit maximizing financial intermediaries a'la Gertler and Karadi (2009). I quantitatively assess how important shocks to banks' capital (net worth shocks) are relative to standard productivity shocks in driving the fluctuations of aggregate real and financial variables and what role financial sector leverage plays in the amplification of these shocks. A model with productivity shocks alone can explain all of the observed variation in debt, 47% of the fluctuations in net worth and 25% of the fluctuations in leverage ratio, and generates procyclicality in financial variables. The model with the addition of net worth shocks accounts for all of the variation in debt, 81% of the fluctuations in net worth and all of the fluctuations in the leverage ratio, and generates an acyclical leverage ratio. Leverage amplifies productivity shocks while it dampens net worth shocks.

**Keywords:** Financial intermediaries, Leverage Ratio, Net Worth Shocks, Business Cycles

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# 1 Introduction

The recent global financial crisis has increased the interest in understanding how disruptions in financial intermediation can cause a substantial downturn in real economic activity. The macro literature studying the role of the financial accelerator in macroeconomic fluctuations emphasizes credit constraints on non-financial firms and models financial intermediaries as passive players that simply transfer funds from savers to firms.<sup>1</sup> This paper builds on the idea that the financial intermediaries themselves could be an important source of business cycle fluctuations since they are also dependent on external financing.<sup>2</sup> Moreover, the behavior of balance sheet items of financial intermediaries and how they interact with real variables over the business cycle have not been fully explored in the literature. Most previous studies on financial frictions have not tried to match fluctuations in both standard macro variables and aggregate financial variables at the same time. In this paper, I construct a model with a financial sector capable of matching both real and financial fluctuations.

I first document the business cycle properties of aggregate financial variables (debt, equity and the leverage ratio) of the U.S. financial sector, which are not widely known in the macro literature.<sup>3</sup> The following stylized facts emerge from the empirical analysis: (1) Leverage cycles in the U.S. financial sector are apparent and are associated with major crises. (2) The cyclical volatility of the financial leverage ratio is nearly 4 times larger than that of output. (3) The financial leverage ratio is mildly procyclical. (4) The cyclical volatility of the aggregate liabilities of the U.S. financial sector is roughly equal to that of output while the cyclical volatility of aggregate equity is approximately 3 times larger than that of output. (5) Variation in the financial leverage ratio is mainly due to variation in the aggregate equity (aggregate net worth) of the U.S. financial sector. (6) Aggregate liabilities are strongly procyclical, while aggregate equity is acyclical.

I then build a DSGE model augmented with profit maximizing financial intermediaries a'la Gertler and Karadi (2009), in which business cycles are driven by productivity and net worth shocks.<sup>4</sup> Financial intermediaries borrow funds from households and their ability to

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<sup>1</sup>Recent papers by Woodford and Curdia (2008), Gertler and Karadi (2009), Gertler and Kiyotaki (2009), Brunnermeier and Sannikov (2009) have attempted to model the financial sector as an active player.

<sup>2</sup>Carlson et al. (2008), Adrian and Shin (2009).

<sup>3</sup>The leverage ratio is defined as the ratio of total liabilities to total shareholders' equity.

<sup>4</sup>The net worth shock represents any kind of shock to bank capital such as loan losses, asset write-downs

borrow from households is limited due to a moral hazard (costly enforcement) problem. Non-financial firms acquire capital in each period by equity financing. In particular, non-financial firms issue state contingent claims (equity), whose number is equal to the number of units of capital acquired, and sell these shares to financial firms. Financial intermediaries have perfect information about non-financial firms.<sup>5</sup> Thus, in our model, only financial firms face endogenous borrowing constraints in obtaining funds from households.

I ask two quantitative questions: (1) How important are net worth shocks relative to productivity shocks in explaining the cyclical properties of real and financial variables? (2) What is the role of banks' leverage in the amplitude of productivity and net worth shocks? We show that a model driven solely by standard productivity shocks explains all of the observed variation in debt, 47% of the fluctuations in net worth and 25% of the fluctuations in leverage ratio, and implies procyclicality in all financial variables. The model with the addition of net worth shocks accounts for all of the variation in debt, 81% of the fluctuations in net worth and all of the fluctuations in the leverage ratio, and can match the observed acyclical leverage ratio. The model with both kinds of shocks matches most of the observed volatilities and cross-correlations among real and financial variables.

Financial sector leverage amplifies productivity shocks while it dampens net worth shocks. For example, in the model with productivity shocks, if the leverage ratio increases from 1 to 3, the volatilities of output, investment, net worth and liabilities increase by 19%, 26%, 72% and 10%, respectively. In the model with net worth shocks, if the leverage ratio increases from 1 to 3, the volatilities of output, investment, net worth and liabilities fall by 92%, 80%, 43%, and 144%, respectively.

The basic mechanism works as follows: Negative productivity shocks reduce investment and output, leading to a lower aggregate demand. This depresses asset prices and erodes net worth of banks. Deterioration in banks' net worth tightens their borrowing constraints and forces them to fire-sale their assets. Tightening financing conditions for banks and

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and reductions in banks' profits. These shocks capture shocks to bank health that originate solely in the financial sector.

<sup>5</sup>In order to focus on the dynamics driven by financial intermediaries, we simplify the model in this sense. In future work, we are planning to extend the model to include financial frictions between financial and non-financial firms, since we think that having both types of friction may increase the propagation and the amplification generated in the model.

fire-sale of assets means less shares purchased by banks, which further reduces investment and output of non-financial firms. In case of a negative net worth shock, the deterioration in net worth of banks tightens their borrowing constraints directly and triggers the fire-sale of their assets. This immediately reduces their equity purchases. As a result, non-financial firms reduce their investment. The fall in their investment causes the price of capital to decline and erodes banks' net worth further.

The financial accelerator between households and financial intermediaries plays a central role for the amplification of shocks. The magnitude of financial sector leverage and the fluctuations in the price of capital are crucial in generating the observed dynamics of aggregate real and financial variables. Leverage amplifies productivity shocks while it dampens net worth shocks. In case of productivity shocks, fluctuations in investment induce movements in net worth through their effect on the price of capital. If the leverage ratio is higher, the volatility of investment is higher and so is the volatility of net worth. In case of net worth shocks, the fluctuations in net worth induce movements in investment. Higher leverage strengthens the net worth smoothing motive for bankers since they might be forced to fire-sell their assets in the future. Incomplete markets due to borrowing constraints of banks create this net worth smoothing incentive. Because the shock directly affects the net worth, they will smooth their net worth more when the leverage ratio is higher, inducing lower volatility of net worth and thus lower volatility of investment.

This paper is different from Gertler and Karadi (2009) in several dimensions. First, their paper assesses the effect of direct central bank intermediation on private lending in a Christiano, Eichenbaum and Evans (2005) -type New Keynesian model with financial intermediaries, and offers optimal credit policy prescriptions. However, my aim is to construct a benchmark model driven by productivity and net worth shocks, and to quantify to what extent these shocks can account for the fluctuations in both aggregate financial variables and standard macro aggregates simultaneously. Unlike Gertler and Karadi (2009), I build a DSGE model with minimal features, abstracting from habit persistence, monopolistic competition, and price stickiness. In addition, this paper empirically documents both business cycle volatilities and cross-correlations of the aforementioned aggregate financial variables, providing some stylized facts against which we can judge the predictions of our model.

Finally, Gertler and Karadi's model features capital quality shocks that directly affect the amount of capital that non-financial firms hold. I remove these shocks and instead introduce net worth shocks into our model in order to study the impact of shocks that originate purely in the financial sector.<sup>6</sup> I calibrate the properties of these net worth shocks to match the empirical observations on financial variables in U.S. data.

The rest of the paper is structured as follows: In Section 2, I relate our paper to the recent literature on financial frictions. In Section 3, I document evidence on the cyclical behavior of the leverage ratio, debt and equity of the U.S. financial sector together with the standard macro aggregates. Section 4 describes the theoretical model. Section 5 presents the model parametrization and calibration together with the quantitative results of the model. Section 6 concludes.

## 2 Related Literature

This paper is related to recent empirical and theoretical literature on financial intermediaries. On the empirical side, Adrian and Shin (2008) show that the active management of balance sheets by financial intermediaries implies procyclical leverage in the financial sector.<sup>7</sup> They argue that if financial intermediaries were passive and did not adjust their balance sheets in response to changes in their net worth, then leverage in the financial sector would fall when total assets increase. However, they show empirically that there is a strong positive relationship between changes in leverage and changes in total assets, implying the active management of balance sheets by financial firms. Moreover, Adrian and Shin (2009) show that fluctuations in the balance sheet items of financial intermediaries forecast standard macro aggregates such as total consumption, total investment and GDP, implying that changes in the leverage ratio of financial institutions are empirically relevant for macroeconomic fluctuations.

On the theoretical side, Gertler and Karadi (2009) build a New Keynesian model that

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<sup>6</sup>One difference between my model and Gertler and Karadi (2009) is that the capital quality shock shifts the demand for bank loans while the net worth shock shifts the supply of bank loans.

<sup>7</sup>They define leverage as the ratio of total assets to total equity. Moreover, their notion of "procyclical" is not with respect to GDP, but with respect to total assets of financial intermediaries. The notion of "procyclical" in the current paper is more standard in the sense that it is with respect to GDP. Hence, we undertake a more standard macro business cycle accounting exercise.

incorporates financial intermediaries facing endogenous collateral constraints, and use the model to evaluate quantitatively the effect of direct central bank intermediation on private lending, and to determine the optimal credit policy in this environment. In their model, standard productivity shocks generate modest fluctuations, while "capital quality" shocks, defined as a 5% decline in the value of capital held by intermediate good firms, create a crisis that resembles the recent economic downturn. Gertler and Kiyotaki (2009) similarly investigates how a disruption in financial intermediation can induce a crisis that affects real activity, and ask the same normative question as Gertler and Karadi (2009). The difference from Gertler and Karadi (2009) is that financial intermediaries can also engage in interbank lending.

Jermann and Quadrini (2009) first document the cyclical properties of equity and debt payouts for U.S. non-financial firms and then construct a model in which non-financial firms use debt and equity financing for their expenditures. In their model, standard productivity shocks can only partially explain the fluctuations in real and financial variables. Introducing stochastic innovations to the enforcement constraint of non-financial firms, which they call "financial shocks", brings the model much closer to the data. This paper is different in the following aspects: (1) they provide empirical evidence on equity and debt payouts of U.S. non-financial firms while I document the cyclical properties of debt, equity and the leverage ratio of U.S. financial firms, and (2) they don't explicitly model a financial sector, since they are only interested in the behavior of debt and equity flows of non-financial firms, while I model the financial intermediation sector in order to study the observed dynamics of financial flows. Finally, Geanakoplos (2009) displays evidence that variation in leverage ratios has a tremendous effect on asset prices, which contributes to economic bubbles and busts.<sup>8</sup> Leverage ratios become too high in good times and too low in bad times, a phenomenon he labels as "leverage cycles", and de-leveraging is the main reason asset prices fall in recessions.

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<sup>8</sup>He doesn't model financial and non-financial sector explicitly. His paper features optimist and pessimist agents, where nature decides who is an optimist and who is a pessimist. Optimists are the investors in the economy and the variation in their leverage ratio creates economic booms and busts.

### 3 Empirical Regularities

This section documents stylized facts on the leverage ratio, debt and equity of U.S. financial firms using quarterly data for the period 1984-2009. In particular, I compute standard business cycle statistics of the aggregate financial variables, such as standard deviations and cross-correlations with standard macro variables.

The stylized facts about the financial variables documented in this paper are not widely known in the macro literature. To the best of my knowledge, the only related work is Adrian and Shin (2008, 2009), who provide evidence on the time series behavior of balance sheet items of some financial intermediaries using Flows of Funds data. However, they do not compute standard business cycle statistics for these financial variables. In addition, they do not analyze the whole financial sector, as they omit bank holding companies, finance companies and insurance companies. On the other hand, there are several papers documenting the behavior of the liabilities and equity of non-financial U.S. firms and a few papers providing empirical facts on the leverage ratio of the financial sector.<sup>9</sup> The closest available study is Chugh (2009), which computes standard business cycle statistics of the leverage ratio of U.S. non-financial firms using quarterly data from Compustat over the past 22 years. My construction of standard business cycle statistics for the financial leverage ratio is in the same spirit as that paper.

I use quarterly balance sheet data from the Flow of Funds Accounts of the Federal Reserve Board. I use aggregate data on financial variables since current paper is mainly theoretical and empirical results just serve for motivation and calibration. The balance sheet data in the levels tables at the Flow of Funds Accounts are not seasonally adjusted and are nominal. I make the seasonal adjustment using Census X12 and deflated the series using GDP deflator. Moreover, the balance sheet items at the Flow of Funds Accounts are market-value based. The time period is 1984-2009.<sup>10</sup> I focus on the period after 1984 since this time period is characterized by significant changes in the U.S. financial sector,

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<sup>9</sup>Covas and den Haan (2006), and Jermann and Quadrini (2009) discuss non-financial firms, while Levin, Natalucci, and Zakrajsek (2004), and Korajczyk and Levy (2003) discuss the leverage ratio of the financial sector.

<sup>10</sup>I perform the same business cycle analysis for the period 1952-2009, with results available upon request. Briefly, the volatility of aggregate financial variables is roughly the same compared to 1984-2009. The aggregate debt and equity are procyclical while the financial leverage ratio is acyclical in this period.

including regulatory changes and increases in financial flexibility as suggested by Jermann and Quadrini (2009).<sup>11</sup> This implies that financial intermediaries started to manage their balance sheets more actively after 1984. This fact is also confirmed by procyclical leverage ratio in U.S. financial sector found in the empirical analysis below.<sup>12</sup>

I focus on both depository and non-depository financial institutions. The depository institutions are U.S. chartered commercial banks, savings institutions, and credit unions. The non-depository institutions are issuers of asset-backed securities, bank holding companies, security brokers and dealers, finance companies, insurance companies, funding corporations, and real estate investment trusts. These institutions perform the majority of activity in the U.S. financial sector as measured by their total assets.<sup>13</sup> The debt measure I use is total liabilities, while the equity measure is total shareholder's equity. For each quarter, I compute the aggregate leverage ratio as the ratio of the aggregate liabilities of aforementioned financial institutions to their aggregate shareholders' equity.

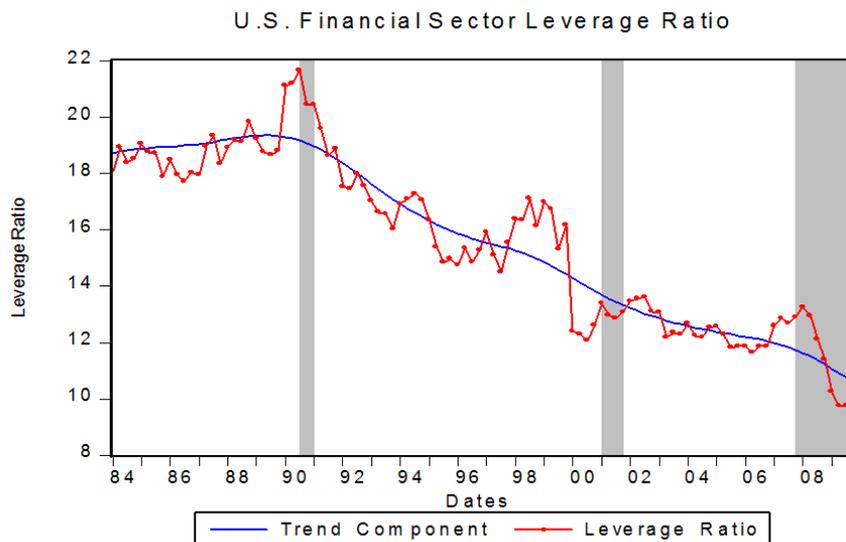


Figure 1: Leverage ratio of U.S. financial firms, 1984-2009. Mean = 15.57.

<sup>11</sup>In particular, important changes in U.S. financial sector after 1984 include the increase in share repurchases, "shelf" offerings under Rule 415, the development of the venture capital market, the introduction of new trading markets such as NASDAQ, the reduction in underwriting costs for equity issues, the increase in asset backed securitization (ABS), and banking liberalization.

<sup>12</sup>Adrian and Shin (2008) argue that active management of balance sheets by financial intermediaries induces a procyclical leverage ratio in financial sector.

<sup>13</sup>The total assets of these institutions is 90% of the total assets of the U.S. financial sector.

Figure 1 displays the time series of the aggregate leverage ratio of financial firms together with its HP trend component. The mean leverage ratio over the sample period is 15.57; The leverage ratio trends downwards until the present. Note that the de-leveraging in the recent financial crisis is consistent with the longer term trend since 1988.

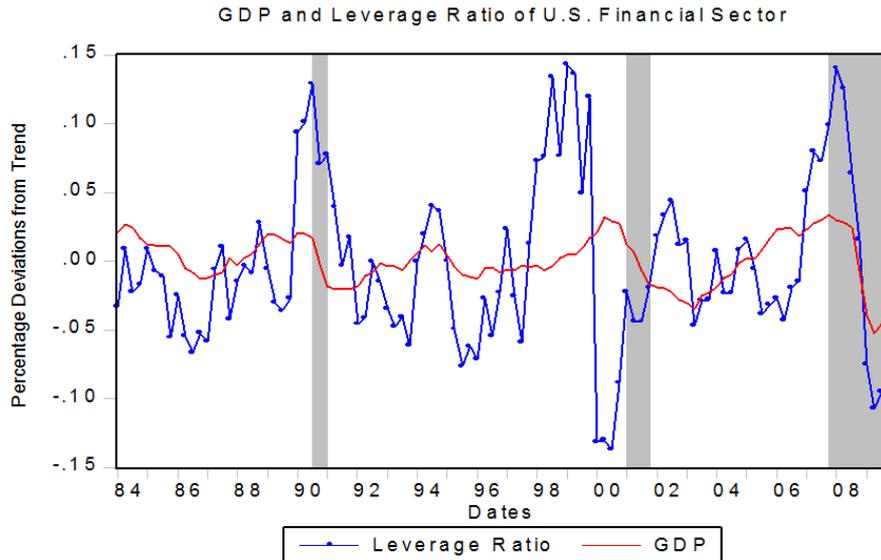


Figure 2: Cyclical Components of GDP and Leverage ratio of U.S. financial sector, 1984-2009

Figure 2 shows the HP-filtered cyclical components of GDP and the financial leverage ratio with NBER recession dates. There are several sharp spikes evident in this figure: in 1987, in 1991, in 1999 and 2000, and in 2006 and 2007. All of these spikes are associated with known economic and financial crises. The 1987 spike corresponds to 1987 Black Monday. The 1991 spike is associated with the Savings and Loan crisis in the U.S. and the burst of Japanese asset price bubble. The 1999 and 2000 episode is associated with the expansion and bursting of the tech bubble, while the final episode in 2006 and 2007 is due to recent global financial crisis preceded by the substantial leveraging before the crisis. Therefore, I can say that leverage cycles in U.S. financial sector are apparent and associated with major crises observed after 1984.

Table 1 presents business cycle statistics for the aggregate leverage ratio for U.S. financial

Table 1: Business Cycle Statistics, Quarterly U.S. Data, 1984-2009: Leverage Ratio

	GDP	C	I	Financial Leverage
Standard deviation (%)	1.55	0.68	4.49	6.14
Quarterly autocorrelation	0.88	0.87	0.86	0.72
Correlation matrix	GDP	1	0.8633	0.2589
	C	-	1	0.2663
	I	-	-	1
	Financial Leverage	-	-	-

<sup>a</sup> Business cycle statistics for GDP, consumption and investment are computed using quarterly data from FRED database. Consumption is the sum of personal consumption expenditures on nondurables and services (PCND + PCESV). Investment is the sum of personal consumption expenditures on durable goods and gross private domestic investment (PCDG + GPDI). GDP is the sum of consumption and investment.

<sup>b</sup> Aggregate leverage ratio of U.S. financial firms is computed using quarterly data from the Flow of Funds Accounts of the Federal Reserve Board. Aggregate leverage ratio in a given year is computed as the ratio of total liabilities to total shareholders' equity.

<sup>c</sup> Business cycle statistics in the table are based on HP-filtered cyclical components over the period 1984-2009.

sector. The volatility of the leverage ratio is nearly 4 times larger than that of output and 10 times that of consumption. The financial leverage ratio is mildly procyclical. The contemporaneous correlation between the financial leverage ratio and output is 0.26.

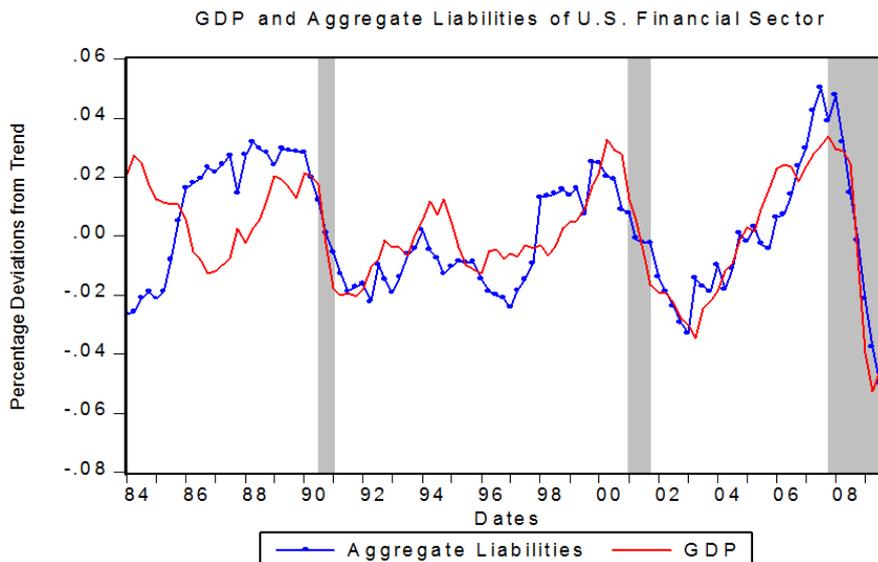


Figure 3: Cyclical Components of GDP and Debt of U.S. Financial Sector, 1984-2009

Figures 3 and 4 display the cyclical components of aggregate total liabilities (aggregate debt), aggregate equity, and GDP from 1984 to 2009 with NBER recession dates. The fluctuations in aggregate equity are larger than those in aggregate liabilities. The quarterly standard deviation of aggregate equity is 5.20 % compared to 1.90% for aggregate liabil-

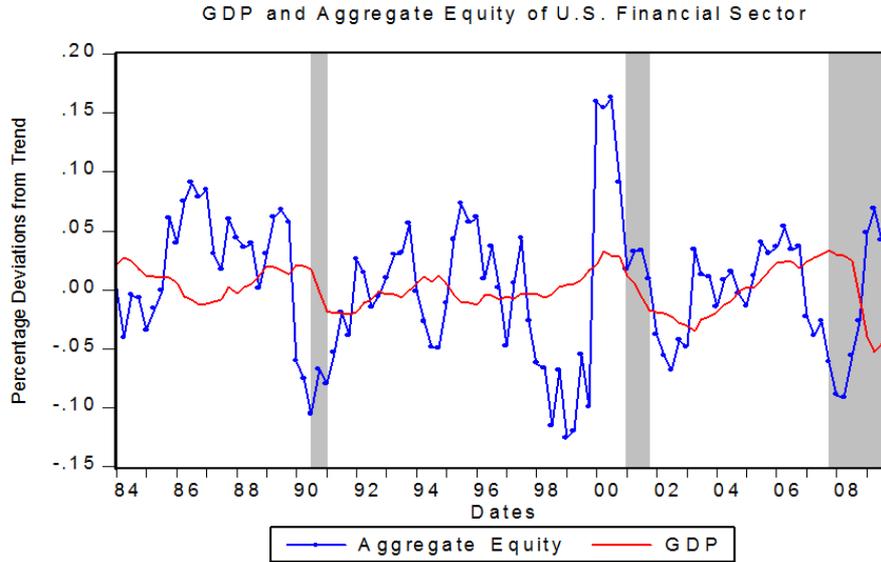


Figure 4: Cyclical Components of GDP and Equity of U.S. Financial Sector, 1984-2009

ities. In addition, if we compare figures 2 and 4, we can observe that movements in the leverage ratio of U.S. financial firms are mainly due to fluctuations in their aggregate net worth (aggregate equity). The contemporaneous correlation between the leverage ratio and aggregate equity is  $-0.92$ .

Table 2: Business Cycle Statistics, Quarterly U.S. Data, 1984-2009: Debt and Equity

		GDP	C	I	Debt	Equity
Standard deviation (%)		1.55	0.68	4.49	2.16	5.77
Quarterly autocorrelation		0.88	0.87	0.86	0.88	0.71
Correlations	GDP	1	0.8633	0.9663	0.6259	-0.0379
	C	-	1	0.7140	0.4681	-0.1149
	I	-	-	1	0.6346	-0.0058
	Debt	-	-	-	1	0.0008
	Equity	-	-	-	-	1

<sup>a</sup> See the footnote (a) in Table 1 for the construction of standard macro aggregates.

<sup>b</sup> Aggregate liabilities and aggregate equity of U.S. financial firms are computed using quarterly data from Flow of Funds Accounts of the Federal Reserve Board. Aggregate liabilities (aggregate equity) in a given year are computed as the sum of total liabilities (total shareholders' equity) of the five different types of financial institutions mentioned in the text.

<sup>c</sup> Business cycle statistics in the table are based on HP-filtered cyclical components over the period 1984-2009.

Table 2 presents the business cycle statistics for aggregate liabilities (debt) and aggregate equity of the U.S. financial sector. The volatility of aggregate equity is 3 times larger than

that of output, while the volatility of aggregate debt is a bit higher than that of output <sup>14</sup>. The contemporaneous correlation between aggregate liabilities and output is 0.88 while that between aggregate equity and output is -0.04, indicating that debt is strongly procyclical while equity is acyclical. <sup>15</sup>.

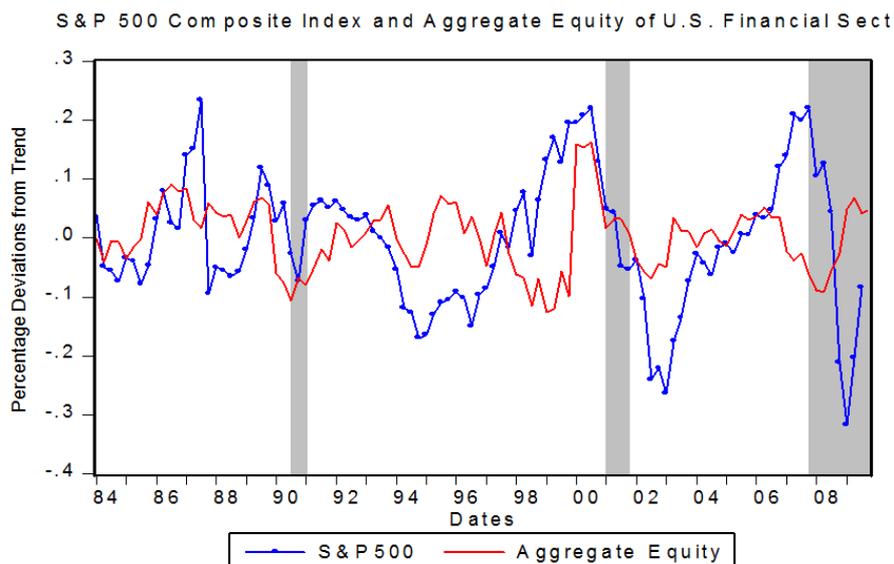


Figure 5: Cyclical Components of SP 500 Composite Index and Equity of U.S. Financial Sector, 1984-2009

Figure 5 displays the cyclical components of SP 500 composite index and aggregate equity of U.S. financial sector. The volatility of SP 500 composite index are larger than that of aggregate equity. The quarterly standard deviation of SP 500 index is 11.31% while that of aggregate equity is 5.77%. The contemporaneous correlation between these two series is 0.04, which shows that fluctuations in equity is not driven by fluctuations in SP 500. Moreover, the contemporaneous correlation between SP 500 and the leverage ratio is 0.07, leading to the same conclusion that the movements in the leverage ratio is not induced by the movements in SP 500.

<sup>14</sup>Using the Flow of Funds database, Jermann and Quadrini (2009) shows that relative volatilities of non-financial sector debt and equity to nonfinancial business sector GDP are 1.29 and 1.05, respectively.

<sup>15</sup>Jermann and Quadrini (2009) find that debt is countercyclical and equity is procyclical for non-financial firms for the same time period. In addition, using Compustat database, Covas and Den Haan (2006) shows that debt and equity issuance is procyclical for the majority of publicly listed firms.

The following stylized facts emerge from the empirical analysis above: (1) Leverage cycles in U.S. financial sector are apparent and associated with major crises. (2) The cyclical volatility of the financial leverage ratio is nearly 4 times larger than that of output. (3) The financial leverage ratio is mildly procyclical. (4) The cyclical volatility of the aggregate liabilities of the U.S. financial sector is roughly equal to that of output while the cyclical volatility of aggregate equity is approximately 3 times larger than that of output. (5) The variation in the financial leverage ratio is mainly due to the variation in the aggregate equity (aggregate net worth) of U.S. financial sector. (6) Aggregate liabilities are strongly procyclical while aggregate equity is acyclical. I will assess the model below by its ability to match these facts.

## 4 The Model

The economy consists of four types of agents: households, financial intermediaries, non-financial firms, and capital producers. The financial intermediation sector is modeled a'la Gertler and Karadi (2009). The ability of financial firms to borrow from households is limited due to a moral hazard (costly enforcement) problem, which will be described below. Non-financial firms acquire capital in each period by selling shares to financial intermediaries. Finally, capital producers are incorporated into the model in order to introduce capital adjustment costs in a tractable way.

Table 3: The Sequence of Events in a Given Time Period

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1. Aggregate productivity shock  $z_t$  is realized.
  2. Non-financial firms hire labor  $H_t$  and production for period  $t$  takes place,  $Y_t = z_t F(K_t, H_t)$ .
  3. Non-financial firms make their wage payments  $w_t H_t$  and dividend payments to shareholders (banks) from period  $t-1$ .
  4. Financial intermediaries make their interest payments on deposits of households from period  $t-1$  and bankers exit with probability  $(1-\theta)$ .
  5. Net worth shock  $\omega_t$  is realized.
  6. Households make their consumption and saving decisions and deposit their resources at banks.
  7. Non-financial firms sell their depreciated capital to capital producers. These agents make investment and produce new capital  $K_{t+1}$ .
  8. Non-financial firms choose  $K_{t+1}$  and banks' incentive constraints bind.
  9. Non-financial firms issue equity [ $s_t = K_{t+1}$ ] and sell these shares to banks to finance their capital expenditures.
  10. Non-financial firms purchase capital  $K_{t+1}$  from capital producers at the price of  $q_t$  with borrowed funds.
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## 4.1 Households

There is a continuum of identical households of measure unity. Households are infinitely-lived with preferences over consumption ( $c_t$ ) and leisure ( $1-L_t$ ) given by

$$\sum_{t=0}^{\infty} \beta^t U(c_t, 1 - L_t) \quad (1)$$

Each household consumes and supplies labor to non-financial firms at the market clearing real wage  $w_t$ . In addition, they save by holding deposits at a riskless real return  $r_t$  at competitive financial intermediaries.

There are two types of members within each household: workers and bankers. Workers supply labor and return the wages they earn to the household while each banker administers a financial firm and transfers any earnings back to the household. Hence, the household owns the financial intermediaries that its bankers administer. However, the deposits that the household holds are put in financial intermediaries that it doesn't own.<sup>16</sup> Moreover, there is perfect consumption insurance within each household.

At any point in time the fraction  $1-\zeta$  of the household members are workers and the remaining fraction  $\zeta$  are bankers. An individual household member can switch randomly between these two jobs over time. A banker this period remains a banker next period with probability  $\theta$ , which is independent of the banker's history. Therefore, the average survival time for a banker in any given period is  $\frac{1}{1-\theta}$ . The bankers are not infinitely-lived in order to make sure that they don't reach a point where they can finance all investment from their own net worth. Hence, every period  $(1-\theta)\zeta$  bankers exit and become workers while the same mass of workers randomly become bankers, keeping the relative proportion of workers and bankers constant. Period  $t$  bankers learn about survival and exit at the beginning of period  $t+1$ . Bankers who exit from the financial sector transfer their accumulated earnings to their respective household. Furthermore, the household provides its new bankers with some start-up funds.

The household budget constraint is given by:

$$c_t + b_t = w_t L_t + (1 + r_t) b_{t-1} + \Pi_t \quad (2)$$

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<sup>16</sup>This assumption ensures independent decision-making. Depositors are not the owners of the bank, so the bank does not maximize their utility, but its own net worth.

The household's subjective discount factor is  $\beta \in (0,1)$ ,  $c_t$  denotes the household's consumption,  $b_t$  is the total amount of deposits that the household gives to the financial intermediary,  $r_t$  is the non-contingent real return on the deposits from  $t-1$  to  $t$ ,  $w_t$  is the real wage rate, and  $\Pi_t$  is the profits to the household from owning both non-financial and financial firms net of the transfer that it gives to its new bankers plus the accumulated net worth of exiting bankers.

The household chooses  $c_t$ ,  $L_t$ , and  $b_t$  to maximize (1) subject to the sequence of flow budget constraints in (2). The resulting first order conditions for labor supply and deposit holdings are given by:

$$\frac{U_l(t)}{U_c(t)} = w_t \quad (3)$$

$$U_c(t) = \beta(1 + r_{t+1})E_t U_c(t+1) \quad (4)$$

The first condition states that the marginal rate of substitution between consumption and leisure is equal to the wage rate. The second condition is the standard consumption-savings Euler equation, which equates the marginal cost of not consuming and saving today to the expected discounted marginal benefit of consuming tomorrow.

## 4.2 Financial Intermediaries

### 4.2.1 Balance Sheets

Financial intermediaries transfer the funds that they obtain from households to the non-financial firms. The balance sheet of financial intermediary  $j$  at the end of period  $t$  is given by

$$q_t s_{jt} = \omega_t \widetilde{n}_{jt} + b_{jt} \quad (5)$$

where  $\widetilde{n}_{jt}$  is the net worth of financial firm  $j$  at the beginning of period  $t$  before the net worth shock hits,  $b_{jt}$  is the amount of deposits that the intermediary obtains from the households,  $q_t$  is the price of shares of non-financial firms and  $s_{jt}$  is the quantity of these shares. Banks undertake equity investment and non-financial firms finance their capital expenditures by issuing shares. Therefore, the financial contract between the intermediary

and the non-financial firm is an equity contract (or equivalently a state-dependent debt contract).

Moreover,  $\omega_t$  is an i.i.d. net worth shock that I introduce into the model to capture exogenous movements in the net worth of financial intermediaries.<sup>17</sup> The shock  $\omega_t$  represents any kind of shock to bank capital such as loan losses, asset writedowns, and reductions in banks' profits or shifts in public sentiment about the value of financial firms (or the financial system). Examples of suck shocks include the collapse of the market for mortgage-based securities, termination of brokerage relationships between banks and their clients, and the sudden demise of Bear Stearns and Lehmann Brothers in 2008. Therefore, we can think of  $\omega_t \widetilde{n}_{jt}$  as the effective net worth of the financial intermediary. For notational convenience, I write down  $\omega_t \widetilde{n}_{jt}$  as  $n_{jt}$ . Hence,  $n_{jt}$  is the net worth of financial firm  $j$  at the beginning of period  $t$  after the net worth shock hits. Furthermore, even though the net worth shock is i.i.d., it endogenously persists through its effect on net worth accumulation.

The current paper is not the only paper that introduce net worth shocks. Meh and Moran (2008) also consider shocks that originate within the banking sector and produce sudden shortages in bank capital. They suggest that these shocks reflect periods of financial distress and weakness in financial markets. Moreover, Brunnermeier and Pedersen (2009) introduce net worth shocks to banks' capital and interpret them as independent shocks arising from other activities like investment banking. They give as an example that Bear Stearns' clients terminated their brokerage relationships and ran on the investment bank in March 2008.

The households put their deposits into the financial intermediary at time  $t$  and obtain the non-contingent real return  $r_{t+1}$  at  $t+1$ . Therefore, we can think of  $b_{jt}$  as the financial firm's total liabilities (debt) and  $n_{jt}$  as its equity capital.<sup>18</sup> The financial intermediaries receive  $r_{kt+1}$  for their equity investment. The fact that  $r_{kt+1}$  is potentially greater than  $r_{t+1}$  creates an incentive for bankers to engage in financial intermediation.

The financial intermediary's net worth at the beginning of period  $t+1$  (before the time  $t+1$  net worth shock hits) is given by the difference between the earnings on equity in-

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<sup>17</sup>I model this shock as an i.i.d. process because I assume that financial intermediaries immediately write off their losses in a given period when they realize it.

<sup>18</sup>In U.S. financial data, household deposits constitute 70% of total liabilities of banks.

vestment in non-financial firms (assets of financial intermediary) and interest payments on deposits obtained from the households (liabilities of financial intermediary). Thus the law of motion for the net worth of the financial firm is given by

$$\widetilde{n}_{jt+1} = (1 + r_{kt+1})q_t s_{jt} - (1 + r_{t+1})b_{jt} \quad (6)$$

Using the balance sheet of the financial firm given by (5), we can re-write (6) as follows:

$$\widetilde{n}_{jt+1} = (r_{kt+1} - r_{t+1})q_t s_{jt} + (1 + r_{t+1})n_{jt} \quad (7)$$

The financial intermediary's net worth at time  $t+1$  depends on the premium  $r_{kt+1} - r_{t+1}$  that it earns on shares purchased as well as the total value of these shares,  $q_t s_{jt}$ .

The profits of the financial intermediary will be affected by the premium given above. That is, the banker will not have any incentive to buy shares of non-financial firms if the return on these shares is less than the cost of deposits. Thus the financial firm will continue to operate in period  $t+i$  if the following inequality is satisfied:

$$E_{t+i}\beta\Lambda_{t,t+1+i}(r_{kt+1+i} - r_{t+1+i}) \geq 0 \quad \forall i \geq 0 \quad (8)$$

where  $\beta\Lambda_{t,t+1+i}$  is the stochastic discount factor that the financial firm applies to its earnings at  $t+1+i$ . Imperfect capital markets lead to a positive premium due to the constraints on the financial firm's ability to obtain deposits from the households.

#### 4.2.2 Profit Maximization

In the households section, I assume that the deposits of the household are put in financial intermediaries that it doesn't own. This assumption ensures that depositors are not the owners of the bank, so the bank does not maximize depositors' expected discounted life-time utility, but rather its expected discounted terminal wealth upon exiting the financial intermediation sector. Thus, the financial intermediary maximizes its expected discounted terminal net worth, given by<sup>19</sup>

$$V_{jt} = \max E_t \sum_{i=0}^{\infty} (1 - \theta)\theta^i \beta^i \Lambda_{t,t+1+i} \widetilde{n}_{jt+1+i} \quad (9)$$

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<sup>19</sup>The detailed profit maximization problem of financial intermediaries is in Appendix 7.1.

$$= \max E_t \sum_{i=0} (1 - \theta) \theta^i \beta^i \Lambda_{t,t+1+i} [(r_{kt+1+i} - r_{t+1+i}) q_{t+i} s_{jt+i} + (1 + r_{t+1+i}) n_{jt+i}]$$

Since the risk premium is positive in any period, the financial intermediary will always have an incentive to buy shares of non-financial firms. Obtaining additional funds (deposits) from the households is the only way to achieve this. Therefore, in order to introduce a limit on the size of the financial intermediary, a costly enforcement problem is incorporated into the optimization problem: At the end of the period, the financial intermediary may choose to divert a fraction  $\lambda$  of available funds from its shares of nonfinancial firms and give them to the household of which the banker is a member. If the financial intermediary diverts the funds, the assumed legal structure ensures that the households (depositors) are able to force the intermediary to go bankrupt and may recover the remaining fraction  $1-\lambda$  of the assets. The depositors are not able to get the remaining fraction  $\lambda$  of the funds since, by assumption, the cost of recovering these funds is too high. Therefore, for the households to have an incentive to supply funds to the banker, the following incentive compatibility constraint must be satisfied at the end of period  $t$ :<sup>20</sup>

$$V_{jt} \geq \lambda q_t s_{jt} \tag{10}$$

The left-hand side of (10) is the loss to the financial firm from diverting a fraction of assets while the right-hand side is the gain from diverting this fraction of assets. The intuition for this constraint is that in order for the financial intermediary not to divert the funds and for the households to put their deposits into the bank, the loss to the intermediary from diverting assets must be greater than or equal to the gain from doing so.

The costly enforcement problem in the financial intermediation sector creates a social cost in this economy in the following way: The moral hazard problem between households and financial firms generates endogenous collateral constraints for financial firms, which will be described below. Due to these collateral constraints, financial firms cannot obtain as many funds from households as they want, and non-financial firms therefore cannot obtain as much credit from financial intermediaries as they would like to acquire capital. This inefficiency induces  $r_{kt+1}$  to be greater than  $r_{t+1}$ , which is a social cost to the economy and is borne by non-financial firms.

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<sup>20</sup>The incentive constraint binds when the value of  $K_{t+1}$  is decided at the end of period  $t$ .

### 4.2.3 Leverage Ratio and Net Worth Evolution

In the appendix it is shown that  $V_{jt}$  can be expressed as follows:

$$V_{jt} = \nu_t q_t s_{jt} + \eta_t n_{jt} \quad (11)$$

where

$$\nu_t = E_t[(1 - \theta)\Lambda_{t,t+1}(r_{kt+1} - r_{t+1}) + \beta\Lambda_{t,t+1}\theta\frac{q_{t+1}s_{jt+1}}{q_t s_{jt}}\nu_{t+1}] \quad (12)$$

$$\eta_t = E_t[(1 - \theta)\Lambda_{t,t+1}(1 + r_{t+1}) + \beta\Lambda_{t,t+1}\theta\frac{\widetilde{n_{jt+1}}}{n_{jt}}\eta_{t+1}] \quad (13)$$

$\nu_t$  can be interpreted as the expected discounted marginal gain to the financial intermediary of increasing its assets  $q_t s_{jt}$  by one unit, holding its effective net worth  $n_{jt}$  constant (i.e. the benefit of getting one more unit of deposits and using it to buy shares of non-financial firms). This expected discounted marginal gain has two terms: The first term is the discounted value of the net return on shares to the financial firm if it exits the intermediation sector tomorrow. The second term is the continuation value of its increased assets if it survives. Meanwhile,  $\eta_t$  can be interpreted as the expected discounted marginal value of increasing net worth  $n_{jt}$  by one unit, holding  $q_t s_{jt}$  constant (i.e. the benefit of having one less unit of deposits and one more unit of net worth). This expected discounted marginal gain also has two terms: The first term is the discounted value of the return on net worth to the financial firm in case it exits the financial sector tomorrow. The second term is the continuation value of its increased effective net worth if it survives. Therefore, we can write the incentive compatibility constraint as follows:

$$\nu_t q_t s_{jt} + \eta_t n_{jt} \geq \lambda q_t s_{jt} \quad (14)$$

When this constraint binds, the financial intermediary's assets are limited by its net worth.<sup>21</sup> That is, if this constraint binds, the funds that the intermediary can obtain from households will depend positively on its equity capital:

$$q_t s_{jt} = \frac{\eta_t}{\lambda - \nu_t} n_{jt} \quad (15)$$

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<sup>21</sup>As shown in Appendix 7.1., the incentive compatibility constraint will bind as long as the risk premium ( $r_{kt+1} - r_{t+1}$ ) is positive. In numerical simulations, I ensure that the risk premium is always positive under reasonable parameter values.

The constraint (15) limits the leverage of the financial intermediary to the point where its incentive to divert funds is exactly balanced by its loss from doing so. Thus, the costly enforcement problem leads to an endogenous borrowing constraint on the financial firm's ability to acquire assets. When the leverage ratio and/or the net worth of the financial intermediary is high, it can purchase more shares of the non-financial firms. Conversely, de-leveraging or the deterioration in net worth in recessions will limit the ability of the financial firms to buy shares of the non-financial firms. Note that by manipulating this expression using the balance sheet, we can obtain the leverage ratio of financial firms as follows:

$$\frac{b_{jt}}{n_{jt}} = \frac{\eta_t}{\lambda - \nu_t} - 1 \quad (16)$$

The leverage ratio increases in the expected marginal benefit of obtaining one more unit of deposits and using it to buy shares of non-financial firms, and in the expected marginal gain of having one less unit of deposits and one more unit of net worth. Intuitively, increases in  $\eta_t$  or  $\nu_t$  mean that financial intermediation is expected to be more lucrative going forward, which makes it less attractive to divert funds today and thus increases the amount of funds depositors are willing to entrust to the financial intermediary.

Using (16), we can re-write the law of motion for the banker's net worth as follows:

$$\widetilde{n_{jt+1}} = [(r_{kt+1} - r_{t+1}) \frac{\eta_t}{\lambda - \nu_t} + (1 + r_{t+1})] n_{jt} \quad (17)$$

The sensitivity of net worth of the financial intermediary  $j$  at  $t+1$  to the ex-post realization of the premium  $r_{kt+1} - r_{t+1}$  increases in the leverage ratio. Moreover, from (17), one can obtain the following:

$$\frac{\widetilde{n_{jt+1}}}{n_{jt}} = [(r_{kt+1} - r_{t+1}) \frac{\eta_t}{\lambda - \nu_t} + (1 + r_{t+1})] \quad (18)$$

$$\frac{q_t \widetilde{s_{jt+1}}}{q_t s_{jt}} = \frac{\frac{\eta_{t+1}}{\lambda - \nu_{t+1}} \widetilde{n_{jt+1}}}{\frac{\eta_t}{\lambda - \nu_t} n_{jt}} \quad (19)$$

Financial firms have identical expected growth rates of assets and net worth.<sup>22</sup> Therefore, none of the components in the leverage ratio depends on firm-specific factors. Since

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<sup>22</sup>This immediately implies that  $\eta_t$  and  $\nu_t$  are independent of  $j$ . In Appendix 7.1, I use this result in explicit derivation of  $\eta_t$  and  $\nu_t$ .

all financial firms have identical leverage ratios, we can sum demand for assets across  $j$  to obtain the total intermediary demand for assets:

$$q_t s_t = \frac{\eta_t}{\lambda - \nu_t} n_t \quad (20)$$

where  $s_t$  is the aggregate amount of assets held by financial intermediaries and  $n_t$  is the aggregate intermediary net worth. In the equilibrium of the model, movements in the leverage ratio of financial firms and/or in their net worth will generate fluctuations in total intermediary assets.

The aggregate intermediary net worth at the beginning of period  $t+1$  (before the net worth shock hits but after exit and entry),  $\widetilde{n}_{t+1}$ , is the sum of the net worth of surviving financial intermediaries from the previous period,  $\widetilde{n}_{et+1}$ , and the net worth of entering financial intermediaries,  $\widetilde{n}_{nt+1}$ . Thus, we have

$$\widetilde{n}_{t+1} = \widetilde{n}_{et+1} + \widetilde{n}_{nt+1} \quad (21)$$

Since the fraction  $\theta$  of the financial intermediaries at time  $t$  will survive until time  $t+1$ , their net worth,  $\widetilde{n}_{et+1}$ , is given by

$$\widetilde{n}_{et+1} = \theta[(r_{kt+1} - r_{t+1}) \frac{\eta_t}{\lambda - \nu_t} + (1 + r_{t+1})] n_t \quad (22)$$

Newly entering financial intermediaries receive start-up funds from their respective households. The start-up funds are assumed to be a transfer equal to a fraction of the net worth of exiting bankers. The total final period net worth of exiting bankers at time  $t$  is equal to  $(1 - \theta)n_t$ . The household is assumed to transfer the fraction  $\frac{\epsilon}{(1-\theta)}$  of the total final period net worth to its newly entering financial intermediaries. Therefore, we have

$$\widetilde{n}_{nt+1} = \epsilon n_t \quad (23)$$

Using (21), (22), and (23), we obtain the following law of motion for  $\widetilde{n}_{t+1}$ :

$$\widetilde{n}_{t+1} = \theta[(r_{kt+1} - r_{t+1}) \frac{\eta_t}{\lambda - \nu_t} + (1 + r_{t+1})] n_t + \epsilon n_t \quad (24)$$

### 4.3 Non-financial Firms

There is a continuum of unit mass of non-financial firms that produce the final output in the economy. The production technology at time  $t$  is described by the constant returns to scale function:

$$Y_t = z_t F(K_t, H_t) = z_t K_t^\alpha H_t^{1-\alpha} \quad (25)$$

where  $K_t$  is the firm's purchase of physical capital from capital producers,  $H_t$  is the firm's hiring of labor and  $z_t$  is an aggregate TFP realization.

Non-financial firms acquire capital  $K_{t+1}$  at the end of period  $t$  to produce the final output in the next period. After producing at time  $t+1$ , the firm can sell the capital on the open market.

Non-financial firms finance their capital expenditures in each period by issuing equities and selling them to financial intermediaries. Firms issue  $s_t$  units of state-contingent claims (equity), which is equal to the number of units of capital acquired  $K_{t+1}$ . The financial contract between a financial intermediary and a non-financial firm is an equity contract (or equivalently, a state contingent debt contract). The non-financial firm pays a state-contingent interest rate equal to the ex-post return on capital  $r_{kt+1}$  to the financial intermediary. The non-financial firms set their capital demand  $K_{t+1}$  taking this stochastic repayment into consideration. At the beginning of period  $t+1$  (after shocks are realized), when output becomes available, non-financial firms obtain resources  $Y_{t+1}$  and use them to make repayments to shareholders (or financial intermediaries). The firm prices each financial claim at the price of a unit of capital,  $q_t$ . Thus, we have

$$q_t s_t = q_t K_{t+1} \quad (26)$$

There are no frictions for non-financial firms in obtaining funds from financial intermediaries by issuing shares. The financial intermediary has perfect information about the firm and there is perfect enforcement. Therefore, in the current model, only financial intermediaries face endogenous borrowing constraints in obtaining funds. These constraints directly affect the supply of funds to the non-financial firms.

Non-financial firms choose  $K_t$  and  $H_t$  in order to maximize their profits. The profit maximization problem solved by a representative non-financial firm is given by

$$\max_{K_t, H_t} F(K_t, H_t) + q_t(1 - \delta)K_t - (1 + r_{kt})q_{t-1}K_t - w_t H_t \quad (27)$$

Profit maximization with respect to  $K_t$  and  $H_t$  gives us the following capital and labor demand conditions:

$$r_{kt} = \frac{F_K(K_t, H_t) + q_t(1 - \delta)}{q_{t-1}} - 1 \quad (28)$$

$$w_t = F_H(K_t, H_t) \quad (29)$$

Condition (28) states that the real rate of return on capital is equal to the marginal product of capital plus the capital gain from changed prices. Condition (29) states that the wage rate is equal to the marginal product of labor.

#### 4.4 Capital Producers

Following the literature on financial accelerator, I incorporate capital producers into the model in order to introduce capital adjustment costs in a tractable way. I need capital adjustment costs to introduce some variation in the price of capital; otherwise the price of capital will not respond to the changes in capital stock and will always be equal to 1.<sup>23</sup> I choose the functional form of the capital adjustment cost following previous literature.<sup>24</sup>

I assume that households own capital producers and receive any profits. At the end of period  $t$ , competitive capital producers buy capital from non-financial firms to repair the depreciated capital and to build new capital. Then they sell both the new and repaired capital. The cost of replacing the depreciated capital is unity; thus the price of a unit of new capital or repaired capital is  $q_t$ . The profit maximization problem of the capital producers is given by:

$$\max_{I_t} q_t K_{t+1} - q_t K_t - I_t \quad (30)$$

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<sup>23</sup>There will be no financial accelerator between households and financial intermediaries if there is no variation in the price of capital.

<sup>24</sup>Bernanke, Gertler and Gilchrist (1999), Gertler, Gilchrist, and Natalucci (2007) etc.

$$s.t. \quad K_{t+1} = (1 - \delta)K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t \quad (31)$$

where  $I_t$  is the total investment by capital producing firms and  $\Phi\left(\frac{I_t}{K_t}\right)$  is the capital adjustment cost function. The resulting optimality condition gives the following "Q" relation for investment:

$$q_t = \left[ \Phi_{I_t}\left(\frac{I_t}{K_t}\right) \right]^{-1} \quad (32)$$

where  $\Phi_{I_t}\left(\frac{I_t}{K_t}\right)$  is the partial derivative of the capital adjustment cost function with respect to investment at time  $t$ . The fluctuations in investment expenditures will create variation in the price of capital. A fall in investment at time  $t$  (*ceteris paribus*) will reduce the price of capital at time  $t$ .

#### 4.5 Competitive Equilibrium

A competitive equilibrium of this model economy consists of sequences of allocations  $\{c_t, L_t, K_{t+1}, s_t, n_t, \tilde{n}_t, i_t, \eta_t, \nu_t, H_t\}_{t=0}^{\infty}$ , of prices  $\{w_t, r_{kt}, r_t, q_t\}_{t=0}^{\infty}$  and of exogenous processes  $\{z_t, \omega_t\}_{t=0}^{\infty}$  such that (i) the allocations solve the household's, the non-financial firm's and the financial intermediary's problems at the equilibrium prices and (ii) markets for factor inputs clear. The following equilibrium conditions must be satisfied:

$$\frac{U_l(t)}{U_c(t)} = w_t \quad (33)$$

$$U_c(t) = E_t \beta (1 + r_{t+1}) U_c(t+1) \quad (34)$$

$$r_{kt} = \frac{F_K(K_t, H_t) + q_t(1 - \delta)}{q_{t-1}} - 1 \quad (35)$$

$$w_t = F_H(K_t, H_t) \quad (36)$$

$$n_t = \omega_t \tilde{n}_t \quad (37)$$

$$q_t s_t = \frac{\eta_t}{\lambda - \nu_t} n_t \quad (38)$$

$$\eta_t = E_t \left[ (1 - \theta) \Lambda_{t,t+1} (1 + r_{t+1}) + \beta \Lambda_{t,t+1} \theta \frac{n_{t+1}}{n_t} \eta_{t+1} \right] \quad (39)$$

$$\nu_t = E_t \left[ (1 - \theta) \Lambda_{t,t+1} (r_{kt+1} - r_{t+1}) + \beta \Lambda_{t,t+1} \theta \frac{q_{t+1} s_{t+1}}{q_t s_t} \nu_{t+1} \right] \quad (40)$$

$$\widetilde{n}_{t+1} = \theta[(r_{kt+1} - r_{t+1}) \left( \frac{\eta_t}{\lambda - \nu_t} \right) + (1 + r_{t+1})]n_t + \epsilon n_t \quad (41)$$

$$q_t s_t = q_t K_{t+1} \quad (42)$$

$$K_{t+1} = (1 - \delta)K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t \quad (43)$$

$$q_t = \left[ \Phi_{I_t}\left(\frac{I_t}{K_t}\right) \right]^{-1} \quad (44)$$

$$L_t = H_t \quad (45)$$

$$C_t + I_t = z_t F(K_t, H_t) \quad (46)$$

$$\log(z_{t+1}) = \rho_z \log(z_t) + \epsilon_{t+1}^z \quad (47)$$

$$\log(\omega_{t+1}) = \epsilon_{t+1}^\omega \quad (48)$$

## 5 Quantitative Analysis

This section studies the quantitative predictions of the model by examining the results of numerical simulations of an economy calibrated to U.S. data. The model parameters are calibrated so that the deterministic steady-state of the model matches several average ratios of aggregate financial variables and standard macroeconomic aggregates taken from the U.S. data. Parameters that cannot be directly related to the data are set to values typically used in the literature. The calibration period is set to a quarter since the cyclical properties of the leverage ratio and standard macro aggregates are derived from quarterly data. In order to investigate the dynamics of the model, I compute a first-order approximation to the equilibrium conditions using the perturbation algorithm developed by Schmitt-Grohe and Uribe (2004).

### 5.1 Functional Forms, Parametrization and Calibration

The quantitative analysis uses the following standard functional forms for preferences, production technology and capital adjustment costs:

$$U(c_t, 1 - L_t) = \log(c_t) + v(1 - L_t) \quad (49)$$

$$F(K_t, H_t) = z_t K_t^\alpha L_t^{1-\alpha} \quad (50)$$

$$\Phi\left(\frac{I_t}{K_t}\right) = \frac{I_t}{K_t} - \frac{\varphi}{2} \left[ \frac{I_t}{K_t} - \delta \right]^2 \quad (51)$$

Table 4: Model Parameterization and Calibration

Description	Parameter	Value	Target	Data	Moment
<b>Preferences</b>					
Quarterly discount factor	$\beta$	0.99	Quarterly T-bill rate	1%	1%
Relative utility weight of leisure	$v$	7.02	Hours worked	0.33	0.33
<b>Production Technology</b>					
Share of capital in output	$\alpha$	0.36	N/A	N/A	N/A
Capital adjustment cost parameter	$\varphi$	3	Quarterly standard dev. of investment	4.49%	4.45%
Depreciation rate of capital	$\delta$	0.026	Average annual ratio of investment to capital	9.35%	10%
Steady-state total factor productivity	$\bar{z}$	1	N/A	N/A	N/A
<b>Financial Intermediaries</b>					
Steady-state fraction of assets that can be diverted	$\lambda$	0.07	Average financial leverage ratio	13.06	13.06
Proportional transfer to the entering bankers	$\epsilon$	0.001	Average interest rate spread	2%	2%
Survival probability of the bankers	$\theta$	0.972	Average survival time of banks in years	10	10
Steady-state level of net worth shock	$\bar{\omega}$	1	N/A	N/A	N/A
<b>Shock Processes</b>					
Persistence parameter of productivity shock	$\rho_z$	0.95	Quarterly persistence of TFP	0.95	0.95
Standard deviation of productivity shock	$\sigma_z$	0.00252	Quarterly standard dev. of output	1.55%	1.55%
Standard deviation of net worth shock	$\sigma_\omega$	0.038	Quarterly standard dev. of leverage ratio	5.36%	5.35%

Table 4 lists the parameter values for the model economy. The preference and production parameters are standard in business cycle literature. I take the quarterly discount factor,  $\beta$  as 0.99 to match the 4% U.S. average annual real interest rate. I use log utility in consumption. I pick the relative utility weight of labor  $v$  to fix hours worked in steady state at one third of the available time, i.e.  $\bar{L} = 0.33$ . The share of capital in the production function is set to 0.36, which is conventional in business cycle models. The capital adjustment cost parameter is taken so that the elasticity of the price of capital with respect to the investment to capital ratio is equal to 0.1.<sup>25</sup> The quarterly depreciation rate of capital is set to 2.26% to match the average annual nominal investment to nominal capital ratio. Finally, I set the persistence of TFP shocks to 0.95, i.e.  $\rho_z = 0.95$ . I choose the standard deviation of shocks to TFP such that the model with productivity and net worth shocks generates a standard deviation of output equal to 1.55%, which is the volatility of GDP fluctuations over the past 25 years. The resulting value for  $\sigma_z$  is 0.00252.

The non-standard parameters in our model are the financial sector parameters: the fraction of the revenues that can be diverted,  $\lambda$ , the proportional transfer to newly entering bankers,  $\epsilon$ , and the survival probability of bankers,  $\theta$ . I pick these parameters to match the

<sup>25</sup>Bernanke, Gertler and Gilchrist (1999) suggest that this value should lie between 0 and 0.5.

following three targets: a steady-state interest rate spread of two hundred basis points; a steady-state leverage ratio of 13.06, the average value for the U.S. financial sector over past the 25 years; and an average survival time of 10 years for bankers.<sup>26</sup> The resulting values for  $\bar{\lambda}$ ,  $\epsilon$  and  $\theta$  are 0.07, 0.001 and 0.972, respectively.

The other non-standard parameters to be calibrated in the model are the persistence and the standard deviation of net worth shocks. I assume that  $\omega_t$  is an i.i.d. shock:

$$\log(\omega_{t+1}) = \epsilon_{t+1}^\omega \quad (52)$$

with  $\epsilon_\omega \sim N(0, \sigma_\omega)$ . I choose the standard deviation of the shock,  $\sigma_\omega$ , such that in the model with both shocks, the standard deviation of the leverage ratio is equal to 5.36%, which is the volatility of the leverage ratio over the past 25 years. The resulting value for  $\sigma_\omega$  is 0.038.

## 5.2 Numerical Results

In this section, I first analyze the statistical properties of the model economy with the goal of assessing the importance of net worth shocks for business cycle fluctuations. Then I present the impulse responses to productivity and net worth shocks and I discuss how standard macroeconomic variables and aggregate financial variables respond to productivity and net worth shocks in our basic framework. Finally, I present how the volatilities of real and financial variables are affected by the steady-state leverage ratio.

### 5.2.1 Business Cycle Analysis

This section presents numerical results from stochastic simulations of the economy with productivity and net worth shocks. I simulate the economy 500 times for 1000 periods each and discard the first 500 periods in each simulation. I then compute the business cycle statistics using the simulated series.

Table 5 presents standard business cycle statistics for the model economies with (1) only productivity shocks, (2) only net worth shocks and (3) both productivity and net worth

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<sup>26</sup>The steady-state interest rate spread is taken from Bernanke, Gertler and Gilchrist (1999), while the average survival time for bankers is taken from Gertler and Karadi (2009).

Table 5: Standard Business Cycle Statistics

Statistic	Data	Productivity Shocks	Net Worth Shocks	Both Shocks
$\sigma_Y$	1.55	1.51	0.29	1.55
$\sigma_C$	0.68	1.15	0.19	1.19
$\sigma_I$	4.49	3.90	2.03	4.45
$\sigma_L$	1.73	0.50	0.35	0.62
$\rho_Y$	0.88	0.98	0.76	0.97
$\rho_C$	0.87	0.98	0.80	0.98
$\rho_I$	0.86	0.92	0.64	0.86
$\rho_L$	0.92	0.90	0.63	0.81
$\rho_{Y,I}$	0.96	0.91	0.90	0.87
$\rho_{Y,C}$	0.86	0.95	-0.13	0.91
$\rho_{Y,L}$	0.87	0.80	0.86	0.74
$\rho_{C,I}$	0.71	0.79	-0.55	0.65

<sup>a</sup> The business cycle statistics given in this table are quarterly.

<sup>b</sup> The standard deviation of the shock to TFP process is 0.00252 while its persistence is 0.95 and the standard deviation of the net worth shock is 0.038.

shocks. The third column in Table 5 indicates that the volatility of output in the model with only productivity shocks is higher than the volatility of consumption, which is the case in the data. Moreover, the volatility of investment in the model is nearly three times that of output, as observed in the data, but its absolute value is a bit smaller (4.49 vs. 3.90). This is due to the capital adjustments costs in our model. Since capital adjustment costs generate endogenous changes in the price of capital, this reduces the volatility of investment generated in the model. The autocorrelations and cross-correlations of standard macro aggregates are roughly equal to those in the data. The fourth column shows that a model with only net worth shocks explains 20% of the observed output fluctuations and 50% of the investment fluctuations. In addition, the model with only net worth shocks is quite successful in terms of persistence of macro variables. However, net worth shocks generate counterfactual negative correlations between consumption and output and between consumption and investment, while generating the observed positive cross-correlation between output and investment. This result is due to the fact that investment is very sensitive to fluctuations in net worth. Therefore, net worth shocks alone are not able to generate the observed cross-correlations

among real variables. The last column indicates that the model with both shocks replicates the standard business cycle statistics quite well. If we compare the third and fourth column, we can conclude that productivity shocks are more dominant than net worth shocks in terms of governing the cyclical properties of real variables.

Table 6 summarizes the results for quarterly financial statistics for the model economies with (1) only productivity shocks, (2) only net worth shocks and (3) both productivity and net worth shocks. The third column shows that the model with only productivity shocks generates all of the observed variation in debt, 47% of the fluctuations in net worth and 25% of the fluctuations in the leverage ratio. The observed persistences of debt, equity and the leverage ratio are lower than the model generated persistence. The model overpredicts somewhat the observed degree of procyclicality of debt, and even more so equity and the leverage ratio. In addition, the model matches the strong positive correlation between investment and debt together with weak positive correlation between investment and equity. However, the model also generates a strong positive correlation between debt and equity, contrary to the data.

The fourth column of table 6 shows that the model with only net worth shocks generates 18% of the observed fluctuations in debt and 67% of the fluctuations net worth. Although we don't calibrate net worth shock to match the persistence of equity and the leverage ratio, the model endogenously matches these moments. In addition, the procyclicality of net worth is very high compared to the data while that of debt is lower. Thus, the model generates a very strong countercyclicality in the leverage ratio, contrary to the data. Finally, net worth shocks create a mild negative correlation between debt and equity, which is not found in the data.

The last column of table 6 indicates that the model featuring both shocks explains all of the variation in debt and 80% of the fluctuations in net worth. It matches the volatility of leverage ratio by construction. Debt and net worth are more procyclical than in the data, which leads an acyclical leverage ratio. The model creates a mild negative correlation between investment and leverage, contrary to the data. Moreover, it generates a significant positive correlation between debt and equity, which is not found in the data.

Table 6: Quarterly Financial Statistics

Statistic	Data	Productivity Shocks	Net Worth Shocks	Both Shocks
$\sigma_{Debt}$	1.90	1.93	0.34	1.99
$\sigma_{Equity}$	5.20	2.45	3.46	4.23
$\sigma_{Leverage}$	5.36	1.38	5.18	5.35
$\rho_{Debt}$	0.83	0.99	0.98	0.99
$\rho_{Equity}$	0.66	0.99	0.68	0.78
$\rho_{Leverage}$	0.66	0.98	0.65	0.67
$\rho_{Y,Debt}$	0.70	0.87	0.48	0.87
$\rho_{Y,Equity}$	0.11	0.48	0.97	0.42
$\rho_{Y,Leverage}$	0.14	0.36	-0.93	-0.06
$\rho_{C,Debt}$	0.70	0.98	0.80	0.98
$\rho_{C,Equity}$	0.20	0.70	-0.36	0.34
$\rho_{C,Leverage}$	0.07	0.13	0.47	0.13
$\rho_{I,Debt}$	0.61	0.66	0.05	0.58
$\rho_{I,Equity}$	0.05	0.14	0.98	0.44
$\rho_{I,Leverage}$	0.15	0.64	-0.99	-0.27
$\rho_{Debt,Equity}$	0.04	0.82	-0.26	0.49

<sup>a</sup> Debt is the total liabilities of financial sector while equity is its net worth (total shareholders' equity). Leverage is the financial leverage ratio, which is defined as the ratio of debt to equity.

<sup>b</sup> The standard deviation of the shock to TFP process is 0.00252 while its persistence is 0.95. The standard deviation of TFP is set as to generate the observed output volatility in the model with both shocks.

<sup>c</sup> The standard deviation of the net worth shock is 0.038. The standard deviation of net worth shock is set as to generate the observed volatility in leverage ratio in the model with both shocks.

### 5.2.2 Leverage Ratio and Volatility of Real and Financial Variables

This section compares the volatilities of aggregate real and financial variables in response to productivity and net worth shocks for different leverage ratios. The model in this paper can accommodate different steady-state leverage ratios with appropriate changes in parameterization (in particular, with different values of the fraction of diverted funds  $\lambda$ ). I compute volatilities from numerical simulations of five parameterizations, with steady-state leverage ratios of 1, 3, 7, 13, and 19.

Figure 5 shows the volatilities of output, investment, debt, net worth, and leverage ratio for the leverage ratios of 1, 3, 7, 13, and 19 in the economy driven by productivity shocks. Leverage amplifies productivity shocks. Moreover, the relationship between leverage ratio

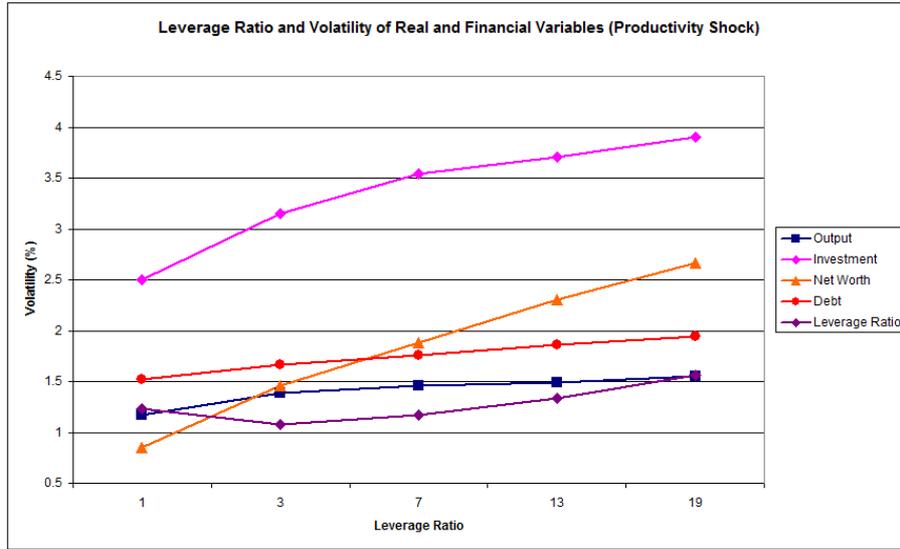


Figure 6: Volatility of Real and Financial Variables across Different Leverage Ratios

and volatilities of output and investment is concave. The intuition is the following: In case of productivity shocks, the fluctuations in investment induce movements in net worth through their effect on the price of capital. If the leverage ratio is higher, the volatility of investment is higher and so is the volatility of net worth. Although higher leverage increases the incentive of bankers to smooth their net worth over time, which can induce lower volatility in net worth, larger fluctuations in investment dominate this smoothing motive. However, as the leverage ratio increases, the net worth smoothing motive becomes more dominant, which reduces the increase in volatilities of real and financial variables. The latter also explains the concave relationship.

Figure 6 shows the volatilities of output, investment, debt, net worth, and the leverage ratio for steady-state leverage ratios of 1, 3, 7, 13, and 19 in the economy driven only by net worth shocks. Leverage dampens net worth shocks. In addition, the relationship between the leverage ratio and volatilities is convex. The reason behind is the following: In case of net worth shocks, the fluctuations in net worth induce movements in investment. Higher leverage increases the incentive for bankers to smooth net worth since they might be forced to fire-sell their assets in the future. Because the shock directly affects the net worth, they will smooth their net worth more when the leverage ratio is higher inducing lower

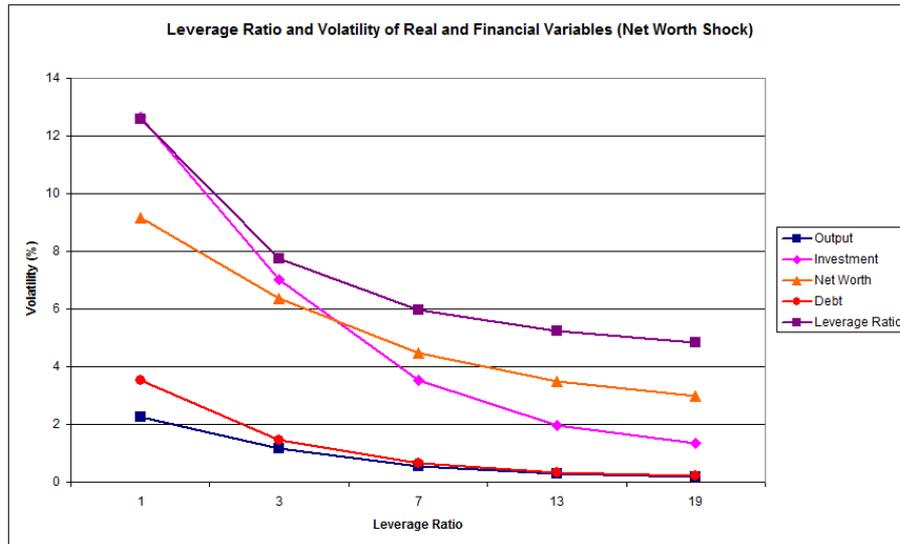


Figure 7: Volatility of Real and Financial Variables across Different Leverage Ratios

volatility. However, as the leverage ratio increases, the net worth smoothing effect become less dominant, which reduces the dampening in volatilities of real and financial variables. This also explains the convex relationship.

### 5.2.3 Impulse Response Analysis

Now I present the dynamics of the model in response to productivity and net worth shocks. The figures below show the percentage deviations of the variables from their steady state values. Figures 7, 8, and 9 present the impulse responses to a one-time, one-standard deviation negative shock to TFP without any shock to net worth. The negative productivity shock immediately reduces output, and this fall in output persists. Households reduce their consumption and savings due to the fall in their income. The reduction in their savings leads to a fall in their deposits. The fall in deposits translates into a sharp fall in return to deposits on impact and then a steady rise. Hours worked decline due to the reduction in wage rates. Total liabilities of financial firms decrease since households reduce their supply of deposits. The return to capital rises as the price of capital declines. The credit spread rises, inducing an increase in the net worth of financial firms. Hence, the model generates countercyclical credit spreads, as observed in the data. Net worth then falls as

the credit spread declines. The decline in total liabilities exceeds the reduction in net worth since the fall in the expected marginal benefit of having one less unit of deposits and one more unit of net worth dominates the increase in the expected marginal gain of getting one more unit of deposits and buying shares of non-financial firms. This leads to a fall in the financial leverage ratio. Therefore, the model with only productivity shocks creates mildly procyclical leverage ratio. This reduction in the financial leverage ratio together with the fall in net worth reduce the amount of shares of non-financial firms purchased by banks. This causes a fall in investment expenditures of non-financial firms.

Figures 10, 11, and 12 present the impulse responses to a one-time, one-standard deviation negative shock to net worth without any shock to TFP process. The negative net worth shock immediately reduces the net worth of financial intermediaries. The fall in their net worth translates into a reduction in equity investment undertaken by banks. The decline in equity investment is severe because financial firms are highly leveraged. Since non-financial firms finance their capital expenditures via equity purchases by intermediaries, they cut back their investment severely. The drop in investment reduces the price of capital, which causes a rise in return to capital. Hours worked decrease due to the fall in demand for labor, which raises the wage rate. The reduction in hours worked increases consumption on impact, but then it starts to decrease. The initial rise in consumption raises the return to deposits. The increase in credit spreads doesn't fully offset the initial fall in net worth, and overall there is a decrease in total liabilities. The expected marginal benefit of getting one more unit of deposits and buying shares of non-financial firms and the expected marginal gain of having one more unit of net worth rise, inducing an increase in the financial leverage ratio. Therefore, the model with only net worth shocks generates a countercyclical leverage ratio, contrary to the data. However, the addition of net worth shocks move the model in the right direction, towards an acyclical leverage ratio.

## 6 Conclusion

This paper documents the observed dynamics of aggregate financial variables (debt, equity, and the leverage ratio) of U.S. financial firms as well as those of standard macro aggregates

(output, consumption and investment) using quarterly data for the period 1984-2009. I found that (i) debt is strongly procyclical while the leverage ratio and equity are acyclical, and (2) aggregate financial variables are more volatile than output. Then I develop a standard real business cycle model augmented with profit maximizing financial intermediaries a'la Gertler and Karadi (2009), in which business cycles are driven by "productivity" and "net worth" shocks. The financial accelerator between households and financial intermediaries plays a central role for the amplification of both shocks. Standard productivity shocks partially explain the observed dynamics of aggregate financial variables and the addition of net worth shocks brings the model much closer to the data. The net worth shocks are also an important source of business cycle fluctuations.

In the current paper, equilibrium dynamics and amplification effects in response to macro shocks are analyzed around steady-state. Brunnermeier and Sannikov (2009) argue that steady-state analysis does not capture important dynamics. In particular, volatility effects and precautionary motives create substantial deleveraging, making the system less stable and more volatile. Since these features significantly amplify shocks, the severity of crisis episodes will not be captured by a steady-state analysis. Hence, I need to use a numerical solution method that allows for occasionally binding constraints to capture these important dynamics. Using a penalty function for the borrowing constraint of banks, the SGU algorithm can be still implemented to make such global analysis (instead of a strict steady-state analysis).

In this paper, adverse macro shocks create an adverse feedback loop of falling asset prices, deteriorating balance sheets and tightening borrowing constraints for financial intermediaries. Banks do not internalize that changes in their net worth affect asset prices, which in turn determine the tightness of their borrowing constraints. As Jeanne and Korinek (2009) suggests, they do not internalize their contribution to aggregate volatility and take on excessive leverage inducing systemic externalities. The model can quantify the systemic externalities originating from excessive borrowing of bankers. Moreover, the following normative question arises in this environment: Which macroprudential regulations can be introduced to make the banks internalize the systemic externalities? What is the optimal tax rate to impose on borrowing by financial intermediaries to prevent them from taking

on excessive leverage? I can answer this question using the model in this paper extended to accommodate occasionally binding borrowing constraints for banks.

In this paper, in the model without productivity shocks the variation in net worth of financial intermediaries is triggered exogenously. However, in the data, most of the fluctuations in net worth of financial intermediaries originate from the movements in asset prices of these institutions. In the current model, the assets of banks are shares of non-financial firms. However, in reality, non-financial firms use a mix of debt and equity financing depending on external financing conditions. Thus the assets of financial intermediaries are debt and equity of non-financial firms. Variability in the flows of debt and equity financing of non-financial firms (or the variation in the fraction of debt and equity financing) may induce greater variability in assets of financial firms, which can explain the observed volatility in net worth of these firms. Jermann and Quadrini (2009) show that in the past 30 years, debt and equity flows of non-financial firms have become more volatile due to financial innovations in firm financing. We believe that this could explain the high observed volatility of debt, equity and the leverage ratio of the U.S. financial sector over the past 30 years. Therefore, for further research, I may introduce explicit roles for non-financial firm's debt and equity financing into the current version of the model and assess the performance of the model against the observed behavior of financial aggregates (including debt, equity and the leverage ratio) of U.S. non-financial firms.

For further research, I can also study optimal credit and monetary policy in this framework. In order to start thinking about how optimal credit and monetary policy should be conducted in an environment in which the financial sector is crucial for business cycle fluctuations, I need a model capable of matching real and financial fluctuations simultaneously. I think that the model proposed in this paper is quite successful in this dimension. Therefore, when the central bank determines the policy rate, it should also monitor the leverage ratio of the financial sector on top of inflation and output gaps. The optimal Taylor rule (which gives the same allocations as the optimal Ramsey policy) may feature a pronounced positive reaction to the financial leverage ratio.

## 7 Appendix

### 7.1 Profit Maximization Problem of Financial Intermediaries

$$V_{jt} = \max_{s_{jt}} E_t \sum_{i=0} (1-\theta)\theta^i \beta^i \Lambda_{t,t+1+i} [(r_{kt+1+i} - r_{t+1+i})q_{t+i}s_{jt+i} + (1+r_{t+1+i})n_{jt+i}]$$

$$s.t. \quad V_{jt} \geq \lambda q_t s_{jt} \quad (\mu_t)$$

where  $\mu_t$  is the Lagrange multiplier associated with the incentive compatibility constraint. The first order conditions w.r.t.  $s_{jt}$  and  $\mu_t$  are given respectively by

$$(1-\theta)\theta\beta\Lambda_{t,t+1}(r_{kt+1} - r_{t+1})q_t - \mu_t\lambda q_t = 0 \quad (53)$$

$$V_{jt} - \lambda q_t s_{jt} = 0 \quad (54)$$

From (6), we establish that  $\mu_t$  is greater than zero as long as  $r_{kt+1} > r_{t+1}$ . Since, by assumption,  $\beta^i \Lambda_{t,t+i} [(r_{kt+1+i} - r_{t+1+i})]$  is greater than zero, the incentive compatibility constraint will hold with equality. Thus, we have

$$V_{jt} = \lambda q_t s_{jt} \quad (55)$$

Now, we will write  $V_{jt}$  in a recursive form to obtain the leverage ratio of financial intermediaries. Let's define the following variables:

$$\nu_t q_t s_{jt} = E_t \sum_{i=0} (1-\theta)\theta^i \beta^i \Lambda_{t,t+1+i} [(r_{kt+1+i} - r_{t+1+i})q_{t+i}s_{jt+i}] \quad (56)$$

$$\eta_t n_{jt} = E_t \sum_{i=0} (1-\theta)\theta^i \beta^i \Lambda_{t,t+1+i} (1+r_{t+1+i})n_{jt+i} \quad (57)$$

Thus, we have the following expression:

$$V_{jt} = \nu_t q_t s_{jt} + \eta_t n_{jt} \quad (58)$$

Now we need to write  $\nu_t$  and  $\eta_t$  recursively in order to get rid of infinite sums. Let's begin with  $\nu_t$ . To ease the notation, let's drop expectations for now.

$$\nu_t = \sum_{i=0} (1-\theta)\theta^i \beta^i \Lambda_{t,t+1+i} [(r_{kt+1+i} - r_{t+1+i})x_{t,t+i}] \quad (59)$$

where  $x_{t,t+i} = \frac{q_{t+i}s_{jt+i}}{q_t s_{jt}}$ .

$$\nu_t = (1 - \theta)\Lambda_{t,t+1}(r_{kt+1} - r_{t+1}) + \sum_{i=1} (1 - \theta)\theta^i \beta^i \Lambda_{t,t+1+i} [(r_{kt+1+i} - r_{t+1+i})x_{t,t+i}] \quad (60)$$

$$\nu_t = (1 - \theta)\Lambda_{t,t+1}(r_{kt+1} - r_{t+1}) + \beta\Lambda_{t,t+1}\theta x_{t,t+1} \sum_{i=1} (1 - \theta)\theta^{i-1} \beta^{i-1} \Lambda_{t+1,t+i} [(r_{kt+1+i} - r_{t+1+i})x_{t+1,t+i}] \quad (61)$$

By updating equation (43) one period, we obtain the following expression for  $\nu_{t+1}$

$$\nu_{t+1} = \sum_{i=0} (1 - \theta)\theta^i \beta^i \Lambda_{t+1,t+i} [(r_{kt+1+i} - r_{t+1+i})x_{t+1,t+i}] \quad (62)$$

where  $x_{t+1,t+i} = \frac{q_{t+i}s_{jt+i}}{q_{t+1}s_{jt+1}}$ .

We can write (46) in the following way:

$$\nu_{t+1} = \sum_{i=1} (1 - \theta)\theta^{i-1} \beta^{i-1} \Lambda_{t+1,t+i} [(r_{kt+1+i} - r_{t+1+i})x_{t+1,t+i}] \quad (63)$$

Thus, (47) is equal to the expression in the second term of RHS of equation (45). Hence, we can re-write (45) with the expectations as follows:

$$\nu_t = E_t[(1 - \theta)\Lambda_{t,t+1}(r_{kt+1} - r_{t+1}) + \beta\Lambda_{t,t+1}\theta x_{t,t+1}\nu_{t+1}] \quad (64)$$

Let's continue with  $\eta_t$ . To ease the notation, let's drop expectations for now.

$$\eta_t = \sum_{i=0} (1 - \theta)\theta^i \beta^i \Lambda_{t,t+i}(1 + r_{t+1+i})z_{t,t+i} \quad (65)$$

where  $z_{t,t+i} = \frac{n_{jt+i}}{n_{jt}}$

$$\eta_t = (1 - \theta)\Lambda_{t,t+1}(1 + r_{t+1}) + \sum_{i=1} (1 - \theta)\theta^i \beta^i \Lambda_{t,t+i}(1 + r_{t+1+i})z_{t,t+i} \quad (66)$$

$$\eta_t = (1 - \theta)\Lambda_{t,t+1}(1 + r_{t+1}) + \beta\Lambda_{t,t+1}\theta z_{t,t+1} \sum_{i=1} (1 - \theta)\theta^{i-1} \beta^{i-1} \Lambda_{t+1,t+i}(1 + r_{t+1+i})z_{t+1,t+i} \quad (67)$$

By updating equation (49) one period, we obtain the following expression for  $\eta_{t+1}$

$$\eta_{t+1} = \sum_{i=0} (1 - \theta)\theta^i \beta^i \Lambda_{t+1,t+i}(1 + r_{t+1+i})z_{t+1,t+i} \quad (68)$$

where  $z_{t+1,t+i} = \frac{n_{jt+i}}{n_{jt+1}}$

We can write (52) in the following way:

$$\eta_{t+1} = \sum_{i=1} (1-\theta)\theta^{i-1}\beta^{i-1}\Lambda_{t+1,t+i}(1+r_{t+1+i})z_{t+1,t+i} \quad (69)$$

Thus, (53) is equal to the expression in the second term of RHS of equation (51). Hence, we can re-write (51) with the expectations as follows:

$$\eta_t = E_t[(1-\theta)\Lambda_{t,t+1}(1+r_{t+1}) + \beta\Lambda_{t,t+1}\theta z_{t,t+1}\eta_{t+1}] \quad (70)$$

When the incentive constraint binds, we have

$$\nu_t q_t s_{jt} + \eta_t n_{jt} = \lambda q_t s_{jt} \quad (71)$$

$$q_t s_{jt} = \frac{\eta_t}{\lambda - \nu_t} n_{jt} \quad (72)$$

By manipulating this expression using the balance sheet of financial firm, we can obtain the leverage ratio of financial firms as follows:

$$b_{jt} + n_{jt} = \frac{\eta_t}{\lambda - \nu_t} n_{jt} \quad (73)$$

$$\frac{b_{jt} + n_{jt}}{n_{jt}} = \frac{\eta_t}{\lambda - \nu_t} \quad (74)$$

$$\frac{b_{jt}}{n_{jt}} = \frac{\eta_t}{\lambda - \nu_t} - 1 \quad (75)$$

Therefore,  $\frac{\eta_t}{\lambda - \nu_t} - 1$  is the leverage ratio of the financial sector.

## 8 Bibliography

- [1] Adrian, Tobias and Hyun Song Shin. (2008), "Liquidity and Leverage," *Journal of Financial Intermediation*, forthcoming
- [2] Adrian, Tobias and Hyun Song Shin. (2009), "Financial Intermediaries and Monetary Economics," *Federal Reserve Bank of New York Staff Reports*, No. 398
- [3] Bernanke, Ben S., Mark Gertler, and Simon Gilchrist. (1999), "The Financial Accelerator in a Quantitative Business Cycle Framework," In *Handbook of Macroeconomics*, edited by John B. Taylor and Michael Woodford, Vol. 1C. Elsevier
- [4] Carlson, Mark A., Thomas B. King, and Kurt F. Lewis. (2008), "Distress in the Financial Sector and Economic Activity", *Finance and Economics Discussion Series*, Federal Reserve Board, Washington, D.C., 2008-43.
- [5] Carlstrom, Charles T. and Timothy S. Fuerst. (1998), "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis", *American Economic Review*, Vol.87, pp. 893-910
- [6] Carlstrom, Charles T. and Timothy S. Fuerst. (1998), "Agency Costs and Business Cycles", *Economic Theory*, Vol.12, pp. 583-597
- [7] Chugh, Sanjay. (2009), "Firm Risk and Leverage Based Business Cycles", manuscript, University of Maryland.
- [8] Covas, Francisco and Wouter J. Den Haan. (2006), "The Role of Debt and Equity Finance over the Business Cycle" *Bank of Canada Working Paper*, 2006-45.
- [9] Fostel, Ana and John Geanakoplos. (2008), "Leverage Cycles and the Anxious Economy." *American Economic Review*, Vol. 98(4), pp. 1211-1244.
- [10] Geanakoplos, John. (2009), "The Leverage Cycle" *NBER Macroeconomics Annual 2009*, forthcoming

- [11] Gertler, Mark, Simon Gilchrist, and Fabio M. Natalucci. (2007), "External Constraints on Monetary Policy and the Financial Accelerator." *Journal of Money, Credit, and Banking*, Vol. 39(2-3), pp. 295-330.
- [12] Gertler, Mark and Peter Karadi. (2009), "A Model of Unconventional Monetary Policy", manuscript, New York University.
- [13] Gertler, Mark and Nuhri Kiyotaki (2009), "Financial Intermediation and Credit Policy in Business Cycle Analysis", manuscript, Princeton University
- [14] Jeanne, Oliver and Anton Korinek. (2009), "Managing Credit Booms and Busts: A Pigouvian Taxation Approach", manuscript, University of Maryland.
- [15] Jermann, Urban and Vincenzo Quadrini. (2009), "Macroeconomic Effects of Financial Shocks", manuscript, University of Southern California.
- [16] Jermann, Urban and Vincenzo Quadrini. (2009), "Financial Innovations and Macroeconomic Volatility", manuscript, University of Southern California.
- [17] Kiyotaki, Nuhri and John Moore. (1997), "Credit Cycles", *Journal of Political Economy*, Vol.105, pp. 211-248.
- [18] Korajczyk, Robert A. and Amnon Levy. (2003), "Capital Structure Choice: Macroeconomic Conditions and Financial Constraints", *Journal of Financial Economics*, Vol.68, pp. 75-109.
- [19] Korinek, Anton. (2010), "Systemic Risk-Taking: Amplification Effects, Externalities, and Regulatory Responses", manuscript, University of Maryland.
- [20] Levin, Andrew T., Fabio M. Natalucci, and Egon Zakrajsek. (2004), "The Magnitude and Cyclical Behavior of Financial Market Frictions", *Finance and Discussion Series*, Federal Reserve Board of Governors, No.70.
- [21] Meh, Cesaire and Kevin Moran. (2008), "The Role of Bank Capital in the Propagation of Shocks" *Bank of Canada Working Paper*, 2008-36.

- [22] Schmitt-Grohe, Stephanie and Martin Uribe (2004), "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function", *Journal of Economic Dynamics and Control*, Vol.28, pp. 755-775.
- [23] Townsend, Robert M. (1979), "Optimal Contracts and Competitive Markets with Costly State Verification", *Journal of Economic Theory*, Vol.21, pp. 265-293.
- [24] Williamson, Stephen D. (1987), "Financial Intermediation, Business Failures, and Real Business Cycles", *Journal of Political Economy*, Vol.95, pp. 1196-1216.

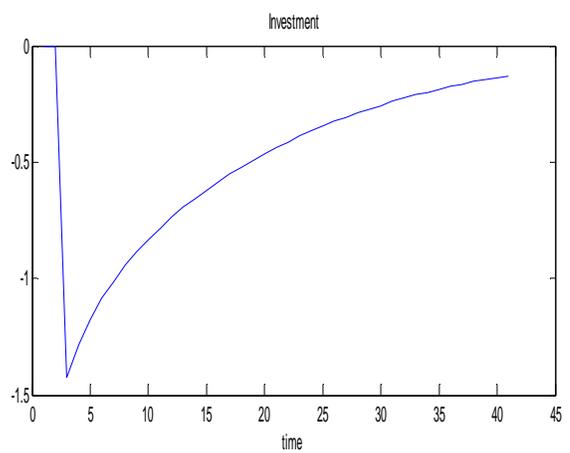
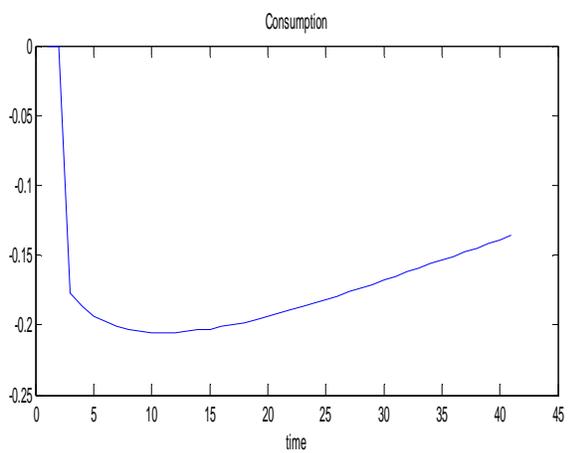
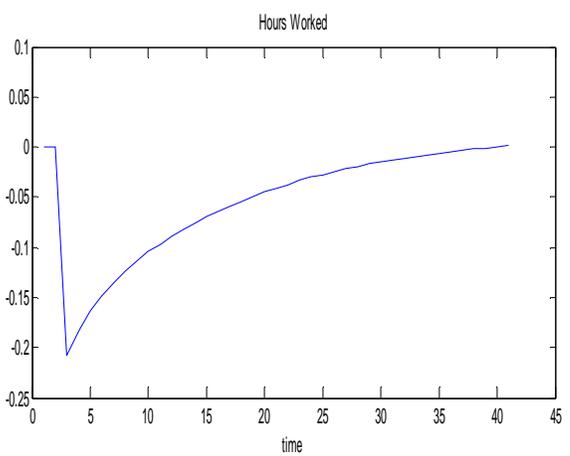
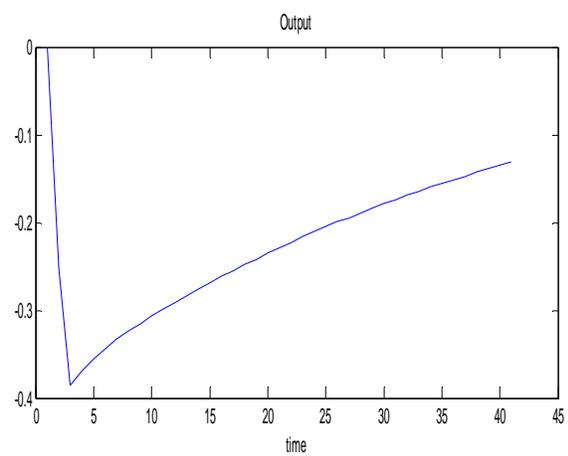
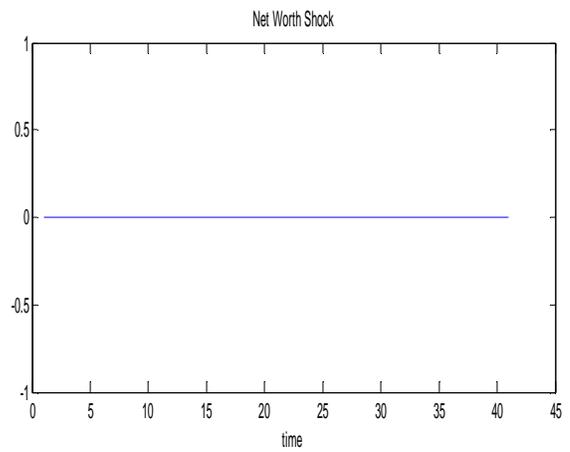
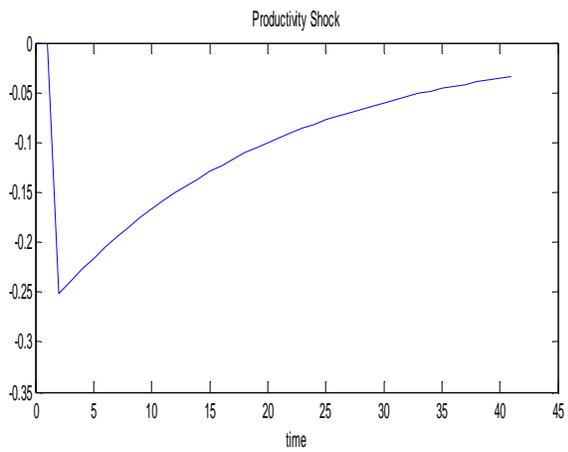


Figure 7: Impulse responses to negative productivity shock

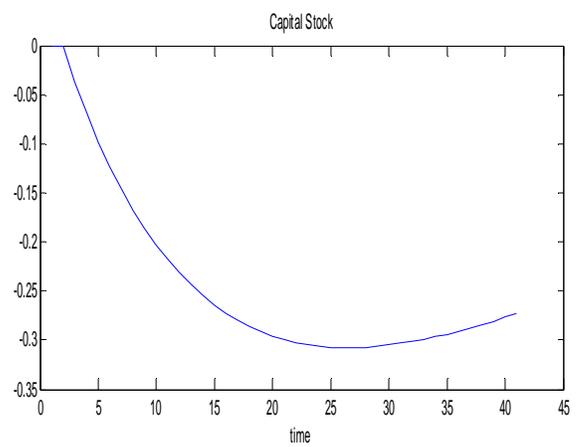
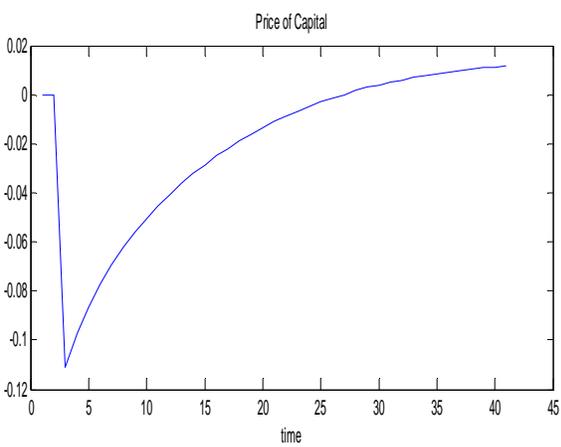
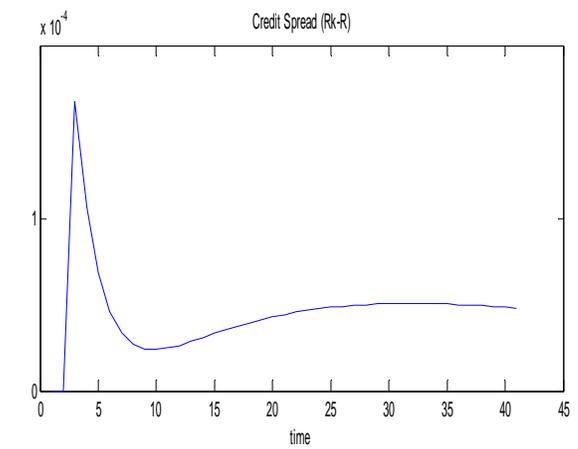
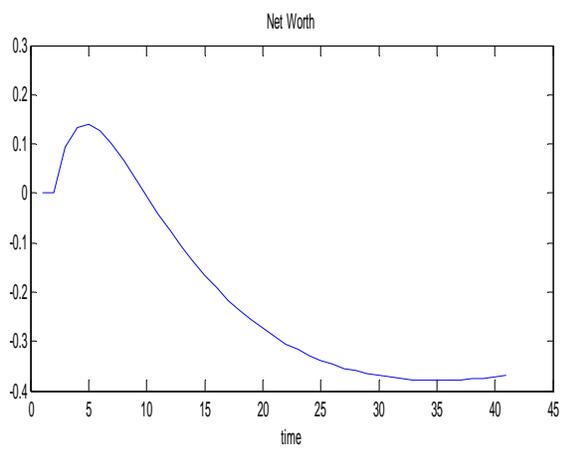
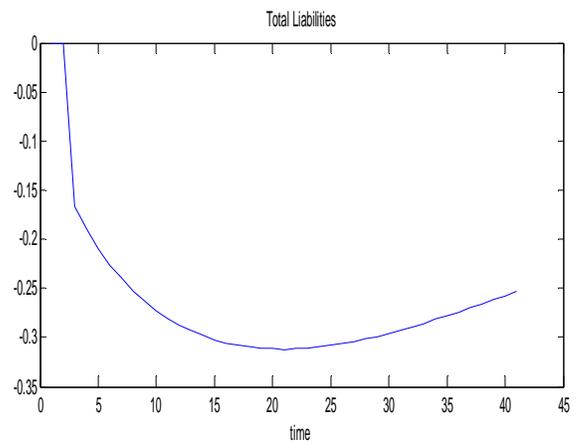
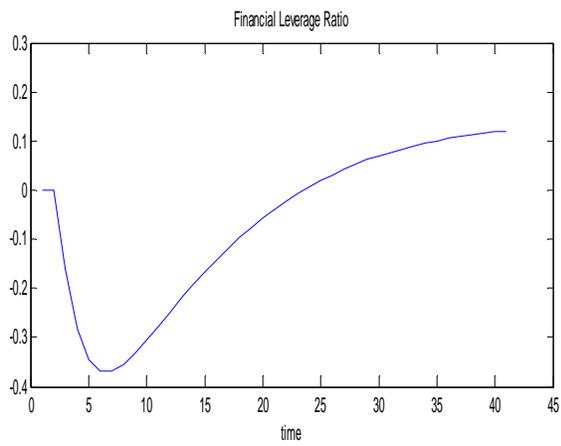


Figure 8: Impulse responses to negative productivity shock

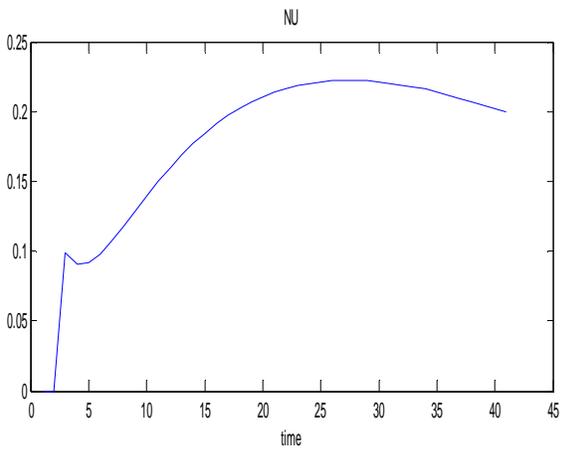
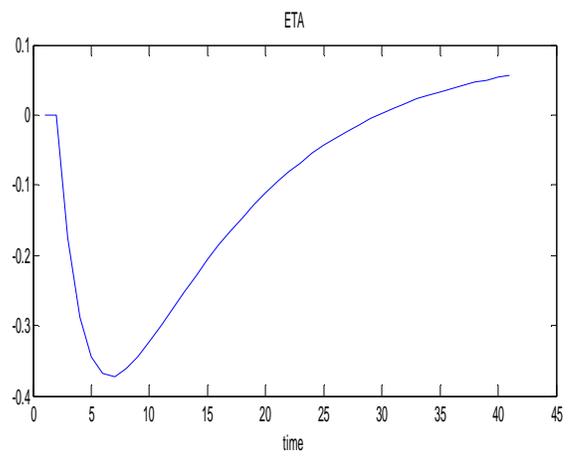
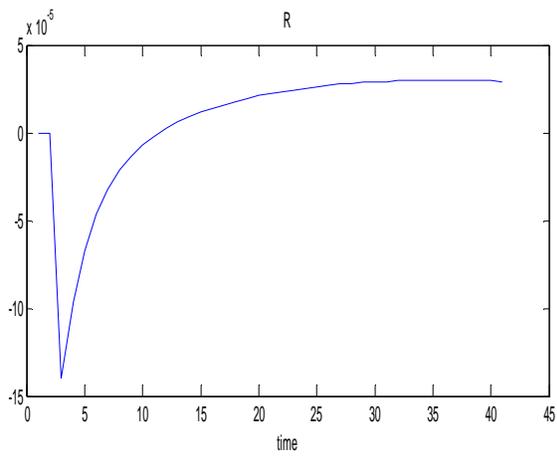
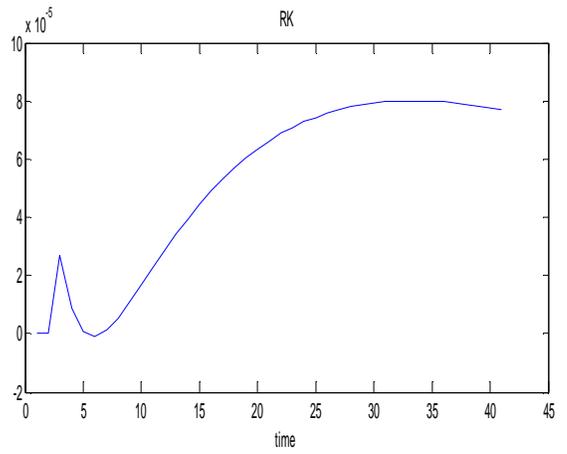
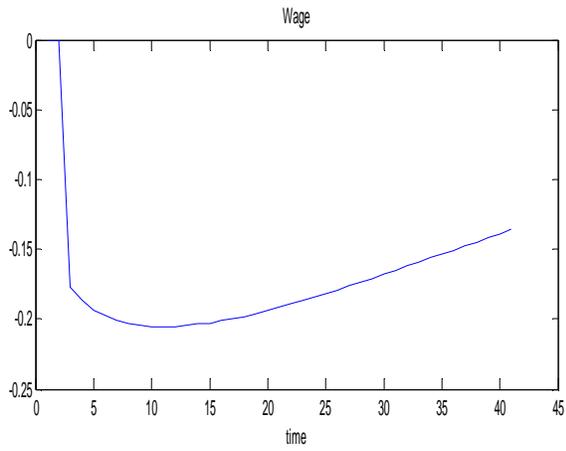


Figure 9: Impulse responses to negative productivity shock

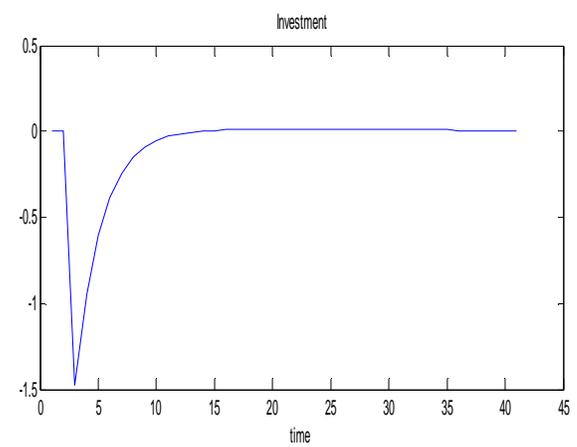
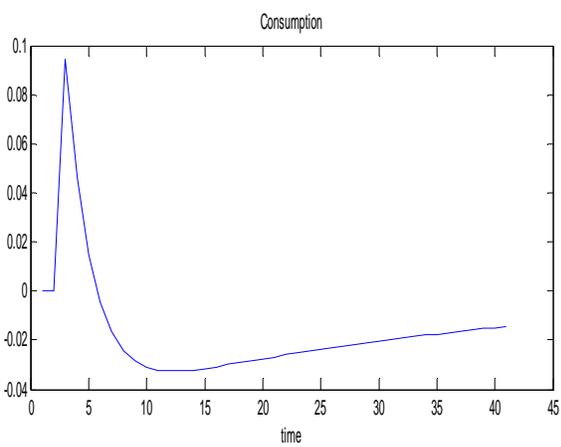
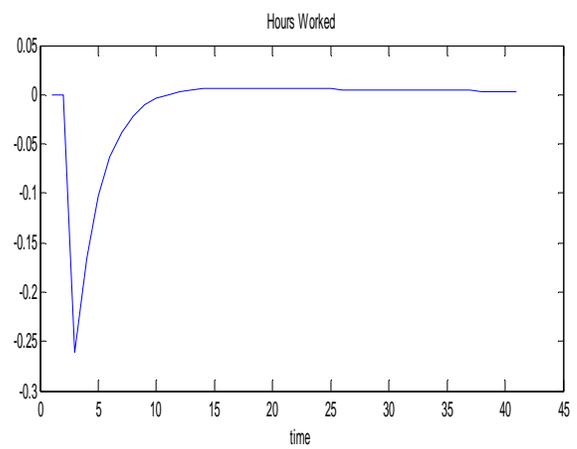
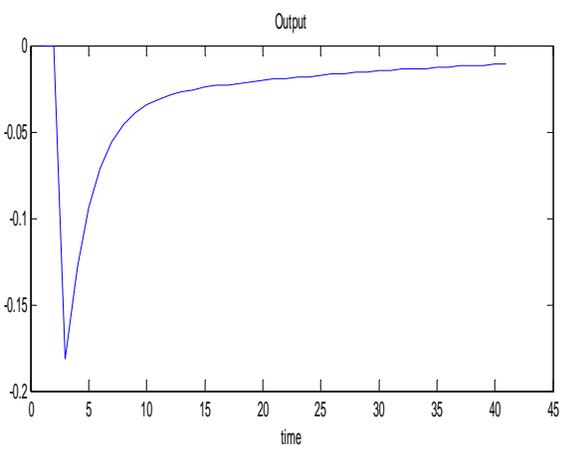
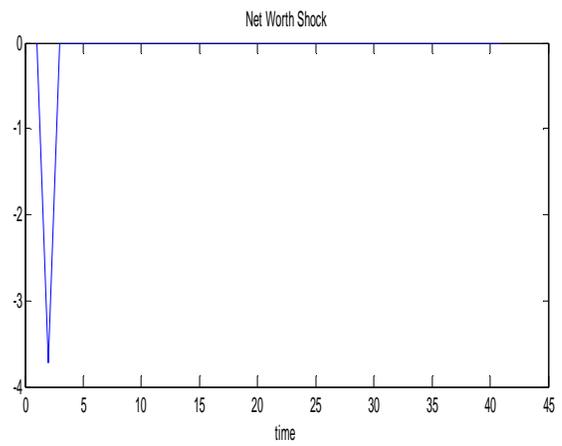
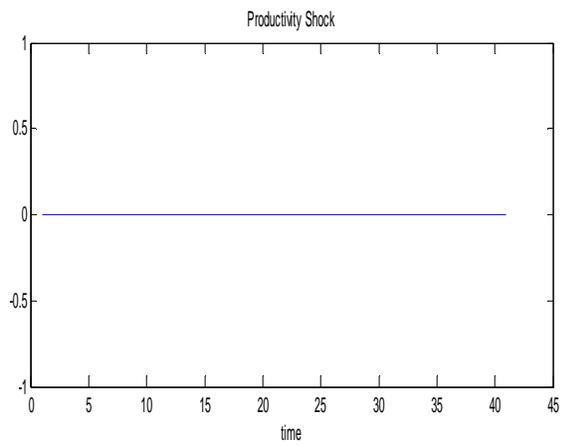


Figure 10: Impulse responses to negative net worth shock

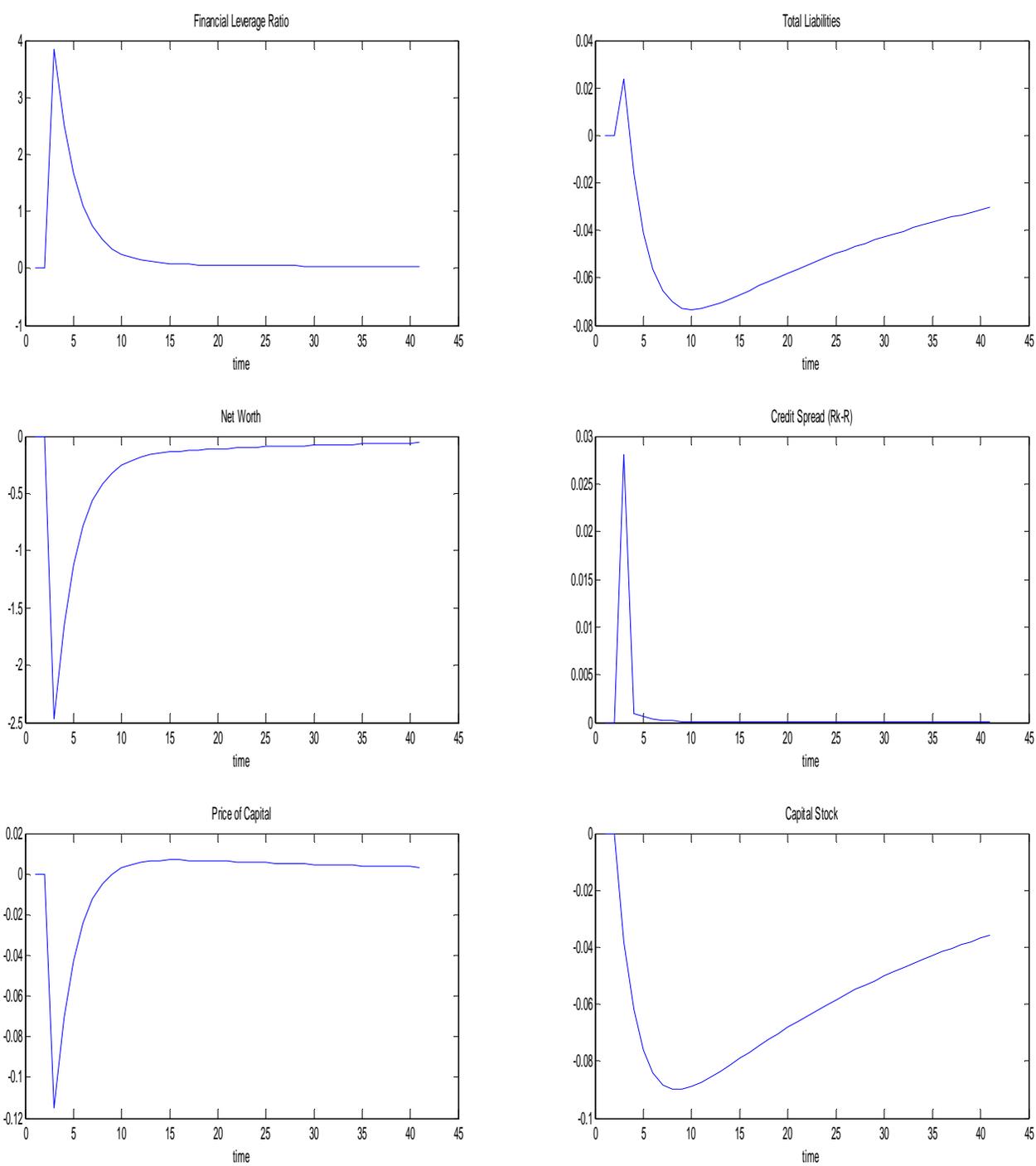


Figure 11: Impulse responses to negative net worth shock

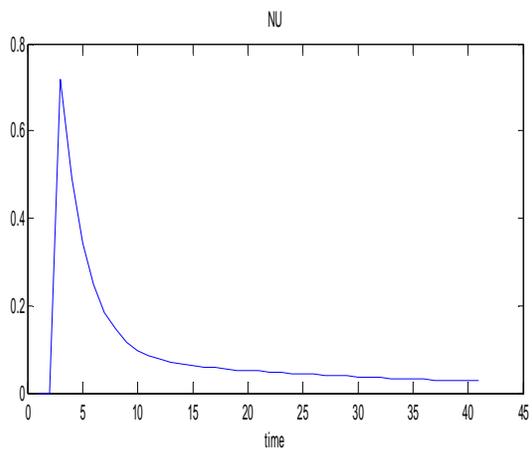
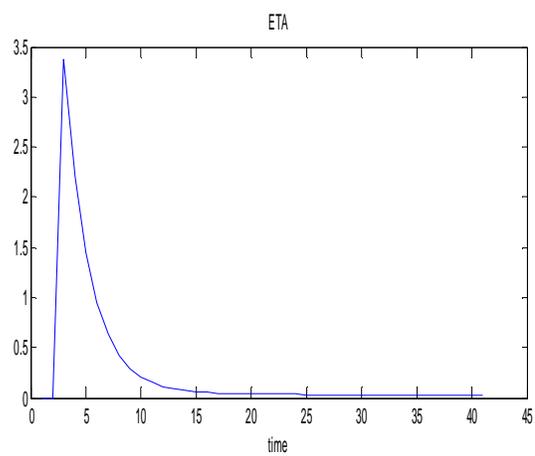
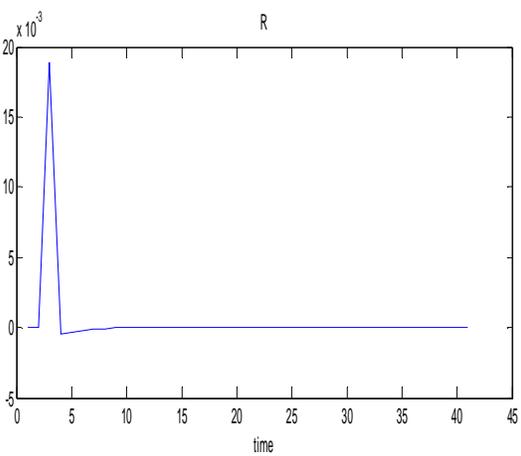
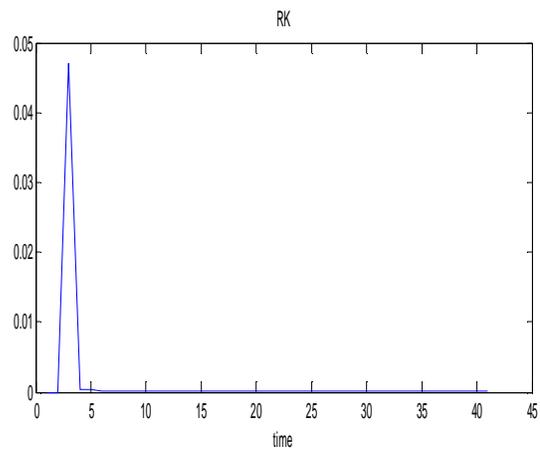
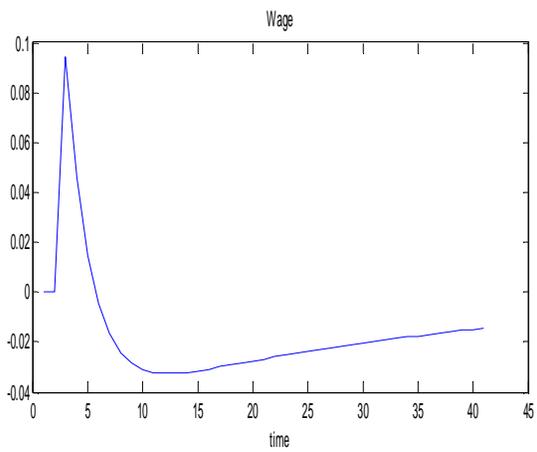


Figure 12: Impulse responses to negative net worth shock

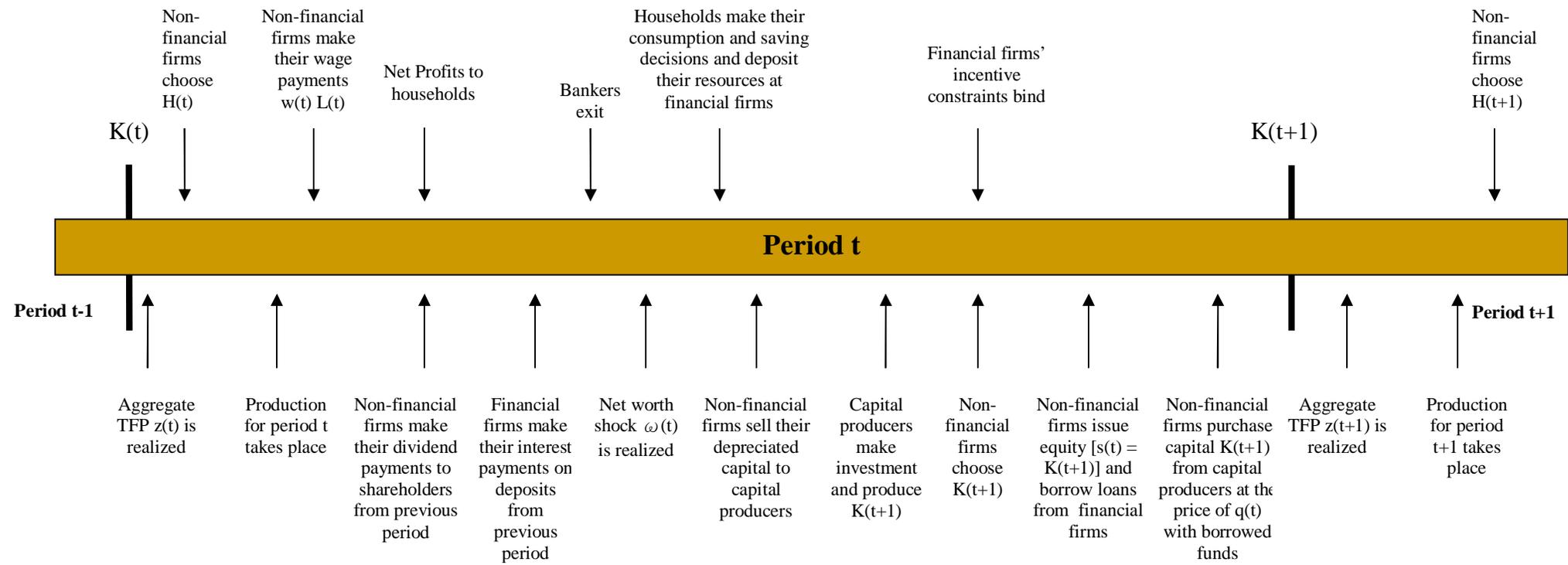


Figure 13: Timing of events in model