The Green Paradox and the Choice of Capacity

Marc Gronwald[1] Darko Jus[2] Markus Zimmer[3]

Abstract

A number of recent papers extend traditional Hotelling frameworks by the topical issue of climate change. In fact, they study the effects of environmental taxes on the resource extraction path of carbon resource and derive important and far-reaching policy implications. Of particular relevance is Sinn (2008) who introduces the *green paradox*as a possible outcome of today's environmental policy. He points out that the resource owner will come to the logical conclusion that shifting extraction quantities to the presence increases his expected total cash-flow if an over time increasing tax imposes a threat on profits of future extraction. Consequently such an environmental policy would even be counterproductive. We show, however, that this result may not prevail anymore if the capacity building decision is endogenous and costly instead of costless and therefore not considered as in Sinn (2008). By deriving necessary conditions for the increasing taxes to be an effective instrument we show that the evaluation of the*green paradox*changes between the pre-peak oil and the post-peak oil period.

Keywords: green paradox, resource extraction, global warming, carbon taxation, production capacity

JEL Classification: Q38, Q54, H21

[1]Ifo Institute for Economic Research at University of Munich
Address: Poschingerstr. 5, 81679 Munich, Fon: +49 (0) 89 92 24 14 00, gronwald@ifo.de
[2]Center for Economic Studies, University of Munich
Address: Schackstr. 4, 80539 Munich, Fon: +49 (0) 89 21 80 31 04, E-Mail: darko.jus@lrz.uni-muenchen.de
[3]Ifo Institute for Economic Research at University of Munich
Address: Poschingerstr. 5, 81679 Munich, Fon: +49 (0) 89 92 24 12 60, zimmer@cesifo.de

1 Introduction

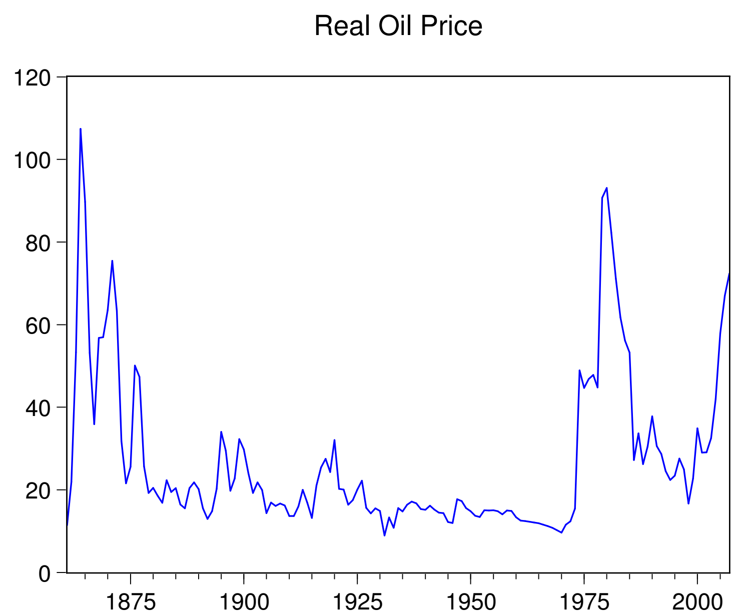
A vast literature suggests that Hotelling (1931) is the proper framework for modeling the extraction decision of a non-renewable resource. In his seminal paper Hotelling derived the fundamental equilibrium condition, by which in the simple setting the price of a non-renewable resource has to grow with the interest rate. Various extensions of this framework have been considered by economists in the following decades. The main purpose of this literature, however, changed substantially. At the beginning and through the 1970s and 1980s a large number of papers emerged that were focussing on the scarcity problem of non-renewable resources. Long (1975), for instance, investigates the impact of insecure property rights. Long and Sinn (1985) analyse consequences of sudden price shifts on resource extraction. Dasgupta and Heal (1979) provide a comprehensive treatment of the resource extraction problem including imperfect competition and uncertainty issues. Obviously the literature at that time found its motivation in concerns regarding the future availability of oil initiated by the first oil crisis. Therefore the dominant question was to find the optimal allocation of a scarce resource while possible negative externalities, e.g. global warming, were neglected or considered to be not important. Beginning in the 1990s, a new literature started to be written, having in mind the idea of extending the traditional framework by the topical issue of climate change. Early examples include Ulph and Ulph (1994), Withagen (1994) as well as Hoel and Kverndokk (1996). Today the problem of climate change has reached the top of the political agenda and moves more and more into the public eye. Consequently research in this area has never been more important and more in demnad than today. Examples of recent papers include Chakravorty et al. (2006, 2008), who consider pollution ceilings and the order of extraction of different carbon resources.

The general problem of today's environmental policies lies in the inability to effectively restrict the supply of carbon resources since policy instruments are limited to demand side interventions. Of particular importance, however, is Sinn's (2008) formulation of the*green paradox*. The article assumes that resource owners are confronted with carbon taxes that increase over time - a plausible assumption given the increasing immediacy of the climate change problem. It is shown that if resource owners include this policy development into their maximisation, a faster rather than a slower extraction of the carbon resource results. The rationale for this paradox lies in the dynamics of the resource extraction decision. If a over time increasing tax imposes a threat on future profits of extraction, the resource owner will come to the logical conclusion that shifting extraction quantities to the presence increases his expected total cash-flow. Thus an oil-sheikh - the typically given example of a resource owner - wants to sell his oil as long as it is still relatively low-taxed in order to maximise his profits. It is obvious that this result has important and far-reaching policy implications. The *green paradox*literature generally concludes that a binding global certificate system covering all carbon dioxide sources is the only practical solution and that attempts of implementing greener policies in the transition process are non-effective or even counterproductive. Therefore in addition to the quite well understood carbon leakage phenomenon, i.e. the fact that a unilateral emissions reduction may lead to more emissions by other countries, a second major problem arisis with demand side environmental policies. In case of carbon leakage Eichner and Pethig (2009), Smulders et al. (2009) and van der Werf (2009) have analytically derived how large the (negative) effect might actually be; however, for the *green paradox*this exercise still needs to be done.

We add an important issue to the existing literature by explicitly modelling the process of capacity building. As resource extraction is a highly capital intensive production process, the size and the costs of capacity play a crucial role. Even so, previous articles have assumed that the production capacity is sufficiently large or that it can be build up at zero costs. The contributions by Campbell (1980), Holland (2003) as well as Ghoddusi (2009) form notable exceptions in this regard. Still we are far away from having a complete understanding of the role of capacities in resource extraction problems. Therefore, we focus on futher developing the resource extraction decision as it is modelled by Sinn (2008) by adding an endogenous capacity choice under the assumption of convex adjustment costs. Generally the inclusion of this decision leads to a in two parts devided extraction path. Initially there is a phase in which extraction capacity is build up and where extraction quantities are rising. This pre-peak oil phase is followed by the post-peak oil phase in which extraction rates are declining and the (costly) overcapacities are reduced accordingly. Thus, our results regarding both, the optimal extraction path and the associated price path, are in line with important streams of the literature. The peak-oil literature can be traced back to Hubbert (1956), who correctly predicted the peak in U.S. crude oil production in the 1970s. Later on this issue has been addressed in a large number of papers. Holland's (2008) contribution is one of the most interesting ones as it shows that Hotelling models and peak-oil settings are compatible with each other. Also the resulting U-shaped price path has been addressed by the literature. Slade's (1983) seminal paper attempts to empirically test this type of price path assumption. In Figure 1 we illustrate the time series of historical oil prices which obviously points towards the U-shaped price path scenario. Dvir and Rogoff (2008) argue that in the years before 1900 and in those after 1970 uncertainty regarding the availability of sufficient amounts of oil was present which resulted in a higher volatility of the price. In the first of these two periods the concerns about oil scarcity could be attributed to insufficient capacities while at the moment the finite nature of oil supply appears to be the main issue.

Our analysis shows that for usually made assumptions about the cost structure, policies becoming greener over time, in contrast to the *green paradox* literature, remain a valid instrument in the transition process.We are able to derive the necessary conditions for the policy measures to be effective and show that the evaluation with regard to the *green paradox*differs for pre and post oil-peak regimes. The remainder of the paper is organized as follows: Section 2 presents the theoretical model based on Sinn (2008); in Section 3 we illustrate the results, before we offer concluding remarks in Section 4.

Figure 1: History of the oil price



Source: BP

2 Model

Sinn (2008) shows in an extension of the standard Hotelling model that an increasing tax over time leads to a faster extraction of the resource. This paper employs the extended Hotelling resource extraction model used by Sinn (2008) and makes the following essential additional assumption: The resource owner faces a binding capacity constraint R which is always fully utilized but can but adjusted (increased or decreased) by V for the cost $F(V)$. Hence, capacity is either increased or decreased, but never both at the same time. The variable extraction costs are denoted $C(S(t))$ rather than $g(S(t))$ as in Sinn (2008). In fact, Sinn's model is a special case of our generalized model in which the adjustment costs are zero, i.e. for $F(V) = 0$. The assumption of full utililization of the extraction capacity is a useful simplification. Relaxing it would not change the results with regard to the existence of the *green paradox* in a deterministic analysis. The variable R can either be interpreted as the capacity or as a simple form of a production function where $f(R) = R$. We allow the marginal costs $C(S)$ to be stock-dependent, i.e. to increase as the resources faces exhaustion.

Following Sinn (2008), $\hat{\Theta}$ is the growth rate of a cash flow tax, $i+\pi$ is the sum of interest rate and the probability of immediate expropriation of the resource owner, Θ is the cash flow tax, P is the consumer price for a unit of the resource,

R is the production capacity and S the stock of the resource. We introduce V as the adjustment of capacity and $F(V)$ as the adjustment costs of adding (or subtracting) V units of capacity .

$$\underset{V}{max} \int_0^\infty \Theta(0) \cdot [[\Theta \cdot P(t) - C(S)] \cdot R - F(V)] \cdot e^{-[i+\pi-\hat{\Theta}]\cdot t} \mathrm{d}t \tag{1}$$

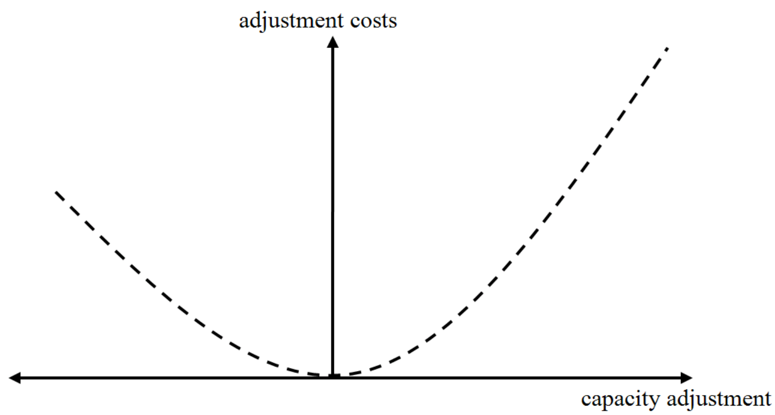
$$s.t. \qquad \dot{S} = -R(t) \tag{2}$$

$$\dot{R}(t) = V \tag{3}$$

$$S(0) = S_0 \tag{4}$$

It is worth noting that the tax function is defined as $\Theta(t) = \Theta(0) \cdot e^{\hat{\Theta}\cdot t}$ where $\Theta(0) = 1 - \tau(0)$ and $\tau(\cdot)$ is the tax rate in percent. Thus Θ is the share of the consumer price $P(t)$ that the producer receives before the cash-flow tax applies. An increasing tax would therefore be expressed by $\hat{\Theta} < 0$. The adjustment cost function $F(V) \geq 0$ is convex and twice continuously differentiable, and has its minimum at $V = 0$.

Figure 2: Adjustment cost function



Source: own illustration

Figure 1 illustrates the shape of the adjusment cost function under this mild set of assumptions. As long as these assumptions hold we do not to explicitly impose any symmetry restriction for the costs of positive or, respectively, negative capacity adjustments. Thus the slope for capacity reductions can be different from the one for building up of capacity .

3 Results

The following section presents only the main findings. The complete analytical solution can be found in the Appendix. The optimal control problem stated above can be formulated by the following current value Hamiltonian:

$$H = \Theta(0) \cdot [[\Theta \cdot P(t) - C(S)] \cdot R(t) - F(V)] - \lambda_1(t) \cdot R(t) + \lambda_2(t) \cdot V \quad (5)$$

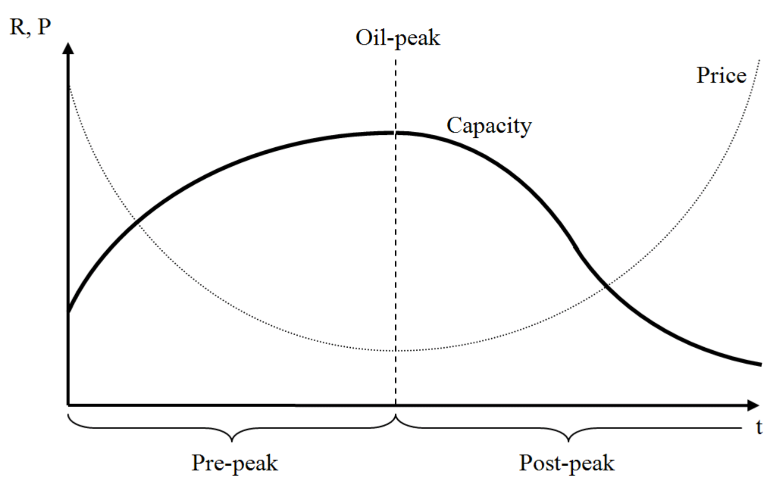
Solving the optimal control problem analytically and displaying it in the conventional form yields the resulting equivalent to the Hotelling rule:

$$i + \pi - \hat{\Theta} = \frac{\Theta \cdot \dot{P}(t)}{\Theta \cdot P(t) - C(S) - \left[i + \pi - \hat{\Theta}\right] \cdot F_V(V)} \quad (6)$$

First we can observe that this rule deviates from the condition in Sinn (2008) only in the last term of the denominator on the right-hand-side of the equation. As was stated above the original analysis is a special case of our analysis for which $F(V) = 0$ in which case obviously also $F_V(V) = 0$ and thus equation (6) reduces to the condition in Sinn (2008):

$$i + \pi - \hat{\Theta} = \frac{\Theta \cdot \dot{P}(t)}{\Theta \cdot P(t) - C(S)} \quad (7)$$

Figure 3: regime type



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According to Sinn (2008) the growth rate of marginal profits (right-hand-side of equation (7)) has to be equal to the time preference rate (which is the sum of the interest rate i and π, an additional discounting term that accounts for the probability of immediate expropriation) plus the growth rate of the cash-flow tax. Therefore the introduction of an increasing tax rate ($\hat{\Theta} = 0$ changes to $\hat{\Theta} < 0$) increases the left-hand-side of the equation so that in consequence extraction is brought forward as the growth rate of marginal profits needs to be higher. Thus the carbon resource will be extracted in shorter time, with the previously explained negative impact on the problem of global warming.

This clear-cut result cannot be supported by our analysis as shown in equation (6) which is our equilibrium condition after including the choice of capacity. The additional term entering our condition ($\left[i + \pi - \hat{\Theta}\right] \cdot F_V(V)$, in the denominator of the right-hand-side) itself depends on the growth of the tax rate. In fact - as we will show - this might oppose the *green paradox* hypothesis. The above discussed effect, however, remains all the same true since the increasing tax rate still gives an incentive to bring forward resource extraction; but at the same time an increasing tax rate reduces the present value of the whole project so that in particular the incentive for building-up of capacity becomes smaller. As building-up of capacities is not that favourable anymore, theresource owners will require the marginal capacity expansion to have a higher rate of return which implies in total less investment in capacities. So we have identified an additional effect (the "capacity effect") which in any case will decrease the intensity of the green paradox and possibly even neutralise it completely.

In the following we analyse the conditions under which the capacity effect dominates the original *green paradox*conjecture. First of all it useful to think in two regimes as clearly at beginning capacity has to build-up while it eventually has to be reduced after peaking at some point in between. So the order is given by the fact that the resource owner starts with zero capacity which leads to a pre-peak oil regime in which capacity increases.[4] After the point in time with maximum oil production is reached, the post-peak oil regime begins in which capacity decreases. Once this regime is entered, it is never optimal to switch back to higher extraction rates as shown in an example by Feichtinger and Hartl (1986).

In our further analysis we distinguish when necessary between these two regimes which were already illustrated by Figure 3: As our framework is deterministic the resource owner starts with building-up of the capacity in the first regime and then switches once and for all to the second regime at some point in time. Therefore,

- regime 1: $V \geq 0$
- regime 2: $V < 0$.

To examine whether the *green paradox* still occurs when the capacity choice is endogenous we start by using the condition that the markets have to clear and thus supply has to equal demand. Thus substituting the inverse demand

[4]If marginal adjustment costs were not increasing, the peak in oil production would be reached immediately.

function $P = P(R(t))$ into the optimal extraction rule yields equation (8).

$$i + \pi - \hat{\Theta} = \frac{\Theta \cdot P_R(R(t)) \cdot V}{\Theta \cdot P(R(t)) - C(S) - \left[i + \pi - \hat{\Theta}\right] \cdot F_V(V)} \tag{8}$$

Equation (8) could be solved if a specific demand function was assumed such that we could determine the optimal building-up of capacity V depending only on the current capacity level $R(t)$ and exogenous factors. Thus let $V^* = V\left(R(t), \left[i + \pi - \hat{\Theta}\right], \Theta, C(S)\right)$ be the solution to equation (8). The question of interest is now: How does a change of $\hat{\Theta}$ affect V for any given level of $R(t)$? In other words, will a faster increasing cash-flow tax lead to more extraction in the short-run, which would be a confirmation of the *green paradox*, or will it slow-down down the pace of extraction? If the latter case is true, increasing tax rates would remain a valid instrument for reducing the speed of extraction and consequently also the process of global warming.

The answer to this question delivers the sign of the derivative of V^* with respect to $\hat{\Theta}$ which we derive by totally differentiating equation (8). Then we obtain:

$$\frac{\mathrm{d}V^*}{\mathrm{d}\hat{\Theta}} = \frac{\Omega(V, R(t))}{V \cdot \left[\left[i + \pi - \hat{\Theta}\right] \cdot F_{VV}(V) + \frac{\Theta \cdot P_R(R(t))}{i + \pi - \hat{\Theta}}\right]}, \tag{9}$$

and we can show that $\Omega(V, R(t))$ is strictly positive so that we do not need to worry about this term in the following.[5] We can rearrange our optimal extraction rule (equation (6)) such that:

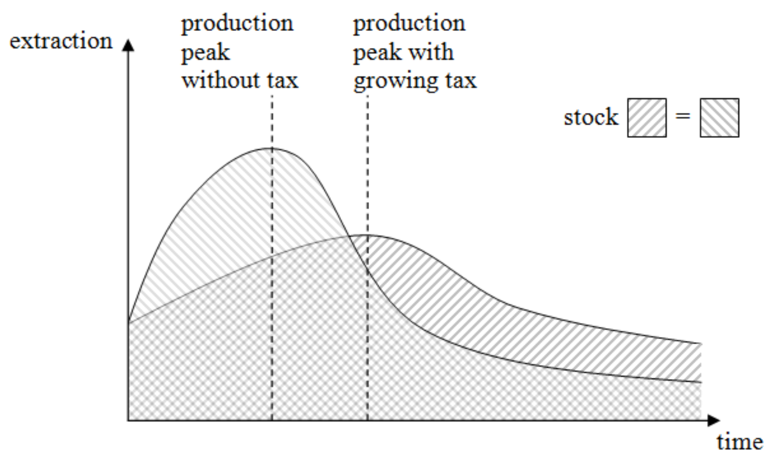
$$\Theta \cdot P(R(t)) - C(S) = V \cdot \left[\left[i + \pi - \hat{\Theta}\right] \cdot \frac{F_V(V)}{V} + \frac{\Theta \cdot P_R(R(t))}{i + \pi - \hat{\Theta}}\right]. \tag{10}$$

It shows that - given the assumptions about the cost function - that the denominator of equation (9) is positive if $F_{VV}(V) = \frac{F_V(V)}{V}$ which is the case if the cost function is quadratic. An increasing tax rate is present if $\hat{\Theta} < 0$ and therefore a further increase in the tax growth requires a decrease in $\hat{\Theta}$. Since for a quadratic cost function the derivative in equation (9) is positive for an increase$\hat{\Theta}$ it will be negative for a further increase of the tax growth. Thus for an increase in the tax growth, the capacity adjustment V will decrease. That is good news for the mitigation of climate change since in the first regime where the capacity increase will be slowed down (the positive value of V will be reduced for any given Value of R) and capacity reduction will be speeded up (the negative value of V will be further reduced for any given Value of R). The effect of an introduction of a growing tax rate on the extraction path given quadratic adjustment costs is illustrated in figure 4. Imagine that in both scenarios we start from some arbitrary small but positive initial value of R. In both scenarios

[5] $\Omega(V, R(t)) = V \cdot \left[\left[i + \pi - \hat{\Theta}\right] \cdot F_V(V) - \frac{\Theta \cdot P_R(R(t)) \cdot V}{i + \pi - \hat{\Theta}}\right] > 0$, as derived in the Appendix.

the total stock will be extracted eventually. Therefore the area under the shown capacity curve has to be the same in both scenarios. Since the capacity will increase slower and decrease faster in the tax scenario for any given level of R, the production peak has to be delayed in order to allow for the total extraction to be the same.

Figure 4: path comparision before and after the introduction of increasing taxes with quadratic adjustment costs



Source: own illustration

If the cost function is not quadratic we can derive sufficient conditions for the validity of the *green paradox* from the relation of $F_{VV}(V)$ vs. $\frac{F_V(V)}{V}$. First it is helpful to note that for $F_{VVV}(V) \geq 0$ the marginal cost function is convex and given the assumptions about the cost function it holds true that $F_{VV}(V) \geq \frac{F_V(V)}{V}$. Consequently for $F_{VVV}(V) \leq 0$ the marginal cost function is concave and thus $F_{VV}(V) \leq \frac{F_V(V)}{V}$. Therefore we can record that a convex marginal cost function is sufficient condition for:

$$\frac{\mathrm{d}V}{\mathrm{d}\hat{\Theta}} \geq 0 \qquad \text{in regime 1} \tag{11}$$

Meaning that for any given capacity R the increase in capacity will be lower after the increase of the tax rate growth $\hat{\Theta} < 0$. This is equivalent to a slower extraction in that regime and thus the green paradox is not valid. Likewise a concave marginal cost function is a sufficient condition for

$$\frac{\mathrm{d}V}{\mathrm{d}\hat{\Theta}} \geq 0 \qquad \text{in regime 2} \tag{12}$$

Meaning that for any given capacity R the decrease in capacity will be higher after the increase of the tax rate growth $\hat{\Theta} < 0$. This is also equivalent to a slower extraction in that regime and thus the *green paradox* is not valid. To derive the neccessary condition for the validity of the *green paradox* we can directly analyze what is the condition for the denominator to be positive. From rearranging the Hotelling rule

$$\Theta \cdot P(R(t)) - C(S) = \left[i + \pi - \hat{\Theta}^*\right] \cdot F_V(V) + \frac{\Theta \cdot P_R(R(t)) \cdot V}{i + \pi - \hat{\Theta}^*} \tag{13}$$

And defining

$$\Upsilon \equiv \left[i + \pi - \hat{\Theta}^*\right] \cdot [F_V(V) - F_{VV}(V) \cdot V] \tag{14}$$

then it is true for the for the denominator above that

$$\Theta \cdot P(R(t)) - C(S) - \Upsilon = \left[i + \pi - \hat{\Theta}^*\right] \cdot F_{VV}(V) \cdot V + \frac{\Theta \cdot P_R(R(t)) \cdot V}{i + \pi - \hat{\Theta}^*} \tag{15}$$

Then the *green paradox* is not valid (the denominator is positive) if:

$$\Theta \cdot P(R(t)) - C(S) > \Upsilon \tag{16}$$

which is equivalent to

$$\Theta \cdot P(R(t)) - C(S) > \left[i + \pi - \hat{\Theta}^*\right] \cdot [F_V(V) - F_{VV}(V) \cdot V] \tag{17}$$

or

$$\frac{\Theta \cdot P_R(R(t)) \cdot V}{i + \pi - \hat{\Theta}^*} > \left[i + \pi - \hat{\Theta}^*\right] \cdot [-F_{VV}(V) \cdot V] \tag{18}$$

which we can interpret for the two regimes seperatly:

- Regime 1 ($V > 0$)

$$\frac{\Theta}{i + \pi - \hat{\Theta}^*} \cdot \underbrace{|P_R(R(t))|}_{\text{slope of inverse demand curve}} < \left[i + \pi - \hat{\Theta}^*\right] \cdot \underbrace{F_{VV}(V)}_{\text{slope of marginal cost curve}} \tag{19}$$

- Regime 2 ($V < 0$)

$$\frac{\Theta}{i + \pi - \hat{\Theta}^*} \cdot |P_R(R(t))| > \left[i + \pi - \hat{\Theta}^*\right] \cdot F_{VV}(V) \tag{20}$$

4 Conclusions

A number of recent papers extend traditional Hotelling frameworks by the topical issue of climate change. In fact, they study the effects of environmental taxes on the resource extraction path of carbon resource and derive important and far-reaching policy implications. Of particular relevance is Sinn (2008) who introduces the *green paradox*as a possible outcome of today's environmental policy. He points out that the resource owner will come to the logical conclusion that shifting extraction quantities to the presence increases his expected total cash-flow if an over time increasing tax imposes a threat on profits of future extraction. Consequently such an environmental policy would even be counterproductive. We show, however, that this result may not prevail anymore if the capacity building decision is endogenous and costly instead of costless and therefore not considered as in Sinn (2008). By deriving necessary conditions for the increasing taxes to be an effective instrument we show that the evaluation of the*green paradox*changes between the pre-peak oil and the post-peak oil period.

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6 Appendix

$$\underset{V}{max} \int_0^\infty \Theta(0) \cdot [[\Theta \cdot P(t) - C(S)] \cdot R - F(V)] \cdot e^{-[i+\pi-\hat{\Theta}]\cdot t} \mathrm{d}t \tag{21}$$

$$s.t. \qquad \dot{S} = -R(t) \tag{22}$$

$$\dot{R}(t) = V \tag{23}$$

$$S(0) = S_0 \tag{24}$$

Assumptions about the adjustment cost function:

$$\begin{aligned} F(V) &> 0 \\ F'(V) = 0 &\qquad \text{for} \qquad V = 0 \\ F'(V) > 0 &\qquad \text{for} \qquad V > 0 \end{aligned}$$

$$\begin{aligned} F'(V) < 0 &\qquad \text{for} \qquad V < 0 \\ F''(V) > 0 & \\ F'''(V) \text{ is monotonic} & \end{aligned} \tag{25}$$

Adjustment costs:

$$H = \Theta(0) \cdot [[\Theta \cdot P(t) - C(S)] \cdot R(t) - F(V)] - \lambda_1(t) \cdot R(t) + \lambda_2(t) \cdot V \tag{26}$$

Partial derivate of equation (26) with respect to V:

$$\lambda_2(t) = \Theta(0) \cdot F_V(V) \tag{27}$$

Ddifferentiate equation (27) with respect to time:

$$\dot{\lambda}_2 = 0 \tag{28}$$

$$\dot{\lambda}_1 = \left[i + \pi - \hat{\Theta}\right] \cdot \lambda_1(t) - H_S \tag{29}$$

$$\dot{\lambda}_1 = \left[i + \pi - \hat{\Theta}\right] \cdot \lambda_1(t) + \Theta(0) \cdot C_S(S) \cdot R(t) \tag{30}$$

$$\dot{\lambda}_2 = \left[i + \pi - \hat{\Theta}\right] \cdot \lambda_2(t) - H_R \tag{31}$$

with

$$-H_R = -\Theta(0) \cdot [[\Theta \cdot P(t) - C(S)]] + \lambda_1(t) \tag{32}$$

$$\dot{\lambda_2} = \left[i + \pi - \hat{\Theta}\right] \cdot \lambda_2(t) - \Theta(0) \cdot [\Theta \cdot P(t) - C(S)] + \lambda_1(t) \tag{33}$$

Use equation (27) and equation (28) in equation (33):

$$0 = \left[i + \pi - \hat{\Theta}\right] \cdot \Theta(0) \cdot F_V(V) - \Theta(0) \cdot [\Theta \cdot P(t) - C(S)] + \lambda_1(t) \tag{34}$$

$$\lambda_1(t) = \Theta(0) \cdot [\Theta \cdot P(t) - C(S)] - \left[i + \pi - \hat{\Theta}\right] \cdot \Theta(0) \cdot F_V(V) \tag{35}$$

Differentiate equation (34) with respect to time:

$$\dot{\lambda_1} = \Theta(0) \cdot \left[\Theta \cdot \dot{P}(t) - C_S(S) \cdot \dot{S}\right] \tag{36}$$

$$\dot{\lambda_1} = \Theta(0) \cdot \left[\Theta \cdot \dot{P}(t) + C_S(S) \cdot R(t)\right] \tag{37}$$

Use equation (34) and equation (36) in equation (30):

$$\dot{\lambda_1} = \left[i + \pi - \hat{\Theta}\right] \cdot \lambda_1(t) + \Theta(0) \cdot C_S(S) \cdot R(t) \tag{38}$$

$$\begin{aligned} &\Theta(0) \cdot \left[\Theta \cdot \dot{P}(t) + C_S(S) \cdot R(t)\right] \\ &= \left[i + \pi - \hat{\Theta}\right] \cdot \left[\Theta(0) \cdot [\Theta \cdot P(t) - C(S)] - \left[i + \pi - \hat{\Theta}\right] \cdot \Theta(0) \cdot F_V(V)\right] + \Theta(0) \cdot C_S(S) \cdot R(t) \end{aligned} \tag{39}$$

simplify

$$\Theta \cdot \dot{P}(t) = \left[i + \pi - \hat{\Theta}\right] \cdot \left[\Theta \cdot P(t) - C(S) - \left[i + \pi - \hat{\Theta}\right] \cdot F_V(V)\right] \tag{40}$$

rearrange

$$i + \pi - \hat{\Theta} = \frac{\Theta \cdot \dot{P}(t)}{\Theta \cdot P(t) - C(S) - \left[i + \pi - \hat{\Theta}\right] \cdot F_V(V)} \tag{41}$$

Use the inverse demand function $P = P(R(t))$ in equation (41):

$$i + \pi - \hat{\Theta} = \frac{\Theta \cdot P_R(R(t)) \cdot \dot{R}}{\Theta \cdot P(R(t)) - C(S) - \left[i + \pi - \hat{\Theta}\right] \cdot F_V(V)} \tag{42}$$

Use equation (23) in equation (42):

$$i + \pi - \hat{\Theta} = \frac{\Theta \cdot P_R(R(t)) \cdot V}{\Theta \cdot P(R(t)) - C(S) - \left[i + \pi - \hat{\Theta}\right] \cdot F_V(V)} \tag{43}$$

Which can obviosly be solved to determine the optimal capacity buildup V depending only on the current capacity $R(t)$ and exogenous factors, thus $V^* = V\left(R(t), \left[i + \pi - \hat{\Theta}\right], \Theta, C(S)\right)$. The obvious question for the climate effect (faster extraction bad for climate) is now how a change in $\hat{\Theta}$ will change V for any given level of $R(t)$ (higher V faster extraction). The answer to that question delivers the sign of the derivative of $V*$ with respect to $\hat{\Theta}$ which we derive by totally differentiating equation (43):

$$\left[i + \pi - \hat{\Theta}\right] \cdot F_V(V) = \Theta \cdot P(R(t)) - C(S) - \frac{\Theta \cdot P_R(R(t)) \cdot V}{\left[i + \pi - \hat{\Theta}\right]} \tag{44}$$

$$\begin{aligned} &F_V(V) \cdot \mathrm{d}\hat{\Theta} - \left[i + \pi - \hat{\Theta}\right] \cdot F_{VV}(V) \cdot \mathrm{d}V \\ &= \frac{\Theta \cdot P_R(R(t)) \cdot V}{\left[i + \pi - \hat{\Theta}\right]^2} \cdot \mathrm{d}\hat{\Theta} + \frac{\Theta \cdot P_R(R(t))}{\left[i + \pi - \hat{\Theta}\right]} \cdot \mathrm{d}V \end{aligned} \tag{45}$$

$$\begin{aligned} &\left[\left[i + \pi - \hat{\Theta}\right] \cdot F_{VV}(V) + \frac{\Theta \cdot P_R(R(t))}{\left[i + \pi - \hat{\Theta}\right]}\right] \cdot \mathrm{d}V \\ &= \left[F_V(V) - \frac{\Theta \cdot P_R(R(t)) \cdot V}{\left[i + \pi - \hat{\Theta}\right]^2}\right] \cdot \mathrm{d}\hat{\Theta} \end{aligned} \tag{46}$$

$$\frac{\mathrm{d}V}{\mathrm{d}\hat{\Theta}} = \frac{V \cdot \left[F_V\left(V\right) - \frac{\Theta \cdot P_R(R(t)) \cdot V}{\left[i+\pi-\hat{\Theta}\right]^2}\right]}{V \cdot \left[\left[i+\pi-\hat{\Theta}\right] \cdot F_{VV}\left(V\right) + \frac{\Theta \cdot P_R(R(t))}{\left[i+\pi-\hat{\Theta}\right]}\right]} \tag{47}$$

$$\frac{\mathrm{d}V}{\mathrm{d}\hat{\Theta}} = \frac{\overbrace{V \cdot \left[\left[i+\pi-\hat{\Theta}\right] \cdot F_V\left(V\right) - \frac{\Theta \cdot P_R\left(R\left(t\right)\right) \cdot V}{i+\pi-\hat{\Theta}}\right]}^{\text{positive}}}{V \cdot \left[\left[i+\pi-\hat{\Theta}\right] \cdot F_{VV}\left(V\right) + \frac{\Theta \cdot P_R(R(t))}{i+\pi-\hat{\Theta}}\right]} \tag{48}$$

with

$$\Omega\left(V, R\left(t\right)\right) = V \cdot \left[\left[i+\pi-\hat{\Theta}\right] \cdot F_V\left(V\right) - \frac{\Theta \cdot P_R\left(R\left(t\right)\right) \cdot V}{i+\pi-\hat{\Theta}}\right] \tag{49}$$

$$\frac{\mathrm{d}V}{\mathrm{d}\hat{\Theta}} = \frac{\Omega\left(V, R\left(t\right)\right)}{V \cdot \left[\left[i+\pi-\hat{\Theta}\right] \cdot F_{VV}\left(V\right) + \frac{\Theta \cdot P_R(R(t))}{i+\pi-\hat{\Theta}}\right]} \tag{50}$$

Where $\Omega\left(V, R\left(t\right)\right) > 0$ and we can rearrange the Hotelling rule from equation (43) to:

$$\Theta \cdot P\left(R\left(t\right)\right) - C\left(S\right) = V \cdot \left[\left[i+\pi-\hat{\Theta}^*\right] \cdot \frac{F_V\left(V\right)}{V} + \frac{\Theta \cdot P_R\left(R\left(t\right)\right)}{i+\pi-\hat{\Theta}^*}\right] \tag{51}$$