MEMBER: Multi-Country Euro area Model with Boundedly Estimated Rationality*

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Abstract

Rational expectations has been the dominant way to model expectation, but the literature has quickly moved to a more realist assumption of boundedly rational learning where agents are assumed to use only a limited set of information to form their expectations and to not know the complete structure of the model. However, learning is often criticised for being arbitrary. In contrast, our approach assumes agents optimise their learning based on unknown driving stochastic processes but without uncertainty of the deep parameters. Hence, uncertainty concerning the correct parameterisation of the model is replaced by the uncertainty concerning the process driving the future developments. Using a new Multi-country Euro area Model with Boundedly Estimated Rationality (MEMBER) where agents are asssumed to form their expected with learning we show that there are strong differences in the adjustment path to the shocks to the economy compared to a expectations formed under stronger rationality assumptions. Furthermore, we find that an expansionary fiscal policy is more effective in periods of downturns than in a period of boom.

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1 Introduction

The dominant way to model expectations has been via model-consistent rational expectations (strong rationality). Whilst rational expectations (RE) can be taken as a theoretically well founded polar case, resorting to only them is not unproblematic. It is well known that rational expectations can give rise to a multiplicity of solutions, sometimes terminal or transversality conditions may be enough to produce a unique solution but these conditions are always somewhat arbitrary. Rational expectations has also been criticised as it assume too much information on the part of agents. Furthermore, it is well known that there have been major difficulties in using large models that incorporate RE for forecasting.

While rational expectation has been the dominant way to model expectation over the last forty years the literature on learning goes back almost as far as the rational expectations literature. Early work on learning includes Friedman(1975), Townsend (1978,1983), Frydman(1982), Bray(1983) and Bray and Kreps(1984). This work focused almost exclusively on the stability properties of very small models, usually only one market. These models investigated a situation often referred to as 'rational learning' as it is assumed that agents know the true structure of the model being investigated but simply have to learn the parameter values. Given the extreme nature of the rational learning assumption the literature quickly moved to a more realist assumption of boundedly rational learning where agents are assumed to use only a limited set of information to form their expectations and to not know the complete structure of the model. Some early example from this literature include DeCanio(1979), Radner(1982) and Bray and Savin(1986), these examples focused on a case where the learning rule was the full reduced form of the model. Later papers began to use a learning rule which contained only a subset of the full set of reduced form variables and to define the idea of E-stability, when the parameters of the learning process converge to a fixed point (Evans(1989), Evans and Honkapohja(1994, 1992, 1995, 1994)). Marcet and Sargent(1988, 1989) make the important link between E-stability and a conventional rational expectation equilibria (REE) when E-stability is attained the model has also reached a REE.

The learning literature (Evans 1986, Woodford(1990)) has pointed out that often a particular learning specification will produce a unique solution and of course given the association of an E-equilibria with a REE, this implies that a particular REE is being chosen without recourse to these arbitrary transversality conditions. This illustrates the important point that learning can bring positive advantages from an analytical standpoint. However it is important to point out that if the specific form of the learning process produces different solutions then the choice between these solutions (and implicitly the corresponding REE) is still being made on the basis of a largely arbitrary decision. This motivates much of the argument presented in this paper that the choice of the form of the learning rule itself can be crucially important.

In a policy setting it has become increasingly obvious that learning as opposed to rational expectations can impose a different set of constraints on how policy should be formulated. For recent surveys of this literature see Evans and Honkapohja(2008) or Bullard(2006). It is interesting that this literature is having a clear impact on the thinking of policy makers, e.g. Trichet(2005) or Bernanke(2007). It is now clear that the performance of some interest rate policy rules may be very different under rational expectations and learning. It is also possible that some monetary policy rules can give rise to multiple REE and that the economy may not settle on the most desirable one, see Honkapohja and Mitra(2004), Carltrom and Fuerst(2004) and Evans and McGough(2005). The key insight of this literature is that monetary policy can have, in effect, a new role; to facilitate the learning process itself. Policy should help agents to learn about the economic environment and to settle on the desirable equilibrium. Of course in some ways this would be an obvious point in most central banks, where great emphasis has always been given to maintaining credibility specifically so that market participants believe official announcements and in effect learn more quickly.

Most of the literature above has focused on very small models, often single markets and where macroeconomic models have been used they are generally fairly small comprising just 4 or 5 equations. For example in their recent survey Evans and Honkapohja(2008) use what they describe as 'the new Keynesian model that has become the workhorse in the analysis of monetary policy'. This model comprises 2 equations and an interest rate rule. Similarly, Slobodyan and Wouters(2007) have incorporated learning into a standard DSGE model. There is however a branch of literature which has focused on introducing learning into large scale empirical models. This literature begins with work at the London Business School which had a large empirical model comprising of around 300 equations and learning was introduced there into a number of sectors, see Hall and Garratt(1997). Later applications to large models include Hall and Symanski(1999) which applied learning to one country block of the IMF's large world model MULTIMOD and Barrell, Caporale, Garratt and Hall(1992) which applied learning to the national institutes large world model NIGEM (comprising of around 2000 equations). These applications have established the feasibility of applying learning even in a large model context. A common feature in all these applications, however, is that that learning expectations are applied ad-hoc only in a few key equations.

Orphanides and Williams (2002), show that a modelling approach based on learning accommodates the Lucas critique in the sense that expectations formation is endogenous and adjusts to changes in policy or structure; and, although expectations are 'imperfectly' rational, in that agents are required to estimate the reduced form processes needed to form expectations, the resulting expectations are close to being efficient.

In this paper we consider alternative information assumptions, and present our approach of learning based on unknown driving stcohastic processes but without uncertainty of the deep parameters. We apply this approach to a Muli-Country Euro area Model with Boundedly Estimated Rationality (MEMBER). The model can be characterised as a optimising agent - new keynesian model, but in contrast to standard DSGE models, we assume limited-information. The key framework of the model is outlined in the appendix. We illustrate the implications by studing the impact of a fiscal policy expansion under an assumption of announced and credible change in policy under perfect foresight compared to unannounced and uncredible change in policy or one where agents learn about the policy change. Finally, we consider how policy expansion changes depending on the state of the economy.

2 Boundedly Rational Expectations

A typical forward looking equation contains at least one lead and, possibly, lags of the dependent variable, a set of driving variables determining the long run-development of the dependent variable and a stochastic error term. For instance, by using the specification corresponding to the Hybrid New Keynesian Phillips Curve as an example, we can write the following equation:

$$\Delta y_t = \gamma^f \Delta E_t y_{t+1} + \gamma^b \Delta y_{t-1} + \zeta \left(y_t^* - y_t\right) \tag{1}$$

where $E_t y_{t+1}$ is at time t, expected y at time t+1; y_t^* is the long-run determinant, μ_t is the i.i.d. error term and $\gamma^f, \gamma^b, \zeta$ are estimated parameters.
Under *model consistent rational expectations* (strong rationality) the expected value is simply replaced by the future realisations and the model solved iteratively, such that the expectations are fully model consistent. Hence, in equation (1)

$$E_t y_{t+1} = y_{t+1} \tag{2}$$

Underlying information assumptions are very strong. Agents know the structure and all the parameters of the model and they are able to do unbiased forecasts on the future development of the economy.
Under *boundedly rational learning expectations* (weak rationality), agents do not know the complete structure of the model and in forming their expectations they use only a limited set of information. Instead, in forming their expectations they revise their forecast rules over time in response to new data. Hence, the assumption of rational expectations is replaced by explicit models of forecasting and model updating. Among different approaches to modelling learning, the most common is 'adaptive learning'. It views economic agents as econometricians who repeatedly estimate parameters of their model and make forecasts using their updated estimates.
The advantage of the learning expectations is that they are consistent with the core model, but they do not retain the strong assumption of rationality that there are no errors in expectation formation. Instead, whilst there are errors there are no long lasting systematic errors and in the medium run it should converge to the strong rationality solution. Indeed, the core of the model is estimated with GMM that implicitly assumes boundedly rational expectations. It is for these reasons that it is our preferred approach as the main specification for the model.
Another key advantage of bounded rationality approach to expectations over strong rationality is that it can easily be used to produce forecasts with judgement. With strong rationality, the model would typically need to be solved iteratively over a long

horizon. However as the learning rule is essentially based on past information, the forecast can be updated in the same way as with a backward model.

2.1 Information assumptions of learning

In the adaptive learning literature a standard assumption is that agents do not know the exact parameter values of equation (1) (or the underlying deep parameters), but they know the stochastic processes of exogenous variables driving the fundament y_t^* as well as the functional form of the implied REE solution to (1), i.e. the minimal state variable solution (MSV), McCallum (1983).

Case a: Correctly specified Perceived Law of Motion (PLM)

Under boundedly rational learning, where agents use only a limited set of information to form their expectations, a reasonable assumption is that the agents treat y_t^* as if it were generated by a stochastic ARMA process. Now the agents know that the MSV solutions of (1) are of the form:

$$y_t = \delta_0 + \delta_1 y_{t-1} + \delta_2 y_{t-2} + \delta_3 y_t^* + \phi(L)\, v_{t-1} + \mu_t \tag{3}$$

where $\phi(L)$ denotes polynomial lag operator and v_{t-1} the lagged shocks in the ARMA of y_t^*. However, as the agents do not know the parameterisation of equation (3), not, at least, fully – their Perceived Law of Motion (PLM) is of the form:

$$y_t = \delta_{0t} + \delta_{1t} y_{t-1} + \delta_{2t} y_{t-2} + \delta_{3t} y_t^* + \varepsilon_t \tag{4}$$

where parameters δ_{it} are updated by data available up to the point in time t. This implies the following forecast equation:[1]

$$E_t y_{t+1} = \delta_{0t} + \delta_{1t} y_t + \delta_{2t} y_{t-1} + \delta_{3t} y_t^* \tag{5}$$

In a single equation framework, y_t^* can be considered to be exogenously given to individual optimising agents, and hence can be estimated. However, in a full-information multi-equation world, equation (5) is replaced by a system where y_t^* is reduced to truely pre-determined variables of the whole model. For small models, the form can be known, whereas for large non-linear models the system is too complex to determine, so some form of approximation is needed.

Case b: Misspecified PLM:

A topic in the recent learning literature has been that instead of using the correctly specified PLM equation (4) for expectations formation agents use a misspecified model as e.g.:

$$E_t y_{t+1} = \delta_{0t} + \delta_{1t} y_t + \delta_{2t} y_{t-1} \tag{6}$$

[1] In the context of the full model, this equation may cause a simultaneity problem. A way to circumvent the problem is to assume that at period t y_t is not observable while y_t^* (or its determinants) are observable (see Evans and Honkapohja, 2001 Chapter 8.6). Hence, an alternative would be: $E_t y_{t+1} = \delta_{0t} + \delta_{1t} y_{t-1} + \delta_{2t} y_{t-2} + \delta_{3t} y_t^*$

or

$$E_t y_{t+1} = \delta_{0t} + \delta_{3t} y_t^* \tag{7}$$

We see that in equations (6) and (7) information bases are narrower than in (8). In equation (6) the development of the fundament variable plays no role in expectation formation and in (7) the fundament variable captures the whole information base while the information contained by the lags of expectation variable is neglected. Evans and Honkapohja (2001) showed that the learning solution may still converge, but to a restrictive perception equilibrium, rather than to the RE equilibrium. To learn better to understand the importance of the information assumption we could compare our learning results based on forecasting equation (8) implied by correctly specified PLM to those based on forecasting equations (6) and (7) implied by misspecified PLMs.

2.2 Learning without uncertainty on deep parameters

Under the correctly specified perceived law of motion, equation (4), the dynamic properties of the model depend crucially on the estimates δ_{it} that, in practice, may introduce non-voluntary arbitrariness into the dynamics of the estimated model. This arbitrariness can be reduced by retaining the assumption used in deriving equation (1), that optimizing agents know the relevant deep parameters and, hence, they also know parameters δ_1, δ_2 and δ_3.[2] They do not, however, know the intercept parameter δ_0, which is determined by the expected future growth rate of the fundament variable, and the parameterisation of $\phi(L)$. Hence, uncertainty concering the correct parameterization of the model is replaced by the uncertainty concerning proceses driving the future development. These assumptions allow us to define the predetermined part $F_t = \delta_1 y_{t-1} + \delta_2 y_{t-2} + \delta_3 y_t^*$ with δ_1, δ_2 and δ_3 solved in terms of the parameter estimates $\gamma^f, \gamma^b, \zeta$ of (1) and, as an alternative to (5), to rewrite the forecast equation as follows:

$$E_t y_{t+1} = \alpha_{0t} + \alpha_{1t} F_t \tag{8}$$

The main difference between equation (8) and (5) is that in (8) the relative weights of information related to the autoregressive part and to the fundament are the same as in the MSV solution (3), while in (5) this is not the case. Evans and Honkapohja (2001) showed that specification (5) fulfils the expectational stability (E-stability) conditions. Therefore it is self-evident that also (8) fulfils them.

As an illustration of the derivation, we consider a hybrid NKPC equation, which can be written as follows:

$$\Delta p_t = \gamma^f E_t \Delta p_{t+1} + \gamma^b \Delta p_{t-1} + \eta\left(p_t^* - p_t\right) \tag{9}$$

[2]Conventional ways to solve these parameters is the unobserved coefficient method or to express them in terms of the roots of the homogenous part of equation (1) as presented e.g. Sargent (1979, CH IX).

The derivation equation (9) is based on the assumption that optimising firms know the deep parameters and implied composed parameters γ^f, γ^b, η. However, it is quite likely that firms do not know the exact stochastic process driving the development of the fundament variable p_t^* . This just the opposite with the standard assumption in the learning literature, i.e. agents do not know exact parameterization of equation (9) but they know the stochastic process driving the fundament. Hence, in the following we study how the learning process should be specified, when theses information assumptions are reversed

We start by writing in equation (9) as a difference equation for levels:

$$E_t p_{t+1} - \Gamma_0 p_t + \Gamma_1 p_{t-1} - \Gamma_2 p_{t-2} = -\frac{\eta}{\gamma^f} p_t^* \tag{10}$$

where $\Gamma_0 = \left(1 + \gamma^f + \eta\right)/\gamma^f$; $\Gamma_1 = \left(1 + \gamma^b\right)/\gamma^f$; $\Gamma_2 = \gamma^b/\gamma^f$

The homogenous part of difference equation (10) implies the following identities for its three roots λ_i (*i=1,2,3*)

$$\Gamma_0 = \lambda_1 + \lambda_2 + \lambda_3; \Gamma_1 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3; \Gamma_2 = \lambda_1\lambda_2\lambda_3$$

It can be shown that the real parts of two roots (e.g. λ_1 and λ_2) are inside the unit circle and one outside (λ_3)

As discussed by Sargent (1979), equation (10) can be equivalently presented in terms of the roots:

$$\left(1 - \lambda_1 L\right)\left(1 - \lambda_2\right) E_t p_{t+1} = -\frac{\eta}{\left(1 - \lambda_3 L\right)\gamma^f} p_t^* \tag{11}$$

where L refers to the lag operator. Equation (11) can be rewritten in the form:

$$\begin{aligned} p_t &= \left(\lambda_1 + \lambda_2\right) p_{t-1} - \lambda_1\lambda_2 p_{t-2} + \tfrac{1}{\lambda_3} \textstyle\sum_{i=0}^{\infty} \left(\tfrac{1}{\lambda_3}\right)^i \tfrac{\eta}{\gamma^f} E_t \underbrace{p_{t+1+i}^*}_{p_t^* + \sum_{j=0}^{i} \Delta p_{t+1+j}^*} \\ &= \underbrace{\left(\lambda_1 + \lambda_2\right) p_{t-1} - \lambda_1\lambda_2 p_{t-2} + \frac{1}{\left(\lambda_3 - 1\right)} \frac{\eta}{\lambda^f} p_t^*}_{x\left(p_{t-1}, p_{t-2}, p_t^*\right)} \\ &\quad \underbrace{+ \frac{1}{\lambda_3 - 1} \sum_{i=0}^{\infty} \left(\frac{1}{\lambda_3}\right)^i \frac{\eta}{\gamma^f} E_t \Delta p_{t+1+i}^*}_{E_t \Delta PV_{t+1}} \end{aligned} \tag{12}$$

In equation (12) the evolvement of pt is determined by two components, i.e. the predetermined component $x\left(p_{t-1}, p_{t-2}, p_t^*\right)$ and the expectations component $E_t \Delta PV_{t+1}$. If optimizing firms know equation (9), then it is reasonable to assume that they fully know also $x\left(p_{t-1}, p_{t-2}, p_t^*\right)$, whilst the exact form and parameters of the stochastic process driving the fundament and, hence, the expectation component $E_t \Delta PV_{t+1}$ may be unknown. Hence, all expectational and model uncertainty is related to the

$E_t \Delta PV_{t+1}$ component. Now, if variable p_t^* follows an ARIMA(p,1,q), then its difference has an ARMA presentation. Now equation (12) implies the following MSV solution of equation (9) :

$$p_t = x\left(p_{t-1}, p_{t-2}, p_t^*\right) + \varphi\left(L\right)\nu_t \tag{13}$$

where the exact form and parameterisation of $\varphi\left(L\right)\nu_t$ are not known. Anyway equation (13) suggests the following state-space presentation for learning.

$$p_t = \alpha_{0t} + \alpha_{1t}\left[x\left(p_{t-1}, p_{t-2}, p_t^*\right)\right] + u_t \tag{14}$$

$$\alpha_{it} = \alpha_{it-1} + v_{it} \tag{15}$$

with u_t ~$iid\left(0, \sigma_u^2\right)$ and v_{it} ~$iid\left(0, \sigma_i^2\right)$ and $\alpha_{it} \geq 0$. Parameter α_{0t} accounts for the drift (and changes in the drift parameter reflecting the inflation regime and its shifts) and the deviations of parameter α_{1t} from unity are related to the learning process of the parameters of $\varphi\left(L\right)\nu_t$. Equation (13) however, suggests that $\lim_{t->\infty}\alpha_{1t} = 1$. This constraint can be imposed by respecifying equation (13) regarding α_{1t} as follows:

$$\alpha_{1t} = c + (1-c)\cdot\alpha_{1t-1} + v_{1t} \tag{16}$$

where c, is speed of convergence parameter. Now applying the implied forecasting equation in (9) $E_t p_{t+1} = \alpha_{0t} + \alpha_{1t}\left[x\left(p_{t-1}, p_{t-2}, p_t^*\right)\right]$ it is straightforward to see that the learning equilibrium converges towards the RE equilibrium implied by (12) or the MSV solution of (13).

2.3 Kalman filter estimation

We estimate our learning without uncertainty on deep parameters with the following equation:

$$E_t y_{t+1} = \alpha_{0t} + \alpha_{1t}F_t + \varepsilon_t \tag{17}$$

and ε_t ~N(0,H).

The time-varying parameters can be updated in various ways, but for our purposes we consider an autoregressive approach for the law of motion:

$$\alpha_t = \alpha_{t-1} + \eta_t \tag{18}$$

where $\alpha_t = \left(\alpha_{0t}\ \alpha_{1t}\right)'$ and η_t is the error vector used to update the parameter vector, η_t ~N(0,Q).

However, under the assumption there is no uncertainty on deep parameters but on unknown driving process (See section 2.2), then we constrain the coefficient on the predetermined part F_t to converge to 1, that is:

$$\alpha_{1t} = c + (1-c) * \alpha_{1t-1} + \eta_t \tag{19}$$

where c, is speed of convergence parameter.

The system of equations (1), (17), (18), and (19) describes the expectations framework where equations (17), and (18) / (19) are respectively the measurement and transition

equations of a system. Equation (17) describes the expectations formations mechanism where the time varying parameters α_t evolve according to equations (18) for the constant, and (19) for the information parameter α_{1t}. The matrix H is the variance of the disturbance in the measurement equation and Q is the variance of the parameters. The whole system can be estimated by a Kalman filter recursion.
Following Harvey 1992 and Rockinger and Urga 2000, the variance-covariance matrix associated to $\hat{\alpha}_t$ is: $P_t = E_t[(\alpha_t - \hat{\alpha}_t)(\alpha_t - \hat{\alpha}_t)']$. The best estimates of P_t conditional on information at t-1 is

$$P_{t|t-1} = T_t P_{t-1} T_t^{'} + R_t Q_t R_t^{'} \tag{20}$$

where T and R are matrices linked to c, the speed of convergence parameter. The Kalman updating equations then become:

$$\hat{\alpha}_t = \hat{\alpha}_{t|t-1} + \frac{P_{t|t-1} F_t' (y_t - F_t' \hat{\alpha}_{t|t-1})}{F_t P_{t|t-1} F_t' + H_t} \tag{21}$$

$$P_t = (I_m - \frac{P_{t|t-1} F_t' F_t}{F_t P_{t|t-1} F_t' + H_t}) P_{t|t-1} \tag{22}$$

2.4 Optimising parameter estimation

However, there is a lot of arbitrariness in setting up the Kalman filter estimates, as it depends on the assumptions for initial starting values (priors) for $\hat{\alpha}$ and P_0, and assumptions for Q_t and H_t - i.e. the degree of variability of the parameters, and the degree to which agents discount past observations. Overall, the dynamics of the model are sensitive to these assumptions and this adds a sense of arbitrariness, that isn't present in the rational expectation solution.
One algorithm used in the literature, is recursive least squares estimation. This is a subset of Kalman filter estimates, where $\hat{\alpha}$=0; Q_t =0; H_t =1, and P_0, is set to a diagonal infinity matrix and the normalisation of $H_t = 1$, implies Q_t becomes the signal to noise ratio. In this case agents have infinite memory, with each observation being given equal weight and uninformative prior on the parameters. Another algorthim often used in the literature is constant gain, where agents give more weight to more recent observations. However, typically the discount rate is chosen arbitrarily.
Instead we follow Evans and Branch (2005), in calculating the mean square forecast errors:

$$MSE(y_i) = \frac{1}{T} \sum_{t=0}^{T} (y_{i,t} - \hat{y}_{i,t})^2 \tag{23}$$

where $\hat{y}_{i,t}$is the forecast of the ith component based on t-1 information.
We then compute the Q matrix that minimises the in-sample MSE by doing a grid search over a range from 0.01 to 1.9. As one of the aims of the model is to be used to for forecasting, this method should provide the optimal forecasts given the model.

The higher this parameter, the more variable is α_t and thereby provides a reflection of the extent of structural change in that variable. Another way to think of the Q matrix is as a measure of the rate at which the past is discounted, if the elements of Q are all zero then the past is not discounted at all. As Q rises we effectively discount past observations more rapidly. The value of Q which matters is its relative value compared with H, which is normalized to be 1. As both are normal distributions a relative value of Q twice as large as H will provide very rapid discounting and hence our search of Q over the range 0.01 to 1.9. Furthermore, given our assumption that there is no uncertainty on deep parameters but unknown driving forces, we use starting values initialised as though agents knew the parameters i.e. α_0 =0 and α_1=1,

3 The Multi-Country Model Overview

The model presented here is a new estimated large-scale multi-country model (NMCM) developed at the European Central Bank which is being used to produce the country projections and to aid policy analysis. The model currently covers the 5 biggest euro area countries. The model has firm micro-economic foundations with the theoretic core of the model containing one exportable domestic good and one imported good. All central behavioural relations are based on the optimisation behaviour of three private sector decision making units (i.e. households, labour unions and firms) and the reaction functions of the government sector and the central bank. Expectation formation is treated explicitly and the model can be characterised as a limited-information - optimising agent - New Keynesian model.

As the available data does not disaggregate government into a separate institutional sector, the theoretical core of the model assumes a single domestic good produced by aggregated production function with total employment and total capital stock as inputs. Hence, the optimisation framework derives "true" behavioural relations for total employment, investment, private consumption and corresponding deflators and factor prices. For forecasting purpose, however, the accounting framework of the model is markedly more disaggregated, but with no feed-back effects on the longer-run adjustment dynamics of the model.

The real world data which we have to confront stands in stark contrast to the predictions of many simple macroeconomic models. In particular we would point out that a simple model with a balanced growth path (BGP), as adopted e.g. by DSGE and all other models related to the real business cycle (RBC) paradigm, would predict that the GDP-shares of capital, labour and total factor income as well as the capital-output ratio are stationary. In the real data for our five countries this is clearly not the case. Therefore, as discussed by McAdam and Willman (2008), Solow (2000) and Blanchard (1997) we adopt a medium-run view regarding the underlying "trend" developments of our data in the sample period. Accordingly, the medium run developments, towards which the short-run dynamics converges, are allowed to deviate from the BGP. However, this view does not exclude the possibility that many processes, which from the medium run perspective may be advisable to treat as exogenous, are from the very long-run perspective endogenous and drive the medium run development eventually to converge to the BGP. Acemoglu (2002, 2003) gives an excellent example by showing that while technical progress is necessarily labour-augmented along the BGP, it may become capital-augmented in periods of transition reflecting the interplay of innovation activities, factor intensities and profitability. Given a below-unitary substitution elasticity this pattern promotes the asymptotic stability of income shares while precisely allowing them to fluctuate in the medium run. Accordingly, we allow non-unitary elasticity of substitution, non-constant augmenting technical progress and heterogeneous sectors with differentiated price and income elasticities of demand across sectors. We achieve this by following McAdam and Willman (2007 and 2008).

In addition to the relaxations concerning the medium-run development, the optimisation frameworks of agents contain a lot of frictional elements which are needed for explaining realistically the observed stylised short-run features. Labour is indivisible with important implications for be behaviour for all optimising agents. Regarding households' utility maximisation problem the indivisibility assumption simplifies the analysis, because the labour supply adjusts to the demand for labour conditional on the wage contract set by unions maximising either the utility of member households or targeting the warranted wage rate consistent with a desired employment rate. The basic framework in household's utility maximisation is Blanchard's (1985) overlapping generation framework that, however, is supplemented in many ways. Firstly to incorporate income uncertainty in a tractable way into the utility maximization framework we assume a two-stage approach in utility maximisation, Willman (2007). In the first stage, the consumer evaluates her risk-adjusted non-human and human wealth conditional on uncertain lifespan and labour income. Thereafter, in the second stage, conditional on the risk adjusted life-time resource constraint, the consumer is assumed to determine her optimal planned path of consumption.

In the profit maximising framework of the firm the assumption of indivisible labour, adjustment costs with respect to number of workers and convex costs with respect to work intensity introduce the discrepancy between paid hours and efficient hours. This explains the observed pro-cyclicality in labour productivity, when labour input is measured in heads or paid hours. It also introduces the ratio of efficient hours (per worker) to normal hours into optimal price setting on the top of the conventionally defined marginal cost of labour. The price setting of firms and the wage setting of unions are staggered with three-valued Calvo-signal, McAdam and Willman (2007). Part of firms (unions) keep prices (wages) fixed, another part changes prices (wages) following backward-looking rule and the rest set them optimally. To capture the observed inertia in capital formation, capital stock and its rate of change are coupled with adjustment costs. Regarding the stock formation firms minimise quadratic losses induced by the deviations of inventories, on the one hand, and production, on the other hand, from their respective target levels related to the level of production implied by the production function, when existing inputs are utilised at their normal (cost minimising) rates.

All euro area countries are open economies and, therefore, also in our theoretic single domestic good framework a part of output is exported. However, firms face separate demand functions in domestic and export market leading to the pricing to market behaviour. This effectively separates the optimal price setting of exports from the rest of the firm's optimisation problem. We assume that the volume of exports is determined by the almost ideal demand system (AIDS) function. The advantage of this functional form compared to the conventional iso-elastic form is that now, compatibly with empirical evidence, the foreign competitors' price affect optimal export-price setting. The export demand and the optimal price setting form a two-equation system with cross-equation parameter constraints. This allows a model consistent way to estimate the price elasticity of export demand. Import side of the model is more conventional being determined by domestic demand and the relative price of imports to domestic good.

The steady state form of the first-order conditions of profit maximising firms and the utility of member households maximising unions imply the 5-equation medium-run supply system that allows a consistent two-step estimation of the underlying deep parameters of the model. As the supply system contains cross-equation parameter constrains it is estimated with the method of non-linear SUR that León-Ledesma et al. (2009) have proven to be a very efficient estimation approach outperforming all single-equation methods. This system defines all parameters related to technology, production function and the mark-up allowing to define optimal frictionless prices, wages, labour demandand marginal cost and product concepts needed in estimating in the second stage the dynamic first order optimisation conditions of firms and unions. All dynamic equations containing the expectations of variables are estimated by the generalised method of moment (GMM).

In order to close the model the following additional relationships are required: a monetary policy rule, a fiscal policy rule and an exchange rate UIP rule. The model may be operated either with or without these rules, typically in a forecast the rules would be turned of while in simulation they would be used. The fiscal policy rule is based on a reaction of taxes to the Government's debt to GDP ratio. The fiscal rule determines in the first place the path of the personal income tax rate. Furthermore transfers and other taxes follow this rate and other taxes follow this rate.Transfers to households are modelled as a function of the unemployment rate. The monetary policy rule follows a simple Taylor rule specification in which the short term interest rate is determined by the inflation gap (where the inflation gap measures the distance between the actual inflation rate and its target level) and the output gap along with the lagged interest effect. The exchange rate follows a standard forward-looking UIP equation.

4 Estimation of learning equations

We now proceed to present the estimation results for the six main stochastic equations where expectations are applied are: consumption, wages, GDP deflated inflation, and investment, employment and inventories. Expectations are also in the policy rule and in determination of exchange rates, however in the current version they are treated as outlined above. We have estimated the equation (8) above:

$$E_t y_{t+1} = \alpha_{0t} + \alpha_{1t} F_t \tag{24}$$

$$F_t = \delta_1 y_{t-1} + \delta_2 y_{t-2} + \delta_3 y_t^* \tag{25}$$

The tables below present the key parameters for the estimated equations. For the time-varying parameters, α_{0t} and α_{1t} the table shows the end point of the parameter estimate (i.e. 2007Q2)[3]. We report δ^1, δ^2 and δ^3, which are based on the parameter estimates of the main model. The speed of convergence parameter in (19) was selected as 0.03. We also report the hyper-parameter for the Q matrix, obtained by minimizing the in-sample MSE. The higher this parameter, the higher the variability in α_t and provides a reflection of structural change in those estimations. Finally, we report the in-sample R^2, which gives an indication of fit of the equations.

4.1 Employment expectations

Labour demand has a one lagged and a long-run (desired) number of workers, which is derived from the inverted production function.

$$\Delta n_{t+1}^e = \alpha_{0t} + \alpha_{1t} \underbrace{\Delta(\delta^1 n_{t-1} + \delta^2 n_t^*)}_{\text{information}} + \varepsilon_{nt} \tag{26}$$

where $n_t = \log$ (number employed), and $n_t^* = \log(N_t^*)$ (inverted production function – see equation 33).

Learning Employment Estimation

	DE	FR	IT	ES	NL
α_0	0.0000	0.0003	-0.0002	0.0010	0.0011
α_1	0.8999	0.9518	0.8645	0.8908	0.8499
δ^1	0.6803	0.7475	0.6292	0.4767	0.7475
δ^2	0.3197	0.2525	0.3708	0.5233	0.2525
Q	0.0000	0.0010	0.0000	0.0200	0.0010
$\mathbf{R^2}$	0.9543	0.9990	0.9853	0.9982	0.9976

4.2 Investment expectations

Capital accumulation reflects time to build considerations see section 8.3.2 .

$$\Delta k_{t+1}^e = \alpha_{0t} + \alpha_{1t} \underbrace{\left(\delta^1 \Delta k_t + \delta^2 \Delta k_t^*\right)}_{information} + \varepsilon_{ksrt} \tag{27}$$

[3] Although clearly they continue to evolve as new information comes into place

where kt is log of capital stock, $\Delta k_t^* = MPK_t - (1-\eta)\, UC_t + (1-d)\,(1-\delta)\,[\phi - (1-\eta)]$, MPK is marginal product of capital and UC is the real user cost of capital.

Learning Investment Estimation

	DE	FR	IT	ES	NL
α_0	-0.0007	0.0022	-0.0001	0.0034	0.0002
α_1	0.9991	1.0035	0.9903	0.8315	0.9133
δ^1	0.4830	0.6207	0.6078	0.6138	0.3928
δ^2	1.6950	1.7879	1.7796	1.7834	1.6267
Q	1.9000	1.9000	1.9000	1.9000	0.7000
$\mathbf{R^2}$	0.9545	0.9756	0.8120	0.7482	0.7674

4.3 Consumption expectations

Household consumption follows an optimized framework with overlapping generations – see section (8.4). The expectation equation consistent with this equation is:

$$C_{t+1}^e = \alpha_{0t} + \alpha_{1t} \log \underbrace{\left(\delta^1 C_{t-1} + \delta^2 C_t^*\right)}_{\text{information}} + \varepsilon_{ct} \tag{28}$$

where $C_t^* = (1/(rc-1))(\gamma^y Y_t + \gamma^v V_t)$, Y_t is real labour income, and V_t is private sector total real wealth.

Learning Consumption Estimation

	DE	FR	IT	ES	NL
α_0	0.7006	0.0040	0.0033	0.0031	0.0598
α_1	0.9986	1.0000	0.9993	1.0003	0.9974
δ^1	0.2105	0.8464	0.8718	0.7286	0.8071
δ^2	3.8192	17.1154	7.6845	10.3162	8.8344
Q	0.0700	0.0150	0.0050	1.9000	0.0050
$\mathbf{R^2}$	0.8156	0.9975	0.9784	0.9983	0.9945

4.4 Price expectations

Price equations follow the three-valued Calvo-signal, with expectations so that the expectation equation becomes:

$$\Delta p_{t+1}^e = \alpha_{0t} + \alpha_{1t} \underbrace{\Delta(\delta^1 p_t + \delta^2 p_{t-1} + \delta^3 p_t^*)}_{\text{information}} + \varepsilon_{pt} \tag{29}$$

where p_t = gdp factor cost prices (log); $p_t^* = w_t - mpn_t + a_h\,(n_t^* - n_t) + \mu_t$ = frictionless equilibrium price level (log); w_t = compensation per worker (log); mpn_t= marginal product of labour (log); n_t^* = optimal number of workers (log) and n_t = actual employment (log);

Learning Price Estimation

	DE	FR	IT	ES	NL
α_0	0.0008	0.0013	-0.0014	0.0013	0.0083
α_1	0.7270	0.8690	0.7130	0.7895	0.5586
δ^1	1.2267	1.0592	0.9704	0.7631	0.8421
δ^2	0.8290	0.7062	0.6470	0.5087	0.5614
δ^3	0.1030	0.1901	0.2388	0.3663	0.3155
Q	1.2000	0.2000	0.8000	0.1000	0.0030
R^2	0.9742	0.9883	0.9887	0.9936	0.9947

4.5 Wage expectations

As with prices, wages follow the three-valued Calvo-signal, with expectations based on frictionless equilibrium price level:

$$\Delta w_{t+1}^e = \alpha_{0t} + \alpha_{1t} \underbrace{\Delta(\delta^1 w_t + \delta^2 w_{t-1} + \delta^3 w_t^*)}_{\text{information}} + \varepsilon_{wt} \tag{30}$$

where $w_t^* = (p_t^c + c_t) - n_t^f + \log\left(\sigma - 1 + \frac{MPN_t}{Y_t/N_t}\right) - h(time) + b_h\left(n_t^* - n_t^f\right)$,$w_t$=log of compensation per worker; MPN_t= marginal product of labour; n_t^* = optimal (desired) number of workers (log); n_t^F = labour force (log).

Learning Wage Estimation

	DE	FR	IT	ES	NL
α_0	0.0008	0.0004	0.0049	0.0022	-0.0013
α_1	0.9570	0.8809	0.7206	0.9321	0.5914
δ^1	0.4280	1.0190	0.9917	1.0667	0.8998
δ^2	0.0407	0.2307	0.2185	0.2529	0.1799
δ^3	0.6127	0.2118	0.2268	0.1861	0.2801
Q	0.0010	1.9000	0.0001	0.0010	0.0300
R^2	0.9875	0.9982	0.9867	0.9976	0.9975

4.6 Inventories expectations

Labour demand has a one lagged and a long-run (desired) number of workers, which is derived from the inverted production function.

$$\Delta KII^{e}_{t+1} = \alpha_{0t} + \alpha_{1t}\underbrace{\Delta(\delta^{1}KII_{t-1} + \delta^{2}KII^{*}_{t})}_{\text{information}} + \varepsilon_{nt} \tag{31}$$

whereKII_t = inventory stock.

Learning Inventories Estimation

	DE	FR	IT	ES	NL
α_0	162.9	499.7	442.9	624.5	359.0
α_1	1.0646	0.8544	1.1062	1.0864	0.9572
δ^1	0.1516	0.4986	0.1678	0.1153	0.2139
δ^2	0.8714	3.9621	0.9615	0.6781	1.2353
Q	0.9875	0.9875	0.9875	0.9875	0.9875
R^2	0.9545	0.9756	0.8120	0.7482	0.7674

4.7 Learning Estimation observations

Overall, the α_{1t} coefficient at 2007Q2 was close to 1 supporting the choice of using the present value learning expectation rule. The main exceptions were price equations where the parameters were below 0.9 (wages for the Netherlands also is far from the prior.
There is some variability in the Q hyper-parameter across countries and equations, with Investment having a high Q hyper-parameter. In general, the estimates suggest a forgetting half-life of between 5 and 70 quarters.
Changes in the coefficients appears to be small over time (e.g. since 2000Q1 the α_{1t} parameter for the Italian consumption equation has a minimum of 0.9995 and a maximum of 1.0003), However, the variation in this parameter along with the variation in the constant, α_{ot} parameter has a significant impact on the simulation results. In general, the estimation by Kalman filter provided a very good fit of data. This can be seen as being attributable to two factors, firstly the movement in parameters provides an additional degree of freedom, which captures variation, and secondly, the equations are optimised by minimising the squared error. The worse fits tend to be for the the investment and inventory equations, with some R^2 of less than 0.8.

5 Scenario analysis

To understand the model it is informative to do shock/scenario - analysis. However, in undertaking scenario analysis it is important to consider exactly what underlying assumptions are required, for example, there is a need to consider what information set agents have, i.e. do households and firms have the same information set as Central Banks and the Governments, or are there information asymmetries. Another important aspect is to distinguish between an anticipated or an unanticipated change in the

economy. A shock to the economy, is by definition always unanticipated. However, policy changes, could be anticipated, if agents are changing their behaviour in advance, or unanticipated, if the event is both not pre-announced and not anticipated by agents. Furthermore it is important to distinguish between transitory and permanent shocks, as clearly the response of the economy will depend on the duration of the shock hitting the economy. In the case of a change of policy, then consideration is needed as to whether the changes in policy are credible or uncredible.
We will consider in this section, three alternative approaches to expectation formation. The first approach is the case of perfect foresight (model consistent) rationality where all agents have the same information set and know the duration of the shock and how the Central Bank and the economy will react to the shock. In this approach, agents adjust their behaviour as soon as a change is anticipated, we call this 'announced and credible' shock. In the second approach of unanticipated rationality, the information set is the same for all agents, but is one of protracted surprises, where expectations will repeatedly not be fulfilled and agents will be surprised every period (for example a government continously surprising firms and households). This may to some extent be considered at odds with the rational expectations approach that does not allow systematic recurrant errors, but is a quite common approach in DSGE modelling, we term this unannounced-uncredible shock, but could also be interpreted as rational inattention. The third approach is the bounded rationality approach outlined above, where agents learn about the shocks and how the economy responds to those shocks. In some ways, the learning approach is between these approaches, in that agents are surprised, but are not continiously surprised and gradually learn about the shocks (e.g. if it is permanent or temporary) and adjust their behaviour accordingly. This approach would be expected to converge to the anticipated rationality.

In the case where expectations are assumed to be fully model consistent forward-looking, rational expectations (i.e. announced and credible or unannounced/uncredible-scenario the future expected value is simply replaced by the model future realisations and the model solved iteratively, such that the expectations are fully model consistent[4], i.e. expected outcomes are replaced with model outcomes i.e:

$$y^e_{t+1} = y_{t+1} \tag{32}$$

For each country we have created a steady state baseline over a sufficiently large time horizon to study model properties and perform standard shock simulations. The comparative simulation analysis is undertaken over a horizon of 250 years[5]. This has enabled us to simulate the model under alternative modelling of expectations, namely model-consistent rational expectations and learning expectations. We can do scenarios in single country mode, where there are no cross-country trade linkages and monetary policy and exchange rates react to domestic developments. In linked mode there are

[4]We use the Stacked Time algorithm in Troll to solve out the forward-looking solution.

[5]Under rational expectations this ensures limited effect of terminal conditions. Our approach in creating the baseline is to consider 2010q1 to be at steady state.

cross-country linkages and monetary policy and exchange rates react to euro area developments. For comparability purposes, under learning we keep the exchange rate a forward-looking, model-consistent UIP[6].

5.1 One period shock to Interest Rates

We start by considering the reaction of the economy to a 1 period shock to the short-term interest rate followed by the Taylor rule. We consider both the announced case - i.e. rational model-consistent agents know it is only a one period shock and compare this with model simulated under learning. This simulation is for Germany only, in single country mode, where interest rates and exchange rate react to the German economy. Figure 1 shows the response of the economy to this shock, where results are presented as percentage deviations from baseline (except for Interest rates, fiscal deficit and unemployment which are expressed as absolute deviations (either in percentage points of percent of GDP). In both cases, both demand and prices initially fall, but then start to return back to base as increased competitiveness aided by lower interest rates boost the economy. Under the announced, rational-model consistent scenario, both prices and demand reacts quicker, however a key aspect to note is convergence of the learning to the Rational expectations equilibrium.

5.2 Fiscal policy expansion under alternative expectations

As noted by Van Brusselen (2009), there is no consensus on the response of the economy to a fiscal policy expansion. Indeed estimates in the literature vary widely depending on the model used. The fiscal block is quite extensive, and we can consider expansionary fiscal policy through alternative instruments, but in this section we focus purely on an expansion in government expenditure. However, using our core framework, we are able to illustrate the sensitivity to different expecation assumptions as well as how the impact varies depending on the state of the economy.

5.2.1 Impact of alternative expectations

In the second simulation we consider a shock to government expenditure of 0.5% of GDP for 3 years and then government expenditure returns to back to base. In all cases, the initial shock is unanticipated. Under the announced and credible scenario, both households, firms the central bank and the government know the timing of the shock. They also anticipate that debt-financed expenditures will lead to higher taxes. The second case we consider is where the change in government policy is unannounced/uncredible. In this case agents expect the government to return expenditure to their previous level of government spending, and agents are continuously surprised

[6] The learning model can currently be simulated using a learning based UIP rule, but this clearly has different properties to the rational, model-consistent UIP. Using this could be interpreted as assuming rationality in the asset market.

by government, i.e. sequentially surprised. In this case, we also assume that the central bank has the same information set as the private agents and is also continously surprised, and monetary policy reactions reflects this. The third case is where instead of expecting government expenditure to follow an AR1 process with parameter 0.9. Finally, the fourth case consider is the bounded rationality, where agents don't know the duration of the shock, but learning about it, and subsequently are not continuously suprised. Figure 2 shows reactions under the four approaches. In all three cases, the impact of increasing government consumption by 0.5% of GDP is to initially increase GDP by around 0.5% of GDP, but subseqently there is strong crowding out due to higher interest rates, and hence higher cost of capital which reduces both investment and consumption as well as helps to reduce the surge in inflation. Furthermore, consistent with the uncovered interest rate parity condition, the shock triggers a depreciation of the domestic currency, thus boosting export demand to some extent. The initial depreciation of the currency reflects the fact that in spite of the initial rise of the interest rate it later decrease below the baseline and forces the exchange rate to appreciate back to the baseline level.

However, the dynamics are quite different across the four different cases. In the first case, of announced and credible change in government spending under model-consistent rationality, agents react immediately to the shock and adjust their behaviour according, with a quick crowding out, particularly for investment, higher interest rates, but also crowding out by households due to negative wealth effect on private consumption in anticipation of higher (future) tax burdens. Furthermore there are strong inflationary pressures in the first year. In the second case, where the increase in government spending is unannounced/uncredible then under model-consistent rationality, where agents are repeatedly surprised, the impact on the economy is different, with initally less inflationary pressures, and less crowding out in the second and third years. In the learning expectations case where agents learn about the duration of the shock, interest rates remain above base for longer as inflation is more presistent and consequently the slump in output is more severe,after 3 years. One of the first things to notice from these simulations is that the learning simulations are converging to the rational expectations solution. This is in line with the literature, e.g. Beeby et al. (2001), however, as also previously shown, the model with the learning process exhibits very different properties from a model with rational expectations with clear differences in the adjustment path to equilibrium. Indeed, the model seems to support the work of Orphanides and Williams (2002), who showed in a small model for inflation that policies which are efficient under rational expectations are not when agents use a learning process. In our scenario, monetary policy takes time to react, as the Central Bank learns about the duration of the shock, but then interest rates remain higher for longer. Further work is needed on studying the optimal policy responses alternative expectation formation.

5.2.2 Cross-country impact

The previous scenarios were undertaken by one country. In our third scenario, we use the full linked model, with trade-linkages, and interest rates and exchange rate reacting to the aggregate. Figure 3 shows the impact of a synchronized, permanent shock to Government consumption of 0.5% of GDP for all countries under the announced-credible rationality assumption. Figure 4 shows the same shock but under bounded rationality. We see that the crowding out effects are much quicker and stronger in the anticipated rational expecations approach than under learning. There are cross-country differences, both in the initial impact as well as the dynamics. The differences are due to a variety of factors including openness of the economy, responsiveness to monetary policy, degree of financing constraints, perceived response of the Central Bank and perceived state of the economy.

5.2.3 Time-variation in scenarios

As well as differences in treatment of expectations the impact of an expansionary fiscal policy, potentially clearly depends on the state of the economy, and the perceived (expected) state of the economy. In this respect, using the bounded-rationality framework, Figure 5 shows the impact of a permanent increase in government expenditure, with monetary accomodation interest rates (and hence exogenous exchange rates). The figure shows the impact starting the shock in 1999Q1, to starting the shock in 2009Q4. The timeframe is 8 quarters. The responsiveness of the economy to fiscal stimulus seems to decline as the economy is doing well (2000, 2006-2007), but then the responsiveness increases during a downturn or recession, particularly during the recent period (2009). This is because the economy has some slack, i.e. is not being fully utlized, so there are less inflationary pressures, and less crowding out. Therefore, government expansion has more impact. The initial impact of an increase in government spending has declined in all countries since 1999Q1, but the change has been strongest in Spain, where the initial impact on GDP was estimated to be of 0.9% of GDP, but then by 2008Q1, the initial impact reduced to 0.6% of GDP. This is in line with the other 4 euro area countries, and reflects the transformation the economy has undergone since 1999 as well as the robust growth of the Spanish economy prior to 2009.
The scenarios analysis presented here is illustrative, and as our fiscal block is quite extensive we are currently investigating more alternative, perhaps more realistic scenarios, e.g. a reduction in income tax, consumption tax or firm's social security payments or via an increase in government transfers. We also plan to consider the implications of co-ordinated versus un-co-ordinated fiscal expansion.

6 Conclusions

In this paper, we have presented a new Multi-country model consisting of three optimising private sector decision making units, i.e. firms, trade unions and households, where output is in the short run demand-determined and monopolistically competing firms set prices and factor demands. Labour is indivisible and utilitarian monopoly-unions set wages and households make consumption/saving decisions. The core of the model is estimated with the GMM approach that assumes boundedly rational expectations. In this paper we have presented agents expectation formation using a Kalman filter learning approach without uncertainty of deep parameters but with unknown driving stochastic processes. The learning approach is criticised for the arbitrary approach, and as Milani (2004a), shows the estimates can vary strongly over different gain coefficients. We address this cricitism by optimising over a key gain parameter, with the assumption agents choose the 'best performing rule'. We have shown that under these assumptions, expectations based on learning (weak rationality), converges to the anticipated model-consistent rational expectations solution. However, there are strong differences in the adjustment path of the economy to an expansionary fiscal policy. Furthermore, under our framework, the behaviour of the economy varies depending on the state of the economy and agents perceptions of the future. In particular, we find an expansionary fiscal policy is more effective in periods of downturns than in a period of boom.

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8 The Model Framework

The theoretical core of the model consists of three optimising private sector decision making units, i.e. utility maximising households, profit maximising firms and trade unions, which minimise the quadratic loss function under the staggered wage adjustment assumption. Monopolistically competing firms set prices, inventories and factor demands under the assumptions of indivisible labour. Output is in the short run demand-determined. Monopoly unions set wages and overlapping generation households make consumption/saving decisions. In the rest of this section we give a concise overview of the main features of the model. The detailed theoretical framework is in Dieppe et al 2009.

8.1 The Normalised CES production function

Our technology assumption is the "normalized" CES function allowing for time-varying factor-augmenting technical progress. The importance of explicitly normalizing CES functions was discovered by La Grandville (1989) and first implemented empirically by Klump, McAdam and Willman (2007). Normalization starts from the observation that a family of CES functions whose members are distinguished only by different substitution elasticities need a common benchmark point. Since the elasticity of substitution is defined as a point elasticity, one needs to fix benchmark values for the level of production, factor inputs and marginal rate of substitution, or equivalently for per-capita production, capital deepening and factor income shares. The normalized CES production function corresponding to the general function 33 is given by[7],

$$\frac{Y_t}{Y_0} = \left\{ \pi_0 \left[\Gamma_K (t, t_0) \frac{K_t}{K_0} \right]^{\frac{\sigma-1}{\sigma}} + (1-\pi_0) \left[\Gamma_N (t, t_0) \frac{h_t N_t}{h_0 N_0} \right]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}} \tag{33}$$

where σ is the elasticity of substitution between capital and labour, π_0 distribution parameter equalling the capital share evaluated at the normalization point (subscript 0) and $\Gamma_i (t, t_0)$ define the (indexed) level of technical progress associated to factor i (with $\Gamma_i (t_0, t_0) = 1$).

8.1.1 Technical progress

As it is not obvious that growth rates should always be constant, we follow an agnostic approach and model technical progress drawing on a well-known flexible, functional form (Box and Cox, 1964):

$$\log \left[\Gamma_i (t, t_0, \gamma_i, \lambda_i) \right] = \frac{\gamma_i t_0}{\lambda_i} \left[\left(\frac{t}{t_0} \right)^{\lambda_i} - 1 \right] \tag{34}$$

where $i = N, K$. The log level of technical progress, $\Gamma_i (\bullet)$ is, therefore, a function of time, t (around its normalization point, t_0), a curvature parameter, λ_i, and has a growth rate of γ_i at the representative point of normalization.[8] When λ_i =1 (=0) [<0], technical progress displays linear (log-linear) [hyperbolic] dynamics:

$$\log \Gamma_{\mathrm{i}}(t) \Rightarrow \begin{cases} \lim_{t\to\infty} \left[\log \Gamma_i(\mathrm{t}) \right] = \infty & \text{if } \lambda_i \geqslant 0 \\ \lim_{t\to\infty} \left[\log \Gamma_i(\mathrm{t}) \right] = -\frac{\gamma_i t_0}{\lambda_i} > 0 & \text{if } \lambda_i < 0 \end{cases} \tag{35}$$

$$\frac{\partial \log \Gamma_i(\mathrm{t})}{\partial \mathrm{t}} = \gamma_i \left(t/t_0 \right)^{\lambda_i - 1}$$

$$\Rightarrow \begin{cases} = \gamma_i \left(t/t_0 \right)^{\lambda_i - 1} > 0; \ \lim_{t\to\infty} \frac{\partial \log \Gamma_i(\mathrm{t})}{\partial \mathrm{t}} = \infty & \text{if } \lambda_i > 1 \\ = \gamma_i, \ \forall t & \text{if } \lambda_i = 1 \\ \geqslant 0 \, ; \ \lim_{t\to\infty} \frac{\partial \log \Gamma_i(\mathrm{t})}{\partial \mathrm{t}} = 0 & \text{if } \lambda_i < 1 \end{cases} \tag{36}$$

Thus, if $\lambda_i \geq 0$, the level of technical progress accruing from factor i tends to infinity but is bounded otherwise, (see equation 35). If $\lambda_i = 1$ the factor growth of technical

[7] León-Ledesma, McAdam and Willman (2009) discuss and evaluate normalization more extensively.

[8] Note we scaled the Box-Cox specification by t0 to interpret γ_N and γ_K as the rates of labour- and capital-augmenting technical change at the fixed (i.e., representative) point.

progress is constant (i.e., the "text-book" case) but asymptotes to zero from above for any $\lambda_i < 1$, (see equation 36)

8.2 The supply system

The behaviour of profit maximizing firms and the member households 'utility maximizing trade unions determine the long-run supply of the model as defined by the following 5-equation suppy system 37 - 41. The first four equations are the steady-state forms of the first-order conditions of maximization of the firms with costly adjustment of inputs and calvo stickiness in price setting, McAdam and Willman (2008) and Dieppe et al (2009). Equation 41 is the first order condition of maximization of the trade union, Dieppe et al (2009):

$$\log\left(\frac{P_t^y Y_t}{w_t N_t + q_t K_t}\right) - \log\left(1+\mu\left(t\right)\right) = 0 \tag{37}$$

$$\log\left(\frac{w_t N_t}{P_t^y Y_t}\right) - \log\left(1-\bar{\pi}\right) - \frac{1-\sigma}{\sigma}\left[\log\left(\frac{Y_t/\bar{Y}}{N_t/\bar{N}}\right) - \log\xi - \frac{\bar{t}\gamma_N}{\lambda_N}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_N} - 1\right)\right] + \log\left(1+\mu\left(t\right)\right) = 0 \tag{38}$$

$$\log\left(\frac{q_t K_t}{P_t^y Y_t}\right) - \log\left(\bar{\pi}\right) - \frac{1-\sigma}{\sigma}\left[\log\left(\frac{Y_t/\bar{Y}}{K_t/\bar{K}}\right) - \log\xi - \frac{\bar{t}\gamma_K}{\lambda_K}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_K} - 1\right)\right] + \log\left(1+\mu\left(t\right)\right) = 0 \tag{39}$$

$$\log\left(\frac{Y_t/\bar{Y}}{N_t/\bar{N}}\right) - \log\left(\xi\right) - \frac{\bar{t}\gamma_N}{\lambda_N}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_N} - 1\right) + \frac{\sigma}{1-\sigma}\log\left[\bar{\pi}e^{\frac{1-\sigma}{\sigma}\left[\frac{\bar{t}\gamma_N}{\lambda_N}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_N}-1\right)-\frac{\bar{t}\gamma_K}{\lambda_K}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_K}-1\right)\right]}\left(\frac{K_t/\bar{K}}{N_t/\bar{N}}\right)^{\frac{\sigma-1}{\sigma}} + \left(1-\bar{\pi}\right)\right] = 0 \tag{40}$$

$$\log\left(\frac{N_t^F w_t}{P_t^c C_t}\right) + \log\left\{\sigma - 1 + \left(1-\bar{\pi}\right)\left[\frac{Y_t/\left(\xi\cdot\bar{Y}\right)}{\frac{N_t}{\bar{N}}e^{\frac{\bar{t}\gamma_N}{\lambda_N}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda}-1\right)}}\right]^{\frac{\sigma-1}{\sigma}}\right\} - \log\left(\frac{\sigma\kappa}{\bar{h}_t}\right) = 0 \tag{41}$$

where Y, N and K refer to output, employment and capital. P^y, w and q are their respective prices; C, P^c and N^F are consumption, consumption deflator and labour force. Bars above the variables refer to the sample averages.[9] Normalised production function implies that $\overrightarrow{\pi} = \frac{\bar{q}\bar{K}}{\bar{w}\bar{N}+\bar{q}\bar{K}}$ is the capital share evaluated at the fixed point (sample mean).

Aggregation across heterogeneous sectors facing differentiated price and income elasticities of demand introduces a trend component into the mark-up as shown in equation 37. The task of this equation is to control this common trend component in equations 38 and 39 which are the steady state forms of the first order conditions of profit maximisation with respect of labour and capital. Equation 40 is the production function 33 after taking logs on its both sides. Equation 41 determining the optimal frictionless wage rate is the first order condition of maximisation of the utilitarian trade union under the right-to-manage structure. It is part of the supply side system as it is conditional to the same production technology as the firm's maximisation conditions. In estimating of 41 variable $\bar{h}_t$, referring to average normal working hours per employee, was also allowed to include a box-cox trend to account for observed decrease in total hours per employee ratio.

In actual estimation we allowed a break in the speed of factor augmenting technical progress to capture effect of the IT-boom on technical progress in the latter part of 1990s. Likewise to improve the data compatibility of the system we allowed breaks in the trends of mark-up and normal working hours per employee.The steady state form of the first-order conditions of the profit maximising firms and the union maximising utility of member households imply the 5-equation medium-run supply system that allows a consistent two-step estimation of the underlying deep parameters of the model. As the supply system contains cross-equation parameters constraints, it is estimated with the method of non-linear SUR, see León-Ledesma et al. (2009), and which has proven to be a very efficient estimation approach outperforming all single-equation methods. Estimates of the key technology parameter, the elasticity of substitution between capital and labour, are presented below. It very uniformly shows markedly below unit substitution elasticity in all 5 countries, thus rejecting a Cobb-Douglas production function. Also strong capital augmenting technical progress component were identified (not shown in the table) in all countries.

Production Function Estimates

	FR	DE	IT	ES	NL
Elasticity of Substitution*	0.532 (0.0005)	0.614 (0.0006)	0.614 (0.0006)	0.550 (0.0036)	0.575 (0.0080)
Long-run Estimation sample	81q2	93q1	87q1	83q1	81q1

* Standard errors of estimates in brackets.

[9] We have defined the point of normalisation so that $t_0 = \bar{t}$, $N_0 = \bar{N}$. $K_0 = \bar{K}$, $Y_0 = \xi\bar{Y}$ and $\pi_0 = \bar{\pi}$. Parameter ξ is a normalization constant (close to unity) that resulting from nonlinearities means the sample average of production need not exactly coincide with the level of production implied by the sample averages of inputs and time, Klump et al. (2007).

This system defines all parameters related to technology, production function and the mark-up allowing to define optimal frictionless prices, wages, labour demand and marginal cost and product concepts needed in estimating in the second stage the dynamic first order optimisation conditions of firms and unions. All dynamic equations containing the leads of variables are estimated by the generalised method of moment (GMM) that is compatible with the assumption of bounded rationality[10].

8.3 Firm behaviour

The firm maximizes its expected discounted stream of dividends, V_t. Without loss of generality, for notational simplicity, we assume that bank loans are the only form of external financing. We first define the determination of dividends in terms of income and cost components. Regarding cost, as in McAdam and Willman (2008), we allow a careful modelling of factor adjustment costs. Labour participation decisions are modeled along the intensive and extensive margins. In so doing, we introduce the concept of "effective labour hours". An innovative aspect is that the former margin turns out to have a key spillover onto firms' pricing decisions.
Capital accumulation, in turn, reflects time-to-build considerations. To capture price and inflation stickiness we adopt three valued Calvo-signalling mechanism, which as discussed in McAdam and Willman (2007), implies that each firm faces ex ante exactly the same optimization problem independently from the ex post outcome of the Calvo-signal. In addition to conventional demand and technology constraints the present value maximisation is constrained by non-negativity of gross investment and by the upper bound for the debt-to-capital stock ratio. Hence, the estimated aggregate level equations assume that for a certain percent of firms these constrains are binding. In addition, our aggregation accounts for the effects of structural changes in sectoral output shares from competitive (low mark-up) manufacturing to less-competitive (high-mark-up) services sectors. This phenomenon introduces secularly growing mark-up on aggregate price level as discussed by Willman (2002) and McAdam and Willman (2004).

8.3.1 Labour Demand

The desired (optimal) number of workers is derived from the inverted production function equation 33 such that:

$$N_t^* = \frac{\bar{N}\left(1-\bar{\pi}\right)^{\frac{\sigma}{\sigma-1}}}{\Gamma_N\left(t\right)}\left[\left(\frac{Y_t}{\xi\bar{Y}}\right)^{\frac{\sigma-1}{\sigma}} - \bar{\pi}\left(\frac{K_{t-1}}{\bar{K}}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma}{\sigma-1}} \tag{42}$$

From the point of view of production relevant labour input concept is effective (or intensive) hours worked. Hence, a firm can increase labour input either by recruiting

[10]The instruments used in estimation are lags of dependent, driving and other relevant variables.

new employees or increasing the number of effective hours (via work intensity or overtime) per worker of existing employees. If wage agreements are basically fixed time contracts with overtime premium and, in addition, recruiting new employees and firing existing ones are associated with costs, then wage costs per efficient hour are above the contracted straight (normal) time rate both when effective hours h_t are either below (as hours with low intensity are paid at the same rate as hours with normal intensity) or above normal hours $\bar{h} = 1$. Hence, total wage costs can be presented as a convex function of the deviation of effective hours from normal hours.

$$W_t = \overline{W}_t \left[h_t + \frac{a_h}{2} \cdot (h_t - 1)^2 \right] \tag{43}$$

Changes in employment are coupled with adjustment costs. For estimation of the dynamic labour demand, we need to define function forms for the adjustment cost function $A_N(N_t, N_{t-1})$:

$$A_N(N_t, N_{t-1}) = \frac{a_N}{2} \cdot \Delta N_t \Delta n_t \tag{44}$$

where n=log(N).
Now the dynamic system of first order conditions imply the following labour demand:

$$n_t = \frac{D_t}{(1 + D_t + a_h/a_N)} n_{t+1} + \frac{1}{(1 + D_t + a_h/a_N)} n_{t-1} + \frac{a_h/a_N}{(1 + D_t + a_h/a_N)} n_t^* \tag{45}$$

$n_t^* = \log(N_t^*)$ (inverted production function)
$D_t = \frac{(1+(w_{t+1}-w_t))}{(1+r_t)_t} \cdot \frac{\left(1+(n_t^*-n_t)+a_h(n_t^*-n_t)^2\right)}{\left(1+\left(n_{t+1}^*-n_{t+1}\right)+a_h\left(n_{t+1}^*-n_{t+1}\right)^2\right)} \cdot$ = discounting factor (≈ 1)

Labour Demand Estimation results

	FR	DE	IT	ES	NL
a_h/a_N	0.0225	0.0388	0.0559	0.1414	0.0396
	(0.044)	(0.011)	(0.020)	(0.037)	(0.084)
1/Root 1 (forward)	0.8564	0.8171	0.7855	0.6848	0.8173
Root 2 (backward)	0.8646	0.8248	0.7932	0.6904	0.8219
p-value of J-test	0.85	0.974	0.84	0.839	0.955

8.3.2 Investment formation

Capital accumulation reflects time-to-build considerations. In addition to conventional demand and technology constraints the present value maximisation is constrained by non-negativity of gross investment and by the upper bound for the debt-to capital stock ratio. Hence, the estimated aggregate level equations assume that for a certain percent of firms these constraints are binding.

As with employment, we define the adjustment cost function $A(K_t, K_{t-1}, K_{t-2})$, as follows:

$$A(K_t, K_{t-1}, K_{t-2}) = \frac{a_K}{2} \cdot \Delta K_t \Delta k_t + \frac{a_K b_K^2}{2} \cdot \Delta K_{t-1} \Delta k_{t-1} - a_K b_K \cdot \Delta K_t \Delta k_{t-1} \quad (46)$$

where $k = \log K$ and $b_K \in [0,\ 1]$.

Now the dynamic system of first order conditions implies the investment equation:

$$\begin{aligned}
&\frac{\left(1-\Lambda^B\right)^2 b_K}{(1+r_t)(1+r_{t+1})} \Delta k_{t+2} - -\left(\frac{\left(1-\Lambda^B\right)^2 b_K^2}{(1+r_t)(1+r_{t+1})} + \frac{\left(1-\Lambda^B\right)(1+b_K)}{(1+r_t)} \right) \Delta k_{t+1} \\
&\qquad + \left(\frac{\left(1-\Lambda^B\right) b_K(1+b_K)}{(1+r_t)} + 1 \right) \Delta k_t - b_K \Delta k_{t-1} \\
&= \frac{1}{a_K} \left(\frac{P_t}{(1+\mu_t) P_t^I} MPK_t - \left\{ \left(1-\Lambda^I\right) UC_t + \Lambda^B (1-\delta) \left[\alpha - \left(1-\Lambda^I\right)\right] \right\} \right) \quad (47)
\end{aligned}$$

Λ^B = LG-multiplier related to the borrowing constraint; α= the debt to capital stock ceiling ratio $0 \leq \alpha \leq 1$; MPK = marginal product of capital; UC = real user cost of capital; Λ^I= LG-multiplier related to the irreversibility of investment ; and a_K and b_K are adjustment cost parameters.

Estimated parameters and the implied roots of the homogenous part of the difference equation (47) are presented in the table below. In all cases the backward root (equalling parameter b_k) is high implying slow adjustment to shocks. However, also the inverse of forward roots are quite high, although well below unity, implying forward lookingness in investment decisions. From the roots, we see that Spain is the most forward looking and the Netherlands the least forward looking.

Estimated Investment parameters*

	FR	DE	IT	ES	NL
$1-\Lambda^B$	0.5087 (0.012)	0.4517 (0.005)	0.5067 (0.008)	0.5876 (0.011)	0.376 (0.015)
b_K	0.7879 (0.010)	0.695 (0.008)	0.7796 (0.012)	0.7834 (0.014)	0.6267 (0.026)
$1/a_K$	0.0126 (0.0007)	0.0122 (0.0006)	0.0129 (0.0010)	0.0139 (0.0016)	0.0163 (0.0016)
Λ^I	0.25	0.26	0.2	0.25	0.33
a	0.8	0.8	0.855	0.8	0.75
1/Root 1 (forward)	0.3961	0.311	0.3918	0.4585	0.2306
1/Root 2 (forward)	0.5027	0.4475	0.5026	0.5852	0.3679
Root 3 (backward)	0.7879	0.695	0.7796	0.7834	0.6267
p-value of J-test	0.999	0.999	0.99	0.996	0.993

*Standard errors of estimates in brackets

Total investment has been further disaggregated via bridge equations to housing (incorporating short/long-term interest rates), and non-residential, but all feedback effects to the rest of the model go through aggregate fixed investment.

8.3.3 Price formation

Price and wage setting are staggered with three-valued Calvo-signal resulting in a conventional hybrid New Keynesian Phillips curve as in Gali and Gertler (1999):

$$\begin{aligned}\{\theta_p + \omega_p [1 - \theta_p (1-\beta)]\} \Delta p_t - \omega_p \Delta p_{t-1} - \beta\theta_p \Delta p_{t+1} \\ - \ (1-\omega_p)(1-\theta_p)(1-\beta\theta_p)(p_t^* - p_t) = 0 \quad (48)\end{aligned}$$

where p_t = log of gdp deflator at factor costs; $p_t^* = w_t - mpn_t + a_h (n_t^* - n_t) + \mu_t$ = log of the frictionless equilibrium price level; w_t = log of compensation per worker; mpn_t= log of the marginal product of labour (<= production function); n_t^* = optimal number of workers (log), n_t = actual employment (log); a_h is the overtime premium parameter determined by (43) and aggregate mark-up $\mu(t)$ is determined by the system 37-41. θ_p is the probability that firms don't change their prices, and ω_p is the probability prices are changed following a backward-looking rule.

In estimation we assumed the four per cent annual discount rate, which in quarterly data implies $\beta = 0.99$.

Estimated parameters of the hybrid NKPC[11]*

	FR	DE	IT	ES	NL
θ_p	0.7455	0.7955	0.7251	0.6705	0.6926
	(0.030)	(0.020)	(0.013)	(0.017)	(0.043)
ω_p	0.3531	0.3976	0.3235	0.2543	0.2807
	(0.045)	(0.042)	(0.018)	(0.022)	(0.054)
a_h	0.7515	1.5411	0.1797	0.4212	0.4109
	(0.237)	(0.308)	(0.222)	(0.204)	(0.156)
Duration	3.93	4.89	3.64	3.03	3.25
Root 1 (forward)	1.355	1.2697	1.393	1.5065	1.4583
Root 2 (backwards)	0.597	0.729	0.5712	0.5067	0.5298
Root 3 (backwards)	0.5914	0.5455	0.5663	0.502	0.5298
p-value of J-test	0.867	0.953	0.967	0.896	0.922

*Standard errors of estimates in brackets

[11] For simulation purposes, the a$_h$ parameter for Germany has been calibated to the average of the other 4 countries, as the estimated parameter resulted in excessive sensitivity to changes in employment.

Disaggregated Price Equations The GDP deflator at factor costs is an integral part of the supply side and the key determinant of other price variables in the model. In this regard, it is the central price in the new MCM. However, adjustment in other prices also matter for differences in the models' response. Below we present the estimation results for Harmonized consumer price indices (HICP), which are represented by two equations, one for the energy HICP and one for non-energy HICP.
The post-tax HICP deflator is defined as:

$$p_t^{HX} = \frac{1 - tcir}{1 - tci_t} p_t^{HXT} \tag{49}$$

where p_t^{HXT} is the pre-tax HICP excluding energy, tci is the current implicit tax rate and tcir is the tax rate in the base year of price indices. We model the seasonal adjusted version of p_t^{HXT}, so-called p_t^{HXST} where the seasonal factors are estimated using a time-varying airline estimation procedure and kept fixed over the forecast horizon. We retain the Calvo price framework and parameters from above:

$$\begin{aligned}\{\theta_p + \omega_p [1 - \theta_p (1 - \beta)]\} \Delta p_t^{HXST} - \omega_p \Delta p_{t-1}^{HXST} - \beta\theta_p \Delta p_{t+1}^{HXST} \\ - \ (1 - \omega_p)(1 - \theta_p)(1 - \beta\theta_p)\left(p_t^{HXST*} - p_t^{HXST}\right) = 0\end{aligned} \tag{50}$$

where p_t^{HXST*} is the long-run optimal non-energy HICP and is weighted average of the optimal GDP deflator p_t^* including indirect energy prices and imports deflator (excluding energy) where the weights ϕ_1 are estimated by OLS and ϱ is set to 0.015 based on input-output tables. In addition, as with the Calvo price equation, we include a labour adjustment factor:

$$p_t^{HXST*} = \phi_1 p_{MN,t} + (1 - \phi_1)(\varrho p_t^* + (1 - \varrho) p_{EI}) + a_h (n_t^* - n_t) \tag{51}$$

HICP energy (PHE_t) is modelled as a mark-up of energy prices (or oil) and GDP deflator (pt):

$$p_t^{HE} = \delta_1 p_{EIt} + (1 - \delta_1)(p_t) \tag{52}$$

HICP energy and excluding energy and taxes seasonally adjusted

	DE	FR	IT	ES	NL
ϕ_1	0.21	0.15	0.1	0.033	0.17
	(0.071)	.	(0.014)	(0.005)	(0.018)
δ_1	0.33	0.21	0.15	0.21	0.35
	(0.07)	(0.098)	(0.006)	(0.008)	(0.011)

*Standard errors of estimates in brackets

The overall HICP p_t^H,then becomes a weighted average of HICP non-energy (post-tax), p_t^{HX} and HICP energy, p_t^{HE} where w_{et} is the weight of HICP energy in the overall HICP[12].

[12] In 2004 energy HICP weight varied across the countries in the range of 8-10 per cent

$$p_t^H \;=\; w_{et} \cdot p_t^{HE} + \; (1 - w_{et}) \cdot p_t^{HX} \tag{53}$$

The consumption deflator is linked via a simple bridge equation to seasonally adjusted HICP.

All other domestic deflators (e.g. investment deflator) are specified as quasi-identities, i.e. modelled as weighted averages of domestic costs (measured by the value-added deflator defined above) and import prices (measured by the import deflator). This feature ensures static homogeneity in all price equations. For pre-tax deflators we assume that imports are 'cost, insurance and freight at the importer's border' (cif) and the exports are 'free on board at the exporter's border' (fob). For this reason indirect taxes are levied only on total consumption (private and public) and total investment. Since there is no distinction between indirect taxes on consumption goods and on investment goods, both tax rates will be equal in sample but will be kept under different denominations for simulation purposes.

8.3.4 Wage Setting

Consider an imperfectly competitive labour market, where a great number of monopoly unions determine real wages of their members under a right to manage structure (i.e. firms determine the employment level given the wage determined by unions). In renewing wage contracts each union sets the wage rate knowing its effects on employment determined by firms. Labour is indivisible so that variations in demand for labour determined by firms are transmitted to the number of unemployed instead of the hours worked per employee. In addition, assume that contracts are binding until they are renegotiated. That introduces stickiness in wage formation. As with the price setting, wages are also set via a staggered with three-valued Calvo-signal where part of unions keep wages fixed, another part changes wages following backward-looking rule and the rest set them optimally:

$$\begin{aligned}\left\{\theta_w + \omega_w \left[1 - \theta_w \left(1 - \beta\right)\right]\right\} \Delta w_t = {} & \omega_w \Delta w_{t-1} + \beta \theta_w E_t w_{t+1} \\ & + \; \left(1 - \omega_w\right) \left(1 - \theta_w\right) \left(1 - \beta \theta_w\right) \left\{w_t^* - w_t\right\}\end{aligned} \tag{54}$$

where w_t = log of compensation per worker, and β,the discount factor, = 0.99. The optimal frictionless wage rate, w_t^* is based on two alternative behavioural assumptions where we assume that part of the unions are utilitarian, maximising the utility of member households, whist the rest are non-utilitarian keeping wage development in line with productivity development coupled with a high desired employment rate. i.e.

$$w_t^* = a_{wu}\left[\left(p_t^C + c_t - n_t^F\right) - \log\left(\sigma - 1 + \frac{F_N^{CES}}{Y_t/N_t}\right) + \log\frac{\sigma\kappa}{\bar{h}(time)}\right] + (1-a_{wu})\left[p_t + \log\left(\frac{F_N^{CES}}{1+\mu}\right) + \chi\log\left(\frac{F^{-1}(K_t,Y_t)}{\varpi\cdot N_t^F}\right)\right] \quad (55)$$

$$F_N^{CES} = (1-\pi_0)\left(\frac{Y_0}{N_0}\Gamma_N(t,t_0)\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{Y_t}{N_t}\right)^{\frac{1}{\sigma}} \quad (56)$$

where c_t = consumption (log), p_t^C= consumption deflator (log) ; $F^{-1}(\cdot)$ = inverted CES production function (desired number of workers); N_t^F = labour force and the gap between optimal labour demand and supply measures the wage drift effect.
Note that if all unions are non-utilitarian, then in the full model context it implies a constant long-run natural rate of unemployment. If instead, all or part of unions are utilitarian, then shocks that affect the demand structure, e.g. a permanent government expenditure shock affecting the GDP share of private consumption, affects also the long-run equilibrium unemployment rate.
The weight parameter a_{wu} is estimated in the context of the estimation of equation (54). Naturally, for scenario analysis its value can be imposed to unity or zero, depending if we want assume all unions to be utilitarian or non-utilitarian[13].

Estimated parameters of the hybrid wage-NKPC*

	FR	DE	IT	ES	NL
θ_w	0.7364	0.5483	0.7301	0.7471	0.7077
	(0.028)	(0.034)	(0.024)	(0.021)	(0.022)
ω_w	0.3396	0.1427	0.3306	0.3555	0.2999
	(0.041)	(0.024)	(0.034)	(0.032)	(0.029)
χ	0.1	0.1675	0.1	0.15	0.11
	-	(0.121)	-	-	-
a_{wu}	0.3	0.3358	0.5366	0.2234	0.1745
	-	(0.076)	(0.049)	(0.061)	(0.048)
Duration	3.79	2.21	3.71	3.95	3.42
Root 1 (forward)	1.3717	1.842	1.3835	1.3521	1.4273
Root 2 (backwards)	0.5591	0.3795	0.5769	0.5963	0.5488
Root 3 (backwards)	0.5561	0.3759	5730	0.5963	0.5465
p-value of J-test	0.914	0.981	0.912	0.775	0.983

*Standard errors of estimates in brackets

[13] For the simulations in the paper we assume all unions to be non-utilitarian.

8.3.5 Inventory investment

Following general practice and to neglect excessive complexity of the profit maximization framework, inventory formation is left outside that framework. Instead inventory formation is derived on the basis of second stage optimization by assuming that firms minimise a quadratic loss function specified in terms of the deviations of inventories from the optimal level, on one hand, and the deviations of output (sales of storable goods) from the level corresponding to the optimal use of existing input.
The desired equilibrium inventory stock KII* is based on the estimated CES production function:

$$KII = a + bF(K, N, t) \tag{57}$$

and inventories from the dynamic equation:

$$\begin{aligned}(1 - r \cdot A)\Delta KII_t &= (1 - 2A)\, \Delta KII_t^* \\ &- A\left[\Delta S_t - \Delta KII_t - \Delta F(\cdot)\right] + (1 - r)\, A\left[\Delta S_{t+1} - \Delta KII_{t+1} - \Delta F(\cdot, t+1)\right]\end{aligned} \tag{58}$$

S= Sales (Private consumption + exports)

Inventory Estimation results*

	FR	DE	IT	ES	NL
A	0.4705 (0.027)	0.3374 (0.032)	0.3501 (0.072)	0.3038 (0.058)	0.3802 (0.047)
1/Root 1 (forward)	0.7372	0.3855	0.4055	0.3361	0.4579
Root 2 (backward)	0.7447	0.3893	0.4096	0.3395	0.4625
p-value of J-test	0.731	0.889	0.92	0.972	0.987

*Standard errors of estimates in brackets

Estimation results indicate that all 5-countries inventories show quite fast adjusted to their optimal level. The adjustment process is fastest in Spain and slowest in France.

8.4 Households

The basic framework in household's utility maximisation is Blanchard's (1985) overlapping generation framework that, however, is supplemented in many ways. Firstly to incorporate income uncertainty in a tractable way into the utility maximization framework we assume a two-stage approach in utility maximisation, Willman (2007). In the first stage, the consumer evaluates her risk-adjusted non-human and human wealth conditional on uncertain lifespan and labour income. Thereafter, in the second stage, conditional on the risk adjusted life-time resource constraint, the consumer is assumed to determine her optimal planned path of consumption. This approach gives a closed form consumption function with precautionary saving depending positively on the income risk death probability. The explicit treatment of wealth allows us also to account for asset price (i.e. stock and house prices) effects on the perceived wealth relevant for consumption decisions. Further, the assumption of imperfect-front loaded information on future income realisations changes also the weight structure in defining the present value of the expected income stream (more front-loaded increasing the dependency of consumption on current income). Hence, we need not split households artificially into Ricardian utility maximisers and into non-optimising, income constrained consumers to introduce the observed strong dependency of consumption on current income. Instead, all consumers are non-Ricardian utility maximisers. Finally, the assumption of habit persistency introduces the dependency of current consumption on lagged consumption being able to generate a hump-shaped response profile of consumption to shocks. This result in a forward looking aggregate consumption function with strong backward-looking frictional elements.

$$\begin{aligned} & E_t\left\{1+\frac{\gamma}{1.01}\left[(1-\pi)^2 aR_t-(0.01+\pi)\left(1-\frac{(1-\pi)\,a}{1.01}\right)\right]\right\}\frac{C_t}{Y_t} \\ & \quad -(1-\pi)\,\gamma\,\frac{R_tC_{t+1}}{Y_t}-a\,(1-\pi)\,\frac{C_{t-1}}{Y_t}-\left(\frac{1.01-(1-\pi)\,a}{1.01}\right)\left(\frac{0.01+\pi}{1.01}\right) \\ & \quad \left\{\left(\frac{1}{1-\pi}-\gamma\right)\left(\frac{V_{t-1}}{Y_t}+1\right)+\left(\frac{\Lambda\,(1.01-(1-\pi)\,\gamma)}{0.01+\pi}-\frac{1}{1-\pi}\right)\right\}z_t=0 \end{aligned} \tag{59}$$

where C_t is consumption, Y_t labour income net of payroll taxes minus transfers, V_{t-1}total wealth in the beginning of period. Paremeter $\pi=$ death probability; $\gamma=$ forward information parameter; $a=$ habit persistence parameter; $\Lambda=$ income risk parameter and z_t refers to the set of instruments.
As firms are owned by households, total private sector wealth can be used as operational counterparty of V_{t-1} in (59). In a macroeconomic framework the advantage of this large wealth concept is that its component corresponding to accumulated savings equals the sum of private sector capital stock (at repurchasing prices), net government debt GDN and net foreign assets NFA. However, as also changes in real asset prices may affect the "perceived" wealth relevant for consumption decisions, V_{t-1} is operationalised as follows:

$$V_{t-1}=\frac{P_I}{P_C}\left[\begin{array}{c}\left(\frac{P_S}{P_I}\right)^{b0}(1-s_H)\,(KSR_{t-1}-KGR_{t-1})+ \\ \left(\frac{P_H}{P_I}\right)^{b1}s_H\,(KSR_{t-1}-KGR_{t-1})\end{array}\right]+\frac{GDN_{t-1}+NFA_{t-1}}{P_C} \tag{60}$$

where P_I, P_C , P_S and P_H are investment deflator, consumption deflator, stock prices and the market price of housing, respectively. KSR is total capital stock, KGR is government sector capital stock and s_H is the share of housing stock of total private capital stock. Now, if elasticity parameters $b0$ and $b1$ equal zero, then asset prices have no effects on consumption and "perceived" wealth equals total private sector wealth at repurchasing prices.

Correspondingly, if *b0*=*b1*=1, then variations in stock and housing prices are fully transmitted into the "perceived" wealth with maximal effects on consumption. These parameters were estimated jointly with other parameters of the dynamic consumption function specified for the five biggest euro area countries. In estimated equations *b0* and *b1* were constrained to be equal, because data was not able to credibly identify possible differentiated elasticity effects. These equations implied the following long-run marginal properties to consume out of wealth (mcr_w) and labour income (mcr_y) as well as elasticity parameters (*b0* and *b1*).

The long-run marginal propensity to consume out of total wealth is, according to these estimates, 7 cents per euro in Germany and 6 cents in the other four countries. Positive stock and housing price effects on consumption were estimated for all countries except for France (where they were found not to be significant). According to these estimates across countries, in the long-run about 20-34 percent of changes in stock and housing prices are transmitted to the respective wealth components and further to consumption.

Consumption equation*

	FR	DE	IT	ES	NL
π	0.005	0.008	0.0053	0.0053	0.0053
γ	0.8138	0.5318	0.6144	0.7192	0.6665
	(0.138)	(0.052)	(0.055)	(0.09)	(0.052)
a	0.9303	0.464	0.9467	0.864	0.9098
	(0.036)	(0.061)	(0.012)	(0.069)	(0.013)
Λ	0.9031	0.7931	0.6711	0.8522	0.7196
	(0.038)	(0.004)	(0.021)	(0.017)	(0.013)
b0,b1	-	0.3198	0.2036	0.2388	0.336
		(0.019)	(0.101)	(0.1)	(0.048)
mcr_w	0.06	0.07	0.06	0.06	0.06
mcr_y	0.91	0.79	0.61	0.71	0.67
1/Root 1 (forward)	0.805	0.526	0.606	0.711	0.658
Root 2 (backward)	0.92	0.459	0.934	0.854	0.898
p-value of J-test	0.908	0.911	0.701	0.946	0.831

*Standard errors of estimates in brackets

8.5 External Sector Behaviour

8.5.1 Export formation

The first-order conditions of an optimising firm that produces export goods, through the combination of labour, capital and imported goods, allow us to write export prices as a mark-up over marginal costs. We assume that the demand for exports takes an

AIDS form, depending on world demand for exports, Deaton and Muellbauer (1980). The advantage of this representation is that the elasticity of the demand is no longer constant but depends on the relative competitor export prices.
Profit maximisation under the AIDS type of export demand function results in the following 2-equation system for the export volume and export price:

$$(\frac{P_X X}{P_{cx} MF}) = a + b \cdot f(time) - (\phi - 1)\,(p_X - p_{CX}) \tag{61}$$

$$p_X = a + \frac{1 + (a + b \cdot f\,(time)) / \,(\phi - 1)}{2 + (a + b \cdot f\,(time)) / \,(\phi - 1)}\,((1 - a_x)\,(w - mpn) + a_x p_M) + \frac{1}{2 + (a + b \cdot f\,(time)) / \,(\phi - 1)} p_{CX} \tag{62}$$

Where P_X = Export deflator (lower case refers to log); X = Export volume; P_{CX}= the external competitor export prices (lower case refers to log); MF = the world demand for exports; w = compensation per worker (log); mpn = marginal product of labour (log); p_M = import deflator (log), a = point market share (with indexed data close to unity); $\phi > 1$ is the representative point price elasticity of exports; b if different from zero measures the deviation of income elasticity of export demand from unity; and a_X = import content of exports (input-output estimate). In estimation an additional free trend variable was allowed in the price equation.

2-equation export system estimates*

	DE	FR	IT	ES	NL
a	1.08 (0.01)	1.131 (0.01)	1.051 (0.01)	1.05 (0.01)	1.025 (0.01)
ø	1.021 (0.03)	1.056 (0.06)	1.22 (0.09)	1.314 (0.09)	1.345 (0.07)
a_X	0.385	0.465	0.4	0.465	0.672
Log-det	-16.49	-16.83	-14.61	-16.65	-14.49
ADF volume-eq.	-3.81	-4.954	-2.095	-3.16	-3.798
ADF price-eq.	-3.735	-2.812	-2.918	-4	-2.482

*Standard errors of estimates in brackets

The dynamic export volume equation follow a conventional error correction equation for the log change of exports.

Dynamic export equation estimates*

	DE	FR	IT	ES	NL
MF	0.846 (0.182)	0.745 (0.11)	0.663 (0.171)	0.68 (0.186)	0.926 (0.078)
P_{CX}/P_X	-0.288 (0.154)	-0.239 (0.1)	-0.573 (0.093)	-0.399 (0.19)	-0.285 (0.087)
Δx^*_{t-1}		-		0.271 (0.12)	
$EC - term$	-0.152 (0.06)	-0.42 (0.06)	-0.157 (0.053)	-0.165 (0.091)	-0.088 (0.042)
R-square D-W	0.41 2.218	0.559 1.948	0.486 1.775	0.281 1.873	0.378 2.174

*Standard errors of estimates in brackets

The dynamic export price equation also follows a conventional error correction equation for the log change in export prices..

Dynamic export price equation estimates*

	DE	FR	IT	ES	NL
ΔP^{x*}		0.795 (0.0817)	0.529 (0.0955)	0.726 (0.0970)	0.93 (0.1092)
ΔP^{x*}_{t-1}		0.177 (0.0801)			
ΔP^{x}_{t-1}			0.2661 (0.0697)		
$\Delta P^{x}_{t-2} - \Delta P^{x}_{t-3}$	0.1333 (0.0353)				
$EC - term$	-0.328 (0.054)	-0.097 (0.0482)	-0.171 (0.0606)	-0.548 (0.1125)	-0.11 (0.0494)
R-square	0.449	0.633	0.468	0.663	0.489
D-W	1.343	1.984	2.005	1.906	2.002

*Standard errors of estimates in brackets

8.5.2 Import formation

As mentioned above, the modelling approach of imports is conventional where the import supply curve is assumed to be horizontal and, hence, import volume is demand determined. Accordingly two driving variables are the domestic demand indicator and the relative price of domestic production and imports. Hence, we specify the long-run equilibrium aggregate demand for imports to depend on the demand indicator (with unit elasticity), the ratio of exports to the demand indicator and the relative domestic-to-import price:

$$m^* - e_{MR} = k(p_{MD} - p_{MN}) + b(x - e_{MR}) \tag{63}$$

where eMR is the demand indicator for imports (import content weighted index of domestic demand);
PMD is domestic prices (gdp deflator); and PMN is import prices excluding energy. All variables in equation 63 are measured in logs. The dynamic equation follows a standard EC specification.

Import equation*

	DE	FR	IT	ES	NL
k	-0.782	-0.711	-1.001	-0.979	-0.576
	(0.01)	(0.01)	-	(0.04)	(0.02)
b	0.312	0.284	-	0.293	0.072
	(0.01)	(0.004)		(0.01)	(0.002)
$Constant$	-0.772	-0.558	-0.47	0.223	-0.522
	(0.04)	(0.04)	(0.01)	(0.03)	(0.01)
R-square	0.974	0.982	0.793	0.981	0.936
ADF	-3.981	-3.704	-4.124	-3.207	-4.032
Dynamic Import equation					
Δm^*	-	0.813	0.568	0.561	0.722
		(0.1)	(0.09)	(0.08)	(0.1)
Δm_{t-2}	-	-	0.132	-	-
			(0.09)		
Δe_{MR}	0.642	-	-	-	-
	(0.08)				
$EC - term$	-0.235	-0.121	-0.117	0.164	-0.229
	(0.06)	(0.05)	(0.05)	(0.05)	(0.07)
R-square	0.446	0.402	0.339	0.394	0.35
D-W	1.825	1.644	1.901	1.827	2.098

*Standard errors of estimates in brackets

Import price equation The equation for the import deflator excluding energy is also quite traditional. It depends on the GDP deflator net of indirect taxes (to capture possible pricing to market effects), and the competitors' import price with static homogeneity condition imposed

$$p_{MN} = \phi_1(p + \log(1 - TX_1)) + (1 - \phi_1)p_{CM} \tag{64}$$

The dynamic equation follows a standard EC specification.
The trade balance and net factor income equal the current account balance, which in turn is cumulated to give the stock of net foreign assets. Disaggregated trade (intra and extra) imports, exports both real and prices are modelled via bridge equations.

Import price (excluding energy equation)*

	DE	FR	IT	ES	NL
ϕ_1	0.333	0.416	0.225	0.312	0.667
$1-\phi_1$	0.667	0.584	0.775	0.688	0.333
	(0.03)	(0.03)	(0.05)	(0.05)	
R-square	0.957	0.99	77	0.991	
D-W	0.561	0.296	0.389	0.741	
Dynamic Import price equation					
Δp^*_{MN}	0.498	0.673	0.427	0.669	0.72
	(0.07)	(0.11)	(0.14)	(0.15)	
$\Delta p^*_{MN,t-1}$	0.344	0.237	-	-	0.26
	(0.08)	(0.14)			
$\Delta p^*_{MN,t-2}$	0.179	-	-	-	-
	(0.07)				
$\Delta p_{MN,t-1}$	-	0.189	0.158	-	-
		(0.1)	(0.09)		
$EC-term$	-0.118	-0.148	-0.235	-0.417	-0.16
	(0.06)	(0.05)	(0.06)	(0.08)	
R-square	0.649	0.487	0.243	0.375	
D-W	1.978	2.167	1.968	2.053	

*Standard errors of estimates in brackets

8.6 Government, Central Banks and Financial Markets

We close the model with the inclusion of the Central Bank which sets monetary policy, governments with a fiscal policy reaction function, and financial markets which are forward-looking and determine exchange rates and long-term interest rates. Although the activation of a monetary policy rule, a fiscal policy rule and an exchange rate UIP rule is required for long-run stability, there are scenarios where the model can be simulated when these aspects are exogenised (e.g. in under scenario of monetary accomodation).

8.6.1 Governments

The fiscal block of the model comprises a set of identities in expenditure and revenue categories. Government receipts are split into a number of components: direct taxes on households earned income (T_f) which includes social contributions split into employers 'and employee's and is determined via a fiscal rule (see below); direct taxes on firms (T_o), and indirect taxes (T_I), which include VAT and excises duties are calculated as exogenous implicit tax rates, and other public income (OI_G), which includes the gross operating surplus. On the expenditure side, the fiscal authority has (net) transfers (TR_F), which includes pensions and unemployment payments. As these vary significantly over the business cycle, transfers as a proportion of nominal GDP (tr) are modelled as a function of the unemployment rate, mainly reflecting the dependency of unemployment compensation to unemployed:

$$TR_F = tr.PY - \varkappa W(N - N_{bas}) \tag{65}$$

where $\varkappa$ is calibrated to 0.7. This means transfers are counter-cyclical - increasing with the unemployment rates.In addition, the fiscal authority has net interest payments on government debt (IN_G) and different types of primary expenditure categories, namely, government consumption (G_N) and government investment (I_{NG}) which are exogenous in real terms but can be shocked as part of a fiscal policy expansion. Finally, the government consumption deflator follows both the price of home produced goods with a weight of δ^G and of imported goods with a weight of (1-δ^G) .
The public deficit (D) each period is then the difference between the recipts and expenditures:

$$D = T_F + T_o + T_I + OI_G - TF_F - IN_G - G_N - I_{NG} \tag{66}$$

The fiscal authority's is faced by a budget constraint which says that public debt B_t is the cumulative sum of past public deficits (D) i.e.

$$B_t = B_{t-1} + D_t \tag{67}$$

As households are non-Ricardian, the path of government debt and taxes matter for the evolution of the economy. Therefore, governments aim to insure stability of the public debt stock. This is modelled via a fiscal policy rule based on a reaction of personal income taxes to the deviation of the government's debt to GDP ratio from its predetermined target and which contributes to adjustment towards the stock-flow equilibrium in the long-run.[14]

$$\Delta\tau_t = \varphi_1(b_{t-1} - \bar{b}) + \varphi_2\Delta b_{t-1} \tag{68}$$

where τ_t is the personal income tax rate (T_F/Y), and b_t is the government debt to GDP ratio (B/Y), and $\bar{b}$ is the target. The parameters $\varphi_1$and φ_2 are set at 0.01 and 0.1 respectively.

8.6.2 Monetary Authority

Households and firms adjust their plans by taking into account the expected response by monetary authorities. The endogenous monetary policy rule provides the nominal anchor to the model and incorporate a smooth interest rate reaction to shocks in the short-run. The monetary policy rule follows a simple Taylor rule specification in which

[14] As the focus in the govenment sector stability target is more in the long than short-run and to strengthen the short run effects of fiscal policy, the fiscal policy reaction function could have longer lags from the debt ratio to the change of the income tax rate, but this would also increase the cyclicality of the model.

the short term interest rate is determined by the inflation gap, where the inflation gap measures the distance between the actual inflation rate and its target level and the output gap $(y_t - \bar{y}_t)$ along with the lagged interest effect. For the purpose of the simulations this target level of inflation $(\Delta\hat{\bar{p}}_t)$ is set to 2 per cent per annum, but it could, in principle, be set to any other *reasonable* level.

$$i_t = 0.7 * i_{t-1} + (1 - 0.7) * (4 * 1.5 * (\Delta p_{t+1} - \Delta\hat{\bar{p}}_t) + 0.5 * (y_t - \bar{y}_t)) + \varepsilon_t \qquad (69)$$

where i_t is the nominal interest rate and ε_t is a serially uncorrelated shock to the interest rate. Regarding the choice of parameters, they are calibrated on the basis of available relevant estimates in the literature and with the view of ensuring a sensible profile of the models' dynamic behaviour[15]. In particular, the parameter governing the speed of reaction to the interest rate gap is set to 0.7. Following general convention, the inflation gap and output gap parameters are set to 1.5 and 0.5 respectively.

8.6.3 Financial Markets

Financial markets are foward-looking. The specification of the long-term interest rate equation is:

$$l_t = 0.9 * l_{t+1} + 0.1 * i_t \qquad (70)$$

The exchange rate follows a standard UIP equation.

$$e_t = e_{t+1} + (i_t^f - i_t)/400 \qquad (71)$$

where e is the (log of) the exchange rate and i_t^f.is the foreign interest rate.

8.7 Linked Model

Cross-country linkages occur through four channels: trade volumes; trade prices; common monetary policy and a common exchange rate. For each given country, all international trade variables related to other country blocks that are part of the linked model should be endogenized in the simulation. The linked variables for each country are import demand, competitor's prices on the import side, foreign demand and competitors prices on the export side. Note that trade variables for countries outside the euro area, such as US, Japan, China and the UK remain exogenous in the linked mode, thereby reflecting the distinction between intra- and extra-euro area trade, where the intra euro-area trade involves trade between euro-area countries. For foreign demand, the demand for country k's exports is expressed in the form of a world demand index, mf, calculated as a weighted average of the import volumes of the trading partners, j, of country k:

$$mf_{kt} = \sum_j x_{k,j,t} \cdot m_{jt} \qquad (72)$$

[15] In principle, a more refined approaches could be utilized here, for instance direct estimation

where $x_{k,j,t}$ is the three-year moving average of the share of total exports going from country k to country j. The weight can be interpreted as the elasticity of export demand of country k with respect to the imports of trading partner j. In equation 72 we can restrict the summation to only include countries belonging to the euro area, we can define the intra euro area export demand index for country k as follows:

$$mf_{kt}^{intra} = \sum_{j \in euro \ area} x_{k,jt}^{in} \cdot m_{jt} \tag{73}$$

In this way, in the linked-block this becomes endogenous. Similarly, we can define an extra euro area export demand index for country k by only including countries outside the euro area, which remains exogenous.
Whilst there is a distinction between modelled intra-euro area countries, and non-modelled (extra-euro area countries), the linkage is currently based on total country imports volumes, mjt. Therefore fully consistency is not achieved. A further extension is to break the trade into intra and extra components, and link directly the intra trade component, and thereby ensure fully consistency. This will be investigated in the future. Similarly, on the price side linkages are currently on total export prices, via competitor's prices using a double weighting scheme[16].

$$p_{cxut} = \sum_j x_{k,jt} \cdot p_{xu} \tag{74}$$

where p_{xu} = log of export prices in dollars, p_{cxu} = log of external competitor export prices in dollars.
Just as in the case of the export demand index, we can decompose competitors' prices on the export side, for country k, into intra and extra euro area components. A similar computation is done for competitor's prices on the import side. Note that the competitors' prices, for country k, are measured in US dollars and need to be converted back into euros.
The other linkages occur via monetary policy and exchange rate rules which then follow a common euro area rule, i.e. interest rates follow the Taylor based rule reacting to a weighted average of the 5 countries expected inflation and output gaps.

9 Figures

[16]See Karlsson and Hubich (2008) for further information on trade linkage framework.

Figure 1: Short−Term Interest Rate shock − Germany

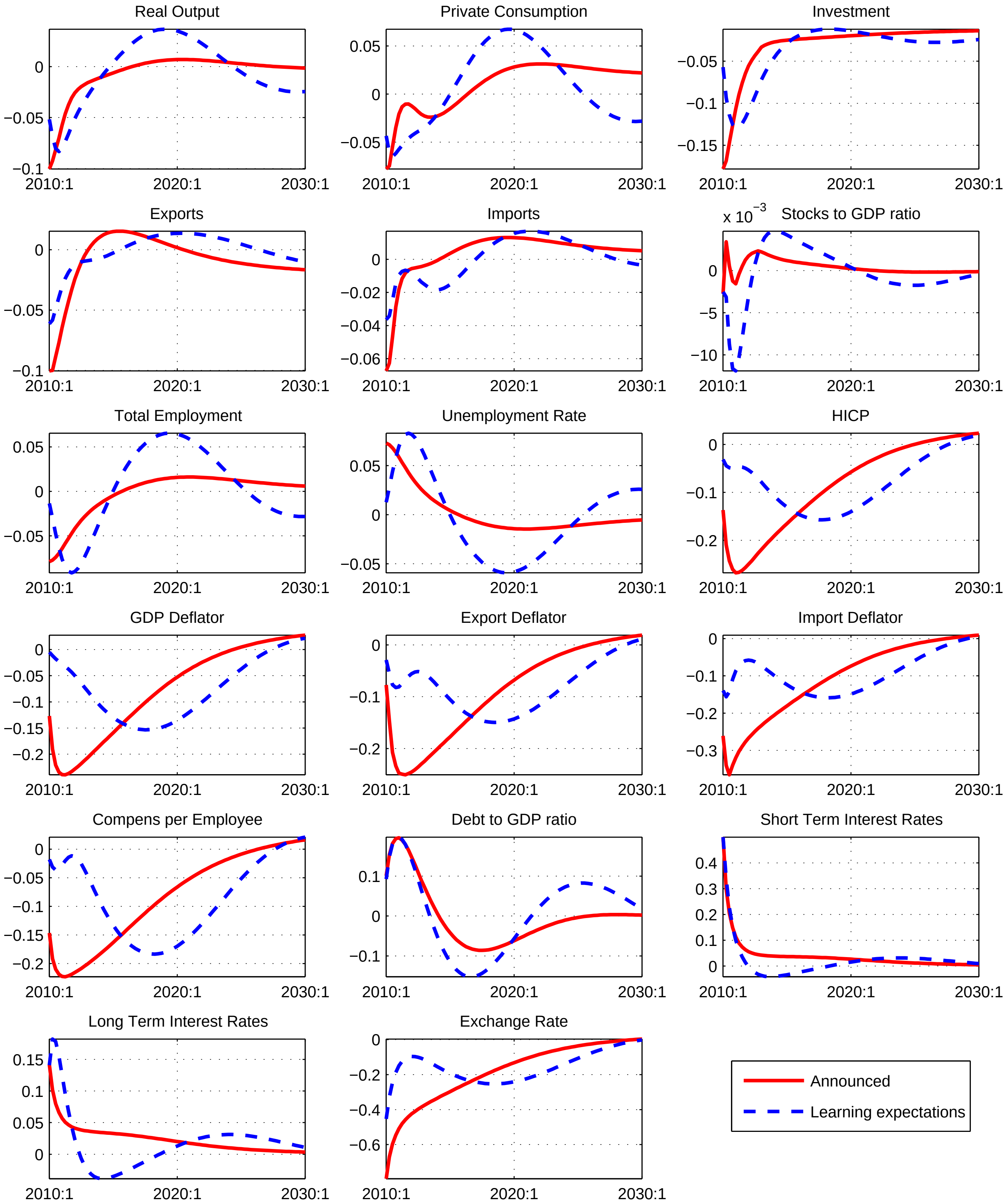

Figure 2: Announced, unannounced and learning 3−year shock
to Government Consumption (0.5 % of GDP): Germany

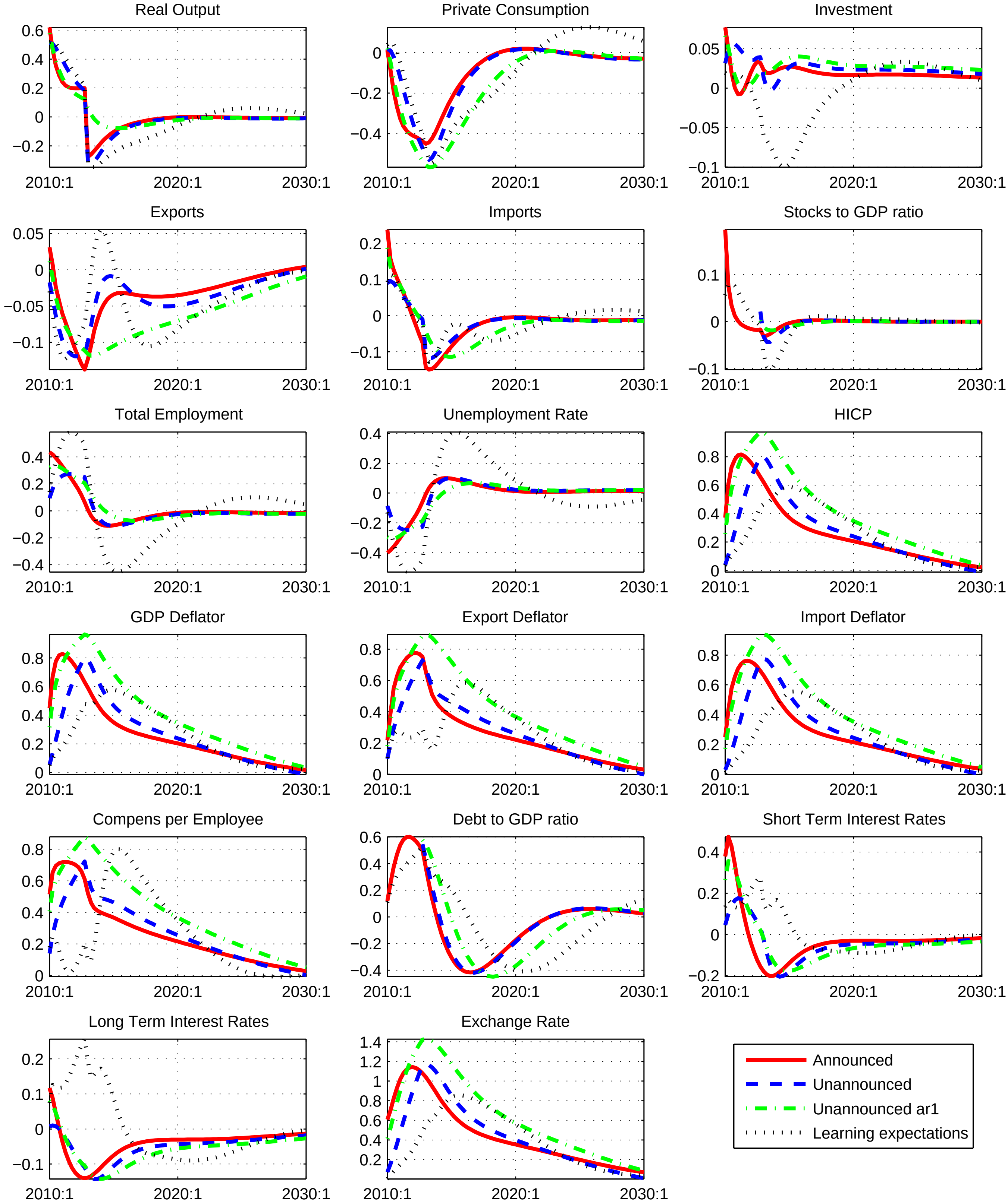

Figure 3: Announced Permanent Government Consumption

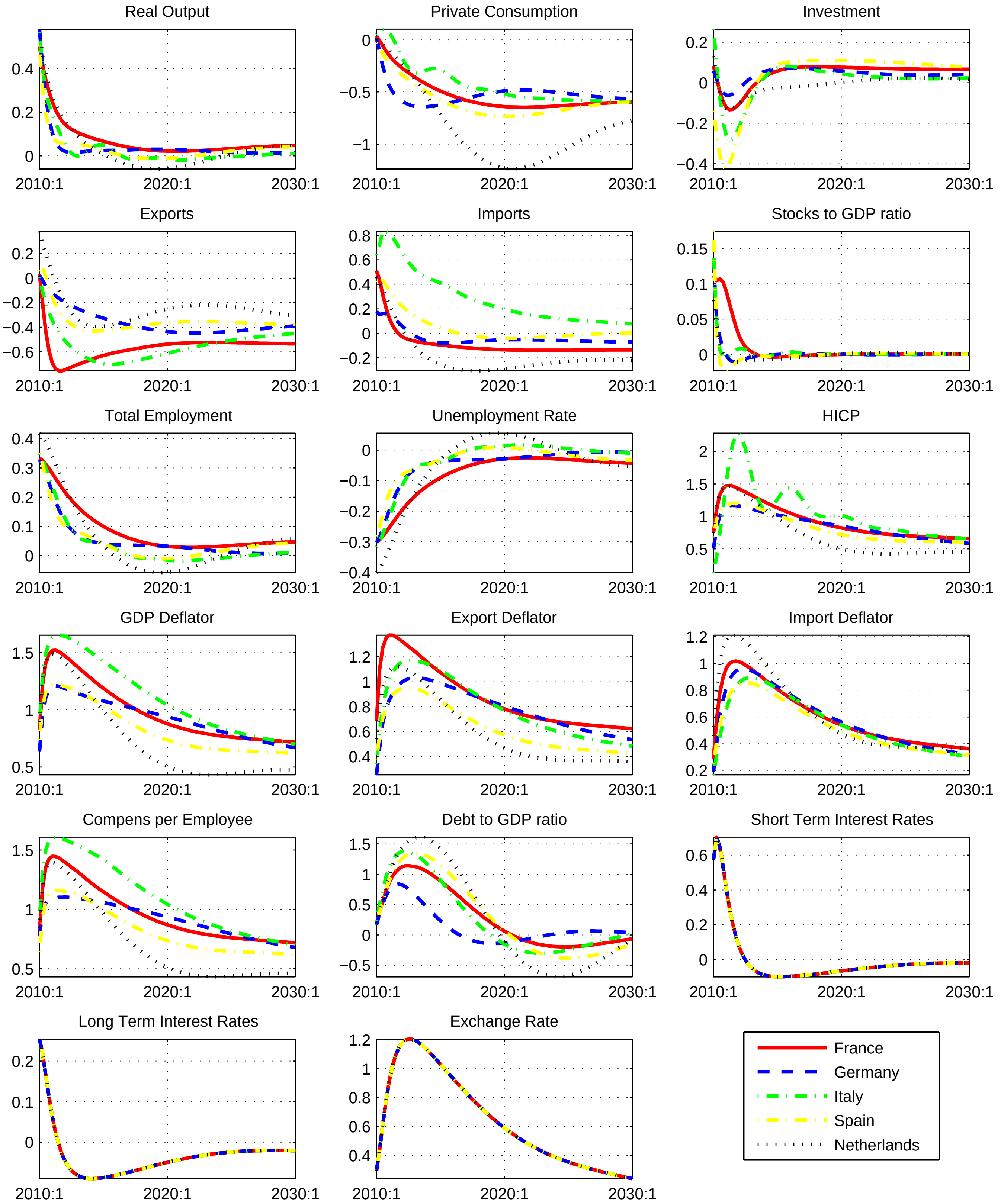

Figure 4: Learning expectations Permanent Government Consumption

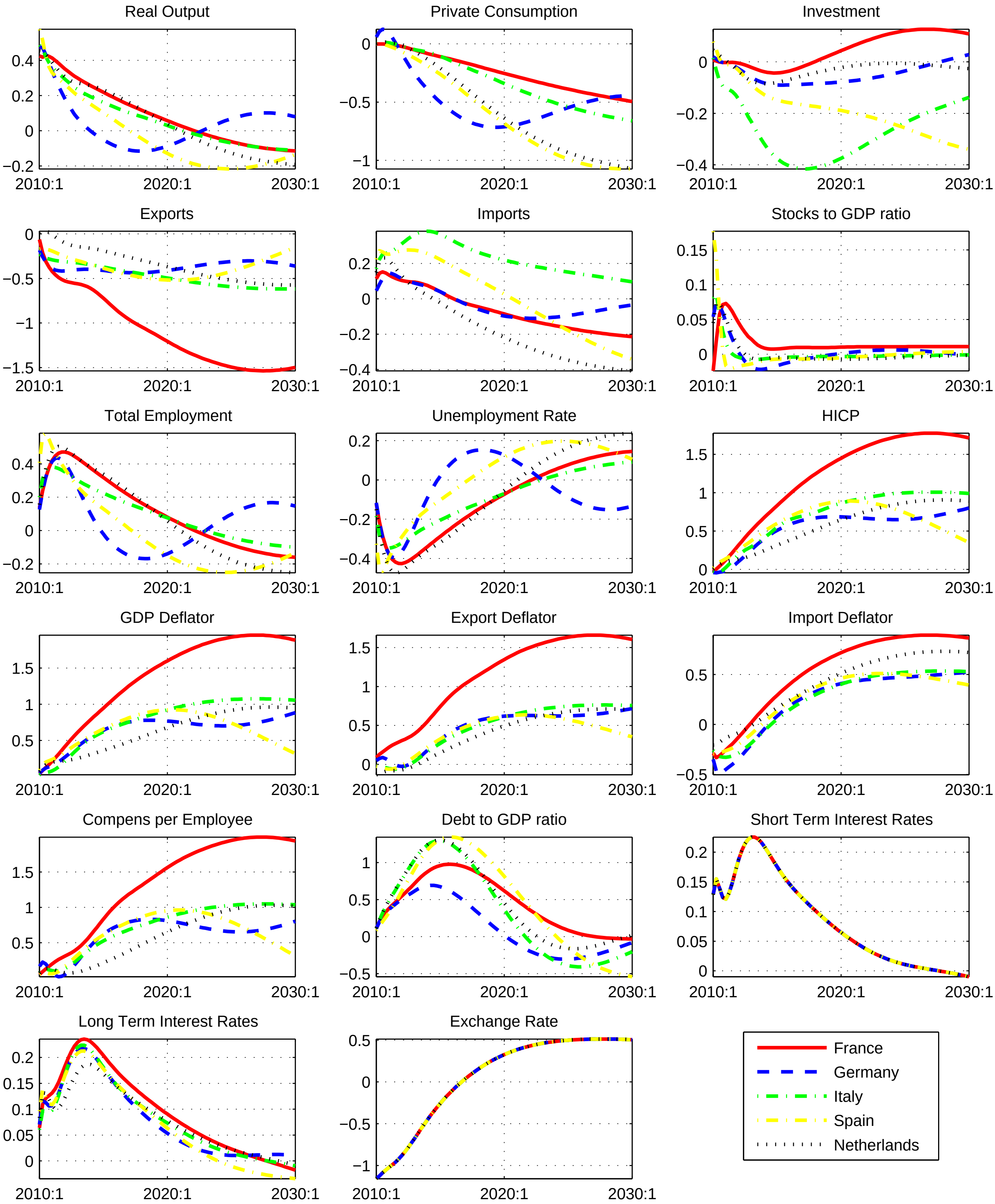

Figure 5: Permanent shock to Government Consumption (0.5 % of GDP): Exogenous interest rates; Impact on Real Output