Structured portfolio analysis under SharpeOmega ratio

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Preliminary version (Mai 2010)

Abstract

This paper deals with performance measurement of financial structured products. For this purpose, we introduce the SharpeOmega ratio, based on put as downside risk measure. This allows to take account of the asymmetry of the return probability distribution. We provide general results about the optimization of some standard structured portfolios with respect to the SharpeOmega ratio. We determine in particular the optimal combination of Risk free, stock and Call/Put instruments with respect to this performance measure. We show that, contrary to Sharpe ratio maximization (Goetzmann *et al.,* 2002), the payoff of the optimal structured portfolio is not necessarily increasing and concave.

Key words: Structured portfolio, Performance measure, SharpeOmega ratio.

JEL Classification: C 61, G 11.

1 Introduction

Structured investments have been initially introduced by firms that searched for cheaper issue debt. For instance, convertible bonds can be sold instead of standard bonds to allow the conversion to equity. Structured products have been further extended to combinations of derivatives and financial instruments in order to provide funds with better risk/return profiles that are not always directly available on the financial market. They have became rather popular in the US in the 1980s and further introduced in Europe since the mid-1990s.

These products are intoduced to provide investors with highly targeted investments related to their performance objectives and risk profiles. They are created to satisfy specific needs that cannot be provided by standardized financial instruments, usually available in the financial markets. One of their main characteristics is fixed maturity. They are based on combinations of financial assets such as bonds, shares or indices, commodities,...and derivatives. The

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derivative component is often an option (Put or Call) chosen to provide some specific portfolio profile at maturity, while the other component is generally a note that delivers interest payments. Some structured products are linked to portfolio insurance. They guarantee a predetermined amount at maturity (usually a fixed percentage of the initial invesment) whatever market fluctuations.

Structured products allow complex positions in options without the need for access to option market. There exists a large variety of such products, since a large number of underlyings and options can be introduced. The main classes are asset-linked notes and equity-linked notes and deposits, where the financial asset may be interest rate, equity, hybrid product, credit product, FX and commodity...(see Das (2000) for classifications of structured products). These products can incorporate plain-vanilla options 'corridor, turbo...) or exotic options such as barrier and rainbow products.

The value of options, swaps... is determined from underlying asset prices. However, some mispricing may occur. Chen and Kensinger (1990) have examined Market-Index-Certificates of Deposit (MICD) in the US market, during a period of two months in 1988 and 1989. Using a comparison of the implied volatility of the S&P 500 option with the implied volatility of the MICDs option components, they have shown that significant differences exist between theoretical and market values. Chen and Sears (1990) also illustrate this feature for the S&P 500 Index Note (SPIN). Wasserfallen and Schenk (1996) has been led to the same conclusion when examining the pricing of capital-protected products issued in 1991/1992 in the Swiss market.

Wilkens *et al.* (2003) have analyzed the German market through a large data set of classic structured products, traded in November 2001. They find "evidence of an overpricing of structured products, which can mostly be interpreted as in favor of the issuing institution." Stoimenov and Wilkens (2005) have also studied the German structured products, including in particular implicit exotic option components such as barrier and rainbow options. Their results suggest that "all types of equity-linked structured products are, on average, priced above their theoretical values and thus favor the issuing institution."

Main potential benefits of structured products are guarantees according to the kind of structured product and enhanced returns depending on portfolio profiles. But structured products may also be illiquid, may include credit risk and be not daily priced. Additionally, they are often quite complex and their performance and risk evaluations are note easy to handle. Payoffs of structured products are non linear with respect to the underlying asset. This feature implies asymmetric return distributions. Their risks are similar to those of options and their return distributions are far from being lognormal. Therefore, we must be careful when evaluating their risks and performances. We must search for new performance measures, alternative to standard Sharpe ratio or Jensen alpha, to overcome shortcomings of performance measures based only on the first moments of the return distributions. Such performance measures are usually defined as "reward/risk" but, contrary for example to the Sharpe ratio, the risk measure is downside and tries to take the whole return distribution into account (see e.g. Pedersen and Satchell, 1998; Artzner *et al.*, 1999; Szegö, 2002).

Keating and Shadwick (2002) have introduced such kind of risk measure to define a new performance measure based on a gain-loss approach. This one is called the Omega measure. It takes account of investor loss aversion, which is in line with results of Tversky and Kahneman (1992). It has been applied in finance to examine or instance equities or hedge funds. The Omega measure is equal to the ratio of the expected gains and the expected losses, defined with respect to a given threshold. As noted by Kazemi *et al.* (2004), it corresponds to the ratio of the expectations of a call option divivided by a put option written on the underlying asset. The strike price is the given threshold. The SharpeOmega measure, introduced by Kazemi *et al.* (2004), is equal to the Omega measure minus 1. Such measure has been previously introduced to examine performance of some structured products, such as those related to portfolio insurance. Bertrand and Prigent (2008) use the Omega performance measure to compare standard portfolio insurance strategies. They show that the CPPI method provides better results than the OBPI one for "rational" thresholds. Non normal distributions can also be proposed to model structured product returns, for example Johnson distributions. In this framework, Perez (2004) have used the Omega approach to test adeaquacy of these distributions. Passow (2005) provide explicit representations for Omega and SharpeOmega with all four Johnson distributions. Using a Hedge fund index is back-testing, he shows that Johnson-Omega provides significantly higher returns.

Others researchers have focused on the problem of portfolio allocations in order to maximize Omega. (see Avouyi-Dovi *et al.* 2004). Empirical results show that this measure is more stable than other risk measures such as RoC-VaR , RoVaR and Sharpe (see Hentati *et al.* 2010) but it has many local solutions because of the non-convexity of Omega function. The resolution of the global optimum is proposed by Bartholomew-Biggs *et al.* (2009) by using a NAG library implementation of the Huyer & Neumaier MCS method. Based on another approach, Hentati and Prigent (2010) introduce Gaussian mixtures to model empirical distributions of financial assets and solve the portfolio optimization problem in a static way, taking account of discrete time portfolio rebalancing.

The search for "optimal" structured products has been previously examined both from the insurance portfolio and the financial optimal positioning point of views. Leland and Rubinstein (1976) have introduced the option based portfolio insurance (OBPI). It consists of a portfolio invested in a risky underlying asset S (usually a financial index such as the $S\&P$) covered by a listed put written on it. Whatever the value of S at maturity T, the portfolio value will be always higher than the strike K of the put. At maturity, the investor can limit downside risk while participating in upside markets. More specific insurance constraints can be considered and utility maximization can be solved (see e.g. Bertrand *et al.* (2001), El Karoui *et al.* (2005) and Prigent (2006) for quite general insurance constraints). The optimal positioning can be also examined within rank dependent expected utilities (RDEU) as in Jin and Zhou (2008) for the dynamic case and Prigent (2008) for the static case. In this framework, the choice of the threshold can be further examined (see Pfiffelmann, 2005;

Pfiffelmann and Roger, 2008; Roger, 2008). Hens and Riger (2008) examine various features of structured products from the customer's perspective.

In this paper, we propose to analyze structured products by using the SharpeOmega ratio. It is well known that the Sharpe ratio can be manipulated by option-like strategies. In this context, Goetzmann *et al.* (2002) determine portfolio strategies which maximize the Sharpe ratio. They derive general conditions to achieve the maximum. They prove that appropriate combinations of puts and calls lead to significantly higher Sharpe ratios than "linear" portfolios. Our approch is quite similar, except that we use the SharpeOmega ratio instead of the Sharpe ratio itself. For this purpose, we consider a portfolio manager who invests in three assets: a free risk market account, denoted by B, a risky asset (equity), denoted by S and Call/Put written on this equity. Our aim is to maximize and analyze SharpeOmega ratio under given constraints. We begin by determining the necessary conditions to determine precisely the downside risk component. Subsequently, we study the minimization problem of the Put under the constraint of fixed expectation. The paper is organized as follows. Section 2 recalls definitions and main properties of the Omega and Sharpe Omega measures. Section 3 deals with various portfolio optimizations with respect to these ratios. We prove that, unlike the result of Goetzmann *et al.* (2002) related to the Sharpe ratio maximization, the payoff of the optimal structured portfolio is not always increasing and concave. It can correspond for instance to a straddle. This result is in line with previous results about portfolio optimization within rank dependent utility, as in Prigent (2008).

2 Omega measure

The Omega measure is based on the portfolio return values below and above a given threshold. It is defined as the probability weighted ratio of gains to losses relative to a return threshold. It takes the entire return distribution into account. The Omega measure is compatible with the second order stochastic dominance. This measure can potentially take account of the whole probability distribution of the returns. It requires no parametric assumption on the distribution and is equal to:

$$\Omega_F(L) = \frac{\int_L^b (1 - F(x))\, dx}{\int_a^L F(x)\, dx} = \frac{I_2(L)}{I_1(L)}, \tag{1}$$

where $F(.)$ is the cdf of the random variable (for example equal to the portfolio return) defined on the interval $[a, b]$. The level L is the threshold chosen by the investor: returns smaller than L are viewed as losses (wich correspond to $I_1(L)$) and those higher than L ($I2(L)$) are gains. Thus, for a given threshold L, the investor would prefer the portfolio with the highest Omega measure.

As shown by Kazemi, Schneeweis and Gupta (2003), the Omega function is

equal to:

$$\Omega_F(L) = \frac{\mathbb{E}_{\mathbb{P}}\left[(X-L)^+\right]}{\mathbb{E}_{\mathbb{P}}\left[(L-X)^+\right]}. \tag{2}$$

This is the ratio of the expectations of gains above the given level L upon the expectation of losses below . Therefore, $\Omega_F(L)$ can be interpreted as a ratio Call/Put defined on the same underlying asset X, with strike L and computed with respect to the historical probability $\mathbb{P}$. The Put correspond to the risk measure component. It allows the control of the losses below the threshold L.

Kazemi *et al.* (2003) define the Sharpe Omega by:

$$Sharpe - Omega = \Omega_F(L) - 1 = \frac{\mathbb{E}_{\mathbb{P}}[X] - L}{\mathbb{E}_{\mathbb{P}}\left[(L-X)^+\right]}. \tag{3}$$

3 The model

3.1 Structured portfolio 1

We assume that the portfolio is composed of α money market account, denoted by B, β risky asset (an equity for example) and γ Put option. The time period is $[0, T]$. Thus, the portfolio value V_T is given at maturity T by:

$$V_T = \alpha B_T + \beta S_T + \gamma(K - S_T)^+, \tag{4}$$

where S_T is the value of the risky asset at maturity T and K is the strike price of Put.

The dynamics of the market value of the risky asset S are given by the classic diffusion process:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t, \tag{5}$$

where μ and σ are respectively the drift and the volatility of S_t, W is a standard Brownian motion.

Thus we have:

$$S_T = S_0 \exp[(\mu - \frac{1}{2}\sigma^2)T + \sigma W_T],$$

where $(W)_T$ is a standard Brownian motion.

The return expectation the asset S at time T is given by:

$$\mathbb{E}(S_T) = S_0 \exp[\mu T]. \tag{6}$$

The value of the riskless asset B evolves according to:

$$dB_t = B_t r dt. \tag{7}$$

In this context, the value of the initial investment amount is given by:

$$V_0 = \alpha B_0 + \beta S_0 + \gamma P_0(K), \tag{8}$$

The quantity γ invested in the Put can then be written as follows:

$$\gamma = \frac{V_0 - \alpha B_0 - \beta S_0}{P_0(K)}. \tag{9}$$

where $P_0(K)$ is the value of the Put option at $t = 0$.

3.1.1 Sharpe-omega's risk component analysis

As mentioned in the introduction, the risk component correspond to the expectation $\mathbb{E}_{\mathbb{P}}\left[(L - V_T)^+\right]$. In what follows, we calculate $\mathbb{E}_{\mathbb{P}}\left[(L - V_T)^+\right]$.

Let f be the portfolio payoff. We have: $V_T = f(S_T)$. Then we get:

$$\begin{aligned} \mathbb{E}_{\mathbb{P}}\left[(L - V_T)^+\right] &= \int [L - f(s)]^+ \, d\mathbb{P}_{S_T}(s), \\ &= \int_{f(S_T) \leq L} [L - f(s)] \, d\mathbb{P}_{S_T}(s). \end{aligned}$$

If the portfolio is the asset combination given in (8), than we have:

$$\mathbb{E}_{\mathbb{P}}\left[(L - V_T)^+\right] = \int_{f(S_T) \leq L} \left(L - \alpha B_T - \beta s - \gamma (K - s)^+\right) d\mathbb{P}_{S_T}(s).$$

We begin by determining the integration domain for S_T, defined by set D.

$$D = \{s \,|\, f(s) \leq L\}.$$

Thus, according to the values of α, β and γ, f has three different shapes: We distinguish two cases:

- $\beta < \gamma$
- $\beta > \gamma$

<u>CASE 1 :$\beta > \gamma$</u>

If $\beta > \gamma$, then f becomes an increasing function and D is not empty if $L \geq \alpha B_T + \gamma K$.

So we have two sub-cases:

a.1 $L \leq \alpha B_T + \beta K$ then $S_T \leq \frac{L - \alpha B_T - \gamma K}{\beta - \gamma}$;

a.2 $L > \alpha B_T + \beta K$ and so $S_T \leq \frac{L - \alpha B_T}{\beta}$.

In what follows, we set:

$$k_1 = \frac{L - \alpha B_T - \gamma K}{\beta - \gamma}$$

and

$$k_2 = \frac{L - \alpha B_T}{\beta}$$

In the (a.1) case, $\mathbb{E}_{\mathbb{P}}\left[(L - V_T)^+\right]$ is written as follows :

$$\mathbb{E}_{\mathbb{P}}\left[(L - V_T)^+\right] = \int_0^{k_1} (L - \alpha B_T + (\gamma - \beta)\, s - \gamma K)\, d\mathbb{P}_{S_T}(s). \tag{10}$$

We then get the equation :

$$\mathbb{E}_{\mathbb{P}}\left[(L - V_T)^+\right] = (L - \alpha B_T - \gamma K) \int_0^{k_1} d\mathbb{P}_{S_T}(s) + (\gamma - \beta) \int_0^{k_1} s d\mathbb{P}_{S_T}(s). \tag{11}$$

where

$$\int_0^{k_1} d\mathbb{P}_{S_T}(s) = \phi\left(-\frac{\overline{z} - \ln k_1}{\sigma\sqrt{T}}\right),$$

and

$$\int_0^{k_1} s d\mathbb{P}_{S_T}(s) = \exp\left(\overline{z} + \frac{\sigma^2}{2}T\right) \phi\left(-\frac{\overline{z} - \ln k_1 + \sigma^2 T}{\sigma\sqrt{T}}\right),$$

Notations: $\overline{z} = \ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right) T$ and $\phi(.)$ is the standard cumulative normal function.[1]

In conclusion,

If $\beta > \gamma$ and $\alpha B_T + \gamma K \leq L \leq \alpha B_T + \beta K$

$$\begin{aligned} \mathbb{E}_{\mathbb{P}}\left[(L - V_T)^+\right] &= (L - \alpha B_T - \gamma K)\left[\phi\left(-\frac{\overline{z} - \ln k_1}{\sigma\sqrt{T}}\right)\right] \\ &\quad + (\gamma - \beta) \exp\left(\overline{z} + \frac{\sigma^2}{2}T\right) \phi\left(-\frac{\overline{z} - \ln k_1 + \sigma^2 T}{\sigma\sqrt{T}}\right) \end{aligned} \tag{12}$$

let:

$$A = \phi\left(-\frac{\overline{z} - \ln k_1}{\sigma\sqrt{T}}\right)$$

and

$$B = \phi\left(-\frac{\overline{z} - \ln k_1 + \sigma^2 T}{\sigma\sqrt{T}}\right)$$

Equation (12) becomes:

$$\mathbb{E}_{\mathbb{P}}\left[(L - V_T)^+\right] = (L - \alpha B_T - \gamma K)\, A + (\gamma - \beta) \exp\left(\overline{z} + \frac{\sigma^2}{2}T\right) B$$

[1] We suppose that S is lognormally distributed.

However, in (a.2) case we get :

$$\begin{aligned}
\mathbb{E}_{\mathbb{P}}\left[(L-V_T)^+\right] &= \int_0^K (L-\alpha B_T-\beta s-\gamma K+\gamma s)\, d\mathbb{P}_{S_T}(s) \\
&\quad + \int_K^{k_2} (L-\alpha B_T-\beta s)\, d\mathbb{P}_{S_T}(s) \\
&= \int_0^K (L-\alpha B_T-\gamma K)\, d\mathbb{P}_{S_T}(s) + \int_0^K (\gamma-\beta)\, s d\mathbb{P}_{S_T}(s) \\
&\quad + \int_K^{k_2} (L-\alpha B_T)\, d\mathbb{P}_{S_T}(s) + \int_K^{k_2} (-\beta s)\, d\mathbb{P}_{S_T}(s).
\end{aligned}$$

See Appendix 1 for more details.

CASE 2 :$\beta \leq \gamma$

When we have $\beta \leq \gamma$, inequation $f(S_T) \leq L$ can have solutions only if $L \geq \alpha B_T + \gamma K$. Similarly, two sub-cases are treated:

b.1 $L \leq \alpha B_T + \gamma K \Rightarrow$

$$k_1 \leq S_T \leq k_2;$$

b.2 $L > \alpha B_T + \gamma K \Rightarrow$

$$S_T \leq k_2.$$

Thus, we obtain the limits of S_T according the the value of β et γ.
We perform the same calculation for the case (b.1) and (b.2).
The first case gives the following value of $\mathbb{E}_{\mathbb{P}}\left[(L-V_T)^+\right]$:

$$\begin{aligned}
\mathbb{E}_{\mathbb{P}}\left[(L-V_T)^+\right] &= \int_{k_1}^{k_2} \left(L-\alpha B_T-\beta s-\gamma (K-s)^+\right) d\mathbb{P}_{S_T}(s) \\
&= \int_{k_1}^{K} (L-\alpha B_T+(\gamma-\beta)\, s-\gamma K)\, d\mathbb{P}_{S_T}(s) \\
&\quad + \int_K^{k_2} (L-\alpha B_T-\beta s)\, d\mathbb{P}_{S_T}(s)
\end{aligned}$$

Finally, in (b.2) case, we have:

$$\mathbb{E}_{\mathbb{P}}\left[(L-V_T)^+\right] =$$

$$\int_0^{k_2} (L-\alpha B_T-(\beta-\gamma)\, s-\gamma K)\, d\mathbb{P}_{S_T}(s) + \int_K^{k_2} (L-\alpha B_T-\beta s)\, d\mathbb{P}_{S_T}(s).$$

Detailed calculation is provided in Appendix 1.

3.2 Structured portfolio 3

The last case is a portfolio formed by β risky instrument combined with α Call and γ Put written on the risky instrument.

The time period is $[0, T]$. Thus, the portfolio value V_T is given at maturity T by:

$$V_T = \alpha(S_T - K_C)^+ + \beta S_T + \gamma(K_P - S_T)^+, \tag{13}$$

where K_C is the stike price of Call and K_P is the strike price of the Put.

The value of the initial investment amount is given by:

$$V_0 = \alpha C_0(K_c) + \beta S_0 + \gamma P_0(K_P), \tag{14}$$

4 Maximizing the Sharpe-Omega

<u>Maximizing under the constraint of fixed expectation</u>

$$\underset{(\alpha,\beta,\gamma)}{Max}\ S\Omega_L\left(V_T\right) = \underset{(\alpha,\beta,\gamma)}{Max}\ \frac{\left(\overline{V_T} - L\right)}{\mathbb{E}\left[\left(L - V_T\right)^+\right]}. \tag{15}$$

The parameter L is a return parameter $(L > 0)$ satisfying $L < \overline{V_T}$. We maximize this function under the budget constraint V_0.

4.1 Structured Portfolio 1

4.1.1 Conditions on portfolio weights

Fixing M yields to

$$\alpha B_T + \beta S_0 e^{\mu T} + \gamma \mathbb{E}_{\mathbb{P}}\left[K - S_T\right]^+ = M > L \tag{16}$$

where $\mathbb{E}_{\mathbb{P}}\left[K - S_T\right]^+$is like a Put à la Black and Scholes without the discount factor. We denote it by:

$$BS\left(\mu\right)e^{\mu T}. \tag{17}$$

This is a problem with a single variable:β and γ are driven by:

$$\begin{cases} \alpha B_0 + \beta S_0 + \gamma P_0\left(K\right) = V_0 \\ \beta S_0 e^{\mu T} + \gamma BS\left(\mu\right)e^{\mu T} = M - \alpha B_T \end{cases}$$

Thus, β and γ could be expressed as function of α.

$$\beta = \alpha\left(\frac{-BS\left(\mu\right)e^{\mu T}B_0 + P_0\left(K\right)B_0}{\Delta}\right) + \frac{BS\left(\mu\right)V_0 e^{\mu T} - P_0\left(K\right)M}{\Delta} \tag{18}$$

Details of expression are provided in Appendix 2

β is then written as $a_{11}\alpha+b_{11}$, where

$$a_{11} = \frac{-BS(\mu)\, e^{\mu T} B_0 + P_0(K)\, B_0 e^{rT}}{\Delta} \tag{19}$$

and

$$b_{11} = \frac{BS(\mu)\, V_0 e^{\mu T} - P_0(K)\, M}{\Delta} \tag{20}$$

Similarly we determine γ:

$$\gamma = \alpha \left(\frac{-S_0 B_0 e^{rT} + B_0 S_0 e^{\mu T}}{\Delta} \right) + \frac{S_0 M - S_0 e^{\mu T} V_0}{\Delta}.$$

γ is then written as $a_{21}\alpha+b_{21}$, where:

$$a_{21} = \frac{S_0 B_0}{\Delta} \left(e^{\mu T} - e^{rT} \right), \tag{21}$$

and

$$b_{21} = \frac{S_0}{\Delta} \left(M - S_0 e^{\mu T} V_0 \right). \tag{22}$$

Proposition 1 *Parameters a_{11} and a_{21} are non positive. The function $b_{11}(M)$ is always increasing and $b_{21}(M)$ is decreasing.*

See Appendix 3 for proof.

4.1.2 Constraints on portfolio optimization

The maximization problem is subject to the following constraints:

- <u>Condition of positivity of V_T</u> : $V_T \geqslant 0$
  This condition is expressed as follows:
  - $\alpha B_T + \gamma K \geq 0$, we obtain :

    $$\alpha B_T + a_{21} \times K \times \alpha + K b_{21} \geq 0 \tag{23}$$

    $\Rightarrow$

    $$\alpha \left(-B_T - a_{21} \times K \right) \leq K b_{21}$$
  - The second constraint is written as follows:

    $$\alpha B_T + \beta K \geq 0$$

    and so:

    $$\alpha B_T + a_{11} \times K \times \alpha + K b_{11} \geq 0 \tag{24}$$

    $\Rightarrow$

    $$\alpha \left(-B_T - a_{11} \times K \right) \leq K b_{11}$$
  - $\beta \geq 0$, then $-a_{11}\alpha \leq b_{11}$

4.2 Structured Portfolio 2

4.2.1 Conditions on weights

We determine the optimal combination of Call/Put and Stock whin respect the Sharpe Omega measure. We search α, β and γ parameters. We solve this problem under the budget constraint V_0 :

$$V_0 = \alpha C_0(K_c) + \beta S_0 + \gamma P_0(K_P), \tag{25}$$

In this case, we find (β, γ) such that:

$$\begin{cases} V_0 - \alpha C_0(K_c) = \beta S_0 + \gamma P_0(K_P), \\ M - \alpha \mathbb{E}_{\mathbb{P}} \left[S_T - K_C\right]^+ = \beta S_0 e^{\mu T} + \gamma \mathbb{E}_{\mathbb{P}} \left[K_P - S_T\right]^+ \end{cases}$$

Thus, we find β :

$$\beta = \frac{\begin{vmatrix} M - \alpha \mathbb{E}_{\mathbb{P}} \left[S_T - K_C\right]^+ & \mathbb{E}_{\mathbb{P}} \left[K_P - S_T\right]^+ \\ V_0 - \alpha C_0(K_c) & P_0(K_P) \end{vmatrix}}{\Delta}, \tag{26}$$

where

$$\Delta = \begin{vmatrix} S_0 e^{\mu T} & \mathbb{E}_{\mathbb{P}} \left[K_P - S_T\right]^+ \\ S_0 & P_0(K_P) \end{vmatrix}$$

We obtain :

$$\beta = \alpha \left(\frac{C_0(K_c)\mathbb{E}_{\mathbb{P}} \left[K_P - S_T\right]^+ - P_0(K_P)\mathbb{E}_{\mathbb{P}} \left[S_T - K_C\right]^+}{\Delta} \right) \tag{27}$$

$$+ \frac{MP_0\left(K\right) - V_0 \mathbb{E}_{\mathbb{P}} \left[K_P - S_T\right]^+}{\Delta} \tag{28}$$

β is then written as $a_{12}\alpha + b_{12}$, where

$$a_{12} = \frac{C_0(K_c)\mathbb{E}_{\mathbb{P}} \left[K_P - S_T\right]^+ - P_0(K_P)\mathbb{E}_{\mathbb{P}} \left[S_T - K_C\right]^+}{\Delta} \tag{29}$$

and

$$b_{12} = \frac{MP_0\left(K\right) - V_0 \mathbb{E}_{\mathbb{P}} \left[K_P - S_T\right]^+}{\Delta} \tag{30}$$

Similarly we determine γ:

$$\gamma = \frac{\begin{vmatrix} S_0 e^{rT} & M - \alpha \mathbb{E}_{\mathbb{P}} \left[S_T - K_C\right]^+ \\ S_0 & V_0 - \alpha C_0(K_c) \end{vmatrix}}{\Delta}, \tag{31}$$

and γ is then written as $a_{22}\alpha + b_{22}$, where:

$$a_{22} = \frac{S_0 \mathbb{E}_{\mathbb{P}} \left[S_T - K_C\right]^+ - C_0(K_c) S_0 e^{\mu T}}{\Delta}, \tag{32}$$

and

$$b_{22} = \frac{V_0 S_0 e^{\mu T} - S_0 M}{\Delta}. \tag{33}$$

4.2.2 Constraints on portfolio optimization

We maintain the same constraints as in the first structured portfolio:

- <u>Condition of positivity of V_T</u> : $V_T \geqslant 0$

  This yields to these conditions on alpha :

  - $$\gamma \geq 0$$

    Thus :

    $$a_{22} \times \alpha + b_{22} \geq 0 \tag{34}$$

  - $$\alpha \left(K_P - K_C\right)^+ + \beta K_P \geq 0 \tag{35}$$

    $\Rightarrow$

    $$\left(\left(K_P - K_C\right)^+ + a_{12} K_P\right) \alpha + K_P b_1 \geq 0$$

  - And, the last constraint is:

    $$\beta K_C + \gamma \left(K_P - K_C\right)^+ \geq 0 \tag{36}$$

5 Empirical illustration

5.1 Portfolio with One Put

5.1.1 Portfolio including risk-free instrument

In this section, we calculate the Sharpe-Omega ratio of a portfolio composed of risk-free asset B, risky asset S and a Put on S (Stuctered portfolio 1). We assume a single period optimization problem with a statistic allocation (determined at the beginnig of the period).

We examine optimal portfolio payoffs according to various values of strike. We consider these parameters values:

$$S_0 = 100, B_0 = 1, \sigma = 0.15, r = 0.03, T = 1, V_0 = 1000, L = 1000$$

We search for W_α^*, W_β^* and $W\gamma^*$, the weighting which maximizes Sharpe omega. For this purpose, first we determine the optimal solution for fixed expectation M. Then, in a second step, we search the optimal value W_α^* by varying the level of M . The optimal solution corresponds to $W_\alpha = W_\alpha^*$ that maximizes $S\Omega^*$.

The first result emerging from the analysis of optimal ratio sensitivity to the Strike K is illustrated in Figure 1: the optimal ratio decreases for values of K varying from 90 to 100 and then increases for K below 90.

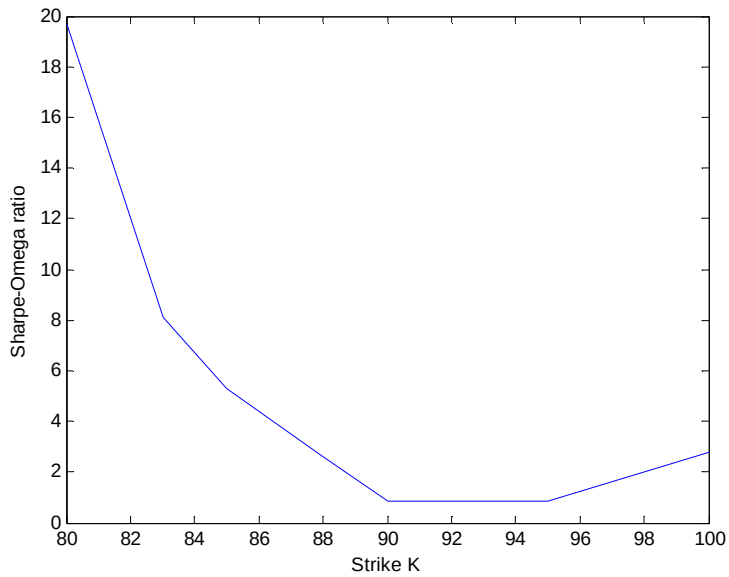

Figure 1: Sharpe-Omega ratio for K varying between 80 and 100

Indeed, the optimal portfolio structure changes dramatically when K becomes slightly smaller than 100. Optimal portfolio when K is ranged between 90 and 100 have the same shape doesn't contain risky instrument (beta is zero). First nonzero allocation to risky asset appears for K below 90.

For Strike $K = 100$, the maximum Sharpe-Omega ratio is ($S\Omega^*_{K=100} = 2.75$) (cf. Fig 2) compared to 0.76 for the risky asset. The maximum ratio is reached for $M = 1026$.

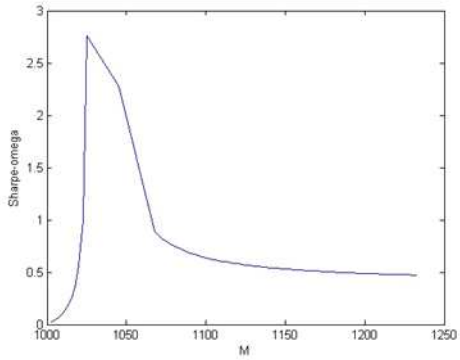

Figure 2 : Sharpe-Omega ratio function of M (K=100)

The resultant portfolio has an expected return of 2.6% with minimum value at the end of the period of 993.7 ($0,63\%$ of the initial value V_0).

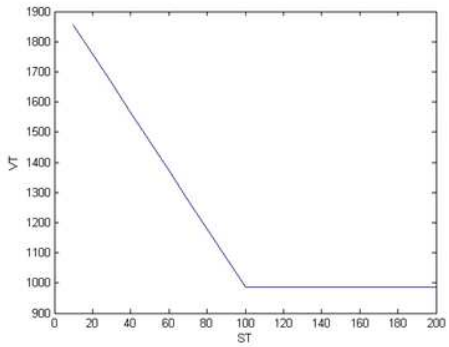

Figure 3 : Portfolio Payoff for $K = 100$

The obtained payoff is similar to a naked put.The optimal allocation shows 95.6% invested in risk-free, 0% in risky asset and 4.4% used as a hedge.

In summary, the maximisation of Sharpe-omega ratio yields to an optimal portfolio which limits the downside risk and enhances the performance profile on the left side.

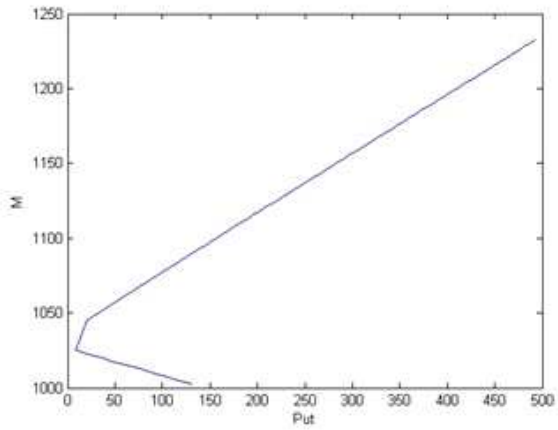

Figure 4 : Efficient frontier M – Put(L, VT)

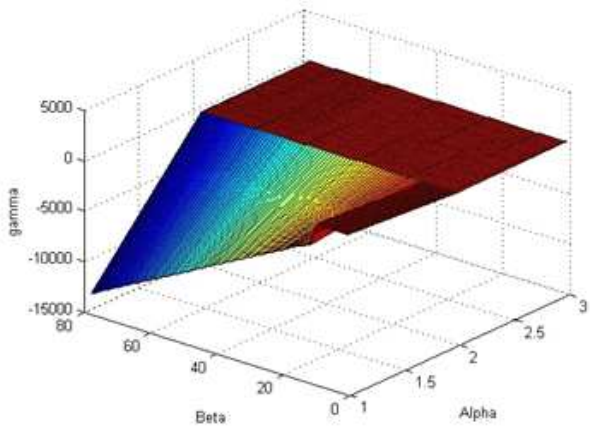

Figure 5 Optimal allocations for different value of M

When we focus on the impact of the μ value on the shape of the Payoff V_T we obtain the same result concerning the optimal allocation. (we fixed $K = 100$).

Note that the maximal Sharpe-ratio return is obtained for M^* equal to 1026 less than $V_0 \exp^{\mu T}$. Moreover, we note that the value of M^* is relatively low. In addition the shape of V_T^* is not always increasing and concave as prouved by Goetzmann *et al.* (2002) for the Sharpe ratio maximization. And for the efficient frontiers (Put , Expected return), maximization at given expected return M, the Put price shows the following variations: decreasing when M varies between 1000 and 1026 and increasing for M greater than 1026.

μ	M^*	Put^*	$S\Omega^*$	$W\alpha^*$	$W\beta^*$	$W\gamma^*$	α^*	β^*	γ^*
0,03	1032,5	24,295	1,3377	0,16569	0,73	0,10294	165,69	7,31	22,726
0,05	1026	9,0626	2,7586	0,95629	0,00	0,04371	956,29	0,00	9,6498
0,08	1020	2,5232	7,9263	0,96686	0,00	0,033141	966,86	0,00	7,3166
0,11	1015	2,3193	6,4674	0,96744	0,00	0,032558	967,44	0,00	7,1877
0,15	1008	4,8028	1,6657	0,96482	0,00	0,035177	964,82	0,00	7,7661
0,2	1004	4,6601	0,85836	0,96542	0,00	0,03458	965,42	0,00	7,6341

Table 1: Optimal parameters for different values of μ

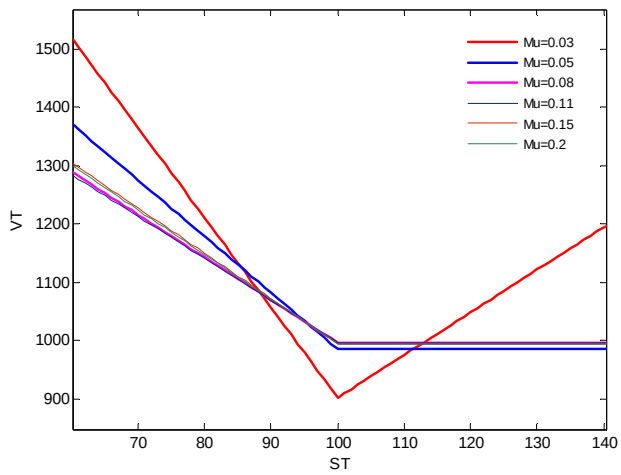

Figure 6: Portfolio Payoff for different value of μ

The portfolio payoff changes significantly when μ becomes greater than the risk-free rate ($r = 0.03$). Furthermore, optimal portfolio allocation is concentrated on risk-free and put instruments once μ is greater than 0.03.

In former cases, optimal portfolio displays a put payoff whose slope is not a monotone function of μ.

5.1.2 Portfolio excluding risk-free instrument

We run the same optimisation as in section 5.1.1. Sharpe-Omega ratio is minimum for $K = 93$. It is interesting to note that the optimal ratio calculated for $K = 100$ is very close to value in the case of portfolio including risk-free instrument (2.74).

Unlike the previous case, optimal ratio exceeds 2.74 only when K becomes much smaller, i.e. below 85.

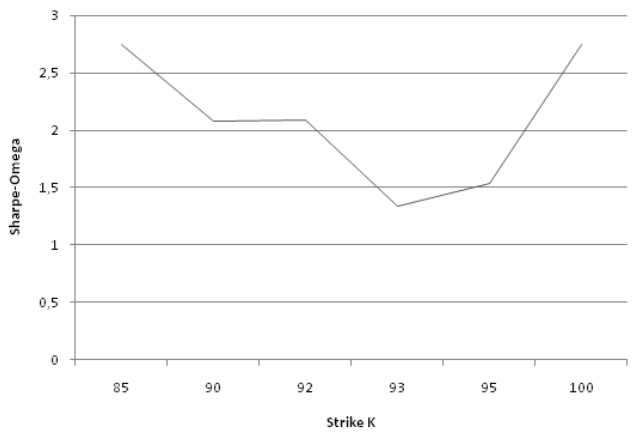

Figure 7 : Sharpe-Omega ratio for K varying between 80 and 100

Optimal allocation for $K = 100$ shows 94% invested in the risky instrument and 6% as a hedge on the downside. Portfolio expected return (M) is greater than in the previous case with a lower minimum at 6%.

5.2 Portfolio with one Put and one Call

In this section we add a Call option to the precedent portfolio (excluding risk-free instrument).

As shown in Figure 8, optimal ratio variations are very similar to those in the first structured portfolio case. Moreover, the Sharpe-Omega ratio calculated for $K_P = 100$ is insensitive to K_C. Looking more closely to optimal portfolio weightings, Call option allocation varies between -7% (when K_P is deeply out-of-the money) to 20%. When Call option weight is positive, it provides additional "beta" to the portfolio for high values of the risky instrument, in exchange of lower portfolio minimum. In the opposite case, it neutralises the portfolio "beta".

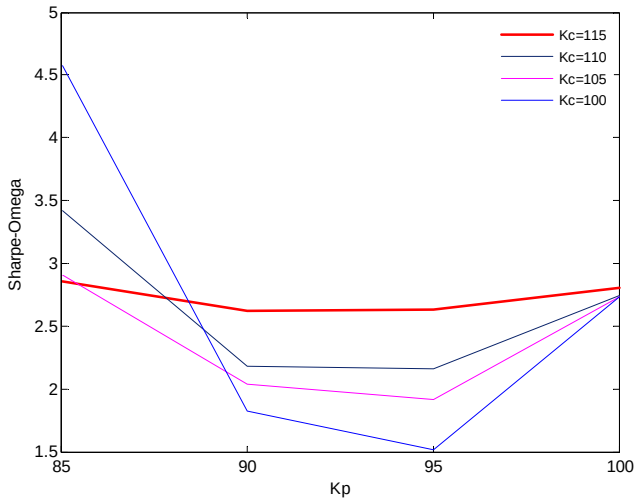

Figure 8 : Sharpe-Omega ratio for different values of K_C

In what follows, we analyse in more details the case for $K_P = 100$ (in the money Put) and $K_C = 115$ (out-of-the money call). Maximum Sharpe-Omega ratio is reached for $M^* = 1112$. Portfolio payoff remains convex with higher slope on its right side. Thus, the portfolio expected return is 11.2% with a limited downside from its initial value at 10.6%.

The optimal allocation is: 4.4% to Call option, 89% to risky instrument and 5.6% to Put option.

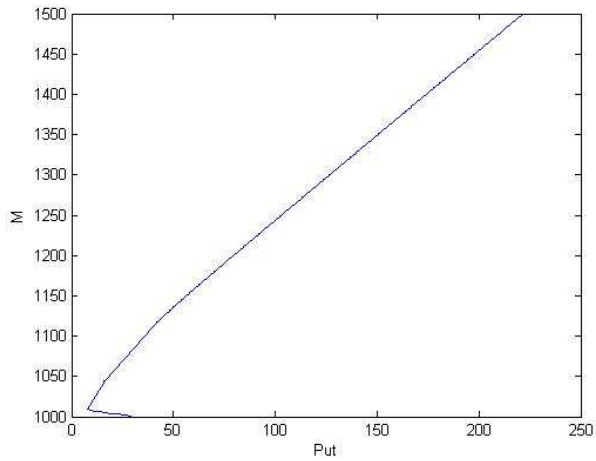

Figure 8 : Efficient frontier M – Put(L, VT)

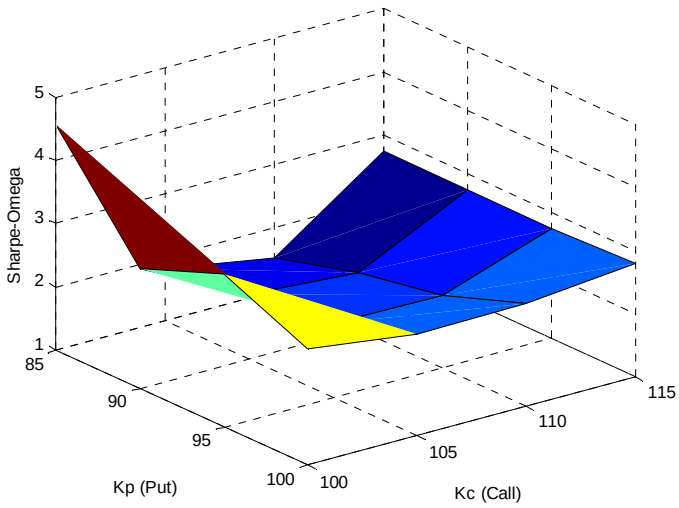

Figure 9 : Optimal allocations for different value of M

6 Conclusion

In this paper, we have examined performance maximization of plain-vanilla structured products. For this purpose, we have considered the Omega or SharpeOmega performance ratios introduced by Keating and Shadwick (2002) and by Kazemi et al. (2004). Calculated payoff are not always increasing and concave as for the Sharpe ratio maximization case illustrated by Goetzmann et al. (2002). This is in line with previous results on portfolio optimization within rank dependent utility, as quoted by Prigent (2008).

Further extensions can take account of more complex derivatives, such as exotic options and also dynamic portfolio strategies.

APPENDIX

APPENDIX 1: $E_{\mathbb{P}}\left[(L-V_T)^+\right]$

- $E_{\mathbb{P}}\left[(L-V_T)^+\right]$ **in a.2 case:**

$$\begin{aligned}
\mathbb{E}_{\mathbb{P}}\left[(L-V_T)^+\right] &= \int_0^K (L-\alpha B_T-\beta s-\gamma K+\gamma s)\, d\mathbb{P}_{S_T}(s) \\
&\quad + \int_K^{k_2} (L-\alpha B_T-\beta s)\, d\mathbb{P}_{S_T}(s) \\
&= \int_0^K (L-\alpha B_T-\gamma K)\, d\mathbb{P}_{S_T}(s) + \int_0^K (\gamma-\beta)\, s d\mathbb{P}_{S_T}(s) \\
&\quad + \int_K^{k_2} (L-\alpha B_T)\, d\mathbb{P}_{S_T}(s) + \int_K^{k_2} (-\beta s)\, d\mathbb{P}_{S_T}(s)
\end{aligned}$$

we have:

$$\begin{aligned}
\int_0^K (L-\alpha B_T-\beta s)\, d\mathbb{P}_{S_T}(s) &= \int_K^{k_2} (L-\alpha B_T)\, d\mathbb{P}_{S_T}(s) - \beta \int_K^{k_2} s d\mathbb{P}_{S_T}(s) \\
&= (L-\alpha B_T)\left[\phi\left(-\frac{\overline{z}-\ln k_2}{\sigma\sqrt{T}}\right) - \phi\left(-\frac{\overline{z}-\ln K}{\sigma\sqrt{T}}\right)\right] \\
&\quad -\beta \exp\left(\overline{z}+\frac{\sigma^2}{2}T\right) W_{a2}
\end{aligned}$$

where W_{a_2} is equal to:

$$\left[\phi\left(-\frac{\overline{z}-\ln k_2+\sigma^2 T}{\sigma\sqrt{T}}\right) - \phi\left(-\frac{\overline{z}-\ln K+\sigma^2 T}{\sigma\sqrt{T}}\right)\right]$$

and

$$\begin{aligned}
\int_0^K (L-\alpha B_T-\beta S_T-\gamma K+\gamma S_T)\, dS_T &= (L-\alpha B_T-\beta S_T)\, \phi\left(-\frac{\overline{z}-\ln K}{\sigma\sqrt{T}}\right) \\
&\quad + (\gamma-\beta)\left[\phi\left(-\frac{\overline{z}-\ln K+\sigma^2 T}{\sigma\sqrt{T}}\right)\right]
\end{aligned}$$

- $E_{\mathbb{P}}\left[(L-V_T)^+\right]$ **in b.1 case:**

$$\begin{aligned}
\mathbb{E}_{\mathbb{P}}\left[(L-V_T)^+\right] &= \int_{k_1}^K (L-\alpha B_T+(\gamma-\beta)\, s-\gamma K)\, d\mathbb{P}_{S_T}(s) \\
&\quad + \int_K^{k_2} (L-\alpha B_T-\beta s)\, d\mathbb{P}_{S_T}(s)
\end{aligned}$$

The first term of this integration is written as follows:

$$
\begin{aligned}
i_1 &= (L-\alpha B_T-\gamma K)\int_{k_1}^{K} d\mathbb{P}_{S_T}(s)+(\gamma-\beta)\int_{k_1}^{K} s d\mathbb{P}_{S_T}(s) \\
&= (L-\alpha B_T-\gamma K)\left[\phi\left(-\frac{\overline{z}-\ln K}{\sigma\sqrt{T}}\right)-\phi\left(-\frac{\overline{z}-\ln k_2}{\sigma\sqrt{T}}\right)\right] \\
&\quad +(\gamma-\beta)\exp\left(\overline{z}+\frac{\sigma^2 T}{2}\right)\left[\phi\left(-\frac{\overline{z}-\ln K+\sigma^2 T}{\sigma\sqrt{T}}\right)-\phi\left(-\frac{\overline{z}-\ln k_1+\sigma^2 T}{\sigma\sqrt{T}}\right)\right]
\end{aligned}
$$

and

$$
\begin{aligned}
i_2 &= (L-\alpha B_T)\int_{K}^{k_2} d\mathbb{P}_{S_T}(s)-\beta\int_{K}^{k_2} s d\mathbb{P}_{S_T}(s) \\
&= (L-\alpha B_T)\left[\phi\left(-\frac{\overline{z}-\ln k_2}{\sigma\sqrt{T}}\right)-\phi\left(-\frac{\overline{z}-\ln K}{\sigma\sqrt{T}}\right)\right] \\
&\quad -\beta\exp\left(\overline{z}+\frac{\sigma^2 T}{2}\right)\left[\phi\left(-\frac{\overline{z}-\ln k_2+\sigma^2 T}{\sigma\sqrt{T}}\right)-\phi\left(-\frac{\overline{z}-\ln K+\sigma^2 T}{\sigma\sqrt{T}}\right)\right]
\end{aligned}
$$

- $E_{\mathbb{P}}\left[(L-V_T)^+\right]$ **in b.2 case:**

$$
\begin{aligned}
\mathbb{E}_{\mathbb{P}}\left[(L-V_T)^+\right] &= \int_0^{k_2}(L-\alpha B_T-(\beta-\gamma)\,\mathbb{P}_{S_T}(s)-\gamma K)\,d\mathbb{P}_{S_T}(s) \\
&\quad +\int_K^{k_2}(L-\alpha B_T-\beta s)\,d\mathbb{P}_{S_T}(s)
\end{aligned}
$$

The first term is expressed as follows:

$$
\begin{aligned}
&= (L-\alpha B_T-\gamma K)\int_0^{K} d\mathbb{P}_{S_T}(s)-(\beta-\gamma)\int_0^{K} s d\mathbb{P}_{S_T}(s) \\
&\quad (L-\alpha B_T-\gamma K)\,\phi\left(-\frac{\overline{z}-\ln K}{\sigma\sqrt{T}}\right)-(\beta-\gamma)\,\phi\left(-\frac{\overline{z}-\ln K+\sigma^2 T}{\sigma\sqrt{T}}\right)
\end{aligned}
$$

The second term is expressed as follows:

$$
\begin{aligned}
&= (L-\alpha B_T)\int_K^{k_2} d\mathbb{P}_{S_T}(s)-\beta\int_K^{k_2} s d\mathbb{P}_{S_T}(s) \\
&= (L-\alpha B_T)\left[\phi\left(-\frac{\overline{z}-\ln k_2}{\sigma\sqrt{T}}\right)-\phi\left(-\frac{\overline{z}-\ln K}{\sigma\sqrt{T}}\right)\right] \\
&\quad -\beta\exp\left(\overline{z}+\frac{\sigma^2 T}{2}\right)\left[\phi\left(-\frac{\overline{z}-\ln k_2+\sigma^2 T}{\sigma\sqrt{T}}\right)-\phi\left(-\frac{\overline{z}-\ln K+\sigma^2 T}{\sigma\sqrt{T}}\right)\right]
\end{aligned}
$$

APPENDIX 2: calculation of β et γ:

$$\begin{aligned}
\beta &= \frac{\begin{vmatrix} V_0 - \alpha B_0 & P_0(K) \\ M - \alpha B_0 e^{rT} & BS(\mu) e^{\mu T} \end{vmatrix}}{\Delta} \\
&= \frac{BS(\mu) e^{\mu T}(V_0 - \alpha B_0) - P_0(K)\left(M - \alpha B_0 e^{rT}\right)}{\Delta} \\
&= \frac{-BS(\mu) e^{\mu T} \alpha B_0 + BS(\mu) e^{\mu T} V_0 - P_0(K)(M - \alpha B_0)}{\Delta} \qquad (37) \\
&= \alpha \left(\frac{-BS(\mu) e^{\mu T} B_0 + P_0(K) B_0 e^{rT}}{\Delta} \right) + \frac{BS(\mu) V_0 e^{\mu T} - P_0(K) M}{\Delta}
\end{aligned}$$

where:

$$\begin{aligned}
\Delta &= \begin{vmatrix} S_0 & P_0(K) \\ S_0 e^{\mu T} & BS(\mu) e^{\mu T} \end{vmatrix} \\
&= S_0 \times BS(\mu) e^{\mu T} - S_0 e^{\mu T} P_0(K) \\
&= S_0 \left(BS(\mu) - P_0(K) e^{\mu T} \right) \qquad (38)
\end{aligned}$$

β is then written as $a_{11}\alpha + b_{11}$, where

$$a_{11} = \frac{-BS(\mu) e^{-\mu T} B_0 + P_0(K) B_0 e^{rT}}{\Delta}$$

and

$$b_{11} = \frac{BS(\mu) V_0 e^{\mu T} - P_0(K) M}{\Delta}$$

Similarly we determine γ:

$$\begin{aligned}
\gamma &= \frac{\begin{vmatrix} S_0 & V_0 - \alpha B_0 \\ S_0 e^{\mu T} & M - \alpha B_0 e^{rT} \end{vmatrix}}{\Delta} \\
&= \frac{S_0 \left(M - \alpha B_0 e^{rT} \right) - S_0 e^{\mu T}(V_0 - \alpha B_0)}{\Delta} \\
&= \alpha \left(\frac{-S_0 B_0 e^{rT} + B_0 S_0 e^{\mu T}}{\Delta} \right) + \frac{S_0 M - S_0 e^{\mu T} V_0}{\Delta}
\end{aligned}$$

γ is then written as $a_{21}\alpha + b_{21}$:

$$\begin{aligned}
a_{21} &= \frac{-S_0 e^{rT} B_0 + B_0 S_0 e^{\mu T}}{\Delta} \\
&= \frac{S_0 B_0}{\Delta} \left(e^{\mu T} - e^{rT} \right)
\end{aligned}$$

and

$$
\begin{aligned}
b_{21} &= \frac{S_0 M - S_0 e^{\mu T} V_0}{\Delta} \\
&= \frac{S_0}{\Delta}\left(M - S_0 e^{\mu T} V_0\right)
\end{aligned}
$$

APPENDIX 3 : Sign of $\mathbf{a_{11}}$, $\mathbf{a_{21}}$, $b_1(M)$ et $b_2(M)$

Parameters a_{11} and a_{21} are non positive.

Proof. We have:

$$\begin{aligned} a_{11} &= \frac{-BS(\mu)e^{\mu T}B_0 + P_0(K)B_0e^{rT}}{\Delta}, \\ &= \frac{B_0}{\Delta}\left[P_0(K)e^{rT} - BS(\mu)e^{\mu T}\right]. \end{aligned}$$

The sign of Δ depends on $[BS(\mu) - P_0(K)]$.

However, to look for the sign of a_{11}, we first determine the sign of $\frac{\partial P(r)}{\partial r}$.

We have:

$$\begin{aligned} \frac{\partial P}{\partial r} &= \frac{\partial C}{\partial r} - TKe^{-rT}, \\ &= Ke^{-rT}\left[N(d_2) - 1\right] \end{aligned}$$

$[N(d_2) - 1] < 0$ thus $TP(r) + \frac{\partial P}{\partial r} = T\left(P(r) + Ke^{-rT}\left[N(d_2) - 1\right]\right)$

or

$$P(r) = S_0N(d_1) - Ke^{-rT}\left[N(d_2) - 1\right] - S_0 + Ke^{-rT}$$

$\Rightarrow$

$$P(r) + Ke^{-rT}\left[N(d_2) - 1\right] = S_0\left[N(d_1) - 1\right]$$

$[N(d_1) - 1] < 0$. And so, $e^{-rT}\ P(r)$ is decreasing.

If $\mu > r$ then $BS(\mu) < P_0(K)$. Hence, the sign of a_1 is negative since $P_0(K)e^{rT} > BS(\mu)e^{\mu T}$ and $\Delta < 0$.

Similarly we determine the sign of a_2.

$$a_{21} = \frac{S_0B_0}{\Delta}\left(e^{\mu T} - e^{rT}\right),$$

If $\mu > r \Rightarrow e^{\mu T} > e^{rT}$, and $\Delta < 0$. Therefore, $a_2 < 0$. ■

The function $b_{11}(M)$ is always increasing and $b_{21}(M)$ is decreasing.

Proposition 2 ***Proof.***

$$b_{11} = \frac{BS(\mu)V_0e^{\mu T} - P_0(K)M}{\Delta}$$

If $\mu > r$ and $M > V_0e^{\mu T}$ (100% in S_T) then $BS(\mu) < P_0(K)$ and $b_1 > 0$.

■

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