

# Off-the-Record Target Zones: Theory with an Application to Hong Kong's Currency Board

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## Abstract

This paper provides a modelling framework for evaluating the exchange rate dynamics of a target zone regime with undisclosed bands. We generalize the literature to allow for asymmetric one-sided regimes. Market participants' beliefs concerning an undisclosed band change as they learn more about central bank intervention policy. We apply the model to Hong Kong's one-sided currency board mechanism. In autumn 2003, the Hong Kong dollar appreciated from close to 7.80 per US dollar to 7.70, as investors feared that the currency board would be abandoned. In the wake of this appreciation, the monetary authorities finally revamped the regime as a symmetric two-sided system with a narrow exchange rate band.

**Keywords:** Currency Board Arrangement, Target Zone Model, Hong Kong

**JEL-Classification:** C61, E42, F31, F32

# 1 Introduction

Recent years have seen a resurgence of interest in exchange rate regimes for emerging economies. In the aftermath of the Asian crisis in 1997-98, crisis prevention was viewed as a key criterion for choosing an exchange rate regime. Much attention focused on the "hardness degrees" of peg systems, and the standard textbook dichotomy of float versus peg was replaced by a more continuous grading of exchange rate regimes: free floats, various intermediate regimes, and hard pegs.<sup>1</sup> After the partial collapse of Europe's exchange rate mechanism (ERM) in September 1992, a standard proposition attracting widespread support was that corner solutions such as free floats or super-strict pegs (currency board, dollarisation) are preferable to intermediate regimes (soft peg, band, crawling peg) on the grounds that they are less crisis-prone given that today's financial markets are too powerful and volatile. According to this view, investors will overwhelm intermediate regimes like band systems sooner or later. Thus the options for exchange rate regimes have been hollowed out. Governments should either let exchange rates float or fix them permanently via a currency board or monetary union.<sup>2</sup>

In fairness, one should say that academics do not agree on the best solution. Until the mid-1990s the mainstream literature argued essentially the opposite. Several objectives and trade-offs (insulation from external nominal and real shocks, flexibility vs. commitment, inflation stabilisation, exposure to international capital flows) pointed to intermediate regimes with limited flexibility, i.e. a compromise between hard peg and free float. Balancing risks and benefits has led to the conclusion that intermediate regimes were appropriate for many countries, especially those without large exposure to international capital flows. Another fundamental reason why economists then considered the extremes - pure float or absolute fix - to be overdone is that economics generally prefers interior to corner solutions.

A prime example for a hard peg is Hong Kong's currency board system.<sup>3</sup> Hong Kong's currency board stands out among such arrangements around the world as one of the oldest existing ones.<sup>4</sup> Hong Kong's currency board was established in 1983 with the Hong Kong dollar (HKD) peg to the US dollar (USD) at 7.80 to 1 as a firm anchor for the external value of the HKD. The USD was an obvious choice, as the US was a major trading partner of Hong Kong. In September 1998, the convertibility rate was changed to 7.75 to 1. Subsequently the rate moved gradually over the period between April 1999 and August 2000 back to the linked rate of 7.80, where it has since remained. A distinctive feature of the system was that up to May 2005 no strong-side boundary existed. Thus, the currency board system was asymmetric. Finally, in May 2005 the Hong Kong Monetary Authority (HKMA) decided to bite the bullet on appreciation and replaced the previous implicit strong band with something less muddled. More precisely, the HKMA introduced a symmetric target zone with a band of [7.75, 7.85].

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<sup>1</sup>More attention was also given to the distinction between *de facto* and *de jure* exchange rate regimes, as revealed by the ex-post behaviour of exchange rates.

<sup>2</sup>But free floats have a big drawback: they can overshoot and become highly unstable, especially in open emerging economies with large capital flows.

<sup>3</sup>An in-depth discussion of Hong Kong's currency board including technical details, is available at [http://www.info.gov.hk/hkma/eng/currency/link\\_ex/index.htm](http://www.info.gov.hk/hkma/eng/currency/link_ex/index.htm). For a perceptive and thorough discussion, see Latter (2007).

<sup>4</sup>In the empirical exchange rate literature, the performance of currency board systems has been discussed *ad nauseam*. For example, Ghosh et al. (2000) have found that currency boards exhibit better inflation performance than soft pegs, mostly due to a credibility effect. Regarding growth, currency boards also do better than soft pegs.

In this paper we provide new theoretical insights in the working of Hong Kong's currency board system since January 2001. While there is a huge literature on symmetric target zones, extensive research on one-sided regimes with implicit bands is lacking. Based upon Klein (1992), we expand the scope of the existing literature and offer a new analytical framework for one-sided target zones with implicit (unknown) bands and derive the exchange rate dynamics of such regimes. The paper is structured as follows: Section 2 provides a brief self-contained overview of Hong Kong's currency board system. Section 3 then presents an analytical framework for modelling Hong Kong's currency board system since January 2001. The theoretical modelling approach provides the input for the calibration exercise in Section 4. The final section offers concluding remarks.

## 2 Main Features of Hong Kong's Currency Board System Since 2001

We begin by discussing the main elements of Hong Kong's currency board system, to provide an anchor for our modelling work. Contrary to actively managed fixed exchange rate systems, a currency board system is a passive "hard-fixed" peg system. The predictability and rule-based nature of a currency board are its key advantages. Like a traditional peg, a currency board offers the prospect of stable exchange rates, which can promote both investment and trade.

Hong Kong's currency board has survived a number of booms and busts, including a massive speculative attack during Asia's financial crisis of 1997-98. Given the speculative outflow of US dollars, HKMA sold large amounts of US dollars in October 1998, to defend the peg. Furthermore, HKMA pursued a defensive interest rate strategy, which was partly responsible for bringing on a severe recession.<sup>5</sup> Despite the presumed rigidity of the currency board system, the convergence between the exchange rate in the interbank market and the fixed rate for the currency was not realised. Thus the first major set of reforms of the operational framework after the Asian crisis involved the introduction of a weak-side Convertibility Undertaking (CU). This was an asymmetric weak-side commitment in that HKMA was ready to sell US dollars at 7.80, but was not obliged to purchase them at a pre-announced rate.<sup>6</sup> This weak-side commitment is shown by the red line in Figure 1. Since late 2003, speculative pressure for a revaluation of the Chinese Renminbi has grown, resulting in large speculative inflows.<sup>7</sup> The HKD appreciated from 7.80 to about 7.70 in autumn 2003, fuelling speculation that the currency board link to the USD

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<sup>5</sup>The result of this surprise move was that interbank interest rates jumped and the overnight rate hit 280 percent. This successfully stemmed the speculative outflow of US dollars. Overnight rates dropped back to about 5 percent within a few days.

<sup>6</sup>As a historical note, while no formal strong-side intervention point was introduced, the Subcommittee on Currency Board Operations already considered the options in this area in meetings in October 1999 and July 2000 and "agreed that there would be scope to review the arrangement again, should the need arise" [HKMA (2000)].

<sup>7</sup>One reason for the strengthening of the HKD was the signing of the Mainland China and Hong Kong Closer Economic Partnership Agreement (CEPA) in June 2003 which was expected to provide stimulus to Hong Kong's economy. This has led some market participants to believe that any appreciation of the renminbi against the US dollar could lead to an appreciation of the HKD. Another reason why Hong Kong's "iron peg" has come under attack despite strong economic fundamentals might be that HKMA has begun to look a bit too much like a de facto central bank, intervening in money markets to smooth interest rates. This kind of intervention undermines the advantages of the currency board since the system is no longer regarded as automatic.



Figure 1: Daily HKD spot exchange rate against USD and exchange rate band

Source: Bloomberg

might be abandoned.<sup>8</sup> During these years, the exact exchange rate triggering HKMA strong-side intervention was unknown to the public. Only the latest interventions would provide hints as to HKMA's intentions. Lastly, the link was defended with a combination of market interventions, including direct foreign exchange operations and manipulation of liquidity and interest rates. An assessment stemming from the 2003-2004 episode is that HKMA has had a tried-and-trusted method for halting intermittent appreciation pressure: beyond some undisclosed threshold they have intervened. In other words, there was a flexible strong-side regime that was managed as if it were fixed but without an explicit strong-side precommitment.

Over and above these actions, on 18 May 2005 HKMA finally revamped the one-sided currency board mechanism as a symmetric two-sided system with a narrow exchange rate band of  $\pm 0.6$  percent. The strong-side Convertibility Undertaking (green line in Figure 1) was fixed at HKD 7.75/USD.<sup>9</sup> At the same time, the weak-side CU was changed from HKD 7.80/USD to HKD 7.85/USD. These "refinements" were intended to anchor market expectations and to prevent speculative attacks. Figure 1 shows that the HKD spot rate stayed close to the strong-side CU (green line) most of the time after May 2005. Finally, note that the validity of the current arrangement has not been called into question by the current financial crisis.

### 3 Modelling Discretion on the Strong Side

Until May 2005 Hong Kong's exchange rate system comprised a credible weak-side CU and an undisclosed strong-side CU. Ultimately, the system was thus an undisclosed one-sided target

<sup>8</sup>For an analysis of the strong-side pressure on the HKD and particularly HKMA's response, see IMF (2005). This paper also offers a simple second-generation currency crisis framework for modelling trade-offs faced by HKMA.

<sup>9</sup>In other words, the HKMA moved from a single-edge shield towards a double-edged shield and thus the idea of target zones made a comeback. One underlying motivation was that, if appreciation pressures were to recur often, this could undermine the credibility of the currency board arrangement. For a theoretical analysis of this lightning-bolt solution, see Chen et al. (2010).

zone model.<sup>10</sup> Keeping in mind the specific one-sidedness of the currency board system, we study the associated credibility issues and the exchange rate dynamics in a formal model. Roughly speaking, we attempt to model the working inside the clock. The new aspects and insights of this paper are to extend the previous work by Klein (1992). Klein (1992) analysed the dynamics of the exchange rate in a target zone with unknown band width. In our model, past exchange rate interventions convey information about the undisclosed bands and affect the exchange rate dynamics via the changed fundamentals and triggered revision of exchange rate expectations.

### 3.1 Basic Model

It is useful to preface a detailed discussion of the one-sided target zone model with some reference to Krugman's (1991) seminal paper. Although the review pretends to be brief, it will be adequate to demonstrate the conceptual and research framework. His model starts from the observation that due to the forward-looking nature of rational expectations, the presence of a credible and perfectly known band exerts an influence on the dynamics of the exchange rate. The model starts with the log-linear asset pricing equation that expresses the log exchange rate,  $s(t)$ , as the sum of the logarithm of the fundamental,  $f$ , and its own expected change:

$$(1) \quad s(t) = f(t) + \tau \frac{E(ds(t))}{dt},$$

where  $E[\cdot]$  denotes the rational expectations operator and  $\tau > 0$ . The factors affecting the exchange rate are the fundamentals and financial markets' expectations of the future movement of the exchange rate. The fundamental  $f$  consists of the logarithm of money supply,  $m$ , and velocity,  $v$ :

$$(2) \quad f(t) = m(t) + v(t).$$

Except for intervention periods,  $m(t)$  is constant. In the case of an intervention at  $t = T$ , the money supply takes a new value in  $T$ . Beyond intervention periods, the driving force of  $f$  is  $v$ . Therefore, the values of the function  $f$  will be denoted by  $V$  for possible strong/weak band thresholds. It is assumed that the log of the velocity follows an arithmetic Brownian motion without drift:

$$(3) \quad dv(t) = \sigma dz,$$

where  $\sigma$  is the risk parameter and  $z$  a standard Brownian motion. To handle this process, we introduce a function  $g$  with

$$(4) \quad g(f) = s(t).$$

Applying Itô's lemma to the expectations term yields

$$(5) \quad \frac{E(ds)}{dt} = \frac{\sigma^2}{2} g'',$$

which means that the logarithm of the exchange rate is subject to the second-order differential equation

$$(6) \quad s(t) = f(t) + \tau \frac{\sigma^2}{2} g''.$$

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<sup>10</sup>An implicit strong-side band is likely because history has shown that Hong Kong's policymakers place a heavy emphasis on exchange rate stability.

The innovation in the paper is that we solve the second-order ordinary differential equation (6) describing the dynamics of the exchange rate in the target zone for the special case of a one-sided target zone with an undisclosed strong-side band. This provides a sound modelling framework with considerable rigour enabling an understanding of HKD dynamics since January 2001.<sup>11</sup>

### 3.2 One-Sided Target Zone Framework

In Krugman's (1991) model the central bank credibly commits to maintaining the symmetric target zone regime. Given Hong Kong's asymmetric exchange rate regime, the question becomes: How do we introduce such asymmetric dynamics into the target zone model? In our one-sided target zone model, the weak-side band  $\bar{S}$  is credibly fixed. On the contrary, the central bank's strong-side band  $\underline{S}$  is undisclosed. In other words, we assume a "high" confidence weak-side bank and a "low" confidence strong-side band. Market participants form expectations of the undisclosed strong-side band  $\underline{S}$ . Their expectations depend on the unknown threshold that triggers a central bank intervention.

Assume that the exchange rate that triggers intervention against appreciation pressure is expected to be somewhere in the interval  $[S_1, S_2]$ ,  $S_1, S_2 \in \mathbb{R}$  with  $S_1 < S_2$ , and the corresponding fundamental values form the interval  $[V_1, V_2]$  with  $V_1 < V_2$ .

In order to obtain a solution for equation (6) in the case of a one-side target zone, we implement the value-matching and smooth-pasting conditions and substitute the fundamental value  $V^l$ ,  $V^l \in [V_1, V_2]$ , which triggers the next intervention on the strong side:<sup>12</sup>

$$(7) \quad s(t) = f(t) + A_1(V^l)e^{rf(t)} + A_2(V^l)e^{-rf(t)},$$

where

$$\begin{aligned} r &= \sqrt{\frac{2}{\tau\sigma^2}} \\ A_1(V^l) &= -\frac{\tau\sigma^2}{2\omega(V^l)} \left( e^{-rV^l} - e^{-r\bar{V}} \right) r \\ A_2(V^l) &= \frac{\tau\sigma^2}{2\omega(V^l)} \left( e^{r\bar{V}} - e^{rV^l} \right) r \\ \omega(V^l) &= e^{r\bar{V}-rV^l} - e^{rV^l-r\bar{V}}. \end{aligned}$$

The three terms  $A_1(V^l)$ ,  $A_2(V^l)$  and  $\omega(V^l)$  typify the uncertainty inherent in the model since they depend on the uncertain fundamental value  $V^l$ .

Rearranging  $A_1(V^l)$  and  $A_2(V^l)$  yields

$$(8) \quad \begin{aligned} A_1(V^l) &= -\frac{r\sigma^2\tau}{2(e^{rV^l} + e^{r\bar{V}})} \\ A_2(V^l) &= \frac{e^{r(V^l+\bar{V})}r\sigma^2\tau}{2(e^{rV^l} + e^{r\bar{V}})}. \end{aligned}$$

Next, we describe the sequence of events and the strategic interactions between the central bank and market participants.

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<sup>11</sup>Unfortunately, Klein's (1992) modelling approach does not lend itself naturally to the asymmetric one-sided band exchange rate regime case. That may explain why the current paper is the first to model an asymmetric one-sided target zone. It also explains why we depart from Klein's (1992) approach on technical grounds.

<sup>12</sup>A thorough description of the approach is provided by Sarno and Taylor (2003, pp. 177-184).

### 3.3 The Situation at the Outset

This section provides an outline of the initial situation. Market participants' expectations depend on their perception of central bank behaviour. We postulate that market participants act on the assumption of a uniform distribution of possible (unknown) trigger values of fundamentals in the range  $[V_1, V_2]$ . The uniform distribution makes sense because no a priori information about intervention trigger points is available. This implies that the probability of an intervention at time  $t$  is

$$(9) \quad P(V_t = V_T) = \frac{1}{V_2 - V_1} \mathbb{1}_{\{V_t \in [V_1, V_2]\}},$$

where  $V_T$  is the intervention triggering fundamental value. At the outset, the exchange rate has not yet moved outside the range  $(S_2, \bar{S}]$ . The value-matching condition implies

$$(10) \quad s(t_0) = E(s(t_0)),$$

where

$$(11) \quad E(s(t_0)) = f(t_0) + E(A_1(V^l(t_0)))e^{rf(t_0)} + E(A_2(V^l(t_0)))e^{-rf(t_0)}$$

and

$$(12) \quad E(A_1(V^l)) = - \int_{-\infty}^{\infty} \frac{r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} dP = - \int_{V_1}^{V_2} \frac{r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} \frac{1}{V_2 - V_1} dv$$

and

$$(13) \quad E(A_2(V^l)) = \int_{-\infty}^{\infty} \frac{e^{r(v+\bar{V})}r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} dP = \int_{V_1}^{V_2} \frac{e^{r(v+\bar{V})}r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} \frac{1}{V_2 - V_1} dv.$$

Inserting equations (12) and (13) into equation (11) yields

$$(14) \quad s(t_0) = f(t_0) - \left( \int_{V_1}^{V_2} \frac{r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} \frac{1}{V_2 - V_1} dv \right) e^{rf(t_0)} + \left( \int_{V_1}^{V_2} \frac{e^{r(v+\bar{V})}r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} \frac{1}{V_2 - V_1} dv \right) e^{-rf(t_0)}.$$

The general closed-form solution of equation (14) is

$$(15) \quad s(t_0) = f(t_0) - e^{-r(f(t_0)+\bar{V})}\sigma^2\tau \frac{\left( e^{2f(t_0)r}r(V_1 - V_2) - \left( e^{2f(t_0)r} + e^{2r\bar{V}} \right) \left( \ln(e^{rV_1} + e^{r\bar{V}}) - \ln(e^{rV_2} + e^{r\bar{V}}) \right) \right)}{2(V_1 - V_2)}.$$

Equation (15) describes the dynamics of the exchange rate when the central bank has discretion on the strong side.<sup>13</sup>

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<sup>13</sup>The interested reader is referred to Appendix A.1 for the derivation. Note that the closed-form solution in Klein (1992) is more appealing because it is derived for the symmetric case where the integrals are characteristic.

To add some intuition, consider the graphical illustration of the exchange rate against the fundamentals in Figure 2. The segment  $ab$  displays the relationship between the exchange rate and the fundamentals in case that the exchange has not moved outside the range  $(S_2, \bar{S}]$ . The segment  $ac$  represents the exchange rate dynamics for the unattained velocity interval  $[V_1, V_2)$ . In a nutshell, all possible exchange rate curves are located in the area  $abc$ .

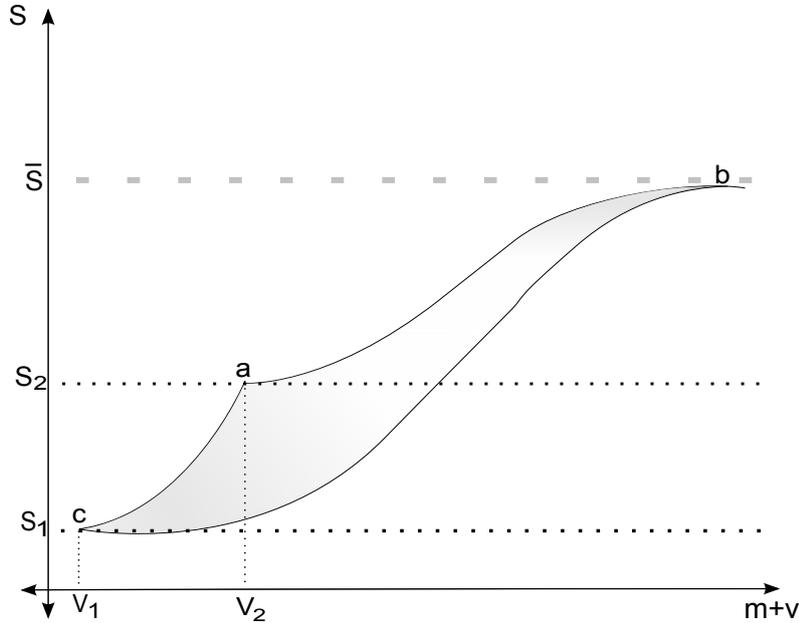


Figure 2: Relationship between fundamentals and exchange rate with a continuum of possible intervention triggering exchange rates on the strong side

Once the exchange rate has moved outside the range  $(S_2, \bar{S}]$  taking the value  $\tilde{S}$ ,  $S_1 < \tilde{S} < S_2$ , without an intervention response, the system is changed in two ways. First, the range of expected intervention triggering exchange rates degenerates to  $[S_1, \tilde{S}]$ . Second, the relation between fundamentals and exchange rate undergoes a change. Denoting  $\mathbf{f}$ ,  $V_1 < \mathbf{f} < V_2$ , as the smallest value of the fundamentals that has so far occurred, the new relationship is

$$(16) \quad s(t) = f(t) - e^{-r(f(t)+\bar{V})} \sigma^2 \tau \frac{\left( e^{2f(t)r} r (V_1 - \mathbf{f}) - (e^{2f(t)r} + e^{2r\bar{V}}) \left( \ln(e^{rV_1} + e^{r\bar{V}}) - \ln(e^{r\mathbf{f}} + e^{r\bar{V}}) \right) \right)}{2(V_1 - \mathbf{f})}.$$

Substituting  $\mathbf{f}$  for  $V_2$  in equation (16) implies that fundamental values larger than  $\mathbf{f}$  are irrelevant. Analogously, the probability of an intervention at time  $t$ , given the fundamental value  $\mathbf{f}$  has been observed, becomes

$$(17) \quad P(V_t = V_T | \mathbf{f}) = \frac{1}{\mathbf{f} - V_1} \mathbb{1}_{\{V_t \in [V_1, \mathbf{f}]\}} \quad \forall \mathbf{f} \in (V_1, V_2],$$

where

$$(18) \quad \lim_{\mathbf{f} \searrow V_1} P(V_t = V_T | \mathbf{f}) = 1.$$

The expectation terms in (12) and (13) are altered accordingly. The updated relationship between exchange rate and fundamentals also alters the segments in Figure 2. More precisely, the

segment  $ab$  becomes steeper and Krugman's (1991) honeymoon effect weakens.

The framework described above works on the assumption that the non-occurrence of an intervention signals the unknown edge of the strong-side band. It reveals the central bank's true preferences and alters market participants' expectations. Upon observing the exchange rate  $\tilde{S}$ , market participants expect that the intervention triggering exchange rate is located in the smaller range  $[S_1, \tilde{S})$  and no intervention will occur as long as the exchange rate remains within the range  $[\tilde{S}, \bar{S}]$ . In addition, the more the exchange rate appreciates, the higher the expected intervention probability.

In the next subsection, we offer a proper assessment of the post-intervention exchange rate dynamics.

### 3.4 Post-Intervention Exchange Rate Dynamics

As time evolves, an intervention takes place at  $t = T_1$ . This activates the undisclosed strong-side band. But at the same time the problem becomes more complicated. One clear implication is that market participants set a higher probability on central bank intervention close to  $S_{T_1}$ .<sup>14</sup> Thus we need to replace the uniform distribution in equation (9) and (17) by a density function which puts more weight on  $V_{T_1}$  and thus on  $S_{T_1}$ . On the other hand, the first intervention may not be a landmark decision for the entire future, i.e. the intervention triggering point may still be a moving target.<sup>15</sup> Clearly expectations concerning this target depend on the success of the last intervention. Therefore we define market participants' expectations of the intervention triggering exchange rate in  $t = T_2$  conditional on the actual exchange rate being located in the upper,  $(S_{T_1}, S_2]$ , or lower interval,  $[S_1, S_{T_1}]$ .<sup>16</sup>

Starting with the exchange rate dynamics in the lower interval, we consider the conditional probability function  $P(V_t = V_{T_2} | S_1 \leq s(t) \leq S_{T_1})$ . As mentioned above, the distribution function should put more weight on  $S_{T_1}$ . To simplify the problem somewhat, we assume that the density is convex and defined by

$$(19) \quad \varphi(v) = \lambda e^{2(v-V_{T_1})} \mathbf{1}_{\{v \in [V_1, V_{T_1}]\}},$$

where  $0 < \lambda \leq 1$  controls how much weight is put on  $V_{T_1}$  and thus on  $S_{T_1}$ .<sup>17</sup>

When the exchange rate appreciates beyond  $S_T$  without triggering another intervention, this newly acquired information serves as feedback to market participants and provides the basis

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<sup>14</sup>In dynamic economic models backward-looking expectations with systematic forecasting errors are inconsistent with rational behaviour. In nonlinear dynamic models, exhibiting seemingly unpredictable breaks due to the sporadic nature of the interventions, however, simple "rule of thumb" backward-looking expectation rules may yield non-systematic forecasting errors. Furthermore, numerous survey studies, such as Cheung and Chinn (2001) and Menkhoff (1998), uniformly confirm that speculators in foreign exchange markets generally do not rely on mathematically well-defined econometric or economic models, but instead follow simple backward-looking trading rules.

<sup>15</sup>In Klein's (1992) model, the first intervention is such a landmark decision for the future. This implies that after the first intervention the model with undisclosed band width collapses to the standard Krugman (1991) model with full faith in the target zone.

<sup>16</sup>Whether  $S_{T_1}$  is determined to belong to the upper or lower interval, which influences the conditions in the conditional probability functions, is negligible for the exchange rate movements, as only a null set is integrated.

<sup>17</sup>Convex functions are typically used in macroeconomic models with adjustment costs to penalise swift changes in variables and thereby to induce gradual movements over time. Among the many models with convex adjustment costs, quadratic functions have been by far the most common specification, essentially for tractability reasons. Without loss of generality and for mathematical convenience, we also assume a quadratic specification.

for updating prior expectations. To be precise, we alter the probability function by using the smallest observed fundamental  $\mathbf{f}$  to update the prior beliefs. In our notation, we therefore write  $P(V_t = V_{T_2} | \mathbf{f}, S_1 \leq s(t) \leq S_{T_1})$ , in order to show the relation to the updating of  $\mathbf{f}$ , so that equation (19) becomes

$$(20) \quad \varphi(v, \mathbf{f}) = \lambda e^{2(v-\mathbf{f})} \mathbf{1}_{\{v \in [V_1, \mathbf{f}]\}} \quad \forall \mathbf{f} \in (V_1, V_{T_1}).$$

The quintessence is that the lower the value of  $\mathbf{f}$  is, the higher the intervention probability. Ultimately, it is reasonable to assign measure 1 to  $P(V_t = V_{T_2} | \mathbf{f}, S_1 \leq s(t) \leq S_{T_1})$  on the set  $\{V_1, \dots, V_{T_1}\}$ . In other words, the central bank will definitely intervene in the lower range  $[S_1, S_{T_1}]$ . This is tantamount to

$$(21) \quad \int_{V_1}^{\mathbf{f}} \lambda e^{2(v-\mathbf{f})} dv = 1.$$

Equation (21) signifies that  $\lambda$  is a function of  $\mathbf{f}$ .<sup>18</sup>

An illustration of the updating-mechanism is provided in Figure 3. The convex curves show the density in equation (20) for different fundamentals  $\mathbf{f}$ . The horizontal curves show the density of the uniform distribution over the same interval  $(V_1, \mathbf{f}]$ . Apparently, the convex density functions puts more weight on  $\mathbf{f}$  and penalises more distant values in the interval  $(V_1, \mathbf{f}]$ .<sup>19</sup>

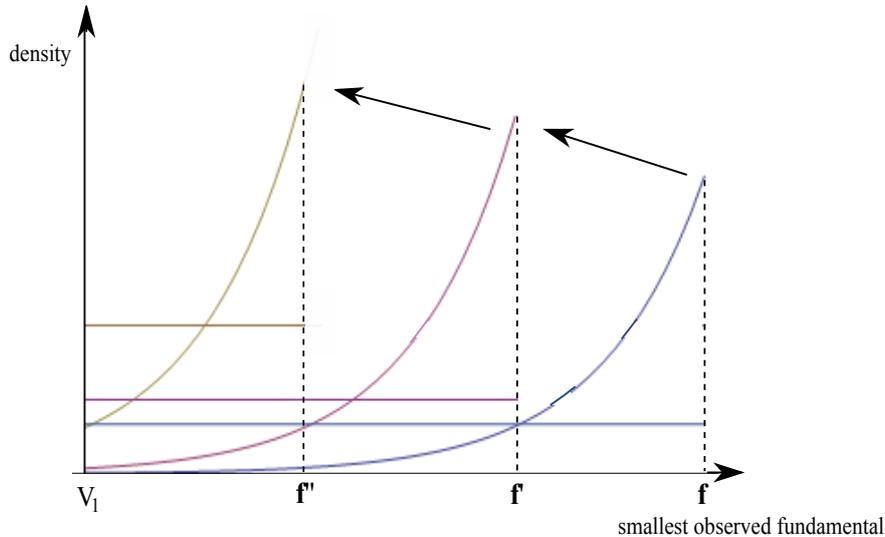


Figure 3: Density function in equation (20) and density function of the uniform distribution on the interval  $(V_1, \mathbf{f}]$  for different fundamentals  $\mathbf{f}$

<sup>18</sup>In our framework, we approximate the relevant considerations with the simplest functional forms to keep the model tractable and the conclusions less susceptible to certain twists in the functions. The derivation of  $\lambda$  is shown in Appendix A.2. Another functional shape for  $\lambda$  is technically feasible if it obeys condition (21).

<sup>19</sup>In other words, Figure 3 highlights the role of choosing  $V_1$ . The larger the interval  $[V_1, \mathbf{f}]$  is set, the flatter the density in (19) and the uniform distribution. Therefore a different choice of  $V_1$  implies a different exchange rate curve progression. If  $V_1$  takes a value close to  $\mathbf{f}$ , the curve pastes smoothly near the exchange rate  $\mathbf{S}$  corresponding to  $\mathbf{f}$ . Otherwise, the more distant  $V_1$  is from  $\mathbf{f}$ , the stronger is the smooth pasting effect departing from  $\mathbf{S}$ . This provides a helpful instrument, as the size of the interval  $[V_1, \mathbf{f}]$  is the mirror-image of the publicly perceived need for an intervention. The following formulas show how this mirror-image is transferred first to the expectations of a monetary operation, which then affects the exchange rate curve progression and the smooth pasting behaviour.

Next we derive the closed-form expression for the exchange rate in the lower range along the lines in subsection 3.3. For  $t \in (T_1, T_2]$  we obtain

$$(22) \quad s(t) = f(t) + \underline{E}(A_1(V^l(t)))e^{rf(t)} + \underline{E}(A_2(V^l(t)))e^{-rf(t)} \quad \forall s(t) \in [S_1, S_{T_1}],$$

where

$$(23) \quad \underline{E}(A_1(V^l)) = - \int_{V_1}^{\mathbf{f}} \frac{r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} \lambda e^{2(v-\mathbf{f})} dv$$

and

$$(24) \quad \underline{E}(A_2(V^l)) = \int_{V_1}^{\mathbf{f}} \frac{e^{r(v+\bar{V})}r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} \lambda e^{2(v-\mathbf{f})} dv.$$

Rearranging (23) and (24) using Hypergeometric function  ${}_2F_1$ , we obtain the closed form solution for the exchange rate dynamics in the lower range  $s(t) \in [S_1, S_{T_1}]$ :<sup>20</sup>

$$(25) \quad s(t) = f(t) + \frac{1}{2(2+r)} e^{-2\mathbf{f}-f(t)r} r \lambda \sigma^2 \tau \left( e^{\mathbf{f}(2+r)} {}_2F_1 \left[ 1, \frac{2+r}{r}; 2 + \frac{2}{r}; -e^{r(\mathbf{f}-\bar{V})} \right] - e^{V_1(2+r)} {}_2F_1 \left[ 1, \frac{2+r}{r}; 2 + \frac{2}{r}; -e^{r(V_1-\bar{V})} \right] \right) - \frac{1}{4} e^{-2\mathbf{f}-r\bar{V}+f(t)r} r \lambda \sigma^2 \tau \left( e^{2\mathbf{f}} {}_2F_1 \left[ 1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(\mathbf{f}-\bar{V})} \right] - e^{2V_1} {}_2F_1 \left[ 1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(V_1-\bar{V})} \right] \right).$$

In the upper range of exchange rates  $(S_{T_1}, S_2]$ , however, we face a different situation concerning the public's expectations. As long as the exchange moves above  $S_{T_1}$ , the market participants may remember the last intervention and consider it the off-the-record strong-side band. This means they implement Krugman's model (1991) with target zone  $[S_{T_1}, \bar{S}]$ . However, including an expectation updating process is reasonable when the exchange rate does not approach  $S_{T_1}$  for a longer time. This might be rationalised by changed economic developments. In this situation the market participants update by taking into account their observations after the first intervention. For implementation, we fix a period of time in which exchange rate behaviour is assumed to fluctuate as in Krugman's basic target zone model with  $V^l = V_{T_1}$  in equation (7). After this period, the public updates its expectations if the exchange rate has departed from  $S_{T_1}$ . If the exchange rate has come close to  $S_{T_1}$ , no updating occurs and the basic Krugman model holds.<sup>21</sup> For an expectations update after a fixed period of time  $\hat{t}$ , the smallest observed fundamentals  $\mathbf{f}$  between  $T_1$  and  $T_1 + \hat{t}$  is used to recondition the relationship between exchange rate and fundamentals. The exchange rate  $\mathbf{S}$  corresponding to  $\mathbf{f}$  is thus used to divide the upper range

<sup>20</sup>The interested reader might look at the derivation in Appendix A.3. A Hypergeometric function can be defined in the form of a convergent Hypergeometric series. Many functions can be expressed as special cases of a Hypergeometric function (eg Exponential, Gamma, Trigometrical and the Bessel functions).

<sup>21</sup>HKMA's foreign currency market interventions are carried out in an open and transparent manner and so are public knowledge. In all cases, the interventions are announced on the day they occur. Agents can therefore distinguish between movements in  $f$  due to interventions and fluctuations due to equation (3)

$(S_{T_1}, S_2]$  into two subsets  $(S_{T_1}, \mathbf{S})$  and  $[\mathbf{S}, S_2]$ , where the exchange rate dynamics in the lower subset  $(S_{T_1}, \mathbf{S})$  are the same as explained above for the lower range  $[S_1, S_{T_1}]$ . For consistency, the exchange rate fluctuations in the subset  $[\mathbf{S}, S_2]$  are modelled as in Krugman's basic target model.

In our mathematical framework, the exchange rate dynamics in the upper interval  $(S_{T_1}, S_2]$  are expressed as follows. As long as no expectation update is implemented, the exchange rate behaves according to

$$(26) \quad s(t) = f(t) + A_1(V_{T_1})e^{rf(t)} + A_2(V_{T_1})e^{-rf(t)} \quad \forall s(t) > S_{T_1}, \quad \forall t: T_1 < t < T_1 + \hat{t} < T_2,$$

where  $A_1$  and  $A_2$  arise from (8).

The smallest observed fundamental between the  $i$ 'th and  $i + 1$ 'th,  $i \in \mathbb{N}$ , expectation update is denoted by  $\mathbf{f}_i$ . After the first expectation update, the exchange rate (if  $s(t) \geq \mathbf{S}_i$ ) moves according

$$(27) \quad s(t) = f(t) + A_1(\mathbf{f}_i)e^{rf(t)} + A_2(\mathbf{f}_i)e^{-rf(t)} \quad \forall t: i(T_1 + \hat{t}) \leq t < (i + 1)(T_1 + \hat{t}) < T_2.$$

In contrast, the dynamics of the exchange rate  $s(t) < \mathbf{S}_i$  accord with the mechanism in equation (25).

### 3.5 Information Content of Further Interventions

In the last subsection, we analysed the exchange rate dynamics assuming that an intervention only occurs once. This setup may be unrealistic for economies for which (a) the structure of the economy is constantly evolving in ways that are imperfectly understood by both the public and policymakers and (b) the policymakers' objective function may change over time and is not fully known by private agents.<sup>22</sup> What happens once further interventions are carried out? Where does that leave us? For the sake of simplicity we assume that market participants use a weighted average of past intervention triggering exchange rates as a predictor of future interventions. Hence, in terms of our notation we define

$$(28) \quad \mathcal{S}_{\mathcal{T}_N} = \sum_{i=1}^N a_i S_{T_i} \quad \forall N \in \mathbb{N},$$

where  $a_i \in [0, 1]$  and  $\sum_{i=1}^N a_i = 1$ .

The weighted average exchange rate  $\mathcal{S}_{\mathcal{T}_N}$  yields the intervals  $[S_1, \mathcal{S}_{\mathcal{T}_N}]$  and  $(\mathcal{S}_{\mathcal{T}_N}, S_2]$  for the mechanism in section 3.4. Corresponding to  $\mathcal{S}_{\mathcal{T}_N}$  is the fundamental  $\mathcal{V}_{\mathcal{T}_N}$ . Equation (28) implies that the extent to which intervention expectations are anchored can change, depending on economic developments and (most important) the current and past conduct of monetary policy.<sup>23</sup>

<sup>22</sup>Market participants realise, however, that a currency board is no free lunch. For example, countries with currency boards imported the loose monetary conditions of the advanced economies. This policy put a floor under asset prices and eventually created the conditions for the credit crunch of 2008.

<sup>23</sup>Technically expressed, the coefficients  $a_i$  in equation (28) show the importance of the last interventions. However, expectations are also influenced by  $V_1$  (compare the effects of different  $V_1$  by means of Figure 3). Therefore, there is also room for expectations, which are not primarily anchored by past interventions.

### 3.6 Introduction of a Symmetric Band

On 18 May 2005 the currency board arrangement was altered when finally a narrow symmetric target zone of 0.6 percent was introduced with a strong-side CU at HKD 7.75/USD. For the first time, this added a ceiling to the floor by which it had traditionally managed the currency, in a move to discourage investors from using the HKD to speculate on RMB appreciation. At the same time, the weak-side CU was shifted from HKD 7.80/USD to HKD 7.85/USD. These "refinements" were intended to anchor market expectations and promote smooth functioning of the currency board arrangement.<sup>24</sup>

Viewed in retrospect, it is reasonable to say that different considerations and assessments may have prevailed after May 2005. Judged by the HKD exchange rate since January 2004, market participants may have assessed the newly introduced symmetric band as generously dimensioned. Alternatively, one can well imagine that financial markets may not have based their expectations on a blind faith in the effectiveness of the currency board mechanism and the will and commitment of the monetary authorities to defend the edges of the band.<sup>25</sup> If markets can figure out the potential fragility of the edges and perform the requisite backward induction, then a target zone may lose its reputation and stabilising power.<sup>26</sup>

We implement the credibility issues arising in the new regime via a new version for equation (28). Credibility is defined as the capacity of the policymakers to announce a policy which is trusted by market participants. For analytical convenience we modify the model such that  $\mathcal{S}_{\mathcal{T}_N}$  is given by

$$(29) \quad \mathcal{S}_{\mathcal{T}_N} = \sum_{i=1}^N a_i S_{T_i} + a_A S_A, \quad \forall N \in \mathbb{N},$$

where  $S_A$  is the announced strong-side band, the coefficients  $a_A$ ,  $a_i \in [0, 1]$  and  $\sum_{i=1}^N a_i = 1 - a_A$ . In other words,  $\mathcal{S}_{\mathcal{T}_N}$  is the weighted average of the past  $N$  interventions and the announced strong-side band  $S_A$ , and the coefficient  $a_A$  gauges the extent to which the announcement is seen as credible. A larger  $a_A$  coefficient ties  $\mathcal{S}_{\mathcal{T}_N}$  closer to  $S_A$ . Subsequently, the full credibility scenario is given by  $\mathcal{S}_{\mathcal{T}_N} = S_A$  and  $\mathcal{V}_{\mathcal{T}_N} = V_1$  (see Figure 2). For  $\mathcal{S}_{\mathcal{T}_N} > S_A$  the credibility constraint is not binding. Lastly, the imperfect credibility case, where the public doubts the monetary authority's ability to defend the announced band, is given by  $\mathcal{S}_{\mathcal{T}_N} < S_A$ .

In the next section we conduct an analysis of the model by resorting to numerical methods.

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<sup>24</sup>For a graphical display, see Figure 1.

<sup>25</sup>The 1-year forward rate of the HKD was consistently outside the convertibility zone between October 2005 and the start of 2007. This is known as Svensson's (1992, 1994) 100% credibility test and indicates that financial market participants have initially revealed scepticism about the ability of the new strong side CU to limit exchange rate fluctuations. Intermittent upward pressure on the HKD occurred again in autumn 2007 when HKMA interventions again were aimed at anchoring market expectations. Related empirical analysis of the credibility of the two-sided system is scarce, exceptions being Fung and Yu (2007) and Hui and Fong (2007).

<sup>26</sup>The HKMA would not be the first central bank to do this. For example, in the European Exchange Rate Mechanism (ERM) currencies were initially allowed to fluctuate no more than 2.25 % above or below their fixed bilateral rates. The UK joined in 1990 but was forced to leave in 1992 when sterling came under speculative pressure. Fluctuation bands were then widened to  $\pm 15$  % in 1993 to avoid defending the indefensible.

## 4 Putting it all Together

The idea is to make our model match exchange rate data of interest.<sup>27</sup> Figure 4 illustrates the interventions of the HKMA over the period 2001 to 2007.<sup>28</sup> Contrary a common view, Hong Kong's currency board is not a simple rule-based monetary policy but rather involves some discretion. Figure 4 also shows that the currency board was one-sided until May 2005, i.e. there was a commitment to sell, but not to buy, US dollars at 7.80 HKD/USD. The spot exchange rate of HKD against USD remained on the strong side from autumn 2003 through January 2004, with repeated foreign exchange market interventions by the HKMA to stem the upward pressure amidst continued capital inflows.<sup>29</sup> This stepping in and "leaning against the wind" in late 2003 to defend the HKD peg to the US dollar highlights the fear that an appreciation of HKD might hurt the reputation of the currency board system. Ultimately, there was a slight easing in the strength of the HKD in late January 2004, amidst growing concern about the spread of avian flu in East Asia.

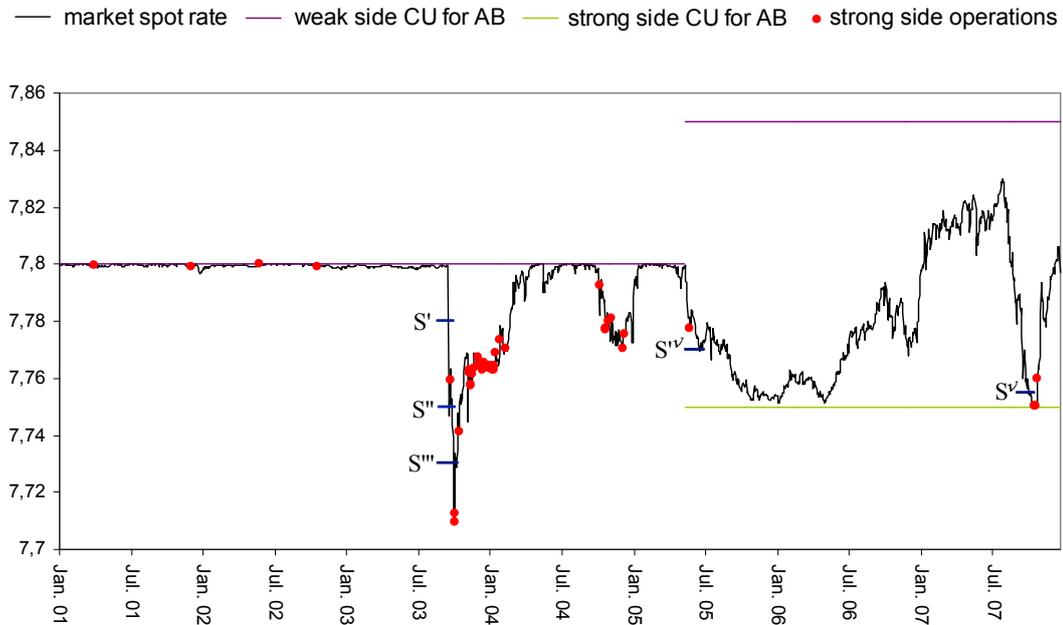


Figure 4: Strong side operations

The graph also highlights the five points indicated by  $S'$ -  $S''''$ . Below we calibrate the dynamics of the exchange rate at these instants.

<sup>27</sup>At the end of the day, it is always useful to acknowledge that the theoretical framework and calibrations primarily serve as a communication device. The modelling approach introduced in this paper goes some way towards achieving the purpose of understanding the dynamics of the HKD. Yet we do not claim empirical accuracy for the model but use it rather for qualitative features and predictions.

<sup>28</sup>Interventions refer to net injections or withdrawals of funds by HKMA in the interbank money market. For the daily market operations data, see <http://www.info.gov.hk/hkma/eng/statistics/msb/index.htm>.

<sup>29</sup>Interventions were necessary because markets believed that the HKD would appreciate alongside the RMB made the automatic adjustment mechanisms of the currency board system ineffective. For an econometric logit analysis of monetary operations conducted by the HKMA, using daily data for the one-sided regime between September 1998 and December 2001, see Gerlach (2005).

First we derive the exchange rate dynamics prior to the strong-side interventions at  $S_{T_1}$ . The benchmark set of parameters for the simulation is  $\sigma = 0.01$ ,  $\tau = 0.9$ ,  $V_1 = 2.035 \approx \ln(7.65)$ .

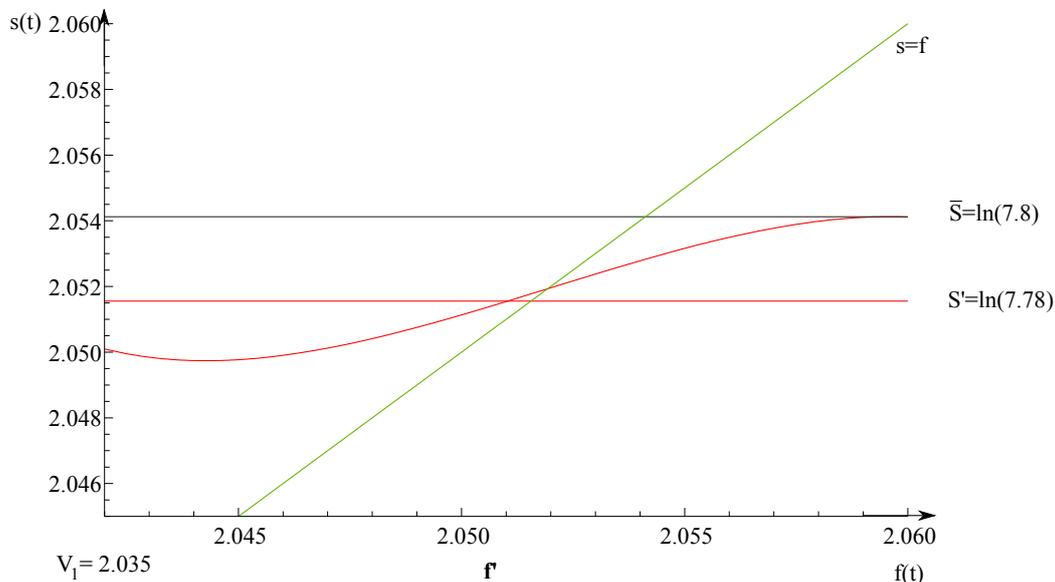


Figure 5:  $S$ -shaped curve for  $S' = \ln(7.78)$

The first point to note is that the relationship between fundamentals and exchange rate is an  $S$ -shaped curve, i.e. the exchange rate is a function of the fundamentals and the expected exchange rate, leading to a disconnect between fundamentals and nominal volatility. Two properties of the solution are apparent from Figure 5. First, the upper weak-side band is fully credible. Second, as long as no intervention occurs, market participants expect a further appreciation of the HKD beyond  $S'$ . Where does that leave us? For  $S' = \ln(7.78)$ , for example, the "lens" below the horizontal line indicates that the perceived exchange rate band ranges up to approximately  $2.05 \approx \ln(7.768)$ .

Figure 5 shows the dynamics of the exchange rate for a given level of  $V_1$  determining the lower bound of the perceived exchange rate interval prior to the first intervention.<sup>30</sup> Next we explore the sensitivity of the conclusions presented above to assumptions about  $V_1$ . The effect of alternative  $V_1$  parameters is indicated in Figure 6. The  $S$ -shaped curves illustrate that the appreciation pressure is perceived to be less severe for larger  $V_1$  parameters. This leads to a narrower perceived target zone range. The intuition for the result is straightforward and can be sketched as follows. The moderating honeymoon effect is the stronger, the better the reputation of the policymaker, which leads to a narrower interval for the fundamentals that can trigger an intervention. Formally stated, given the uniform distribution in equations (9) and (17) the probability of an intervention increases with larger values of  $V_1$ .

<sup>30</sup>In the theoretical modelling framework,  $V_1$  is assumed to be exogenous, neglecting central bank incentives to influence expectations with announcements. Rational central banks choose "verbal interventions" as a toolkit since it has the ability to enhance the predictability of monetary policy decisions and potentially to help achieve central banks' macroeconomic objectives. On the other hand, when optimal policies are dynamically inconsistent, announcements will only be considered cheap talk. For a survey of this partially credible commitment device, see Blinder et al. (2008).

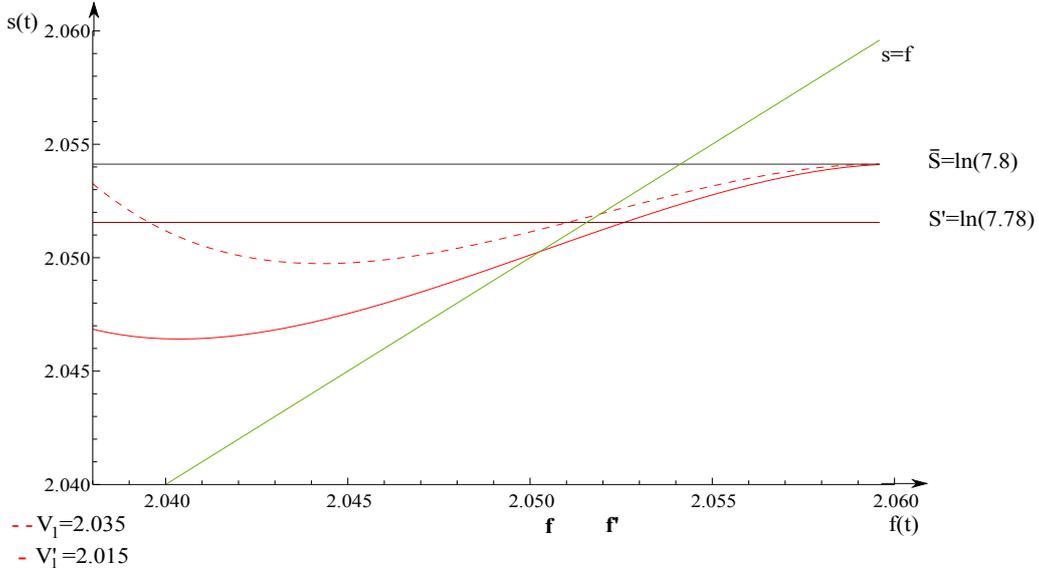


Figure 6:  $S$ -shaped curves for  $S' = \ln(7.78)$  and  $V_1 \approx \ln(7.65)$  vs.  $V_1' \approx \ln(7.50)$

We now return to our main theme and consider the exchange rate dynamics after the first intervention, i.e.  $S''$  and  $S'''$  in Figure 4. Interventions induce market participants to make inferences about HKMA preferences, i.e. to predict the implicit strong-side band. How does the intervention affect the belief of the public? As demonstrated in subsection 3.5 of the theoretical framework, credibility increases discretely with successive interventions. Furthermore, interventions influence future behaviour until the learning process brings beliefs closely in line with reality. The resulting exchange rate dynamics in  $S''$  and  $S'''$  after the first intervention can be studied with the help of Figure 7.

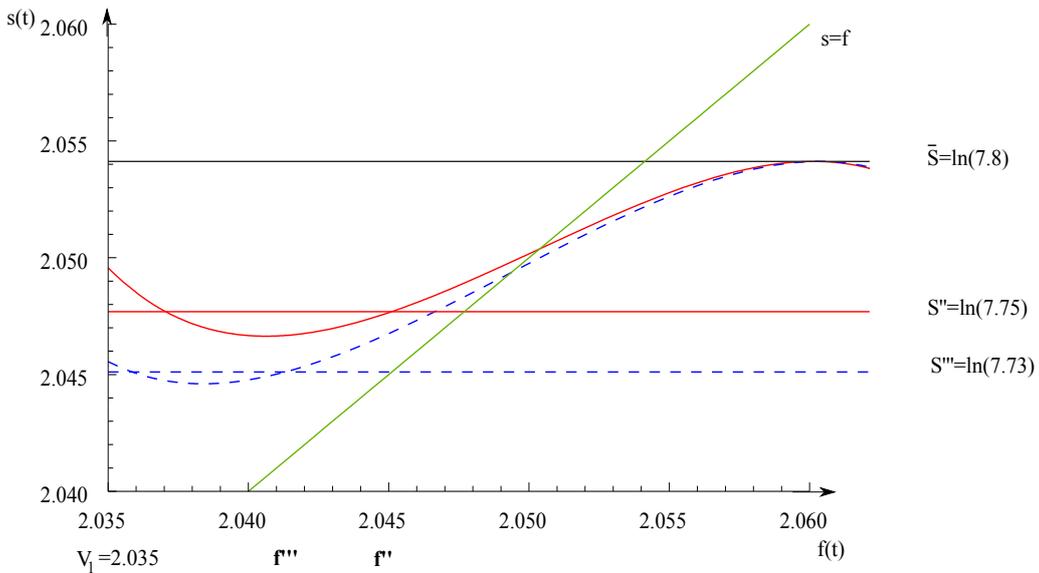


Figure 7:  $S$ -shaped curves with learning about the first intervention

Again we obtain a family of  $S$ -shaped curves. Comparing the exchange rate dynamics for  $S'' = \ln(7.76)$  and  $S''' = \ln(7.74)$ , three differences are apparent. First, the relationship be-

tween fundamentals and exchange rate becomes steeper, and the non-linear effect is reduced for  $S''' = \ln(7.74)$ . Second, since the exchange rate has appreciated beyond  $S'' = \ln(7.76)$ , the perceived bandwidth has increased. Third, and not less importantly, central bank interventions are expected to be more likely and more intensive at  $S''' = \ln(7.74)$ . This means that expectations of further appreciations are lower and the stabilizing effect of the undisclosed target zone increases.<sup>31</sup>

Over the last decade, central banks have implemented new versions of target zone exchange rate regimes. In Hong Kong, a symmetric band  $(\underline{S}, \bar{S})$  forming upper and lower limits for HKD fluctuations around the central parity was adopted in May 2005, as an integral part of the currency board regime (see Figure 1). By way of example, we finally calibrate the dynamics of the exchange rate according to equation (25) and (29) for  $S'^v$  (5 July 2005) and  $S^v$  (25 October 2007).

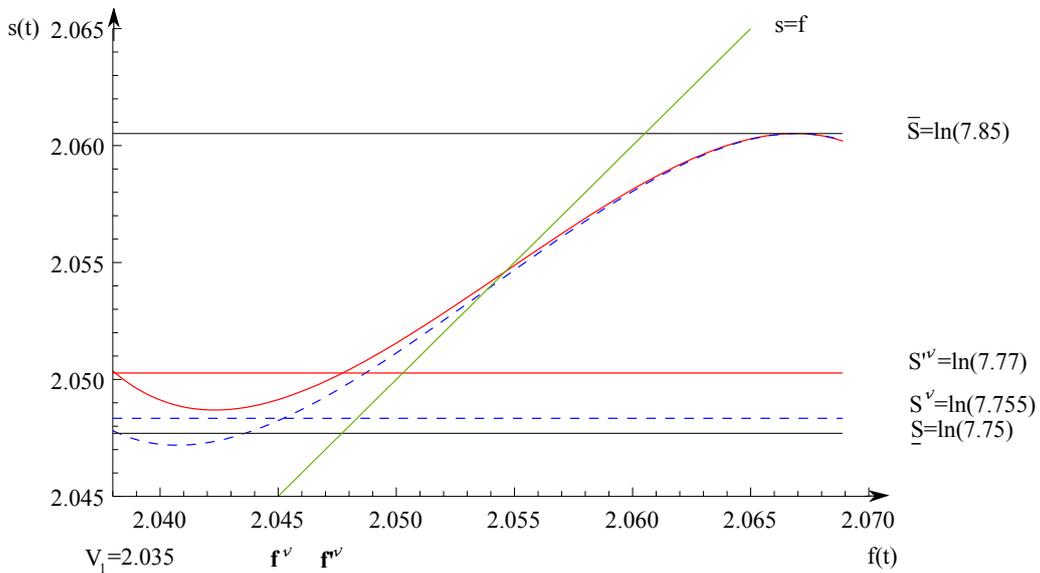


Figure 8:  $S$ -shaped exchange rate dynamics in the symmetric target zone

Viewed in retrospect, Figure 8 displays the exchange rate dynamics for  $S'^v = \ln(7.77) \approx 2.0502$  and  $S^v = \ln(7.755) \approx 2.0483$ . We have assumed  $N = 8$  in equation (29). The resultant  $a_A$ -coefficients for  $S'^v$  and  $S^v$  are  $a_A \approx 0.309$  and  $a_A \approx 0.827$ , respectively (see Appendix A.4).

The calibration exercise suggests several conclusions. As a start, Figure 8 tells essentially the same qualitative story from the exchange rate dynamics point of view. Again we obtain  $S$ -shaped behaviour of the exchange rate. The difference is in the detail. First, the exchange rate is now stabilised at the credible upper edge  $\bar{S} = \ln(7.85) \approx 2.0605$ .<sup>32</sup> Second, at  $S'^v$  the lower (strong-side) edge of the band is perceived to be credible according to the calibrations. Market participants form beliefs that the automaticity of the currency board system and/or market operations conducted in light of market conditions are effective. This calibration result is consistent with the empirical fact that on 5 July 2005 the 1-year forward rate  $S^{1-Year} = 7.7528$  was slightly above the strong-side band at 7.75 HKD/USD, i.e. no HKD appreciation beyond

<sup>31</sup>Further points after the next interventions yielded qualitatively similar, although quantitatively different, results. For brevity of exposition, only the exchange rate dynamics for  $S''$  and  $S'''$  is presented here. Interested readers may obtain further calibrations of the model dynamics from the authors upon request.

<sup>32</sup>Please note that the upper band of the target zone hasn't been tested to the limit during the sample period.

the strong-side band was expected by the market. Third, in autumn 2007 another episode with strong pressure for revaluation occurred (see Figure 4).<sup>33</sup> Subsequently, the HKMA started to conduct operations in the foreign exchange market with the intent to stabilise the rate. In the end, stability was maintained. The simulated  $S$ -shaped dynamics for 25 October 2007 indicates that this time appreciations of the HKD beyond the strong-side band  $\underline{S} = \ln(7.75)$  were expected by the market, i.e. market participants had doubts about the ability and/or commitment of the HKMA to defend the narrow band in the face of exchange rate shocks.<sup>34</sup>

## 5 Summary and Conclusion

The modelling framework in the paper provides a variation of the standard hard edge target zone concept. The theoretical literature provides few clues to understanding the phenomenon of one-sided target zones. The classical model of Krugman (1991) describes the behaviour of the exchange rate in a perfectly known and credible symmetric target zone. Klein (1992) presents a model in which the width of the symmetric band is unknown to the public. The exchange rate is thus a function of fundamentals and expected bandwidth. Unfortunately, he only analyses the dynamics within symmetric bands up to the first intervention.

We have considered a model of an asymmetric target zone with an undisclosed strong-side band and learning by market participants. Depending upon the occurrence or non-occurrence of interventions, market participants revise their intervention probabilities and the location of the off-the-record strong-side band. To study the expectations-updating scheme and the mechanisms that give rise to the dynamics of the exchange rate, we consider Hong Kong's exchange rate regime since the turn of the millenium. We believe the model allows an elegant and parsimonious treatment of undisclosed asymmetric target zones. The model in this paper is stylised, but the results are rich and may be of relevance to other target zone economies. Nevertheless, the analysis may be pressed further, to show how uncertainty about exchange rate policy might result from uncertainty about broader government objectives such as the Pearl River Delta integration and how this uncertainty can be reduced by precommitments.

## A Appendix

### A.1 Derivation of equation (15)

At first we solve the integrals in (14). Manipulations of the integrand provide the primitive  $\ln(e^{rv} + e^{r\bar{V}})$  of  $\frac{re^{rv}}{e^{rv} + e^{r\bar{V}}}$ . Therefore we obtain

$$E(A_1(V^l)) = \int_{V_1}^{V_2} -\frac{r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} \frac{1}{V_2 - V_1} dv$$

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<sup>33</sup>Our analysis invites a number of questions. One of these is why the HKMA cannot take actions which would provide an unambiguous signal of its intentions. For a model with Markov switching and recurrent interventions to keep the HKD within the band, see Chen et al. (2010). The regime-dependent toolkit makes the perceived nonlinear swings in exchange rate pressure tractable.

<sup>34</sup>This calibration result is consistent with the empirical fact that on 25 October 2007 the 1-year forward rate  $S^{1-Year} = 7.728$  was below the strong-side band at 7.75 HKD/USD. One perceived risk was that the newly introduced symmetric bands might be widened in the future.

$$\begin{aligned}
&= \frac{\sigma^2 \tau}{2(V_1 - V_2)} \int_{V_1}^{V_2} \frac{r e^{-r\bar{V}} e^{r\bar{V}}}{e^{rv} + e^{r\bar{V}}} dv \\
&= \frac{e^{-r\bar{V}} \sigma^2 \tau}{2(V_1 - V_2)} \int_{V_1}^{V_2} \frac{r e^{r\bar{V}} + r e^{rv} - r e^{rv}}{e^{rv} + e^{r\bar{V}}} dv \\
&= \frac{e^{-r\bar{V}} \sigma^2 \tau}{2(V_1 - V_2)} \int_{V_1}^{V_2} r \frac{(e^{rv} + e^{r\bar{V}})}{e^{rv} + e^{r\bar{V}}} - \frac{r e^{rv}}{e^{rv} + e^{r\bar{V}}} dv \\
&= \frac{e^{-r\bar{V}} \sigma^2 \tau}{2(V_1 - V_2)} \int_{V_1}^{V_2} r - \frac{r e^{rv}}{e^{rv} + e^{r\bar{V}}} dv \\
&= \frac{e^{-r\bar{V}} \sigma^2 \tau}{2(V_1 - V_2)} \left( r v - \ln(e^{rv} + e^{r\bar{V}}) \right) \Big|_{V_1}^{V_2} \\
&= \frac{-e^{-r\bar{V}} \sigma^2 \tau \left( r(V_1 - V_2) + \ln(e^{rV_2} + e^{r\bar{V}}) - \ln(e^{rV_1} + e^{r\bar{V}}) \right)}{2(V_1 - V_2)}
\end{aligned}$$

and

$$\begin{aligned}
E(A_2(V^l)) &= \int_{V_1}^{V_2} \frac{e^{r(v+\bar{V})} r \sigma^2 \tau}{2(e^{rv} + e^{r\bar{V}})} \frac{1}{V_2 - V_1} dv \\
&= -\frac{e^{r\bar{V}} \sigma^2 \tau}{2(V_1 - V_2)} \int_{V_1}^{V_2} \frac{r e^{rv}}{e^{rv} + e^{r\bar{V}}} dv \\
&= -\frac{e^{r\bar{V}} \sigma^2 \tau}{2(V_1 - V_2)} \ln(e^{rv} + e^{r\bar{V}}) \Big|_{V_1}^{V_2} \\
&= \frac{e^{r\bar{V}} \sigma^2 \tau \left( \ln(e^{rV_1} + e^{r\bar{V}}) - \ln(e^{rV_2} + e^{r\bar{V}}) \right)}{2(V_1 - V_2)}.
\end{aligned}$$

Now we derive the closed form expression (15) using both expectation values in equation (14).

$$\begin{aligned}
s(t_0) &= f(t_0) - \frac{e^{f(t_0)r-r\bar{V}} \sigma^2 \tau \left( r(V_1 - V_2) + \ln(e^{rV_2} + e^{r\bar{V}}) - \ln(e^{rV_1} + e^{r\bar{V}}) \right)}{2(V_1 - V_2)} \\
&\quad + \frac{e^{-f(t_0)r+r\bar{V}} \sigma^2 \tau \left( \ln(e^{rV_1} + e^{r\bar{V}}) - \ln(e^{rV_2} + e^{r\bar{V}}) \right)}{2(V_1 - V_2)} \\
&= f(t_0) - e^{-r(f(t_0)+\bar{V})} \sigma^2 \tau \\
&\quad \frac{\left( e^{2f(t_0)r} r (V_1 - V_2) - \left( e^{2f(t_0)r} + e^{2r\bar{V}} \right) \left( \ln(e^{rV_1} + e^{r\bar{V}}) - \ln(e^{rV_2} + e^{r\bar{V}}) \right) \right)}{2(V_1 - V_2)}
\end{aligned}$$

## A.2 Derivation of $\lambda$

An easy way to choose  $\lambda$  properly is to derive it from condition (21), which claims that

$$\begin{aligned}
1 &\stackrel{!}{=} \int_{V_1}^{\mathbf{f}} \lambda e^{2(v-\mathbf{f})} dv = \lambda \frac{1}{2} e^{2v-2\mathbf{f}} \Big|_{V_1}^{\mathbf{f}} = \lambda \frac{1}{2} - \lambda \frac{1}{2} e^{2V_1-2\mathbf{f}} \\
&\Rightarrow \lambda = \frac{1}{\frac{1}{2} - \frac{1}{2} e^{2V_1-2\mathbf{f}}}.
\end{aligned}$$

### A.3 Derivation of equation (25)

Before we prove equation (25), we provide a short introduction to the Hypergeometric function. The Hypergeometric function  ${}_2F_1$  is the convergent Gauss Hypergeometric series

$${}_2F_1[a, b; c; z] = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{z^n}{n!},$$

where the circle of convergence is the unit circle  $|z| = 1$  and  $\Gamma(\cdot)$  denotes the Gamma function. The relationship between the factorial and the Gamma function is defined as  $\Gamma(n+1) = n!$  for all  $n \in \mathbb{N}$ . The functional equation of the Gamma function is  $x\Gamma(x) = \Gamma(x+1)$  for all  $x \in \mathbb{R}^+$ . An important property of Hypergeometric functions is that the six functions  ${}_2F_1[a \pm 1, b; c; z]$ ,  ${}_2F_1[a, b \pm 1; c; z]$  and  ${}_2F_1[a, b; c \pm 1; z]$  are contiguous to  ${}_2F_1[a, b; c; z]$ . They are used to express one of them as a linear combination of any two of the other contiguous functions and are derived by Gauss. The two relations that are applied in the following are

**Property 1:**  $\frac{bz}{c} {}_2F_1[a+1, b+1; c+1; z] = {}_2F_1[a+1, b; c; z] - {}_2F_1[a, b; c; z]$

**Property 2:**  $a {}_2F_1[a+1, b; c; z] = b {}_2F_1[a, b+1; c; z] - (b-a) {}_2F_1[a, b; c; z]$

Another important property is

**Property 3:**  ${}_2F_1[a, b; b; z] = (1-z)^{-a}$

A useful overview of the linear combinations and other interesting relations is given in Abramowitz and Stegun (1972).

Being equipped with this short introduction to Hypergeometric functions  ${}_2F_1$ , we turn to the calculation of the expectation  $\underline{E}(A_1(V^l))$ .

$$\begin{aligned}
\underline{E}(A_1(V^l)) &= \int_{V_1}^{\mathbf{f}} -\frac{r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} \lambda e^{2(v-\mathbf{f})} dv \\
&= -\frac{1}{2} r\sigma^2\tau\lambda \int_{V_1}^{\mathbf{f}} \frac{e^{2(v-\mathbf{f})}}{e^{rv} + e^{r\bar{V}}} dv \\
&= -\frac{1}{4} r \lambda \sigma^2 \tau e^{-2\mathbf{f}+2v-r\bar{V}} {}_2F_1\left[1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(v-\bar{V})}\right] \Big|_{V_1}^{\mathbf{f}} \\
&= -\frac{1}{4} e^{-2\mathbf{f}-r\bar{V}} r \lambda \sigma^2 \tau \\
&\quad \left( e^{2\mathbf{f}} {}_2F_1\left[1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(\mathbf{f}-\bar{V})}\right] - e^{2V_1} {}_2F_1\left[1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(V_1-\bar{V})}\right] \right)
\end{aligned}$$

In order to prove the third equality, we show that the derivative of  $\frac{1}{2} e^{-2\mathbf{f}+2v-r\bar{V}} {}_2F_1 \left[ 1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(v-\bar{V})} \right]$  equals the integrand in the second equation. As a preliminary we consider the derivative of the Hypergeometric function. The derivation of the Hypergeometric function is

$$\begin{aligned}
& \frac{d}{dv} \left( {}_2F_1 \left[ 1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(v-\bar{V})} \right] \right) \\
&= \frac{d}{dv} \left( \frac{\Gamma(\frac{2}{r}+1)}{\Gamma(1)\Gamma(\frac{2}{r})} \sum_{n=0}^{\infty} \frac{\Gamma(1+n)\Gamma(\frac{2}{r}+n)}{\Gamma(\frac{2}{r}+1+n)} \frac{-e^{nr(v-\bar{V})}}{n!} \right) \\
&= \frac{\Gamma(\frac{2}{r}+1)}{\Gamma(1)\Gamma(\frac{2}{r})} \sum_{n=0}^{\infty} \frac{\Gamma(1+n)\Gamma(\frac{2}{r}+n)}{\Gamma(\frac{2}{r}+1+n)} \frac{nr(-e^{nr(v-\bar{V})})}{n!} \\
&= \frac{\Gamma(\frac{2}{r}+1)}{\Gamma(1)\Gamma(\frac{2}{r})} \sum_{n=0}^{\infty} \frac{\Gamma(2+n)\Gamma(\frac{2}{r}+n+1)}{\Gamma(\frac{2}{r}+n+2)} \frac{r(-e^{r(v-\bar{V})})^{n+1}}{n!} \\
&= -re^{r(v-\bar{V})} \frac{2}{2+r} \frac{\Gamma(\frac{2}{r}+2)}{\Gamma(2)\Gamma(\frac{2}{r}+1)} \sum_{n=0}^{\infty} \frac{\Gamma(2+n)\Gamma(\frac{2}{r}+1+n)}{\Gamma(\frac{2}{r}+2+n)} \frac{(-e^{r(v-\bar{V})})^n}{n!} \\
&= -re^{r(v-\bar{V})} \frac{2}{2+r} {}_2F_1 \left[ 2, \frac{2}{r}+1; \frac{2}{r}+2; -e^{r(v-\bar{V})} \right].
\end{aligned}$$

The third equality holds because the first summand in the line above is zero. For the fourth equality we use the functional equation of the Gamma function, which results in a Hypergeometric function with new parameters.

According to the order of the equal signs the properties 1-3 are applied:

$$\begin{aligned}
& r \frac{-2e^{r(v-\bar{V})}}{2+r} {}_2F_1 \left[ 2, \frac{2}{r}+1; \frac{2}{r}+2; -e^{r(v-\bar{V})} \right] \\
&= r \left( {}_2F_1 \left[ 2, \frac{2}{r}; \frac{2}{r}+1; -e^{r(v-\bar{V})} \right] - {}_2F_1 \left[ 1, \frac{2}{r}; \frac{2}{r}+1; -e^{r(v-\bar{V})} \right] \right) \\
&= r \left( \frac{2}{r} {}_2F_1 \left[ 1, \frac{2}{r}+1; \frac{2}{r}+1; -e^{r(v-\bar{V})} \right] - \frac{2}{r} {}_2F_1 \left[ 1, \frac{2}{r}; \frac{2}{r}+1; -e^{r(v-\bar{V})} \right] \right) \\
&= 2 \left( \frac{1}{1+e^{r(v-\bar{V})}} - {}_2F_1 \left[ 1, \frac{2}{r}; \frac{2}{r}+1; -e^{r(v-\bar{V})} \right] \right).
\end{aligned}$$

Now we have obtained all the ingredients for differentiating the above mentioned primitive.

$$\begin{aligned}
& \frac{d}{dv} \left( \frac{1}{2} e^{-2\mathbf{f}+2v-r\bar{V}} {}_2F_1 \left[ 1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(v-\bar{V})} \right] \right) \\
&= e^{-2\mathbf{f}+2v-r\bar{V}} {}_2F_1 \left[ 1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(v-\bar{V})} \right] \\
& \quad + e^{-2\mathbf{f}+2v-r\bar{V}} \left( \frac{1}{1+e^{r(v-\bar{V})}} - {}_2F_1 \left[ 1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(v-\bar{V})} \right] \right) \\
&= e^{-2\mathbf{f}+2v-r\bar{V}} \left( \frac{1}{1+e^{r(v-\bar{V})}} - {}_2F_1 \left[ 1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(v-\bar{V})} \right] + {}_2F_1 \left[ 1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(v-\bar{V})} \right] \right) \\
&= \frac{e^{2(v-f)}}{e^{r\bar{V}}(1+e^{r(v-\bar{V})})} \\
&= \frac{e^{2(v-f)}}{e^{r\bar{V}}+e^{rv}}.
\end{aligned}$$

The other expectation is given without proof of the primitive, as it is derived in a like manner.

$$\begin{aligned}
\underline{E}(A_2(V^l)) &= \int_{V_1}^{\mathbf{f}} \frac{e^{r(v+\bar{V})} r \sigma^2 \tau}{2(e^{rv} + e^{r\bar{V}})} \lambda e^{2(v-\mathbf{f})} dv \\
&= \frac{e^{-2\mathbf{f}+(2+r)v} r \lambda \sigma^2 \tau {}_2F_1 \left[ 1, \frac{2+r}{r}; 2 + \frac{2}{r}; -e^{r(v-\bar{V})} \right]}{2(2+r)} \Bigg|_{V_1}^{\mathbf{f}} \\
&= \frac{1}{2(2+r)} e^{-2\mathbf{f}} r \lambda \sigma^2 \tau \\
&\quad \left( e^{\mathbf{f}(2+r)} {}_2F_1 \left[ 1, \frac{2+r}{r}; 2 + \frac{2}{r}; -e^{r(\mathbf{f}-\bar{V})} \right] - e^{V_1(2+r)} {}_2F_1 \left[ 1, \frac{2+r}{r}; 2 + \frac{2}{r}; -e^{r(V_1-\bar{V})} \right] \right)
\end{aligned}$$

Hence the closed form solution results in

$$\begin{aligned}
s(t) &= f(t) + \frac{1}{2(2+r)} e^{-2\mathbf{f}-f(t)r} r \lambda \sigma^2 \tau \\
&\quad \left( e^{\mathbf{f}(2+r)} {}_2F_1 \left[ 1, \frac{2+r}{r}; 2 + \frac{2}{r}; -e^{r(\mathbf{f}-\bar{V})} \right] - e^{V_1(2+r)} {}_2F_1 \left[ 1, \frac{2+r}{r}; 2 + \frac{2}{r}; -e^{r(V_1-\bar{V})} \right] \right) \\
&\quad - \frac{1}{4} e^{-2\mathbf{f}-r\bar{V}+f(t)r} r \lambda \sigma^2 \tau \\
&\quad \left( e^{2\mathbf{f}} {}_2F_1 \left[ 1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(\mathbf{f}-\bar{V})} \right] - e^{2V_1} {}_2F_1 \left[ 1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(V_1-\bar{V})} \right] \right).
\end{aligned}$$

#### A.4 Calculation of the coefficient $a_A$

Suppose that market participants assign the same weight to the last  $N = 8$  interventions in their expectations formation process. In the case where  $S^{lv} = \ln(7.77)$  and  $S^v = \ln(7.755)$ , these market operation dates and the corresponding HKD spot exchange rates are:

12.10.04	7.7925	10.11.04	7.7812
25.10.04	7.7771	08.12.04	7.7705
27.10.04	7.7777	10.12.04	7.7753
01.11.04	7.7799	27.05.05	7.7775

As a start, this enables us to calculate the logarithmized exchange rates  $S_{T_1} - S_{T_8}$ . The value of  $a_A$  for  $S^{lv} = \ln(7.77)$  can then be solved from

$$(1 - a_A) \frac{1}{8} \sum_{i=1}^8 S_{T_i} + a_A S_A = S^{lv}.$$

This equation is derived from equation (29). However, the question arises as to why the left hand side is equal to  $S^{lv}$ . One has to take into consideration that whenever a smaller fundamental  $\mathbf{f}$  is observed, the interval of possible intervention triggering exchange rates is truncated. Where does that leave us? In case of  $S^{lv}$ , the original interval  $[S_1, \mathcal{S}_{T_N}]$  is reduced to  $[S_1, S^{lv}]$  and so we can use  $S^{lv}$  to calculate the unknown coefficient  $a_A$ . The resulting parameter is  $a_A \approx 0.309$ . In an analogous manner,  $a_A \approx 0.827$  is obtained for  $S^v$ .

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