In Dubio, pro CES: Technical Progress Specification and the Elasticity of Substitution

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Abstract

Capital-labor substitution and total factor productivity (TFP) estimates are essential features of growth and income distribution models. We show that the estimation of the elasticity of substitution can be substantially biased if the form of technical progress is misspecified. For plausible parameter values, when factor shares are relatively constant, there is an inherent bias towards the Cobb-Douglas case. The implied estimates of TFP growth also yield significantly different results depending on the specification of technical progress. A CES function is then estimated within a normalized system approach for the US economy for the 1960:1–2004:4 period. Our results show that the estimated substitution elasticity tends to be substantially lower using a factor augmenting specification and it is well below one. We are able to reject Hicks-, Harrod- and Solow-neutral specifications in favor of a more general factor augmenting one. The model is able to capture the acceleration of TFP growth throughout the 1990s.

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1 Introduction

The shape of the production function plays a key role in many macroeconomic models of growth, distribution, and the business cycle. Perhaps the most important parameter determining the functional form relating inputs to output is the elasticity of substitution (ES) between capital and labor. The ES has important implications for growth theory, income distribution, dynamic stability, unemployment, tax policy, etc. However, most of the standard growth and business cycle models work under the assumption that a Cobb-Douglas describes the supply side, hence imposing a unitary ES. This is because the Cobb-Douglas (CD) conveniently yields stable factor shares making all productivity growth factor neutral. When the elasticity of substitution is one, production function-based estimates of TFP growth are also much simpler to obtain, since all technical progress degenerates to the Hicks-Neutral representation.

Recent empirical studies, however, have challenged the validity of the unitary ES assumption implicit in the CD function and favoring a Constant Elasticity of Substitution (CES) production function. Antràs (2004) and Klump et al. (2007) find support for an ES significantly below unity for aggregate time series for the USA.[1] Given the difficulties encountered in empirical research to separately identify the ES and factor-augmenting technical progress, researchers commonly assume that technical change is either Hicks- or Harrod-neutral. This is because, for instance, in non-linear least squares (NLL) estimates of CES production functions, extra parameters greatly increase convergence problems and the finding of global maxima. The curvature of the CES function also adds to these estimation problems. Recently, León-Ledesma et al. (2010) also show that linearization methods such as the Kmenta (1967) approximation or the use of first order conditions (FOCs) are unable to identify jointly the ES and biased technical change. León-Ledesma et al. (2010) show, however, that the so-called normalized supply side system of joint estimation of the FOCs and production function is capable of correctly estimating these deep production parameters. Our work draws directly on these results.

Antràs (2004) suggests that, if the true nature of technical progress deviates from Hicks-neutrality, estimating a CES function with Hicks-neutral technology could potentially bias the estimate of the elasticity of substitution towards one if factor shares are relatively constant. In other words, the choice of specification of technical change

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[1]See Chirinko (2008) for an overview of empirical results on the ES. He concludes that, despite large differences depending on sample period, type of data, and estimation technique, most evidence favors elasticities ranging between 0.4 and 0.6. Duffy and Papageorgiou (2000) also provide cross-country evidence rejecting the Cobb-Douglas specification.

could substantially bias the estimate of the ES towards accepting a CD functional form. This may explain, for instance, the influential results in Berndt (1976), which is often cited in support of a CD functional form in aggregate models. A similar reasoning may also apply to other forms of neutrality of technical change. Most often, researchers assume either Hicks- or labor (Harrod)-neutrality when estimating the production function. However, depending on the observed evolution of factor shares, assuming a particular form of neutrality could dramatically change our estimates of the ES. Capital-augmentation, when the ES is different from one, can generate increasing or decreasing labor share depending on whether the ES is above or below one. Factor income shares may also, independently from the magnitude of the ES, remain relatively stable when capital augmentation is coupled with a non-steady user cost. Hence, if we restrict a priori the direction of technical change, then our estimates of the ES and the implied estimates of TFP may be forced away from their true value. These arguments are especially relevant when considering recent theory debates on directed technical change (Acemoglu (2007)) that views balanced growth as the consequence of induced bias in technological change in combination with a less than unitary ES.

In this study we first provide Monte Carlo evidence on the bias in the estimated ES generated by mis-specification of the nature of technical change. In order to minimize estimation biases, we use the normalized system approach advocated in Klump et al. (2007) and León-Ledesma et al. (2010). We find that our general factor augmenting specification identifies correctly the true nature of the technical progress with practically unbiased ES estimates. Alternative neutrality specifications work well only when they correspond to the true DGP. For parameter configurations that yield (relatively) constant factor shares, the ES is biased upwards (downwards), when its true value is below (above) unity. For plausible true values of the ES, this can most often lead to biases in the estimated ES towards 1.

We then estimate the system for the US economy for the 1960:1–2004:4 period under general factor-augmenting, Hicks-neutral, Harrod-neutral, and Solow-neutral specifications. The results yield very different values for the ES under these alternative specifications. In all cases, our tests support the general factor-augmenting specification. Using the latter, the ES for the US is substantially below unity (0.49-0.63). We then derive estimates of TFP growth for all cases and also show relevant differences between all specifications. Our preferred general factor-augmenting system is also able to capture an acceleration of TFP growth during the second half of the 1990s decade coinciding with the IT-led productivity boom (see Basu et al.

(2003), Fernald and Ramnath (2004) and Jorgenson (2001)).

The paper is organized as follows. We first present some relevant theory concepts and discuss the potential biases arising from mispecification. In Section 3 we present the Monte Carlo setup and discuss the results. In Section 4 we present empirical results for the USA. Section 5 concludes.

2   Theory background.

The CES production function allows the elasticity of capital and labor with respect to their relative price to be any constant between zero and infinity. This special type of production functions, was introduced into economics by Solow (1956) and pioneered by Pitchford (1960), Arrow et al. (1961) and David and van de Klundert (1965). We present here the "normalized" form, i.e. indexed form, of the production function following La Grandville (1989), since its parameters have a direct economic interpretation.[2] Normalization also turns out to be important for estimation as emphasized by León-Ledesma et al. (2010). The normalized CES takes he form:

$$
Y_t = F\left(\Gamma_t^K K_t,\, \Gamma_t^N N_t\right) = Y_0 \left[\pi_0 \left(\frac{\Gamma_t^K K_t}{\Gamma_0^K K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi_0)\left(\frac{\Gamma_t^N N_t}{\Gamma_0^N N_0}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \quad (1)
$$

where the point of time $t = 0$ has been chosen as the point of normalization. Y_t represents potential output, K_t is capital and N_t labor input. The terms Γ_t^K and Γ_t^N capture capital and labor-augmenting technical progress respectively. The capital income share at the point of normalization $\pi_0 = \frac{r_0 K_0}{Y_0}$ reflects capital intensity in production; the elasticity of substitution (σ) between capital and labor inputs is given by the percentage change in factor proportions due to a change in the factor price ratio:

$$
\sigma \in (0,\infty) = \frac{d\log\left(K/N\right)}{d\log\left(F_N/F_K\right)}. \quad (2)
$$

Without loss of generality, we assume a normalization $Y_0 = N_0 = \Gamma_0^N = \Gamma_0^K = 1$:

$$
Y_t = \left[\pi_0 \left(\Gamma_t^K \frac{K_t}{K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi_0)\left(\Gamma_t^N N_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \quad (3)
$$

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[2]See also Klump and Preissler (2000), Klump and de La Grandville (2000), and La Grandville (2009) for an analysis of the relevance of normalized production functions for growth theory.

where (by definition) $K_0 = \frac{\pi_0}{r_0}$.[3] Equations (1) and (3) nest Cobb-Douglas when $\sigma = 1$; the Leontief function (i.e., fixed factor proportions) when $\sigma = 0$; and a linear production function (i.e., perfect factor substitutes) when $\sigma \longrightarrow \infty$. Finally, when $\sigma < 1$, factors are gross complements in production and gross substitutes when $\sigma > 1$. It can be easily shown that capital deepening, ceteris paribus, assuming gross complements (gross substitutes) reduces (increases) capital's income share. Likewise, it is only in the gross-substitutes case that, capital (labor) augmenting technical progress implies capital-biased (labor-biased) technical progress (i.e., raising its relative marginal product and income share). Naturally, these relations between the substitution elasticity, technical bias and factor shares evaporate under Cobb-Douglas.[4]

In the competitive goods market, profit maximization with respect to capital and labor results in the following first order conditions:

$$r_t = \pi_0 \left(\frac{\Gamma_t^K}{K_0} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{K_t} \right)^{\frac{1}{\sigma}}, \tag{4}$$

$$w_t = (1 - \pi_0) \left(\Gamma_t^N \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{N_t} \right)^{\frac{1}{\sigma}}. \tag{5}$$

If a neo-classical growth model is to posses an asymptotic balanced growth path,[5] then either the substitution elasticity must be unity or technical change must exhibit Harrod-Neutrality, $\Gamma_t^K = 1$, Uzawa (1961). Nevertheless, when we analyze particular periods of time, other forms of technical neutrality can be present in historical data. Indeed, in some types of growth models, factor augmentation may result from the interplay of innovation activities, factor intensities and the substitution elasticity, raising the possibility of persistent capital augmentation (i.e., departures from balanced growth) and directed technical change (Kennedy (1964), Acemoglu (2002) and Acemoglu (2003)). For a closer examination, multiply condition (4) by the capital-output ratio and solve for the capital income share:

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[3]In addition, this indexation scheme, perfect competition and profit maximization imply that the wage rate $w_0 = 1 - r_0 K_0 = 1 - \pi_0$.

[4]See Acemoglu (2009) and La Grandville (2009).

[5]I.e. the capital-output ratio, factor income shares, and user cost of capital are constants.

$$\frac{r_t K_t}{Y_t} = \pi_0 \left(\frac{\Gamma_t^K K_t}{K_0 Y_t} \right)^{\frac{\sigma-1}{\sigma}} \tag{6}$$

$$= \pi_0 \left(\frac{\Gamma_t^K r_0}{r_t} \right)^{\sigma-1}. \tag{7}$$

Equation (6) expresses capital share in terms of the capital-output ratio, and (7), equivalently, in terms of the real user cost of capital. Both forms show that the balanced growth path requires Harrod-neutral technical progress. However, independent from the size of the ES, factor income shares remain constant also outside the balanced growth path if the user cost r_t has the same trend as the capital-augmenting technical progress. Alternatively, if the user-cost only partially absorbs the capital-augmenting technical progress, there will be trends also in the the factor income shares, but these trends may be quite weak when coupled with a moderate pace of capital augmentation. For example, assuming that the capital augmenting-technical progress is 0.5% (1%) per year and it is fully absorbed by the real user cost, then the real user cost would rise from an initial value of 0.05 to 0.064 (0.082) within a period of 50 years. The allowance of weak trends also in factor income shares would, correspondingly, require smaller trends in the real user cost. Hence, the relative stability of factor income shares is not a sufficient condition for the correctness of either the CD production function or the CES production function with Harrod-neutral technical progress. Misspecification of technical progress, in turn, may seriously disturb the estimation of the ES.

As argued by Antràs (2004), the Hicks neutral specification when the true DGP is Harrod-neutral, can result in a bias towards unity in the ES estimates.[6] Correspondingly, we can show that quite generally (although not universally) also the Harrod-neutral specification can result estimates of the ES that are biased towards unity when the true DGP contains capital-augmenting technical progress.[7] A misspecified Harrod-neutral specification implies:

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[6] Under Hicks neutrality (4) and (5) imply that relative factor shares are given by $\frac{r_t K_t}{w_t N_t} = \frac{\pi_0}{1-\pi_0} \left(\frac{K_t/K_0}{N_t} \right)^{\frac{\sigma-1}{\sigma}}$. However, with balanced growth, under Harrod neutrality (or when $r_t = \Gamma_t^K r_0$) the left-hand side is constant, whilst on the right-hand side capital intensity K/N grows at the same rate as labor-augmenting technical progress. To eliminate this trend the estimate of σ must be strongly biased towards unity.

[7] Equivalently, one can show the potential biases for a Solow-neutral specification. This, however, is not common in the literature, although we also use it in our simulation experiments.

$$\pi_0 \left(\frac{\Gamma_t^K K_t}{K_0 Y_t} \right)^{\frac{\sigma-1}{\sigma}} = \pi_0 \left(\frac{K_t}{K_0 Y_t} \right)^{\frac{\widehat{\sigma}^H - 1}{\widehat{\sigma}^H}}. \tag{8}$$

The left hand-side of (8) corresponds to the "true" DGP and the right-hand side to the misspecified Harrod-neutral version with $\widehat{\sigma}^H$ the implied estimate of σ. Equation (8) then implies

$$\frac{\sigma - 1}{\sigma} \log \Gamma_t^K = \frac{\widehat{\sigma}^H - \sigma}{\widehat{\sigma}^H \sigma} \log \left(\frac{K_t}{K_0 Y_t} \right). \tag{9}$$

In the data, $\frac{K_t}{K_0 Y_t} = \left(\Gamma_t^K \right)^{\sigma-1} \left(\frac{r_0}{r_t} \right)^{\sigma}$. Assume that $r_t = r_0 \left(\Gamma_t^K \right)^{\alpha}$, which implies that in the interval $\alpha \in (0, 1]$ the real user cost partly absorbs the trend in capital-augmenting technology. It can be shown that with values of $\alpha > \frac{\sigma-1}{\sigma}$, the negative trend in the capital-output ratio corresponds to the positive trend of Γ_t^K. When this condition holds, then in the interval $\alpha \in (0, 1]$ the estimate $\widehat{\sigma}^H$ is larger than σ and with $\sigma > 1$, in turn, $\widehat{\sigma}^H$ is smaller than σ. However, when $\alpha = 0$ and $\sigma > 1$, then the capital-output ratio has a positive trend and $\widehat{\sigma}^H > \sigma > 1$. Hence, our conclusion is that misspecified technical progress results in biased ES estimates. For plausible true values of σ this would quite often lead to a bias towards unity, i.e upwards (downwards) biased when the true ES is below (above) unity.

As mentioned earlier, TFP growth estimates will also be dependent on the interaction between the ES and technical change. As Acemoglu (2003) points out, the relative contributions of technological change and factor accumulation in accounting for long-run growth depend on σ. To show this in our framework, one can obtain TFP by means of a linearisation of the CES production function assuming given estimated values of the relevant parameters. We assume that, as is common in the literature, the technical progress functions follow exponential processes: $\Gamma_t^K = \Gamma_0 e^{\gamma_K (t-t_0)}$ and $\Gamma_t^N = \Gamma_0 e^{\gamma_N (t-t_0)}$. Starting from a normalized CES production function (1), following Kmenta (1967) and Klump et al. (2007), we apply a Taylor-series log expansion of around $\sigma = 1$:

$$y_t = \pi k_t + a k_t^2 \tag{10}$$
$$+ \underbrace{\pi \left[1 + \frac{2a}{\pi} k_t \right] \gamma_K \cdot \bar{t} + (1 - \pi) \left[1 - \frac{2a}{(1 - \pi)} k_t \right] \gamma_N \cdot \bar{t} + a \left[\gamma_K - \gamma_N \right]^2 \cdot \bar{t}^2}_{Log \ (TFP)}$$

where, in the normalized context, $\bar{t} = t - t_0$, $y_t = \log[(Y_t/Y_0)/(N_t/N_0)]$, $k_t = \log[(K_t/K_0)/(N_t/N_0)]$, $a = \frac{(\sigma-1)\pi(1-\pi)}{2\sigma}$, and $\pi = \pi_0$.

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A useful feature of approximation (10) is that it separates the output contribution of TFP from that of inputs. Conditional on estimates for π, σ, γ_N and γ_K, the level and growth of TFP can be calculated. In addition, (10) helps distinguish the contribution of labor and capital augmenting technical progress to TFP developments. To illustrate, for the special cases of Hicks neutrality ($\gamma_K = \gamma_N = \gamma$) and Cobb-Douglas, log(TFP) reduces to $\gamma \cdot t$ and $(\pi \gamma_K + (1 - \pi) \gamma_N) \cdot t$, respectively.[8] In addition, (10) shows that, when the elasticity of substitution is non-unitary, factor-augmentation, ($\gamma_K \neq \gamma_N > 0$ plus $a \neq 0$), introduces additional curvature into the estimated production function.

One can also obtain a second, simpler, approximation for TFP. For values of K_t and N_t close to their normalization points, $k_t \simeq 0$, and we can simplify the expression above for TFP:

$$log(TFP)_{simple} = \pi \gamma_K \cdot \bar{t} + (1 - \pi) \gamma_N \cdot \bar{t} + a \left[\gamma_K - \gamma_N\right]^2 \cdot \bar{t}^2. \qquad (11)$$

This simplified expression for TFP, of course, will not consider the effect of capital deepening. In a general factor augmenting CES function, productivity growth depends not only on the (exogenous) rate of labor and capital-augmenting technology, but on changes in factor accumulation. Since technical progress is factor biased when $\sigma \neq 1$, growth of the factor in favor of which technological progress is biased would increase TFP growth. If, for instance, labor-augmenting technical progress is biased in favor of capital because $\sigma < 1$, then the higher the rate of capital accumulation, the faster the rate of TFP growth as the share of this factor is increasing.[9] This is why capital intensity appears in the TFP part of (10). This would exacerbate the effect of biases in estimates of the ES on TFP, since the bias in technical change depends on σ and not only on the estimated technical progress parameters.

3   Monte Carlo evidence on the specification bias

In this section, we use a Monte Carlo simulation for a variety of parameter values of the production function to quantitatively analyze the potential bias arising from misspecification of technical progress discussed in the previous section. We simulate a consistent DGP for factor inputs, output, and factor payments and then estimate

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[8]Although, in the CD case, strictly speaking γ_K and γ_N cannot be separately identified.

[9]This is different from the concept of embodied technical progress, although it does bear an empirical relationship with it.

it using the normalized system approach of Klump et al. (2007), and León-Ledesma et al. (2010) imposing particular forms of factor neutrality. We first give a brief overview of the normalized system and then proceed to explain the Monte Carlo experiments and analyzing the results.

3.1 The normalized system estimation

The normalized system estimator of the parameters of the CES production function follows León-Ledesma et al. (2010). It consists of the joint estimation of (log-version of) the CES function (1) and the FOCs for K and N (4)-(5). Normalization allows us to fix parameter π_0 to its observed value (share of capital in income in period 0) also simplifying the estimation problem. The system of equations (1)-(4)-(5) is then estimated jointly using a Generalised Nonlinear Least Squares (GNLLS) system estimator.

3.2 The Monte Carlo experiment

In order to understand the potential biases arising from misspecification, we generate data in a consistent way corresponding to a particular evolution of factor inputs, technical progress and output. This data is the true DGP which is then estimated under both, correctly specified and misspecified systems. Hence, we draw M simulated stochastic processes for labor (N_t), capital (K_t), labor- (Γ_t^N) and capital-(Γ_t^K) augmenting technology. Using these, we then derive what we term "equilibrium" output (Y_t^*), observed output (Y_t) and real factor payments (w_t and r_t), for a range of parameter values and shock variances. The simulated system is consistent with the normalized approach, so that we ensure our parameters are *deep*, i.e. can be given an economic interpretation and are not the result of a combination of other parameters.

Given our interest in realistic settings where the economy does not deviate in a significant way from the case of constant factor shares, we first need to devise a way to set parameter values such that we exclude unrealistic strong trends in factor shares. This is in contrast to the DGP of León-Ledesma et al. (2010), who also consider formulations where factor shares display trends. We can do this by looking at the expression for the capital-to-labor income share under profit maximization in competitive markets,

$$\frac{rK}{wN} = \frac{\pi}{1-\pi} \left(\frac{\Gamma^K K/K_0}{\Gamma^N N/N_0} \right)^{\frac{\sigma-1}{\sigma}}. \tag{12}$$

From this expression we can see that, if $\sigma \neq 1$, capital- and labor-augmenting technical change can lead to ever increasing or decreasing relative factor shares for a given evolution of the K/N ratio. It is logical to assume an exogenous rate of growth of labor. Hence, for given rates of technical progress, to obtain almost constant shares, we set the rate of growth of K in such a way that we avoid any counter-factual heavy trends in shares. One simple mechanism to achieve this, following our earlier discussion on potential biases, is to allow the real interest rate (r) to absorb some fraction (α) of capital augmentation (assuming $\Gamma_0^K = \Gamma_0^N = 1$),

$$r = r_0 e^{\alpha(\gamma_K \cdot t)}, \tag{13}$$

with the capital stock then solved from the first order condition for capital

$$K = Y \left(\frac{\pi}{r}\right)^\sigma \left[\frac{Y_0}{K_0} e^{\gamma_K \cdot t}\right]^{\sigma-1}. \tag{14}$$

If $\alpha = 0$ and/or $\gamma_K = 0$, the Harrod-neutral case, the real interest rate and capital-output ratio are constant (at $r = r_0$ and $K/Y = \left(\frac{\pi}{r_0}\right)^\sigma \left(\frac{Y_0}{K_0}\right)^{\sigma-1}$, $\forall t$, respectively). If $\alpha \neq 0$ and $\gamma_K > 0$, $r \to \infty$. Given that it controls the rate of change of r, i.e. α, however, a sufficiently small value of α can be set to mimic empirically relevant paths for r and hence K/Y, as discussed above.

Hence, once we decide α, for given technology parameters, we obtain r from (13). Given an exogenous law of motion for N (which we shall explain below) the CES function and (14) solve for K and Y. Using the value of K from this recursive system, we obtain the *average* rate of growth of K that we then use to build our stochastic DGP. This is the value compatible with factor shares and real interest rates that do not display counterfactual trends. In our experiments, we chose a value for α of 0.5. Depending on the different configurations for technical progress, this value yields very reasonable evolutions for r and Θ for the sample sizes we are dealing with.

Hence, once the rate of growth of K has been derived, we can then describe the full DGP for the MC simulations. Labor and capital evolve as stationary stochastic processes around a deterministic trend:

$$K_t = K_0 e^{\left(\kappa(t-t_0)+\varepsilon_t^K\right)}, \tag{15}$$

$$N_t = N_0 e^{\left(\eta(t-t_0)+\varepsilon_t^N\right)}, \tag{16}$$

where η and κ represent their mean growth rates respectively. The initial value

for N values was set to $N_0 = 1$, and the one for capital $K_0 = \pi_0/r_0$, with $r_0 = 0.05$.[10] Both ε_t^K and ε_t^N are assumed to be normally distributed i.i.d error terms with zero mean and standard errors $se\left(\varepsilon_t^K\right)$ and $se\left(\varepsilon_t^N\right)$.[11]

The technical progress functions, as described before, are also assumed to be exponential with a deterministic and stochastic component (around a suitable point of normalization):

$$\Gamma_t^K = \Gamma_0^K e^{\left(\gamma_K(t-t_0)+\varepsilon_t^{\Gamma^K}\right)}, \quad \Gamma_t^N = \Gamma_0^N e^{\left(\gamma_N(t-t_0)+\varepsilon_t^{\Gamma^N}\right)},$$

where Γ_0^K and Γ_0^N are initial values for technology which we also set to unity. Shocks to technical progress are assumed to be normally distributed i.i.d. with standard errors $se\left(\varepsilon_t^{\Gamma^K}\right)$ and $se\left(\varepsilon_t^{\Gamma^N}\right)$ respectively.

We then obtain equilibrium output from the normalized CES function:

$$Y_t^* = Y_0^* \left[\pi_0 \left(\frac{K_t}{K_0}e^{\left(\gamma_K(t-t_0)+\varepsilon_t^{\Gamma^K}\right)}\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi_0)\left(\frac{N_t}{N_0}e^{\left(\gamma_N(t-t_0)+\varepsilon_t^{\Gamma^N}\right)}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

(17)

with $Y_0^* = 1$. This "equilibrium" output is then used to derive the real factor payments from the FOCs, to which we add a multiplicative shock.

$$r_t = \frac{\partial Y_t^*}{\partial K_t} = \pi_0 \left(\frac{Y_0^*}{K_0}e^{\left(\gamma_K(t-t_0)+\varepsilon_t^{\Gamma^K}\right)}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{Y_t^*}{K_t}\right)^{\frac{1}{\sigma}}e^{\varepsilon_t^r}, \tag{18}$$

$$w_t = \frac{\partial Y_t^*}{\partial N_t} = (1-\pi_0)\left(\frac{Y_0^*}{N_0}e^{\left(\gamma_N(t-t_0)+\varepsilon_t^{\Gamma^N}\right)}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{Y_t^*}{N_t}\right)^{\frac{1}{\sigma}}e^{\varepsilon_t^w}. \tag{19}$$

Equations (18) and (19) imply that real factor returns equal their marginal product times a multiplicative i.i.d error term that represent shocks that temporarily deviate factor payments from equilibrium, $\varepsilon_t^r \sim N\left(0, se\left(\varepsilon_t^r\right)\right)$, $\varepsilon_t^w \sim N\left(0, se\left(\varepsilon_t^w\right)\right)$.

Because we need to ensure that our artificial data is consistent with national accounts identities, we then obtain the "observed" output series using the identity:

$$Y_t \equiv r_t K_t + w_t N_t. \tag{20}$$

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[10]For estimation, initial values for r_0 and K_0 do not affect the results if the system is appropriately normalized.

[11]For all the experiments we also simulated K_t and N_t such that they displayed stochastic, rather than deterministic, trends as in León-Ledesma et al. (2010). We report here the case of deterministic trends because it makes the discussion above about factor shares more transparent. However, the conclusions of the analysis did not change. Results are available on request.

We use the "observed" output series for estimation purposes. This ensures that, regardless of the shocks, factor shares sum up to unity, which has to be ensured in this artificial setting with no mark-ups.

Hence, the experiment consists of, first, simulating the time series for factor inputs, technical progress, and equilibrium output. Second, from these we obtain factor payments and observed output. Finally, we estimate the normalized system, (1)-(4)-(5), imposing Hicks-, Harrod- and Solow-neutrality in technical progress. We repeat these steps M times and analyze the possible biases arising from mispecification by looking at the difference between the true and estimated σ.

Table 1 lists the parameters used to generate the simulated series. We fixed the distribution parameter to a reasonable value of 0.4.[12] The substitution elasticity ranges from 0.5 to a near Cobb-Douglas (0.9) value and a value exceeding unity, 1.3.

Labor supply growth is set to 1.5% per year and capital stock growth to the values implied from our earlier discussion, so that κ changes for each experiment. We use a wide variety of values for technical progress. We set $\gamma_N = 2\%$ and $\gamma_K = 0\%$, the Harrod-neutral case; a Solow-neutral case with $\gamma_N = 0\%$ and $\gamma_K = 2\%$; and a Hicks-neutral with $\gamma_N = \gamma_K = 1\%$. Finally, we have two intermediate cases where technical progress is of the general factor augmenting form.

The standard errors of the shocks need to be chosen so that they also generate series with sufficiently realistic behavior. We chose a value of 0.1 for the capital and labor stochastic shocks.[13] For the technical-progress parameters, following León-Ledesma et al. (2010), we used a value of 0.01 when the technical progress parameter is set to zero, so that the stochastic component of technical progress does not dominate. When technical progress exceeds zero we used a value of 0.05 so when technical progress is present it is also subject to larger shocks.[14] Finally, for shocks to factor payments, we resorted to real data. We used the standard deviation of the de-trended real wages and the standard deviation of demeaned user cost of capital for the US economy over 1950-2000.[15] These take values of 0.05 and 0.1 respectively, reflecting the larger volatility of user cost.

We used a sample size of 50 (years).[16] Finally, since nonlinear system estimators

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[12]In practice, setting different values for π_0 did not affect the results.

[13]This is approximately the standard error of labor and capital equipment around a trend with US data from 1950 to 2005. The results, however, remained invariant when we used values of 0.2 and 0.05.

[14]For robustness purposes, we also replicated the results assuming no shock when technical progress is zero and also equal shocks for both components. The results were not affected by this changes either.

[15]From the Bureau of Economic Analysis.

[16]Using values of 100 and 30 led to very similar results, although, as expected, the range of

used require initial guesses for the parameters, which we set these to their true value following León-Ledesma et al. (2010).

3.3 Monte Carlo results

Table 2 reports the results from the Monte Carlo for the Hicks-neutral specification. We report the median value of the estimated coefficient across he 5,000 draws.[17] We also report the 10 and 90% percentiles for σ. Parameter $\hat{\gamma}$ is the Hicks-neutral technology growth coefficient, and we also report its median.

The Hicks-neutral case with $\gamma_K = \gamma_N = 0.01$ can be used as a benchmark as in this case the estimated model corresponds to the true DGP. We can see that the parameters of interest are very precisely estimated, which reflects the power of the normalized system to identify deep production function coefficients as shown in León-Ledesma et al. (2010). The bias that we focus our paper on can be observed by looking at the other cases. For $\gamma_K = 0.00$ and $\gamma_N = 0.02$, the estimated σ is heavily biased towards unity (Cobb-Douglas) irrespective of whether the true σ is above or below unity. As we approach the true Hicks-neutral case ($\gamma_K = \gamma_N = 0.01$) the bias diminishes. For cases in which technical progress is capital augmenting, we observe similar biases, although for $\sigma = 1.3$ we obtain an upwards bias rather than a bias towards one. As the estimator appears to be unbiased when the true DGP is Hicks-neutral, the bias found in the factor augmenting case can only be attributed to misspecification.

Tables 3 and **4** report the results for the Harrod and Solow-neutral specifications. As with the Hicks specification, the system captures well the elasticity and technical progress parameters when the true DGP corresponds to the specified model. A useful way of looking at the results for the estimated σ is to move from the case where the true DGP corresponds to the specified model towards the opposite case. In both cases we can observe that as we move away from the true DGP, the bias in the estimated ES increases substantially, and in most cases yields $\hat{\sigma}$ values biased towards one (the Cobb-Douglas case). This provides support for our conjecture in Section 2 that misspecification with roughly constant factor shares tends to bias the estimated ES up (down) when the true σ is below (above) unity. For values of the ES like the ones used here, this bias is commonly towards the unitary case

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estimated values for the parameters increased as we decrease the sample size.

[17]Although the mean values were very close to the median, we prefer to report the latter as, with a non-linear estimator, one cannot rule out abnormal estimation results that would skew the distribution of estimated parameters.

(Cobb-Douglas).

Figures 1 and **2** show the distribution of the estimated σ across the 5,000 draws for four cases of technical progress augmentation and for $\sigma = 0.5$ and $\sigma = 1.3$. Focusing on the 0.5 case, we can see that the general factor augmenting specification is always tightly distributed around the true value of the ES. It is interesting to note that the Solow-neutral specification yields a clearly bi-modal distribution for the two cases in which technical progress is labor-augmenting. To a smaller degree, the Harrod-neutral specification also shows bi-modality in two cases. The distributions also tend to be more skewed when the specified model differs from the true DGP. For the case of $\sigma = 1.3$ the distributions are much flatter, except for the Solow-neutral specification, which tends to be tighter. In the case where $\gamma_N = 0$ and $\gamma_K = 0.02$, the Hicks-neutral specification is very flat, although the scale of the graph makes it difficult to show any frequency variation. This explains the high values for the median σ reported in **Table 2** for that particular case. This value, though, is hardly representative. It is worth noting that the distribution plots also allows to see that the factor augmenting specification, despite capturing very well the true values of σ, also display a small local maximum around a value of one. This result may be indicating that the model's likelihood experiences a change at this point.[18]

As a conclusion, the MC experiments show that misspecified technical progress can lead to important biases in estimates of the ES. In general, there is an upward bias in the estimated ES as the estimated model departs from the true DGP. This bias, for reasonable parameter values, tends to yield values of σ closer to one. The next step is then to analyze how these potential biases affect estimates of the ES using real data and how they might also affect estimates of TFP growth.

4   CES estimation for the US economy

4.1   Data and specification

We use quarterly series for the U.S. economy for the 1960:1–2004:4 period. Our principal source was the NIPA Tables (National Income and Product Accounts) for production and income.[19] The output series is calculated as Private non-residential Sector Output – this is total output minus Indirect Tax Revenues, Public-Sector output, and Housing-Sector Output. After these adjustments, the output concept

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[18]We do not pursue this point further in this paper, which is being investigated more in-depth elsewhere by the authors.

[19]These series can be found at http://www.bea.doc.gov/bea/dn/nipaweb/index.asp

used is compatible with that of our private, non-residential capital stock series. Employment is defined as a sum of self-employed persons and the private sector full-time equivalent employees. To create quarterly private non-residential capital stock compatible with both the annual index of constant replacement cost capital stock, Herman (2000), and the accumulated NIPA net investment, we first estimated the base value for the capital stock as a ratio:

$$KB = \frac{\sum_{t=0}^{T} \text{Net Investment}}{KI_T - KI_0}$$

where KI_T and KI_0 refer to the values of the capital stock index at the end and the beginning of the sample respectively, and $t = 0, 2, \ldots, T$ is the sample period. The quarterly constant price non-residential private capital stock was then calculated by accumulating (de-cumulating) the base level KB from the midpoint of the sample by using the quarterly NIPA series of non-residential private net investment. This procedure ensures that the constructed quarterly capital stock has the same trend as the annual capital stock index. Time-varying depreciation rate was calculated as the ratio of NIPA consumption of non-residential capital to the capital stock lagged one period. The nominal user cost is defined as the product of the investment deflator and the real user cost, the latter being the sum of real interest rate (defined in terms of investment deflator inflation) and the depreciation rate. The underlying interest rate is the U.S. 5-year government securities rate.

Figures 3 and **4** present some relevant ratios. The Y/K ratio appears to show a declining trend during this period, although it is not very pronounced and is more relevant during the first half of the sample. Both the Y/N and K/N show clear upward trends that are close to each other, although the Y/N ratio has grown slightly faster. The share of labor in income shows important variations during the period of analysis. It declines up to 1965 to its lowest value and then increases to remain at a higher level during the 1970s. The share then declines from the mid-1980s onwards, but not monotonically.[20] **Figure 4** also shows the evolution of real wages and the user cost of capital. The user cost shows high volatility from the mid-1970s to the mid-1980s mostly due to the volatility of inflation and changes in the monetary policy stance.[21]

Given the practical existence of a markup over factor costs in the data, the estimated model includes an extra parameter μ which proxies an estimated average

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[20]The capital and labor shares do not sum up to unity in this dataset owing to the existence of a mark-up. This is later on introduced as a parameter in the supply side normalized system.

[21]Standard ADF tests on capital and labor shares rejected the null of non-stationarity.

mark-up. Also, as the user cost of capital takes negative values in some observations, the FOC for capital enters the system without log-linearization. Also, with real data, to diminish the size of stochastic component in the point of normalization we prefer to define the normalization point in terms of sample averages (geometric averages for growing variables and arithmetic ones for factor shares). The nonlinearity of the CES function, in turn, implies that the sample average of production need not exactly coincide with the level of production implied by the production function with sample averages of the right hand variables. Following Klump et al. (2007), we introduce an additional parameter ζ whose expected value is around unity. Hence, we can define $Y_0 = \zeta \bar{Y}$, $K_0 = \zeta \bar{K}$, $N_0 = \zeta \bar{N}$ and $t_0 = \bar{t}$ where the bar refers to the respective sample average. The estimated system, allowing for factor augmentation, is then:

$$r = \left(\frac{\bar{\pi}}{1+\mu} \frac{\zeta \bar{Y}}{\bar{K}} \right) \left[\frac{Y/(\zeta \bar{Y})}{K/\bar{K}} \right]^{\frac{1}{\sigma}} e^{\frac{\sigma-1}{\sigma} \gamma_K (t-\bar{t})}, \tag{21}$$

$$\log(w) = \log \left(\frac{(1-\bar{\pi})}{1+\mu} \frac{\zeta \bar{Y}}{\bar{N}} \right) + \frac{1}{\sigma} \log \left(\frac{Y/(\zeta \bar{Y})}{N/\bar{N}} \right) + \frac{\sigma-1}{\sigma} \gamma_N (t - \bar{t}), \tag{22}$$

$$\log \left(\frac{Y}{\bar{Y}} \right) = \log \zeta + \frac{\sigma}{\sigma-1} \log \left[\bar{\pi} \left(\frac{e^{\gamma_K (t-\bar{t})} K}{\bar{K}} \right)^{\frac{\sigma-1}{\sigma}} + (1-\bar{\pi}) \left(\frac{e^{\gamma_N (t-\bar{t})} N}{\bar{N}} \right)^{\frac{\sigma-1}{\sigma}} \right]. \tag{23}$$

The "Hicks-neutral system" is estimated by imposing the constraint $\gamma_K = \gamma_N = \gamma$ on (21)-(23). The Harrod- and Solow-nutral systems are estimated imposing the constraints $\gamma_K = 0$ and $\gamma_N = 0$ respectively. These testable restrictions can then be used to select between the factor augmenting and the restricted specifications by means of a Likelihood Ratio (LR) test. For the estimation of the system we fix parameter $\bar{\pi}$ to its sample average, which is one of the empirical advantages of normalization. We also obtained the results estimating $\bar{\pi}$ freely, but it made no difference to the other relevant parameters.

The system is estimated using two methods. First we use a Generalized Nonlinear Least Squares (GNLLS) estimator which is equivalent to a nonlinear SUR model, allowing for cross-equation error correlation. We also used a nonlinear-3SLS (NL3SLS) estimator using a constant, a trend, and the first two lags of all the

variables as instruments.[22] In both cases we report heteroscedasticity and autocorrelation consistent standard errors. Initial guesses for the parameters were set as follows: for ζ we set an initial value of 1, as we have a strong prior; for the ES and the technical progress parameters, we used a range of initial values from 0.2 to 1.2 for σ and 0.00 to 0.05 for technical progress. We will report robustness tests to the change in initial values for σ; finally, for μ we used a value of 0.1.

4.2   Estimation results

The results of the two estimation methods for the four specifications are reported in **Tables 5a-b** and **6a-b**. The tables also report, together with the Log-Determinant of the system, an LR test for the null of Hicks-, Harrod-, or Solow-neutrality against general factor augmentation and, in the NL3SLS case, a J-test for instrument validity. We also report ADF-type unit root tests on the residuals of the three equations of the system. Given that we do not know the distribution of the statistic under the no-cointegration null, we resorted to the use of bootstrapped p-values following Park (2003) and Chang and Park (2003).

The results from both estimation methods appear close to each other and show similar patterns across specifications. In all cases, the null of no-cointegration for each equation is rejected according to the bootstrapped p-values. For the NL3SLS estimator, the J-test rejects instrument validity only for the Solow-neutral specification. The estimate of the mark-up parameter μ is very close to 0.11 in all cases, which is consistent with national accounts data. The scale parameter ζ is practically indistinguishable from one, again consistent with our priors. All variables are statistically significant.[23] The estimate of the ES in the factor augmenting specification is 0.63 for the GNLLS estimator and 0.49 for the NL3SLS estimator. These estimates are well below and significantly different from unity, clearly rejecting Cobb-Douglas. These values are similar to those obtained recently by Klump et al. (2007), who use a flexible specification for technical progress. Chirinko (2008) also claims that the weight of evidence favors values of σ between 0.4 and 0.6, very close to our estimates. The values for labor-augmenting technical progress in both estimation methods imply an annual rate of growth of 1.6%. Those for capital augmenting technical progress imply a non-negligible 0.56-0.72% per year. Technical progress is, on average, labor-saving but there is positive and significant capital augmentation.

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[22]This is a particular case of a more general GMM estimator.

[23]The high t-ratios for the ζ parameter are common in these specifications. Pre-fixing this parameter to 1, did not lead to any change in the results.

Turning now to the other specifications, we see that the estimates obtained for σ are substantially different from those obtained with general factor augmentation. The point estimate of σ with Hicks neutral technology is between 0.2 and 0.3 points higher than the one with factor augmentation. This is consistent with the results from the MC experiment. Although still significantly below one, the Hicks specification biases the estimate of the ES towards one. This results from the fact that technical progress contains also a positive capital-augmenting component while it deviates from Hicks-neutrality. The Solow-neutral specification leads to an even sharper bias towards Cobb-Douglas. Again, looking back at the results in **Table 4** this is consistent with our simulations, which showed that the more the DGP deviates from Solow-neutrality, the stronger the bias towards unity. In the case of the Harrod-neutral specification which, together with Hicks-neutral, is most commonly used for estimation, we observe that the results are strongly biased upwards. The NLLS estimator yields a very high 1.7, whilst the NL3SLS estimator yields 1.3. According to our experiments, and for the estimated values of the technical progress parameters, we would expect a much smaller upwards bias if the true σ is in the region of 0.5. However, the density graphs showed that the Harrod-neutral specification can lead to multi-modal distributions. In fact, changing the initial conditions for the estimator in this case led to substantially different results and, in some cases, the nonlinear algorithm was unable to converge. This sensitiveness to initial conditions calls for caution when interpreting the results for this specification.

Finally, the log-determinant (our goodness of fit measure) always showed the same ordering of across specifications. The general factor-augmenting specification had the best fit, followed by Hicks-neutral and then Solow-neutral specifications. Harrod-neutral always resulted in the worst fit. In addition, the results from the LR test for the restrictions implied by specific forms of factor augmentation, always reject the restrictions in favor of the general factor augmenting specification. Hence, our results support the use of a more general specification for technical progress and confirm our claim that misspecification of technical progress can lead to important biases in the estimated ES.

Figure 5 plots the actual and fitted values for the user cost of capital, wages and output. The model is able to capture very precisely the trend evolution of wages and output. The average user cost is also fitted successfully, although it shows a greater (relative) divergence due to the large cyclical swings driven by changes in the nominal interest rate in the mid-1970 to mid-1980s decade. **Figure 6** plots the residuals of the model for the four specifications. For the user cost, the three models yield

almost the same fit. A similar picture emerges for output, although differences start widening from 1990 onwards. The main difference emerges in the way the models fit wages, especially for the Harrod-neutral specification which, unsurprisingly given the high values of $\hat{\sigma}$, produces substantially different fitted values for wages.[24] Of course, even if the three models yield similar fit for variables such as output, the implications of the different estimates of the ES and technical progress to explain the evolution of factor shares are still markedly different. As we will see later, this is also the case for estimates of TFP growth. In this sense, our aim is to find a model that, whilst explaining the data correctly, offers insights on the mechanisms behind the workings of the supply side of the economy.

We carried out further analysis of the results for the four specifications. The nonlinear algorithms can potentially be sensitive to the initialization of of the parameter values. We hence paid particular attention to this aspect. The model was re-estimated for an initialization of the σ parameter ranging from 0.2 to 1.2 at increments of 0.05, so as to analyze its robustness to initial conditions. **Figures 7** and **8** present the plots of the sensitivity analysis for both estimation methods. We plot the estimated σ vs. the initialization value and also the log-determinant of the system against the estimated σ and the initial value of σ (lower panel). The preferred specification is the one that minimizes the Log-determinant. For the GN-LLS estimator, initial values below 0.5 tend to yield lower $\hat{\sigma}$'s, however, the value always ranges from 0.48 to 0.65, the exception being an initial value of 1, which is an inflexion point for the CES function (see La Grandville and Solow (2009)), and finds an estimated σ close to one. However, the log-determinant for this case is the highest, and the model performs substantially worse. For the rest of the initialization points and estimated σ's the log-determinant is quite flat, and usually finds a minimum in the neighborhood of 0.6, as the one reported in **Table 5a**. Similar conclusions ca be reached with the NL3SLS estimator, but in this case changes in initial conditions lead to almost no change to the estimated σ (with the exception of an initial value of one). For reasons of space, we only present here the sensitivity analysis for our preferred factor-augmenting specification. Similar results are found for the other specifications with the notable exception of the Harrod-neutral one, as already commented above.

Finally, we analyze how the different specifications can affect estimates of TFP growth. In order to do so, we obtained estimates of TFP *growth* arising from the ap-

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[24]Interestingly, this is a result that Fisher et al. (1977) also obtain in a simulation experiment about production function aggregation. Despite many specifications providing a good fit for output, wages proved much more sensitive to the estimated values of σ.

proximations (10) and (11) discussed in Section 2. **Figure 9** plots the two estimates of TFP together, and **Figure 10** separately for each specification. For reasons of space, we only report here the GNLLS estimates, but the NL3SLS ones delivered very similar conclusions. The Hicks-neutral specification, obviously, always yields constant growth of TFP, and hence we do not plot it separately. The rest of specifications will always yield increasing or decreasing TFP *growth*. This can be seen in expression (11), whose rate of growth is going to be trended owing to the quadratic trend in levels. Whether the trend is positive or negative depends on parameter "a", whose sign is a function of whether $\sigma \gtrless 1$ (except in the Hicks case when the trend is zero). We can see that the differences are important. The simple form of the Harrod specification gives an increasing TFP growth, with a substantial slope going from 0.3% per quarter at the beginning of the sample to almost 0.4% towards the end. This is due to the high value for the ES obtained in this specification, making coefficient "a" positive. The Solow-neutral specification gives slightly decreasing TFP growth. The factor-augmenting specification also yields decreasing TFP growth, although the slope is even flatter than in the Solow-neutral case.

Perhaps the main differences can be observed when we take the evolution of K/N into account as in equation (10). In this case, as mentioned earlier, the impact of capital deepening will depend on the factor bias in technology, since it affects the weight of each factor in the calculation of TFP. Driven by an acceleration of capital deepening in the mid-1990, all the specifications except Hicks-neutral, display larger deviations from the simple Kmenta approximation towards the end of the sample. Because in the Harrod-neutral specification $\sigma > 1$ and $\gamma_N > \gamma_K = 0$, capital deepening appears to have reduced substantially the rate of TFP growth during this period, which is incompatible with the observed acceleration of TFP in this period as reported, amongst others, by Basu et al. (2003), Fernald and Ramnath (2004), and Jorgenson (2001). The factor-augmenting specification, however, shows an acceleration of TFP growth, although it is quantitatively small. This is consistent with the idea that investment in IT led to an economy-wide productivity increase. In our model, of course, we do not separate types of capital and we cannot say anything about the specific source of this acceleration. However, as far as this capital deepening is related with investment in new technologies, our results seem to support the contention that there was a productivity acceleration in the USA from the mid-1990s until the early 2000s. From our perspective of specification bias, it is illustrating to note that the differences in annualized TFP growth around year 2001 implied by these three specifications are substantial. They range from about 0.8%

per year for the Harrod-neutral specification, to 1.4% for the factor-augmenting specification.

5   Conclusions

The elasticity of substitution (ES) between capital and labor is the key parameter that shapes the relationship between factor inputs and output in an economy. It is important for models of growth, income distribution, dynamic stability, business cycles, tax policy, labor market outcomes, etc. With a few exceptions, most macroeconomic models work under the assumption that the ES is equal to one, implying that the production function is Cobb-Douglas. Increasingly, however, empirical evidence runs against the Cobb-Douglas specification in favor of a more general Constant Elasticity of Substitution (CES) production function.

We analyzed the effect of specification of biased technical change on estimates of the ES. Estimates of the CES function are usually carried out assuming that technical progress is either Hicks- or Harrod-neutral. This is either because of the complications arising from the nonlinear functional form of the CES, or because of theoretical considerations relating to a balanced growth path. We argue that, when technical progress takes a more general factor augmenting form, misspecifying technical change can lead to substantial biases in estimates of the ES. That is, when technical progress has both a capital- and a labor-augmenting component, specifying the CES as Hicks-, Harrod-, or Solow-neutral, will yield biased estimates of the ES. We then provide quantitative evidence using a Monte Carlo experiment and show that, when the ES is below (above) unity, the estimated ES is biased upwards (downwards). For reasonable parameter values, this bias will tend towards one, the Cobb-Douglas case. When the "true" factor augmentation of technical progress is either Hicks-, Harrod-, or Solow-neutral, estimating the CES as general factor augmenting yields no cost in terms of biases in estimated parameters. We also show that the bias arising in the ES estimate can affect estimates of total factor productivity (TFP) growth.

We then estimate a "normalized" supply side system for the US economy for the 1960:1–2004:4 period. We use a general factor augmenting specification and also Hicks-, Harrod-, and Solow-neutral specifications. In all the cases we can reject the latter three specifications in favor of a factor augmenting one. We find that capital-augmenting technical progress is non-negligible and takes values of 0.56-0.72% per year. Importantly, the ES is found to be in the vicinity of 0.5, emphatically rejecting

the Cobb-Douglas case. That is, our results question the use of Cobb-Douglas production functions for aggregate studies of the US economy in favor of a general CES function. We also provide evidence that the implied TFP growth estimates for the various specifications used is substantially different. Our preferred specification is also able to capture a TFP growth acceleration in the mid 1990's.

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Tables

Table 1. Parameter values for the Monte Carlo

Parameter	Parameter values
π_0: distribution parameter	0.4
σ: substitution elasticity	0.5, 0.9, 1.3
γ_K: Capital-Augmenting Technical Progress	0.00, 0.005, 0.01, 0.015, 0.02
γ_N: Labor-Augmenting Technical Progress	0.02, 0.015, 0.01, 0.005, 0.00
η: Labor growth rate	0.015
κ: Capital growth rate	See text
$Y_0^* = N_0$: initial values for output and labor	1
K_0: initial value for capital	π_0 / r_0
r_0: initial value for the user cost of capital	0.05
$se(\varepsilon_t^N), se(\varepsilon_t^K)$: Standard Error in the Labor and Capital DGP shock	0.10
$se(\varepsilon_t^{\Gamma^K})$: Standard Error in Capital-Augmenting Technical Progress shock	0.01 for $\gamma_K = 0$; 0.05 for $\gamma_K \neq 0$
$se(\varepsilon_t^{\Gamma^N})$: Standard Error in Labor-Augmenting Technical Progress shock	0.01 for $\gamma_N = 0$; 0.05 for $\gamma_N \neq 0$
$se(\varepsilon_t^w)$: Standard Error of Real Wage shock	0.05
$se(\varepsilon_t^r)$: Standard Error of Real Interest Rate shock	0.10
T: Sample Size	50
M: Monte Carlo Draws	5,000

Table 2. Monte Carlo results. Hicks-neutral specification

	$\sigma = 0.5$		$\sigma = 0.9$		$\sigma = 1.3$	
	$\gamma_K = 0.00, \gamma_N = 0.02$					
$\hat{\sigma}$	0.8670		0.9893		1.0458	
[10% 90%]	[0.7679	1.0078]	[0.9135	1.0850]	[0.9460	1.1467]
$\hat{\gamma}$	0.0120		0.0120		0.0121	
	$\gamma_K = 0.005, \gamma_N = 0.015$					
$\hat{\sigma}$	0.6966		0.9603		1.1442	
[10% 90%]	[0.6151	0.8609]	[0.8617	1.0773]	[0.9989	1.3009]
$\hat{\gamma}$	0.0109		0.0110		0.0110	
	$\gamma_K = \gamma_N = 0.01$					
$\hat{\sigma}$	0.5198		0.9144		1.3084	
[10% 90%]	[0.4688	0.5940]	[0.8068	1.0550]	[1.1177	1.5612]
$\hat{\gamma}$	0.0101		0.0100		0.0100	
	$\gamma_K = 0.015, \gamma_N = 0.005$					
$\hat{\sigma}$	0.7257		0.8992		1.5341	
[10% 90%]	[0.6009	0.9180]	[0.7921	1.0253]	[1.2407	2.0363]
$\hat{\gamma}$	0.0099		0.0092		0.0090	
	$\gamma_K = 0.02, \gamma_N = 0.00$					
$\hat{\sigma}$	0.9597		0.9814		1.5198	
[10% 90%]	[0.8060	1.4691]	[0.8617	1.1450]	[1.2185	2.1014]
$\hat{\gamma}$	0.0081		0.0082		0.0077	

Table 3. Monte Carlo results. Harrod-neutral specification

	$\sigma = 0.5$		$\sigma = 0.9$		$\sigma = 1.3$	
	$\gamma_K = 0.00, \gamma_N = 0.02$					
$\hat{\sigma}$	0.5206		0.8998		1.2949	
[10% 90%]	[0.4815	0.5568]	[0.8045	1.0183]	[1.0962	1.5780]
$\hat{\gamma}$	0.0198		0.0201		0.0200	
	$\gamma_K = 0.005, \gamma_N = 0.015$					
$\hat{\sigma}$	0.5873		0.9155		1.2535	
[10% 90%]	[0.5317	0.7315]	[0.8290	1.0276]	[1.0581	1.4891]
$\hat{\gamma}$	0.0163		0.0186		0.0177	
	$\gamma_K = \gamma_N = 0.01$					
$\hat{\sigma}$	0.8187		0.9726		1.1109	
[10% 90%]	[0.7100	0.9642]	[0.8686	1.0889]	[0.9149	1.3236]
$\hat{\gamma}$	0.0171		0.0171		0.0158	
	$\gamma_K = 0.015, \gamma_N = 0.005$					
$\hat{\sigma}$	0.9503		1.0091		0.9299	
[10% 90%]	[0.8517	1.1824]	[0.9196	1.1308]	[0.7776	1.1582]
$\hat{\gamma}$	0.0156		0.0153		0.0149	
	$\gamma_K = 0.02, \gamma_N = 0.00$					
$\hat{\sigma}$	1.0670		1.0315		0.8504	
[10% 90%]	[0.9370	1.3455]	[0.9469	1.1329]	[0.7213	0.9992]
$\hat{\gamma}$	0.0128		0.0134		0.0140	

Table 4. Monte Carlo results. Solow-neutral specification

	$\sigma = 0.5$		$\sigma = 0.9$		$\sigma = 1.3$	
	$\gamma_K = 0.00, \gamma_N = 0.02$					
$\hat{\sigma}$	0.7685		1.0049		1.0007	
[10% 90%]	[0.7122	0.9988]	[0.9651	1.0431]	[0.9530	1.0403]
$\hat{\gamma}$	0.0212		0.0299		0.0301	
	$\gamma_K = 0.005, \gamma_N = 0.015$					
$\hat{\sigma}$	0.8946		0.9943		1.0338	
[10% 90%]	[0.7220	0.9676]	[0.9483	1.0393]	[0.9794	1.0802]
$\hat{\gamma}$	0.0275		0.0276		0.0271	
	$\gamma_K = \gamma_N = 0.01$					
$\hat{\sigma}$	0.8360		0.9808		1.0872	
[10% 90%]	[0.7485	0.9258]	[0.9282	1.0348]	[1.0215	1.1465]
$\hat{\gamma}$	0.0262		0.9980		0.0241	
	$\gamma_K = 0.015, \gamma_N = 0.005$					
$\hat{\sigma}$	0.6911		0.9561		1.1682	
[10% 90%]	[0.5715	0.8117]	[0.8875	1.0263]	[1.0754	1.2556]
$\hat{\gamma}$	0.0217		0.0228		0.0219	
	$\gamma_K = 0.02, \gamma_N = 0.00$					
$\hat{\sigma}$	0.5274		0.9138		1.3080	
[10% 90%]	[0.4764	0.5722]	[0.8332	1.0006]	[1.1809	1.4385]
$\hat{\gamma}$	0.0201		0.0201		0.0201	

Table 5a. GNLLS estimates of the normalized system, US, 1960:1-2004:4

	Factor Augmenting specification		
	Parameter Estimate	Standard error	T-ratio
σ	0.6301	0.0407	15.4674
γ_N	0.0040	0.0002	19.4031
γ_K	0.0018	0.0004	4.9988
ζ	0.9988	0.0055	183.024
μ	0.1072	0.0090	11.8900
Log-determinant	-21.172		
FOC˙K ADF	-2.6350 (0.0112)		
FOC˙N ADF	-3.4668 (0.0001)		
CES ADF	-2.9666 (0.0028)		
	Hicks Neutral specification		
σ	0.8000	0.0426	18.7548
γ	0.0034	0.0001	25.9222
ζ	1.0013	0.0065	153.5386
μ	0.1106	0.0085	12.9320
Log-determinant	-21.137		
FOC˙K ADF	-2.6524 (0.0072)		
FOC˙N ADF	-3.6366 (0.0000)		
CES ADF	-3.0749 (0.0012)		
LR-test for Hicks Neutrality	6.6112 (0.0101)		

Notes: p-values in parentheses. Auto-correlation and heteroskedasticity robust standard errors reported. The p-values for the residual ADF (co-integration) tests were obtained from 2,500 bootstrap draws.

Table 5b. GNLLS estimates of the normalized system, US, 1960:1-2004:4. Contd.

	Harrod-Neutral Specification		
	Parameter Estimate	Standard error	T-ratio
σ	1.7020	0.3565	4.7739
γ	0.0046	0.0002	23.2089
ζ	1.0035	0.0080	124.7402
μ	0.1106	0.0059	18.6950
Log-determinant	-20.936		
FOC˙K ADF	-2.7551 (0.0044)		
FOC˙N ADF	-2.5239 (0.0100)		
CES ADF	-3.1665 (0.0000)		
LR-test for Harrod vs Factor augmenting	45.3494 (0.0000)		
	Solow-Neutral specification		
σ	0.9504	0.0171	55.6004
γ	0.0131	0.0005	26.3396
ζ	1.0045	0.0072	140.008
μ	0.1131	0.0085	13.3727
Log-determinant	-21.111		
FOC˙K ADF	-2.6490 (0.0064)		
FOC˙N ADF	-3.3522 (0.0000)		
CES ADF	-3.0052 (0.0032)		
LR-test for Solow vs Factor augmenting	11.7390 (0.0000)		

Notes: see Table 5a.

Table 6a. NL3SLS estimates of the normalized system, US, 1960:1-2004:4

	Factor Augmenting specification		
	Parameter Estimate	Standard error	T-ratio
σ	0.4914	0.0254	19.3835
γ_N	0.0041	0.0002	27.6293
γ_K	0.0014	0.0002	6.4195
ζ	0.9979	0.0042	239.2997
μ	0.1049	0.0088	11.8532
Log-determinant	-21.153		
J-test	36.216	(0.5060)	
FOC˙K ADF	-2.5744	(0.0064)	
FOC˙N ADF	-3.2001	(0.0001)	
CES ADF	-2.9582	(0.0028)	
	Hicks Neutral specification		
σ	0.7845	0.0425	18.4753
γ	0.0034	0.0001	25.8856
ζ	1.0019	0.0065	153.1776
μ	0.1115	0.0087	12.7974
Log-determinant	-21.134		
J-test	17.035	(0.9987)	
FOC˙K ADF	-2.6346	(0.0096)	
FOC˙N ADF	-3.6622	(0.0000)	
CES ADF	-3.0984	(0.0016)	
LR-test for Hicks Neutrality	2.7272 (0.0986)		

Notes: see Table 5a.

Table 6b. NL3SLS estimates of the normalized system, US, 1960:1-2004:4. Contd.

	Harrod-Neutral Specification		
	Parameter Estimate	Standard error	T-ratio
σ	1.3382	0.2159	6.1970
γ	0.0047	0.0002	23.5301
ζ	1.0005	0.0080	125.1103
μ	0.1083	0.0067	16.1153
Log-determinant	-20.917		
J-test	24.466 (0.9563)		
FOC˙K ADF	-2.7669 (0.0056)		
FOC˙N ADF	-2.7090 (0.0060)		
CES ADF	-3.0390 (0.0024)		
LR-test for Harrod vs Factor augmenting	44.481 (0.0000)		
	Solow-Neutral specification		
σ	0.9497	0.0172	55.3298
γ	0.0131	0.0005	26.3164
ζ	1.0046	0.0072	139.5456
μ	0.1133	0.0085	13.3537
Log-determinant	-21.111		
J-test	14993.2 (0.0000)		
FOC˙K ADF	-2.6457 (0.0068)		
FOC˙N ADF	-3.3533 (0.0016)		
CES ADF	-3.0073 (0.0040)		
LR-test for Solow vs Factor augmenting	7.2402 (0.0071)		

Notes: see Table 5a.

Figures

Figure 1. Distribution of estimated σ. True σ = 0.5.

$\gamma_K = 0.00, \gamma_N = 0.02$

Distribution of estimated sigma

FACTORAUG — HICKS — HARROD — SOLOW

$\gamma_K = 0.005, \gamma_N = 0.015$

Distribution of estimated sigma

FACTORAUG — HICKS — HARROD — SOLOW

$\gamma_K = \gamma_N = 0.01$

Distribution of estimated sigma

FACTORAUG — HICKS — HARROD — SOLOW

$\gamma_K = 0.02, \gamma_N = 0.00$

Distribution of estimated sigma

FACTORAUG — HICKS — HARROD — SOLOW

Figure 2. Distribution of estimated σ. True σ = 1.3.

$\gamma_K = 0.00, \gamma_N = 0.02$

Distribution of estimated sigma

$\gamma_K = 0.005, \gamma_N = 0.015$

Distribution of estimated sigma

$\gamma_K = \gamma_N = 0.01$

Distribution of estimated sigma

$\gamma_K = 0.02, \gamma_N = 0.00$

Distribution of estimated sigma

Figure 3. Great ratios for the US economy.

Great Ratios: K/Y, Y/N and K/N

Labor income share
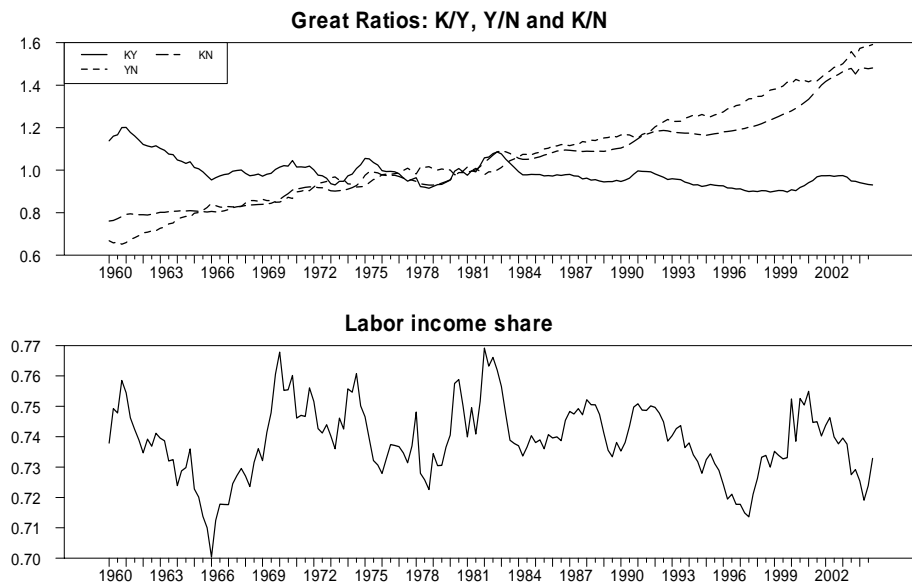
Figure 4. Real wages and real user cost.

Wages

User Cost
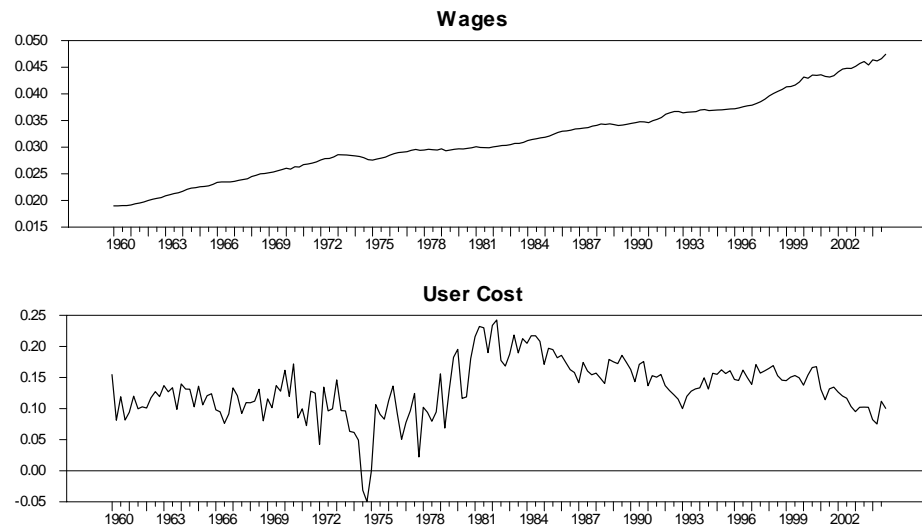
Figure 5. Actual (solid) and fitted (dashed) values for *w*, *Y* and *r*. Factor augmenting specification (NLGLS).
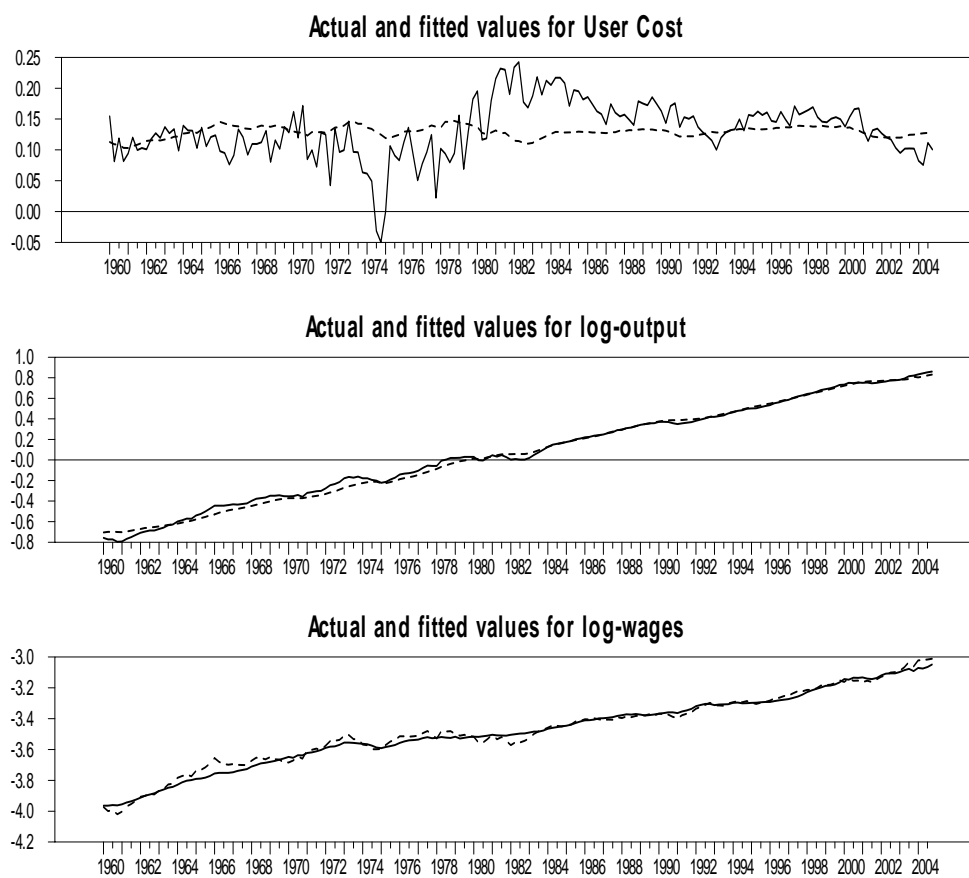
Figure 6. Residuals for the r, w and Y equations for the four specifications (NLGLS).

Rental price

Wage

Output
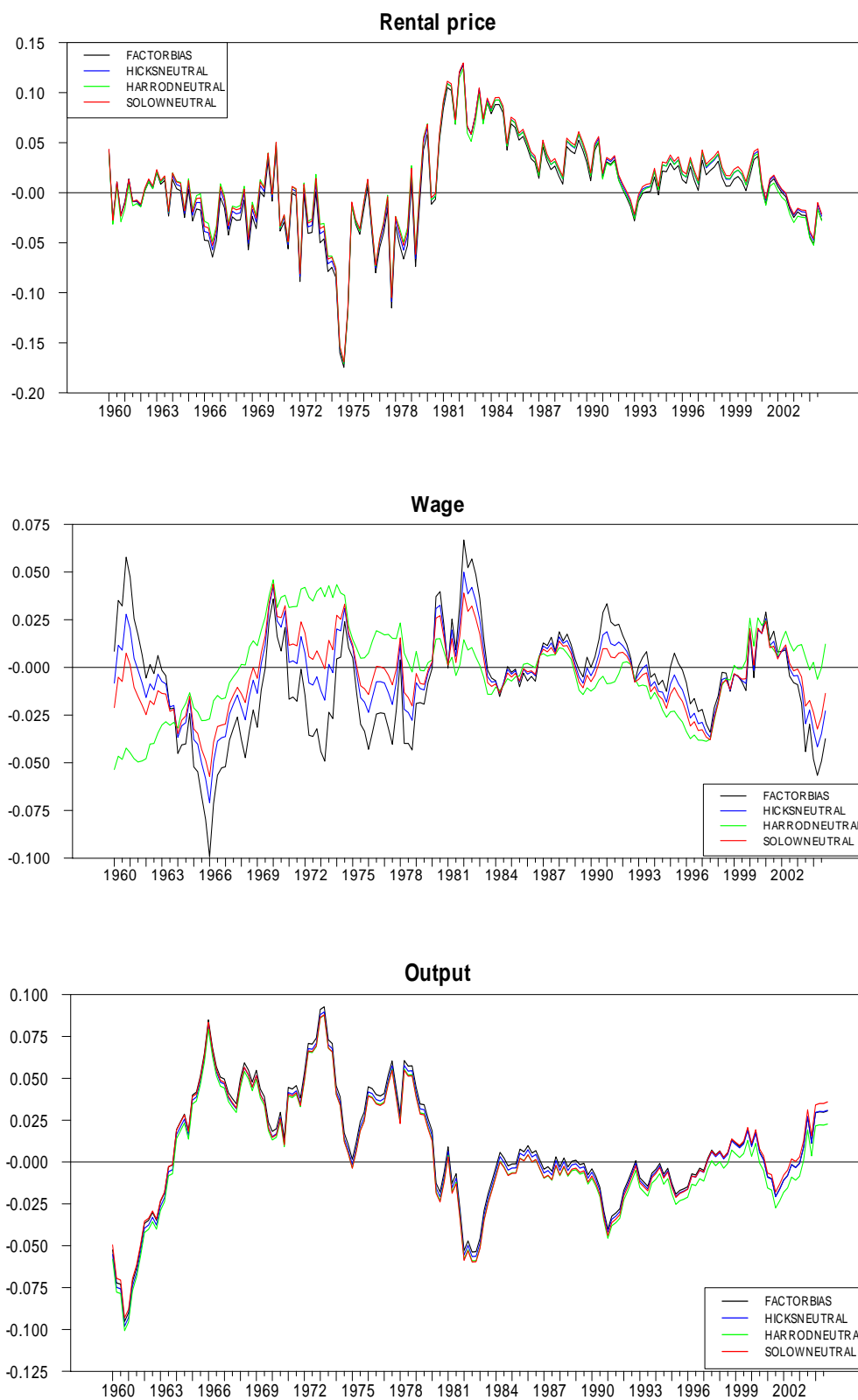
Figure 7. Sensitivity to changes in initialization of σ (GNLLS).

Estimated sigma vs initial condition

Log-determinants vs estimated sigma

Log-determinants vs initial condition
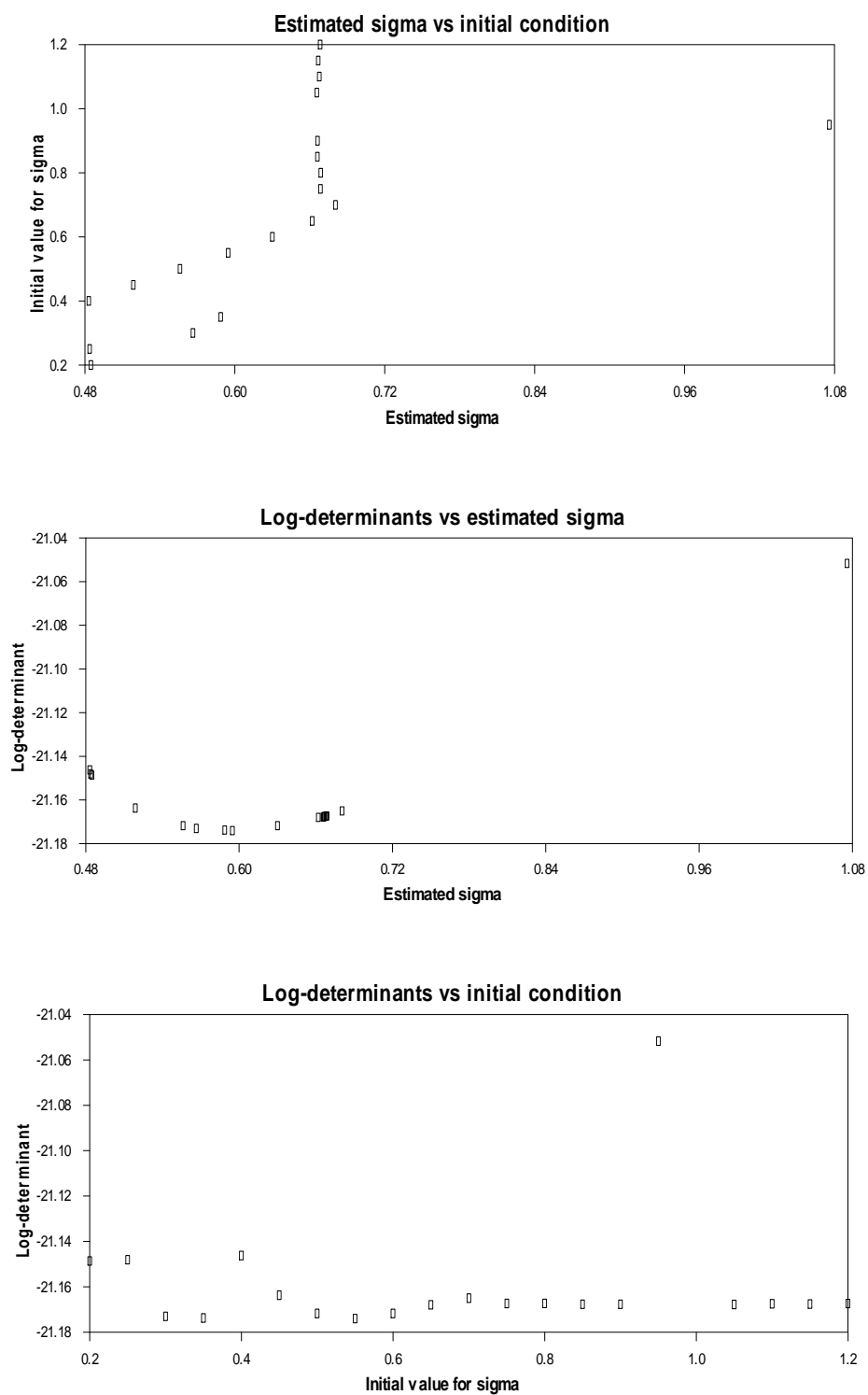
Figure 8. Sensitivity to changes in initialization of σ (NL3SLS).

Estimated sigma vs initial condition

Log-determinants vs estimated sigma

Log-determinants vs initial condition
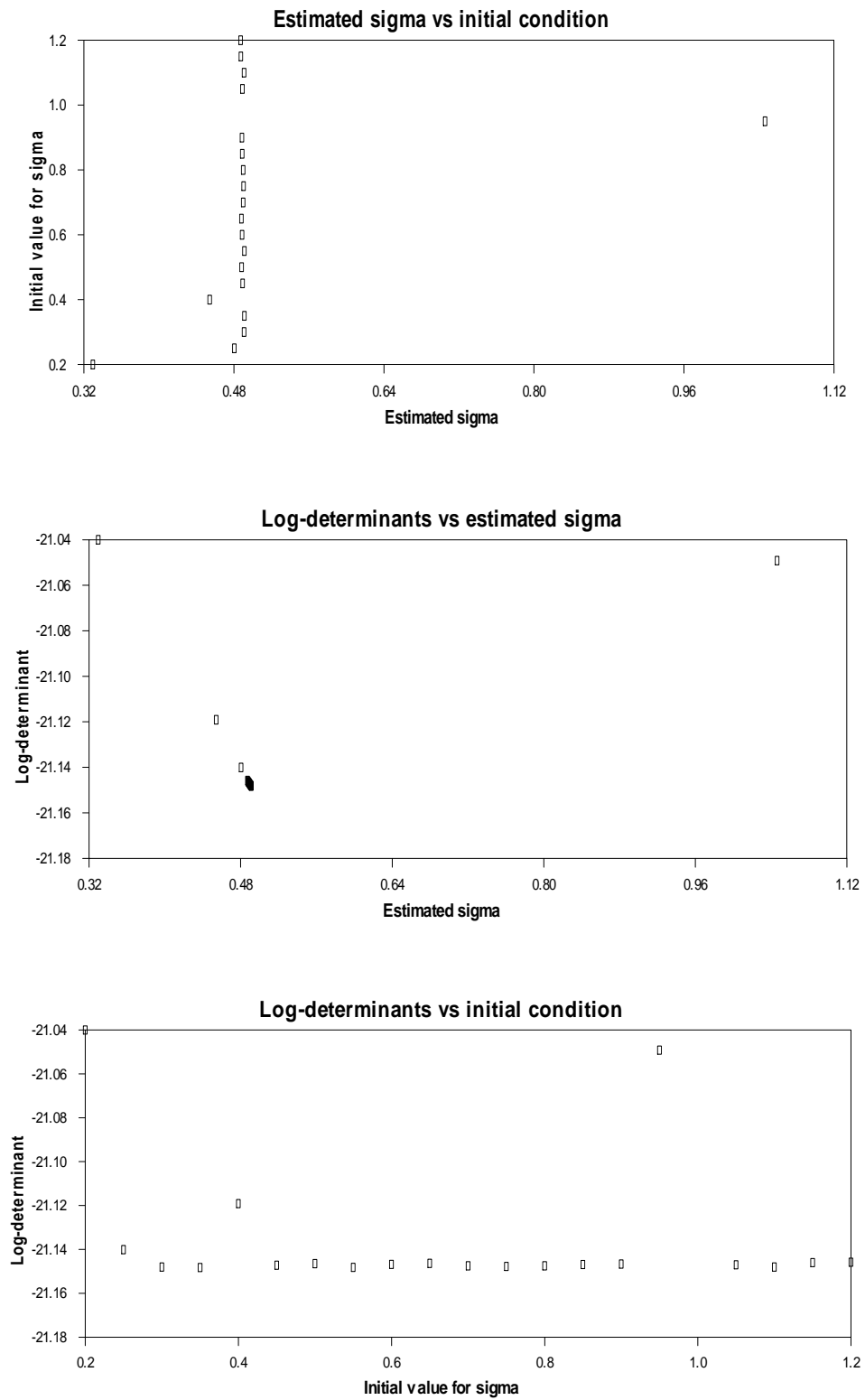
Figure 9. Total Factor Productivity growth from Kmenta approximations (GNLLS).

TFP growth from four specifications. Simple Kmenta

FACTOR_AUG — HICKS_ — HARROD_ — SOLOW_

TFP growth from four specifications. Extended Kmenta

FACTORAUG — HICKS__ — HARROD__ — SOLOW__
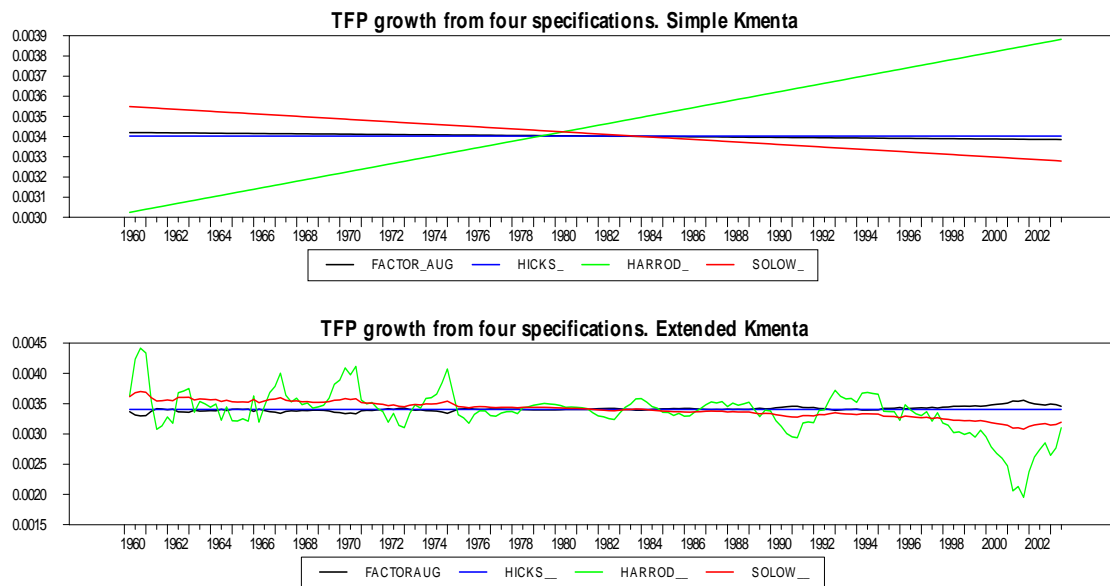
Figure 10. Total Factor Productivity and K/L ratio Growth, simple (solid) and extended (dashed) Kmenta approximations (GNLLS).

Factor augmenting specification

Solow-neutral specification

Harrod-neutral specification

K/L ratio growth