# Asset Pricing in the Presence of Background Risk

*By Andrei Semenov*[*]

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**Abstract**

We assume an economy in which an agent faces, in addition to uncertainty about the return on risky asset holdings, an independent non-hedgeable zero-mean background risk. Within this framework, the pricing kernel is a function of aggregate consumption per capita and the size of the background risk. The introducing of the independent non-hedgeable background risk in the otherwise standard consumption CAPM enables to break the link between the utility curvature parameter, relative risk aversion to the financial investment risk, and the elasticity of intertemporal substitution. The proposed model does well in explaining the observed equity premia and risk-free rate.

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[*]Department of Economics, York University, Vari Hall 1028, 4700 Keele Street, Toronto, Ontario M3J 1P3, Canada, Phone: (416) 736-2100 ext.: 77025, Fax: (416) 736-5987, E-mail: asemenov@econ.yorku.ca. The author thanks Melanie Cao, Mark J Kamstra, Clarence C.Y. Kwan, Pauline Shum, and Yisong Tian for helpful comments and suggestions. The author also benefitted from discussions with conference and seminar participants at Schulich School of Business, John Molson School of Business, DeGroote School of Business, and the 2009 European Financial Management Association Annual Meeting. The author gratefully acknowledges financial support from the Social Sciences and Humanities Research Council of Canada (SSHRC).

# 1 Introduction

The Lucas (1978) and Breeden (1979) consumption CAPM (standard consumption CAPM, hereafter) relates asset prices to the consumption and savings decisions of a single investor, interpreted as a representative for a large number of identical infinitely-lived investors. The power utility maximizing representative investor is assumed to freely trade in perfect capital markets without transaction costs, limitations on borrowing or short sales, and taxes. The stochastic discount factor (SDF), or pricing kernel, in this model is the discounted aggregate per capita consumption growth rate raised to the power of the negative utility curvature parameter (relative risk aversion (RRA) coefficient).

As argued in the literature, a problem with the standard consumption CAPM is that, when reasonably parameterized, this model yields the average equity premium, which is substantially lower than the observed excess return on the market portfolio over the risk-free rate, and the representative agent must be assumed implausibly averse to risk (the agent's utility function must be unrealistically concave) for the model to fit the observed mean equity premium. This is the equity premium puzzle discussed by Mehra and Prescott (1985) and Hansen and Jagannathan (1991), among others. Another problem with the same model is that it yields the mean risk-free rate, which is much greater than the observed average return on the risk-free asset. The model explains the mean return on the risk-free asset only if the subjective time discount factor is greater than one (the representative investor has a negative rate of time preference). This is the risk-free rate puzzle as described in Weil (1989).

Since the Mehra and Prescott (1985) seminal paper, several ways to improve the standard consumption CAPM have been proposed in the literature. One response to the difficulties of this model is to generalize the utility function. Another way to improve the ability of the standard representative-agent consumption CAPM to fit empirical data on asset returns is to take into account the effects of psychological biases on asset prices. Some authors argue that various market frictions (such as transactions costs, limits on borrowing or short sales, etc.) may make aggregate consumption an inadequate proxy for the consumption of stock market investors and develop asset pricing models that do not rely so heavily on the aggregate consumption of the economy.

A possible explanation for the empirical failure of the standard representative-agent model is that this model abstracts from the lack of certain types of insurance such as insurance against idiosyncratic shocks to the agent's income like loss of employment or divorce, for example. The potential for the incomplete consumption insurance model to explain the equilibrium behavior of stock and bond returns, both in terms of the level of equilibrium rates and the discrepancy between equity and bond returns, was first suggested by Bewley (1982), Mehra and Prescott (1985), and Mankiw (1986).

The empirical evidence for the importance of the incomplete consumption insurance hypothesis for explaining asset returns is somewhat mixed. Aiyagari and Gertler (1991), Huggett (1993), Telmer (1993), Lucas (1994), Heaton and D. Lucas (1996, 1997), Jacobs (1999), and Cogley (2002), for example, find no evidence that the assumption of incomplete consumption insurance can improve substantially the asset-pricing implications of the standard representative-agent consumption CAPM. However, Brav *et al.* (hereafter, BCG) (2002), Balduzzi and Yao (2007), and Kocherlakota and Pistaferri (2009) take the opposite view. Despite the difference in approaches, they find empirically that incomplete consumption insurance plays an important role in explaining the market premium. BCG (2002), for example, obtain that, when incomplete consumption insurance is taken into account, the consumption CAPM can explain the mean equity premium with a RRA coefficient between three and four. Kocherlakota and Pistaferri (2009) show that the coefficient of RRA must be in the range from five to six for the incomplete consumption insurance model to explain the market excess return. The SDF proposed by Balduzzi and Yao (2007) enables to fit the equity premium with a value of the RRA coefficient, which is substantially lower than the estimate obtained for the standard representative-agent model, but still remains unrealistically high (greater than nine).

Although the hypothesis of incomplete consumption insurance allowed to make substantial progress in explaining the equity premium, there is still a problem with a joint explanation of the equity premium and the risk-free rate of return. For example, the pricing kernels proposed by BCG (2002) and Kocherlakota and Pistaferri (2009) fit the observed excess return on the market portfolio over the risk-free rate at low values of RRA, but fare poorly when used to explain the risk-free rate. The both SDFs yield implausibly low estimates of the subjective time discount factor.[1]

In this paper, we also investigate the role of the hypothesis of incomplete consumption insurance in explaining asset returns. However, our approach is different from the approaches in BCG (2002), Balduzzi and Yao (2007), and Kocherlakota and Pistaferri (2009). We assume that an agent is subject, in addition to the financial investment risk, to a non-hedgeable background risk (such as labor income uncertainty, for instance) that is independent of the risk associated with the investment in a risky asset. We also follow Franke *et al.* (1998) and Kimball (1993) and assume that the background risk has a zero expected value (is a pure risk). Within this framework, the SDF is a function of not only aggregate per capita consumption (as in the standard representative-agent model), but also the size of the background risk.

We argue that in the presence of background risk, one must distinguish the agent's original utility function and the indirect utility function, as defined by Kihlstrom *et al.* (1981), Nachman (1982), and Gollier. One of the problems with the standard representative-agent consumption CAPM is that the agent's utility is required to be unrealistically concave for the model to account for the observed equity premium. This model is implicitly based on the assumption of complete

[1] Balduzzi and Yao (2007) do not test the ability of their SDF to explain the return on the risk-free asset.

consumption insurance and hence the absence of the non-hedgeable background risk. Our argument is that the utility function in the standard representative-agent model is, in fact, the indirect utility function, which may differ from the agent's original utility function. In the absence of background risk, the two utility functions coincide. Gollier (2001) shows that risk vulnerability of preferences implies that, in the presence of an independent non-hedgeable background risk with a non-positive mean, the indirect utility is more concave than the original utility. Because the CRRA utility exhibits risk vulnerability, this suggests that the model with background risk has the potential to explain the observed equity premium with an economically realistic value of the agent's original utility function curvature parameter and the so-called equity premium puzzle may simply be due to the misinterpretation (in the presence of background risk) of the utility curvature parameter in the standard representative-agent consumption CAPM, which is, in fact, the curvature parameter of the indirect utility function and not that of the original utility function.

The introducing of the independent non-hedgeable background risk in the otherwise standard consumption CAPM enables to disentangle the utility curvature parameter, the agent's aversion towards the market risk, and the elasticity of intertemporal substitution in consumption (hereafter, EIS). If the decision maker's preferences exhibit risk vulnerability, then the increase in the size of the background risk raises the RRA to uncertainty about the return on risky asset holdings making it different from the original utility curvature parameter (the coefficient of RRA in the absence of background risk in consumption), like in the Campbell and Cochrane (1999) difference habit model. The attractive feature of our approach is that, in contrast with Campbell and Cochrane (1999), we can disentangle RRA and the utility curvature parameter under weaker assumptions and can avoid getting unrealistically volatile estimates of the RRA coefficient. Within our framework, the RRA coefficient is a function of the utility curvature parameter as well as of the first two unconditional moments of the distribution of background risk and therefore may vary with the state of the economy even if the agent is assumed to have a time-invariant utility function. This relates our approach to the strand of the literature that postulates that the attitudes towards financial risk are not fixed, but rather contingent upon the state of the world. The advantage of our approach is that the choice of the factors deemed to be relevant in explaining the investor's risk aversion and the functional form relating the coefficient of RRA to these factors obtain endogenously from the theoretical restrictions implied by a structural model.

Another important result is that, in the presence of an independent non-hedgeable background risk with a non-positive mean, an increase in the consumption of an agent with power utility lowers the agent's RRA to financial risk, whereas the coefficient of RRA is constant at different levels of consumption in the no background risk environment (in the standard consumption CAPM). We show that the RRA coefficient in the model with background risk may differ from the indirect utility curvature parameter, meaning that the interpretation of the utility curvature parameter in the standard representative-agent consumption CAPM as the agent's RRA is false if, in fact, the agent is subject to an independent non-hedgeable background risk.

Our result is that the EIS for the agent with the original utility function under background risk is identical to the EIS for the agent with indirect utility in the no background risk environment and is therefore the reciprocal of the indirect utility function curvature parameter. Since, in the presence of background risk, the indirect utility function is more concave than the original utility function, this implies that, in contrast with the standard representative-agent model as well as the models proposed in BCG (2002), Balduzzi and Yao (2007) and Kocherlakota and Pistaferri (2009), the model with background risk makes it possible to get a low value of the EIS even if the agent's utility function is not very concave.

The rest of the paper is organized as follows. In Section 2, we first derive the consumption CAPM with the independent non-hedgeable zero-mean background risk and show how this model breaks the link between the utility curvature parameter, risk aversion, and the EIS. Then, we investigate the potential of the proposed model to solve the equity premium and risk-free rate puzzles. In Section 3, we describe data and explore the empirical relevance of background risk for asset pricing. The results for the model with background risk are compared with the results for the standard representative-agent model and the asset-pricing model with the SDF proposed in BCG (2002). Section 4 concludes.

# 2 The consumption CAPM with background risk

## 2.1 The consumption choice problem

Consider the discrete-state intertemporal consumption choice problem of an infinitely living investor who maximizes the expected present value of discounted lifetime utility of consumption. Assume that uncertainty about the return on risky asset holdings is traded in a complete market. Suppose furthermore that the agent faces, in addition to the market risk, another risk, which is non-hedgeable (i.e., the market for this risk is incomplete) and independent of the risk associated with the investment in a risky asset. The examples of such risks are the labor income risk, loss of employment, divorce, etc. Following Stapleton (1999) and Gollier (2001), we refer this risk to as a background risk.

In the absence of certain contingent-claims markets, the agent's total consumption is the sum of his hedgeable consumption (the consumption that may be hedged using the capital markets) and an unexpected deviation from the hedgeable consumption (the non-hedgeable consumption) caused by background risk factors. We assume that the background risk is independent of any other risk and hence the agent's non-hedgeable consumption is independent of both the hedgeable consumption and the market risk.

The agent is unable to fully insure his consumption against the background risk, but (knowing the distribution of the non-hedgeable consumption) can choose the consumption he is able to hedge in the capital markets (the hedgeable consumption) that maximizes the expected present value of discounted lifetime utility of consumption[2]

$$U = \sum_{\tau=0}^{\infty} \delta^{\tau} E_t \left[ E^{\Phi} \left[ u \left( C_{i,t+\tau} + \Phi_{i,t+\tau} \right) \right] \right], \tag{1}$$

where $\delta$ is the subjective time discount factor, $C_{i,t+\tau}$ is the agent $i$'s $(i = 1, ..., N)$ hedgeable consumption in period $t + \tau$, $\Phi_{i,t+\tau} = \sigma_{i,t+\tau}\varepsilon_{i,t+\tau}$ is the non-hedgeable consumption ($\varepsilon_{i,t+\tau}$ is a random variable with zero mean and unit variance and $\sigma_{i,t+\tau}$ is a constant measuring the size of the background risk),[3] $u(\cdot)$ is the agent's period utility function,[4] $E_t$ denotes the expectation over the hedgeable consumption conditional on the information available to the agent at time $t$. The notation $E^{\Phi}$ indicates the expectation over the non-hedgeable consumption. The non-hedgeable consumption $\Phi_{t+\tau}$ is independent of both the hedgeable consumption $C_{t+\tau}$ and the risky payoff.

One of the first-order conditions, or Euler equations, describing the agent's optimal hedgeable consumption plan is

$$E^{\Phi} \left[ u' \left( C_{i,t} + \Phi_{i,t} \right) \right] = \delta E_t \left[ E^{\Phi} \left[ u' \left( C_{i,t+1} + \Phi_{i,t+1} \right) \right] R_{j,t+1} \right], \tag{2}$$

where $u'(\cdot)$ denotes the first derivative of utility with respect to the hedgeable consumption and $R_{j,t+1}$ is the real gross return between time $t$ and $t+1$ on asset $j$ in which the agent holds a non-zero position. Observe that the decision on the hedgeable consumption must be taken prior to the realization of the background risk and the risk associated with the investment in a risky asset.

The left-hand side of equation (2) is the expected (over the background risk states) loss in utility if the investor buys another unit of asset $j$ at time $t$ and the right-hand side of (2) is the increase in discounted, expected utility the investor obtains from the extra payoff at time $t+1$ under background risk. If there are no arbitrage opportunities, the investor equates the expected marginal loss and the expected marginal gain from holding asset $j$. Denote as $\Phi_{is,t}$ the value of $\Phi_{i,t}$ in the background risk state $s$ $(s = 1, ..., S)$. In order for marginal utility to be well-defined in any state $s$, assume that the agent $i$'s time $t$ total consumption $C^{*}_{is,t} = C_{i,t} + \Phi_{is,t}$ is positive in any state $s$.

[2] See also Franke *et al.* (1998) and Poon and Stapleton (2005).

[3] We follow Kimball (1993) and Franke *et al.* (1998) and assume that $\Phi_{i,t+\tau}$ has a zero expected value.

[4] We suppose that all agents in the economy have identical preferences and assume that the first four derivatives of $u(\cdot)$ exist. As is conventional in the literature, we also assume that the agent is risk averse (i.e., $u'(\cdot) > 0$ and $u''(\cdot) < 0$) and prudent ($u'''(\cdot) > 0$). Kimball (1990) defines prudence as a measure of the sensitivity of the optimal choice of a decision variable to risk (of the intensity of the precautionary saving motive in the context of the consumption-saving decision under uncertainty). A precautionary saving motive is positive when $-u'(\cdot)$ is concave ($u'''(\cdot) > 0$) just as an individual is risk averse when $u(\cdot)$ is concave. Intuitively, the willingness to save is an increasing function of the expected marginal utility of future consumption. Since marginal utility is decreasing in consumption, the absolute level of precautionary savings must also be expected to decline as consumption rises. The condition $u''''(\cdot) < 0$ is necessary for decreasing absolute prudence.

Since $E^{\Phi}\left[u'\left(C_{i,t}+\Phi_{i,t}\right)\right]$ is known to the agent at time $t$, we can divide both the left- and right-hand sides of equation (2) by $E^{\Phi}\left[u'\left(C_{i,t}+\Phi_{i,t}\right)\right]$ to get

$$E_t\left[\delta\frac{E^{\Phi}\left[u'\left(C_{i,t+1}+\Phi_{i,t+1}\right)\right]}{E^{\Phi}\left[u'\left(C_{i,t}+\Phi_{i,t}\right)\right]}R_{j,t+1}\right]=1. \tag{3}$$

This is the consumption CAPM with background risk. In this model, the SDF is the discounted ratio of expectations of marginal utility over the background risk states at two consecutive dates:

$$M_{t+1}=\delta\frac{E^{\Phi}\left[u'\left(C_{i,t+1}+\Phi_{i,t+1}\right)\right]}{E^{\Phi}\left[u'\left(C_{i,t}+\Phi_{i,t}\right)\right]}. \tag{4}$$

In the absence of background risk,

$$E^{\Phi}\left[u'\left(C_{i,t}+\Phi_{i,t}\right)\right]=u'\left(C_{i,t}\right) \tag{5}$$

for all $t$ and hence model (3) reduces to the conventional consumption CAPM with no background risk:

$$E_t\left[\delta\frac{u'\left(C_{i,t+1}\right)}{u'\left(C_{i,t}\right)}R_{j,t+1}\right]=1 \tag{6}$$

in which the SDF is the discounted ratio of the marginal utility of consumption at time $t+1$ to the marginal utility of consumption at time $t$:

$$M_{t+1}=\delta\frac{u'\left(C_{i,t+1}\right)}{u'\left(C_{i,t}\right)}. \tag{7}$$

## 2.2 The precautionary premium and the pricing kernel

In $E^{\Phi}\left[u'\left(C_{i,t}+\Phi_{i,t}\right)\right]$ for all $t$, $C_{i,t}$ is a certain quantity and $\Phi_{i,t}$ is a random variable. Following Kimball (1990), we can hence write $E^{\Phi}\left[u'\left(C_{i,t}+\Phi_{i,t}\right)\right]$ as

$$E^{\Phi}\left[u'\left(C_{i,t}+\Phi_{i,t}\right)\right]=u'\left(C_{i,t}-\Psi_{i,t}\right), \tag{8}$$

where $\Psi_{i,t}=\Psi\left(C_{i,t},u\left(\cdot\right),\Phi_{i,t}\right)$ is an equivalent precautionary premium (the certain amount by which the agent is ready to reduce his consumption in order to escape the background risk). Franke *et al.* (1998) and Poon and Stapleton (2005) show that, in the case of HARA utility functions, the precautionary premium $\Psi_{i,t}$ is positive, strictly increasing in the variability of $\Phi_{i,t}$, and (except for exponential utility) strictly decreasing and convex in $C_{i,t}$.[5]

Substituting (8) into equation (4) gives

$$M_{t+1}=\delta\frac{u'\left(C_{i,t+1}-\Psi_{i,t+1}\right)}{u'\left(C_{i,t}-\Psi_{i,t}\right)}. \tag{9}$$

[5] The precautionary premium is a constant in the case of the exponential utility function.

Notice that, as follows from condition (8), the precautionary premium $\Psi_{i,t}$ is equivalent to the risk premium of $\Phi_{i,t}$ for the agent with utility function $-u'(\cdot)$, i.e.,

$$\Psi_{i,t} = \Pi\left(C_{i,t}, -u'(\cdot), \Phi_{i,t}\right). \tag{10}$$

Assume that the background risk is "small". Hence, by analogy with the risk premium,

$$\Psi_{i,t} \approx \frac{1}{2}\sigma_{i,t}^2 \left(-\frac{u'''(C_{i,t})}{u''(C_{i,t})}\right), \tag{11}$$

where $\sigma_{i,t}^2$ is the variance of the non-hedgeable consumption $\Phi_{i,t}$,

$$\sigma_{i,t}^2 = E[(\Phi_{i,t} - E[\Phi_{i,t}])^2] = E[\left(C_{i,t}^* - (C_{i,t} + E[\Phi_{i,t}])\right)^2] = \frac{1}{S}\sum_{s=1}^{S}\left(C_{is,t}^* - C_{i,t}\right)^2. \tag{12}$$

Suppose that the agent's utility function in (1) is CRRA:

$$u = \frac{C_{i,t}^{1-\gamma} - 1}{1-\gamma}, \tag{13}$$

where the utility curvature parameter $\gamma > 0$, $\gamma \neq 1$.[6]

For this utility specification, $u'(C_{i,t}) = C_{i,t}^{-\gamma}$, $u''(C_{i,t}) = -\gamma C_{i,t}^{-\gamma-1}$, $u'''(C_{i,t}) = \gamma(\gamma+1)C_{i,t}^{-\gamma-2}$, and $u''''(C_{i,t}) = -\gamma(\gamma+1)(\gamma+2)C_{i,t}^{-\gamma-3}$.

The precautionary premium $\Psi_{i,t}$ for an agent with CRRA utility is hence a function of the agent's hedgeable consumption $C_{i,t}$ and the background risk scale $\sigma_{i,t}$:

$$\Psi_{i,t} = \Psi_{i,t}(C_{i,t}, \sigma_{i,t}) \approx \frac{1}{2}\sigma_{i,t}^2 \frac{\gamma+1}{C_{i,t}}. \tag{14}$$

This implies that, with CRRA utility, for all $t$

$$u'(C_{i,t} - \Psi_{i,t}) = (C_{i,t} - \Psi_{i,t})^{-\gamma} = C_{i,t}^{-\gamma}\left(1 - \frac{\gamma+1}{2}\varsigma_{i,t}^2\right)^{-\gamma}, \tag{15}$$

where $\varsigma_{i,t}^2$ is the normalized variance of $\Phi_{i,t}$:

$$\varsigma_{i,t}^2 = \frac{\sigma_{i,t}^2}{(C_{i,t} + E[\Phi_{i,t}])^2} = \frac{1}{S}\sum_{s=1}^{S}\left(\frac{C_{is,t}^*}{C_{i,t}} - 1\right)^2. \tag{16}$$

Because $\varsigma_{i,t}^2$ is small,

$$1 - \frac{\gamma+1}{2}\varsigma_{i,t}^2 \approx exp\left(-\frac{\gamma+1}{2}\varsigma_{i,t}^2\right) \tag{17}$$

and therefore

$$u'(C_{i,t} - \Psi_{i,t}) = C_{i,t}^{-\gamma} exp\left(\frac{\gamma(\gamma+1)}{2}\varsigma_{i,t}^2\right). \tag{18}$$

[6] As $\gamma$ approaches one, the power utility function (13) approaches the logarithmic utility $u = log(C_{it})$.

Substituting (18) into (9) gives the following expression for the SDF in the consumption CAPM with background risk:

$$M_{t+1} = \delta \left( \frac{C_{i,t+1}}{C_{i,t}} \right)^{-\gamma} exp \left( \frac{\gamma (\gamma + 1)}{2} \bigtriangleup \varsigma_{i,t+1}^2 \right) \tag{19}$$

with $\bigtriangleup \varsigma_{i,t+1}^2 = \varsigma_{i,t+1}^2 - \varsigma_{i,t}^2$.

When the agent's utility function is (13), the consumption CAPM with background risk is therefore

$$E_t \left[ \delta \left( \frac{C_{i,t+1}}{C_{i,t}} \right)^{-\gamma} exp \left( \frac{\gamma (\gamma + 1)}{2} \bigtriangleup \varsigma_{i,t+1}^2 \right) R_{j,t+1} \right] = 1. \tag{20}$$

If there is no background risk (i.e., $\Phi_{is,t} = 0$ for all $s$ and $t$ and therefore $\sigma_{i,t}^2 = 0$ for all $t$ implying $\varsigma_{i,t}^2 = 0$ for all $t$), then the marginal utility of consumption in (15) becomes

$$u' (C_{i,t} - \Psi_{i,t}) = u' (C_{i,t}) = C_{i,t}^{-\gamma} \tag{21}$$

and thus, as expected, model (20) reduces to the conventional complete consumption insurance model

$$E_t \left[ \delta \left( \frac{C_{i,t+1}}{C_{i,t}} \right)^{-\gamma} R_{j,t+1} \right] = 1. \tag{22}$$

### 2.3 Risk vulnerability, aversion to risk and the elasticity of intertemporal substitution

The intuition is that the presence of an independent non-hedgeable background risk increases the aversion of a decision maker to other independent risks. The preferences with such a property are said to exhibit risk vulnerability.[7]

As introduced by Kihlstrom *et al.* (1981), Nachman (1982), and Gollier (2001), define the following indirect utility function:[8]

$$g (C_{i,t}) = E^{\Phi} [u (C_{i,t} + \Phi_{i,t})] . \tag{23}$$

Since $\Phi_{i,t}$ is independent of $C_{i,t}$, as assumed in Section 2.1, observe that

$$g^{(n)} (C_{i,t}) = E^{\Phi}[u^{(n)} (C_{i,t} + \Phi_{i,t})], \tag{24}$$

where $g^{(n)}$ and $u^{(n)}$ are the $n$th derivatives of the indirect utility function $g(\cdot)$ and the utility function $u(\cdot)$, respectively.

[7] See Gollier (2001), for example.

[8] The indirect utility function $g(\cdot)$ inherits decreasing absolute risk aversion (see Pratt (1964)) and decreasing absolute prudence (see Kimball (1993)) from utility $u(\cdot)$. Gollier (2001) argues that if at least one of the two utility functions $u_i(\cdot)$ and $u_{i'}(\cdot)$ $(i \neq i')$ is characterized by non-increasing absolute risk aversion, then comparative risk aversion is also preserved in the presence of background risk.

As shown by Gollier (2001), in the case of the background risk with a non-positive mean (i.e., $E[\Phi_{i,t}] \leqslant 0$), preferences exhibit risk vulnerability if and only if the indirect utility function $g(\cdot)$ is more concave than the utility function $u(\cdot)$, i.e., for all $C_{i,t}$

$$-\frac{g''(C_{i,t})}{g'(C_{i,t})} \geqslant -\frac{u''(C_{i,t})}{u'(C_{i,t})}. \tag{25}$$

Gollier (2001) points out that this inequality holds if at least one of the following two conditions is satisfied: (i) absolute risk aversion is decreasing and convex and (ii) both absolute risk aversion and absolute prudence are positive and decreasing in wealth.[9] The latter condition is referred to as standard risk aversion, the concept introduced by Kimball (1993).[10]

The CRRA utility function exhibits decreasing and convex absolute risk aversion and decreasing absolute prudence and is therefore risk vulnerable. Thus, the presence of an independent background risk should increase the aversion to financial risk of the agent with CRRA utility. To see this, suppose that at time $t$ the agent faces the background risk and a lottery with an uncertain payoff $Z_{i,t}$. In the presence of background risk, for any distribution functions $F_Z$ and $G_\Phi$,

$$E^{Z,\Phi}[u(C_{i,t}+Z_{i,t}+\Phi_{i,t})] = E^{\Phi}[u(C_{i,t}+E[Z_{i,t}]+\Phi_{i,t}-\Pi_{i,t})], \tag{26}$$

where $\Pi_{i,t}$ is the uncertain payoff's risk premium the agent is ready to pay in order to escape the market risk.

Assume further that the lottery is actuarially neutral (i.e., $E[Z_{i,t}] = 0$) and the risk associated with the lottery is "small". Using Taylor series approximations of the both sides of equation (26) around $C_{i,t} + \Phi_{i,t}$, under the assumption that the background risk and the market risk are independent (and therefore $\Phi_{i,t}$ is independent of $Z_{i,t}$)[11] we obtain

$$\Pi_{i,t} \approx \frac{1}{2}\eta_{i,t}^2\left(-\frac{E^{\Phi}[u''(C_{i,t}+\Phi_{i,t})]}{E^{\Phi}[u'(C_{i,t}+\Phi_{i,t})]}\right), \tag{27}$$

where $\eta_{i,t}^2$ is the variance of $Z_{i,t}$ and the term in parentheses is the coefficient of absolute risk aversion.

In the presence of an independent non-hedgeable background risk, the RRA coefficient is then

$$\gamma_{i,t}^* = -\frac{E^{\Phi}[u''(C_{i,t}+\Phi_{i,t})]}{E^{\Phi}[u'(C_{i,t}+\Phi_{i,t})]}C_{i,t}. \tag{28}$$

When studying the properties of the RRA coefficient under background risk, let us relax for the moment the assumption that the background risk has a zero mean value and consider a more general setting in which $E[\Phi_{i,t}] \leqslant 0$, as in Gollier and Pratt (1990), for example.

[9] Under the assumption that the agent is risk averse (i.e., $u'(\cdot) > 0$ and $u''(\cdot) < 0$), the conditions $u'''(\cdot) > 0$ and $u''''(\cdot) < 0$ are necessary (but not sufficient) for respectively decreasing and convex absolute risk aversion. The condition $u'''(\cdot) > 0$ is also the necessary and sufficient condition for positive absolute prudence and the condition $u''''(\cdot) < 0$ is necessary (but not sufficient) for decreasing absolute prudence.

[10] Kimball (1993) shows that standard risk aversion implies risk vulnerability.

[11] In Section 2.1, we assumed that the non-hedgeable consumption $\Phi_{i,t}$ is independent of both the hedgeable consumption $C_{i,t}$ and the risky asset payoff $Z_{i,t}$.

Kimball (1992) defines the temperance premium $\Lambda_{i,t}$ by the following condition:

$$E^{\Phi}\left[u''\left(C_{i,t}+\Phi_{i,t}\right)\right]=u''\left(C_{i,t}+E\left[\Phi_{i,t}\right]-\Lambda_{i,t}\right). \tag{29}$$

By analogy with the risk premium,

$$\Lambda_{i,t}\approx\frac{1}{2}\sigma_{i,t}^{2}\left(-\frac{u''''\left(C_{i,t}+E\left[\Phi_{i,t}\right]\right)}{u'''\left(C_{i,t}+E\left[\Phi_{i,t}\right]\right)}\right). \tag{30}$$

The conditions $u''''(\cdot)<0$ and $u'''(\cdot)>0$ imply the positiveness of $\Lambda_{i,t}$.

Combining equation (28) with conditions (29) and (8), we obtain

$$\gamma_{i,t}^{*}=-\frac{u''\left(C_{i,t}+E\left[\Phi_{i,t}\right]-\Lambda_{i,t}\right)}{u'\left(C_{i,t}+E\left[\Phi_{i,t}\right]-\Psi_{i,t}\right)}C_{i,t}. \tag{31}$$

When utility has the power form (13),

$$\Lambda_{i,t}\approx\frac{1}{2}\sigma_{i,t}^{2}\frac{\gamma+2}{C_{i,t}+E\left[\Phi_{i,t}\right]}, \tag{32}$$

$$u'\left(C_{i,t}+E\left[\Phi_{i,t}\right]-\Psi_{i,t}\right)=\left(C_{i,t}+E\left[\Phi_{i,t}\right]-\frac{1}{2}\sigma_{i,t}^{2}\frac{\gamma+1}{C_{i,t}+E\left[\Phi_{i,t}\right]}\right)^{-\gamma}, \tag{33}$$

$$u''\left(C_{i,t}+E\left[\Phi_{i,t}\right]-\Lambda_{i,t}\right)=-\gamma\left(C_{i,t}+E\left[\Phi_{i,t}\right]-\frac{1}{2}\sigma_{i,t}^{2}\frac{\gamma+2}{C_{i,t}+E\left[\Phi_{i,t}\right]}\right)^{-\gamma-1}, \tag{34}$$

and therefore the RRA coefficient is

$$\gamma_{i,t}^{*}=\gamma\frac{C_{i,t}}{C_{i,t}+E\left[\Phi_{i,t}\right]}\left(1-\frac{\gamma+2}{2}\frac{\sigma_{i,t}^{2}}{\left(C_{i,t}+E\left[\Phi_{i,t}\right]\right)^{2}}\right)^{-\gamma-1}\left(1-\frac{\gamma+1}{2}\frac{\sigma_{i,t}^{2}}{\left(C_{i,t}+E\left[\Phi_{i,t}\right]\right)^{2}}\right)^{\gamma}. \tag{35}$$

It can be seen from formula (35) that when the agent bears an independent background risk on his consumption, the RRA coefficient is a function of not only the utility curvature parameter $\gamma$, but also the agent's hedgeable consumption, as well as the first two unconditional moments of the distribution of the non-hedgeable consumption. Therefore, in the model with background risk, the measure of RRA may vary over time even if the agent is assumed to have time-invariant utility ($\gamma$ is constant over time). This relates our approach to the strand of the literature, which argues that the attitudes towards portfolio risk are not fixed, but rather contingent upon the state of the world. The intuition here is that an agent adjusts his aversion to financial risk given the problem that he faces. Within this approach, the RRA coefficient is usually restricted to be a function of some factors deemed to be relevant in explaining the investor's attitudes towards risk.[12] However, the choice of such factors and the particular functional form relating the coefficient of RRA to proxies for the state of the world are generally somewhat arbitrary. The attractive feature of our

[12] Bakshi and Chen (1996) and Gordon and St-Amour (2004), for example, suppose that the RRA coefficient is a decreasing function of the individual's wealth.

model is that the set of factors and the form of the relationship between the RRA coefficient and these factors obtain endogenously from the theoretical restrictions implied by a structural model.

If there is no background risk, then the RRA coefficient coincides with the utility $u(\cdot)$ curvature parameter ($\gamma^*_{i,t} = \gamma$) and is therefore constant over time. Because of $E[\Phi_{i,t}] \leqslant 0$ (implying $C_{i,t} + E[\Phi_{i,t}] \leqslant C_{i,t}$) and

$$\frac{\gamma+1}{2} \frac{\sigma^2_{i,t}}{(C_{i,t} + E[\Phi_{i,t}])^2} < \frac{\gamma+2}{2} \frac{\sigma^2_{i,t}}{(C_{i,t} + E[\Phi_{i,t}])^2} < 1, \tag{36}$$

the presence of an independent non-hedgeable background risk raises the agent's aversion to financial risk compared with the no background risk case, so that the RRA coefficient becomes greater than the utility curvature parameter ($\gamma^*_{i,t} > \gamma$). This result is in the line with the finding in the beginning of this section that the presence of an independent background risk increases the aversion of a decision maker with CRRA utility to financial risk.

The values of the RRA coefficient $\gamma^*_{i,t}$ for different levels of the agent's hedgeable consumption $C_{i,t}$, the first two unconditional moments of the distribution of the non-hedgeable consumption, $E[\Phi_{i,t}]$ and $\sigma^2_{i,t}$, and the utility curvature parameter $\gamma$ are shown in Figure 1. Any risk with a non-positive mean can be decomposed into a sure reduction in consumption and a pure (zero-mean) risk. Panel A of Figure 1 shows the values of the RRA coefficient in the case of a sure reduction in consumption ($\sigma^2_{i,t} = 0$, $E[\Phi_{i,t}] < 0$) for $C_{i,t}$ normalized to one for simplicity. The values of the RRA coefficient in the presence of a pure independent non-hedgeable background risk ($E[\Phi_{i,t}] = 0$, $\sigma^2_{i,t} > 0$, and $C_{i,t} = 1$) are plotted in Panel B. Panels C and D illustrate the influence of $C_{i,t}$ on the RRA coefficient for $\sigma^2_{i,t} = 0.1$ and $E[\Phi_{i,t}]$ set to -0.1 and 0, respectively. For each case, the values of the RRA coefficient $\gamma^*_{i,t}$ are plotted for three different values of the curvature parameter in utility: $\gamma = 1$, $\gamma = 2$, and $\gamma = 5$.

As predicted by economic theory, an independent non-hedgeable background risk with a non-positive mean raises the agent's aversion to financial risk. It is seen in Figure 1 that risk aversion of an agent with CRRA utility is an increasing and convex function of the sure reduction in consumption and the variance of the non-hedgeable consumption. An increase in the hedgeable consumption $C_{i,t}$ lowers the agent's aversion to financial risk, so that the RRA coefficient approaches its value in the no background risk case (approaches the utility curvature parameter $\gamma$) as both the expected value and variance of the non-hedgeable consumption become smaller relative to $C_{i,t}$. Thus, in the presence of an independent non-hedgeable background risk with a non-positive mean, the agent's behavior towards financial risk exhibits decreasing RRA, whereas RRA is constant at different levels of consumption in the no background risk environment. Finally, not surprisingly, the greater the agent's aversion to financial risk in the absence of background risk (i.e., the more concave the utility function), the more vulnerable the agent's attitudes towards financial risk to an independent non-hedgeable background risk.

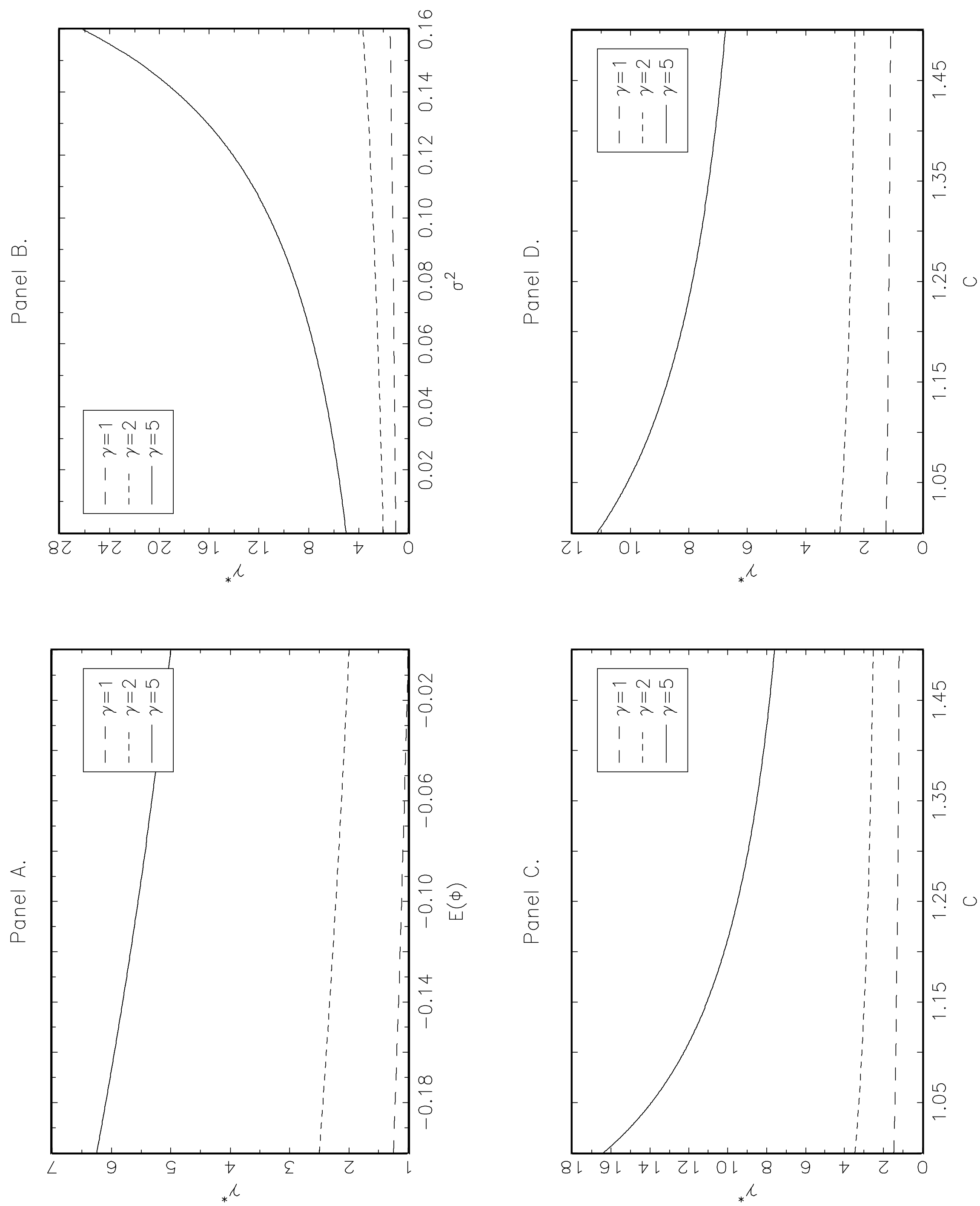


Figure 1: The coefficient of RRA in the case of a sure reduction in consumption ($\sigma^2_{i,t} = 0$, $E[\Phi_{i,t}] < 0$, and $C_{i,t} = 1$) (Panel A) and in the case of a pure independent non-hedgeable background risk ($E[\Phi_{i,t}] = 0$, $\sigma^2_{i,t} > 0$, and $C_{i,t} = 1$) (Panel B). The effect of the hedgeable consumption $C_{i,t}$ on the RRA coefficient for $\sigma^2_{i,t} = 0.1$ and $E[\Phi_{i,t}]$ set to -0.1 (Panel C) and 0 (Panel D).

As we emphasized above, in the presence of an independent non-hedgeable background risk with a non-positive mean, the RRA coefficient $\gamma_{i,t}^{*}$ differs from the utility curvature parameter $\gamma$. Campbell and Cochrane (1999) also disentangle attitudes towards financial risk from the utility curvature parameter. They assume that an agent's utility is a power function of the difference between the agent's total consumption and subsistence or habit requirements. When habit is external, the agent's aversion to financial risk in the Campbell and Cochrane (1999) model is given by the ratio of the utility curvature parameter $\gamma$ to the surplus consumption ratio, which is defined as the fraction of total consumption that is surplus to habit. With this model, one can get a time-varying RRA coefficient as consumption rises or declines toward habit. However, Campbell and Cochrane (1999) need consumption to always be above habit requirements for marginal utility to be well-defined. This might be a problem in microeconomic models with exogenous consumption.[13] Another drawback of the Campbell and Cochrane (1999) difference model is that the RRA coefficient goes to infinity when consumption is close to habit even if $\gamma$ is low. As a result, the model can produce unrealistically volatile estimates of RRA.

The results presented in Figure 1 suggest that, in contrast with the Campbell and Cochrane (1999) model, the estimate of RRA in the model with background risk is unlike to be highly volatile over time unless the utility curvature parameter $\gamma$ is quite high. Although the Campbell and Cochrane (1999) habit formation model and the consumption CAPM with background risk are based on different assumptions, in the both models the agent's attitudes towards financial risk are time varying, whereas RRA is constant over time in the standard consumption CAPM.

Risk vulnerability implies that in the presence of an independent non-hedgeable background risk with a non-positive mean, the risk premium $\Pi_{i,t}$ can be decomposed into two components. The first component is the risk premium due to the curvature of the the agent's utility function $u(\cdot)$. The background risk makes the agent more risk averse to the financial investment risk and hence raises the risk premium that the agent is willing to pay for the elimination of the market risk. The increase in the risk premium due to the presence of background risk may be seen as the second component of the total risk premium.

Denote as $\widetilde{\Pi}_t$ the risk premium we would observe if the agent's RRA were $\gamma$. The proportion of the risk premium due to the background risk in the total risk premium $\Pi_{i,t}$ is then

$$\frac{\Pi_{i,t} - \widetilde{\Pi}_{i,t}}{\Pi_{i,t}} \approx \frac{\gamma_{i,t}^{*} - \gamma}{\gamma_{i,t}^{*}}. \tag{37}$$

Since the RRA increases in the sure reduction in consumption and the variance of the non-hedgeable consumption, the lower the expected value of the non-hedgeable consumption and/or the greater the variance of the non-hedgeable consumption, the larger the RRA coefficient relative to the utility curvature parameter and hence, as follows from equation (37), the greater the

[13] See Campbell *et al.* (1997).

proportion of the risk premium attributed to the background risk (the smaller the fraction of the risk premium that may be explained by the curvature of the agent's utility function).

Property (24) could induce us to think that the RRA coefficient of the agent with utility $u(\cdot)$ in the presence of background risk, $\gamma^*_{i,t}$, equals the coefficient of RRA $\beta$ of the agent with the indirect utility function $g(\cdot)$ when there is no background risk, i.e.,

$$-\frac{g''(C_{i,t})}{g'(C_{i,t})}C_{i,t} = -\frac{E^{\Phi}\left[u''(C_{i,t}+\Phi_{i,t})\right]}{E^{\Phi}\left[u'(C_{i,t}+\Phi_{i,t})\right]}C_{i,t}. \tag{38}$$

It is not true. To see this, consider, for example, a risk-averse agent with utility $u(\cdot)$ given by (13) and utility $g(\cdot)$ given by

$$g = \frac{C_{i,t}^{1-\beta}-1}{1-\beta}. \tag{39}$$

Set $C_{i,t}$ to 100 and suppose that the individual faces the zero-mean independent background risk $\Phi_{i,t} = (-20, +20; 1/2, 1/2)$. Assume further that the utility $g(\cdot)$ curvature parameter $\beta$ equals 4. From (23)[14] and (28), we obtain that the corresponding value of $\gamma^*_{i,t}$ is 4.73, which is greater than the value of $\beta$. This example shows that the value of the RRA coefficient $\gamma^*_{i,t}$ may substantially differ from the indirect utility curvature parameter $\beta$. It may be shown that the greater the variance and lower the expected value of $\Phi_{i,t}$, the greater the difference between the values of the two parameters.

An important determinant of the investor's consumption and savings decisions is the EIS in planned consumption, which measures the sensitivity of changes in the investor's expected consumption growth rate between two periods to changes in interest rates. The EIS can be computed as the derivative of planned log consumption growth with respect to the log return on the risk-free asset.

Using property (24), we can write the consumption CAPM with background risk (3) in terms of the indirect utility function $g(\cdot)$ as

$$E_t\left[\delta\frac{g'(C_{i,t+1})}{g'(C_{i,t})}R_{j,t+1}\right] = 1. \tag{40}$$

For the agent with CRRA utility (39), we can write condition (40) as

$$E_t\left[\delta\left(\frac{C_{i,t+1}}{C_{i,t}}\right)^{-\beta}R_{j,t+1}\right] = 1. \tag{41}$$

This is the standard consumption CAPM. In this model, the EIS, which we write as $\psi$, is the reciprocal of the utility $g(\cdot)$ curvature parameter $\beta$, $\psi = 1/\beta$. As follows from equations (3) and (40), the agent with utility $u(\cdot)$ under background risk has the same hedgeable consumption plan

[14] We need this equality to find the utility $u(\cdot)$ curvature parameter $\gamma$ that corresponds to the assumed value of $\beta$.

as the agent with utility $g(\cdot)$ who faces no background risk. This suggests that the willingness of the investor with utility $u(\cdot)$ to move consumption between time periods in response to changes in the interest rate under background risk is identical to that of the investor with utility $g(\cdot)$ in the absence of background risk, implying that the EIS in planned consumption in the model with an independent non-hedgeable background risk and utility $u(\cdot)$ is the same as the EIS in model (41) and equals $1/\beta$.

Since, as we showed above, the RRA coefficient $\gamma_{i,t}^{*}$ may substantially differ from the indirect utility curvature parameter $\beta$, this implies that, when the background risk in consumption is taken into account, we break the link between risk aversion and intertemporal substitution, the two concepts that are related so tightly in the standard consumption CAPM. The CRRA utility function exhibits risk vulnerability and hence, in the presence of background risk, utility $u(\cdot)$ is less concave than the indirect utility function $g(\cdot)$, $\gamma < \beta$. Therefore, under background risk, we may get low elasticity even if the agent's the utility function $u(\cdot)$ is not very concave, whereas in the conventional consumption CAPM we need a large value of the utility curvature parameter $\gamma$ to get a low value of the EIS.

The results in this section show that, in contrast with the no background risk environment, under the assumption of background risk we are able to disentangle the utility curvature parameter $\gamma$, aversion to risk (presented by the parameter $\gamma_{i,t}^{*}$), and the EIS ($\psi = 1/\beta$), the concepts that are tightly linked in the standard consumption CAPM. In the presence of background risk, the agent's aversion to financial risk $\gamma_{i,t}^{*}$ diverges from the utility curvature parameter $\gamma$ and is no longer constant. It may change over time with the state of the economy even if the agent is assumed to have time-invariant preferences. Importantly, the model with background risk enables to get a low estimate of the EIS even when the agent's utility function is not very concave. Finally, under background risk, the EIS is no longer necessarily the reciprocal of the RRA coefficient, as it is the case in the consumption CAPM with no background risk.

## 2.4 The equity premium and risk-free rate puzzles

In Section 2.1, we assumed that uncertainty about the return on risky asset holdings is traded in a complete market, whereas the market for the background risk is incomplete. In model (41), there is no background risk. In the absence of an independent non-hedgeable background risk, agents can use financial markets to fully insure their consumption and hence are able to equalize, state-by-state, their intertemporal marginal rates of substitution. As a consequence, in equation (41) the aggregate per capita consumption growth rate can be used in place of the consumption growth rate of any particular agent, so that we get

$$E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\beta} R_{j,t+1}\right] = 1, \tag{42}$$

where $C_t$ is aggregate consumption per capita at time $t$. This is the standard representative-agent consumption CAPM.

It has been shown empirically that, at economically realistic values of the utility curvature parameter $\beta$ and the subjective time discount factor $\delta$, the standard representative-agent consumption CAPM (42) substantially underestimates the average equity premium (the equity premium puzzle) and overestimates the average risk-free return (the risk-free rate puzzle). Model (42) is able to fit the observed mean excess return on the market portfolio over the risk-free rate and the observed mean risk-free rate of return only if the utility function of the typical investor is implausibly concave (the agent is too averse to risk)[15] and the subjective time discount factor is greater than one (the agent has a negative rate of time preference). In this section, we examine whether the consumption CAPM with background risk can help solve these asset pricing anomalies.

The intuition is as follows. Leland (1968), Sandmo (1970), and Drèze and Modigliani (1972), for example, argue that if the agent's absolute risk aversion is decreasing (i.e., the third derivative of the utility function is positive), then the presence of a non-hedgeable background risk leads the agent to save more in order to insure his future consumption against the additional variability caused by the non-hedgeable background risk.[16] The precautionary saving induced by incompleteness of the market for the background risk drives down the equilibrium rate of return on both the risky and risk-free assets. If the agent's preferences exhibit risk vulnerability, then the nonavailability of insurance against an additional independent non-hedgeable background risk makes the agent less willing to bear the financial risk and the equilibrium risky asset expected premium rises relative to the no background risk case. Since CRRA utility exhibits decreasing and convex absolute risk aversion and decreasing absolute prudence and is therefore risk vulnerable, the intuition above suggests that the model, in which the agent with CRRA preferences faces in addition to aggregate dividend risk an independent non-hedgeable background risk, should predict a smaller bond return and a larger equity premium than would the standard representative-agent consumption CAPM with no background risk.

In Section 2.3, it was shown that the hedgeable consumption plan of the agent with utility $u(\cdot)$ under background risk is identical to the consumption plan the same agent would choose if he has utility $g(\cdot)$ in the no background risk environment. Since the aggregate per capita consumption growth rate can be used in place of the consumption growth rate of any agent with the utility function $g(\cdot)$ in the absence of background risk, the same may therefore be done in the consumption CAPM with background risk.

[15] In model (42), the RRA coefficient coincides with the utility curvature parameter and hence a high concavity of the utility function implies a high value of the coefficient of RRA.

[16] Courbage and Rey (2007) stress that a positive third derivative of the utility function is still a necessary and sufficient condition for a positive precautionary saving motive when a non-financial background risk and the financial market risk are independent. They show that the set of sufficient conditions is more complex otherwise.

Thus, we can write the consumption CAPM with background risk (20) as

$$E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} exp\left(\frac{\gamma(\gamma+1)}{2}\triangle\varsigma^2_{i,t+1}\right)R_{j,t+1}\right]=1. \tag{43}$$

Hence, in the presence of a pure background risk, the pricing kernel in the consumption CAPM is no longer a function of aggregate consumption per capita alone (as in the standard consumption CAPM), but is also a function of the change in the variance of the non-hedgeable consumption.

Recall that under the assumption that $E[\Phi_{i,t}]=0$, the RRA coefficient is

$$\gamma^*_{i,t}=\gamma\left(1-\frac{\gamma+2}{2}\varsigma^2_{i,t}\right)^{-\gamma-1}\left(1-\frac{\gamma+1}{2}\varsigma^2_{i,t}\right)^{\gamma}\approx\gamma\cdot exp\left((\gamma+1)\varsigma^2_{i,t}\right). \tag{44}$$

If there is no background risk ($\sigma^2_{i,t}=0$ and hence $\varsigma^2_{i,t}=0$ for all $t$), this model reduces to the standard representative-agent consumption CAPM (42). In this case, $\gamma=\beta$, $\gamma^*_{i,t}=\gamma=\beta$, and $\psi=1/\beta=1/\gamma$. Another special case is when there is background risk and $\varsigma^2_{i,t+1}=\varsigma^2_{i,t}$ for all $t$, implying $\triangle\varsigma^2_{i,t+1}=0$ for all $t$. If this is the case, the model with background risk also coincides with the representative-agent model (42), but, although $\gamma=\beta$ and $\psi=1/\beta=1/\gamma$ (as for the standard representative-agent model), because of $\sigma^2_{i,t}\neq 0$ and hence $\varsigma^2_{i,t}\neq 0$, the coefficient of RRA $\gamma^*_{i,t}$ is greater than the RRA coefficient in the no background risk environment, $\gamma^*_{i,t}>\gamma$, as expected given the presence of background risk.

To see whether the asset-pricing model (43) helps solve the equity premium and risk-free rate puzzles, assume joint conditional lognormality and homoskedasticity of $exp(\gamma(\gamma+1)\triangle\varsigma^2_{i,t+1}/2)$, $C_{t+1}/C_t$, and $R_{j,t+1}$. By taking logs of (43), we then obtain for the log risk premium on asset $j$ over the risk-free interest rate:

$$E_t[r_{j,t+1}]-r_{F,t+1}=-\frac{\sigma^2_j}{2}+\gamma\sigma_{jc}-\frac{\gamma(\gamma+1)\sigma_{j\varsigma}}{2}, \tag{45}$$

where $r_{j,t+1}=ln(R_{j,t+1})$, $r_{F,t+1}=ln(R_{F,t+1})$, $\triangle c_{t+1}=ln(C_{t+1}/C_t)$, $\sigma^2_j=var(r_{j,t+1})$, $\sigma_{jc}=cov(r_{j,t+1},\triangle c_{t+1})$, $\sigma_{j\varsigma}=cov(r_{j,t+1},\triangle\varsigma^2_{i,t+1})$. The first two terms on the right-hand side of this equation are familiar from the standard representative-agent consumption CAPM, while the last term is new. If $\varsigma^2_{i,t+1}=\varsigma^2_{i,t}$ for all $t$ and hence $\sigma_{j\varsigma}=0$, then equation (45) reduces to the risk premium formula in the no background risk framework and therefore $\gamma$ will coincide with $\beta$, as we showed above.

Recall that $\varsigma^2_{i,t}=\sigma^2_{i,t}/C^2_{i,t}$. Since the background risk is independent of the risk associated with the investment in a risky asset, $\sigma^2_{i,t}$ is not correlated with $r_{j,t+1}$. However, it is observed that consumption growth is positively correlated with the market return, implying that $\triangle\varsigma^2_{i,t+1}$ and $r_{j,t+1}$ should be negatively correlated ($\sigma_{j\varsigma}<0$). From this, it follows that, in the presence of an independent non-hedgeable background risk, the last term in (45), $-\gamma(\gamma+1)\sigma_{j\varsigma}/2$, is positive and hence the model with background risk should yield the expected equity premium, which is larger than the expected equity premium generated by model (42) at the same value of the utility curvature parameter. Alternatively, the consumption CAPM with background risk has the potential to explain the observed equity premium with a lower value of the utility curvature parameter compared with the standard representative-agent consumption CAPM (i.e., $\gamma < \beta$).[17] This shows the potential of the model with background risk to solve the equity premium puzzle.

To see how the introducing of an independent non-hedgeable background risk in the otherwise standard consumption CAPM affects the risk-free rate implied by the model, note that joint conditional lognormality and homoskedasticity imply the following restriction on the risk-free interest rate:

$$r_{F,t+1} = -log\left(\delta\right) + \gamma E_t\left[\triangle c_{t+1}\right] - \frac{\gamma^2\sigma_c^2}{2} - \frac{\gamma\left(\gamma+1\right)}{2}\left(E_t\left[\triangle\varsigma_{i,t+1}^2\right] + \frac{\gamma\left(\gamma+1\right)\sigma_\varsigma^2}{4} - \gamma\sigma_{c\varsigma}\right), \quad (46)$$

where $\sigma_c^2 = var\left(\triangle c_{t+1}\right)$, $\sigma_\varsigma^2 = var(\triangle\varsigma_{i,t+1}^2)$, and $\sigma_{c\varsigma} = cov(\triangle c_{t+1}, \triangle\varsigma_{i,t+1}^2)$.

This equation specifies that, in the presence of background risk, the risk-free rate equals its value implied by the standard consumption CAPM, less the third term, which is new. This term reflects the positive precautionary motive. As we argued above, in the presence of an independent non-hedgeable background risk consumers save more in order to insure their future consumption against the additional variability caused by the non-hedgeable background risk. This decreases the equilibrium risk-free rate. In the context of model (43), this requires the term $(E_t[\triangle\varsigma_{i,t+1}^2] + \gamma\left(\gamma+1\right)\sigma_\varsigma^2/4 - \gamma\sigma_{c\varsigma})$ to be positive. Because, as stated above, $\triangle\varsigma_{i,t+1}^2$ is expected to be negatively correlated with $r_{j,t+1}$ and $\triangle c_{t+1}$ is positively correlated with $r_{j,t+1}$, we may expect negative correlation between $\triangle\varsigma_{i,t+1}^2$ and $\triangle c_{t+1}$, $\sigma_{c\varsigma} < 0$. In the presence of background risk, the term $\gamma\left(\gamma+1\right)\sigma_\varsigma^2/4$ is always positive. These results provide some evidence of the plausibility of the expectation that the term $(E_t[\triangle\varsigma_{i,t+1}^2] + \gamma\left(\gamma+1\right)\sigma_\varsigma^2/4 - \gamma\sigma_{c\varsigma})$ is positive. If so, as expected, the model with background risk generates the risk-free rate that is lower than in the case of the standard representative-agent consumption CAPM. This demonstrates the potential of model (43) to solve the risk-free rate puzzle.

# 3 Tests of alternative pricing kernels

In this section, we perform an empirical test of the consumption CAPM with background risk and examine the ability of this model to explain the observed excess returns on risky assets and risk-free rate with plausible values of the utility curvature parameter, risk aversion, and intertemporal elasticity of substitution. The results for the model with background risk are compared with the results for the standard representative-agent model and the consumption CAPM with the pricing kernel proposed in BCG (2002).

[17] This finding is in the line with the result in Section 2.3 that risk vulnerability implies that utility $u\left(\cdot\right)$ is less concave than utility $g\left(\cdot\right)$.

### 3.1 The empirical SDF in the model with background risk

In Section 2.2, we showed that the SDF in the consumption CAPM with background risk is

$$M_{t+1} = \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} exp \left( \frac{\gamma (\gamma + 1)}{2} \triangle \varsigma_{i,t+1}^2 \right). \tag{47}$$

Because the values of the normalized variance of the non-hedgeable consumption at $t$ and $t+1$, $\varsigma_{i,t}^2$ and $\varsigma_{i,t+1}^2$, are unobservable, we have to estimate these values in order to make the model with the SDF, given by equation (47), testable. To estimate the $\varsigma_{i,t}^2$ for all $t$, we adopt the following approach.

Recall that

$$\varsigma_{i,t}^2 = \frac{1}{S} \sum_{s=1}^{S} \left( \frac{C_{is,t}^*}{C_{i,t}} - 1 \right)^2. \tag{48}$$

As stated in Section 2.1, the agent $i$'s time $t$ total consumption in the background risk state $s$ is $C_{is,t}^* = C_{i,t} + \Phi_{is,t}$. Since in this paper the background risk is assumed to have a zero expected value, $E\left[\Phi_{i,t}\right] = 0$, we get $E[C_{i,t}^*] = \frac{1}{S} \sum_{s=1}^{S} C_{is,t}^* = C_{i,t}$ and therefore

$$\varsigma_{i,t}^2 = \frac{1}{S} \sum_{s=1}^{S} \left( \frac{C_{is,t}^*}{\frac{1}{S} \sum_{s=1}^{S} C_{is,t}^*} - 1 \right)^2. \tag{49}$$

Denote as $\lambda_{iss',t}$ the agent $i$'s total consumption growth rate between $t-1$ and $t$, $\lambda_{iss',t} = C_{is,t}^* / C_{is',t-1}^*$. This implies

$$\varsigma_{i,t}^2 = \frac{1}{S} \sum_{s=1}^{S} \left( \frac{C_{is',t-1}^* \lambda_{iss',t}}{\frac{1}{S} \sum_{s=1}^{S} C_{is',t-1}^* \lambda_{iss',t}} - 1 \right)^2 = \frac{1}{S} \sum_{s=1}^{S} \left( \frac{\lambda_{iss',t}}{\frac{1}{S} \sum_{s=1}^{S} \lambda_{iss',t}} - 1 \right)^2. \tag{50}$$

For seek of simplicity, let us assume that all the agents in the economy are subject to the same independent non-hedgeable background risk, so that the distribution of the non-hedgeable consumption is identical across all consumers and hence $\varsigma_{i,t}^2 = \varsigma_{i't}^2 = \varsigma_t^2$ for any two agents $i$ and $i'$. Thus, we can omit the subscript $i$ and rewrite the above equality as

$$\varsigma_t^2 = \frac{1}{S} \sum_{s=1}^{S} \left( \frac{\lambda_{ss',t}}{\frac{1}{S} \sum_{s=1}^{S} \lambda_{ss',t}} - 1 \right)^2. \tag{51}$$

In Section 2.1, we made the assumption of market completeness for uncertainty about the return on the risky asset and market incompleteness for the independent background risk. In the absence of the independent non-hedgeable background risk the agents are able to equalize, state-by-state, their intertemporal marginal rates of substitution in consumption (consumption growth rates in the case of the agents with CRRA preferences) and therefore variations in the observed consumption growth rate for individual investors may be attributed to non-hedgeable background risks the agents faced in the time period under consideration. This makes it reasonable to assume that the distribution of $\lambda_{ss',t}$ may be well-proxied by the observed distribution of the individual consumption growth rate and to estimate the $\varsigma_t^2$ for all $t$ as

$$\widehat{\varsigma}_t^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{q_{i,t}}{q_t} - 1 \right)^2, \tag{52}$$

where $q_{i,t}$ is the observed consumption growth rate for agent $i$ between $t-1$ and $t$, and $q_t$ is the time $t$ cross-sectional mean of the consumption growth rate, $q_t = \frac{1}{N} \sum_{i=1}^{N} q_{i,t}$.

Substituting the $\widehat{\varsigma}_t^2$ for the $\varsigma_{i,t}^2$ in (47) yields

$$M_{t+1} = \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} exp \left( \frac{\gamma(\gamma+1)}{2N} \sum_{i=1}^{N} \left\{ \left( \frac{q_{i,t+1}}{q_{t+1}} - 1 \right)^2 - \left( \frac{q_{i,t}}{q_t} - 1 \right)^2 \right\} \right). \tag{53}$$

## 3.2 The data

### 3.2.1 The consumption data

We use quarterly consumption data from the US Consumer Expenditure Survey (CEX), produced by the US Bureau of Labor Statistics (BLS). The CEX data available cover the period from 1980:Q1 to 2003:Q4. It is a collection of data on approximately 5000 households per quarter in the US. Each household in the sample is interviewed every three months over five consecutive quarters (the first interview is practice and is not included in the published data set). As households complete their participation, they are dropped and new households move into the sample. Thus, each quarter about 20% of the consumer units are new. The second through fifth interviews use uniform questionnaires to collect demographic and family characteristics as well as data on quarterly consumption expenditures for the previous three months made by households in the survey (demographic variables are based upon heads of households). Various income information and information on the employment of each household member is collected in the second and fifth interviews. As opposed to the Panel Study of Income Dynamics (PSID), which offers only food consumption data on an annual basis, the CEX contains highly detailed data on quarterly consumption expenditures.[18] The CEX attempts to account for an estimated 70% of total household consumption expenditures. Since the CEX is designed with the purpose of collecting consumption data, measurement error in consumption is likely to be smaller for the CEX consumption data compared with the PSID consumption data.

As suggested by Attanasio and Weber (1995), BCG (2002), and Vissing-Jorgensen (2002), we drop all consumption observations for the years 1980 and 1981 because the quality of the CEX consumption data is questionable for this period. Thus, our sample covers the period from 1982:Q1 to 2003:Q4. Following BCG (2002), in each quarter we drop households that do not

[18] Food consumption is likely to be one of the most stable consumption components. Furthermore, as is pointed out by Carroll (1994), 95% of the measured food consumption in the PSID is noise due to the absence of interview training.

report or report a zero value of consumption of food, consumption of nondurables and services, or total consumption. We also delete from the sample nonurban households, households residing in student housing, households with incomplete income responses, households that do not have a fifth interview, and households whose head is under 19 or over 75 years of age.

In the fifth (final) interview, the household is asked to report the end-of-period estimated market value of all stocks, bonds, mutual funds, and other such securities held by the consumer unit on the last day of the previous month as well as the difference in this estimated market value compared with the value of all securities held a year ago last month. Using these two values, we calculate each consumer unit's asset holdings at the beginning of a 12-month recall period in constant 2005 dollars. We consider the following sets of households based on the reported amount of asset holdings at the beginning of a 12-month recall period. The first set consists of all households regardless of the reported amount of asset holdings. To take into account the limited participation of households in the capital markets, we also consider two subsets of households defined as asset holders according to a criterion of asset holdings at the beginning of a 12-month recall period above a certain threshold. The first subset consists of all consumer units that report an estimated market value of all stocks, bonds, mutual funds, and other such securities held a year ago last month equal to or greater than \$1000 (in 2005 dollars). The second subset consists of households that report total assets equal to or exceeding \$5000 (in 2005 dollars) at the beginning of a 12-month recall period. Since the CEX reports only some limited information about asset holdings,[19] we consider consumer units that report asset holdings equal to or exceeding \$1000 and \$5000 (in 2005 dollars), rather than households that report a positive amount of total asset holdings, in order to reduce the likelihood of including households, who do not participate in the capital markets.

As is conventional in the literature, the consumption measure used in this paper is consumption of nondurables and services. For each household, we calculate quarterly consumption expenditures for all the disaggregate consumption categories offered by the CEX. Then, we deflate obtained values in 2005 dollars with the CPI's (not seasonally adjusted, urban consumers) for appropriate consumption categories.[20] Aggregating the household's quarterly consumption across these categories is made according to the National Income and Product Account (NIPA) definition of consumption of nondurables and services. The household's per capita consumption growth between two quarters $t$ and $t+1$ is defined as the ratio of the household's per capita consumption in quarters $t+1$ and $t$.[21] To mitigate measurement error in individual consumption, we subject the households to a consumption growth filter and use the conventional $z$-score method to detect outliers. Following common practice for highly skewed data sets, in each time period we

[19] See Cogley (2002) and Vissing-Jorgensen(2002), for example, for more details.

[20] The CPI series are obtained from the BLS through CITIBASE.

[21] The quarterly consumption growth between dates $t$ and $t+1$ is calculated if consumption is not equal to zero for both of the quarters (missing information is counted as zero consumption).

consider the consumption growth rates with $z$-scores greater than two in absolute value to be due to reporting or coding errors and remove them from the sample. The household's per capita consumption growth is deseasonalized using the multiplicative adjustments obtained from the per capita consumption growth rate, as explained below.

Aggregate consumption per capita within each set of households is calculated as the average per capita consumption expenditures of the households in the set. For each set of households, the per capita consumption growth is seasonally adjusted by using multiplicative adjustments obtained from the X-12 procedure.

### 3.2.2 The returns data

The nominal quarterly equally weighted and value-weighted returns on the CRSP indices and the nominal quarterly value-weighted market capitalization-based decile index returns (capital gain plus all dividends) on all stocks listed on the NYSE, AMEX, and Nasdaq are from the Center for Research in Security Prices (CRSP) of the University of Chicago. Smallest stocks are placed in portfolio 1 and the largest in portfolio 10. The nominal quarterly value-weighted returns on the ten NYSE, AMEX, and Nasdaq industry portfolios ((i) consumer nondurables, (ii) consumer durables, (iii) manufacturing, (iv) oil, gas, and coal extraction and products, (v) business equipment, (vi) telephone and television transmission, (vii) wholesale, retail, and some services (laundries, repair shops), (viii) healthcare, medical equipment, and drugs, (ix) utilities, and (x) other) are from Kenneth R. French's web page.

The nominal quarterly risk-free rate is the 3-month US Treasury Bill secondary market rate on a per annum basis obtained from the Federal Reserve Bank of St. Louis. In order to convert from the annual rate to the quarterly rate, we raise the 3-month Treasury Bill return on a per annum basis to the power of 1/4.

The real quarterly returns are calculated as the nominal quarterly returns divided by the 3-month inflation rate based on the deflator defined for consumption of nondurables and services. We calculate the equity premium as the difference between the real equity return and the real risk-free rate.

Table I reports summary statistics for the data set used in the estimation. Table II displays the estimates of the correlation of $\triangle\widehat{\varsigma}_t^2$ with the real returns on the CRSP equally weighted and value-weighted indices as well as with the value-weighted market capitalization-based decile index real returns and with the value-weighted real returns on the ten NYSE, AMEX, and Nasdaq industry portfolios.

## 3.3 The estimation results

We begin by examining the ability of each candidate SDF to explain the mean excess return on the CRSP indices (both the equally weighted and value-weighted) and the return on the risk-free asset.

Then, we use Hansen and Jagannathan's (1991) volatility bounds to investigate the potential of each of the considered pricing kernels to explain a cross-section of the excess returns on risky assets (the value-weighted market capitalization-based decile portfolios and the value-weighted industry portfolios) and the risk-free rate of return.

### 3.3.1 The model calibration results

First, we test the hypothesis of complete consumption insurance by investigating whether the SDF given by the intertemporal marginal rate of substitution of the representative agent

$$M_{t+1} = \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\beta} \tag{54}$$

is a valid pricing kernel.

Then, we relax the assumption that a complete set of markets exists and suppose that the agents are unable to insure themselves against some risks. Here, we test two SDFs. The first one is the SDF expressed in terms of the cross-sectional mean, variance, and skewness of the household consumption growth rate, as in BCG (2002):

$$M_{t+1} = \delta q_{t+1}^{-\alpha} \left\{ 1 + \frac{\alpha(\alpha+1)}{2N} \sum_{i=1}^{N} \left( \frac{q_{i,t+1}}{q_{t+1}} - 1 \right)^2 - \frac{\alpha(\alpha+1)(\alpha+2)}{6N} \sum_{i=1}^{N} \left( \frac{q_{i,t+1}}{q_{t+1}} - 1 \right)^3 \right\}, \tag{55}$$

where $\alpha$ is the CRRA utility curvature parameter.

When assessing the empirical performance of the model with background risk proposed in this paper, we test the pricing kernel as in equation (53). We restate this SDF here for convenience:

$$M_{t+1} = \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} exp \left( \frac{\gamma(\gamma+1)}{2N} \sum_{i=1}^{N} \left\{ \left( \frac{q_{i,t+1}}{q_{t+1}} - 1 \right)^2 - \left( \frac{q_{i,t}}{q_t} - 1 \right)^2 \right\} \right). \tag{56}$$

For each candidate SDF, we test the conditional Euler equations for the excess return on the CRSP index

$$E_t \left[ M_{t+1} \left( R_{M,t+1} - R_{F,t+1} \right) \right] = 0 \tag{57}$$

and the risk-free rate

$$E_t \left[ M_{t+1} R_{F,t+1} \right] = 1. \tag{58}$$

Denote the error term in the Euler equation for the excess market portfolio return as $\epsilon_{t+1}$, $\epsilon_{t+1} = M_{t+1} \left( R_{M,t+1} - R_{F,t+1} \right)$ and the error term in the Euler equation for the risk-free rate as $\varepsilon_{F,t+1}$, $\varepsilon_{F,t+1} = R_{F,t+1} M_{t+1} - 1$. Equations (57) and (58) imply that the error terms in the Euler equations are orthogonal to any variable $x_t$ in the agent's time $t$ information set and hence, at the true parameter vector,

$$E \left[ \epsilon_{t+1} x_t \right] = 0, \tag{59}$$

$$E\left[\varepsilon_{F,t+1}x_t\right]=0. \tag{60}$$

Since the true Euler equation error is contemporaneously uncorrelated with any lagged variable, we take as instruments $x_t$ a constant, the aggregate per capita consumption growth rate, the returns on the CRSP equally weighted and value-weighted indices, and the risk-free rate of return. All the variables are lagged one and two periods.[22]

To test the orthogonality condition (59), we calculate the statistic $v\left(x_t\right)$ as

$$v\left(x_t\right)=\frac{1}{T}\sum_{t=0}^{T-1}\widetilde{M}_{t+1}(R_{M,t+1}-R_{F,t+1})x_t, \tag{61}$$

where $\widetilde{M}_{t+1}=M_{t+1}/\delta$.

Denote as $\theta$ the utility curvature parameter in the SDF $M_{t+1}$.[23] For a given instrument $x_t$, we calculate the statistic $v\left(x_t\right)$ for the values of $\theta$ increasing from zero to 50 with increments of 0.0001. Under the null hypothesis that the orthogonality condition for the equity premium (59) holds, $v\left(x_t\right)$ is zero. For each value of $\theta$, we test the null hypothesis $v\left(x_t\right)=0$ against an unspecified alternative, based on the $t$-statistic. The standard error of $v\left(x_t\right)$ is calculated as the sample standard deviation of the time-series observations of the quantity $\widetilde{M}_{t+1}(R_{M,t+1}-R_{F,t+1})x_t$ divided by $\sqrt{T}$. The value of $\theta$ at which the statistic $v\left(x_t\right)$ becomes not significantly different from zero at the 5% significance level is considered as the value of the parameter $\theta$ at which the orthogonality condition (59) holds for the instrument $x_t$. Denote such a value of $\theta$ as $\widehat{\theta}\left(x_t\right)$. It is observed that $v\left(x_t\right)$ gets less significant as $\theta$ increases. It then follows that at

$$\widehat{\theta}^{opt}=\max_{\{x_t\}}\widehat{\theta}\left(x_t\right) \tag{62}$$

$v\left(x_t\right)$ is statistically zero (the orthogonality condition (59) holds) for any instrument $x_t$ in the agent's set of instrumental variables.

Denote as $x_t^{opt}$ the instrument $x_t$, for which $\widehat{\theta}\left(x_t\right)=\widehat{\theta}^{opt}$. Since at the true parameter vector the orthogonality condition (60) holds for any $x_t$, it also holds for $x_t^{opt}$ and therefore this orthogonality condition can be written as

$$E[(M_{t+1}R_{F,t+1}-1)\,x_t^{opt}]=0 \tag{63}$$

or, equivalently,

$$\delta E[\widetilde{M}_{t+1}R_{F,t+1}x_t^{opt}]=E[x_t^{opt}]. \tag{64}$$

[22] As emphasized by Hansen and Singleton (1982), time aggregation can make instruments correlated with the error term in the Euler equation. Hall (1988) argues that the use of variables lagged two periods helps to reduce this correlation as well as the effect of the mismatching of measurement time periods with planning time periods. As argued by Epstein and Zin (1991), the use as instruments variables lagged two periods require weaker assumptions on the information structure of the problem compared with the case of variables lagged one period. Ogaki (1988) demonstrates that the use of the second lag is consistent with the information structure of a monetary economy with cash-in-advance constraints.

[23] $\theta\equiv\beta$ in the SDF, given by equation (54), $\theta\equiv\alpha$ in the SDF, given by equation (55), and $\theta\equiv\gamma$ in the SDF, given by equation (56).

Because we use a time series of the return on the risk-free asset, we estimate the subjective time discount factor $\delta$ as

$$\widehat{\delta} = \frac{\sum_{t=0}^{T-1} x_t^{opt}}{\sum_{t=0}^{T-1} \widetilde{M}_{t+1}(\widehat{\theta}^{opt}) R_{F,t+1} x_t^{opt}}, \tag{65}$$

where $\widetilde{M}_{t+1}(\widehat{\theta}^{opt})$ is the value of $\widetilde{M}_{t+1}$ calculated at $\theta = \widehat{\theta}^{opt}$.

In the standard representative-agent consumption CAPM and the model with the BCG (2002) SDF, the RRA coefficient coincides with the utility curvature parameter and hence, for these two models, $\widehat{\theta}^{opt}$ is also the estimate of the coefficient of RRA. Since in the both these models the EIS is the reciprocal of the utility curvature parameter, for the both models we estimate the EIS as $1/\widehat{\theta}^{opt}$.

As we argued in Section 2.4, in the model with background risk the RRA coefficient differs from the utility curvature parameter and is given by (44). Thus, we estimate the coefficient of RRA in this model as

$$\widehat{\gamma}_t^* \approx \widehat{\gamma}^{opt} \cdot exp\left(\left(\widehat{\gamma}^{opt} + 1\right) \widehat{\varsigma}_t^2\right) = \widehat{\gamma}^{opt} \cdot exp\left(\left(\widehat{\gamma}^{opt} + 1\right) \frac{1}{N} \sum_{i=1}^{N} \left(\frac{q_{i,t}}{q_t} - 1\right)^2\right), \tag{66}$$

where $\widehat{\gamma}^{opt}$ is $\widehat{\theta}^{opt}$ obtained for the pricing kernel in equation (56) as explained above.

To calculate the value of the EIS in the model with background risk, we need to find the value of the parameter $\beta$ in the asset-pricing model with the SDF, given by equation (54), corresponding to the estimate of the utility curvature parameter $\gamma$, $\widehat{\gamma}^{opt}$, in the consumption CAPM with background risk. Since the estimate of $\gamma$ at which the model with background risk explains the observed mean excess return on the market portfolio implies a value of the subjective time discount factor $\delta$ required for the same model to explain the risk-free rate of return, we consider as the estimate of $\beta$ in the model with the pricing kernel in equation (54) the value of $\beta$ at which this model yields the same estimate of $\delta$ as the model with background risk. The EIS in the model with background risk is then calculated as $1/\beta$.

The calibration results for the CRSP value-weighted index are presented in Table III.[24] The results are displayed for the three sets of households defined in Section 3.2.1. When all households regardless of the reported amount of asset holdings are considered, the estimate of the utility curvature parameter $\beta$ in the standard representative-agent consumption CAPM is unrealistically high (49.42),[25] whereas the estimate of the subjective time discount factor is implausibly low (0.53).[26] When the limited participation of households in the capital markets is taken into account,

[24] For the CRSP equally weighted index, we obtain qualitatively similar results. These results are not reported in the paper in order to save space, but are available upon request.

[25] For this model, the RRA coefficient coincides with $\beta$.

[26] BCG (2002) also obtain low estimates of the subjective time discount factor in the standard representative-agent consumption CAPM and argue that this result is due to error in the observed per capita consumption, which severely biases downward the estimated subjective time discount factor in this model.

as expected, we obtain the estimates of the utility curvature parameter $\beta$ that are lower than the estimates obtained for the whole set of households. These estimates are 8.12 for the set of consumer units that report total assets equal to or exceeding \$1000 and 9.97 for the set of households with total assets equal to or exceeding \$5000 and hence are still too high to be recognized as economically plausible. The estimates of the subjective time discount factor for the two sets of households classified as asset holders (0.97 and 0.94, respectively) are greater and more realistic compared with the estimate of this parameter obtained for the whole set of households.

The model with the SDF expressed in terms of the cross-sectional mean, variance, and skewness of the household consumption growth rate, as proposed in BCG (2002), explains the mean excess return on the CRSP value-weighted index with the values of the agent's utility curvature parameter that are lower than the values implied by the standard representative-agent consumption CAPM, but are still unrealistically high. These estimates are 13.28 for the whole set of consumer units, 7.33 for the set of households with total assets equal to or exceeding \$1000, and 6.56 for the set of households with total assets equal to or exceeding \$5000.[27] Another drawback of this model is that it yields an estimate of the subjective time discount factor, which ranges from 0.53 (for the whole set of consumer units) to 0.63 (for the households with total assets equal to or exceeding \$5000) and is hence implausibly low.[28]

In Section 2.4, we showed that if $\triangle\varsigma^2_{t+1}$ and asset returns are negatively correlated, then the model with background risk has the potential to resolve the equity premium and risk-free rate puzzles. The negative values of the estimates of the correlation of $\triangle\widehat{\varsigma}^2_t$ with the returns on the CRSP equally weighted and value-weighted indices reported in Table II suggest that the consumption CAPM with background risk should yield the estimate of the utility curvature parameter $\gamma$ that is lower than the estimate of $\beta$ in the standard representative-agent consumption CAPM. The results reported below support this intuition. We find that, in contrast with the standard representative-agent model and the model proposed in BCG (2002), the consumption CAPM with

[27] The potential problem with the Taylor series approximation used in BCG (2002) is that, in order for the approximation to be valid, by the Cauchy–Hadamard theorem, the ratio of each individual's consumption growth rate to the cross-sectional mean of the consumption growth rate should not exceed two. Otherwise, the Taylor series does not converge and considering higher-order moments makes the approximation worse. To mitigate observation error, BCG (2002) keep in the sample an individual's consumption growth rate if it is less than five. For the reason mentioned above, the use of such a filter may lead to a non-convergence of the Taylor series expansion. In the present paper, we use a different consumption growth filter (see Section 3.2.1) that allows to mitigate this problem. The use of this filter makes lower the volatility of the cross-sectional skewness of the household consumption growth rate and, as a consequence, makes the estimate of the utility curvature parameter in the BCG (2002) pricing kernel closer to the estimate of this parameter in the standard representative-agent model. This may explain why in our empirical investigation the estimates of the utility curvature parameter in the BCG (2002) pricing kernel are slightly greater than the estimates obtained by BCG (2002).

[28] In BCG (2002), the estimated subjective time discount factor in the pricing kernel that captures the mean, variance, and skewness of the cross-sectional distribution of the household consumption growth is also very low and below the estimate of the subjective time discount factor obtained for the standard representative-agent model. The authors explain this by the fact that the observation error in household consumption is substantially higher than the observation error in per capita consumption.

background risk explains the mean excess return on the market portfolio and the risk-free rate at much lower values of the utility curvature parameter and at more realistic values of the subjective time discount factor. Under the assumption of the limited capital market participation, the estimates of the utility curvature parameter are 3.24 for the households with total assets equal to or exceeding \$1000 and 3.62 for the consumer units that report total assets equal to or exceeding \$5000. The estimates of $\delta$ for these two sets of asset holders are both within the acceptable range of values and are respectively 0.99 and 0.97.

In the consumption CAPM with background risk, the estimate of the RRA coefficient differs from the estimate of the utility curvature parameter $\gamma$ and may change over time. We calculate the coefficient of RRA in this model for each quarter over the 1982:Q3 - 2003:Q4 sample period. As expected, the estimates of RRA are greater than the estimates of the utility curvature parameter and are not highly volatile. Table III reports the average over the period from 1982:Q3 to 2003:Q4 estimate of the RRA coefficient.

The null hypothesis of serial correlation of the RRA coefficient is rejected statistically at any conventional level of significance for the entire set of households regardless of the reported amount of asset holdings. Under the limited capital market participation, both the first- and second-order autocorrelation coefficients are positive and become greater and more statistically significant as the threshold value in the definition of asset holders is raised. The autocorrelation coefficient of order one is statistically different from zero at the 10% level of significance for the set of households with total assets equal to or exceeding \$5000, while the second-order autocorrelation coefficient is always low in absolute value and not statistically different from zero at any conventional significance level.

When estimating the proportion of the risk premium due to the background risk, we use the time average estimate of the RRA coefficient under background risk as the value of $\gamma_t^*$ in (37). Empirical evidence is that, under the limited capital market participation, the background risk accounts for nearly one third of the risk premium.

Using the property that the EIS in the consumption CAPM with background risk equals the reciprocal of $\beta$, we estimate the EIS to be around 0.15 for the both sets of households classified as asset holders. This value is lower than the reciprocal of the RRA coefficient and the reciprocal of the utility curvature parameter. This is an example of the possibility for the model with background risk to yield a low estimate of the EIS when the agent's utility function is not very concave that we mentioned in Section 2.3.

### 3.3.2 Hansen and Jagannathan's volatility bounds

In this section, we test the candidate SDFs using two alternative sets of portfolios: (i) the CRSP market capitalization-based decile portfolios (we also consider two subsets of this set of risky assets, namely the small stock portfolios (deciles 1-5) and the large stock portfolios (deciles 6-10)) and (ii) the ten NYSE, AMEX, and Nasdaq industry portfolios.

For each SDF, we test the conditional Euler equations for the excess portfolio returns

$$E_t\left[M_{t+1}\mathbf{Z}_{t+1}\right]=\mathbf{0} \tag{67}$$

and the risk-free rate

$$E_t\left[M_{t+1}R_{F,t+1}\right]=1, \tag{68}$$

where $\mathbf{Z}_{t+1}$ is the $K$-vector of time $t+1$ excess returns with elements $Z_{j,t+1}=R_{j,t+1}-R_{F,t+1}$, $j=1,...,K$, and $\mathbf{0}$ is the $K$-vector of zeros.

Denote the vector of error terms in the Euler equation for the excess returns as $\boldsymbol{\epsilon}_{t+1}$, $\boldsymbol{\epsilon}_{t+1}=\mathbf{Z}_{t+1}M_{t+1}$, and the error term in the Euler equation for the risk-free rate as $\varepsilon_{F,t+1}$, $\varepsilon_{F,t+1}=R_{F,t+1}M_{t+1}-1$. Equalities (67) and (68) imply that the true Euler equation errors are contemporaneously uncorrelated with any variable $x_t$ in the agent's time $t$ information set and therefore, at the true parameter vector,

$$E\left[\boldsymbol{\epsilon}_{t+1}x_t\right]=\mathbf{0}, \tag{69}$$

$$E\left[\varepsilon_{F,t+1}x_t\right]=0. \tag{70}$$

As instruments $x_t$, we take the same variables we used in the calibration exercise described in Section 3.3.1.

A lower volatility bound for admissible SDFs $M_{t+1}^{a}(m)$, which have unconditional mean $m$ and satisfy the orthogonality condition (69), can then be calculated as

$$\sigma\left(M_{t+1}^{a}(m)\right)=\left(m^2E\left[\mathbf{Z}_{t+1}x_t\right]'\boldsymbol{\Sigma}^{-1}E\left[\mathbf{Z}_{t+1}x_t\right]\right)^{1/2}, \tag{71}$$

where $\boldsymbol{\Sigma}$ is the unconditional variance-covariance matrix of $Z_{j,t+1}x_t$.

As in the previous section, denote as $\theta$ the utility curvature parameter in the SDF $M_{t+1}$. Following Hansen and Jagannathan (1991), we first treat $m$ as an unknown parameter and look for the value of $\theta$ at which a considered SDF $M_{t+1}(m)$ satisfies the volatility bound (71), i.e.,

$$\frac{\sigma\left(M_{t+1}(m)\right)}{\sigma\left(M_{t+1}^{a}(m)\right)}>1 \tag{72}$$

or, equivalently,

$$\frac{\sigma\left(M_{t+1}(m)\right)}{\sigma\left(M_{t+1}^{a}(m)\right)}=\frac{\sigma(\widetilde{M}_{t+1}(\widetilde{m}))}{\left(\widetilde{m}^2E\left[\mathbf{Z}_{t+1}x_t\right]'\boldsymbol{\Sigma}^{-1}E\left[\mathbf{Z}_{t+1}x_t\right]\right)^{1/2}}>1, \tag{73}$$

where, as in Section 3.3.1, $\widetilde{M}_{t+1}=M_{t+1}/\delta$ and and $\widetilde{m}=T^{-1}\sum_{t=0}^{T-1}\widetilde{M}_{t+1}$.

For each instrument $x_t$, we look for the smallest value of the utility curvature parameter $\theta$, say $\widehat{\theta}(x_t)$ at which inequality (73) holds. Then, we find the optimal value of the estimate of the curvature parameter as the largest such value of $\widehat{\theta}(x_t)$:[29]

$$\widehat{\theta}^{opt}=\max_{\{x_t\}}\widehat{\theta}(x_t). \tag{74}$$

[29] This approach is somewhat similar to that in Bekaert and Liu (2004).

This value of $\widehat{\theta}(x_t)$ is optimal in the sense that at $\theta = \widehat{\theta}^{opt}$ the orthogonality condition (69) for the given set of risky assets holds for any instrument in the chosen instrument set.[30]

Using the orthogonality condition (70), we estimate the subjective time discount factor $\delta$ as

$$\widehat{\delta} = \frac{\sum_{t=0}^{T-1} x_t^{opt}}{\sum_{t=0}^{T-1} \widetilde{M}_{t+1}(\widehat{\theta}^{opt}) R_{F,t+1} x_t^{opt}}, \tag{75}$$

where, as in the previous section, $\widetilde{M}_{t+1}(\widehat{\theta}^{opt})$ refers to the value of $\widetilde{M}_{t+1}$ calculated at $\theta = \widehat{\theta}^{opt}$ and $x_t^{opt}$ refers to the instrument $x_t$ for which $\widehat{\theta}(x_t) = \widehat{\theta}^{opt}$.

The estimates of the pricing kernel parameters ($\widehat{\theta}^{opt}$ and $\widehat{\delta}$) obtained in this fashion are the values of the utility curvature parameter $\theta$ and the subjective time discount factor $\delta$ at which the conditional Euler equations (67) and (68) hold for the given set of instrumental variables and therefore the candidate SDF is consistent with the given set of the excess returns on risky assets and the risk-free rate of return.

As the value of the parameter $\beta$ that corresponds to the estimate of the utility curvature parameter $\gamma$, $\widehat{\gamma}^{opt}$, in the consumption CAPM with background risk, we consider the value of $\beta$ at which the asset-pricing model with the SDF, given by equation (54), yields the same estimate of $\delta$ as the model with background risk. The EIS in the model with background risk is then calculated as $1/\beta$. Similarly to the calibration exercise, we calculate the RRA coefficient as in equation (66) for each quarter over the period 1982:Q3 - 2003:Q4.

The estimation results are reported in Table IV. When testing the ability of each candidate pricing kernel to jointly explain the cross-section of excess returns on the CRSP market capitalization-based decile portfolios and the NYSE, AMEX, and Nasdaq industry portfolios, we find that the estimation results are quite similar to the results we obtained in Section 3.3.1. As in the calibration exercise, for any set of households the standard representative-agent consumption CAPM (see Panel A of Table IV) yields implausibly high estimates of the utility curvature parameter, whereas the estimate of the subjective time discount factor is within the range of acceptable values when the limited capital market participation is taken into account. The model with the BCG (2002) SDF (see Panel B of Table IV) again requires the agent's utility function to be too concave to explain the equity premia. The subjective time discount factor required to explain the observed risk-free rate is implausibly low.

The negative estimates of the correlation of $\triangle\widehat{\varsigma}_t^2$ with the real capitalization-based decile portfolio and industry portfolio returns suggest that, as argued in Section 2.4, the model with background risk has the potential to explain the equity premium puzzle with a lower value of the utility curvature parameter compared with the standard representative-agent model. The

[30] Because $\widehat{\theta}(x_t) \leq \widehat{\theta}^{opt}$ for any instrument $x_t$ and for any instrument the value of $\widehat{\theta}(x_t)$ is such that the pricing kernel satisfies the lower volatility bound at any $\theta \geq \widehat{\theta}(x_t)$, it also satisfies the lower volatility bound at $\theta = \widehat{\theta}^{opt}$ for any variable in the set of instruments.

results reported in Panel C of Table IV support this intuition. When no provision is made for the limited capital market participation, the estimates of the utility curvature parameter are quite lower and the estimates of the subjective time discount factor are quite greater than the estimates obtained in Section 3.3.1. Despite this, the curvature parameter is still too high to be recognized as economically plausible. Under the limited capital market participation, the estimates of the utility curvature parameter are slightly greater and the estimates of the subjective time discount factor are slightly lower than the estimates of the corresponding parameters obtained in the calibration exercise. In contrast with the standard representative-agent consumption CAPM and the model with the BCG (2002) pricing kernel, in the model with background risk the estimates of the both parameters are in the acceptable range of values.

To investigate whether the estimation results are sensitive to the size of stocks under consideration, we test the three models separately for the decile 1-5 and decile 6-10 portfolios. We find no evidence that the stock size significantly affects the estimates of the parameters.

### 3.4 The measurement error issue

A well documented potential problem with using individual-level data is the large measurement error in reported individual consumption.[31] In this section, we illustrate the implications of an additive and unbiased measurement error in consumption growth for the estimation results, in the context of the consumption CAPM with background risk.

Recall that the normalized variance of the non-hedgeable consumption $\widehat{\varsigma}_t^2$, which we use in the estimation of the model with background risk, is calculated as

$$\widehat{\varsigma}_t^2 = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{q_{i,t}}{q_t} - 1\right)^2. \tag{76}$$

Assume that the observed consumption growth rate is measured with error, so that

$$q_{i,t} = q_{i,t}^* + u_{i,t}, \tag{77}$$

where $q_{i,t}^*$ is the agent $i$'s true consumption growth rate and $u_{i,t}$ is the measurement error that has a zero mean ($E\left[u_{i,t}\right] = 0$) and is independent of $q_{i,t}^*$ and of all other variables in the Euler equation as well as any instruments used in the estimation.

Thus,

$$\widehat{\varsigma}_t^2 = E\left[\left(\frac{q_{i,t}^* + u_{i,t} - E[q_{i,t}^* + u_{i,t}]}{E[q_{i,t}^* + u_{i,t}]}\right)^2\right] = \frac{var(q_{i,t}^* + u_{i,t})}{(E[q_{i,t}^* + u_{i,t}])^2}. \tag{78}$$

Because $u_{i,t}$ and $q_{i,t}^*$ are independent,

$$var(q_{i,t}^* + u_{i,t}) = var(q_{i,t}^*) + var\left(u_{i,t}\right). \tag{79}$$

[31] See Zeldes (1989) and Runkle (1991), for example.

Rewriting $var\,(u_{i,t})$ as $var\,(u_{i,t}) = \mu_t var(q^*_{i,t})$, we obtain

$$var(q^*_{i,t} + u_{i,t}) = (1 + \mu_t)\, var(q^*_{i,t}). \tag{80}$$

Since $E\,[u_{i,t}] = 0$, we then have

$$\widehat{\varsigma}_t^2 = (1 + \mu_t)\, \frac{var(q^*_{i,t})}{(E[q^*_{i,t}])^2} = (1 + \mu_t)\, \widehat{\varsigma}_t^{*2} \tag{81}$$

and hence

$$\widehat{\varsigma}_t^{*2} = \frac{\widehat{\varsigma}_t^2}{1 + \mu_t}. \tag{82}$$

The true Euler equation for asset $j$ is

$$E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} exp\left(\frac{\gamma\,(\gamma+1)}{2} \triangle\, \widehat{\varsigma}^{*2}_{t+1}\right) R_{j,t+1}\right] = 1. \tag{83}$$

Substituting in for $\widehat{\varsigma}^{*2}_{t+1}$, this becomes

$$E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} exp\left(\frac{\gamma\,(\gamma+1)}{2\,(1+\mu)} \triangle\, \widehat{\varsigma}^{2}_{t+1}\right) R_{j,t+1}\right] = 1. \tag{84}$$

To examine the effect of the measurement error in consumption growth on the estimates of the parameters $\delta$ and $\gamma$ in (84), we, for simplicity, treat the ratio of the variance of the error to the cross-sectional variance of the true household consumption growth rate as time-invariant, i.e., $\mu_t = \mu$ for all $t$, and test the model with $\mu = 0.1$, 0.2, and 0.5. Evidence is that the measurement error of the particular form assumed here biases downward the estimates of both the utility curvature parameter $\gamma$ and the subjective time discount factor $\delta$.[32] However, the bias is small even if the error is quite large ($\mu = 0.5$).

# 4 Concluding Remarks

Consistent with earlier results, we reject empirically the model based on the assumption of complete consumption insurance (i.e., the standard representative-agent consumption CAPM). The both incomplete consumption insurance SDFs (the BCG (2002) pricing kernel and the SDF proposed in this paper) outperform the pricing kernel in the standard representative-agent consumption CAPM in explaining the observed equity premium. A problem with the SDF in BCG (2002) is that the model with this pricing kernel is able to account for the risk-free rate of return only with an unrealistically low value of the subjective time discount factor. We present empirical evidence that, in contrast with the previously proposed incomplete consumption insurance models,

[32] To save space, we do not report these results here. They are available from the author upon request.

the asset-pricing model with the SDF calculated as the discounted ratio of expectations of marginal utilities over the non-hedgeable consumption states at two consecutive dates jointly explains the observed risky asset excess returns and risk-free rate with economically plausible values of the utility curvature parameter, the EIS, and the subjective time discount factor.

This evidence is robust across different estimation techniques, different sets of stock-returns and threshold values in the definition of asset holders. This supports the hypothesis that an independent non-hedgeable zero-mean background risk can account for the market premium and the return on the risk-free asset and shows that the size of the background risk is an important asset pricing factor.

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**Table I. Summary Statistics**

| Variable | Minimum | Median | Maximum | Mean | Standard Deviation | Skewness | Excess Kurtosis |
|---|---|---|---|---|---|---|---|
| | | | 1. Consumption | | | | |
| | | | All Households | | | | |
| $C_t/C_{t-1}$ | 0.9379 | 0.9963 | 1.0702 | 0.9965 | 0.0194 | 0.6035 | 2.2370 |
| $\varsigma_t^2$ | 0.0460 | 0.0693 | 0.1380 | 0.0697 | 0.0121 | 2.0647 | 10.8590 |
| | | | Households with Total Assets $\geqslant$ \$1000 | | | | |
| $C_t/C_{t-1}$ | 0.9227 | 0.9989 | 1.1197 | 1.0037 | 0.0413 | 0.6221 | 0.4524 |
| $\varsigma_t^2$ | 0.0285 | 0.0674 | 0.1296 | 0.0694 | 0.0198 | 0.6020 | 0.4136 |
| | | | Households with Total Assets $\geqslant$ \$5000 | | | | |
| $C_t/C_{t-1}$ | 0.9124 | 1.0000 | 1.1379 | 1.0034 | 0.0458 | 0.4611 | 0.5543 |
| $\varsigma_t^2$ | 0.0313 | 0.0692 | 0.1442 | 0.0728 | 0.0231 | 0.8569 | 1.1304 |
| | | | 2. Asset Returns | | | | |
| | | | Market Portfolio Returns and the Risk-Free Rate | | | | |
| $R_{VW,t}$ | 0.7633 | 1.0346 | 1.2059 | 1.0269 | 0.0872 | -0.4703 | 0.5029 |
| $R_{EW,t}$ | 0.7033 | 1.0257 | 1.3176 | 1.0321 | 0.1161 | -0.0395 | 0.3164 |
| $R_{F,t}$ | 0.9933 | 1.0056 | 1.0178 | 1.0053 | 0.0048 | 0.2153 | 0.3662 |
| | | | Decile Portfolio Returns | | | | |
| $R_{D1,t}$ | 0.6440 | 1.0244 | 1.4993 | 1.0494 | 0.1615 | 0.5395 | 0.7105 |
| $R_{D2,t}$ | 0.6714 | 1.0127 | 1.3813 | 1.0357 | 0.1407 | 0.4936 | 0.4085 |
| $R_{D3,t}$ | 0.6494 | 1.0118 | 1.3733 | 1.0298 | 0.1276 | 0.2263 | 0.6815 |
| $R_{D4,t}$ | 0.6691 | 1.0072 | 1.3460 | 1.0281 | 0.1246 | 0.1792 | 0.5111 |
| $R_{D5,t}$ | 0.6738 | 1.0156 | 1.3228 | 1.0289 | 0.1203 | -0.0432 | 0.4332 |
| $R_{D6,t}$ | 0.7044 | 1.0229 | 1.2969 | 1.0283 | 0.1148 | -0.0857 | 0.2744 |
| $R_{D7,t}$ | 0.7023 | 1.0234 | 1.2821 | 1.0272 | 0.1098 | -0.1854 | 0.5067 |
| $R_{D8,t}$ | 0.7272 | 1.0311 | 1.2786 | 1.0284 | 0.1085 | -0.1505 | 0.3921 |
| $R_{D9,t}$ | 0.7471 | 1.0334 | 1.2893 | 1.0282 | 0.1022 | -0.1738 | 0.3954 |
| $R_{D10,t}$ | 0.7716 | 1.0354 | 1.2146 | 1.0270 | 0.0850 | -0.5105 | 0.5690 |

Note.- $C_t/C_{t-1}$ is the aggregate per capita consumption growth rate, $\varsigma_t^2$ is the normalized variance of the non-hedgeable consumption, $R_{EW,t}$ and $R_{VW,t}$ are the returns on the CRSP equally weighted and value-weighted indices, respectively, $R_{F,t}$ is the risk-free rate of return, $R_{Dj,t}$ $(j = 1, ..., 10)$ are the returns on the CRSP market capitalization-based decile portfolios, and $R_{Ij,t}$ $(j = 1, ..., 10)$ are the returns on the industry portfolios. The sample period is 1982:Q1 to 2003:Q4.

**Table I (continued)**

| Variable | Minimum | Median | Maximum | Mean | Standard Deviation | Skewness | Excess Kurtosis |
|---|---|---|---|---|---|---|---|
| | | | Industry Portfolio Returns | | | | |
| $R_{I1,t}$ | 0.7644 | 1.0434 | 1.2164 | 1.0363 | 0.0880 | -0.4554 | 0.3587 |
| $R_{I2,t}$ | 0.7199 | 1.0262 | 1.2855 | 1.0302 | 0.1152 | -0.3173 | 0.4944 |
| $R_{I3,t}$ | 0.7584 | 1.0364 | 1.2256 | 1.0297 | 0.0883 | -0.5788 | 0.9908 |
| $R_{I4,t}$ | 0.7658 | 1.0242 | 1.2436 | 1.0236 | 0.0780 | -0.6265 | 2.0133 |
| $R_{I5,t}$ | 0.6518 | 1.0381 | 1.3968 | 1.0303 | 0.1436 | -0.1897 | 0.8175 |
| $R_{I6,t}$ | 0.7652 | 1.0398 | 1.2570 | 1.0266 | 0.1031 | -0.2717 | -0.2402 |
| $R_{I7,t}$ | 0.7027 | 1.0321 | 1.3129 | 1.0338 | 0.1065 | -0.1787 | 0.9889 |
| $R_{I8,t}$ | 0.7544 | 1.0320 | 1.2409 | 1.0341 | 0.0983 | -0.2926 | 0.1397 |
| $R_{I9,t}$ | 0.8023 | 1.0352 | 1.2684 | 1.0243 | 0.0742 | 0.0048 | 1.1137 |
| $R_{I10,t}$ | 0.7606 | 1.0354 | 1.2076 | 1.0309 | 0.0958 | -0.6319 | 0.5109 |

## Table II. Correlation of $\triangle\widehat{\varsigma}_t^2$ with Portfolio Returns

| Returns | All Households | | Total Assets $\geqslant$ \$1000 | | Total Assets $\geqslant$ \$5000 | |
|---|---|---|---|---|---|---|
| | $\rho$ | $t$-stat. | $\rho$ | $t$-stat. | $\rho$ | $t$-stat. |
| | | | Market Portfolio Returns | | | |
| $R_{VW,t}$ | -0.1392 | (-1.30) | -0.1561 | (-1.46) | -0.1323 | (-1.23) |
| $R_{EW,t}$ | -0.2398 | (-2.24) | -0.1934 | (-1.80) | -0.2257 | (-2.11) |
| | | | Decile Portfolio Returns | | | |
| $R_{D1,t}$ | -0.2657 | (-2.48) | -0.2349 | (-2.19) | -0.3062 | (-2.86) |
| $R_{D2,t}$ | -0.2524 | (-2.35) | -0.1700 | (-1.59) | -0.2379 | (-2.22) |
| $R_{D3,t}$ | -0.2407 | (-2.25) | -0.1688 | (-1.57) | -0.2181 | (-2.03) |
| $R_{D4,t}$ | -0.2466 | (-2.30) | -0.1774 | (-1.65) | -0.2189 | (-2.04) |
| $R_{D5,t}$ | -0.2478 | (-2.31) | -0.1714 | (-1.60) | -0.2168 | (-2.02) |
| $R_{D6,t}$ | -0.2138 | (-1.99) | -0.1420 | (-1.32) | -0.1660 | (-1.55) |
| $R_{D7,t}$ | -0.1969 | (-1.84) | -0.1271 | (-1.19) | -0.1425 | (-1.33) |
| $R_{D8,t}$ | -0.1710 | (-1.60) | -0.1142 | (-1.07) | -0.1229 | (-1.15) |
| $R_{D9,t}$ | -0.1471 | (-1.37) | -0.1306 | (-1.22) | -0.1177 | (-1.10) |
| $R_{D10,t}$ | -0.1262 | (-1.18) | -0.1564 | (-1.46) | -0.1254 | (-1.17) |
| | | | Industry Portfolio Returns | | | |
| $R_{I1,t}$ | -0.1418 | (-1.32) | -0.1849 | (-1.72) | -0.1445 | (-1.35) |
| $R_{I2,t}$ | -0.1033 | (-0.96) | -0.1510 | (-1.41) | -0.1439 | (-1.34) |
| $R_{I3,t}$ | -0.1250 | (-1.17) | -0.1974 | (-1.84) | -0.1738 | (-1.62) |
| $R_{I4,t}$ | -0.2115 | (-1.97) | -0.1986 | (-1.85) | -0.2432 | (-2.27) |
| $R_{I5,t}$ | -0.0758 | (-0.71) | -0.0679 | (-0.63) | -0.0391 | (-0.36) |
| $R_{I6,t}$ | -0.0787 | (-0.73) | -0.0653 | (-0.61) | -0.0342 | (-0.32) |
| $R_{I7,t}$ | -0.1968 | (-1.84) | -0.2024 | (-1.89) | -0.1816 | (-1.69) |
| $R_{I8,t}$ | -0.1745 | (-1.63) | -0.1728 | (-1.61) | -0.1302 | (-1.21) |
| $R_{I9,t}$ | 0.0560 | (0.52) | -0.0368 | (-0.34) | -0.0141 | (-0.13) |
| $R_{I10,t}$ | -0.1542 | (-1.44) | -0.1724 | (-1.61) | -0.1662 | (-1.55) |

Note.- $\rho$ is the coefficient of correlation of $\triangle\widehat{\varsigma}_{t+1}^2$ with a portfolio return. $R_{EW,t}$ and $R_{VW,t}$ are the returns on the CRSP equally weighted and value-weighted indices, respectively, $R_{Dj,t}$ $(j = 1, ..., 10)$ are the returns on the CRSP market capitalization-based decile portfolios, and $R_{Ij,t}$ $(j = 1, ..., 10)$ are the returns on the industry portfolios. $t$-statistics are in parentheses. The sample period is 1982:Q1 to 2003:Q4.

## Table III. Calibration Results

| | $x_t^1$ | $x_t^2$ | $x_t^3$ | $x_t^4$ | $x_t^5$ | $x_t^6$ | $x_t^7$ | $x_t^8$ | $x_t^9$ | $x_t^{opt}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 1. All Households | | | | | |
| | | | | | The Standard Consumption CAPM | | | | | |
| $\beta$ | 47.1224 | 46.4847 | 46.5436 | 49.4213 | 43.3044 | 41.4390 | 31.4325 | 48.1532 | 47.5262 | 49.4213 |
| $\delta$ | 0.5659 | 0.5737 | 0.5728 | 0.5334 | 0.6283 | 0.6356 | 0.7617 | 0.5539 | 0.5615 | 0.5334 |
| $\psi$ | 0.0212 | 0.0215 | 0.0215 | 0.0202 | 0.0231 | 0.0241 | 0.0318 | 0.0208 | 0.0210 | 0.0202 |
| | | | | | The Model with the BCG (2002) Pricing Kernel | | | | | |
| $\alpha$ | 12.9080 | 12.8904 | 12.8583 | 13.1636 | 12.3089 | 13.2791 | 11.0447 | 12.9410 | 12.9296 | 13.2791 |
| $\delta$ | 0.5259 | 0.5255 | 0.5255 | 0.5353 | 0.5191 | 0.5284 | 0.5159 | 0.5275 | 0.5269 | 0.5284 |
| $\psi$ | 0.0775 | 0.0776 | 0.0778 | 0.0760 | 0.0812 | 0.0753 | 0.0905 | 0.0773 | 0.0773 | 0.0753 |
| | | | | | The Model with Background Risk | | | | | |
| $\gamma$ | 6.4172 | 6.3976 | 6.4039 | 6.3592 | 6.5182 | 6.2495 | 6.0080 | 6.4290 | 6.4152 | 6.5182 |
| $\delta$ | 0.8353 | 0.8373 | 0.8362 | 0.8393 | 0.8283 | 0.8555 | 0.8732 | 0.8332 | 0.8351 | 0.8283 |
| $\beta$ | 23.2292 | 23.0189 | 23.1331 | 23.1274 | 24.9795 | 21.4718 | 19.9769 | 23.4792 | 23.2876 | 24.9795 |
| $\psi$ | 0.0430 | 0.0434 | 0.0432 | 0.0432 | 0.0400 | 0.0466 | 0.0501 | 0.0426 | 0.0429 | 0.0400 |
| $\overline{\gamma}_t^*$ | 12.7163 | 12.6457 | 12.6683 | 12.5084 | 13.0859 | 12.1237 | 11.3136 | 12.7589 | 12.7090 | 13.0859 |
| | (34.77) | (35.07) | (34.97) | (35.66) | (33.25) | (37.38) | (41.34) | (34.59) | (34.80) | (33.25) |
| $\rho_1$ | 0.0766 | 0.0772 | 0.0770 | 0.0784 | 0.0735 | 0.0819 | 0.0893 | 0.0763 | 0.0767 | 0.0735 |
| | (0.70) | (0.71) | (0.71) | (0.72) | (0.67) | (0.75) | (0.82) | (0.70) | (0.70) | (0.67) |
| $\rho_2$ | -0.0432 | -0.0432 | -0.0432 | -0.0431 | -0.0434 | -0.0427 | -0.0418 | -0.0432 | -0.0432 | -0.0434 |
| | (-0.40) | (-0.40) | (-0.40) | (-0.39) | (-0.40) | (-0.39) | (-0.38) | (-0.40) | (-0.40) | (-0.40) |
| $\mu$ | 0.4954 | 0.4941 | 0.4945 | 0.4916 | 0.5019 | 0.4845 | 0.4690 | 0.4961 | 0.4952 | 0.5019 |

Note.- The column labeled $x_t^l$, $l = 1, ..., 9$, shows the results for the $l$th set of instruments. The sets of instruments are: $x_t^1$ = a constant, $x_t^2$ = the aggregate per capita consumption growth rate lagged one period, $x_t^3$ = the aggregate per capita consumption growth rate lagged two periods, $x_t^4$ = the return on the CRSP value-weighted index lagged one period, $x_t^5$ = the return on the CRSP value-weighted index lagged two periods, $x_t^6$ = the return on the CRSP equally weighted index lagged one period, $x_t^7$ = the return on the CRSP equally weighted index lagged two periods, $x_t^8$ = the risk-free rate of return lagged one period, and $x_t^9$ = the risk-free rate of return lagged two periods. The column labeled $x_t^{opt}$ reports the results for the optimal instrumental variables set. $\beta$ is the curvature parameter of the indirect utility function, $\alpha$ is the utility curvature parameter in the model with the BCG (2002) pricing kernel, $\gamma$ is the curvature parameter of the utility function in the model with background risk, $\delta$ is the subjective time discount factor, $\psi$ is the EIS, $\overline{\gamma}_t^*$ is the sample mean of the RRA coefficient $\gamma_t^*$, $\rho_k$ is the $k$th-order autocorrelation coefficient of $\gamma_t^*$, $\mu$ is the proportion of the risk premium due to the background risk. $t$-statistics are in parentheses.

## Table III (continued)

| | $x_t^1$ | $x_t^2$ | $x_t^3$ | $x_t^4$ | $x_t^5$ | $x_t^6$ | $x_t^7$ | $x_t^8$ | $x_t^9$ | $x_t^{opt}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 2. Households with Total Assets $\geqslant$ \$1000 | | | | | |
| | | | | | The Standard Consumption CAPM | | | | | |
| $\beta$ | 8.0356 | 7.9874 | 7.9939 | 8.0184 | 7.8236 | 7.2544 | 6.2907 | 8.1243 | 8.0614 | 8.1243 |
| $\delta$ | 0.9732 | 0.9737 | 0.9737 | 0.9759 | 0.9779 | 0.9808 | 0.9867 | 0.9728 | 0.9734 | 0.9728 |
| $\psi$ | 0.1244 | 0.1252 | 0.1251 | 0.1247 | 0.1278 | 0.1378 | 0.1590 | 0.1231 | 0.1240 | 0.1231 |
| | | | | | The Model with the BCG (2002) Pricing Kernel | | | | | |
| $\alpha$ | 7.0405 | 7.0108 | 7.0160 | 7.0151 | 7.3266 | 6.5466 | 6.0142 | 7.0819 | 7.0520 | 7.3266 |
| $\delta$ | 0.6078 | 0.6090 | 0.6089 | 0.6112 | 0.5988 | 0.6303 | 0.6550 | 0.6063 | 0.6076 | 0.5988 |
| $\psi$ | 0.1420 | 0.1426 | 0.1425 | 0.1425 | 0.1365 | 0.1528 | 0.1663 | 0.1412 | 0.1418 | 0.1365 |
| | | | | | The Model with Background Risk | | | | | |
| $\gamma$ | 3.2180 | 3.2081 | 3.2053 | 3.2133 | 3.1817 | 3.0780 | 2.7962 | 3.2402 | 3.2269 | 3.2402 |
| $\delta$ | 0.9858 | 0.9860 | 0.9861 | 0.9869 | 0.9883 | 0.9880 | 0.9919 | 0.9855 | 0.9858 | 0.9855 |
| $\beta$ | 6.3362 | 6.2963 | 6.2931 | 6.4711 | 6.3139 | 6.1353 | 5.3430 | 6.4197 | 6.3844 | 6.4197 |
| $\psi$ | 0.1578 | 0.1588 | 0.1589 | 0.1545 | 0.1584 | 0.1630 | 0.1872 | 0.1558 | 0.1566 | 0.1558 |
| $\overline{\gamma}_t^*$ | 4.5333 | 4.5151 | 4.5099 | 4.5246 | 4.4666 | 4.2787 | 3.7860 | 4.5743 | 4.5497 | 4.5743 |
| | (77.72) | (77.98) | (78.05) | (77.84) | (78.68) | (81.50) | (89.93) | (77.15) | (77.49) | (77.15) |
| $\rho_1$ | 0.1575 | 0.1577 | 0.1577 | 0.1576 | 0.1580 | 0.1595 | 0.1633 | 0.1572 | 0.1574 | 0.1572 |
| | (1.44) | (1.44) | (1.45) | (1.44) | (1.45) | (1.46) | (1.50) | (1.44) | (1.44) | (1.44) |
| $\rho_2$ | 0.0503 | 0.0504 | 0.0504 | 0.0504 | 0.0505 | 0.0512 | 0.0527 | 0.0502 | 0.0503 | 0.0502 |
| | (0.46) | (0.46) | (0.46) | (0.46) | 0.46 | (0.47) | 0.48 | (0.46) | (0.46) | (0.46) |
| $\mu$ | 0.2901 | 0.2895 | 0.2893 | 0.2898 | 0.2877 | 0.2806 | 0.2614 | 0.2916 | 0.2907 | 0.2916 |

## Table III (continued)

| | $x_t^1$ | $x_t^2$ | $x_t^3$ | $x_t^4$ | $x_t^5$ | $x_t^6$ | $x_t^7$ | $x_t^8$ | $x_t^9$ | $x_t^{opt}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 3. Households with Total Assets $\geqslant$ \$5000 | | | | | |
| | | | | | The Standard Consumption CAPM | | | | | |
| $\beta$ | 9.8640 | 9.8084 | 9.8330 | 9.5700 | 9.7168 | 8.6738 | 7.8590 | 9.9663 | 9.8788 | 9.9663 |
| $\delta$ | 0.9366 | 0.9374 | 0.9372 | 0.9435 | 0.9429 | 0.9548 | 0.9634 | 0.9354 | 0.9368 | 0.9354 |
| $\psi$ | 0.1014 | 0.1020 | 0.1017 | 0.1045 | 0.1029 | 0.1153 | 0.1272 | 0.1003 | 0.1012 | 0.1003 |
| | | | | | The Model with the BCG (2002) Pricing Kernel | | | | | |
| $\alpha$ | 6.3977 | 6.3751 | 6.3873 | 6.2331 | 6.5646 | 5.9483 | 5.7642 | 6.4244 | 6.3997 | 6.5646 |
| $\delta$ | 0.6371 | 0.6381 | 0.6377 | 0.6464 | 0.6320 | 0.6595 | 0.6690 | 0.6361 | 0.6373 | 0.6320 |
| $\psi$ | 0.1563 | 0.1569 | 0.1566 | 0.1604 | 0.1523 | 0.1681 | 0.1735 | 0.1557 | 0.1563 | 0.1523 |
| | | | | | The Model with Background Risk | | | | | |
| $\gamma$ | 3.5946 | 3.5855 | 3.5806 | 3.5671 | 3.5974 | 3.4286 | 3.1850 | 3.6200 | 3.6054 | 3.6200 |
| $\delta$ | 0.9735 | 0.9739 | 0.9740 | 0.9766 | 0.9763 | 0.9793 | 0.9840 | 0.9728 | 0.9733 | 0.9728 |
| $\beta$ | 6.7412 | 6.7001 | 6.6973 | 6.5867 | 6.7896 | 6.2519 | 5.6431 | 6.8379 | 6.7964 | 6.8379 |
| $\psi$ | 0.1483 | 0.1493 | 0.1493 | 0.1518 | 0.1473 | 0.1600 | 0.1772 | 0.1462 | 0.1471 | 0.1462 |
| $\overline{\gamma}_t^*$ | 5.3894 | 5.3704 | 5.3602 | 5.3322 | 5.3952 | 5.0492 | 4.5714 | 5.4426 | 5.4120 | 5.4426 |
| | (54.35) | (54.53) | (54.62) | (54.89) | (54.30) | (57.69) | (63.04) | (53.86) | (54.14) | (53.86) |
| $\rho_1$ | 0.2094 | 0.2095 | 0.2096 | 0.2098 | 0.2094 | 0.2114 | 0.2141 | 0.2091 | 0.2093 | 0.2091 |
| | (1.92) | (1.92) | (1.92) | (1.92) | (1.92) | (1.94) | (1.96) | (1.92) | (1.92) | (1.92) |
| $\rho_2$ | 0.0617 | 0.0617 | 0.0617 | 0.0617 | 0.0617 | 0.0617 | 0.0617 | 0.0616 | 0.0617 | 0.0616 |
| | (0.57) | (0.57) | (0.57) | (0.57) | (0.57) | (0.57) | (0.57) | (0.57) | (0.57) | (0.57) |
| $\mu$ | 0.3330 | 0.3324 | 0.3320 | 0.3310 | 0.3332 | 0.3210 | 0.3033 | 0.3349 | 0.3338 | 0.3349 |

## Table IV. The Conditional Volatility Bounds Results
## A. The Standard Consumption CAPM

| | $x_t^1$ | $x_t^2$ | $x_t^3$ | $x_t^4$ | $x_t^5$ | $x_t^6$ | $x_t^7$ | $x_t^8$ | $x_t^9$ | $x_t^{opt}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 1. All Households | | | | | |
| | | | | | CRSP Market Capitalization-Based Decile Portfolios 1-10 | | | | | |
| $\beta$ | 25.8347 | 25.8421 | 25.8328 | 25.9816 | 25.9612 | 25.7854 | 25.0234 | 25.8766 | 25.8570 | 25.9816 |
| $\delta$ | 0.8093 | 0.8092 | 0.8092 | 0.8104 | 0.8187 | 0.8131 | 0.8268 | 0.8092 | 0.8095 | 0.8104 |
| $\psi$ | 0.0387 | 0.0387 | 0.0387 | 0.0385 | 0.0385 | 0.0388 | 0.0400 | 0.0386 | 0.0387 | 0.0385 |
| | | | | | CRSP Market Capitalization-Based Decile Portfolios 1-5 | | | | | |
| $\beta$ | 19.2210 | 19.2184 | 19.2082 | 19.2395 | 19.1853 | 19.2024 | 18.9250 | 19.2359 | 19.2248 | 19.2395 |
| $\delta$ | 0.8728 | 0.8727 | 0.8729 | 0.8759 | 0.8812 | 0.8762 | 0.8822 | 0.8729 | 0.8731 | 0.8759 |
| $\psi$ | 0.0520 | 0.0520 | 0.0521 | 0.0520 | 0.0521 | 0.0521 | 0.0528 | 0.0520 | 0.0520 | 0.0520 |
| | | | | | CRSP Market Capitalization-Based Decile Portfolios 6-10 | | | | | |
| $\beta$ | 15.8583 | 15.8474 | 15.8190 | 16.5340 | 15.8395 | 16.0995 | 15.2646 | 15.8948 | 15.8699 | 16.5340 |
| $\delta$ | 0.9015 | 0.9015 | 0.9018 | 0.8992 | 0.9084 | 0.9026 | 0.9117 | 0.9015 | 0.9017 | 0.8992 |
| $\psi$ | 0.0631 | 0.0631 | 0.0632 | 0.0605 | 0.0631 | 0.0621 | 0.0655 | 0.0629 | 0.0630 | 0.0605 |
| | | | | | Industry Portfolios | | | | | |
| $\beta$ | 20.7009 | 20.7116 | 20.6920 | 20.6666 | 20.9547 | 20.6737 | 20.9080 | 20.7486 | 20.7316 | 20.9547 |
| $\delta$ | 0.8593 | 0.8592 | 0.8594 | 0.8628 | 0.8658 | 0.8629 | 0.8650 | 0.8592 | 0.8594 | 0.8658 |
| $\psi$ | 0.0483 | 0.0483 | 0.0483 | 0.0484 | 0.0477 | 0.0484 | 0.0478 | 0.0482 | 0.0482 | 0.0477 |

Note.- See Table III.

## Table IV (continued)

| | $x_t^1$ | $x_t^2$ | $x_t^3$ | $x_t^4$ | $x_t^5$ | $x_t^6$ | $x_t^7$ | $x_t^8$ | $x_t^9$ | $x_t^{opt}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| **2. Households with Total Assets ⩾ \$1000** | | | | | | | | | | |
| CRSP Market Capitalization-Based Decile Portfolios 1-10 | | | | | | | | | | |
| $\beta$ | 13.8945 | 13.8994 | 13.8932 | 13.9919 | 13.9783 | 13.8619 | 13.3624 | 13.9222 | 13.9092 | 13.9919 |
| $\delta$ | 0.9029 | 0.9029 | 0.9031 | 0.9061 | 0.9057 | 0.9068 | 0.9103 | 0.9030 | 0.9033 | 0.9061 |
| $\psi$ | 0.0720 | 0.0719 | 0.0720 | 0.0715 | 0.0715 | 0.0721 | 0.0748 | 0.0718 | 0.0719 | 0.0715 |
| CRSP Market Capitalization-Based Decile Portfolios 1-5 | | | | | | | | | | |
| $\beta$ | 9.8063 | 9.8048 | 9.7989 | 9.8170 | 9.7857 | 9.7956 | 9.6355 | 9.8149 | 9.8085 | 9.8170 |
| $\delta$ | 0.9561 | 0.9562 | 0.9564 | 0.9592 | 0.9597 | 0.9583 | 0.9582 | 0.9564 | 0.9565 | 0.9592 |
| $\psi$ | 0.1020 | 0.1020 | 0.1021 | 0.1019 | 0.1022 | 0.1021 | 0.1038 | 0.1019 | 0.1020 | 0.1019 |
| CRSP Market Capitalization-Based Decile Portfolios 6-10 | | | | | | | | | | |
| $\beta$ | 8.0000 | 8.0000 | 8.0000 | 8.2885 | 8.0000 | 8.0496 | 8.0000 | 8.0000 | 8.0000 | 8.2885 |
| $\delta$ | 0.9735 | 0.9736 | 0.9736 | 0.9736 | 0.9764 | 0.9747 | 0.9740 | 0.9738 | 0.9739 | 0.9736 |
| $\psi$ | 0.1250 | 0.1250 | 0.1250 | 0.1206 | 0.1250 | 0.1242 | 0.1250 | 0.1250 | 0.1250 | 0.1206 |
| Industry Portfolios | | | | | | | | | | |
| $\beta$ | 10.6747 | 10.6810 | 10.6694 | 10.6543 | 10.8261 | 10.6585 | 10.7982 | 10.7031 | 10.6930 | 10.8261 |
| $\delta$ | 0.9463 | 0.9463 | 0.9466 | 0.9501 | 0.9481 | 0.9489 | 0.9450 | 0.9464 | 0.9466 | 0.9481 |
| $\psi$ | 0.0937 | 0.0936 | 0.0937 | 0.0939 | 0.0924 | 0.0938 | 0.0926 | 0.0934 | 0.0935 | 0.0924 |
| **3. Households with Total Assets ⩾ \$5000** | | | | | | | | | | |
| CRSP Market Capitalization-Based Decile Portfolios 1-10 | | | | | | | | | | |
| $\beta$ | 12.1257 | 12.1298 | 12.1246 | 12.2080 | 12.1965 | 12.0982 | 11.6759 | 12.1491 | 12.1381 | 12.2080 |
| $\delta$ | 0.9010 | 0.9010 | 0.9013 | 0.9033 | 0.9046 | 0.9053 | 0.9096 | 0.9010 | 0.9013 | 0.9033 |
| $\psi$ | 0.0825 | 0.0824 | 0.0825 | 0.0819 | 0.0820 | 0.0827 | 0.0856 | 0.0823 | 0.0824 | 0.0819 |
| CRSP Market Capitalization-Based Decile Portfolios 1-5 | | | | | | | | | | |
| $\beta$ | 8.6514 | 8.6501 | 8.6451 | 8.6606 | 8.6337 | 8.6422 | 8.5051 | 8.6588 | 8.6533 | 8.6606 |
| $\delta$ | 0.9527 | 0.9527 | 0.9530 | 0.9550 | 0.9569 | 0.9552 | 0.9558 | 0.9529 | 0.9531 | 0.9550 |
| $\psi$ | 0.1156 | 0.1156 | 0.1157 | 0.1155 | 0.1158 | 0.1157 | 0.1176 | 0.1155 | 0.1156 | 0.1155 |
| CRSP Market Capitalization-Based Decile Portfolios 6-10 | | | | | | | | | | |
| $\beta$ | 7.0265 | 7.0214 | 7.0080 | 7.3468 | 7.0177 | 7.1405 | 6.7474 | 7.0437 | 7.0320 | 7.3468 |
| $\delta$ | 0.9708 | 0.9708 | 0.9711 | 0.9695 | 0.9742 | 0.9714 | 0.9748 | 0.9708 | 0.9710 | 0.9695 |
| $\psi$ | 0.1423 | 0.1424 | 0.1427 | 0.1361 | 0.1425 | 0.1400 | 0.1482 | 0.1420 | 0.1422 | 0.1361 |
| Industry Portfolios | | | | | | | | | | |
| $\beta$ | 9.3936 | 9.3990 | 9.3891 | 9.3762 | 9.5228 | 9.3798 | 9.4989 | 9.4178 | 9.4092 | 9.5228 |
| $\delta$ | 0.9431 | 0.9430 | 0.9434 | 0.9460 | 0.9456 | 0.9459 | 0.9429 | 0.9431 | 0.9433 | 0.9456 |
| $\psi$ | 0.1065 | 0.1064 | 0.1065 | 0.1067 | 0.1050 | 0.1066 | 0.1053 | 0.1062 | 0.1063 | 0.1050 |

**Table IV (continued)**
**B. The Model with the BCG (2002) Pricing Kernel**

| | $x_t^1$ | $x_t^2$ | $x_t^3$ | $x_t^4$ | $x_t^5$ | $x_t^6$ | $x_t^7$ | $x_t^8$ | $x_t^9$ | $x_t^{opt}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 1. All Households | | | | | |
| | | | | | CRSP Market Capitalization-Based Decile Portfolios 1-10 | | | | | |
| $\alpha$ | 8.1810 | 8.1830 | 8.1810 | 8.2130 | 8.2080 | 8.1710 | 8.0070 | 8.1900 | 8.1860 | 8.2130 |
| $\delta$ | 0.5531 | 0.5530 | 0.5533 | 0.5549 | 0.5534 | 0.5524 | 0.5597 | 0.5533 | 0.5533 | 0.5549 |
| $\psi$ | 0.1222 | 0.1222 | 0.1222 | 0.1218 | 0.1218 | 0.1224 | 0.1249 | 0.1221 | 0.1222 | 0.1218 |
| | | | | | CRSP Market Capitalization-Based Decile Portfolios 1-5 | | | | | |
| $\alpha$ | 6.7520 | 6.7510 | 6.7490 | 6.7560 | 6.7440 | 6.7480 | 6.6870 | 6.7550 | 6.7530 | 6.7560 |
| $\delta$ | 0.6015 | 0.6015 | 0.6018 | 0.6037 | 0.6028 | 0.6012 | 0.6056 | 0.6018 | 0.6017 | 0.6037 |
| $\psi$ | 0.1481 | 0.1481 | 0.1482 | 0.1480 | 0.1483 | 0.1482 | 0.1495 | 0.1480 | 0.1481 | 0.1480 |
| | | | | | CRSP Market Capitalization-Based Decile Portfolios 6-10 | | | | | |
| $\alpha$ | 6.0020 | 6.0000 | 5.9930 | 6.1550 | 5.9980 | 6.0570 | 5.8660 | 6.0100 | 6.0050 | 6.1550 |
| $\delta$ | 0.6355 | 0.6355 | 0.6360 | 0.6302 | 0.6365 | 0.6325 | 0.6434 | 0.6354 | 0.6355 | 0.6302 |
| $\psi$ | 0.1666 | 0.1667 | 0.1669 | 0.1625 | 0.1667 | 0.1651 | 0.1705 | 0.1664 | 0.1665 | 0.1625 |
| | | | | | Industry Portfolios | | | | | |
| $\alpha$ | 7.0750 | 7.0770 | 7.0730 | 7.0670 | 7.1300 | 7.0690 | 7.1200 | 7.0850 | 7.0810 | 7.1300 |
| $\delta$ | 0.5888 | 0.5887 | 0.5890 | 0.5915 | 0.5877 | 0.5884 | 0.5886 | 0.5888 | 0.5888 | 0.5877 |
| $\psi$ | 0.1413 | 0.1413 | 0.1414 | 0.1415 | 0.1403 | 0.1415 | 0.1404 | 0.1411 | 0.1412 | 0.1403 |

## Table IV (continued)

| | $x_t^1$ | $x_t^2$ | $x_t^3$ | $x_t^4$ | $x_t^5$ | $x_t^6$ | $x_t^7$ | $x_t^8$ | $x_t^9$ | $x_t^{opt}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 2. Households with Total Assets $\geqslant$ \$1000 | | | | | | | | | | |
| CRSP Market Capitalization-Based Decile Portfolios 1-10 | | | | | | | | | | |
| $\alpha$ | 8.7700 | 8.7720 | 8.7690 | 8.8070 | 8.8020 | 8.7570 | 8.5650 | 8.7800 | 8.7750 | 8.8070 |
| $\delta$ | 0.5532 | 0.5532 | 0.5533 | 0.5551 | 0.5545 | 0.5557 | 0.5604 | 0.5531 | 0.5533 | 0.5551 |
| $\psi$ | 0.1140 | 0.1140 | 0.1140 | 0.1135 | 0.1136 | 0.1142 | 0.1168 | 0.1139 | 0.1140 | 0.1135 |
| CRSP Market Capitalization-Based Decile Portfolios 1-5 | | | | | | | | | | |
| $\alpha$ | 7.0180 | 7.0170 | 7.0140 | 7.0230 | 7.0080 | 7.0130 | 6.9350 | 7.0220 | 7.0190 | 7.0230 |
| $\delta$ | 0.6087 | 0.6088 | 0.6089 | 0.6109 | 0.6110 | 0.6107 | 0.6135 | 0.6087 | 0.6089 | 0.6109 |
| $\psi$ | 0.1425 | 0.1425 | 0.1426 | 0.1424 | 0.1427 | 0.1426 | 0.1442 | 0.1424 | 0.1425 | 0.1424 |
| CRSP Market Capitalization-Based Decile Portfolios 6-10 | | | | | | | | | | |
| $\alpha$ | 6.0460 | 6.0420 | 6.0340 | 6.2460 | 6.0400 | 6.1170 | 5.8670 | 6.0560 | 6.0490 | 6.2460 |
| $\delta$ | 0.6522 | 0.6524 | 0.6529 | 0.6446 | 0.6542 | 0.6503 | 0.6624 | 0.6518 | 0.6523 | 0.6446 |
| $\psi$ | 0.1654 | 0.1655 | 0.1657 | 0.1601 | 0.1656 | 0.1635 | 0.1704 | 0.1651 | 0.1653 | 0.1601 |
| Industry Portfolios | | | | | | | | | | |
| $\alpha$ | 7.4270 | 7.4300 | 7.4240 | 7.4170 | 7.4960 | 7.4190 | 7.4830 | 7.4400 | 7.4350 | 7.4960 |
| $\delta$ | 0.5932 | 0.5932 | 0.5934 | 0.5961 | 0.5927 | 0.5954 | 0.5929 | 0.5929 | 0.5932 | 0.5927 |
| $\psi$ | 0.1346 | 0.1346 | 0.1347 | 0.1348 | 0.1334 | 0.1348 | 0.1336 | 0.1344 | 0.1345 | 0.1334 |
| 3. Households with Total Assets $\geqslant$ \$5000 | | | | | | | | | | |
| CRSP Market Capitalization-Based Decile Portfolios 1-10 | | | | | | | | | | |
| $\alpha$ | 7.8450 | 7.8470 | 7.8450 | 7.8790 | 7.8740 | 7.8340 | 7.6560 | 7.8550 | 7.8500 | 7.8790 |
| $\delta$ | 0.5833 | 0.5833 | 0.5835 | 0.5844 | 0.5848 | 0.5851 | 0.5916 | 0.5832 | 0.5836 | 0.5844 |
| $\psi$ | 0.1275 | 0.1274 | 0.1275 | 0.1269 | 0.1270 | 0.1276 | 0.1306 | 0.1273 | 0.1274 | 0.1269 |
| CRSP Market Capitalization-Based Decile Portfolios 1-5 | | | | | | | | | | |
| $\alpha$ | 6.2550 | 6.2540 | 6.2510 | 6.2590 | 6.2460 | 6.2500 | 6.1800 | 6.2580 | 6.2560 | 6.2590 |
| $\delta$ | 0.6435 | 0.6436 | 0.6438 | 0.6453 | 0.6461 | 0.6452 | 0.6488 | 0.6435 | 0.6438 | 0.6453 |
| $\psi$ | 0.1599 | 0.1599 | 0.1600 | 0.1598 | 0.1601 | 0.1600 | 0.1618 | 0.1598 | 0.1598 | 0.1598 |
| CRSP Market Capitalization-Based Decile Portfolios 6-10 | | | | | | | | | | |
| $\alpha$ | 5.3900 | 5.3870 | 5.3790 | 5.5670 | 5.3850 | 5.4530 | 5.2310 | 5.3990 | 5.3930 | 5.5670 |
| $\delta$ | 0.6872 | 0.6874 | 0.6879 | 0.6794 | 0.6895 | 0.6852 | 0.6976 | 0.6868 | 0.6873 | 0.6794 |
| $\psi$ | 0.1855 | 0.1856 | 0.1859 | 0.1796 | 0.1857 | 0.1834 | 0.1912 | 0.1852 | 0.1854 | 0.1796 |
| Industry Portfolios | | | | | | | | | | |
| $\alpha$ | 6.6210 | 6.6240 | 6.6190 | 6.6130 | 6.6840 | 6.6150 | 6.6720 | 6.6330 | 6.6290 | 6.6840 |
| $\delta$ | 0.6274 | 0.6274 | 0.6276 | 0.6297 | 0.6270 | 0.6291 | 0.6273 | 0.6271 | 0.6274 | 0.6270 |
| $\psi$ | 0.1510 | 0.1510 | 0.1511 | 0.1512 | 0.1496 | 0.1512 | 0.1499 | 0.1508 | 0.1509 | 0.1496 |

**Table IV (continued)**
**C. The Model with Background Risk**

| | $x_t^1$ | $x_t^2$ | $x_t^3$ | $x_t^4$ | $x_t^5$ | $x_t^6$ | $x_t^7$ | $x_t^8$ | $x_t^9$ | $x_t^{opt}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 1. All Households | | | | | |
| | | | | | CRSP Market Capitalization-Based Decile Portfolios 1-10 | | | | | |
| $\gamma$ | 5.7348 | 5.7354 | 5.7347 | 5.7458 | 5.7443 | 5.7311 | 5.6731 | 5.7379 | 5.7365 | 5.7458 |
| $\delta$ | 0.8940 | 0.8939 | 0.8937 | 0.8924 | 0.8962 | 0.8967 | 0.8969 | 0.8934 | 0.8938 | 0.8924 |
| $\beta$ | 25.8347 | 25.8421 | 25.8328 | 25.9816 | 25.9612 | 25.7854 | 25.0234 | 25.8766 | 25.8570 | 25.9816 |
| $\psi$ | 0.0387 | 0.0387 | 0.0387 | 0.0385 | 0.0385 | 0.0388 | 0.0400 | 0.0386 | 0.0387 | 0.0385 |
| $\overline{\gamma}_t^*$ | 10.4530 | 10.4548 | 10.4527 | 10.4866 | 10.4820 | 10.4417 | 10.2662 | 10.4625 | 10.4582 | 10.4866 |
| | (46.13) | (46.12) | (46.13) | (45.93) | (45.96) | (46.20) | (47.27) | (46.08) | (46.10) | (45.93) |
| $\rho_1$ | 0.0977 | 0.0976 | 0.0977 | 0.0973 | 0.0974 | 0.0978 | 0.0995 | 0.0976 | 0.0976 | 0.0973 |
| | (0.90) | (0.89) | (0.90) | (0.89) | (0.89) | (0.90) | (0.91) | (0.89) | (0.89) | (0.89) |
| $\rho_2$ | -0.0404 | -0.0404 | -0.0404 | -0.0405 | -0.0405 | -0.0404 | -0.0400 | -0.0404 | -0.0404 | -0.0405 |
| | (-0.37) | (-0.37) | (-0.37) | (-0.37) | (-0.37) | (-0.37) | (-0.37) | (-0.37) | (-0.37) | (-0.37) |
| $\mu$ | 0.4514 | 0.4514 | 0.4514 | 0.4521 | 0.4520 | 0.4511 | 0.4474 | 0.4516 | 0.4515 | 0.4521 |
| | | | | | CRSP Market Capitalization-Based Decile Portfolios 1-5 | | | | | |
| $\gamma$ | 5.1610 | 5.1607 | 5.1597 | 5.1629 | 5.1573 | 5.1591 | 5.1306 | 5.1625 | 5.1614 | 5.1629 |
| $\delta$ | 0.9260 | 0.9259 | 0.9258 | 0.9257 | 0.9283 | 0.9279 | 0.9258 | 0.9256 | 0.9259 | 0.9257 |
| $\beta$ | 19.2210 | 19.2184 | 19.2082 | 19.2395 | 19.1853 | 19.2024 | 18.9250 | 19.2359 | 19.2248 | 19.2395 |
| $\psi$ | 0.0520 | 0.0520 | 0.0521 | 0.0520 | 0.0521 | 0.0521 | 0.0528 | 0.0520 | 0.0520 | 0.0520 |
| $\overline{\gamma}_t^*$ | 8.8134 | 8.8126 | 8.8099 | 8.8185 | 8.8035 | 8.8083 | 8.7322 | 8.8174 | 8.8145 | 8.8185 |
| | (57.45) | (57.46) | (57.48) | (57.41) | (57.53) | (57.49) | (58.11) | (57.42) | (57.45) | (57.41) |
| $\rho_1$ | 0.1144 | 0.1144 | 0.1145 | 0.1144 | 0.1145 | 0.1145 | 0.1153 | 0.1144 | 0.1144 | 0.1144 |
| | (1.05) | (1.05) | (1.05) | (1.05) | (1.05) | (1.05) | (1.06) | (1.05) | (1.05) | (1.05) |
| $\rho_2$ | -0.0364 | -0.0364 | -0.0364 | -0.0364 | -0.0364 | -0.0364 | -0.0362 | -0.0364 | -0.0364 | -0.0364 |
| | (-0.33) | (-0.33) | (-0.33) | (-0.33) | (-0.33) | (-0.33) | (-0.33) | (-0.33) | (-0.33) | (-0.33) |
| $\mu$ | 0.4144 | 0.4144 | 0.4143 | 0.4145 | 0.4142 | 0.4143 | 0.4125 | 0.4145 | 0.4144 | 0.4145 |

**Table IV (continued)**

| | $x_t^1$ | $x_t^2$ | $x_t^3$ | $x_t^4$ | $x_t^5$ | $x_t^6$ | $x_t^7$ | $x_t^8$ | $x_t^9$ | $x_t^{opt}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | CRSP Market Capitalization-Based Decile Portfolios 6-10 | | | | | |
| $\gamma$ | 4.7821 | 4.7808 | 4.7772 | 4.8648 | 4.7798 | 4.8121 | 4.7064 | 4.7867 | 4.7836 | 4.8648 |
| $\delta$ | 0.9412 | 0.9413 | 0.9412 | 0.9382 | 0.9433 | 0.9418 | 0.9424 | 0.9409 | 0.9412 | 0.9382 |
| $\beta$ | 15.8583 | 15.8474 | 15.8190 | 16.5340 | 15.8395 | 16.0995 | 15.2646 | 15.8948 | 15.8699 | 16.5340 |
| $\psi$ | 0.0631 | 0.0631 | 0.0632 | 0.0605 | 0.0631 | 0.0621 | 0.0655 | 0.0629 | 0.0630 | 0.0605 |
| $\overline{\gamma}_t^*$ | 7.8387 | 7.8355 | 7.8266 | 8.0448 | 7.8330 | 7.9130 | 7.6531 | 7.8500 | 7.8424 | 8.0448 |
| | (66.04) | (66.07) | (66.16) | (64.08) | (66.09) | (65.32) | (67.88) | (65.93) | (66.00) | (64.08) |
| $\rho_1$ | 0.1249 | 0.1249 | 0.1250 | 0.1227 | 0.1250 | 0.1241 | 0.1269 | 0.1248 | 0.1249 | 0.1227 |
| | (1.14) | (1.15) | (1.15) | (1.12) | (1.15) | (1.14) | (1.16) | (1.14) | (1.14) | (1.12) |
| $\rho_2$ | -0.0332 | -0.0332 | -0.0331 | -0.0339 | -0.0332 | -0.0335 | -0.0325 | -0.0332 | -0.0332 | -0.0339 |
| | (-0.30) | (-0.30) | (-0.30) | (-0.31) | (-0.30) | (-0.31) | (-0.30) | (-0.30) | (-0.30) | (-0.31) |
| $\mu$ | 0.3899 | 0.3899 | 0.3896 | 0.3953 | 0.3898 | 0.3919 | 0.3850 | 0.3902 | 0.3900 | 0.3953 |
| | | | | | Industry Portfolios | | | | | |
| $\gamma$ | 5.3057 | 5.3067 | 5.3048 | 5.3025 | 5.3294 | 5.3031 | 5.3251 | 5.3102 | 5.3086 | 5.3294 |
| $\delta$ | 0.9190 | 0.9189 | 0.9188 | 0.9188 | 0.9202 | 0.9212 | 0.9166 | 0.9185 | 0.9188 | 0.9202 |
| $\beta$ | 20.7009 | 20.7116 | 20.6920 | 20.6666 | 20.9547 | 20.6737 | 20.9080 | 20.7486 | 20.7316 | 20.9547 |
| $\psi$ | 0.0483 | 0.0483 | 0.0483 | 0.0484 | 0.0477 | 0.0484 | 0.0478 | 0.0482 | 0.0482 | 0.0477 |
| $\overline{\gamma}_t^*$ | 9.2072 | 9.2100 | 9.2047 | 9.1983 | 9.2729 | 9.2000 | 9.2609 | 9.2196 | 9.2152 | 9.2729 |
| | (54.42) | (54.40) | (54.44) | (54.49) | (53.94) | (54.48) | (54.03) | (54.33) | (54.36) | (53.94) |
| $\rho_1$ | 0.1103 | 0.1103 | 0.1103 | 0.1104 | 0.1096 | 0.1104 | 0.1097 | 0.1102 | 0.1102 | 0.1096 |
| | (1.01) | (1.01) | (1.01) | (1.01) | (1.00) | (1.01) | (1.01) | (1.01) | (1.01) | (1.00) |
| $\rho_2$ | -0.0375 | -0.0375 | -0.0375 | -0.0375 | -0.0377 | -0.0375 | -0.0377 | -0.0376 | -0.0376 | -0.0377 |
| | (-0.34) | (-0.34) | (-0.34) | (-0.34) | (-0.35) | (-0.34) | (-0.35) | (-0.34) | (-0.34) | (-0.35) |
| $\mu$ | 0.4237 | 0.4238 | 0.4237 | 0.4235 | 0.4253 | 0.4236 | 0.4250 | 0.4240 | 0.4239 | 0.4253 |

## Table IV (continued)

| | $x_t^1$ | $x_t^2$ | $x_t^3$ | $x_t^4$ | $x_t^5$ | $x_t^6$ | $x_t^7$ | $x_t^8$ | $x_t^9$ | $x_t^{opt}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 2. Households with Total Assets $\geqslant$ \$1000 | | | | | |
| | | | | | CRSP Market Capitalization-Based Decile Portfolios 1-10 | | | | | |
| $\gamma$ | 4.9991 | 4.9998 | 4.9989 | 5.0141 | 5.0120 | 4.9940 | 4.9151 | 5.0034 | 5.0013 | 5.0141 |
| $\delta$ | 0.9048 | 0.9050 | 0.9051 | 0.9059 | 0.9075 | 0.9055 | 0.9113 | 0.9047 | 0.9049 | 0.9059 |
| $\beta$ | 13.8945 | 13.8994 | 13.8932 | 14.0102 | 13.9783 | 13.9490 | 13.3624 | 13.9222 | 13.9092 | 14.0102 |
| $\psi$ | 0.0720 | 0.0719 | 0.0720 | 0.0714 | 0.0715 | 0.0717 | 0.0748 | 0.0718 | 0.0719 | 0.0714 |
| $\overline{\gamma}_t^*$ | 8.4870 | 8.4889 | 8.4865 | 8.5272 | 8.5216 | 8.4734 | 8.2645 | 8.4986 | 8.4929 | 8.5272 |
| | (44.69) | (44.68) | (44.70) | (44.49) | (44.52) | (44.76) | (45.82) | (44.64) | (44.66) | (44.49) |
| $\rho_1$ | 0.1273 | 0.1273 | 0.1274 | 0.1270 | 0.1271 | 0.1275 | 0.1290 | 0.1273 | 0.1273 | 0.1270 |
| | (1.17) | (1.17) | (1.17) | (1.16) | (1.16) | (1.17) | (1.18) | (1.17) | (1.17) | (1.16) |
| $\rho_2$ | 0.0366 | 0.0366 | 0.0366 | 0.0365 | 0.0365 | 0.0367 | 0.0374 | 0.0366 | 0.0366 | 0.0365 |
| | (0.34) | (0.34) | (0.34) | (0.33) | (0.33) | (0.34) | (0.34) | (0.34) | (0.34) | (0.33) |
| $\mu$ | 0.4110 | 0.4110 | 0.4110 | 0.4120 | 0.4118 | 0.4106 | 0.4053 | 0.4113 | 0.4111 | 0.4120 |
| | | | | | CRSP Market Capitalization-Based Decile Portfolios 1-5 | | | | | |
| $\gamma$ | 4.2699 | 4.2696 | 4.2683 | 4.2721 | 4.2656 | 4.2677 | 4.2344 | 4.2717 | 4.2703 | 4.2721 |
| $\delta$ | 0.9518 | 0.9519 | 0.9520 | 0.9533 | 0.9549 | 0.9517 | 0.9530 | 0.9519 | 0.9519 | 0.9533 |
| $\beta$ | 10.2024 | 10.1924 | 10.1931 | 10.3667 | 10.2322 | 10.4054 | 10.1098 | 10.2277 | 10.2268 | 10.3667 |
| $\psi$ | 0.0980 | 0.0981 | 0.0981 | 0.0965 | 0.0977 | 0.0961 | 0.0989 | 0.0978 | 0.0978 | 0.0965 |
| $\overline{\gamma}_t^*$ | 6.6877 | 6.6871 | 6.6841 | 6.6927 | 6.6780 | 6.6827 | 6.6072 | 6.6918 | 6.6886 | 6.6927 |
| | (55.62) | (55.62) | (55.64) | (55.58) | (55.69) | (55.65) | (56.22) | (55.59) | (55.61) | (55.58) |
| $\rho_1$ | 0.1410 | 0.1410 | 0.1410 | 0.1409 | 0.1410 | 0.1410 | 0.1416 | 0.1409 | 0.1410 | 0.1409 |
| | (1.29) | (1.29) | (1.29) | (1.29) | (1.29) | (1.29) | (1.30) | (1.29) | (1.29) | (1.29) |
| $\rho_2$ | 0.0430 | 0.0430 | 0.0430 | 0.0430 | 0.0431 | 0.0431 | 0.0433 | 0.0430 | 0.0430 | 0.0430 |
| | (0.39) | (0.39) | (0.39) | (0.39) | (0.39) | (0.39) | (0.40) | (0.39) | (0.39) | (0.39) |
| $\mu$ | 0.3615 | 0.3615 | 0.3614 | 0.3617 | 0.3612 | 0.3614 | 0.3591 | 0.3617 | 0.3616 | 0.3617 |

## Table IV (continued)

| | $x_t^1$ | $x_t^2$ | $x_t^3$ | $x_t^4$ | $x_t^5$ | $x_t^6$ | $x_t^7$ | $x_t^8$ | $x_t^9$ | $x_t^{opt}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | CRSP Market Capitalization-Based Decile Portfolios 6-10 | | | | | |
| $\gamma$ | 3.8461 | 3.8447 | 3.8409 | 3.9352 | 3.8436 | 3.8782 | 3.7661 | 3.8510 | 3.8477 | 3.9352 |
| $\delta$ | 0.9693 | 0.9695 | 0.9696 | 0.9675 | 0.9718 | 0.9680 | 0.9715 | 0.9693 | 0.9694 | 0.9675 |
| $\beta$ | 8.4814 | 8.4671 | 8.4586 | 8.9751 | 8.5350 | 8.8155 | 8.2884 | 8.5163 | 8.5090 | 8.9751 |
| $\psi$ | 0.1179 | 0.1181 | 0.1182 | 0.1114 | 0.1172 | 0.1134 | 0.1207 | 0.1174 | 0.1175 | 0.1114 |
| $\overline{\gamma}_t^*$ | 5.7645 | 5.7615 | 5.7536 | 5.9519 | 5.7593 | 5.8316 | 5.5990 | 5.7747 | 5.7678 | 5.9519 |
| | (63.40) | (63.43) | (63.51) | (61.66) | (63.45) | (62.77) | (65.02) | (63.30) | (63.37) | (61.66) |
| $\rho_1$ | 0.1480 | 0.1481 | 0.1481 | 0.1466 | 0.1481 | 0.1475 | 0.1493 | 0.1479 | 0.1480 | 0.1466 |
| | (1.36) | (1.36) | (1.36) | (1.34) | (1.36) | (1.35) | (1.37) | (1.36) | (1.36) | (1.34) |
| $\rho_2$ | 0.0462 | 0.0462 | 0.0463 | 0.0456 | 0.0462 | 0.0460 | 0.0468 | 0.0462 | 0.0462 | 0.0456 |
| | (0.42) | (0.42) | (0.42) | (0.42) | (0.42) | (0.42) | (0.43) | (0.42) | (0.42) | (0.42) |
| $\mu$ | 0.3328 | 0.3327 | 0.3324 | 0.3388 | 0.3326 | 0.3350 | 0.3274 | 0.3331 | 0.3329 | 0.3388 |
| | | | | | Industry Portfolios | | | | | |
| $\gamma$ | 4.4432 | 4.4444 | 4.4421 | 4.4392 | 4.4722 | 4.4400 | 4.4669 | 4.4486 | 4.4467 | 4.4722 |
| $\delta$ | 0.9427 | 0.9428 | 0.9430 | 0.9447 | 0.9442 | 0.9428 | 0.9410 | 0.9426 | 0.9427 | 0.9442 |
| $\beta$ | 10.9763 | 10.9733 | 10.9693 | 11.1148 | 11.1512 | 11.1745 | 11.1226 | 11.0192 | 11.0156 | 11.1512 |
| $\psi$ | 0.0911 | 0.0911 | 0.0912 | 0.0900 | 0.0897 | 0.0895 | 0.0899 | 0.0908 | 0.0908 | 0.0897 |
| $\overline{\gamma}_t^*$ | 7.0896 | 7.0925 | 7.0870 | 7.0802 | 7.1584 | 7.0821 | 7.1458 | 7.1024 | 7.0979 | 7.1584 |
| | (52.77) | (52.75) | (52.79) | (52.83) | (52.31) | (52.82) | (52.39) | (52.68) | (52.71) | (52.31) |
| $\rho_1$ | 0.1379 | 0.1379 | 0.1379 | 0.1380 | 0.1374 | 0.1380 | 0.1375 | 0.1378 | 0.1378 | 0.1374 |
| | (1.26) | (1.26) | (1.26) | (1.26) | (1.26) | (1.26) | (1.26) | (1.26) | (1.26) | (1.26) |
| $\rho_2$ | 0.0416 | 0.0416 | 0.0416 | 0.0417 | 0.0414 | 0.0416 | 0.0414 | 0.0416 | 0.0416 | 0.0414 |
| | (0.38) | (0.38) | (0.38) | (0.38) | (0.38) | (0.38) | (0.38) | (0.38) | (0.38) | (0.38) |
| $\mu$ | 0.3733 | 0.3734 | 0.3732 | 0.3730 | 0.3752 | 0.3731 | 0.3749 | 0.3736 | 0.3735 | 0.3752 |

## Table IV (continued)

| | $x_t^1$ | $x_t^2$ | $x_t^3$ | $x_t^4$ | $x_t^5$ | $x_t^6$ | $x_t^7$ | $x_t^8$ | $x_t^9$ | $x_t^{opt}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 3. Households with Total Assets $\geqslant$ \$5000 | | | | | |
| | | | | | CRSP Market Capitalization-Based Decile Portfolios 1-10 | | | | | |
| $\gamma$ | 5.0547 | 5.0555 | 5.0545 | 5.0712 | 5.0689 | 5.0492 | 4.9628 | 5.0594 | 5.0572 | 5.0712 |
| $\delta$ | 0.8894 | 0.8895 | 0.8896 | 0.8923 | 0.8933 | 0.8929 | 0.8986 | 0.8890 | 0.8892 | 0.8923 |
| $\beta$ | 12.7880 | 12.7807 | 12.7910 | 12.8433 | 12.8463 | 12.8219 | 12.3163 | 12.8321 | 12.8282 | 12.8433 |
| $\psi$ | 0.0782 | 0.0782 | 0.0782 | 0.0779 | 0.0778 | 0.0780 | 0.0812 | 0.0779 | 0.0780 | 0.0779 |
| $\overline{\gamma}_t^*$ | 9.0315 | 9.0339 | 9.0309 | 9.0809 | 9.0740 | 9.0150 | 8.7601 | 9.0455 | 9.0390 | 9.0809 |
| | (32.54) | (32.53) | (32.54) | (32.35) | (32.38) | (32.60) | (33.62) | (32.49) | (32.51) | (32.35) |
| $\rho_1$ | 0.1854 | 0.1854 | 0.1854 | 0.1851 | 0.1851 | 0.1855 | 0.1874 | 0.1853 | 0.1854 | 0.1851 |
| | (1.70) | (1.70) | (1.70) | (1.70) | (1.70) | (1.70) | (1.72) | (1.70) | (1.70) | (1.70) |
| $\rho_2$ | 0.0591 | 0.0591 | 0.0591 | 0.0590 | 0.0590 | 0.0591 | 0.0594 | 0.0590 | 0.0590 | 0.0590 |
| | (0.54) | (0.54) | (0.54) | (0.54) | (0.54) | (0.54) | (0.54) | (0.54) | (0.54) | (0.54) |
| $\mu$ | 0.4403 | 0.4404 | 0.4403 | 0.4416 | 0.4414 | 0.4399 | 0.4335 | 0.4407 | 0.4405 | 0.4416 |
| | | | | | CRSP Market Capitalization-Based Decile Portfolios 1-5 | | | | | |
| $\gamma$ | 4.2639 | 4.2636 | 4.2623 | 4.2663 | 4.2594 | 4.2616 | 4.2259 | 4.2658 | 4.2644 | 4.2663 |
| $\delta$ | 0.9458 | 0.9460 | 0.9460 | 0.9489 | 0.9499 | 0.9477 | 0.9481 | 0.9458 | 0.9459 | 0.9489 |
| $\beta$ | 9.1932 | 9.1810 | 9.1899 | 9.1579 | 9.1943 | 9.2401 | 9.1109 | 9.2164 | 9.2178 | 9.1579 |
| $\psi$ | 0.1088 | 0.1089 | 0.1088 | 0.1092 | 0.1088 | 0.1082 | 0.1098 | 0.1085 | 0.1085 | 0.1092 |
| $\overline{\gamma}_t^*$ | 6.8982 | 6.8975 | 6.8943 | 6.9041 | 6.8873 | 6.8926 | 6.8061 | 6.9029 | 6.8994 | 6.9041 |
| | (42.94) | (42.95) | (42.97) | (42.91) | (43.01) | (42.98) | (43.52) | (42.92) | (42.94) | (42.91) |
| $\rho_1$ | 0.2001 | 0.2001 | 0.2001 | 0.2001 | 0.2002 | 0.2001 | 0.2007 | 0.2001 | 0.2001 | 0.2001 |
| | (1.83) | (1.83) | (1.83) | (1.83) | (1.83) | (1.83) | (1.84) | (1.83) | (1.83) | (1.83) |
| $\rho_2$ | 0.0610 | 0.0610 | 0.0610 | 0.0610 | 0.0610 | 0.0610 | 0.0611 | 0.0610 | 0.0610 | 0.0610 |
| | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) |
| $\mu$ | 0.3819 | 0.3819 | 0.3818 | 0.3821 | 0.3816 | 0.3817 | 0.3791 | 0.3820 | 0.3819 | 0.3821 |

## Table IV (continued)

| | $x_t^1$ | $x_t^2$ | $x_t^3$ | $x_t^4$ | $x_t^5$ | $x_t^6$ | $x_t^7$ | $x_t^8$ | $x_t^9$ | $x_t^{opt}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | CRSP Market Capitalization-Based Decile Portfolios 6-10 | | | | | | | | | |
| $\gamma$ | 3.8134 | 3.8119 | 3.8079 | 3.9075 | 3.8108 | 3.8472 | 3.7291 | 3.8186 | 3.8151 | 3.9075 |
| $\delta$ | 0.9661 | 0.9662 | 0.9664 | 0.9651 | 0.9694 | 0.9662 | 0.9692 | 0.9660 | 0.9661 | 0.9651 |
| $\beta$ | 7.4891 | 7.4728 | 7.4723 | 7.7722 | 7.5134 | 7.6603 | 7.3157 | 7.5208 | 7.5154 | 7.7722 |
| $\psi$ | 0.1335 | 0.1338 | 0.1338 | 0.1287 | 0.1331 | 0.1305 | 0.1367 | 0.1330 | 0.1331 | 0.1287 |
| $\overline{\gamma}_t^*$ | 5.8573 | 5.8541 | 5.8453 | 6.0658 | 5.8517 | 5.9317 | 5.6743 | 5.8688 | 5.8611 | 6.0658 |
| | (50.29) | (50.32) | (50.39) | (48.65) | (50.34) | (49.69) | (51.81) | (50.20) | (50.26) | (48.65) |
| $\rho_1$ | 0.2066 | 0.2067 | 0.2067 | 0.2054 | 0.2067 | 0.2062 | 0.2077 | 0.2066 | 0.2066 | 0.2054 |
| | (1.89) | (1.89) | (1.89) | (1.88) | (1.89) | (1.89) | (1.90) | (1.89) | (1.89) | (1.88) |
| $\rho_2$ | 0.0615 | 0.0615 | 0.0615 | 0.0614 | 0.0615 | 0.0615 | 0.0616 | 0.0615 | 0.0615 | 0.0614 |
| | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) |
| $\mu$ | 0.3490 | 0.3488 | 0.3486 | 0.3558 | 0.3488 | 0.3514 | 0.3428 | 0.3493 | 0.3491 | 0.3558 |
| | Industry Portfolios | | | | | | | | | |
| $\gamma$ | 4.4502 | 4.4515 | 4.4491 | 4.4459 | 4.4815 | 4.4468 | 4.4757 | 4.4561 | 4.4540 | 4.4815 |
| $\delta$ | 0.9351 | 0.9352 | 0.9353 | 0.9389 | 0.9374 | 0.9374 | 0.9340 | 0.9348 | 0.9350 | 0.9374 |
| $\beta$ | 9.9672 | 9.9618 | 9.9659 | 9.9080 | 10.1125 | 10.0108 | 10.1204 | 10.0090 | 10.0071 | 10.1125 |
| $\psi$ | 0.1003 | 0.1004 | 0.1003 | 0.1009 | 0.0989 | 0.0999 | 0.0988 | 0.0999 | 0.0999 | 0.0989 |
| $\overline{\gamma}_t^*$ | 7.3626 | 7.3659 | 7.3598 | 7.3516 | 7.4428 | 7.3539 | 7.4279 | 7.3777 | 7.3723 | 7.4428 |
| | (40.24) | (40.22) | (40.26) | (40.30) | (39.80) | (40.29) | (39.88) | (40.16) | (40.19) | (39.80) |
| $\rho_1$ | 0.1970 | 0.1970 | 0.1971 | 0.1971 | 0.1965 | 0.1971 | 0.1966 | 0.1969 | 0.1970 | 0.1965 |
| | (1.81) | (1.81) | (1.81) | (1.81) | (1.80) | (1.81) | (1.80) | (1.81) | (1.81) | (1.80) |
| $\rho_2$ | 0.0607 | 0.0607 | 0.0607 | 0.0607 | 0.0606 | 0.0607 | 0.0606 | 0.0607 | 0.0607 | 0.0606 |
| | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) | (0.56) |
| $\mu$ | 0.3956 | 0.3957 | 0.3955 | 0.3953 | 0.3979 | 0.3953 | 0.3974 | 0.3960 | 0.3958 | 0.3979 |