Climate Policy and Induced R&D: How Great is the Effect?[1]

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Abstract

Carbon taxes or tradable permit systems to address climate change are likely to induce research and development in energy-related technologies. At present, the literature is divided on whether such effects will have a large impact on the cost of emissions abatement. Goulder and Schneider (1999) and Popp (2004) argue that welfare gains due to induced innovation will likely be modest, due to spillovers in research markets. In contrast, Gerlagh (2008) presents a model in which induced R&D results in significant reductions in abatement costs. The present paper seeks to reconcile these views by focusing on the structure of Popp's (2004) ENTICE model. In particular, we find that when (i) research spillovers are modeled as externalities rather than second-best optimization constraints and (ii) two human capital stocks are included – one for efficiency in the consumption of carbon fuels and another for production of carbon fuels – the ENTICE model can be made to reproduce Gerlagh's more optimistic result. (JEL Q4, Q5; keywords: induced technological change, climate policy modeling)

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I. Introduction

Carbon taxes or tradable permit systems to address climate change are likely to induce research and development in energy-related technologies, which in turn will help reduce the cost of emissions abatement. Earlier climate policy models assumed technological change was exogenous (e.g. Nordhaus and Boyer 2000) but now more models take account of endogenous technical change in one way or another. Edenhofer et al. (2006) provides a useful overview of current models with endogenous technological components.

One of the important lessons of this literature concerns the role of knowledge spillovers and crowding out in the market for research. An *inter-firm* knowledge spillover occurs when private firms are only able to appropriate a small portion of the total benefits of their research. As a result, firms spend too little on R&D, compared with the social optimum. An *inter-temporal* knowledge spillover occurs when production of new knowledge today affects the productivity of research effort in the future. Depending on the nature of the spillover (positive or negative) and whether it is taken into account, firms will either spend too little or too much on R&D as a result. Crowding out occurs when a portion of the induced research represents a displacement of effort from other sectors rather than a net increase. Assuming this effect is not taken into account, firms will overspend on R&D. For all of these reasons, the social benefit of induced R&D may in fact be quite limited.

These points were first raised by Goulder and Schneider (1999). Subsequently, Popp (2004) sought to empirically calibrate the size of these effects, in obtaining an estimate of the benefit of induced R&D associated with climate change policy. Using the ENTICE model, Popp estimates that adding an R&D component increases the welfare impact of the optimal climate policy by only 8 percent in the central scenario – a rather modest change which he attributes to the inefficiencies associated with spillovers and crowding out. This lesson has been taken to heart by most climate policy modelers. For example, Buonanno et al. (2003) overlook the restraining effects of spillovers and crowding out, while Bosetti et al. (2006), presenting a later version of the same model, follow Popp's lead.

Now, however, Gerlagh (2008) has developed a model which contradicts the pessimism of Popp's result. Comprising four R&D sectors and accounting for both spillovers and crowding out, Gerlagh's model reveals an important role for induced R&D to reduce the cost of achieving a given climate target (atmospheric carbon concentration not to exceed 450 ppm).[2] In particular, the author estimates that adding induced R&D (i) reduces the cost of achieving the target by approximately half in the case of full crowding out, and (ii) the cost of achieving the target becomes negligible in the case of full flexibility (partial crowding out).

The comparison of the two models is complicated by the different policy objectives – stabilization of the atmospheric carbon concentration in Gerlagh (2008) compared with optimal climate policy in Popp (2004). However, we note that Popp (2004) also considers a policy of stabilizing emissions at the 1995 level. In this case, induced R&D improves welfare by only 5.6 percent – less than in the case of optimal policy (which restrains the growth of emissions but does not stabilize them). Thus, it appears that the more stringent the climate policy in ENTICE, the less important is the impact of induced R&D.

Gerlagh attributes the difference between the two models to (i) "unfavourable production conditions" for energy-related human capital in ENTICE (decreasing returns to R&D in the production of human capital compared with a constant price of carbon fuel during the first century), and (ii) fewer R&D sectors in ENTICE (in particular no possibility of switching R&D from carbon-energy production to carbon-energy savings). We also observe that whereas ENTICE has an important exogenous trend of de-carbonization of energy, Gerlagh's model does not. Therefore, R&D should be relatively more important in Gerlagh's model than in ENTICE.

The present paper seeks to probe more deeply into the structure of ENTICE in order to explain the divergence with Gerlagh's model. The size of the contribution of induced innovation toward emissions abatement is an important matter for policy makers, and therefore it would be useful to know whether economists can say anything systematic about it.

In undertaking this exercise, we identify three major flaws in the structure of ENTICE which require attention. First, inter-firm knowledge spillovers in the research market are modeled as a second-best optimization constraint rather than as an externality. Second, Popp neglects to impose the spillover constraint in the first period, with the result that spillovers are in fact wholly internalized in the first period. Third, Popp overestimates the opportunity cost of crowding out while neglecting the "opportunity benefit" of R&D inputs transferred from the

[2] Gerlagh (2008) and Popp (2004) are narrowly focused models, in that they leave out other mechanisms of endogenous technical change, such as learning-by-doing or substitution of carbon-free substitutes (e.g. backstop technologies). As such, they are appropriate laboratories for isolating the effects of endogenous R&D. Edenhofer et al. (2006) provide an overview of recent models which include these other mechanisms.

competing sector. We also identify numerous other assumptions and modeling practices which we argue should be changed.

In the end, we conclude that energy-related R&D in ENTICE is driven mainly by (i) the intertemporal knowledge spillover, and (ii) the lack of alternative sectors for investment. In contrast, the effect of climate policy on R&D is very minor, and therefore welfare gains attributable to R&D induced by climate policy are also minor.

We find Gerlagh's inclusion of multiple research sectors much more realistic. In particular, we argue that in the absence of an effective pollution policy, the economy will over-invest in carbon-production R&D, as well as over-investing in conventional carbon-production exploration and development. This feature means that, rather than possessing an opportunity cost, energy-efficiency R&D (carbon-saving) leads instead to a double dividend: the first dividend is technical innovation, and the second dividend is the reduction of the distortion related to excessive investment in carbon production. Further research is needed to model these interactions explicitly in the ENTICE framework. Nonetheless, our preliminary results show that, similar to Gerlagh, the scale of benefits flowing from induced innovation may be significant.

The next section (II) presents the structure, data and calibration of ENTICE. Section III presents our critical assessment of the model and calibration. Section IV presents our revised modeling, section V our revised calibration and section VI our results.

II. ENTICE

1. Model structure

Popp's ENTICE model is based on Nordhaus' DICE 99 model (Dynamic Integrated model of the Climate and Economy) (Nordhaus and Boyer 2000). It is a single-region growth model with an environmental module to account for negative impacts of greenhouse gas emissions.

Time proceeds in discrete steps of 10 years each, with periods denoted $t = 1,...,T$. The initial period corresponds with the decade 1990-2000. The terminal period is 35 (T = 35), for a total planning horizon of 350 years.

In any given period, gross output, Q_t, is given by the Cobb-Douglas production function

$$Q_t = A_t K_t^{\gamma} L_t^{1-\gamma-\beta} EN_t^{\beta} \tag{1}$$

where K_t denotes physical capital, L_t denotes labour, EN_t denotes energy services, and A_t denotes total factor productivity. Labour and total factor productivity are assumed to follow exogenous trends, growing exponentially but at declining rates. Details of these trends are discussed in Nordhaus and Boyer (2000).

Physical capital evolves according to the difference equation

$$K_{t+1} = (1-\delta_K)^{\Delta} K_t + \Delta\left[I_t - R_t \cdot \mathit{oppcost} \cdot \mathit{crowd}\right] \tag{2}$$

where I_t represents investment, R_t represents energy-related research spending, δ_K represents the annual depreciation rate, Δ represents the period duration (10 years), *oppcost* represents additional opportunity cost associated with research spending (discussed below), and *crowd* represents the proportion of energy-related research spending which is obtained by crowding out research spending in other sectors.

Energy services are provided by an effective fuel input, $\hat{F}_t$, which is carbon based, and energy-related human capital, H_t, according to the CES function[3]

$$EN_t = \left[\alpha_H H_t^{\rho} + \alpha_F \hat{F}_t^{\rho}\right]^{1/\rho}. \qquad \left(P_t^E\right) \tag{3}$$

This functional form exhibits the property

$$\lim_{\hat{F}\to 0^+} \frac{\partial EN}{\partial \hat{F}} = \infty, \tag{4}$$

i.e. an exploding marginal product as $\hat{F}$ approaches zero. For this reason, human capital supplements but never eliminates the carbon-based fuel in the provision of energy services.[4] The effective fuel input is derived from gross fuel, F_t, according to the relationship

$$\hat{F}_t = \frac{F_t}{\Phi_t}, \tag{5}$$

where $1/\Phi_t$ represents exogenously increasing technical change.

Energy-related human capital evolves according to the difference equation

$$h_t \equiv H_{t+1} - H_t = \Delta \cdot a R_t^{b} H_t^{\varphi}, \tag{6}$$

where the right-hand side is referred to as the research possibility frontier, or simply the research function. The value of the parameter b characterizes the "duplication externality" in research.

[3] The variable in parentheses after the equation represents the shadow value which will be employed in the sequel.
[4] In contrast, Popp (2006) adds a non-carbon backstop technology to the model.

Since research effort is spread out across many firms, it is likely that doubling the total amount of research spending will less than double the amount of new knowledge, as some firms duplicate results. Therefore, we expect $0 < b < 1$.

The value of the parameter φ characterizes the intertemporal knowledge spillover. When $\varphi > 0$, researchers "stand on the shoulders of giants," as new discoveries become easier with more knowledge. When $\varphi < 0$, research is a process of "fishing out the pond," as new discoveries become more difficult the more is already known. When $\varphi = 0$, the productivity of research is not affected by the amount of existing knowledge. Formally, this characteristic relates to the second derivative of the research function, $\frac{\partial^2 h_t}{\partial R_t \partial H_t}$, which is positive, negative, or zero as φ is positive, negative or zero.

The value of φ also determines returns to the amount of existing knowledge in research. This characteristic relates to the second derivative $\frac{\partial^2 h_t}{\partial H_t^2}$. Assuming $\varphi > 0$, so that the first derivative $\frac{\partial h_t}{\partial H_t}$ is positive (standing on the shoulders), the second derivative is positive if $\varphi > 1$, zero if $\varphi = 1$, and negative if $0 < \varphi < 1$. These cases refer respectively to increasing, constant and decreasing returns to H_t in the research process.

Net output, Y_t, is given by the function

$$Y_t = D_t Q_t - P_t F_t \qquad \left(P_t^Y\right) \tag{7}$$

where D_t denotes the climate-related damage function, F_t denotes the carbon-based fuel (also equal to emissions), and P_t denotes the price of fuel. The damage function is specified as

$$D_t = \frac{1}{1 + \alpha_1 TE_t + \alpha_2 TE_t^2} \tag{8}$$

where TE_t denotes global mean temperature (atmosphere and upper ocean level). The price of carbon fuel reflects global scarcity of the resource, with P_t an increasing function of the cumulative amount of fuel consumed to date, Cm_t. In particular, following Nordhaus and Boyer (2000),

$$P_t = \omega_1 + \omega_2 \left(\frac{Cm_t}{\Omega} \right)^4 \tag{9}$$

and

$$Cm_{t+1} = Cm_t + \Delta \cdot F_t . \qquad \left(P_t^{CM}\right) \tag{10}$$

(Recall that Δ is the period duration – 10 years).

The global climate system is summarized by the following six equations:

$$MAT_{t+1} = b_{11}MAT_t + \Delta \cdot F_t + b_{21}MU_t \qquad \left(P_t^{M}\right) \tag{11}$$

$$FORCE_t = 4.1\left(\frac{\ln(MAT_t / 596.4)}{\ln 2} \right) \tag{12}$$

$$MU_{t+1} = b_{12}MAT_t + b_{22}MU_t + b_{32}ML_t \qquad \left(P_t^{MU}\right) \tag{13}$$

$$ML_{t+1} = b_{33}ML_t + b_{23}MU_t \qquad \left(P_t^{ML}\right) \tag{14}$$

$$TE_{t+1} = TE_t + C_1\left(FORCE_t - \frac{4.1}{C_2}TE_t - C_3(TE_t - TL_t) \right) \qquad \left(P_t^{TE}\right) \tag{15}$$

$$TL_{t+1} = TL_t + C_4(TE_t - TL_t) \qquad \left(P_t^{TL}\right) \tag{16}$$

The variables are: carbon mass in the atmosphere, MAT_t; carbon mass in the "upper reservoir" (biosphere and upper oceans), MU_t; carbon mass in the lower oceans, ML_t; radiative forcing, $FORCE_t$; temperature in the lower ocean level, TL_t. A detailed discussion is provided in Nordhaus and Boyer (2000). In a nutshell, emissions accumulate in the atmosphere (MAT), leading with a lag to higher temperatures (TE).

Consumption is denoted C_t. The model is closed by the budget constraint

$$Y_t = C_t + I_t + R_t . \tag{17}$$

Because individual firms are not able to appropriate the entire return from investments in R&D, they underinvest ceteris paribus. As a consequence, the social marginal product of R&D spending is greater than the marginal product of physical capital investment. To capture this effect, Popp imposes a rate-of-return constraint, linking physical investment, I, with research expenditure, R. The rate of return on R is calculated as follows:

$$\frac{\partial Y_t}{\partial R_t} = \frac{\partial Y_t}{\partial EN_t}\frac{\partial EN_t}{\partial H_t}\frac{\partial h_t}{R_t}$$
$$= \beta D(TE_t)A_t K_t^{\gamma} L_t^{1-\gamma-\beta}\left[\alpha_H H_t^{\rho} + \alpha_F \hat{F}_t^{\rho}\right]^{\frac{\beta-\rho}{\rho}} \alpha_H a b R_t^{b-1} H_t^{\varphi+\rho-1} \tag{18}$$

The rate of return on I (net of depreciation) is calculated in the usual manner:

$$\frac{\partial Y_t}{\partial I_t} = \frac{\partial Y_t}{\partial K_t} - \delta_K = \gamma D(TE_t)A_t K_t^{\gamma-1} L_t^{1-\gamma-\beta}\left[\alpha_H H_t^{\rho} + \alpha_F \hat{F}_t^{\rho}\right]^{\frac{\beta}{\rho}} - \delta_K \tag{19}$$

The rate-of-return constraint takes the form

$$\frac{\partial Y_t}{\partial R_t} = \kappa \frac{\partial Y_t}{\partial I_t}, \tag{20}$$

where $\kappa > 1$ represents the factor by which the social return to R exceeds the return to I.

Preferences are expressed in terms of a representative agent. The agent derives instantaneous utility, U, from per capita consumption, according to the logarithmic function

$$U(C_t/L_t) = \ln(C_t/L_t). \tag{21}$$

This function exhibits the familiar Inada condition,

$$\lim_{C\to 0^+} U = \infty. \tag{22}$$

Global welfare is measured by a utilitarian function

$$W = \sum_{t=1}^{T} \theta_t L_t U(C_t/L_t) \tag{23}$$

where θ_t represents the utility discount factor, defined as

$$\theta_t = \begin{cases} 1, & t = 1 \\ \prod_{s=1}^{t-1}(1+\eta_s)^{-1}, & t > 1 \end{cases}$$

with η_t denoting the pure rate of time preference. The latter equals 3 percent per annum in the first period – i.e. $\eta_1 = 0.03$ – and it declines exponentially at an annual rate of 0.26 percent thereafter. At this pace, η falls to 2.3 percent in 2100, 1.75 percent in 2200, and 1.35 percent in 2300.

The representative agent chooses the time paths of flow variables C, I, R, and F to maximize global welfare (23), subject to (i) the constraints (1) – (3) and (5) – (21), (ii) a transversality condition, and (iii) initial values K_0, H_0, MAT_0, MU_0, ML_0, TE_0, and TL_0. The transversality condition takes the form

$$I_T = \delta_K K_T \tag{24}$$

which indicates that terminal investment in physical capital must equal depreciation. Transversality conditions are not explicitly imposed for the other stocks. However, the optimization software implicitly respects the usual condition $\lambda_T X_T = 0$, where λ_T represents the terminal shadow value and X_T represents the terminal value of a given stock (e.g. H_T or TE_T).

Three policy scenarios are considered: business-as-usual (BAU), optimal policy with endogenous R&D, and optimal policy with exogenous R&D. In BAU, no policy is implemented to control greenhouse gas emissions. Under the two optimal scenarios, emissions control policy is chosen as part of the welfare maximization process. These scenarios correspond with an optimal, time-varying carbon tax, or alternatively with a tradable permits scheme where the level of permits is chosen optimally in each period. Under endogenous R&D, research spending is chosen to maximize welfare in combination with emissions control. In contrast, under exogenous R&D, the level of research spending in each period is constrained to equal the level obtained in the BAU scenario.

2. Initial values and parameters

Initial values of variables Y_1, K_1, L_1, F_1, R_1, MAT_1, MU_1, ML_1, TE_1, TL_1, and P_1 correspond with world values in 1990, while Cm_1 is by definition zero. The price, damage and climate parameters ω_1, ω_2, Ω, α_1, α_2, b_{11}, b_{21}, b_{12}, b_{22}, b_{32}, b_{33}, b_{23}, C_1, C_2, C_3, and C_4 are taken from Nordhaus and Boyer (2000).

Data do not exist for H_1. Even if data did exist, the value could be normalized arbitrarily, which would affect the values of the scale parameters α_H and a. Thus we can pick the value of H_1 arbitrarily and let the calibration take care of the scale parameters. Popp uses a value of 0.0001, whereas we use a value of 0.01, which makes computational scaling easier in our simulations.[5]

Capital's share in gross output, γ, is assumed equal to 0.3, and the annual depreciation rate on capital, δ_K, is assumed to be 10 percent. Both assumptions are taken from Nordhaus and

[5] All simulations are run using the GAMS software (www.gams.com).

Boyer (2000). Energy's share in gross output, β, is calculated as the proportion of initial output spent on fossil fuels, a value equal to 0.07029 (Popp 2004).

Popp sets $\kappa = oppcost = 4$, based on a review of relevant literature. The equality of κ and *oppcost* is most easily understood in the extreme case of complete crowding out ($crowd = 1$), where an extra dollar of energy-related R&D comes at the expense of one less dollar for other types of R&D. Since the social marginal product of R&D is κ times greater than the marginal product of physical capital, it follows that crowding out one dollar of R&D in another sector is equivalent to crowding out κ dollars of physical capital. But since other R&D sectors are not represented explicitly in the model structure, it is necessary to gross up the measurement of opportunity cost in equation (2) by the product $oppcost \cdot crowd$. In fact, evidence does not support the hypothesis of complete crowding out. Rather, Popp sets $crowd = 0.5$ in the central case, based on research using expenditure data.

3. Calibration

Exogenous energy-related technical change, $1/\Phi_t$, is calibrated to reproduce the values in DICE 99. Unlike ENTICE, DICE 99 does not contain an explicit fuel (emissions) variable; rather, output is a function of capital and labour, and emissions are modeled as a byproduct of production. The calibration of $1/\Phi_t$ ensures that both models produce the same emissions path in the business-as-usual scenario, in the absence of R&D. By assumption $\Phi_1 = 1$.

When induced innovation is added to the model, the trend of exogenous technical change must be scaled back by an appropriate amount, since it now represents only non-R&D related changes, such as might result from learning-by-doing, for example. In his central case, Popp reduces the trend of $1/\Phi_t$ by 20 percent (parameter *exgscale* equal to 0.20). In other words, 80 percent of exogenous technical change from DICE 99 remains in ENTICE in this case.

There remain seven parameters to calibrate: the initial value of total factor productivity, A_1; energy parameters α_F, α_H and ρ; and research parameters *a*, b and φ. Popp simplifies his task by assuming $\alpha_F = 1$. (Indeed, his statement of equation 3 does not include this parameter.) The remaining six parameters are then calibrated based on the following four empirical observations:

- The elasticity of energy R&D with respect to energy prices must equal 0.35, based on empirical research in Popp (2002).
- Diminishing returns to energy R&D means that the elasticity must fall over time from the initial value of 0.35.
- There exists an estimated 4:1 ratio of energy savings to energy R&D, based on empirical research in Popp (2001).
- The initial value of total factor productivity, A_1, must verify equations (1), (3), (7) – (9) given initial values of Y_1, K_1, L_1, F_1, H_1, TE_1, and P_1.

Details of his calibration strategy are provided in the appendices of Popp (2003, 2006) (but not Popp 2004).

III. Assessment of the model and calibration

1. The model

a. Opportunity cost and opportunity benefit

We note that the *oppcost* parameter in equation (2) should be equal to 3 rather than 4. As Popp explains, the fact that the social marginal product of R_t is 4 times the value of I_t means that crowding out one unit of R&D from another (non-energy) sector is equivalent to crowding out four units of physical capital. Yet one unit of opportunity cost has already been accounted for in the material balance equation (17), leaving only three additional units to be accounted for by *oppcost* in equation (2).

In addition, while Popp has accounted for the opportunity cost of crowding out research in a non-energy sector, he has neglected to account for an associated benefit – namely that the affected inputs have already been allocated to research and therefore they do not need to be taken from C_t or I_t. We refer to this effect as the "opportunity benefit" of crowding out non-energy R&D. The material balance equation (17) must therefore be adjusted to read

$$Y_t = C_t + I_t + crowd \cdot R_t .$$

For example, assuming Popp's value of $crowd = 0.5$, only half of R_t comes at the expense of C_t and I_t. This discussion indicates that Y_t should be viewed as output net of research inputs to the implicit non-energy sector. Alternatively, we can account for this opportunity benefit by substituting $(oppcost - 1)$ for *oppcost* in (2).

b. Second-best knowledge spillovers and crowding out

Inter-firm knowledge spillovers and additional opportunity costs from crowding out non-energy research (*oppcost*) are properly understood as externalities. Firms will choose their research levels without taking into account the benefits of knowledge spillovers since they cannot charge outsiders for these benefits. Similarly, firms perceive the opportunity cost of an extra dollar of energy research as one less dollar spent elsewhere, even if that dollar could have leveraged 3 additional dollars in social benefits through knowledge spillovers.

However, with the inclusion of *oppcost* in equation (2) and with the rate-of-return condition (20), Popp has modeled these effects as constraints on optimization rather than as externalities. As a consequence, the solution of the model must be understood as a second-best optimum, rather than a market equilibrium with externalities. Viewed another way, this modeling structure means that the representative agent takes these effects into account, when, by definition of an externality, he should not.

The second-best approach locks physical and knowledge capital together into a composite capital good, through the rate-of-return constraint (20). We envision two opposite effects of this approach. On the one hand, the agent would choose *more* I_t in every period than under an externality framework, since increasing I_t in the second best also makes it possible to increase R_t, which has a higher rate of return. Under an externality framework, the agent would not perceive the higher rate of return of R_t. On the other hand, the agent perceives the high opportunity cost of R_t in the second best, through *oppcost*, and therefore he chooses *less* R_t than in an externality framework. In the case of total crowing out (*crowd* = 1), we would expect these two effects to cancel out. However, since there is only partial crowding out in ENTICE (*crowd* = 0.5), the first effect should dominate – i.e. the second-best framework should lead to an over-accumulation of physical capital, compared with an externality framework.

c. Inter-firm knowledge spillovers fully internalized in the first period.

For reasons which are not clear, Popp does not impose the rate-of-return constraint (20) in the first period of the model, and neither does he impose the initial value $R_1 = 0.01$ under the optimal policy scenario.[6] It follows that the policy environment in the first period is first-best: firms can appropriate all the benefits from research in the first period, and they are free to increase R_1 to the point where its marginal product equals that of I_1. Moreover, there is an

[6] These results are only apparent in the computer code, not in his text.

incentive to over-invest in R_1 in anticipation of the re-imposition of the rate-of-return constraint in period 2. As a result, we would expect to see a much higher value of R_1 than we would otherwise.

d. Inter-temporal knowledge spillovers and scarcity: internal or external?

Inter-temporal knowledge spillovers and scarcity of carbon fuels are both internalized in ENTICE, through the value of φ in the research function (6) and the fuel price function (9). Stated another way, the representative agent takes account of both effects when making decisions about fuel use and R&D spending, whether in the business-as-usual scenario or under optimal policy.

We question the appropriateness of this approach. First, we believe that intertemporal knowledge spillovers should be modeled as an externality. In this view, private R&D spending by firms is motivated entirely by the prospect of payoffs from new products or production technologies, not by the possibility that new knowledge may make research more productive in the future. While firms would obviously welcome the latter benefit, the public goods nature of most knowledge makes them unwilling to pay for it alone.

Furthermore, in the fishing-out-the-pond scenario $(\varphi < 0)$, new knowledge makes research less productive in the future, not more. Therefore, firms that internalized this effect would be less inclined to invest in R&D. In reality, most firms probably do not know whether they are facing standing on the shoulders or fishing out the pond in terms of the research process. Therefore uncertainty provides another reason why firms would probably ignore intertemporal knowledge spillovers in their decisions about R&D spending.

Regarding scarcity, we are not convinced that the average consumer takes into account the effect of her fuel use today on future prices, although obviously she may have an expectation that future prices will be higher. The key issue is whether she views the increasing price trend as exogenous or endogenous. We consider it more plausible that she considers the trend exogenous.

Similarly, we suspect that individual fuel producers also view the price trend as exogenous. It could be argued on the contrary that the existence of the OPEC cartel indicates that at least some producers recognize the link between today's consumption and tomorrow's prices. However, the fact that many member countries cheat on their quotas suggests that even these countries consider price trends essentially exogenous in relation to their own production.

e. Ad hoc BAU

We have discussed Popp's ad hoc approach to modeling the business-as-usual scenario (BAU) at length in Shiell and Lyssenko (2008). His approach consists of three steps. First, he solves the model without pollution damages. Second, he calculates the trajectory of the savings rate for this solution. Third, he uses this savings rate to calculate adjusted trajectories of Y_t, C_t and K_t, iterating forward from the first period in the presence of pollution damages. This post-hoc adjustment for damages works remarkably well for these three variables. Unfortunately, there appears to be no coherent way to extend the adjustment to the remaining variables. Thus, the values of EN_t, F_t, R_t, H_t, TE_t and any other variables of interest are based on the assumption of no damages, which is obviously not realistic.

f. Benefits of moderate warming

Popp uses Nordhaus and Boyer's (2000) parameter values $\alpha_1 = -0.0045$ and $\alpha_2 = 0.0035$ in the damage function (8). Due to the quadratic specification, moderate warming results in benefits ($D > 1$) rather than damages ($D < 1$). Figure 1 shows the graph of the function for values of TE between 0.43 (the starting value) and 1.50, based on the parameter values. Maximum benefits of warming are experienced at $TE = 0.64$ and the function does not start generating damages until $TE > 1.29$.

This feature reflects the differentiation of regional experiences in the RICE model, upon which DICE 99 is based. In particular, in RICE four out of eight regions enjoy benefits from moderate warming, associated mainly with reduced costs of winter heating and a longer growing season. The aggregation method used by Nordhaus and Boyer preserves this feature for the world as a whole in DICE 99, and Popp carries it over into ENTICE.

We find this projection of moderate benefits from warming problematic, especially since the business-as-usual solution of ENTICE does not pass the temperature threshold of $TE > 1.29$ until 2050. In other words, for the next fifty years, the world is forecast to experience net benefits from climate change under business-as-usual, rather than damages. Combining this feature with the force of discounting, it should not be surprising that the optimal policy simulation would yield only modest efforts to control GHG emissions over the next several decades. While it seems plausible that some regions may benefit from moderate warming, it seems less clear that this feature should be reproduced for the world as a whole. Certainly the choice of regional weights used in the global aggregation is key for this result, and the entire matter deserves further study.

We note that in DICE 94 (Nordhaus 1994), the damage function takes the form

$$D = \frac{1}{1+\alpha TE^2}, \tag{25}$$

which always yields damages $(D < 1)$ for $\alpha > 0$. We suspect that this represents a more likely form of damages. Ultimately, the question can only be resolved empirically. In section VI, we compare results for both forms. In (25), we use a parameter value of $\alpha = 0.0021$, which minimizes the difference between (8) and (25) in terms of the sum of squared deviations over a temperature range of 0.43 to 4.00 degrees C.

g. Rate of time preference

The declining rate of time preference (η) in ENTICE is reproduced from DICE 99 (Nordhaus and Boyer 2000). This feature follows the trend in recent years to argue for a declining rate of discount for benefits and costs in the distant future. Justifications for this approach include individuals' stated preferences – e.g. hyperbolic discounting (Laibson 1997) – and the effects of uncertainty (Weitzman 2001, Newell and Pizer 2003).

These arguments are properly understood in a normative context; i.e. they relate to the question of how an ethical social planner would discount costs and benefits to maximize social welfare. In contrast, it is not clear that these approaches are relevant in a positive context, such as modeling business-as-usual in a decentralized market economy. By definition, discount rates in a positive context are required to replicate agents' revealed preferences, usually relating to sequences of choices over relatively short horizons. For this purpose, we find the approach of a constant rate of pure time preference more persuasive. As estimated in Nordhaus (1994), a constant rate of pure time preference of 3 percent seems reasonable.

Moreover, even in a normative context, we are not convinced that the arguments for a declining discount rate pertain directly to the rate of time preference, η. The standard form of the consumption discount rate, r, based on utilitarian welfare, is

$$r = \eta + \psi g,$$

where ψ represents the elasticity of marginal utility and g represents the growth rate of per capita consumption (see Lind (1982) and Moore et al. (2004) for comprehensive discussions of discounting in cost-benefit analysis). Newell and Pizer's (2003) argument for a declining discount rate over time, under uncertainty, relates to states of the world in which r is especially low. While they do not specify, this sounds like a story about a low rate of growth, g, rather than the rate of time preference. Indeed, since η is a pure preference parameter, we should expect it to remain constant across all states of the world.

Many authors, beginning with Ramsey (1928), have argued that an ethical social planner would use a value of zero for η. This approach has been applied to the climate change problem by Cline (1992, 2004). Others have suggested a dual rate approach. For example, in Shiell (2003), social preferences are characterized by one rate (e.g. zero) and "the market" is characterized by another (higher). In Yang (2003), preferences for environmental goods are characterized by a lower rate, while preferences for private goods are characterized by a higher rate. Shiell (2003) also introduces an inequality aversion parameter, with the result that the business-as-usual solution and the zero rate solution $(\eta = 0)$ become special cases of a range of plausible optimal policies.

Thus, while the precise form of social preferences remains a matter of debate, it does seem important to distinguish between the process that generates BAU and the process that generates socially optimal policies. As mentioned, we do not believe that the declining rate of time preference (η) employed by Popp is the most compelling way to characterize either social preferences or the market (although a declining *discount rate* (r) would be reasonable in conjunction with a declining growth rate (g)).

h. Transversality condition

The transversality condition (24) imposed on physical capital is a steady state condition in which investment just covers depreciation. This approach, proposed originally by Barr and Manne (1967), is common in finite-horizon approximations of infinite-horizon problems. However, the ENTICE model does not have a steady state, due to the continually increasing fuel price, (9). Therefore (24) is not appropriate. The practical effect of (24) is to depress terminal capital, K_T, more than otherwise, in order to reduce unproductive terminal investment. (It is not optimal to drive K_T to zero, since (i) capital is an essential input in production, and (ii) the putty-clay nature of the model means that capital can be allowed to depreciate over time but it cannot be consumed directly.)

The optimization routine employed (Popp uses the MINOS5 solver in GAMS) implicitly imposes the usual transversality condition on capital for a finite-horizon problem, i.e.

$P_T^K K_T = 0$, where P_T^K is the terminal value of the shadow price. Thus, the imposition of (24) represents a minor issue. However, the correct specification of the transversality conditions becomes vital in the sequel, when we employ a first-order condition approach to solve the model (i.e. solving the system of first-order conditions) rather than an optimization routine.

2. The calibration

First, we observe that Popp does not take advantage of the initial value $R_1 = 0.01$ to calibrate any of the research parameters in the model. Rather, he simply imposes it as a fixed value in his computer code.[7] In light of the dearth of empirical information for calibrating parameters, we find this approach inefficient.

Second, Popp's claim of diminishing returns to energy R&D is not well founded. Rather, it appears to be based on confusion in an earlier paper, Popp (2002), between depreciation of knowledge capital and diminishing returns. As discussed in Section II, a positive value of φ in the research equation (6) corresponds with the "standing on the shoulders" hypothesis, under which the marginal product of research $\partial h_t / \partial R_t$ increases as the knowledge stock H_t increases. At the same time, the payoff to research is reduced if the knowledge stock depreciates over time due to obsolescence. It is this latter effect which the author mistakenly refers to as diminishing returns in Popp (2002). In contrast, there is no depreciation of the knowledge stock in ENTICE. Therefore, both the initial value of the research elasticity (0.35), taken from Popp (2002), and the requirement that it must fall over time may be inappropriate in ENTICE.

Third, we find Popp's calibration of the estimated 4:1 ratio of energy savings to energy R&D inappropriate. At best, this approach should be redundant, since the 4:1 relationship is already guaranteed by the rate-of-return constraint (equation 20). However, Popp's handling of this condition appears to be conceptually flawed as well. The 4:1 ratio is an equilibrium result at the margin; i.e. it should hold for the last units of R and I in every period but it should not be expected to hold, in general, for infra-marginal units. Nonetheless, he imposes this ratio as an average condition, which is not appropriate. The marginal nature of the 4:1 ratio is apparent in (20) and also in Popp (2001) (in particular see equation 13, p.231).[8]

[7] We are grateful to Popp for making his code available to us.

[8] The marginal nature of the 4:1 ratio is obscured in Popp (2001) by the assumption of a linear specification for the input demand functions (quadratic cost function). In this special case, marginal and average effects are equal. But in general, there is no reason to believe that input demand functions are linear.

IV. Revised modeling

Some simple changes include (i) we set the value of *oppcost* to 3, and (ii) we substitute (*oppcost* – 1) for *oppcost* in equation (2) to account for the "opportunity benefit" of switching R&D resources from the non-energy to the energy sector.

In order to treat research spillovers and crowding out as externalities rather than second-best constraints, we have had to abandon the use of an optimization routine in the software. Rather, we derive the first-order conditions for BAU and optimal policies analytically, and then solve the resulting system of equations. We use the CONOPT algorithm in GAMS as an equation solver for this purpose. This "first-order condition" approach is also employed by Gerlagh (2008).

To ensure that the maximizing agent treats crowding out as an externality, we modify equation (2) to become

$$K_{t+1} = (1-\delta_K)^{\Delta} K_t + \Delta \cdot I_t \qquad \left(P_t^K\right) \qquad (2')$$

Similarly, to ensure that the agent treats inter-firm research spillovers as an externality, we modify equation (6) to become

$$h_t = \Delta \cdot (1-\ell) a R_t^b H_t^{\varphi} \qquad \left(P_t^H\right) \qquad (6')$$

where ℓ represents the leakage of research benefits to other firms. Put another way, a firm can only appropriate $(1-\ell)$ of the benefits of research. We set $\ell = 0.75$, following Popp's finding that ¾ of the research value leaks out. In solving the intertemporal allocation problem, the representative agent bases his decisions on $(2')$ and $(6')$, when in fact the aggregate accumulation of K and H is described by the original equations, (2) and (6).

We derive the first-order conditions for a planner's optimal policy first. The planner's problem is to maximize (23) with respect to the flow variables C, EN, F, R, Y, I and stock variables K, H, Cm, MAT, MU, ML, TE, TL, subject to (i) constraints (1), $(2')$, (3), (5), $(6')$, (7) – (21), (ii) initial values of the stocks, and (iii) non-negativity constraints on all the variables.[9] Since the agent takes into account the climate feedback, through equations (11) – (16), and the payoff to R&D, through equation $(6')$, the solution represents the optimal

[9] Equations (1), (5), (8), (9), and (12) are substituted into (3), (7), and (15), and therefore we do not present first-order conditions or shadow prices for variables Q, $\hat{F}$, D, P, and FORCE.

environmental policy with endogenous R&D (although still subject to inter-firm and crowding-out research externalities).

The non-negativity constraints are not binding on H and Cm, since these stocks accumulate but never depreciate. It follows by (3) that EN is strictly positive. The non-negativity constraints also prove to be non-binding on the environmental variables MAT, MU, ML, TE, and TL. The exploding marginal product of fuel at the corner (equation 4) ensures an interior solution with respect to F, and similarly the Inada condition (equation 22) ensures an interior solution with respect to C. It follows through the material balance equation (17) that Y must be strictly positive, and K must also be strictly positive, since it is essential in production (equation 1). We conclude that corner solutions are only possible with respect to R and I; our derivations below reflect that possibility.

We construct the Lagrangian for this problem in the usual manner and obtain the following first-order conditions.

$$P_t^Y = \theta_t \frac{L_t}{C_t} \tag{26}$$

$$\Delta \cdot P_t^K - P_t^Y \le 0 \quad \text{and} \quad I_t\left(\Delta \cdot P_t^K - P_t^Y\right) = 0 \tag{27}$$

$$P_t^E = P_t^Y \beta\left(\frac{Y_t + P_t F_t}{EN_t}\right) \tag{28}$$

$$P_t^E \left(\frac{EN_t}{F_t}\right)^{1-\rho} \alpha_F \Phi^{-\rho} = -\Delta \cdot P_t^M - \Delta \cdot P_t^{Cm} + P_t^Y \frac{P_t}{\Phi_t} \tag{29}$$

$$\Delta \cdot P_t^H (1-\ell) ab R_t^{b-1} H_t^{\varphi} - P_t^Y \le 0 \quad \text{and} \quad R_t\left(\Delta \cdot P_t^H (1-\ell) ab R_t^{b-1} H_t^{\varphi} - P_t^Y\right) = 0 \tag{30}$$

$$P_{t+1}^K (1-\delta_K)^{\Delta} = P_t^K - P_{t+1}^Y \gamma\left(\frac{Y_{t+1} + P_{t+1} F_{t+1}}{K_{t+1}}\right) \tag{31}$$

$$P_{t+1}^H \left[1 + \Delta \cdot (1-\ell) \varphi a R_{t+1}^{b} H_{t+1}^{\varphi-1}\right] = P_t^H - P_{t+1}^E \left(\frac{EN_{t+1}}{H_{t+1}}\right)^{1-\rho} \alpha_H \tag{32}$$

$$P_{t+1}^{Cm} = P_t^{Cm} + P_{t+1}^Y \left(\frac{F_{t+1}}{\Phi_{t+1}}\right) \frac{4\omega_2}{\Omega^4} Cm_{t+1}^3 \tag{33}$$

$$P_{t+1}^M b_{11} = P_t^M - P_{t+1}^{TE} \left(\frac{4.1 \cdot C_1}{\ln 2} \Big/ MAT_{t+1}\right) - P_{t+1}^{MU} b_{12} \tag{34}$$

$$P^{MU}_{t+1}b_{22} = P^{MU}_{t} - P^{M}_{t+1}b_{21} - P^{ML}_{t+1}b_{23} \tag{35}$$

$$P^{ML}_{t+1}b_{33} = P^{ML}_{t} - P^{MU}_{t+1}b_{32} \tag{36}$$

$$P^{TE}_{t+1}\left(1 - \frac{4.1 \cdot C_1}{C_2} - C_1C_3\right) = P^{TE}_{t} - P^{TL}_{t+1}C_4 + P^{Y}_{t+1}\frac{(Y_{t+1} + P_{t+1}F_{t+1})(\alpha_1 + 2\alpha_2 TE_{t+1})}{\left(1 + \alpha_1 TE_{t+1} + \alpha_2 TE^2_{t+1}\right)} \tag{37}$$

$$P^{TL}_{t+1}(1 - C_4) = P^{TL}_{t} - P^{TE}_{t+1}C_1C_3 \tag{38}$$

Condition (30) can be rearranged as follows, assuming an interior solution:

$$\frac{\Delta \cdot P^{H}_{t} ab R^{b-1}_{t} H^{\varphi}_{t}}{P^{Y}_{t}} = \frac{1}{1-\ell}. \tag{39}$$

The numerator on the left represents the marginal value of a unit of R (marginal benefit of R&D spending), while the denominator represents the marginal value of a unit of I (marginal cost of R&D spending). Given the leakage value $\ell = 0.75$, the right-hand side reduces to 4, which verifies Popp's 4:1 benefit-cost ratio of R&D spending.

We impose the standard finite-horizon transversality conditions on non-negative stocks: $P^{K}_{T}K_{T} = P^{H}_{T}H_{T} = P^{Cm}_{T}Cm_{T} = P^{M}_{T}MAT_{T} = P^{MU}_{T}MU_{T} = P^{ML}_{T}ML_{T} = P^{TE}_{T}TE_{T} = P^{TL}_{T}TL_{T} = 0$. We acknowledge that the assumption of a finite horizon may seem less than satisfying. Ideally, we would seek a finite approximation to an infinite horizon problem. However, we have not proven existence of an infinite-horizon solution (nor has Popp). The existence of a steady state would make such a proof easier; however, as noted above, a steady state does not exist in ENTICE, owing to the increasing price of carbon fuel. At the very least, the existence of a local maximum under a finite horizon can be established numerically, and therefore we proceed to check for such a solution. If (i) a solution did exist under an infinite-horizon, and (ii) the problem exhibited the turnpike property, then the finite-horizon solution would approximate the infinite-horizon solution during the early periods. How good an approximation would depend upon the choice of T. For extra assurance, we have increased the horizon to $T = 40$ (400 years) from 35.

Equations (27) and (30) show complementary slackness conditions, which reflect the possibility of corner solutions $I_t = 0$ and $R_t = 0$. Inspection of (6) (and the first-order condition 30) indicates a technical problem with R, as $\lim_{R \to 0^+} \frac{\partial h_t}{\partial R_t} = \infty$. Yet we know that it is not optimal to choose positive R in the terminal period at least, since this spending would have no payoff.

Furthermore, in light of the fact that human capital in the model does not depreciate, it may be preferable to build up H to a certain level and then invest no more in human capital beyond a given period $t' < T$. To accommodate these concerns, we impose $R_T = 0$ exogenously, and we conduct sensitivity analysis with a zero value in other periods as well. For these periods, we do not impose the inequality constraint in (30), as the left-hand side is undefined. In practice, it turns out that is it optimal in most scenarios (higher welfare value) to have $R = 0$ in T-1 as well as T, and in some cases in T-2. In all scenarios, $I_T = 0$ and thus the inequality constraint in (27) applies.

The difference equation (32) shows the two determinants of the value of human capital in any period: first, the inter-temporal knowledge spillover, represented by the marginal product of H in the research function,

$$\Delta \cdot (1-\ell)\varphi a R_{t+1}^{b} H_{t+1}^{\varphi-1}; \tag{40}$$

and second the marginal product of H in the production of energy, represented by the term

$$P_{t+1}^{E}\left(\frac{EN_{t+1}}{H_{t+1}}\right)^{1-\rho}\alpha_H .$$

Inclusion of (40) means that the inter-temporal knowledge spillover is internalized (Popp's approach). We compare this scenario with one in which the spillover is external (our preferred approach). Treating the spillover as an externality requires dropping (40) from (32); i.e. the difference equation becomes

$$P_{t+1}^{H} = P_t^{H} - P_{t+1}^{E}\left(\frac{EN_{t+1}}{H_{t+1}}\right)^{1-\rho}\alpha_H \tag{32'}$$

The planner's optimal policy corresponds with the solution of the first-order conditions (26) – (38), structural equations (1) – (3), (5) – (21), and the transversality conditions, given the initial values of variables.[10] We compare results for the rate of pure time preference at 3 per cent per annum $(\eta = 0.03)$ and 0 per cent $(\eta = 0)$.

To compute the business-as-usual scenario, we omit conditions (34) – (38) and we set $P_t^{M} = P_t^{MU} = P_t^{ML} = P_t^{TE} = P_t^{TL} = 0$ for all t, since the representative agent (RA) ignores the

[10] Although conditions $(2')$ and $(6')$ are substituted for (2) and (6) in deriving the first-order conditions (research and crowding out externalities), the original equations (2) and (6) govern the aggregate accumulation of K and H.

environmental consequences of his actions.[11] Note that it is the agent's perceived shadow values of the environmental stocks which are zero. In contrast, the true marginal effects are expected to be positive and significant. For BAU, we assume a constant rate of pure time preference of 3 per cent per annum $(\eta = 0.03)$. We compare a variety of assumptions for both the planner's optimum and BAU about what is internal and external, in particular whether the agent takes into account knowledge spillovers and the scarcity of carbon fuels.

The optimal environmental policy with exogenous R&D is computed by fixing R_t in all periods to match the level in BAU. This solution requires omitting conditions (30) and (32), since R and H are fixed exogenously in all periods. Otherwise, this solution is the same as the planner's optimal policy described above.

V. Revised calibration

We accept Popp's initial values of variables, his price and climate parameters, as well as his choices for γ, δ_K, β, and *crowd*. We also use his 0.35 elasticity value, notwithstanding our reservations. However, we dispense with the calibration of the 4:1 ratio of energy savings to R&D, as it is already verified by condition (39) (see the previous section).

We rely on conjecture and sensitivity analysis for our values of b, φ, and *exgscale*, since we do not possess any useful empirical information. We expect some duplication of research effort in the economy, and therefore $b < 1$. However, we do not expect too much, and therefore we assume a value of $b = 0.75$. (In contrast, Popp's calibration results in a value of 0.2 for b, which suggests extensive duplication.) For φ, we perform sensitivity analysis at values $\varphi = (0.50, 0.25, 0.10, 0.00, -0.10, -0.25, -0.50)$. (Popp's calibration yields $\varphi = 0.55$.) For *exgscale*, we perform sensitivity analysis at values $(0.2, 0.50)$, i.e. a 20 percent reduction in exogenous technical change compared with DICE 99 (Popp's value), and a 50 percent reduction.

We also employ a different approach to the calibration of A_1, α_F, α_H, *a* and ρ. To calibrate these parameters, we begin by obtaining an empirical measure of initial energy services, EN_1. To do this, we define an efficiency factor, EFF, for the world energy system in 1990, which measures the proportion of energy input converted into useful energy output. We then calculate

$$EN_1 = EFF \cdot F_1. \qquad (41)$$

[11] We use the term "representative agent" in the context of BAU and "planner" in the context of the optimal policy.

F_1 is provided in Popp (2004) and we derive an estimate of $EFF = 0.48$ from Ziagos and Berry (2006) and Borg and Briggs (1991).[12] With this information, we solve (1) and (7) for A_1, given Y_1, K_1, L_1, EN_1, TE_1, F_1, and P_1.

To calibrate α_F, we begin by substituting condition (28) into condition (29) and dividing through by P_t^Y, yielding

$$\beta\left(\frac{Y_t + P_t F_t}{EN^\rho}\right)\alpha_F \frac{F^{\rho-1}}{\Phi^\rho} = -\Delta \cdot \frac{P_t^M}{P_t^Y} - \Delta \cdot \frac{P_t^{Cm}}{P_t^Y} + \frac{P_t}{\Phi_t}. \quad (42)$$

In BAU, $P_t^M = P_t^{Cm} = 0$, in which case the expression above reduces to

$$\beta\left(\frac{Y_t + P_t F_t}{EN^\rho}\right)\alpha_F \left(\frac{F}{\Phi}\right)^{\rho-1} = P_t,$$

which is just the competitive outcome that the marginal product of F (left-hand side) equals its price.[13] Solving for α_F in the initial period, and taking advantage of (41) and $\Phi_1 = 1$, gives us a solution conditional on ρ, β and first-period values:

$$\alpha_F = \frac{P_1 F_1 EFF^\rho}{\beta(Y_1 + P_1 F_1)}. \quad (43)$$

We then rearrange (3) to obtain a calibration of α_H, conditional upon α_F, ρ and first-period values:

$$\alpha_H = \frac{EN_1^\rho - \alpha_F F_1^\rho}{H_1^\rho}. \quad (44)$$

(Note that $\hat{F}_1 = F_1$, since $\Phi_1 = 1$.)

We calibrate *a* and ρ using a nested iterative approach. We start with arbitrary values of *a* and ρ (given values of b, ϕ and *exgscale*). Then we solve for α_F and α_H following (43) and (44). The lower level of the nesting calibrates ρ conditional upon *a*. Holding *a* constant, we iterate on

[12] From Ziagos and Berry (2006), we derive estimates of 0.504 for world energy efficiency and 0.430 for US energy efficiency in 2005. From Borg and Briggs (1991), we derive an estimate of 0.408 for US energy efficiency in 1990. Borg and Briggs do not present world data. To obtain an estimate of world energy efficiency in 1990, we scale our 2005 world estimate by the 1990:2005 ratio of US values. In other words, we assume the same rate of efficiency improvement for the world as for the US.
[13] In fact, Popp assumes the agent takes into account the effect of his use of fuel (F) on the price (P), even in BAU. In that case, $P_t^{Cm} \neq 0$. For the purpose of calibration, we assume that P_t^{Cm} is small enough in the first period that we can ignore it.

ρ, updating α_F and α_H with each iteration, to achieve the desired research elasticity of 0.35. Calculating the elasticity involves comparing the results from BAU with those from the optimal scenario (endogenous R&D). Moving to the upper level, we then iterate on *a*, updating ρ, α_F and α_H with each iteration, until R_1 equals the target value of 0.01.

For the purpose of calculating the elasticity, Popp defines research as the ratio R_t/Y_t, in order to neutralize scale effects on the level of R_t. For simplicity, let r_t represent this normalized value; i.e.

$$r_t \equiv \frac{R_t}{Y_t}. \tag{45}$$

The energy price used to calculate the elasticity includes the market price P_t as well as any shadow prices which influence the agent's choices. From (42), we see that these shadow prices may include P_t^M/P_t^Y and P_t^{Cm}/P_t^Y, depending on the scenario.[14] Thus the full price of carbon fuels includes terms relating to pollution damage and scarcity, as well as the production cost. Under business-as-usual, the pollution damage term, P_t^M/P_t^Y, equals zero, since it represents an externality. Under the optimal policy scenario, however, this term would be positive, representing either the value of a tax on carbon emissions or the price of tradable emissions permits. As for the scarcity rent, P_t^{Cm}/P_t^Y, Popp assumes that this value is internalized in both BAU and optimal policy. We assume this value is internalized in BAU in some scenarios and externalized in others. We always assume it is internalized in optimal policy. For simplicity, let π_t represent the full price of carbon fuel; i.e. the right-hand side of (42).

Popp defines both inter and intra-temporal research elasticities respectively as

$$\frac{r_{t+1}^* - r_t^*}{\frac{1}{2}(r_{t+1}^* + r_t^*)} \bigg/ \frac{\pi_{t+1}^* - \pi_t^*}{\frac{1}{2}(\pi_{t+1}^* + \pi_t^*)} \tag{46}$$

and

$$\frac{r_t^* - \hat{r}_t}{\frac{1}{2}(r_t^* + \hat{r}_t)} \bigg/ \frac{\pi_t^* - \hat{\pi}_t}{\frac{1}{2}(\pi_t^* + \hat{\pi}_t)}, \tag{47}$$

[14] The normalization of each shadow price by the marginal utility of income, P_t^Y, casts these values in terms of the consumption numeraire.

where the asterisk and hat notation indicate alternative policy scenarios, e.g. optimal policy versus BAU. Equation (46) defines the elasticity between periods for the same policy scenario, whereas (47) defines the elasticity across policy scenarios for the same period.

The mechanism of induced innovation follows from the substitutability of H for F. If the price of F increases *ceteris paribus*, we would expect the agent to respond by undertaking more research, thus increasing H in subsequent periods. However, the price of fuel, π, is not the only determinant of R. In a given period, the choice of R will also depend on the existing level of H, through the research function (6), and on the level of Y, through the material balance condition (17). Since these values change between periods and between policy scenarios, there is an important conflict between the *ceteris paribus* requirement and the elasticity definitions (46) and (47).[15]

We acknowledge Popp's effort to control for variations in Y through the normalization (45). However, we are not persuaded that this is adequate to meet the test of *ceteris paribus*. For one thing, the relationship between Y and R may not be linear. Second, there is no evidence that this approach adequately controls for variations in H. For these reasons, we reject Popp's use of the intertemporal elasticity (46) and his calibration strategy based on a diminishing value of this elasticity over time.

Further, we note the importance of only applying the intra-temporal definition (47) when Y and H will be held constant across scenarios. The only period which meets this criterion is the first – due to the exogenous values of the initial stocks. Therefore, for our calibration, we apply (47) to measure the research elasticity between the optimal policy and BAU in the first period only. We do not regard the values obtained in subsequent periods as meaningful, due to the failure of *ceteris paribus*.

We perform the calibration under Popp's assumptions of a declining rate of time preference and damage function (8) (benefits associated with moderate warming).

[15] Inspection of (39) indicates that the choice of R also depends upon the shadow values P^H and P^Y. However, changes in these values do not conflict with the ceteris paribus assumption, since these are the mechanisms through which the price of fuel is expected to act on R.

VI. Results

1. Measuring welfare gain

Popp employs a "difference-in-difference" metric for summarizing the welfare gain of induced innovation. The first difference refers to the improvement in welfare resulting from the optimal environmental policy, with R&D held constant, compared with BAU. The second difference refers to the change in this welfare gain which results from adding induced R&D to the model. In particular, the change in welfare is calculated as

$$\Delta W = \left[\frac{(W_{end,op} - W_{BAU})}{(W_{exg,op} - W_{BAU})} - 1 \right] \times 100 \quad (48)$$

where $W_{end,op}$ represents the welfare value (equation 23) under optimal policy with endogenous technical change, $W_{exg,op}$ represents the welfare value under optimal policy with exogenous technical change (R_t constrained to the BAU level), and W_{BAU} represents welfare in the BAU scenario.[16]

It is also tempting to compare the results of endogenous technical change with a model without any R&D (see Gerlagh 2008 for an example.) However, such a comparison would seem to address the wrong question. We are not aware of any dynamic assessment models which do not take account of technical change in one way or another. The key question is thus not whether there is technical change in the models, but whether it will make much difference for climate policy if it is modeled endogenously or exogenously. In ENTICE, if we were to eliminate R&D, then we would have to return the exogenous trend $1/\Phi_t$ to 100 percent (set *exgscale* = 0). But since $1/\Phi_t$ is speculative to begin with, the comparison would not be particularly informative. For this reason, we do not provide such a comparison. In contrast, Popp's metric (48) gives an assessment of the incremental welfare attributable to R&D in the optimal environmental policy, conditional upon a given trajectory of exogenous progress. Since BAU and the optimal scenarios share the same exogenous trend, we anticipate that changes in this trend may not affect the *incremental* value of R&D in the optimal policy very much. Popp (2004) confirms this expectation with his sensitivity analysis on *exgscale*, and we confirm it again below.

[16] In fact, Popp bases his measure of ΔW on discounted consumption, rather than W. Fussel (2007) shows that this can lead to important inconsistencies, and therefore we use the W values directly.

2. results

Table 1 presents our results under different assumptions about the knowledge parameter, φ, the rate of time preference, η, and the treatment of knowledge spillovers, crowding out, and scarcity. Our first comparison – which we refer to as experiment A – presents only modest departures from Popp's baseline assumptions. In particular:

- *Inter-firm* knowledge spillovers and research crowding out are external for both the planner and the RA.
- *Inter-temporal* knowledge spillovers and scarcity effects are internal for both the planner and the RA; i.e. equations (32) and (33) are operative.
- The damage function is the quadratic form (8) which exhibits benefits from moderate warming.
- $oppcost = 3$
- The planner (optimal policy) and the representative agent (BAU) share the same constant rate of time preference $\eta = 0.03$.

Experiment B differs from experiment A only in that it assumes monotonic damages, corresponding with equation (25). We also refer to this case as "high damages" since there is no benefit associated with moderate warming. We expect that the welfare gain from induced innovation will be greater under high damages since the payoff to substituting for carbon fuel is greater in this case. The results in the table confirm our expectation, as the welfare gain attributable to induced innovation rises from an average value (averaged across values of φ) in experiment A of 3.64 percent to 9.78 percent in experiment B. In contrast, there does not appear to be any relationship between welfare gain and φ in these two experiments.

Experiment C differs from B in assuming that inter-temporal knowledge spillovers and scarcity effects are external for the RA but internal for the social planner; i.e. in BAU equation (32) is replaced by $(32')$ and (33) is inoperative. We observe both a marked increase in the scale of welfare effects and the emergence of a distinct pattern in relation to φ. In particular, the welfare effects range between 154 and -34 percent, in direct relationship with φ.

The negative values of ΔW corresponding with negative values of φ in Experiment C can be explained by the interaction of the inter-firm and inter-temporal knowledge spillovers. The inter-firm spillover causes the RA to underestimate the value of R&D and therefore underinvest in it, compared with a first-best social optimum. Conversely, for negative values of φ, the intertemporal spillover causes the RA to over invest in R&D ceteris paribus. To see this, note that the inter-temporal spillover factor (40) has a negative sign when $\varphi < 0$. An agent who takes this effect into account (e.g. the social planner) will therefore invest less in R&D ceteris paribus, while an agent who does not take it into account (e.g. the RA) will invest more in R&D. But in this case, "over investing" due to ignorance of the inter-temporal spillover counteracts the problem of underinvestment due to the inter-firm spillover. No such counteracting occurs for the social planner, since she takes into account the intertemporal spillover but continues to ignore the inter-firm spillover. Thus the BAU level of R is likely closer to the first-best optimal level, even though the RA fails to take into account the effects of scarcity and pollution.

The marked increase in the scale of ΔW between experiments B and C suggests that the combination of fuel scarcity and the intertemporal knowledge spillover is more important in ENTICE than the climate externality. In order to determine the relative order of importance, we compare levels of R_t between the following scenarios:

- inter-temporal knowledge spillover, scarcity, and pollution **internalized** (optimal policy in experiments B and C),
- inter-temporal knowledge spillover and scarcity **internalized**, pollution **external** (BAU in experiment B),
- inter-temporal knowledge spillover **internalized**, scarcity and pollution **external**,
- inter-temporal knowledge spillover, scarcity and pollution **external**,
- **no scarcity** ($\omega_2 = 0$ in equation 9), inter-temporal knowledge spillover and pollution **external.**

All of these scenarios are based on the assumptions $\varphi = 0.5$, $oppcost = 3$, high damages (equation 25), $\eta = 0.03$, and externalized inter-firm spillovers and research crowding-out. Figure 2 presents the results of the comparisons. We use the first scenario (optimal policy) as the baseline, represented by a value of 100. The remaining scenarios are BAU in that pollution is external.

In Figure 2, we observe that the BAU level of R_t when scarcity and the intertemporal knowledge spillover (ITH) are internalized is only slightly lower than the optimal-policy level.[17] Indeed, at no time during the planning horizon does the BAU value fall below 99 percent of the optimal-policy value. As this comparison isolates the effect of pollution policy, we conclude that accounting for greenhouse gas emissions has only a very minor influence on the level of energy R&D in ENTICE.

The next comparison involves making fuel scarcity external – i.e. turning off the first order condition (33) in the solution of BAU. This change lowers R_t modestly compared with the previous scenario, starting after 2050. But again the effect is slight, as R_t falls no lower than 98.6 percent of the optimal-policy baseline in 2170. Therefore we conclude that awareness of the endogenous nature of fuel scarcity (realizing the effects of our consumption today on scarcity tomorrow) is not a major driver of R&D spending in ENTICE.

The next scenario adds the intertemporal knowledge spillover to the list of externalities in BAU (i.e. removes (40) from condition (32)). Now we observe a more significant drop in the level of R_t – to 88.7 percent of the baseline in 2000, rising gradually over time to 96 percent in 2200. The assumption of $\varphi = 0.5$ means that the intertemporal knowledge spillover is strong and positive. Therefore, on its own it would provide a strong incentive for accumulating human capital. It follows that making it external to the RA's decision making would remove an important incentive for spending on R&D.[18] This comparison indicates that the intertemporal knowledge externality has a much greater influence on R&D spending than either pollution policy or concern about the endogeneity of fuel scarcity. Nonetheless, the level of R_t remains relatively high in BAU even after removing this influence.

Above we considered the effect of externalizing the endogenous nature of fuel scarcity. Nonetheless, the agent may still have an expectation of an increasing fuel price, without realizing his contribution to the increase. In this case, the agent would view the increasing trend as exogenous, and such a trend would in itself provide an incentive to substitute from carbon fuels to energy-related human capital. In order to test the influence of increasing fuel prices on R&D,

[17] Note that the optimal-policy level here is not first-best optimal, since it still treats the inter-firm knowledge spillover and research crowding-out as external. Therefore we refer to it as "the optimal policy level" rather than "the optimal level".

[18] This result would be reversed for negative values of φ. In such cases, internalizing ITH would cause the agent to reduce R&D ceteris paribus, while externalizing it would cause the agent to increase R&D.

we now consider a scenario in which there is no scarcity; i.e. the parameter ω_2 in equation 9 is set to zero, resulting in a constant fuel price in all periods. Figure 2 indicates that this change does not have a significant affect on the level of R_t compared with the previous scenario until after 2100. This delay is explained by the combination of relatively high discounting in BAU ($\eta = 0.03$) and the strong convexity of the price function (9) (i.e. the exponent of 4 on Cm_t). Strong convexity means that the price function is quite flat for the first 100 years of the solution. Combined with high discounting, this feature means that the agent doesn't respond to the scarcity problem vary far in advance. Rather, she waits until scarcity "kicks in" before responding with increased levels of R&D. Gerlagh (2008, p. 444) makes the same observation.

We are left with two very important insights from these comparisons. First, whatever differences we observe in the levels of R&D between BAU and optimal policy in ENTICE are driven mostly by assumptions regarding the inter-temporal knowledge spillover. In comparison, climate change and fuel scarcity have relatively little influence.

Second, we still end up with a surprisingly high level of R&D spending in BAU, even after making the intertemporal knowledge spillover external. In Figure 2, we see that R_t in BAU never goes much below 89 percent of the optimal-policy level, no matter what we do. This result raises the question of what is driving the accumulation of R&D in BAU after all the influences above have been stripped out. A moment's reflection makes the answer clear: diminishing returns to consumption and capital investment (K) make R&D spending increasingly attractive over time simply because there is no other alternative in the model.

This point can be seen clearly in condition (30). Rewriting the condition for an interior solution, we see

$$P_t^Y = \frac{\Delta \cdot P_t^H (1-\ell) ab H_t^{\varphi}}{R_t^{1-b}}.$$

Diminishing marginal utility of income over time (left-hand side) is met in part by increases in R_t. Put another way, the opportunity cost of R&D decreases over time as returns to other activities diminish.

We are not persuaded that this feature represents an accurate description of reality. Not only are there many alternative sectors, but the particular relationships between sectors may prove to be quite important. Gerlagh's (2008) findings in this regard are quite illuminating. He models four research sectors, including carbon energy savings (efficiency of consumption) and carbon energy production. In his model, R&D expenditures on carbon savings displace almost entirely R&D expenditures on carbon energy production. In ENTICE, innovation in carbon energy production would translate into a downward shift in the fuel price equation (9).

There are many examples of technological innovation in the domain of carbon energy production. For example, seismic testing and horizontal drilling techniques have enabled oil and gas firms to locate and develop reserves which otherwise would have been impossible to exploit. Moreover, conventional expenditures on exploration and development (E&D) have the same effect on prices as technical innovation, as both increase the amount of effective energy available to the economy. For this reason, it seems appropriate to consider R&D and E&D expenditures together in the carbon production sector – call the aggregate of the two RE&D.

Total expenditures on RE&D in the energy sector are typically large in any given year. [Provide data here.] Moreover, in a world in which greenhouse gas emissions and other pollutants associated with carbon energy are underpriced (or not priced at all), total expenditure on energy RE&D is likely to be excessive. Figure 3 demonstrates this point. In the absence of (i) a pollution policy and (ii) spending on RE&D, the carbon fuel price path in the economy is represented by line *ab*. The socially optimal path is higher, represented by *cd*, as it includes the prices of the pollution externalities, through for example Pigovian taxes or prices of tradable emissions permits. In contrast, in the absence of effective pollution policies, spending on RE&D has the effect of pushing the price path down to *ef*. Of course, some amount of spending on RE&D is probably desirable in the social optimum. Therefore this analysis should be understood as applying only to a certain portion of RE&D. Nonetheless, this portion may be large, and it represents an important misallocation of capital.

Given this misallocation, displacement of a certain amount of RE&D in energy production by R&D in energy efficiency would entail an additional benefit to the economy rather than an opportunity cost. Stated another way, energy efficiency R&D would provide the economy with a double dividend: the first benefit in the form of technological improvement, and the second benefit in the form of increased economic efficiency associated with reducing the level of RE&D in energy production. In this light, the assumption of 3 additional units of opportunity cost for energy R&D in ENTICE appears inappropriate.

Further research is required to model the interaction between carbon saving R&D and carbon production RE&D in an explicit manner in ENTICE. For now, we consider results under the assumption $oppcost = 0$, which at least eliminates the inappropriate measure of opportunity cost. In Table 1, experiment D replicates experiment B and experiment E replicates experiment C, both with $\text{oppcost} = 0$. We observe in both cases a significant increase in the scale of effects, consistent with expectations. In particular, for positive values of φ in experiment E, we see that the presence of induced innovation can provide a very large increase in social welfare (278 percent in the case of $\varphi = 0.5$), which contrasts with the rather modest result obtained by Popp (8 percent). [Misleading: it is an increase in the increase in social welfare.]

[Comparisons based on $\eta = 0$ here.]

[Comparisons of *exgscale* here.]

VII. Conclusion

Popp (2004) presents ENTICE as an empirically calibrated model to provide meaningful estimates of the impact of induced innovation in climate policy. He concludes that the impact is positive but modest (8 percent gain), owing to the restraining effects of inter-firm knowledge spillovers and crowding out of other research.

In contrast, we have found this result to be largely an artifact of the model structure of ENTICE. Our results show that energy related R&D in ENTICE is driven mainly by (i) the intertemporal knowledge spillover, and (ii) the lack of alternative sectors for investment. In contrast, the effect of climate policy on R&D is very minor, and therefore welfare gains attributable to induced R&D in the model are also minor.

We find Gerlagh's inclusion of multiple research sectors much more realistic. In particular, we argue that in the absence of an effective pollution policy, the economy will over-invest in carbon-production R&D, as well as over-investing in conventional carbon-production exploration and development. This feature means that, rather than possessing an opportunity cost, energy-efficiency R&D (carbon-saving) leads instead to a double dividend: first the benefit of technical innovation, and second the benefit of reducing the distortion of excessive investment in carbon production. Further research is needed to model these interactions explicitly in the ENTICE framework. Nonetheless, our preliminary results show that, similar to Gerlagh, the scale of benefits flowing from induced innovation may be significant.

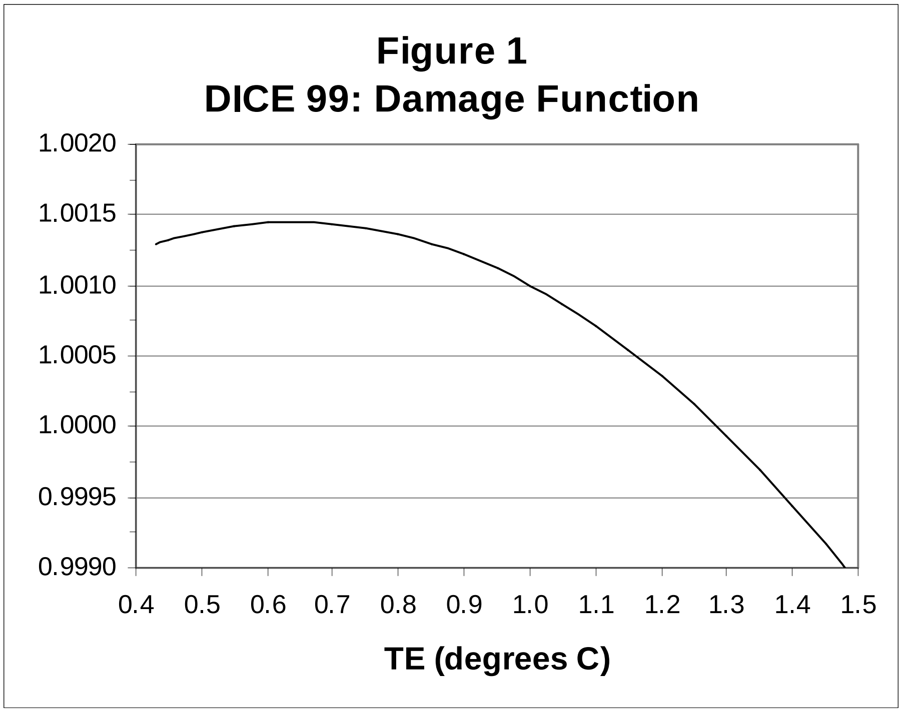
Figure 1
DICE 99: Damage Function
1.0020
1.0015
1.0010
1.0005
1.0000
0.9995
0.9990
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
TE (degrees C)

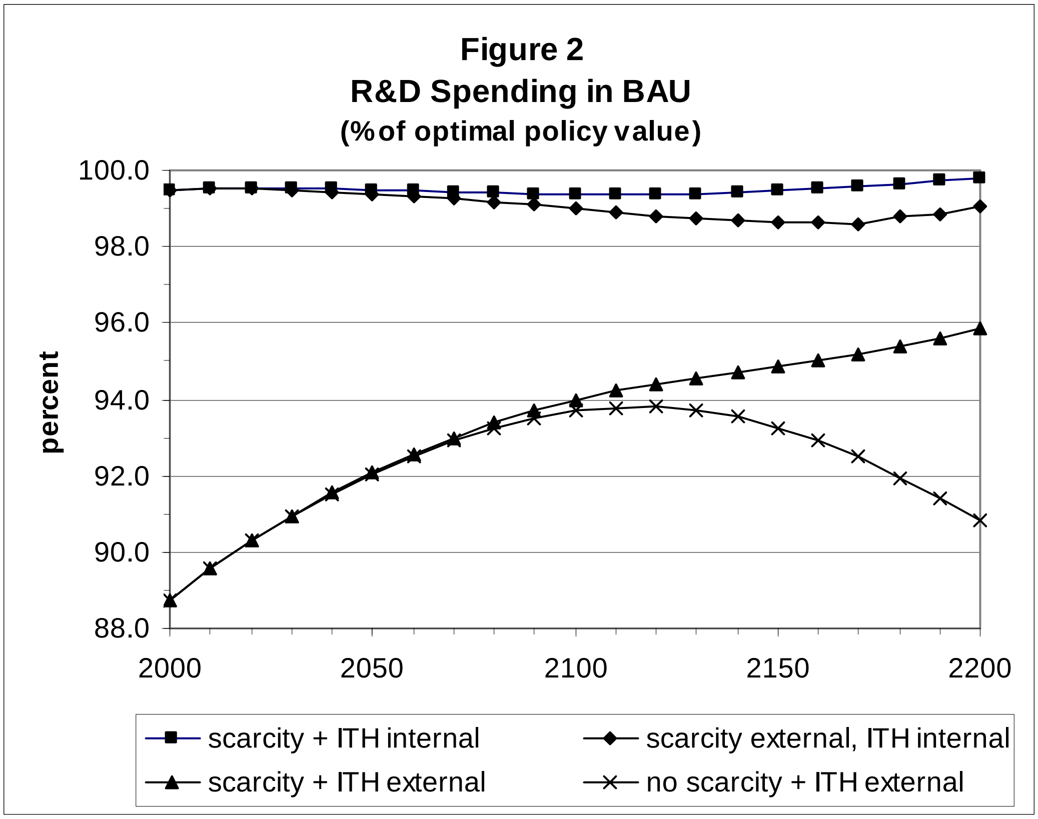

Figure 2
R&D Spending in BAU
(% of optimal policy value)
percent
100.0
98.0
96.0
94.0
92.0
90.0
88.0
2000
2050
2100
2150
2200
scarcity + ITH internal
scarcity external, ITH internal
scarcity + ITH external
no scarcity + ITH external

Figure 3

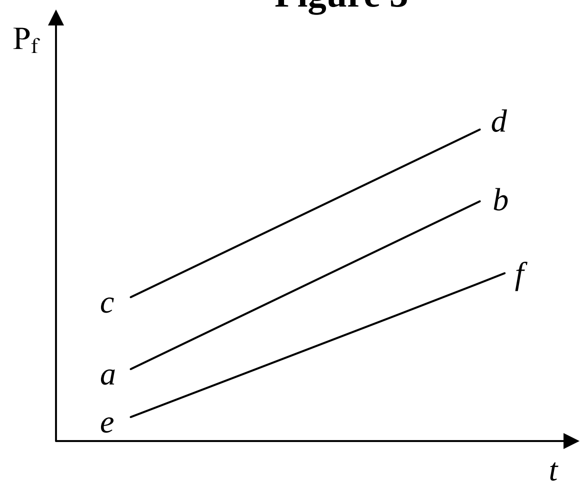

Table 1
Welfare Gain (ΔW) due to Induced R&D in Optimal Policy (%)

φ	A	B	C	D	E
0.50	10.18	13.71	153.67	24.05	278.06
0.25	0.13	7.84	115.09	22.17	82.90
0.10	2.13	6.82	12.63	12.72	30.18
0.00	4.32	13.74	0.90	19.10	6.89
-0.10	3.93	5.01	-5.91	17.27	-7.96
-0.25	3.11	12.90	-11.58	16.37	-28.55
-0.50	1.65	8.43	-34.27	51.71	-42.46
average	3.64	9.78		23.34	

Experiment A:
- *inter-firm* knowledge spillovers and research crowding-out **external** for planner and RA,
- *inter-temporal* knowledge spillovers and scarcity effects **internal** for planner and RA,
- low damages (benefits from moderate warming),
- $oppcost = 3$,
- $\eta = 0.03$ for planner and RA.

Experiment B:
- same as A except:
- high damages (no benefits from moderate warming).

Experiment C:
- same as B except:
- *inter-temporal* knowledge spillovers and scarcity effects **internal** for planner, **external** for RA.

Experiment D:
- same as B except:
- $oppcost = 0$

Experiment E:
- same as C except:
- $oppcost = 0$

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