Dynamic analysis of macroeconomic policies in an asymmetric monetary union. Lessons for the EMU

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Abstract

Most of central banks are currently using the nominal interest rate to decide on the monetary policy, instead of controlling monetary aggregates, as assumed in the IS-LM model. Consequently, alternative static models for the analysis of macroeconomic policies were proposed in the literature, replacing the LM curve by an interest rate monetary rule (Villieu, 2004; Carlin & Soskice, 2006). Our work further develops these studies in two directions: it models an open economy with flexible exchange rate and endogenous long-run product, not detailed before, and secondly, it proposes an extension of the analysis in a dynamic context. We apply this dynamic framework to study monetary and fiscal policies in an asymmetric monetary union, like the Euro Area. From this perspective, our study is close to Clausen & Wohltmann (2005) who provides such a dynamic analysis, but still considers a central bank that controls monetary aggregates. The present study introduces an interest rate monetary rule and also highlights the importance of the public expenditures financing in the model. The policy-mix question is a key issue of our study, which focuses on three main questions: How the central bank must react to stabilize the union after fiscal shocks? What is the impact of structural asymmetries on the stability of member countries when the monetary policy acts to stabilize the union as a whole? How to integrate these asymmetries in the European policy-mix to improve its performance simultaneously at an aggregate and at a national level?

Keywords: open economy, fiscal and monetary policy, interest rate monetary rule, asymmetric monetary union
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The traditional IS-LM model was, until a few years ago, the reference in explaining the macroeconomic policies. However, the work of *Romer (2000)* represented the beginning of a series of papers questioning the pertinence of the IS-LM framework for the study of the macroeconomic policies, since the main instrument of the monetary policy is the interest rate, and not a monetary aggregate as predicted by the IS-LM model.

The main critics regarding the use of the IS-LM model is related to the modelling of the monetary policy which does not describe anymore to the behaviour of the central banks. If the traditional LM curve models a "mass" policy, the central banks use nowadays a "rate" policy. They act directly on the interest rate as instrument of the monetary policy, without considering an intermediary instrument such as a monetary aggregate. Hence, the LM consideration becomes less important for the analysis and a more adequate instrument, such as an interest rate rule, like that proposed by *Taylor (1993),* is favoured.

Authors like *Romer (2000, 2002), Abraham-Frois (2003)* or *Pollin (2003)* conceived alternative models which explain the effects of the macroeconomic policies without using the LM curve. The main conclusions of these different models are considered by *Villieu (2004)* who provides a more general model for the analysis of macroeconomic policies. This static model provides the base of a new approach in analysing the economic policies in a closed economy and an extension is also presented for open economies, assuming that the long run global supply is exogenous and represents the natural product of the economy.

Related to these models, the IS-MR-PC[1] model of *Carlin and Soskice (2006)* represents another alternative of the IS-LM static model, according particular attention to the modelling of the labour market and to the determination of the equilibrium unemployment rate, defined as the unemployment rate allowing a constant inflation in the economy. This equilibrium value of unemployment is unique and exogenous in a closed economy, but it depends on the real exchange rate in open economies. As particular feature in the open economies, the long-run global supply is not anymore exogenous (see *Villieu, 2004*) but it can be stimulated by a real appreciation of the national currency.

Concerning the European economies, there are several reasons to consider this category of models as being the most suitable for the analysis of the economic policies:

1) Unlike the United States where the long-run unemployment rate seems to be around a constant value on a long period, in Europe, persistent trends on the unemployment rates can be found *(Carlin and Soskice, 2006)*. This specificity makes inappropriate to consider a constant long-run global supply, which is synonym to an invariant employment rate on the labour market.

2) In Europe, the power of the labour unions being important, it induces imperfections of the labour market. The determination of the long run level of output by modelling this market should also consider these imperfections, which result in a level of the real wages higher than the equilibrium wage level on a competitive market. These imperfections could come from collective negotiations of the wage level by the labour unions instead of individual negotiations or simply from the companies who search the quality of the labour market.

Modelling an imperfect labour market in an open economy leads to an equilibrium equation in which the employment rate increases with the real appreciation of the national currency *(Carlin and Soskice, 2006)*. As the labour market equilibrium stands only in the medium or in the long run, this solution is equivalent to an endogenous global supply, which can be stimulated by a real appreciation of the national currency. Introducing such a relation in the macroeconomic models leads to more general conclusions, more appropriate to the particular features of the European economies.

In this paper, a simple general equilibrium model is assumed in which the real exchange rate determines the potential level of output. This model is based on the IS-LM alternative models

[1] *Investment Saving equilibrium – Monetary Rule – Phillips Curve*

previously described and proposes an *extension of the analysis of fiscal and monetary policies in a dynamic context, particularly oriented for the case of an asymmetric monetary union*, like the Euro zone.

The asymmetry occurs in the transmission of the interest rate to the real economy, the model being related, hence, to that of *Clausen and Wohltmann (2005)*, denoted hereafter by *CW*. The *CW* model is an example of dynamic analysis of economic policies in an asymmetric union based on the IS-LM model, using a "mass" monetary policy. In the case of non-anticipated macroeconomic shocks (which are also treated in this paper), their conclusions are the following: the asymmetry of the interest rate transmission in the union generates asymmetric adjustments in the member countries and changes in the efficiency of common policies in different countries in time; regarding the coordination of the macroeconomic policies, it seems that, in the case of a symmetric expansionary fiscal policy shock, a restrictive common monetary policy can ensure a perfect stability of the revenues in the union.

Passing to a "rate" policy, similar results apply for the Euro zone. The present model also deals with potential undesirable effects of the fiscal expansion by introducing a second change in the *CW* hypotheses. It considers a sovereign risk premium associated to the public debt, which completes the relation of the current account balance equilibrium in each country and increases the cost of the public debt, depending on the public debt level. It also shows that the effect of a rise of the public expenditures in the economy depends on their financing. If this expansion is financed by an increase of the taxes in the Union, it does not lead to higher inflation or higher interest rate and it can be efficiently used to stimulate the economic activity. On the contrary, if the public spending expansion is financed by additional debt, once an indebting threshold is attained, it induces an increase of the interest rate and of the inflation, which requires a restrictive monetary policy in order to establish the equilibrium. The capacity of the monetary policy to stabilize such a fiscal shock depends also on the debt level, and, in case of high debt ratios it may even become unable to re-establish the equilibrium.

Taking as reference the situation proposed by *CW* model for explaining the effects of a fiscal policy expansion in Europe, the consideration of a "rate" monetary policy and of the sovereign risk premia allows us to revise some of their previous results. The ability of the common monetary policy to ensure national stability after a symmetric fiscal shock is questioned. In the present study, if the global stabilisation of the union is not affected by considering the risk premium, the individual reactions of the national economies are asymmetric inside the union and the national divergences become stronger after the stabilizing intervention of the central bank.

This conclusion is extremely interesting for the management of the macroeconomic policies, in the case of a mix of monetary and fiscal policies compatible with a healthy economic growth in Europe. In the *Section 5* of this paper such a policy mix becomes incompatible with the absence of a fiscal cooperation among the member countries of the union. The optimal solution might be a centralised fiscal policy conducted by a multinational government, who shares the expenditures between the member countries on the bases of a rule taking into account the structural characteristics of each member state. Inducing a unique risk premium in the European zone, such a monetary policy could have more facility to ensure its stability objectives. In order to minimise the costs of a deviation from the optimal behaviour by one of the participants to the policy-mix, the monetary policy should have the prices stability as its main objective, keeping the real activity as a secondary one.

The rest of the paper is organised as follows. In Section 2, we present the model for an asymmetric union of two countries. Section 3 makes a *global* analysis of the effects of fiscal and monetary policies in the union. This study summarizes the behaviour of a single open economy, representative for the union and is an example of how this model can be used in the case of a single country. The Section 4 discusses the implications of the asymmetries in the transmission of policies at a *national level* and the Section 5 raises two main questions on the policy-mix: *how the monetary* *policy must react in the case of an expansionary fiscal policy shock?* and *how taking into account the structural asymmetries in the definition of the policy-mix inside the union?* The last section concludes.

Section 2. The model

Our model grounds on *CW model* but replaces it in a new context, more suitable to characterize the Euro zone. We built a model for a simple monetary union formed of two countries of equal size, with an asymmetric transmission of the interest rate at national level. The first change introduced in our model regards the modelling of the monetary policy by changing the LM equation in the CW model by an interest rate rule equation. The second innovation regards the Governments' policy. In order to keep the national dimension of the conduct of this policy in Europe, the change consists in introducing a risk premia for Governments, with consequences in an important rise of the financing cost of public expenditures by debt in the case of excessive debt level[2]. The main relations of the model are the following:

$$y_1 = a_0 + a_1(y_1 - \tau_1) - a_{21}(i - \dot{p}_1^c) + g_1 + (b_0 - b_1 y_1 + b_2 y_2 + b_3 \tilde{y} - b_4(p_1 - p_2) - b_5 \upsilon_1) \qquad (1a)$$

$$y_2 = a_0 + a_1(y_2 - \tau_2) - a_{22}(i - \dot{p}_2^c) + g_2 + (b_0 - b_1 y_2 + b_2 y_1 + b_3 \tilde{y} - b_4(p_2 - p_1) - b_5 \upsilon_2) \qquad (1b)$$

$$\upsilon_i = p_i - (\tilde{p} + e) \qquad (2)$$

$$y_i = a + \mu\, l_i \qquad (3)$$

$$\dot{p}_i = \dot{w}_i = E(\dot{p}_i^c) + \delta(y_i - \bar{y}_i) \qquad (4)$$

$$\bar{y}_1 = f_0 + f_1\overline{(p_1 - p_2)} + f_2 \overline{\upsilon}_1 \qquad (5a)$$

$$\bar{y}_2 = f_0 + f_1\overline{(p_2 - p_1)} + f_2 \overline{\upsilon}_2 \qquad (5b)$$

$$p_1^c = \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3(\tilde{p} + e) \qquad (6a)$$

$$p_2^c = \alpha_1 p_2 + \alpha_2 p_1 + \alpha_3(\tilde{p} + e) \qquad (6b)$$

$$\dot{i} = \omega\left[\hat{\dot{p}}^c + \bar{r} + \beta_1(\dot{p}^c - \hat{\dot{p}}^c) + \beta_2(y - \hat{y}) - i\right] \ , \ 0 < \omega < 1 \qquad (7)$$

$$i_1 = \tilde{i} + \dot{e} + \zeta_1^g \qquad (8a)$$

$$i_2 = \tilde{i} + \dot{e} + \zeta_2^g \qquad (8b)$$

The first two equations (1a) and (1b) describe the *IS equilibrium* for each of the member states. The main components of the global demand are present. The consummation depends directly on the net revenue after taxation $(y_i - \tau_i)$, the investment is a decreasing function of the real interest rate, the public expenditures (g_i) complete the global demand definition and the last bracket represents the net result of the current account balance. The export of the county *i* depends directly on the revenues of the importer countries (y_j - revenue of the other country in the zone and $\tilde{y}$ - the revenue of the rest of the world) and inversely on the real appreciation of the common currency. The imports depend directly on the national revenue and on the appreciation of the currency. The real appreciation of the common currency is modelled by a variation in time of υ_i, expression of the real exchange rate (relation (2))

[2] This hypothesis uses the result of *Hallerberg and Wolff (2006)*, who found that the fiscal policy represents the most important determinant of the debt risk prime.

and hence of the "external" competitiveness of the country outside the union; the term $(p_i - p_j)$ offers information on the trade exchanges inside the union. The real interest rate (r_i) is modelled as the difference between the nominal interest rate (i_i) and the anticipated inflation, computed in consumer prices: $r_i = i - E(\dot{p}_i^c)$. The rational anticipations hypothesis on the inflation and that of the absence of systematic errors in the anticipation of the agents are assumed, allowing the following relation: $r_i = i - \dot{p}_i^c$.

Each coefficient represents an elasticity or semi-elasticity of the global demand components on the factors specified in their definition. All the coefficients are the same for the two countries, excepting the sensibility of the global demand y_i to the changes of the real interest rate $(a_{21} \neq a_{22})$, which induces an asymmetry of the monetary transmission on the model.

In the equation (2) which defines the external competitiveness of the country, p_i and $\tilde{p}$ are the producer prices index in the country i and in the rest of the world and *e* represents the nominal exchange rate.[3]

In order to model the *global supply* a simple technology function is used, depending only on the labour factor $(Y_i = AL_i^{\mu})$, written in logarithm in the equation (3). *L* represents the labour and A is the global productivity of the factors. The producer prices follow the wages dynamics, as in the relation (4). The indexation of the wages takes into account the economic growth compared to the equilibrium growth and the anticipations on the consumer prices index. Hence, the relation (4) is a Phillips curve augmented by rational inflation expectations.

The long run equilibrium output $(\bar{y}_i)$ is endogenous and represents the potential product resulting from the equilibrium condition on the labour market (equation (5a) and (5b)). In the Appendix 1 it is proven that the equality of the demand and supply on the labour market allows modelling the long run product of each country as a function of its trade competitiveness with other countries of the union $(p_i - p_j)$ and with the rest of the world (υ_i)[4].

The intuitive explanation of this result is the following: on the labour market, the demand depends inversely on the real wage level computed in producer prices, while the labour supply depends directly on the real wage, in consumer prices. In an open economy, the consumer prices index depends on the national consummation structure and hence, on the internal prices (α_1), on the prices of other trade partners in the union (α_2) and on the prices of partner countries in the rest of the world (α_3), as in the relations (6a) and (6b). When these competitiveness terms (υ_i) and/or $(p_i - p_j)$ increase, the importation prices decrease compared to the national prices determining a decrease in the consumer prices inflation. In the case of rational expectations on the consumer prices dynamics, it corresponds to a decrease of the expectations on the consumer prices index which will result in a decrease of the real wages in producer prices, because the indexation of the wages follows the expectations on the consumer prices in the relation (4). In consequence, the employment rate increases at equilibrium and determines in the equation (3) a rise of the potential output.

Regarding the national consumption structure, an usual hypothesis in the models of open monetary union with two countries is adopted: the preferences for final goods produced in the two countries are identical: $\alpha_1 = \alpha_2$[5].

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[3] Defined as X common currency units for one foreign currency units

[4] *Carlin and Soskice (2006)* obtain a similar long-run supply when modeling an imperfect labour market.

[5] *Lane (2001),* makes a review of the papers using this hypothesis. Its utility in our model will be discussed in the following paragraph.

In the alternative approach of IS-LM, the central bank uses an « interest rate » policy, following the rule (7), where $\dot{p}^c = \frac{1}{2}\left(\dot{p}_1^c + \dot{p}_2^c\right)$; $y = \frac{1}{2}(y_1 + y_2)$; $\hat{p}^c$, $\hat{y}$ represent the monetary inflation and output targets, i is the nominal interest rate in the union, $\bar{r}$ is the real equilibrium interest rate and the coefficients β_1 and β_2 respect the Taylor properties $\beta_1 > 1$[6] and $0 < \beta_2 < 1$.

With this rule, the monetary authority pays attention to the mean performances of the union and uses a policy targeting aggregated objectives, without considering what is happening at a national level – behaviour similar to that of ECB in Europe. The presence of the term ω in the rate rule (7) allows adding to the model a manner of smoothing the dynamics of the interest rate in time[7]. This is an important characteristic of the euro zone emphasized in papers as *Clarida and al. (1998), Sack and Wieland (2000), Sibi (2002), Gerlach-Kristen (2003), Sauer and Sturm (2004)* or *Carstensen (2006)*. In fact, due to ω in the equation (7), the interest rate varies less than in a simple Taylor monetary rule confirming its relative dependence on the past levels.

As in *Villieu (2004)*, the targets $\hat{p}^c$, $\hat{y}$ of the monetary policy represent possible instruments of the central bank intervention. In the long-run equilibrium $\left(\dot{i} = 0\right)$ and taking into account the definition of the equilibrium nominal interest rate $\left(\bar{i} \equiv \hat{p}^c + \bar{r}\right)$, the relation (7) allows a relation of the long-run inflation rate $\bar{p}^c = \hat{p}^c + \frac{\beta_2}{\beta_1 - 1}(\bar{y} - \hat{y})$, allowing a discussion on the choice of the monetary instruments. Hence, if the output target chosen by the central bank is the potential product $\left(\hat{y} = \bar{y}\right)$, the long-run inflation corresponds to the inflation target. This is the ideal case when the central bank keeps is commitment in terms of inflation and the economy is in its optimal state. This requests perfect information of the central bank on the evolution of the potential level of output, hypothesis contested by the empirical studies *(Orphanides, 2003; Gerberding and al., 2005)*. In fact, the potential product represents an unobservable variable for the central bank which is obliged to ground its decisions on imperfect estimations, using data often revised after their first publication *(Mishkin, 2007)*. This remark is suitable for a heterogeneous union where everything depends on the quality of the data provided to the central bank by the member states and introduces the issue of the arbitrage between the monetary policy objectives when the revenue target differs from the potential product of the union.

The last equations of the model (8a,b) reflect the perfect capital mobility condition. The equilibrium of the current account balance requests the respect of the Uncovered Interest Parity condition *(UIP)* in each of the countries. In order to take into account the impact of different financing sources of the public expenditures in the model, risk premium are introduced for each country of the union, varying with the national public debt and augmenting the *UIP* relations by an additional term. For rational expectations, without systematic forecast errors of the agents, the exchange rate expecations correspond to the reality and we obtain the relations (8a,b), in which $\tilde{i}$ is the interest rate in the rest of the world, $E(\dot{e})$ represents the exchange rate dynamics expectations and ζ_i^g is the risk premium of the country *i*, which will be defined as an increasing function of the global amount of the public expenses financed by debt $\left(\zeta_i^g = f(g_i)\right)$.[8] The debt is not explicitly modelled, but it appears in the amount of the public expenses non-financed by taxes.

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[6]Using the same reasoning as in *Villieu (2004)* the following relation can be written :
$\dot{i} = \omega\left[\hat{p}^c + \bar{r} + \tilde{\beta}_1\left(\dot{p}^c - \hat{p}^c\right) + \beta_2(y - \hat{y}) - i\right] = \omega\left[\hat{p}^c + \bar{r} + \beta_1\left(\dot{p}^c - \hat{p}^c\right) + \beta_2(y - \hat{y}) - i\right]$, where $\beta_1 = 1 + \tilde{\beta}_1 > 1$.

[7] *Van Aarle and al. (2004)*, discuss the possible reasons of such a smoothing of the monetary instrument in Europe.

[8] The choice of two different financing risk premia for the two countries in the model corresponds to the present estate of the European Union. Each country has its own fiscal policy and has its own risk premium for the excessive debt. A

Short description of the method used in the following sections

In order to analyse the impact of different macroeconomic policies shocks in the union, we use the *Aoki(1981) decomposition method* and we define two sub-systems starting from our basic model: an *aggregated system*, whose variables are described, in a general manner, by : $x = \frac{x_1 + x_2}{2}, \forall x$, very useful in the study of the global behaviour of the union (*section 3*) and a *difference system* with variables such as $x^d = \frac{x_1 - x_2}{2}, \forall x$ which allows analysing the individual behaviour of the member countries by a simple combination of *« aggregated »* variables and *« difference »* variables : $x_1 = x + x^d, \forall x$ et $x_2 = x - x^d, \forall x$ *(sections 4 and 5).*

Section 3. Macroeconomic policies at the Union-wide level

The aim of this section is threefold. Using the model defined in the *section 2*, we seek to obtain the long-run features of the union-wide equilibrium, to analyze the impact of fiscal and monetary policies on this equilibrium and the dynamic adjustments they cause during the return of the economy to equilibrium.

In order to model the Union as a whole, we follow the *Aoki (1981)* decomposition. We use individual equations from our model *(section 2)* to write an *aggregated system*, explicitly developed in the second part of the *Technical Appendix.* Then, this *aggregate system* can be easily reduced to a two-dimensional dynamic system, in which the nominal interest rate i and the external term of trade υ are the *state variables.*

$$\begin{cases} \dot{\upsilon} = X(\upsilon - \overline{\upsilon}) \\ \dot{i} = \omega\Psi(\upsilon - \overline{\upsilon}) + \omega(\beta_1 - 1)(i - \overline{i}) \end{cases} \tag{9}$$

where: $X = -\frac{b_5\delta}{\eta\alpha_3 - a_2(1-\alpha_3)\delta} < 0$ and $\Psi = \frac{X}{\delta}[\beta_1\delta(1-\alpha_3) + \beta_2\alpha_3] < 0$.

The analytical solution of the *aggregate system* conducts to the following features for the union-wide equilibrium **(I)**[9]:

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simplifying hypothesis of this general case using a unique risk premium for all countries in the union would allow analysing the case of the governments with a common management of the national policies, proposal often discussed nowadays in the European Union.

[9] The steady-state value of a variable $x = \frac{x_1 + x_2}{2}, \forall x$ is denoted by $\overline{x}$ in **(I)** , $k = a_0 + b_0 + b_3\tilde{y}$, $\overline{\varsigma}_d^g = \frac{\overline{\varsigma}_1^g - \overline{\varsigma}_2^g}{2}$, $a_2 = \frac{a_{21} + a_{22}}{2}$ and $\breve{a}_2 = \frac{a_{21} - a_{22}}{2}$.

$$
\begin{array}{l}
\bar{\upsilon} = \dfrac{1}{\eta f_2 + b_5}\left[k - a_2\bar{r} - \breve{a}_2\bar{\varsigma}_d^g - a_1\bar{\tau} + \bar{g} - \eta f_0\right] \; ; \; \bar{\dot{p}}^c = \hat{\dot{p}}^c - \dfrac{\beta_2}{\beta_1 - 1}(\bar{y} - \hat{y}) \\
\bar{y} = \dfrac{1}{\eta f_2 + b_5}\left[f_2\left(k - a_2\bar{r} - \breve{a}_2\bar{\varsigma}_d^g - a_1\bar{\tau} + \bar{g}\right) + b_5 f_0\right] \\
\bar{r} = \tilde{r} + \bar{\varsigma}^g \; ; \; \bar{i} = \bar{r} + \bar{\dot{p}}^c \; ; \; \bar{\dot{e}} = \bar{\dot{p}} - \tilde{\dot{p}} = \bar{\dot{p}}^c - \tilde{\dot{p}}^c
\end{array}
\qquad \textbf{(I)}
$$

In the long-run, the output and the competitiveness of the union depend on the real interest rate $(\bar{r})$, implicitly on the foreign interest rate $(\tilde{r})$ and on the aggregate risk premium $(\bar{\varsigma}^g)$, on changes in the governments' policy (government expenditures or taxation) and on the risk premium differentials within the union $(\bar{\varsigma}_d^g)$. To be more precise, there is the external term of trade $\bar{\upsilon}$ who changes to ensure the *IS* equilibrium, and leads to the adjustment of the potential output in the equation describing the labor market equilibrium[10]: $\bar{y} = f_0 + f_2\bar{\upsilon}$.

The long-run inflation in consumer prices depends on the inflation target of the monetary policy and on the deviation of the output target from its potential level. All gap between domestic and foreign rate of inflation results in movements of the nominal exchange rate. Thus, if the domestic inflation is higher than the foreign inflation, the common currency depreciates at a rate $\bar{\dot{e}}$, allowing the equilibrium real interest rates (adjusted by an aggregate sovereign risk premium $\bar{\varsigma}_g$ in the union) to be identical inside and outside the union.

If the output target of the common central bank $(\hat{y})$ corresponds to the potential output $(\bar{y})$, its inflation target $(\hat{\dot{p}}^c)$ will be reached in the steady state. Because the main objective of the monetary policy is the price stability in the Union, the respect of the inflation target is essential, mainly if public expectations are forward-looking. Starting from this result, it seems clear that the central bank has no reason to choose an output target different from the potential output of the union. However, the central bank has not perfect information on the future potential output. The monetary decisions are based on some estimations of the potential output whose quality essentially depend on the quality and precision, often uncertain, of the observable data *(Mishkin, 2007).* This is one of the reasons that the output target of the monetary policy can deviate from the potential output and empirical proofs of such deviations in US or Germany were recently found by *Orphanides (2003)* or *Gerberding & al. (2005).*

The determinant of the Jacobian Matrix corresponding to the *dynamic system* (9) is unambiguously negative and the aggregate system displays stable saddle path toward the steady state. The *Blanchard & Kahn (1980)* Theorem conditions for the stability of the saddle path are fulfilled: there is one stable root $(\lambda_1 = X < 0)$ associated to a predetermined variable, here the *interest rate,* and one unstable root $(\lambda_2 = \omega(\beta_1 - 1) > 0)$ associated to a "jump" variable, here the *real exchange rate*. The predetermined character of the interest rate is explained by the presence of the coefficient ω in the monetary policy rule (7), while the nominal exchange rate e freely fluctuates and gives rise to the jump of τ. This allows the union to join a saddle path with positive slope, result confirmed by the sign of the stable eigenvector component v_{11} of the Jacobian matrix:[11]

[10] These features of the long-run equilibrium can also be found from the traditional analysis of the global demand and supply in the union, as discussed in *Badarau-Semenescu (2008)*.
[11] For details on the Jacobian matrix ans its eigenvalues and eigenvectors expressions, see our technical appendix.

$$v_{11} = \frac{\delta[X - \omega(\beta_1 - 1)]}{X\omega[\beta_1\delta(1-\alpha_3) + \beta_2\alpha_3]} > 0 \quad (10).$$

3.1 Macroeconomic policy analysis

To simplify the discussion on the dynamic adjustments of the union after macroeconomic shocks, we assume an initial steady state of the economy $(\overline{\upsilon}_0, \bar{i}_0)$. When shocks arise, the economy leaves its initial steady state and heads for a new equilibrium modified by the presence of shocks. We will consider in this study the simple case of *unanticipated* monetary and fiscal shocks and we assume that they arise at a moment *T*, so that the impact on the economy appear only after this moment *T* and not before. The solution of the dynamic system (9) describing the adjustments after shocks is:

$$\begin{pmatrix} \upsilon_t \\ i_t \end{pmatrix} - \begin{pmatrix} \overline{\upsilon} \\ \bar{i} \end{pmatrix} = C_1 v_1 \exp(\lambda_1 t), \forall t \geq T \quad (11).$$

Two elements must be taken into consideration when analyzing a shock:

- Its effect on the steady-state, computed by the multipliers of monetary and fiscal variables in the equilibrium **(I).**
- The adjustment of the union towards the new steady state, denoted by $(\overline{\upsilon}_1, \bar{i}_1)$. Written in *T*, the dynamic equation corresponding to the predetermined interest rate in (11) allows us to compute the constant C_1: $C_1 = -d\bar{i}\exp(-\lambda_1 T)$, where $d\bar{i}$ gives the interest rate differential between the initial and the final equilibrium. The jump of the forward variable (υ) in *T* would be: $\upsilon(T+) = \overline{\upsilon}_1 + C_1 v_{11}\exp(\lambda_1 T)$ and, for all $t \geq T$, we could compute: $\upsilon_t = \overline{\upsilon} + C_1 v_{11}\exp(\lambda_1 t)$. This last equation allow us to characterize the adjustment of υ, and of all the others variables of the union, towards the final steady state.

3.1.1. Monetary Policy

In this model, the behavior of the central bank is described by the interest rate monetary rule (7). She must, at every moment, respect this rule, but she could however conduct his monetary policy by modifying some exogenous terms of the rule. As *Villieu (2004)* previously considered, we also use the inflation and output targets chosen by the common central bank $(\hat{y}, \hat{p}^c)$ as instruments of the monetary policy. A restrictive monetary shock will thus be associated either to a cut in the inflation target $(\hat{p}^c)$, or to a cut in the output target $(\hat{y})$, with similar effects on the future adjustment of the economy *(Badarau-Semenescu, 2008)*.

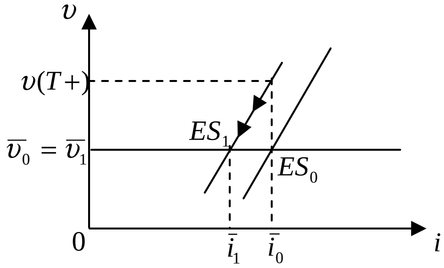

Fig. 1 Dynamic effects of a restrictive monetary shock

The **Figure 1** synthesizes the union-wide global dynamic reaction to a restrictive monetary shock in the system of the steady variables (υ, i), while **Figure 2** describes the individual adjustment of the main aggregates after the shock. The dynamic adjustment of the union towards the new steady state ES_1, in **Fig. 1**, is similar to thus found in the *Dornbusch (1976)* model. There is an initial overreaction of the real exchange rate, corresponding to $\upsilon(T+)$ in **Fig.1**, necessary to ensure the *IS* equilibrium, followed by an adjustment of υ down to its initial equilibrium level.[12]

The sign of monetary multipliers in **(I)** confirms the movement of the steady state after the shock: $\frac{d\bar{y}}{d\hat{y}} = \frac{d\bar{\upsilon}}{d\hat{y}} = 0; \frac{d\bar{y}}{d\hat{\dot{p}}^c} = \frac{d\bar{\upsilon}}{d\hat{\dot{p}}^c} = 0; \frac{d\bar{i}}{d\hat{y}} = \frac{d\bar{\dot{p}}^c}{d\hat{y}} = \frac{\beta_2}{\beta_1 - 1} > 0; \frac{d\bar{i}}{d\hat{\dot{p}}^c} = \frac{d\bar{\dot{p}}^c}{d\hat{\dot{p}}^c} = 1.$

The impact of shock on the real activity y_t is not permanent. The nominal exchange rate depreciates and the inflation goes down in the last graph of **Fig. 2**, explaining the progressive regain of external competitiveness by the Union. The difference with the *Dornbusch (1976)* model comes from the downward adjustment of the inflation rate after the monetary shock, instead of the adjustment of the general level of prices. This kind of reaction seems to be more realistic, corresponding to a positive steady-state inflation rate. In our model, the equilibrium between the global demand and supply takes into account such a “natural” increase in prices, each period.

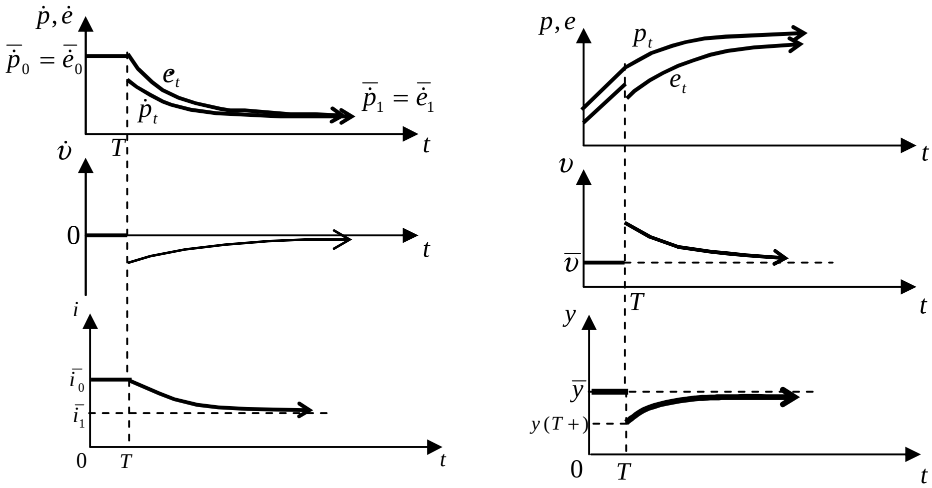

Fig. 2 Dynamic adjustments in the Union after a restrictive monetary shock

The initial overshooting of the exchange rate consequent to the shock results in expectations of a lower consumer price inflation, which implies a smaller indexation of wages and explains the negative jump of the inflation rate $(\dot{p})$ in *T*. During the adjustment path, the inflation continues to fall because the common currency is still strong relative to the equilibrium. However, until the steady-state, wages are under-indexed to domestic inflation; domestic prices fall relative to foreign prices and the common currency depreciates. The exchange rate depreciation is always more important than the fall in the inflation rate and ensures the convergence of the external term of trade υ.

[12] From (11), υ_t is a downward and convex function: $\dot{\upsilon}_t = \lambda_1 C_1 v_{11} \exp(\lambda_1 t) < 0$ et $\ddot{\upsilon}_t = \lambda_1^2 C_1 v_{11} \exp(\lambda_1 t) > 0$.

The common interest rate decreases during the adjustment path[13]; it stimulates the aggregate demand and supports the dynamics of the future exchange rate expectations (in the UIP condition (8)).

3.1.2 Fiscal policy

As fiscal shock, we consider an unexpected increase in governments' spending in the Union and we study the reaction of the Union to the shock, by introducing two different financing for public expenditures: 1) an *autonomous financing* by an increase in taxes within the Union and 2) an *external financing* coming from a new issues of treasury bonds on the international market, for example. This last case corresponds to an upward adjustment of the global external debt of the Union, with potential negative consequences on the cost of the debt, if a higher risk premium is associated to the increased debt.

1) *Governments' expenditures financed by an increase in taxes at the same period:* $dg = d\tau$

Computing the multipliers associated to the public spending in **(I)** conducts to:

$$\frac{d\bar{\upsilon}}{d\bar{g}} = \frac{1-a_1}{nf_2+b_5} > 0; \frac{d\bar{y}}{d\bar{g}} = \frac{f_2}{nf_2+b_5}(1-a_1) > 0; \frac{d\bar{i}}{d\bar{g}} = \frac{d\bar{p}^c}{d\bar{g}} = -\left(\frac{\beta_2}{\beta_1-1}\right)\left(\frac{f_2}{nf_2+b_5}\right)(1-a_1) < 0\,^{14}.$$

They express a positive effect of the governments' spending on the potential output, a real appreciation of the common currency and a fall in inflation to which the common central bank responds by cutting interest rates in the Union. As the financing of the additional public spending comes from taxes, there is no negative impact on the risk premium ς^g, and this term doesn't influence the values of budgetary multipliers. The global adjustment of the union towards the new steady-state (ES_1) is drawn in **Fig. 3.**

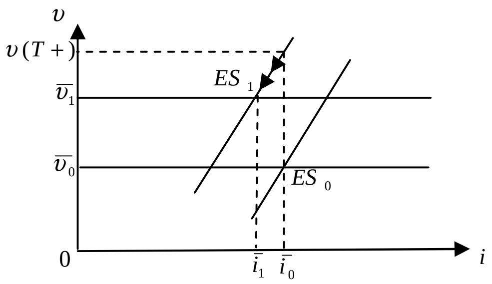

Fig. 3 Union-wide reaction to a public spending expansion with fiscal financing

Like for the restrictive monetary shock, the key point to explain the dynamic adjustment of the union toward the equilibrium after the fiscal shock is the overreaction of the real exchange rate in the sense of an instantaneous appreciation of the common currency after the shock. The responses of the main aggregates of the union to shock are similar to those described in **Fig. 2**, with the only difference

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[13] In (15), for all $t > T$: $i_t - \bar{i}_1 = C_1 \exp(\lambda_1 t) > 0$, $\dot{i} = C_1\lambda_1 \exp(\lambda_1 t) < 0$ and $\ddot{i} > 0$.

[14] In (1a) and (1b), a_1 gives the marginal propensity to consume, satisfying: $0 < a_1 < 1$.

that the final steady-state (ES_1) change, being characterized by a higher level of output and an appreciation of the common currency relative to the initial equilibrium.

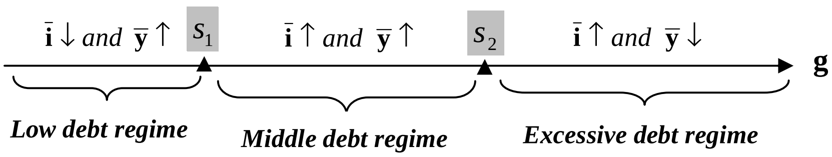
2) Governments' expenditures financed by debt

The impact of this assumption in the model depends on two key elements: *a)* the level of the Union's public debts before the issue of new debt instruments on the market, and *b)* the form of the relation describing the adjustment of the risk premium ς^g when the level of the Union's public debt moves upward.

According to the sign of the fiscal policy multipliers in **(I)**, three situations could be differentiated, that we define as: *low debt regime, middle debt regime* and *excessive debt regime.* We thus write the expression of the two key multipliers describing the long-run reaction of the real activity and of the common interest rate to the fiscal policy:

$$\frac{d\bar{y}}{d\bar{g}}=\frac{f_2}{\eta f_2+b_5}\left(1-a_2\frac{d\bar{\varsigma}^g}{d\bar{g}}-\breve{a}_2\frac{d\bar{\varsigma}_d^g}{d\bar{g}}\right) \text{ and } \frac{d\bar{i}}{d\bar{g}}=-\frac{\beta_2}{\beta_1-1}\frac{f_2}{\eta f_2+b_5}\left(1-a_2\frac{d\bar{\varsigma}^g}{d\bar{g}}-\breve{a}_2\frac{d\bar{\varsigma}_d^g}{d\bar{g}}\right)+\frac{d\bar{\varsigma}^g}{d\bar{g}}.$$

The analysis of the sign of these multipliers gives rise to two threshold levels s_1 and s_2, for $\frac{d\bar{\varsigma}^g}{d\bar{g}}$. These levels mark the separation between the three different regimes, as shown in the following schema: [15]



- In the *low debt regime,* the initial level of the Union's debt is low and the increase in the risk premium is very limited after the public spending expansion $\left(\frac{d\bar{\varsigma}^g}{d\bar{g}}<s_1\right)$. In this situation, the impact of the fiscal policy in the Union would be similar to that exposed in the case of the financing of additional expenditures by taxes. The Governments' policy could stimulate the economic growth in the Union, without having a negative impact on inflation.

- In the *excessive debt regime,* the initial level of debt is already high and the increase in governments' expenditures financed by debt would produce a high increase in the aggregate risk premium $\left(\frac{d\bar{\varsigma}^g}{d\bar{g}}>s_2\right)$. This time, the government policy become counterproductive and conducts to a real depreciation of the common currency in the long-run. Consequently, the real activity is reduced in the long run and the inflation is higher. In dynamics, after the initial over- depreciation of the common

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15 $s_1=\left[1-\breve{a}_2\left(\frac{d\bar{\varsigma}_d^g}{d\bar{g}}\right)\right]/\left[a_2+\frac{\beta_1-1}{\beta_2}\left(\frac{\eta f_2+b_5}{f_2}\right)\right]<s_2=\left[1-\breve{a}_2\left(\frac{d\bar{\varsigma}_d^g}{d\bar{g}}\right)\right]/a_2$

currency relative to the equilibrium, the adjustment of the Union is resumed in a continuous appreciation of the real exchange rate and an increase of the interest rate, explaining the reduction of the output down to its steady state level. Such a fiscal policy expansion would be very dangerous in a period of economic recession because, instead of supporting the real activity, it would conduct to an inevitable worse recession that before (see also *Magud, 2008*). In the same time, a restrictive intervention of the central bank to respond to the higher inflation is not desirable in this case, getting even worse the situation of the real activity.

- In the *middle debt regime*, the previous schema tells us that the long-run impact of a public spending expansion financed by debt corresponds to a real appreciation of the common currency with positive effect on the output, and to an increase of the interest rate in the Union.[16]

Clausen and Wohltmann (2005) recognize, in such adjustments, the specific reaction of the Euro area to an expansionary public expenditures shock. In order to have an uniform base for comparing the results of our dynamic analysis of macroeconomic policies in the presence of an interest rate rule for the central bank, to the results of a similar analysis when the monetary policy is described by a traditional LM curve *(Clausen and Wohltmann, 2005),* we concentrate our attention on this *intermediary regime* for the rest of the paper.

The dynamic reaction of the Union face to a public spending expansion in depicted in **Fig.4**. This time, there is no overreaction of the real exchange rate after the shock, but an insufficient reaction of this term, explained by the raise in the producer price inflation in the relation (4), despite the lower anticipated consumer prices inflation. There is an initial real appreciation of the common currency, but it is insufficient to attain the new steady-state, so that the common currency continues to appreciate until the new equilibrium (ES_1). Consequently, the Union output strongly reacts to the fiscal policy expansion in the short run, and a positive significant effect appears in *T*. Then, the continuous appreciation of the currency explains a loss of external competitiveness for the Union during the adjustment process and its output subsequently goes down until attaining its equilibrium level. In term of inflation, from the point of view of an inhabitant of the Union, the more the common currency is strong, the cheaper are the foreign goods relative to the domestic produced ones, and the consumer prices inflation goes down. The upward adjustment of the interest rate is not due to a higher inflation rate, but to the overreaction of the current output compared to the output target of the monetary policy.

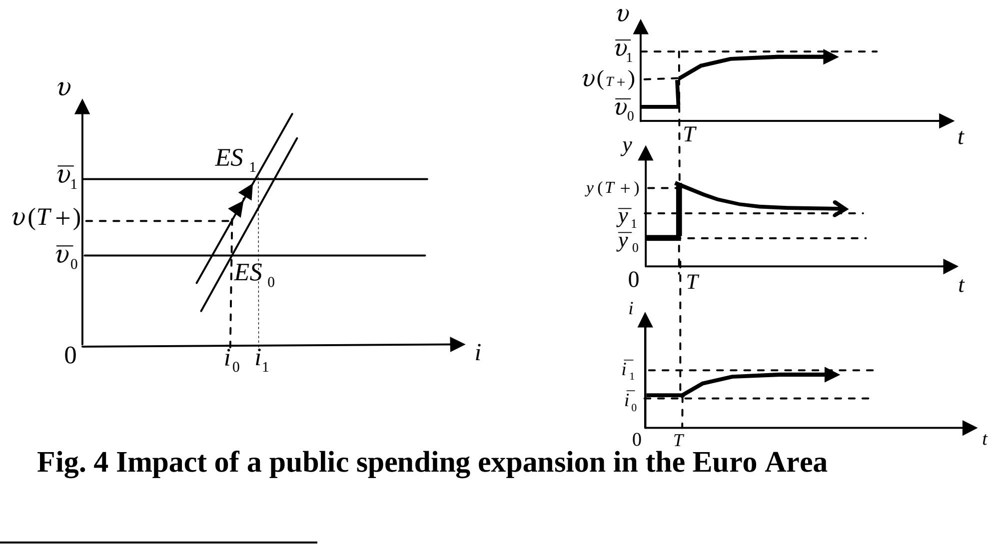

Fig. 4 Impact of a public spending expansion in the Euro Area

[16] Actually, in the interval (s_1, s_2), $\frac{d\bar{y}}{d\bar{g}} > 0$ and $\frac{d\bar{i}}{d\bar{g}} > 0$.

Taking a look on the effects of a restrictive monetary policy shock on the state variables of our economy **(Fig. 1)**, we easily observe some complementarity with the effects of a fiscal expansion developed above. This means that the common central bank could respond to a fiscal expansion by a restrictive monetary policy, in order to ensure the macroeconomic stability of the Union.[17]

Section 4. Structural asymmetries and the transmission of shocks at a national level

Until now, we focused on the behavior of the union as a whole face to macroeconomic shocks. But, since asymmetries exist among member countries, national reactions to shocks are expected to deviate from the union-wide behavior, asking for a particular attention of the authorities in the choice of economic policies. This section highlights these deviations when common symmetric shocks or individual asymmetric shocks hit the member states.

To analyze the asymmetries in the transmission of shocks among countries of the Union, we write the *difference system* by using the *Aoki (1981)* decomposition, we then find the reduced form of this system (see our technical appendix) and we solve the reduced *dynamic system.* Before introducing the discussion on the national dynamic adjustments after shocks, we summarize in the box **(II)** the steady-state solution of the *difference system*, in which: $x_d = (x_1 - x_2)/2,\ \forall x$.

$$\bar{p}_d = \bar{\upsilon}_d = \frac{\bar{g}_d - a_1\bar{\tau}_d - \breve{a}_2\bar{r} - a_2\bar{\varsigma}_d^g}{\mu(2f_1 + f_2) + 2b_4 + b_5};\quad \bar{r} = \tilde{r} + \bar{\varsigma}^g$$
$$\bar{y}_d = \frac{(2f_1 + f_2)(\bar{g}_d - a_1\bar{\tau}_d - \breve{a}_2\bar{r} - a_2\bar{\varsigma}_d^g)}{\mu(2f_1 + f_2) + 2b_4 + b_5};\quad \bar{i}_d = \bar{\varsigma}_d^g \qquad \textbf{(II)}$$

In the long run, the permanent divergences in the Union, synthesized in the model by the *difference variables* x_d, could come either from a divergent behavior of national Governments (in the case of autonomous conduct of the fiscal policy by each member country, giving rise to: $\bar{g}_d, \bar{\tau}_d, \bar{\varsigma}_d^g \neq 0$), or from asymmetries in the real interest rate transmission towards the real economy, introduced in the coefficient[18]. In what follows, we seek to better understand this last source of asymmetry and we will suppose that the national demand in country 1 is more sensitive to variation in the real interest rate than in country 2 $(a_{21} > a_{22})$, so that $\breve{a}_2 > 0$.

A different interpretation of the equilibrium **(II)** is that, permanent structural divergences in the Union cannot be reduced by monetary policy decisions. The monetary policy multipliers issued from **(II)** are equal to zero, and the central bank decisions have no impact on output divergences, in the long run. On the contrary, the fiscal policy could be successfully used to reduce these divergences, by

[17] This kind of reaction will be analyzed in more details in the section 5 of this study.
[18] Intuitively, another source of potential heterogeneity could come from the different real interest rates within the Union, explained by the presence of inflation divergences. However, because the main aim of this study is to understand the role of asymmetries in the transmission of shocks on national output dynamics, we neglect this additional source of divergence by considering, from the beginning: $\alpha_1 = \alpha_2$. In (6a) and (6b), this assumption implies identical inflation rate in consumer prices for the two countries, allowing us to perfectly separate, in **(II),** the effect of the asymmetric interest rate transmission on the macroeconomic divergences in the Union.

deciding on asymmetric changes in public expenditures of each country, with a particular attention paid to the financing sources of these expenditures.

Besides the *permanent effect* of asymmetries on output divergences in the Union, a second *temporary effect* arises during the transmission of macroeconomic shocks. This second effect can be highlighted by solving the *dynamic difference system*, whose solution becomes:

$$\upsilon_{dt} = \bar{\upsilon}_{d_1} - d\bar{\upsilon}_d \exp[\lambda_0(t-T)] + C_1 v_{11} \frac{\lambda_0\lambda_1\tilde{a}_2(1-\alpha_3)}{(\lambda_0-\lambda_1)(2b_4+b_5)}\{\exp(\lambda_1 t) - \exp[(\lambda_1-\lambda_0)T]\exp(\lambda_0 t)\}$$
$$y_d = \bar{y}_{d_1} + \frac{\dot{\upsilon}^d}{\delta} \qquad (12),$$

where λ_0, λ_1 are the eigenvalues of the Jacobian matrix of the system, with $\lambda_0 = -\frac{b_5+2b_4}{\mu}\delta < 0$, $\lambda_1 = X < 0$ and X, C_1, v_{11} are defined in the section 3 of the paper.[19]

To well separate the *temporary effect*, let's consider the simplest case, when the two countries of the Union are symmetrically hit by a common fiscal or monetary shock whose impact on the sovereign risk premium are identical in the two countries.[20]

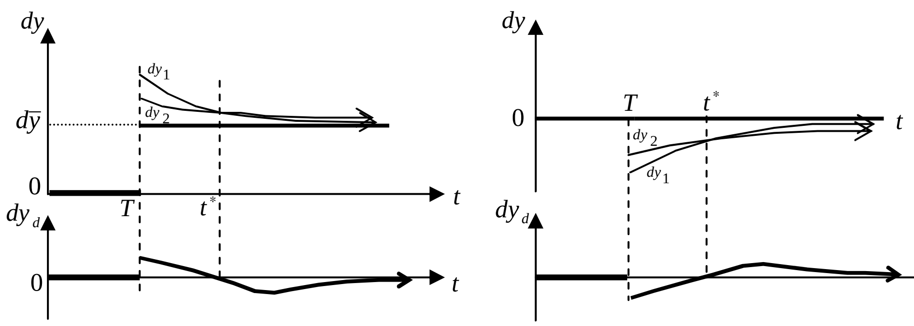

Fig. 5 National dynamic reactions in the Union face to a fiscal policy shock (on the left) or to a monetary policy shock (on the right)

In order to facilitate the comparison between the national adjustments, the following graphs represent the relative deviation of each variable from the initial steady state. Thus, the **Figure 5** depicts the adjustment of dy after a shock, where dy defines the deviation of the variable y from $\bar{y}_0$. With this transformation, we ignore the gap which existed between the initial steady-state of the two economies (the *permanent effect* of asymmetries), without any influence on the form of the adjustment paths towards the final steady-state. **Figure 5** compares the individual dynamic paths within the union after a common expansionary fiscal shock (on the left hand side) or a restrictive monetary shock (on the right hand side). We remark that, in the two cases, asymmetries occur in the union during the adjustment

[19] Details on all intermediary computations are reported in our Technical Appendix.
[20] Consequently, if an initial gap exists between the macroeconomic aggregates of the two countries, this gap persists without being amplified or reduced after the shock. This fact allows us to correctly separate and to compare the dynamic effects of the asymmetric interest rate transmission on national economic performances in the Union.

towards the steady state. The initial positive jump of dy_d at the moment T of an expansionary fiscal shock (negative for a restrictive monetary shock), followed by a gradual movement towards negative (respectively, positive) values, and then, by a return to zero in the long-run, confirms the asymmetric behavior of the two countries to shocks.[21]

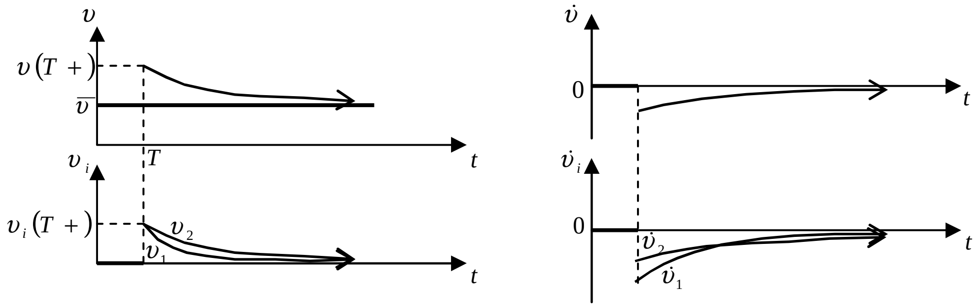

Fig. 6 Dynamics of the real exchange rate after a restrictive monetary shock

This asymmetry of behaviors disappears only in the long run, when the steady state is reached and shocks were completely transmitted in the economy.

The only cause of these different reactions comes from the asymmetric transmission of the interest rate in the Union. To understand this fact, let's take the example of the monetary policy. A restrictive monetary shock impacts the union by means of a spontaneous overshooting of the real exchange rate, followed by a real depreciation of the common currency until the steady state. In *T*, when the shock arises, the jump of υ is the same in the two countries of the union[22] and it corresponds to a symmetric initial real appreciation of the common currency (see **Fig. 6)**.

Subsequently, the real depreciation of the common currency $(\dot{\upsilon} < 0)$ is associated to a real interest rate in the union higher than the steady state level:

$$i - \dot{p}^c = \tilde{r} + \varsigma_g - (1 - \alpha_3)\dot{\upsilon} \qquad (13).$$

The dynamic adjustment of $\dot{\upsilon}$ corresponds to a decrease in the real interest rate during the adjustment process. The asymmetric transmission of this interest rate within the union explains why the output increases more quickly in country 1 than in 2, which is the reason of a reversal in the relative effectiveness of the common policy on output, in t^*. This moment, $t^* = T + \frac{1}{\lambda_1 - \lambda_0} \ln\left(\frac{\lambda_0}{\lambda_1}\right) > T$, [23] is identical to all macroeconomic shocks considered in the model and it is perfectly consistent with the reversal moment in the effectiveness of the common policy on the output of the two countries, highlighted by *Clausen and Wohltmann (2005).*

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[21] Each $dy_d \neq 0$ gives a proof of the existence of asymmetric effect of shocks at a national level.

[22] In (12), $\upsilon_d(T+) = \frac{C_1 \upsilon_{11} \lambda_1 \lambda_0 \tilde{a}_2 (1-\alpha_3)}{(\lambda_0 - \lambda_1)(2b_4 + b_5)} \left(\exp(\lambda_1 T) - \exp(\lambda_1 T)\right) = 0$

[23] For the determination of the expression of the moment t^*, see our technical appendix.

Section 5. Policy-mix and structural asymmetries in the Union

In this section, we seek answers to the following two questions: 1) *How the central bank must react to stabilize the union after expansionary fiscal shocks?* and *2) How these asymmetries could be considered in the policy-mix of the Union?*

To answer the first question, we start from the results of the *section 3* of this paper concerning the dynamic effects of shocks at the union-wide level. We firstly analyze the ability of the monetary policy to stabilize the Union aggregates after expansionary fiscal policy shocks, we then discuss the "optimal" behavior of the central bank from the point of view of the union as a whole and we wonder how member countries individually reacts to such an "optimal" behavior of the common central bank.

In a previous work *(Badarau-Semenescu, 2008)*, where we didn't introduce the discussion on the financing of public expenditures, modifying the output target of the monetary policy to the new level of potential output after the fiscal shock was the optimal reaction of the common central bank, allowing the stability of macroeconomic aggregates at the Union-wide level *and* at a national level too.

This result applies in the new context only if the additional public expenditures are financed by an increase in taxes inside the Union or by additional external debt considered *without risk* by the creditors. Otherwise, modifying the output monetary target to strictly respond to the variation of the potential output is insufficient to ensure the macroeconomic stability of the Union.

Given the neutrality of the monetary policy on the real activity in the long-run, a condition to ensure the stability of all aggregates in the Union is to perfectly suppress the impact of the fiscal expansion on the long run interest rate by an active monetary policy. In the equilibrium **(I)**, this condition amounts to:

$$d\bar{i} = d\hat{\bar{p}}^{c} + d\varsigma^{g} - \frac{\beta_2}{\beta_1 - 1}\left(d\bar{y} - d\hat{y}\right) = 0 \qquad (14).$$

The key elements to be taken into account in the choice of the monetary targets are the estimated reaction of the potential output after the fiscal shock and the effect of this shock on the aggregated sovereign risk premium. Many instruments can be used by the common central bank to satisfy the condition (14). She can modify his output target, change his inflation target, modify the stabilizing coefficients β_1, β_2, or use many of these instruments simultaneously.

If the central bank decides to exclusively modify his output target, his action must respect: $d\hat{y} = d\bar{y} - \frac{\beta_1 - 1}{\beta_2} d\varsigma^{g}$. The Union-wide economy would be in equilibrium after the shocks, due to the complementary effects of the fiscal and monetary policy around the final steady state. But, since the output target of the monetary policy deviates from the potential level of the output, the steady-state level of inflation deviates from inflation target announced by the central bank **(I)**. The central bank credibility could suffer from this, and this is an undesirable consequence mainly in an inflation target monetary regime, when the main objective of the central bank in the price stability.[24]

It seems rather clear that, in order to respect his inflation target, the common central bank should always choose an output target equal to the potential output. But, this is not a sufficient condition to ensure the Union stability when we simultaneously consider in the model a sovereign risk premium. In this context, the inflation target of the monetary policy must also be revised downwards, in such a way that the real interest rate of the Union becomes higher and suppresses the effect of the

[24] A similar reasoning applies when the common central bank decides to react to a fiscal shock by modifying only his inflation target or only the stabilization coefficients of his monetary rule.

higher risk premium on the international capital flows, in the *UIP condition.*[25] In (14), this monetary policy behavior is given by: $d\hat{y} = d\bar{y}$ and $d\hat{p}^c = -d\bar{\varsigma}^g$. Further to the increase in governments' spending in the Union, the central bank should react by revising upwards the output target, in line with the evolution of the potential output, and by simultaneously revising downwards the inflation target – sign of a restrictive monetary policy action necessary to the stabilization **(Figure 7)**.

As consequences of the restrictive monetary policy, υ jumps upwards in *T* (corresponding to d in **Fig. 7**, on the left). After *T*, there is a real depreciation of the common currency during the adjustment process towards the steady-state. After the monetary policy, the adjustment is described by: $\mathbf{ES_0} - d - \mathbf{ES_1'}$, in **Fig. 7**, while the reaction of the Union to the fiscal policy shock is given by: $\mathbf{ES_0} - d - \mathbf{ES_1}$, with an initial positive jump of υ (equal to *d*), followed by a real appreciation of the common currency until the steady state.

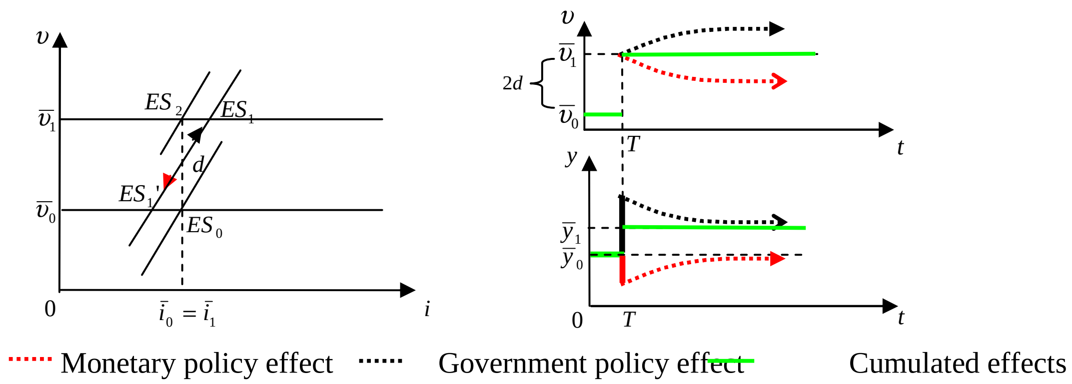


Fig. 7 Stabilizing policy-mix in the Euro-Area

As cumulated effect, the economy of the Union jumps directly to the new steady state $\mathbf{ES_2}$ and it remains to this equilibrium, due to the perfect symmetry of the two policies effects in the Union policy-mix (**Fig. 7,** on the right). The Union output finds instantaneously its equilibrium $\bar{y}_1$, in *T*, confirming the ability of a restrictive monetary policy to stabilize the real activity in the Union, after expansionary actions of Governments.

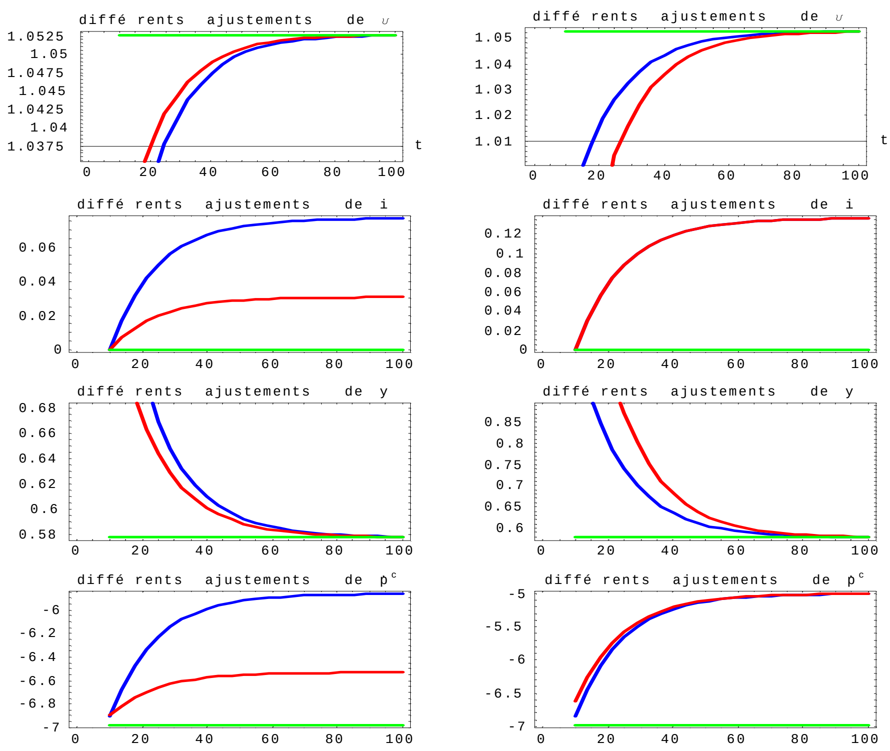
Role of β_1, β_2 coefficients for the macroeconomic stabilization

In the case of the optimal behavior summarized in **Fig. 7**, the central bank is able to ensure the stability of the Union, independently of the values of coefficients β_1, β_2 in the monetary rule. However, this optimal conduct of the monetary policy is not easy to implement, because it asks for a perfect foresight of the sensitivity of the real activity and of the risk premium to fiscal shocks, in order to correctly adjust the monetary targets. As discussed in the previous sections, this task is quite difficult and may conduct to an imperfect adjustment of the monetary targets after fiscal shocks, so that the monetary policy is no more optimal. So, it would be interesting to see what happens in the Union when the monetary policy response to fiscal shocks deviates from the optimum. The **Figure 8** collects the

[25] The central bank ability is however limited in this direction. If the risk premium on the debt can move very quickly in countries with excessive debt level, the fall in the inflation target is limited by the risk of deflation. So, the central bank ability to efficiently react to fiscal shocks is lost in excessively indebted countries.

results of some simulations of our model under two distinct assumptions*: 1)* uncertainty on the “good” level of potential output after shocks, resulting in a deviation of the monetary output target from the optimum (on the left part of the **Figure 8**) and *2)* uncertainty on the risk premium reaction to shocks, resulting in a deviation of the monetary inflation target relative to the optimum (on the right part of **Fig. 8**). In all diagrams, the results of the optimal policy-mix are *green* colored and are taken as reference $(\forall \beta_1, \beta_2)$. The results of a policy-mix who deviates from the optimum, when the stabilizing coefficients β_1, β_2 depict a tight monetary policy $(\beta_1 = 1.5; \beta_2 = 0.5)$, are represented in *red*, while the results of the same simulations when the monetary policy is more accommodating $(\beta_1 = 1.2; \beta_2 = 0.5)$ are *blue* colored.

Non optimal output target ***Non optimal inflation target***



Tight monetary policy $(\beta_1 = 1.5; \beta_2 = 0.5)$ — Accommodating monetary policy $(\beta_1 = 1.2; \beta_2 = 0.5)$ — Optimal policy-mix

Fig. 8 Role of β_1, β_2 coefficients for the macroeconomic stabilization

As deviation from the optimal situation, we consider either an overestimation of the fiscal expansion effect on the potential output, by the central bank, or an underestimation of the effect of the fiscal shock on the aggregate risk premium in the Union, having for consequence the choice of an inflation target by the central bank higher than the optimal target.[26]

To define the sensitivity of the risk premium in each country, the following function is used in the simulations: $\varsigma^g = \exp(g^{0.5}) - 1$, where g introduces the amount of public expenditures financed by

[26] The technical appendix proves analytically that the main results of this section apply also, if we consider contrary sign deviations of the monetary targets from the optimum.

debt. We then simulate a positive shock defined by a 1% symmetric increase in the public spending g in the Union.[27] For the four graphs situated on the left part of **Fig. 8**, we assume that the central bank correctly revises his inflation target, but not his output target, that we take equal to 2.24% instead of 2.231%. On the contrary, for the graphs situated on the right part of **Fig. 8**, we consider the correct output target adjustment, but we assume a *non optimal* inflation target (1.9% instead of 1.86%).

Speaking about the impact of optimal policy-mix on the Union, green colored in **Fig. 8**, we observe a perfect stability of macroeconomic aggregates, according to the previous analytical results. In *the two others situations* considered in the simulations, the response of the monetary policy to the fiscal shock is less restrictive than necessary, and the impact of the fiscal expansion on the Union exceeds the impact of the restrictive monetary response to shock. The dynamic adjustment toward the new steady-state comes from a continuous appreciation of the common currency until the steady-state. The output of the Union overshoots at the moment of the shocks and it comes down during the return to the equilibrium.

If the form of dynamic adjustments is similar under the two situations depicted in **Fig.8**, some differences appear when we analyze the intensity of the reaction to shocks. If the potential output forecasting of the central bank is biased, then a more tight monetary policy, who pays attention more to the inflation stabilization and less to the output stabilization can better stabilize the adjustment paths in the Union (see the red colored adjustments in **Fig.8**). The interest rate volatility and the inflation rate volatility are lower in this case. On the contrary, if the optimal inflation forecasting is biased, a better stabilization of the real variables is ensured by a more accommodating monetary policy, which pays more attention to the output stabilization (blue colored adjustments in **Fig.8**).[28] In terms of interest rate dynamics, the results are unchanged compared to the first situation, because there is no influence of the coefficients β_1, β_2 on the interest rate response to shocks.[29] The inflation rate will reasonably join its steady state value more quickly under a monetary policy more concerned by the price stability.

The main result of this analysis is summarized as follows: To ensure the economic stability in the Union, the central bank must choose the stabilizing coefficients β_1, β_2 *of the monetary rule by taking into consideration the pertinence of the information she has on the future reaction of macroeconomic aggregates to shocks. If the quality of data necessary to a "good" estimation of the long-run output reaction to shocks is doubtful, then, the central bank should favour an objective of inflation stabilization. On the contrary, when the information necessary to estimate the impact of shocks on the risk premium is more doubtful, the central bank should have a more accommodating behavior.*

Lessons for the Euro zone

Currently, the national debt management is ensured by each Member States, who conducts autonomously the fiscal policy and suffers individually the effects of the rise of their risk premium. The European Central Bank (ECB) chooses its policy independently on the decision of the

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[27] We consider $g = 0.05$, as initial level of debt in the simulations. A 1% increase in this debt level will conduct to a 0.14% increase in the risk premium, which demands an optimal adjustment of the inflation target from 2% to 1.86% and an optimal adjustment of the output target from 2.218% to 2.231%.

[28] In the technical appendix, we analytically prove the robustness of the results for all values of stabilization coefficients β_1, β_2.

[29] $\dot{i} = C_1 \lambda_1 \exp(\lambda_1 t)$ and C_1 does not depend on β_1, β_2, for $d\bar{y} = d\hat{y}$.

governments. This policy is not aimed at responding to national fiscal shocks or to an increase of the risk premium associated to the European debt, due to these shocks.

Using the model presented in this paper, if the central bank does not provide a response to fiscal shocks, the effect of the deviation of the inflation target from its equilibrium level will be more important in dynamics than the effect of the deviation of the revenue target from the potential product. This implies adjustments similar to those in the graphics from the second column in **Figure 8**, where a more accommodation monetary regime seems to help to the stabilisation of the real variables adjustment paths. This is a reason in favour of a European monetary policy that pays attention not only to the inflation stabilisation, but also, in a certain measure, to the output stabilisation (see also our appendix on the sensitivity analysis of the initial jumps to the β_1, β_2 coefficients).

The presence of this second objective of the monetary policy, not explicitly declared by the ECB, would also be beneficial if the central bank was supposed to survey the governments' policy and to respond adequately to fiscal shocks. Each member country of the Union currently conducts its own fiscal policy and suffers from its own risk premium.[30] The heterogeneity of the risk premia among countries limits the quality of the data available to the central bank on the impact of a fiscal expansion on the aggregate risk premium of the union and casts doubts on the right adjustments of the inflation target. An output stabilisation oriented monetary objective should be beneficial in this case.

What place for structural asymmetries in defining the policy-mix of the Euro zone?

Our answer to this question is given in two steps. The first one consists in discussing the national impact of the union-wide optimal monetary policy. The second refers to the definition of an optimal policy-mix able to stimulate the real economy and to ensure the macroeconomic stability at an aggregate level, but also at the national level. A solution shall also be proposed to minimize the costs when the behaviour of one of the authorities differs from the optimum.

For the first step, we start from the main result of the last paragraph: *after a symmetric fiscal expansion, the aggregates of the union can be stabilized by using an optimal monetary policy defined by:* $d\hat{y} = d\bar{y}$ and $d\hat{\bar{p}}^c = -d\bar{\varsigma}^g$, where ς^g is the average risk premium computed for the union as a whole. In order to explain the ability of such a policy to ensure the economic stability in each country, we use the results of the *"difference" system*. Starting from (12) we can write:

$$\dot{\upsilon}_{d\,t} = -d\bar{\upsilon}_d \lambda_0 \exp[\lambda_0(t-T)] + C_1 v_{11} \frac{\lambda_0 \lambda_1 \tilde{a}_2 (1-\alpha_3)}{(\lambda_0 - \lambda_1)(2b_4 + b_5)} \{\lambda_1 \exp(\lambda_1 t) - \lambda_0 \exp[(\lambda_1 - \lambda_0)T]\exp(\lambda_0 t)\}$$

Under the optimal monetary policy, $d\bar{i} = 0$ implies $C_1 = 0$ and the second term of the relation $\dot{\upsilon}_{d_t}$ is zero. Unlike the second term, the first term can differ from zero and induces a dynamics of $\dot{\upsilon}_{d_t}$. So, adjustments for the national real variables will be asymmetric, because the term reflecting the asymmetries of the real activity (y_{d_t}) depends on $\dot{\upsilon}_{d_t}$ in the relation: $y_{d_t} = \bar{y}_d + \frac{\dot{\upsilon}_{d_t}}{\delta}$. The stability of national variables is not ensured by this type of optimal monetary response, if $d\bar{\upsilon}_d \neq 0$. Focussing on the determinants of this difference in the equilibrium **(II)**, we remark the influence of the asymmetry of the national fiscal policy (public expenditures or taxes), of the asymmetric transmission of the real

[30] Even if there are some restrictions imposed to national governments by the Stability and Growth Pact in order to insure the coordination of fiscal policies in Europe and to avoid them to become highly indebted, violations of the pact are observed within the Union and the debt ratio of the European countries are quite heterogeneous (from an average debt ratio pf 5% in Luxembourg to more than 100% in Greece, Italy or Netherlands)

interest rate in the union if the average risk premium associated to the debt increases and of the asymmetric evolution of the national risk premia.

$$d\overline{\upsilon}_d = \frac{1}{(2f_1 + f_2)\mu + 2b_4 + b_5}\left(d\overline{g}_d - a_1 d\overline{T}_d - \breve{a}_2 d\overline{\varsigma}^g - a_2 d\overline{\varsigma}_d^g\right) \qquad (15)$$

where $d\overline{x}$ depicts the variation of the steady state value of a variable *x* between the initial and the final equilibrium.

In the case of a symmetric shock on public expenditures, studied under the optimal policy-mix of the union, the role of the financing sources of these expenditures on the economies dynamics can be summarize as follows:

- If the public expenditures are financed by taxes, the variation of the taxes will also be symmetric, without determining any variation of the sovereign risk premium. In consequence, all the differences become zero in the relation (15) and the national variables jump instantly to their steady state value after the shock. The optimal monetary policy would be able to insure the stability of the union and also of each member state.

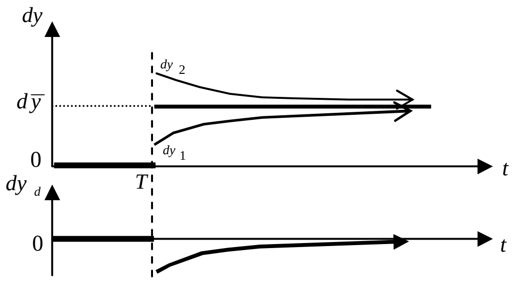

Fig. 9 Adjustments of the national revenue in the union and optimal response of the central bank to a symmetric increase of the public expenditures

- If the public expenditures are financed by debt, the individual risk premia will increase symmetrically after the shock. This implies, in the relation (15), that $d\varsigma_d^g = 0$, but $d\varsigma_d > 0$, implying $d\overline{\upsilon}_d = -\frac{\breve{a}_2}{(2f_1 + f_2)\mu + 2b_4 + b_5} d\overline{\varsigma}^g < 0$ and $\dot{\upsilon}_{d\,t} = -d\overline{\upsilon}_d \lambda_0 \exp[\lambda_0(t-T)] < 0$, because $\lambda_0 < 0$.

Subsequently, the stability of the national variables is not automatic, and the union-wide optimal monetary policy produces asymmetric national adjustments towards the equilibrium. These asymmetries come from initial negative jump of y_d when the shock occurs, and an adjustment towards the equilibrium following an increasing and concave path. At a national level $\left(y_{i_t} = y_t \pm y_{d_t}\right)$, it gives rise to *output divergences* during the adjustment towards the equilibrium. The reaction of the national product of each country to the policy-mix considered in this analysis is presented in the **Figure 9**, where the variables appear always in deviation from their initial steady state values.

For a symmetric shock, the long-run effect on the revenue is the same in the two countries of the union $\left(d\overline{y} = d\overline{y}_1 = d\overline{y}_2\right)$. The representation of the variables in deviation from the steady state brings information only on the amplitude of output divergences during the adjustments of the national variables after the shocks, and ignores the output gap existing between countries at the initial steady state. It is easily to see that the optimal monetary policy for the union has a temporary effect of amplifying the divergences within the union.

The intuitive explanation of this result is the following: an increase of the public expenditures generates a positive effect on the potential output in the two countries and an overshooting of the national output compared to the equilibrium, as in the **Figure 5**. If this fiscal expansion does not imply a rise of the risk premium, its dynamic effects are perfectly neutralized by the optimal monetary policy. This same optimal monetary response would also ensure the stability of the national variables if the monetary transmission was symmetric ($\breve{a}_2 = 0$). But the monetary policy is not able to stabilize, at national level, the dynamic effect induced by the rise of the risk premium ς^g in the union, and that because of the asymmetry of the monetary transmission (the deviation of a_{21} and respectively a_{22} from the average coefficient a_2 considered in the monetary decisions). For $a_{21} > a_2$, the increase of the risk premium will have a negative effect on the output in the country 1 compared to the equilibrium, whereas the country 2 will benefit from the situation, due to the negative variation of a_{22} from the average coefficient.

How to avoid the amplification of the divergences within the union?

If the monetary policy does not take into account the asymmetries and considers only the average aggregates in the union, it will be unable to reduce the divergences previously discussed. But, because the instability affects only the national variables, the ability of the fiscal policies to ensure this objective can be analysed.

The stability of the national economies requests that $d\bar{\upsilon}_d = 0$, in the relation (15). It implies: $d\bar{g}_d = \breve{a}_2 d\bar{\varsigma}^g + a_2 d\bar{\varsigma}_d^g$, if the public expenditures are financed exclusively by debt. Using the equilibrium of the « difference » system **(II)**, the simplest solution in order to insure this condition is to choose: $\bar{g}_d = \breve{a}_2\left(\tilde{r} + \bar{\varsigma}^g\right) + a_2\bar{\varsigma}_d^g$, corresponding to an asymmetric conduct of the fiscal policy within the union. This behaviour has to take into account all the asymmetries among the member states, either in the monetary transmission, or in the risk premium of each country.

Decomposing the *"difference" variables* in the relation above, a fiscal rule is obtained, able to ensuring the stability of the national economies:

$$\bar{g}_i = g_a + a_{2i}\left(\tilde{r} + \bar{\varsigma}^g\right) + a_2\bar{\varsigma}_i^g \qquad (16),$$

where g_a is an autonomous component of public expenditures, symmetric in all countries.

According to this rule, the country which needs the most public expenditures in order to ensure its stability is that where the aggregate demand is the most sensitive to a variation of the real interest rate and where the risk premium is higher. If the fiscal policies are conducted autonomously by each national government, without any involvement of the union, this rule is not easily conceivable. To make it operational, it is necessary to redefine the policy-mix, including a strong cooperation of the member states regarding their fiscal policies or a centralised system of managing the public expenditures and resources within the union[31]. In order to facilitate the task of the authorities in managing the asymmetries, we could imagine using, for all countries, a unique risk premium associated to the debt. In order to prevent certain countries from becoming excessively indebted, a multinational authority can be introduced - a kind of European Government who must manage the public resources,

[31] Cavallari and Di Gioacchino (2005) emphasize the advantages of the fiscal cooperation and those of a centralized fiscal policy, related to a decrease of the volatility of the macroeconomic variables in the union.

expenditures and debt in the union. The role of national governments would be reduced to informing the central decision maker on the specific needs of their countries, to use efficiently the public expenditures allocated by the European Government and to collect taxes for the central authority.

Regarding the monetary policy, the unique risk premium and the existence of a multinational government would allow improving the quality of the expectations of the central bank on the future evolution of the variables included in its decision making strategy, and especially on the sovereign risk premium. In order to limit the risk of a biased estimation of the effects of the fiscal shocks on the potential output of the union, the central bank should choose a strict policy, focussing especially on the prices stability and being less attentive to the output stabilization. We recognize the present choice of the EBC regarding the monetary policy, except that it ignores the amplification effect that its behaviour could have on the national divergences, when there is no fiscal cooperation among national governments.

Conclusion

This paper emphasizes the issue of the policy-mix management in an asymmetric monetary union *via* a dynamic analysis of the macroeconomic policies in an IS-LM alternative model able of better taking into account the actual behaviour of the central banks,. Three fundamental questions are discussed: How the central bank must react in order to stabilize the union after fiscal shocks? What is the impact of structural asymmetries on the stability of member countries when the monetary policy acts to stabilize the union as a whole? How to integrate these asymmetries in the European policy-mix to improve its performance simultaneously at aggregate and at national level?

A first conclusion of the analysis is that in a world with perfect information for the central bank, there is an *optimal monetary policy* able to insure the stability of the union after the fiscal shocks. This policy requests for the central bank to adjust in the meantime the inflation and the output targets to the long-run equilibrium values of these variables. In the optimal case, the choice of the stabilizing coefficients in the monetary rule is not important, but it becomes essential in a "second best" perspective. In fact, under imperfect information, for example, for better stabilizing the union, the choice of the coefficients of the monetary rule should take into account the quality of the central bank information on the evolution of the economic aggregates. If the information necessary to estimate the potential output of the Union is distorted, the central bank should have as main objective the price stability. On the contrary, if the information on the evolution of the risk premium, due to the increase of the debt in order to finance new public expenditures, is less reliable, the central bank should conduct o more accommodating monetary policy.

The analysis at a national level provides some other results. Because of the asymmetric monetary policy transmission within the Union, even the previous optimal monetary policy can lead to an amplification of divergences among member states. So, the policy-mix should take into account these asymmetries. In the last section of the paper, it is shown that, while the central bank focuses only on the union-wide situation, the stabilization of the asymmetries by the national fiscal policies is essential and it can be realized only by a strong cooperation of the national governments or by a centralisation of the management of the public resources, expenditures and debt within the union. The advantage of a centralized fiscal policy is that the central bank would obtain better information on the sovereign risk premium and could choose to conduct a tight policy in order to limit the costs of a potential deviation of its output target from the optimal level.

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Technical Appendix

Part 1. "Potential output" determination from the labor market equilibrium

As in the classical theory, on the labor market of each country, the firms' demand of labor negatively depends on producer prices real wage, while the labor supply positively depends on consumer prices real wage: $L_i^D = \beta_D \left(\frac{W}{P_1} \right)^{-\sigma}$ and $L_i^S = \beta_S \left(\frac{W}{P_i^c} \right)^{\gamma}$. In a logarithmic form, the equilibrium on the national labor market implies: $b_D - \sigma (w - p_i) = b_S + \gamma (w - p_i^c)$, where lower case letters represent the log of the upper case letters used in the definition of labor demand and supply. Then, using the definition of the consumer price index (p_i^c) given by equations (6a) and (6b) in the main text, it is easily to obtain real wages at equilibrium:

$$\overline{w - p_1} = b - \frac{\gamma}{\sigma + \gamma} (\alpha_3 \overline{\upsilon} + \overline{p}_d); \quad \overline{w - p_2} = b - \frac{\gamma}{\sigma + \gamma} (\alpha_3 \overline{\upsilon} - \overline{p}_d),$$

where $b = \frac{b_D - b_S}{\sigma + \gamma}$, $p_d = \frac{p_1 - p_2}{2}$ and $\upsilon = \frac{\upsilon_1 + \upsilon_2}{2}$.

Consequently, the equilibrium levels of employment $(b_D - \sigma(w - p_i))$ are: $\bar{l}_1 = b_D + b\sigma + \frac{\gamma\sigma}{\sigma + \gamma} (\overline{p}_d + \alpha_3 \overline{\upsilon})$ and, respectively $\bar{l}_2 = b_D + b\sigma + \frac{\gamma\sigma}{\sigma + \gamma} (\alpha_3 \overline{\upsilon} - \overline{p}_d)$.

Since: $\overline{y}_i = a + \mu \bar{l}_i$, we find relation (5a) and (5b) of the paper, for: $f_0 = b_D + b\sigma$, $f_1 = \frac{\gamma\sigma}{\sigma + \gamma}$ et $f_2 = \frac{\gamma\sigma\alpha_3}{\sigma + \gamma}$.

Part 2. Analysis to the union-wide level/national level

2.1 The « aggregate system »

The equations of the "<u>*aggregate system*</u>", for: $x = \frac{1}{2}(x_1 + x_2), \forall x$ and $x_d = \frac{1}{2}(x_1 - x_2), \forall x$, are:

$$\eta y^d = k + g - a_1 \tau - b_5 \upsilon - a_2 (i - \dot{p}^c) - \breve{a}_2 \varsigma_d^g \quad \text{(A1)}$$

$$y^s = \overline{y} + \frac{\alpha_3 \dot{\upsilon}}{\delta} \text{ [32]} \quad \text{(A2)}$$

$$\dot{p} = \dot{p}^c + \alpha_3 \dot{\upsilon} \quad \text{(A3)}$$

$$\dot{\upsilon} = \dot{p} - \dot{\tilde{p}} - \dot{e} \quad \text{(A4)}$$

$$i = \tilde{i} + \dot{e} + \varsigma^g \quad \text{(A5)}$$

$$r = i - \dot{p}^c \quad \text{(A6)}$$

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[32] We use: $\tau = p - (\tilde{p} + e)$, $p^c = p - \alpha_3 \tau$ and $\dot{p} = \dot{p}^c + \delta(y^s - \overline{y})$ from the initial system.

$$\dot{i} = \omega\left[\hat{p}^c + \bar{r} + \beta_1\left(\dot{p}^c - \hat{p}^c\right) + \beta_2\left(y - \hat{y}\right) - i\right] \qquad \text{(A7)},$$

where: $\eta = 1 - a_1 + b_1 - b_2$, $k = a_0 + b_0 + b_3\tilde{y}$, $a_2 = \dfrac{a_{21} + a_{22}}{2}$, $\breve{a}_2 = \dfrac{a_{21} - a_{22}}{2}$.

Since υ and i state variables of the analysis, we search a solution of the aggregate system in function of these state variables and of the exogenous variables: $\hat{p}^c, \hat{y}, g, \tilde{i}, \varsigma^g, \varsigma_d^g$.

The ID equilibrium condition on the goods market ($y^d = y^s$) allows us to find the following solution:

$$\dot{\upsilon} = \frac{\delta}{\eta\alpha_3 - a_2(1-\alpha_3)\delta}\begin{pmatrix} k + g - a_2\tilde{i} - \eta\bar{y} - a_1\tau \\ + a_2\tilde{p} - a_2\varsigma^g - \breve{a}_2\varsigma_d^g \end{pmatrix} - \frac{b_5\delta}{\eta\alpha_3 - a_2(1-\alpha_3)\delta}\upsilon \qquad \text{(A8)}$$

$$y = \frac{\alpha_3\begin{pmatrix} k + g - a_2\tilde{i} - a_1\tau \\ + a_2\tilde{p} - a_2\varsigma^g - \breve{a}_2\varsigma_d^g \end{pmatrix} - \delta a_2\left(1 - \alpha_3\right)\bar{y}}{\eta\alpha_3 - a_2(1-\alpha_3)\delta} - \frac{b_5\alpha_3}{\eta\alpha_3 - a_2(1-\alpha_3)\delta}\upsilon \qquad \text{(A9)}$$

$$\dot{e} = i - \tilde{i} - \varsigma^g \qquad \text{(A10)}$$

$$\dot{p} = \dot{\upsilon} + \dot{e} + \dot{\tilde{p}} = \dot{\upsilon} + i - \tilde{r} - \varsigma^g \qquad \text{(A11)}$$

$$\dot{p}^c = (1-\alpha_3)\dot{\upsilon} + i - \tilde{r} - \varsigma^g \qquad \text{(A12)}$$

$$r = i - \dot{p}^c \qquad \text{(A13)}$$

$$\dot{i} = \omega\left[\hat{p}^c + \bar{r} + \beta_1\left(\dot{p}^c - \hat{p}^c\right) + \beta_2\left(y - \hat{y}\right) - i\right] \qquad \text{(A14)}$$

In the *long-run*, the equilibrium (ES) asks for: $\bar{\dot{\upsilon}} = 0$ and $\bar{\dot{i}} = 0$. From (6a) and (6b), we compute the aggregate long-run output in the steady state:

$$\bar{y} = f_0 + f_2\bar{\upsilon} \qquad \text{(A15)}$$

By taking $\bar{\dot{\upsilon}} = 0$ in (A8) and by adding the result to (A15), we obtain the expressions of the external terms of trade υ and of the long-run output in the steady state. In (A14), $\bar{\dot{i}} = 0$ allows us to determinate the steady-state inflation rate based on the consumer price index$\left(\bar{\dot{p}}^c\right)$. We use (A10), (A11), (A12) and (A13) to compute the other aggregate variables which complete the equilibrium **(I)** in the main text of the paper.

$$\begin{aligned} &\bar{\upsilon} = \frac{1}{\eta f_2 + b_5}\left[k - a_2\left(\tilde{r} + \bar{\varsigma}^g\right) - \breve{a}_2\bar{\varsigma}_d^g - a_1\bar{\tau} + \bar{g} - \eta f_0\right];\ \bar{\dot{p}}^c = \hat{p}^c - \frac{\beta_2}{\beta_1 - 1}\left(\bar{y} - \hat{y}\right) \\ &\bar{y} = \frac{1}{\eta f_2 + b_5}\left[f_2\left(k - a_2\left(\tilde{r} + \bar{\varsigma}^g\right) - \breve{a}_2\bar{\varsigma}_d^g - a_1\bar{\tau} + \bar{g}\right) + b_5 f_0\right] \\ &\bar{r} = \tilde{r} + \bar{\varsigma}^g\ ;\ \bar{i} = \bar{r} + \bar{\dot{p}}^c\ ;\ \bar{\dot{e}} = \bar{\dot{p}} - \bar{\dot{\tilde{p}}} = \bar{\dot{p}}^c - \bar{\dot{\tilde{p}}}^c \end{aligned} \qquad \textbf{(I)}$$

2.2 The « aggregate dynamic system »

To study the dynamic adjustment of the Union towards the steady-state, we use the reduced form of the "*aggregate system*".

The first dynamic equation of the reduced form (9) in the paper is similar to the equation used by Clausen & Wolhtmann (2005) and it comes from (A1), (A2) to which we add, in the same time, (A3), (A4) and (A5). The second dynamic equation is deduced from (A7), using (A1), (A2) and (A3):

$$\begin{cases} \dot{\upsilon} = X(\upsilon - \bar{\upsilon}) \\ \dot{i} = \omega\Psi(\upsilon - \bar{\upsilon}) + \omega(\beta_1 - 1)(i - \bar{i}) \end{cases}$$, for: $X = -\frac{b_5\delta}{\eta\alpha_3 - a_2(1-\alpha_3)\delta}$ and $\Psi = \frac{X}{\delta}[\beta_1\delta(1-\alpha_3) + \beta_2\alpha_3]$.

The Jacobian Matrix of the dynamic system is**:** $J = \begin{pmatrix} X & 0 \\ \Psi\omega & \omega(\beta_1 - 1) \end{pmatrix}$.

The sign of the Jacobian Matrix Determinant $Det(J) = X\omega(\beta_1 - 1)$ is given by the sign of X, which depends on the sign of the denominator: $\eta\alpha_3 - a_2(1-\alpha_3)\delta$. From the IS equilibrium condition (A9), we deduce: $\eta\alpha_3 - a_2(1-\alpha_3)\delta > 0$. Actually, this condition corresponds to an upward movement of the IS curve in the (i, y) system, consequent to an exogenous shock (see, for example, the increase in government expenditures).

So, $X < 0$. As $\beta_1 > 1$ and $0 < \omega < 1$, the Jacobian Matrix Determinant is undoubtedly negative. One of the characteristic roots is negative (λ_1), while the second is negative (λ_2).

Effectively, we know that λ_1 and λ_2 are solutions of the following 2nd degree equation: $\lambda^2 - TR(J)\lambda + Det(J) = 0$, see: $\lambda_1 = X = -\frac{b_5\delta}{\eta\alpha_3 - a_2(1-\alpha_3)\delta} < 0$ and $\lambda_2 = \omega(\beta_1 - 1) > 0$, where $Tr(J) = X + \omega(\beta_1 - 1)$ represents the Jacobian Matrix Trace and $Det(J) = X\omega(\beta_1 - 1)$ is the negative determinant described above.

As for the two associated *eigenvectors:* $\left(v_1 = \begin{pmatrix} v_{11} \\ v_{21} \end{pmatrix}; \upsilon_2 = \begin{pmatrix} v_{12} \\ v_{22} \end{pmatrix} \right)$, they represent solutions of the equations: $Jv_i = \lambda_i v_i$.

Let's take: $v_{21} = v_{22} = 1$. The other two components of the eigenvectors are:

$v_{11} = \frac{X - \omega(\beta_1 - 1)}{\Psi\omega} = \frac{\delta[X - \omega(\beta_1 - 1)]}{X\omega[\beta_1\delta(1-\alpha_3) + \beta_2\alpha_3]} > 0$, which gives the slope of the convergent adjustment path for the union towards the steady state, and $v_{12} = 0$. The positive sign of v_{11} justifies the positive slope of the saddle path.

Solution of the aggregate dynamic system

The general solution of such a dynamic system is given by :

$\begin{pmatrix} \upsilon \\ i \end{pmatrix} - \begin{pmatrix} \bar{\upsilon} \\ \bar{i} \end{pmatrix} = C_1 v_1 \exp(\lambda_1 t) + C_2 v_2 \exp(\lambda_2 t)$, where *C₁*, *C₂* are constants which must be calculated, λ_1, λ_2 are eigenvalues (characteristic roots) of the Jacobian Matrix, and v_1, v_2 - the corresponding eigenvectors.

La forme générale de la solution du système dynamique s'écrit :

$\begin{pmatrix} \upsilon \\ i \end{pmatrix} - \begin{pmatrix} \bar{\upsilon} \\ \bar{i} \end{pmatrix} = C_1 v_1 \exp(\lambda_1 t) + C_2 v_2 \exp(\lambda_2 t)$, C_1, C_2 étant des constantes, λ_1 et λ_2 désignant les valeurs propres et v_1, v_2 - les vecteurs propres associés.

As the convergence condition of this solution towards the steady state must be fulfilled, C_2 must be equal to zero.

Effectively, for: $\lim_{t\to\infty}\begin{pmatrix}\upsilon\\ i\end{pmatrix}=\begin{pmatrix}\bar{\upsilon}\\ \bar{i}\end{pmatrix}+C_1 v_1 \exp(\lambda_1 t)+C_2 v_2 \exp(\lambda_2 t)=\begin{pmatrix}\bar{\upsilon}\\ \bar{i}\end{pmatrix}$, we need $C_2 = 0$, since $\lambda_1 < 0$ and $\lambda_2 > 0$. It explains the simplified solution (15) in the main text:

$$\begin{pmatrix}\upsilon\\ i\end{pmatrix}-\begin{pmatrix}\bar{\upsilon}\\ \bar{i}\end{pmatrix}=C_1 v_1 \exp(\lambda_1 t), \forall t \geq T \qquad \text{(A16).}$$

2.3. The « difference system »

Let's note: $x_d = \frac{x_1 - x_2}{2}$, where x_i denotes the *x* variable in country *i* of the union, $\mu = 1 - a_1 + b_1 + b_2$ and $\tilde{a}_2 = \frac{a_{21} - a_{22}}{2}$. We obtain the following equations for the *difference system*:

$$p_d = \upsilon_d \qquad \text{(B1)}$$

$$\mu y_d = g_d - a_1 \tau_d - 2b_4 p_d - b_5 \upsilon_d - \breve{a}_2 \left(i - \dot{p}^c\right) - a_2 \varsigma_d^g \qquad \text{(B2)}$$

$$y_d = \bar{y}_d + \frac{\dot{\upsilon}_d}{\delta} \qquad \text{(B3)}$$

$$\dot{\upsilon}_d = \frac{\delta \tilde{a}_2 (1-\alpha_3)}{\mu}\dot{\upsilon} - \frac{b_5 + 2b_4}{\mu}\delta(\upsilon_d - \bar{\upsilon}_d) \qquad \text{(B4)}$$

To have the (B4) equation, we use (B2) to write $y_d - \bar{y}_d$, knowing that $g_d, \tau_d, \varsigma^g, \varsigma_d^g$ are all exogenous variables given at the moment of the dynamic analysis, and that, from (A3), (A4) and (A5), we can replace $i - \dot{p}^c$ by $\tilde{r} + \varsigma^g - (1-\alpha_3)\dot{\upsilon}$. We then introduce the result in (B3), to have $\dot{\upsilon}_d$.

All in one, we obtain a system of 4 endogenous variables $(p_d, \upsilon_d, y_d, \dot{\upsilon}_d)$ and 4 equations. From $\alpha_1 = \alpha_2$, we deduce: $p_d^c = 0$, $\dot{p}_d^c = \dot{\upsilon}_d = 0$, $g_d, \tau_d, \varsigma^g, \varsigma_d^g, \bar{y}_d, \bar{p}_d$ are exogenous and $i, \dot{p}^c, \dot{\upsilon}$ fulfill the conditions specified for the *aggregate system*, in the 2.2 paragraph of this appendix.

At the steady state: $\dot{\upsilon} = 0$ and $p_d = \bar{p}_d$, so that: $\dot{p}_d^c = 0$. We use : $\bar{p}_d = \bar{\upsilon}_d, \bar{y}_d = (2f_1 + f_2)\bar{\upsilon}_d$ and $\bar{y}_d = \frac{g_d - \tilde{a}_2(\tilde{r} + \varsigma^g) - a_1\tau_d - a_2\varsigma_d^g - (2b_4 + b_5)\bar{\upsilon}_d}{\mu}$, to obtain the following long-run equilibrium **(II)** of the « difference system »:

$$\bar{p}_d = \bar{\upsilon}_d = \frac{\bar{g}_d - \tilde{a}_2(\tilde{r} + \bar{\varsigma}^g) - a_1\bar{\tau}_d - a_2\bar{\varsigma}_d^g}{\mu(2f_1 + f_2) + 2b_4 + b_5};$$

$$\bar{y}_d = \frac{(2f_1 + f_2)\left(\bar{g}_d - \tilde{a}_2(\tilde{r} + \bar{\varsigma}^g) - a_1\bar{\tau}_d - a_2\bar{\varsigma}_d^g\right)}{\mu(2f_1 + f_2) + 2b_4 + b_5}; \quad \bar{i}_d = \bar{\varsigma}_d^g \qquad \textbf{(II)}$$

2.4 Solution of the « difference dynamic system »

Let's note: $\lambda_0 = -\frac{b_5 + 2b_4}{\mu} < 0$ and $X = -\frac{b_5\delta}{\eta\alpha_3 - a_2(1-\alpha_3)\delta} < 0$. The *difference dynamic system* takes the form:

$$\begin{cases} \dot{\upsilon} = -\frac{b_5\delta}{\eta\alpha_3 - a_2(1-\alpha_3)\delta}(\upsilon - \overline{\upsilon}) = X(\upsilon - \overline{\upsilon}) \\ \dot{\upsilon}_d = -\frac{\lambda_0 X\tilde{a}_2(1-\alpha_3)}{2b_4 + b_5}(\upsilon - \overline{\upsilon}) + \lambda_0(\upsilon_d - \overline{\upsilon}_d) \end{cases} \quad \text{(B5).}$$

The general solution of this dynamic system is: $\begin{pmatrix} \upsilon \\ \upsilon_d \end{pmatrix} - \begin{pmatrix} \overline{\upsilon} \\ \overline{\upsilon}_d \end{pmatrix} = A_1 h_1 \exp(\lambda_1 t) + A_2 h_2 \exp(\lambda_0 t), \forall t \geq T$,

where $\lambda_0 = -\frac{b_5 + 2b_4}{\mu} < 0$ and $\lambda_1 = X < 0$ are the characteristic roots of the Jacobian Matrix, while $h_1 = \begin{pmatrix} \frac{(\lambda_0 - \lambda_1)(2b_4 + b_5)}{\lambda_0\lambda_1\tilde{a}_2(1-\alpha_3)} \\ 1 \end{pmatrix}$ and $h_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ represents the eigenvectors of this matrix.

A_1 and A_2 are constants coming from the general solution of (B5) written in the T :

$$A_1 = -d\overline{\upsilon}\frac{\lambda_0\lambda_1\tilde{a}_2(1-\alpha_3)}{(\lambda_0 - \lambda_1)(2b_4 + b_5)}\exp(-\lambda_1 T) = C_1 v_{11}\frac{\lambda_0\lambda_1\tilde{a}_2(1-\alpha_3)}{(\lambda_0 - \lambda_1)(2b_4 + b_5)}$$
$$A_2 = -d\overline{\upsilon}_d \exp(-\lambda_0 T) - A_1 \exp[(\lambda_1 - \lambda_0)T].$$

Using these results, we find for the prices differential between country 1 and country 2, for all $t > T$, the expression (12) in the main text:

$$\upsilon_{dt} = \overline{\upsilon}_{d_1} - d\overline{\upsilon}_d \exp[\lambda_0(t-T)] + C_1 v_{11}\frac{\lambda_0\lambda_1\tilde{a}_2(1-\alpha_3)}{(\lambda_0 - \lambda_1)(2b_4 + b_5)}\{\exp(\lambda_1 t) - \exp[(\lambda_1 - \lambda_0)T]\exp(\lambda_0 t)\} \quad \text{(B6)}$$

Consequently:

$\dot{\upsilon}_{dt} = -\lambda_0 d\overline{\upsilon}_d \exp[\lambda_0(t-T)] + C_1 v_{11}\frac{\lambda_0\lambda_1\tilde{a}_2(1-\alpha_3)}{(\lambda_0 - \lambda_1)(2b_4 + b_5)}\{\lambda_1 \exp(\lambda_1 t) - \lambda_0 \exp[(\lambda_1 - \lambda_0)T]\exp(\lambda_0 t)\}$. This formula helps us to determine y_d in (B3) and $dy_d = y_{d_t} - y_{d_0}$ which is graphically depicted in the **Figure 5** of the main text. We can also easily compute the jump of dy_d in *T,* see, for example: $y_d(T+) - \bar{y} = -\frac{C_1\upsilon_{11}\lambda_0\lambda_1\tilde{a}_2(1-\alpha_3)}{\delta(2b_4 + b_5)}\exp(\lambda_1 T) < 0$, in the case of a restrictive monetary shock.

In $t^* = T + \frac{1}{\lambda_1 - \lambda_0}\ln\left(\frac{\lambda_0}{\lambda_1}\right)$, $dy_{d_t} = 0$; $dy_{d_t} < 0$, if $t < t^*$ and $dy_{d_t} > 0$, if $t > t^*$. Since $\frac{1}{\lambda_1 - \lambda_0}\ln\left(\frac{\lambda_0}{\lambda_1}\right) > 0, \forall \lambda_1, \lambda_0 < 0$, $t^* = T + \frac{1}{\lambda_1 - \lambda_0}\ln\left(\frac{\lambda_0}{\lambda_1}\right) > T$. The first order derivative $\left(\dot{dy}_{d_t}\right)$ is equal to zero in $\hat{t} = T + \frac{2}{\lambda_1 - \lambda_0}\ln\left(\frac{\lambda_0}{\lambda_1}\right)$, it is positive for all $t < \hat{t}$ and negative for $t > \hat{t}$. As for, $\ddot{dy}_{d_t}$, it is equal to zero in $\breve{t} = T + \frac{3}{\lambda_1 - \lambda_0}\ln\left(\frac{\lambda_0}{\lambda_1}\right)$, negative for all $t < \breve{t}$ and positive for $t > \breve{t}$. All these details help us to depict the adjustment of dy_d in the **Figure 5** of the main text.

Part 3. Sensitivity analysis of initials jumps to the β_1, β_2 coefficients

The amplitude of initial jumps in the model is determined by the amplitude of the initial jump of the external term of trade $\upsilon : \upsilon(T+) = \bar{\upsilon}_1 + C_1 \upsilon_{11}\exp(\lambda_1 T)$, which depends on β_1, β_2 coefficients, because $v_{11} = \frac{\delta[X - \omega(\beta_1 - 1)]}{X\omega[\beta_1\delta(1-\alpha_3) + \beta_2\alpha_3]} > 0$ and $C_1 = -d\bar{i}\exp(-\lambda_1 T)$, with :

$$d\bar{i} = d\hat{p}^c + d\bar{\varsigma}^g - \frac{\beta_2}{\beta_1 - 1}\left(\frac{f_2}{\eta f_2 + b_5}\left(d\bar{g} - a_2 d\bar{\varsigma}^g - \breve{a}_2 d\bar{\varsigma}_d^g\right) - d\hat{y}\right) = d\hat{p}^c + d\bar{\varsigma}^g - \frac{\beta_2}{\beta_1 - 1}\left(d\bar{y} - d\hat{y}\right) \quad \text{(C1)}$$

At the optimum, the monetary policy reaction to an increase in the public expenditures financed by debt asks for: $d\bar{y} = d\hat{y}$ and $d\hat{p}^c = -d\bar{\varsigma}^g$.

Outside the optimum, two different cases must be considered:

1) Uncertainty on the future level of potential output, in which case: $d\bar{y} \neq d\hat{y}$ and $d\hat{p}^c = -d\bar{\varsigma}^g$.

From (B1), we obtain:

$d\bar{i} = -\frac{\beta_2}{\beta_1 - 1}(d\bar{y} - d\hat{y})$, $C_1 = -d\bar{i}\exp(-\lambda_1 T)$ and $\upsilon(T+) - \bar{\upsilon}_1 = C_1 v_{11}\exp(\lambda_1 T) = v_{11}\frac{\beta_2}{\beta_1 - 1}(d\bar{y} - d\hat{y})$. Since $(d\bar{y} - d\hat{y})$ is independent from the β_1, β_2 values, analyzing this sensitivity amounts to analyze the sign of the first order derivative of $R = v_{11}\frac{\beta_2}{(\beta_1 - 1)}$ subject to β_1, β_2 . We obtain:

$$\frac{\partial R}{\partial \beta_2} = -\frac{(\alpha_3 - 1)\beta_1\delta^2[X + \omega(1 - \beta_1)]}{X(\beta_1 - 1)[\beta_1\delta + \alpha_3(\beta_2 - \beta_1\delta)]^2\omega} > 0, \text{ since } X < 0 \text{ and } \alpha_3 < 1$$

$$\frac{\partial R}{\partial \beta_1} = -\frac{\beta_2\delta[-\beta_2\alpha_3 X + X(\alpha_3 - 1)(2\beta_1 - 1)\delta - (\alpha_3 - 1)(1 - \beta_1)^2\delta\omega]}{X(\beta_1 - 1)^2[\beta_1\delta + \alpha_3(\beta_2 - \beta_1\delta)]^2\omega} < 0.$$

With these results, for $d\bar{y} > d\hat{y}$, the first order derivative of the jump $C_1 v_{11}\exp(\lambda_1 T)$ to β_1 is negative, while its first order derivative to β_2 is positive. The signs are inversed when $d\bar{y} < d\hat{y}$. Because the initial jump is positive in the first case $\left(d\bar{y} > d\hat{y} \Rightarrow d\bar{i} = -\frac{\beta_2}{\beta_1 - 1}(d\bar{y} - d\hat{y}) < 0 \Rightarrow C_1 > 0 \ et \ \upsilon(T+) - \bar{\upsilon}_1 > 0\right)$ and negative in the second one, we conclude that a more tight monetary policy (higher value of β_1 / β_2) makes the initial jump of υ more close to its optimal value, compared with a more accommodating monetary policy (lower value of β_1 / β_2).

2) Uncertainty on the evolution of the sovereign risk premium: $d\bar{y} = d\hat{y}$ *and* $d\hat{\bar{p}}^c \neq -d\bar{\varsigma}^g$

In this case, we obtain from (B1): $d\bar{i} = d\hat{\bar{p}}^c + d\varsigma^g$, $C_1 = -d\bar{i}\exp(-\lambda_1 T)$ and $\upsilon(T+) - \bar{\upsilon}_1 = C_1 v_{11}\exp(\lambda_1 T) = -\left(d\hat{\bar{p}}^c + d\varsigma^g\right)v_{11}$. Since $\left(d\hat{\bar{p}}^c + d\varsigma^g\right)$ doesn't depend on the β_1, β_2 coefficients, analyzing this sensitivity amount to analyze the sign of the first order derivative of v_{11}, subject to the β_1, β_2 coefficients. We compute:

$$\frac{\partial v_{11}}{\partial \beta_2} = -\frac{\alpha_3 \delta[X + \omega(1-\beta_1)]}{X[\beta_1\delta(1-\alpha_3) + \alpha_3\beta_2]^2\omega} < 0 \text{, since } X < 0 \text{ and } \alpha_3 < 1$$

$$\frac{\partial v_{11}}{\partial \beta_1} = \frac{\delta[X\delta(\alpha_3 - 1) - \omega[\delta(1-\alpha_3) + \alpha_3\beta_2]]}{X[\beta_1\delta(1-\alpha_3) + \alpha_3\beta_2]^2\omega} > 0 \text{, for } \beta_2 > -\frac{\delta(1-\alpha_3)}{\alpha_3\omega}(X + \omega),$$ condition easily fulfilled.

With these results, for $\left|d\hat{\bar{p}}^c\right| < d\varsigma^g$, the first order derivative of the jump $C_1 v_{11}\exp(\lambda_1 T)$ to β_1 is negative, while its first order derivative to β_2 is positive. The signs are inversed when $\left|d\hat{\bar{p}}^c\right| > d\varsigma^g$. Because the initial jump is negative in the first case $\left(d\bar{i} > 0 \Rightarrow C_1 < 0 \; et \; \upsilon(T+) - \bar{\upsilon}_1 < 0\right)$ and positive in the second one, we conclude that a more accommodating monetary policy (lower value of β_1 / β_2) makes the initial jump of υ more close to its optimal value, compared with a more tight monetary policy (higher value of β_1 / β_2).

Sensitivity analysis of $\upsilon(T+) - \bar{\upsilon}_1$ *to* β_1, β_2 *coefficients after a fiscal shock, without any adjustment of the monetary policy targets*

This situation appears as a combination of the two cases previously discussed. Using (C1), we easily compute the initial jump $\upsilon(T+) - \bar{\upsilon}_1$, denoted by *S* hereafter:

$$S = v_{11}\left[\frac{\beta_2}{\beta_1 - 1}\frac{f_2}{\eta f_2 + b_5}\left(d\bar{g} - a_2 d\bar{\varsigma}^g - \breve{a}_2 d\bar{\varsigma}_d^g\right) - d\bar{\varsigma}^g\right] \qquad \text{(C2)},$$

with $v_{11} > 0$ and $d\bar{g} - a_2 d\bar{\varsigma}^g - \breve{a}_2 d\bar{\varsigma}_d^g > 0$, if we assume o positive effect of the fiscal expansion on the real activity, in the long run. The first and second order derivatives of *S* subject to β_1, β_2 are:

$$\frac{\partial S}{d\beta_2} = \frac{\delta[X + \omega(1-\beta_1)]\left[\alpha_3(\beta_1 - 1)(b_5 + \eta f_2)d\bar{\varsigma}^g + \left(a_2 d\bar{\varsigma}^g + \breve{a}_2 d\bar{\varsigma}_d^g - d\bar{g}\right)f_2\beta_1\delta(\alpha_3 - 1)\right]}{X(\beta_1 - 1)[\beta_1\delta + \alpha_3(\beta_2 - \beta_1\delta)]^2\omega(\eta f_2 + b_5)} > 0$$

$$\frac{\partial S}{d\beta_1}=\frac{\delta\begin{Bmatrix} f_2\beta_2\left(a_2 d\bar{\varsigma}^g+\breve{a}_2 d\bar{\varsigma}_d^g-d\bar{g}\right)\left[X\alpha_3\beta_2-X\delta(\alpha_3-1)(2\beta_1-1)+(\alpha_3-1)(\beta_1-1)^2\delta\omega\right]- \\ d\bar{\varsigma}^g(\beta_1-1)^2(\eta f_2+b_5)\left[X\delta(\alpha_3-1)-\omega(\delta(1-\alpha_3)+\alpha_3\beta_2)\right] \end{Bmatrix}}{X(\beta_1-1)\left[\beta_1\delta+\alpha_3(\beta_2-\beta_1\delta)\right]^2\omega(\eta f_2+b_5)}<0,$$

for $\alpha_3<1$ and $\beta_2>-\dfrac{\delta(1-\alpha_3)}{\alpha_3\omega}(X+\omega)$.

In the case that we have favoured for the Euro Area, the fiscal expansion would also conduct to an increase in the long run interest rate: $d\bar{i}>0$. This corresponds to an instantaneous negative deviation of the external term of trade υ from its new steady state level, after the shock. This negative gap, denoted by *S*, would be smaller under a monetary policy that favours the output stability, sustaining the need of a more accommodating policy in order to better stabilize real variables in the Union.