Social Inequalities and Macroeconomic Instability

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Abstract

It is possible to explain macroeconomic volatility through the existence of different transaction cost functions in a model with heterogeneous agenst. Heterogeneity stems from initial capital as well as the transaction cost function. We propose a dynamic equilibrium model in which agents face variable costs that consist of time, brokerage commissions, tax and trasportation costs. Generally, transaction costs create an incentive for consumers to accumulate less capital. We first show that introducing identical cost function in Ramsey model does not have any effect on the economic stability; the steady state is always saddle. In contrast, we demonstrate that the heterogeneity in transaction costs function plays a main role on the appearance of endogenous fluctuation. Moreover, the steady state changes its stability through saddle-node and Hopf cycles.

Key words: Transaction costs, Heterogeneous agents, Saddle-node bifurcation, Endogenous fluctuations.

JEL classification: E20, E21, E30, E32.

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1 Introduction:

It is well known in the literature that transaction costs in asset and financial markets are considered as important factor in determining the investment portfolio. Without these costs agents can take positions in all existing assets, while the introduction of a transaction cost forces agents to reduce the frequency of purchasing this asset. For instance, when there are two different assets without transaction costs, then the portfolio choice would be a segment of these two assets. However, when transaction costs are introduced, the investment choice will mainly move to the assets with low costs. That is, transaction costs have negative effect on the assets' demand.

Transaction costs might consist of communication cost, the opportunity cost of time (real wage), government fees, stamp taxes, insurance, administration cost, brokerage commission and tax. Throughout the literature authors are usually interested in how the transaction costs can influence the investment choices for agents as well as its effect on asset prices. For example, Lo, Mamaysky and Wang (2004) consider a dynamic equilibrium model of trade volume when agents face fixed costs; they show that the presence of these costs allows agents to trade infrequently. Constantinides (1986) argues that proportional costs have only a small impact on asset prices. Alan (2006) supposes that the cost is paid only once over the entire life cycle, once it is paid, the household is free to re-enter the stock market. He finds that the existence of entry costs, the participation rate in the stock market declines from (80-90)% to 30% on average. Vayanos (1998) argues that an increase in transaction costs has two opposite effects on the stock's demand. From one hand, agents buy fewer stocks and from another hand, they hold them for longer periods; he has an ambiguous effect. Empirically, he demonstrates for the stock holding period that if the transaction cost raises from 0 to 2% of the stock prices, agents wait 12,8 years before sell it. Costs in the economic literature have been mentioned in several forms: fixed entry cost, variable cost and per period cost. Moreover, Vissing-Jorgensen (2002) estimates that a 50 dollar of transaction cost is sufficient to explain the choices of half of the stock market nonparticipants using data from 1989-1994, for the same period of years, a 260 dollar cost is enough to explain the choices of 75 percent of nonparticipants.

In this paper, we extend Ramsey by introducing a cost of capital stock in an heterogeneous agents framework. Heterogeneity is considered according to the initial capital as well as the transaction cost function. The objective is to investigate the effect of this cost on local dynamics and on economic stability. Without loss of generality, we suppose that there are two types of households who supply labor inelastically at each period.

Heterogeneity in Ramsey model has been considered in many environments. For instance, Becker (1980), Becker and Foias (1987) and Becker and Foias (1994) consider a Ramsey model with heterogeneous agents consist of initial capital and discount rate and with borrowing constraint. They find that the most patient household hold all the capital stock in the long term. Sarte (1997) proposes a progressive taxation into Becker and Foias (1980) where the taxes are increasing with income, he shows that the most patient agents as well as less patient agents hold positive amount of world capital. Sorger (2002) extends the previous literature through supposing that household's present value of consumption must not exceed its present value of income. Sorger deduces that flip bifurcation and indeterminacy can occur in the long run where all households have positive amount of capital. Non-degenerate capital distribution is obtained even with heterogeneity in the household's labor market productivity Carroll and Young (2008).

It is shown -in the identical cost function case- that if the cost function is monotonic with respect to capital, the economy displays unique steady state equilibrium where all agents have the same amount of wealth and consumption (symmetric steady state). Furthermore, it is found that an identical capital cost function does not influence the local stability of the steady state where it is always stable regardless of the capital cost sensitivity. Using a numerical example, we deduce that for higher sensitivity of capital cost with respect to capital, agents accumulate less capital. This is simply because as the sensitivity raises, then for a small increase in capital accumulation, the cost of capital stock highly increases. Additionally, higher sensitivity makes the capital cost function less concave and so agents purchase less capital. It becomes less profitable for agents to get capital since the marginal cost increases more and more.

In the heterogeneous cost function case, we demonstrate that the introduction of a stock capital cost in an economy with heterogeneous agents plays a crucial role in the appearance of endogenous fluctuations. At the steady state, agents hold different values of capital and consumption. From local dynamic point of view, we confirm that for critical values of the elasticity of cost with respect to capital, the economic system is indeterminate where there are infinite trajectories converge toward the steady state that respect the transversality condition. For other values, the steady state changes its stability through saddle-node bifurcation.

The remainder of the paper is arranged as follows. In section 2, we present the model with identical cost function (the optimization problem of households and firms). The intertemporal equilibrium and the steady state analysis are presented in section 3. We study local dynamics and stability in section 4. In section 5, we present a numerical example. In section 6, we present the model with heterogeneous cost function with its steady state. Local dynamics is presented in section 7. We summarize and discuss the results in section 8 and finally we conclude in section 9.

2 The model with identical cost function

This model consists of a continuous time model with two categories of agents: heterogeneous consumers and a representative firm.

2.1 Households

There are two groups of identical agents i , with $i = 1, 2$. Let n_i be the size of agents of group i. Both types of agents are concerned by their current consumption (c_i). Agent i satisfaction is represented by the following utility function:

$$U = \int_0^{\infty} e^{-\rho t} u(c_i)\, dt \tag{1}$$

This utility function satisfies this assumption:

Assumption (1): *$u(c_i)$ is a continuous function on $c_i, \in [0, +\infty)$ and twice differentiable on $[0, +\infty)$. This function are strictly increasing in its argument c_i, $u'(c_i) > 0$, and strictly concave $u''(c_i) < 0$. Furthermore, this function satisfies Inada conditions: $\lim_{c_i \longrightarrow 0} u'(c_i) = +\infty$, $\lim_{c_i \longrightarrow +\infty} u'(c_i) = 0$.*

Each agent i chooses his capital and consumption to maximize the intertemporal utility function (1) subject to the following budget constraint:

$$\dot{k}_i = r k_i + w l_i - c_i - \xi(k_i) \tag{2}$$

with a given initial endowment of capital $k_{i0} > 0$. ρ is the constant rate of time preference, c_i is the real consumption, k_i is the real stock of capital, r is the real interest rate, W is the real wage and l_i is the agent's i labor supply. Finally, $\xi(k_i)$ is the cost associated with agent's i capital stock. This cost function satisfies that following assumption:[1]

Assumption (2): *The cost function $\xi(k_i)$ for $i = 1, 2$, is a monotonic, continuous function defined on $[0, +\infty)$, twice differentiable and satisfies the following conditions:*

$$\begin{aligned} \xi'(k_i) &> 0 \\ f''(k) &< \xi''(k_i) < 0 \end{aligned}$$

The above assumption is necessary to confirm that the stationary point of Hamiltonian function of agent i represents a local maximum.

Assumption (3): *The instantaneous utility function is given by:*

$$\begin{aligned} u(c_i) &= \frac{c_i^{1-\varsigma}}{1-\varsigma} \text{ iff } \varsigma \neq 1 \\ u(c_i) &= \ln c_i \text{ iff } \varsigma = 1 \end{aligned} \tag{3}$$

[1] We assume that $f''(k) < \xi''(k_i)$ in order to the stationary point of the Hamiltonian function of agents represents a local maximum.

where $1/\varsigma = -u'(c_i)/u''(c_i)\,c_i > 0$ *is the elasticity of intertemporal substitution in consumption for agent* i*, which is equivalent to the inverse of the elasticity of marginal utility with respect to consumption* ς[2]*.*

Then, the current-value Hamiltonian function of the optimization problem is:

$$H = u(c_i) + \lambda\left[rk_i + wl_i - c_i - \xi(k_i)\right] \qquad i = 1,2 \tag{4}$$

The first-order conditions (FOCs) $H_{c_i} = 0$ and $H_{k_i} = \lambda\rho - \dot{\lambda}$ give Euler equation:

$$\dot{c}_i = (1/\varsigma)\,c_i\left[f'(k) - \rho - \xi'(k_i)\right] \tag{5}$$

A rational agent i always takes in consideration the initial condition as well as the final condition. The initial condition consists of the initial capital stock $k_{i0} > 0$ and the final condition is the transversality condition:

$$\lim_{t\longrightarrow\infty} e^{-\rho t}u'(c_i)\,k_i = 0 \tag{6}$$

2.2 Firms:

There are a large number of identical firms that utilize capital and labor to produce goods using constant return to scale production function. We assume that the labor supply is inelastic for both agents $1, 2$. This function satisfies the following assumption:

Assumption (4): *The technology* $F(K,L)$ *is a continuous and differentiable function defined on its arguments* $(K,L) \in [0,+\infty)$*. This function is increasing* $F_K(K,L) > 0$, $F_L(K,L) > 0$ and concave $F_{KK}(K,L) < 0$, $F_{LL}(K,L) < 0$ *and* $F(0,0) = 0$*. Furthermore, it satisfies Inada conditions:* $\lim_{k\longrightarrow 0} f'(k) = +\infty$, $\lim_{k\longrightarrow +\infty} f'(k) = 0$ *where* $f(k) \equiv F(k,1)$ is the production per capita and $k \equiv K/L$ is the capital per capita.

The representative firm takes the factor prices and technology as given and maximizes its profit:

$$\max_{K,L} F(K,L) - rK - wL$$

Then, FOCs imply the equilibrium real interest rate and the real wage:

$$r = f'(k) \tag{7}$$

$$w = f(k) - kf'(k) \tag{8}$$

[2] The rationalized preference (3) satisfies Assumption 1.

3 Equilibrium:

3.1 Intertemporal equilibrium:

Let us begin with the definition of an intertemporal equilibrium:

Definition 1 *An intertemporal equilibrium is a sequence* $\left(r, w, K, L, Y, (k_i, l_i, c_i)_{i=1}^2\right)$ *which satisfies the following conditions:*

1. Given factor prices (r, w)*, then* (K, L) *solves the firm's program.*

2. Given (r, w)*,* (k, l, c) *solves the consumer's program for both types of agents* $i = 1, 2$*.*

3. The capital market clears: $K = n_1 k_1 + n_2 k_2$*.*

4. The labor market clears: $L = n_1 + n_2$*.*

5. The product market clears: $\dot{K} = Y - C$*.*

Define $N_i = n_i / (n_1 + n_2)$ as the mass of agents of type i to the whole number of population, then the capital market equilibrium could be written as:

$$k = N_1 k_1 + N_2 k_2$$

Let

$$\theta_i \equiv \frac{n_i}{n_1 + n_2} \frac{k_i}{k} \in (0, 1)$$

then the above equality could be written as:

$$\theta_1 + \theta_2 = 1$$

3.2 The dynamic system:

At equilibrium, Euler equation (5) for household i optimization problem is:

$$\dot{c}_i = (1/\varsigma)\, c_i \left[f'(k) - \rho - \xi'(k_i)\right] \text{ for } i = 1, 2 \tag{9}$$

In order to complete the description of the competition equilibria, we take in consideration the agent i's budget constraint (2) together with equilibrium conditions (6) and (7). After that, equation (2) could be written as:

$$\dot{k}_i = f'(k)\, k_i + f(k) - k f'(k) - c_i - \xi(k_i) \tag{10}$$

with $k = N_1 k_1 + N_2 k_2$.

Intertemporal equilibrium with perfect foresight is a deterministic sequence $\{c_i, k_i\}_{i=1}^2 >> 0$ that satisfy the dynamic system (9) and (10) respectively. In addition, the solution of this dynamic system respects the initial conditions $\{k_{i0}\}_{i=1}^2$ and the transversality condition (6) simultaneously.

3.3 The steady state:

At the steady state equilibrium, the variables k_i and c_i are constant. Then, equations (9) and (10) become:

$$\rho = f'(k^*) - \xi'(k_i^*) \text{ for } i = 1, 2 \tag{11}$$

$$c_i^* = f'(k^*) k_i^* + f(k^*) - k^* f'(k^*) - \xi(k_i^*) \qquad \text{for } i = 1, 2 \tag{12}$$

with $k^* = N_1 k_1^* + N_2 k_2^*$.

Proposition 2 *There is a unique symmetric steady state in which $k_1^* = k_2^* = k^*$ and $c_1^* = c_2^* = c^*$.*

Proof. Since $f'(k^*)$ is valid for both types of agents, then from condition (11) we get:

$$\xi'(k_1^*) = \xi'(k_2^*)$$

so that the above equality holds if and only if there is a symmetric steady state value such that $k_1^* = k_2^* = k^*$, and so from equation (12), we have $c_1^* = c_2^* = c^*$. Furthermore, since the capital cost function is monotonic (increasing in this case) according to assumption (2), then it is a unique steady state. ■

4 Local dynamics:

Before passing through the stability analysis of the model, let us present some useful elasticities. The elasticities of capital cost $\varepsilon_1 = \xi' k/\xi > 0$ and the elasticity of marginal cost $\varepsilon_2 = \xi'' k/\xi' < 0$[3]. For the production side, the capital share in total income $s = f'k/f > 0$ and the elasticity of marginal product of capital $\varepsilon_r = f''k/f' = -(1-s)/\sigma < 0$, where $\sigma > 0$ is the elasticity of capital-labor substitution.

In this section, we show how the capital cost affects the stability of the economy in a heterogeneous agents model with respect to initial capital. In order to characterize the steady state stability, it is firstly required to linearize the dynamic system (9) and (10) and derive the Jacobian matrix, then we find out the eigenvalues of this Jacobian matrix. The signs of these eigenvalues determine the stability properties of the dynamic system locally. Linearizing the two dynamic equations for $i = 1, 2$ around the symmetric steady state equilibrium yields the following:

$$\begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \dot{k}_1 \\ \dot{k}_2 \end{bmatrix} = J \begin{bmatrix} c_1 - c_1^* \\ c_2 - c_2^* \\ k_1 - k_1^* \\ k_2 - k_2^* \end{bmatrix} \tag{13}$$

[3] Notice that if $\varepsilon_2 = 0$, then the transaction cost is linear with capital. However, if $\varepsilon_2 = \varepsilon_1 = 0$, then the transaction cost does not depend on the amount of capital stock, it is fix.

The characteristic polynomial of J is described as:

$$P(\lambda) = \lambda^4 - T\lambda^3 + W\lambda^2 - Z\lambda + D = 0 \tag{14}$$

Here, T is the trace of the Jacobian matrix, W is the sum of the second-order principal minors, Z is the sum of the third-order principal minors and D is the determinant of the Jacobian matrix. These blocks take the following values:[4]

$$T = 2\rho$$
$$D = \frac{\varepsilon_2\varepsilon_1\psi}{s}\left(\frac{\varepsilon_2\varepsilon_1\psi}{s} + \frac{1-s}{\sigma}\right)$$
$$W = 2\frac{\varepsilon_2\varepsilon_1\psi}{s} + \frac{1-s}{\sigma} + \rho^2$$
$$Z = \rho\left(2\frac{\varepsilon_2\varepsilon_1\psi}{s} + \frac{1-s}{\sigma}\right)$$

where the fraction $\psi = \xi/f \in (0,1)$ is the cost-production ratio. Notice that as ψ belongs to its maximum, i.e. unity, this means that the capital cost is sufficiently high relative to the production. This enforces agents to consume more and hold less capital. By contrary, when ψ is low then agents consume less and hold more capital since capital has low cost relative to its production.

In this model, there are two predetermined variables (k_1, k_2) and two non-predetermined variables (c_1, c_2). So, the model exhibits endogenous fluctuations if and only if there are at least three negative eigenvalues. Otherwise, a saddle path stability occurs and the economy is stable.

Using the second order sufficient conditions, it is easy to show that the determinant is negative regardless the value of the elasticity of marginal cost ε_2. Since the objective is to study the effect of the capital cost on the economic stability, we present some critical values for the elasticity of marginal capital cost ε_2.

$$\begin{aligned} \varepsilon_2^{**} &\equiv -\frac{1}{2}\frac{s}{\varepsilon_1\psi}\left(\rho^2 + \frac{1-s}{\sigma}\right) \\ \ddot{\varepsilon}_2 &\equiv -\frac{1}{2}\frac{s}{\varepsilon_1\psi}\frac{1-s}{\sigma} \end{aligned}$$

It is easy to demonstrate that $\ddot{\varepsilon}_2 > \varepsilon_2^{**}$. Using this result together with the above critical values, we deduce the following proposition:

Proposition 3 *According to the above critical values and assumptions (1-4), the steady state characterization has a stable saddle path with three positive and one negative eigenvalue regardless of the value of the elasticity of marginal cost* ε_2.

[4] It is well known that:
$Z = \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4$,
$W = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4$,
$Det = \lambda_1\lambda_2\lambda_3\lambda_4$,
$T = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$,

Proof. See Appendix A.

This proposition states that the steady state is always locally determinate in the sense that there is a unique saddle-path that converges toward the unique steady state. This implies that stability of the economy does not depend on the elasticity of marginal capital cost ε_2. In all cases, we obtain three positive and one negative eigenvalue which means that this economy is stable under a symmetric steady state.

5 A numerical example:

Let us consider an explicit formulation of the capital cost function as an isoelastic function $\xi(k) \equiv \eta k^{\varepsilon}$, with $\varepsilon \in [0,1]$, $\eta \geq 0$. Then we obtain that $\varepsilon_1 = \xi'(k)k/\xi(k) = \varepsilon$ and $\varepsilon_2 = \xi''(k)k/\xi'(k) = \varepsilon - 1 \leq 0$. The production function $f(k) = Ak^{\alpha}$ with $\alpha \in (0,1)$, where the capital share is $f'(k)k/f(k) = \alpha$. We choose that the discount rate $\rho = 0.05$ as in Benhabib and Farmer (1998), $\alpha = 0.33$, $A = 1$ and $\eta = 0.1$.

5.1 Steady state:

The steady state is defined in (11) and (12) at a symmetric steady state.

$$\rho = f'(k^*) - \xi'(k^*) \tag{15}$$
$$c^* = f(k^*) - \xi(k^*) \tag{16}$$

Additionally, we have two restrictions that should be satisfied. Firstly, the positivity of quantities of capital and consumption. From equation (15) we can determine the value of k^* according to the production and cost function formulations. Then, the steady state value of consumption c^* is determined by equation (16). The second constraint is that the SOC: $f''(k^*) < \xi''(k^*)$[5].

ρ	0.05	0.05	0.05
η	0.1	0.1	0.1
ε	0.05	0.15	0.25
k^*	14.978	9.786 5	2.805 6
c^*	2.328 4	1.982 0	1.276 1

Table (1)

Along with table (1), we show that the steady state of capital and consumption are always positive despite the sensitivity of the capital cost ε. Similarly to Alan (2006) we remark that there is a negative relation between the sensitivity of capital cost with the amount of holding capital. If the sensitivity of cost ε increases, then for a slight increase in capital, the capital cost highly

[5] For given formulations of the production and cost functions, it is easy to show that $f_{\text{J}}(k^*) < \xi''(k^*)$, regardless of the value of the sensitivity of the capital cost with respect to capitalε. For example, for $\varepsilon = 0.1$, $\rho = 0.065$, $\alpha = 0.33$, $A = 1$ and $\eta = 0.1$, we get that $f_{\text{J}}(k^*) = -9.9514 * 10^{-11}$ and $\xi''(k^*) = -2.0906 * 10^{-13}$.

increases. This incentives agents to accumulate low level of capital. Moreover, high ε, makes the cost function less concave, so, accumulating more capital has relatively high capital cost. However, Vayanos (1998) obtains that if the capital cost is a proportional cost with stocks (in this case, $\varepsilon = 1$), then agents purchase less capital than that without capital cost and hold it for longer periods.[6]

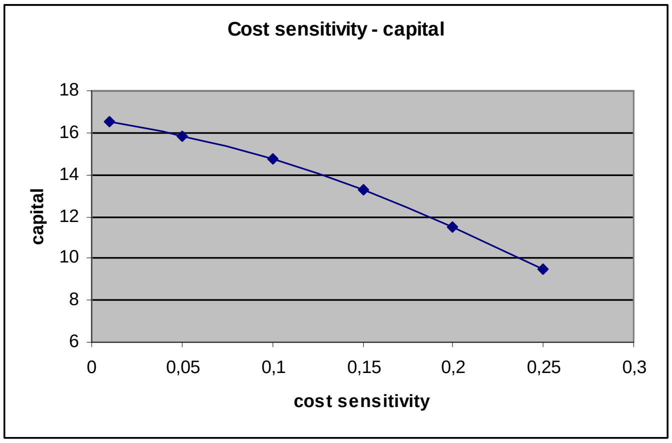

Figure (1) Isoelastic cost function.

This graphic well demonstrates the negative influence of elasticity of capital cost on the amount of capital holding.

5.2 Local dynamics:

Let us now study the impact of the capital cost on local dynamic. We focus on how the elasticity of capital cost ε can affect the stability of the economy by determining the sign of the four eigenvalues of the Jacobian matrix J. In this model, there are two predetermined variables k_1 and k_2. Thus, Indeterminacy occurs when there are three or four negative eigenvalues. Otherwise, the economic is stable and the steady state is saddle.

Parameter / eigenvalues	$\varepsilon = 0.05$	$\varepsilon = 0.15$	$\varepsilon = 0.25$
λ_1	$0.208\,91$	$0.374\,13$	$0.512\,79$
λ_2	$0.025 + 0.797\,61i$	$0.025 + 0.740\,34i$	$0.025 + 0.657\,31i$
λ_3	$0.025 - 0.797\,61i$	$0.025 - 0.740\,34i$	$0.025 - 0.657\,31i$
λ_4	$-0.158\,91$	$-0.324\,13$	$-0.462\,79$

Table (2)

From table (2), we show that once we change ε, the signs of the eigenvalues do not change; there are always three unstable and one stable eigenvalue which denotes that this elasticity has no effect on the stability of the economy and so, it is always stable.

[6] See among others, Lo, Mamaysky and Wang (2004), Jorgensen (2002), Constantinides (1986) and Alan (2006).

6 The model with heterogeneous cost function:

In the previous sections, we study how the capital cost affects the stability of the economy with identical capital cost function where the steady state is symmetric. However, in this section we keep our objective and focus on the case where the capital cost is different across agents.

Similarly, agent i chooses his capital and consumption that maximize (1) subject to

$$\dot{k}_i = rk_i + wl_i - c_i - \xi_i (k_i) \tag{17}$$

Notice that the capital cost function $\xi_i (k_i)$ is different across agents. The FOCs give Euler equation for agent $i = 1, 2$.

$$\dot{c}_i = (1/\varsigma) c_i \left[f'(k) - \rho - \xi_i' (k_i) \right] \tag{18}$$

Agents take in account the transversality condition (6). The capital cost satisfies Assumption (1) in addition to[7]

$$N_i [f''(k) + N_i f'''(k)(k_i - k)] < \xi_i''(k_i) < 0$$

In this case, neither firms nor the intertemporal equilibrium change.

6.1 The dynamic system:

As in the above case, the dynamic system of agent i consists of Euler equation (18) and the budget constraint at *equilibrium.*

$$\dot{k}_i = f'(k) k_i + f(k) - kf'(k) - c_i - \xi_i (k_i) \tag{19}$$

with $k = N_1 k_1 + N_2 k_2$.

Intertemporal equilibrium with perfect foresight is a deterministic sequence $\{c_i, k_i\}_{i=1}^{2} >> 0$ that satisfy (18) and (19) respectively.

6.2 The steady state:

The variables k_i and c_i are constant at the stationary equilibrium. Then, equations (18) and (19) become:

$$\begin{aligned} \rho &= f'(k^*) - \xi_i'(k_i^*) && (20) \\ c_i &= f'(k^*) k_i^* + f(k^*) - k^* f'(k^*) - \xi_i (k_i^*) && (21) \end{aligned}$$

with $k^* = N_1 k_1^* + N_2 k_2^*$.

Proposition 4 *There is asymmetric steady state in which $k_1^* \neq k_2^*$ and $c_1^* \neq c_2^*$.*

[7] As above, we assume that $N_i [f''(k) + N_i f'''(k)(k_i - k)] < \xi_i''(k_i)$ in order to the stationary point of the Hamiltonian function of agents represents a local maximum.

Proof. Since the production function is the same for both agents, then from equation (20) we have:

$$\xi_2' \left(k_2^*\right) = \xi_1' \left(k_1^*\right)$$

But, since the capital cost function is not the same for agents, it is possible to have an asymmetric steady state where $k_1^* \neq k_2^*$ and it is not unique. Given that $k_1^* \neq k_2^*$, equation (21) determines the steady state value of consumption where $c_1^* \neq c_2^*$. ■

7 Local dynamics:

Before passing through the stability analysis of the model, let us present some useful elasticities. The elasticity of capital cost for agent i is $\varepsilon_i = \xi_i' k_i / \xi_i > 0$ and the elasticity of marginal cost for agent i is $\varepsilon_{ii} = \xi_i'' k_i / \xi_i' < 0$. For the production side, the capital share in total income $s = f'k/f > 0$ and the elasticity of marginal product of capital $\varepsilon_r = f''k/f' = -(1-s)/\sigma < 0$, where $\sigma > 0$ is the elasticity of capital-labor substitution.

In this section, we show how the capital cost affects the stability of the economy in a heterogeneous agents model in asymmetric steady state. Linearizing the two dynamic system (18) and (19) for $i = 1, 2$ around an asymmetric steady state equilibrium yields the following:

$$\begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \dot{k}_1 \\ \dot{k}_2 \end{bmatrix} = J \begin{bmatrix} c_1 - c_1^* \\ c_2 - c_2^* \\ k_1 - k_1^* \\ k_2 - k_2^* \end{bmatrix} \tag{22}$$

The characteristic polynomial of J is described as:

$$P(\lambda) = \lambda^4 - T\lambda^3 + W\lambda^2 - Z\lambda + D = 0$$

where

$$T = 2\omega\rho$$

$$D = \omega^2 \left[\frac{\varepsilon_{11}\varepsilon_1\psi_1}{s} \left(\frac{1-s}{\sigma} N_2 + \frac{\varepsilon_{22}\varepsilon_2\psi_2}{s} \right) + \frac{1-s}{\sigma} \frac{\varepsilon_{22}\varepsilon_2\psi_2}{s} N_1 \right]$$

$$W = \omega \left[\frac{1-s}{\sigma} + \frac{\varepsilon_{11}\varepsilon_1\psi_1}{s} + \frac{\varepsilon_{22}\varepsilon_2\psi_2}{s} + \omega\rho^2 \right]$$

$$Z = \omega \left[\frac{1-s}{\sigma}\omega\rho + \frac{\varepsilon_{22}\varepsilon_2\psi_2}{s} a_2 + \frac{\varepsilon_{11}\varepsilon_1\psi_1}{s} a_1 \right]$$

with $a_1 \equiv \frac{1-s}{\sigma} N_2 \Omega_2 + \omega\rho$, $a_2 \equiv \frac{1-s}{\sigma} N_1 \Omega_1 + \omega\rho$, $\omega \equiv 1/f'$, $\Omega_i \equiv (N_i - \theta_i)/N_i$ and $\psi_i \equiv \xi_i / f$.

Since the objective is to study the effect of the capital cost on the economic stability, we present some critical values for the elasticity of marginal capital cost for agent 1, ε_{11} that make D, W and Z equal zero:

$$
\begin{aligned}
\varepsilon_{11}^{D} &= -\frac{s}{\varepsilon_1\psi_1}\frac{\frac{1-s}{\sigma}\frac{\varepsilon_{22}\varepsilon_2\psi_2}{s}N_1}{\frac{1-s}{\sigma}N_2+\frac{\varepsilon_{22}\varepsilon_2\psi_2}{s}} \\
\varepsilon_{11}^{Z} &= -\frac{s}{\varepsilon_1\psi_1}\frac{\frac{1-s}{\sigma}\omega\rho+\frac{\varepsilon_{22}\varepsilon_2\psi_2}{s}a_2}{a_1} \\
\varepsilon_{11}^{W} &= -\frac{s}{\varepsilon_1\psi_1}\left[\frac{\varepsilon_{22}\varepsilon_2\psi_2}{s}+\frac{1-s}{\sigma}+\omega\rho^2\right]
\end{aligned}
\tag{23}
$$

Mostly, we are interested in endogenous fluctuation appearance, so that we search the conditions under which the dynamic system unstable. These conditions are simply summarized in the following proposition.

Proposition 5 *Let*

$$\varepsilon_{22}^{D} \equiv -N_2\frac{1-s}{\sigma}\frac{s}{\varepsilon_2\psi_2}$$

be a critical value of agent's 2 *elasticity of marginal cost. Then according to the above critical values (23) and assumptions (1-4), the steady state has the following characterization:*

- ***Case (1):*** $\varepsilon_{22} > \varepsilon_{22}^{D}$.[8]

(i) $-\infty < \varepsilon_{11} < \varepsilon_{11}^{Z}$*, the steady state is saddle.*
(ii) $\varepsilon_{11}^{Z} < \varepsilon_{11} < \varepsilon_{11}^{W}$*, the steady state is indeterminate.*
(iii) $\varepsilon_{11}^{W} < \varepsilon_{11} < 0$*, the steady state is saddle.*

- ***Case (2):*** $\varepsilon_{22} < \varepsilon_{22}^{D}$.[9]

A- $\max\left\{\varepsilon_{11}^{Z}, \varepsilon_{11}^{D}\right\} < \varepsilon_{11}^{W} < 0$
(i) $-\infty < \varepsilon_{11} < \max\left\{\varepsilon_{11}^{Z}, \varepsilon_{11}^{D}\right\}$*, the steady state is saddle.*
(ii) $\max\left\{\varepsilon_{11}^{Z}, \varepsilon_{11}^{D}\right\} < \varepsilon_{11} < \varepsilon_{11}^{W}$*, the steady state is indeterminate.*
(iii) $\varepsilon_{11}^{W} < \varepsilon_{11} < 0$*, the steady state is saddle.*
B- $\max\left\{\varepsilon_{11}^{Z}, \varepsilon_{11}^{D}\right\} < 0 < \varepsilon_{11}^{W}$
(i) $-\infty < \varepsilon_{11} < \max\left\{\varepsilon_{11}^{Z}, \varepsilon_{11}^{D}\right\}$*, the steady state is saddle.*
(ii) $\max\left\{\varepsilon_{11}^{Z}, \varepsilon_{11}^{D}\right\} < \varepsilon_{11} < 0$*, the steady state is indeterminate.*

Proof. See Appendix B.

This proposition confirms that the elasticity of marginal capital cost for both agents $(\varepsilon_{11}, \varepsilon_{22})$ play an important role in economic stability. Indeterminacy appearance requires a heterogeneity across agents with respect to the capital cost function because in the first part of this paper we demonstrate that there is no room for the endogenous fluctuations for symmetric capital cost for both agents. According to the above proposition, when the elasticity of marginal cost of agent's 2 (ε_{22}) is sufficiently high, multiple equilibria requires that the elasticity of marginal cost of agent's 1 (ε_{11}) is *not* satisfactorily high. However,

[8] Otherwise, for the case where $\varepsilon_{11}^{W} < \varepsilon_{11}^{Z}$, the steady state is saddle and the economy is stable.

[9] Otherwise, for $\varepsilon_{11}^{W} < \min\left\{\varepsilon_{11}^{Z}, \varepsilon_{11}^{D}\right\}$ and $\varepsilon_{11}^{W} \in \left\{\varepsilon_{11}^{Z}, \varepsilon_{11}^{D}\right\}$ the steady state is always stable.

for low values of agent's 2 elasticity, endogenous fluctuation needs that the elasticity of marginal cost is adequately high $\max\left\{\varepsilon_{11}^{Z}, \varepsilon_{11}^{D}\right\} < \varepsilon_{11}$.

In the first case, we show that for very low value of ε_{11}, there is a unique trajectory path converges toward the steady state and respects the transversality condition. Once this value increases and becomes higher than the critical value ε_{11}^{Z}, then the stability of the economy changes and becomes unstable. This involves that there are infinite number of convergence trajectories move toward the steady state.

In the second case, this proposition proves that for low values of ε_{11}, the steady state is saddle and the economy is stable. However, for *not* satisfactorily value the economic system stability changes it stability through saddle-node bifurcation and becomes unstable. Finally, for sufficiently high value of ε_{11} the steady state returns to be stable.

To figure out how the indeterminacy emerges, let us consider first a case where there is no indeterminacy and the system is explosive. For instance, when the elasticity of marginal cost for both agents $(\varepsilon_{11}, \varepsilon_{22})$ are sufficiently low. If agents anticipate that income gets higher at any given date, then current and future consumption increase. This will encourage both agents to accumulate capital and since the elasticity of marginal cost for both agents are low, which means that high increase in capital increase the cost slightly leading to higher income. This means that any equilibrium trajectory starting far from the steady state equilibrium will never locally converge "explosive economy".

Consider that the elasticity of marginal cost for both agents have different values, i.e., ε_{11} is sufficiently high and ε_{22} is adequately low. If agents anticipate that future income will rise, they enhance their current and future consumption, this induces agents to accumulate more capital. Since ε_{11} is high, this means that a small increase in capital will amplify the cost more and so agents 1 will not increase capital stock too much. Conversely, agents 2 will accumulate more capital since ε_{22} is small[10]. This implies that the total amount of capital for both agents do not augment so much and so the economy does converge. Thus indeterminacy requires two contrasting forces: ε_{11} and ε_{22} have completely different values.

8 Discussion:

In this section, we provide the results obtained through this paper and compare it to the literature. We show that when we introduce an identical capital cost function in a model with heterogeneous initial capital across agents, it has no effect on the economic stability and there is no room for the indeterminacy. In this case, there is only a unique trajectory path converges to the symmetric steady state. At the steady state, since the cost function is monotonic then there is a unique symmetric steady state in which all agents hold the same amount of capital. We clarify this point numerically by showing that regardless of the value of elasticity of marginal cost with respect to capital, the model has one

[10] The indeterminacy has the same intuition for small ε_{11} and high ε_{22}.

negative and three positive eigenvalues. Conversely, Mino and Nakamoto (2008) consider a Ramsey model with heterogeneous households in addition to different utility function and a progressive income taxation where the time discount rate of agents is identical. They show that if the marginal tax payment of each agent increases with her relative income, the steady state exhibits a saddle point and the dynamic system is determinate. Further, for certain conditions on taxation function and on utility function, it is possible for indeterminacy to appear.

At the same time, this result is analogous to that with representative agents Ramsey model[11]. It is easy to show that capital cost has no influence on the stability of the economy and the system is always stable. We obtain that the trace is positive and the determinant is negative, so there appears two eigenvalues with different signs. To summarize, introducing this capital cost does not have any effect on the local dynamics in both representative agent as well as in heterogenous agents model with identical capital cost function. Along this line of literature, Becker and Foias (1994) consider a Ramsey model with heterogeneous initial capital across agents as well as the discount factor. They show that the two-period cycles appear when capital income is decreasing in capital stock.

In the second part of the paper, we keep the heterogeneity of initial capital across agents and suppose that the capital cost function is different as well. In proposition (4) it is shown that there is asymmetric steady state where both agents hold different amounts of capital but this proposition does not exclude the existence of a symmetric one. In this case, the stability of the economy depends on the values of the elasticity of marginal cost for both agents. For instance, when the elasticity of marginal cost for agents 2 is low, then endogenous fluctuation requires sufficiently high value of the elasticity of marginal cost for agents 1. On the contrary, for higher values of the elasticity of marginal cost for agents 2, indeterminacy appearance needs not adequately high values of elasticity of marginal cost for agents 1. Furthermore, for certain values of the elasticity of marginal capital cost with respect to capital ε_{11}, the economic system changes its stability through saddle-node bifurcation. This implies that the steady state is no longer hyperbolic and it emerges two new fixed-points in which one is stable and the other is unstable.

9 Conclusion:

Along this paper, we have focused on the stability issue and the local dynamics in an economy with heterogeneous agents. The heterogeneity among agents is according to the initial capital. In line with Vayanos (1998), Amihud and Mendelson (1986), Vayanos and Vila (1997), Vissing-Jorgensen (2002), we introduce a capital cost function that affects the investment choices for agents. In the first part of the paper, the cost function is supposed to be identical across

[11] The representative agents model is briefly summarized in the appendix C, where it is shown that the steady state is stable even with introduction capital cost.

agents. Consistent with this assumption, we show that there is a unique symmetric steady state of capital and consumption under which both agents hold the same amount of capital in the long term. It is demonstrated that the heterogeneity does not matter and does not affect the economic stability; analogous with representative agent model, there exists a unique stable path that converge to the steady state regardless of the elasticity of the capital cost with respect to capital.

After that, we suppose that this cost function is different across agents. We keep our objective and focus on the role of the capital cost on the local stability of the steady state. It is proved that the steady state is asymmetric and is not unique. The stability of the economy is determined by the elasticity of marginal cost for both agents. In contrast to the first part, we demonstrate that for certain values on these elasticities the economy is stable and for some other values the economy changes its stability through saddle-node bifurcation and becomes unstable.

10 Appendix:

(A) Proof of proposition (3):

From the above critical points ε_2^{**}, $\ddot{\varepsilon}_2$, we can directly prove that $\varepsilon_2^{**} < \ddot{\varepsilon}_2 < 0$. So, the stability analysis depends on the value of the elasticity of marginal cost with respect to capital ε_2. Moreover, according to the second order sufficient condition, it is easy to show that the determinant is always negative as well as the trace is positive for all values of the elasticity of marginal cost.

Proof.

(a) If $\varepsilon_2 < \varepsilon_2^{**}$, then both W and Z are negative. Since the determinant is negative, so we have either: (i) three positive eigenvalue and one negative or (ii) three negative eigenvalues and one positive. Here, we show that it is *not* the case (ii). Assume that there are three negative eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ and one positive λ_4. In this case, λ_4 is sufficiently high that makes the trace positive, $(\lambda_4 > \lambda_1 + \lambda_2 + \lambda_3)$. The reader can easily verify that high value of λ_4 leads to a positive value of Z, which is not the case here, $Z < 0$. So, we have three positive eigenvalue and one negative.

(b) If $\varepsilon_2 \in (\varepsilon_2^{**}, \ddot{\varepsilon}_2)$, then W is positive and Z is negative. It has the same analysis as above.

(c) If $\varepsilon_2 \in (\ddot{\varepsilon}_2, 0)$, then both W and Z are positive. As above, we have either (i) three positive eigenvalue and one negative or (ii) three negative eigenvalues and one positive. Here, we prove that it is *not* the case (ii). Assume that there are three negative eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ and one positive λ_4. In this case, λ_4 is sufficiently high which makes the trace positive, $(\lambda_4 > \lambda_1 + \lambda_2 + \lambda_3)$. The reader can easily verify that high value of λ_4 leads to a negative value of W, which is not the case here, $W > 0$. So, there are three positive eigenvalues and one negative.

In all cases, we show that there are three positive eigenvalues and one negative which means that the competitive equilibrium path converging to the steady state is uniquely determined and the economy is stable.

■

(B) Proof of proposition (5):

In this proof, we focus on the sign of eigenvalues. In this model, there are two predetermined variable which means that indeterminacy appears for either three negative eigenvalues and one positive, or the four eigenvalues are negative. Particularly, the economy is unstable once we have $T > 0$, $D < 0$, $W < 0$ and $Z > 0$. This characterization shows that there are three negative eigenvalues and one positive. The second configuration for indeterminacy steady state with four negative eigenvalues is $T < 0$, $D > 0$, $W > 0$ and $Z < 0$. There are infinite number of trajectories that converge to the steady state and satisfy the transversality condition. Otherwise, the economy is stable. In order to characterize these signs we have to use the critical values (23).

Proof.

Case (1): $\varepsilon_{22} > \varepsilon_{22}^{D}$.

Here, $T > 0$ for all values of ε_{22} and ε_{11}. Now, if $\varepsilon_{22} > \varepsilon_{22}^{D}$, then $D' > 0$ in addition to $\varepsilon_{11}^{D} > 0$, which means that $D < 0$ for all $\varepsilon_{11} < 0$. At the same time, $Z' > 0$ and $\varepsilon_{11}^{Z} < 0$ which entails that $Z > 0$ for $\varepsilon_{11} > \varepsilon_{11}^{Z}$ and $Z < 0$ for $\varepsilon_{11} < \varepsilon_{11}^{Z}$. Finally, $W' > 0$ and $\varepsilon_{11}^{W} < 0$, this gives that $W > 0$ for $\varepsilon_{11} > \varepsilon_{11}^{W}$ and $W < 0$ for $\varepsilon_{11} < \varepsilon_{11}^{W}$.

(i) For $\varepsilon_{11} < \varepsilon_{11}^{Z}$, we have $T > 0$, $D < 0$, $W < 0$ and $Z < 0$. This is equivalent a stable economic system with three positive and one negative eigenvalues (the same technique as the above proof).

(ii) For $\varepsilon_{11} \in \left(\varepsilon_{11}^{Z}, \varepsilon_{11}^{W}\right)$, we have $T > 0$, $D < 0$, $W < 0$ and $Z > 0$. This is equivalent to an unstable economy with three negative and one positive eigenvalues.

(iii) For $\varepsilon_{11} \in \left(\varepsilon_{11}^{W}, 0\right)$, we have $T > 0$, $D < 0$, $W > 0$ and $Z > 0$. This denotes that there are three positive and one negative eigenvalues and the steady state is saddle.

Case (2): $\varepsilon_{22} < \varepsilon_{22}^{D}$.

(A) $\max\left\{\varepsilon_{11}^{Z}, \varepsilon_{11}^{D}\right\} < \varepsilon_{11}^{W} < 0$.

- We consider the case where $\varepsilon_{11}^{Z} < \varepsilon_{11}^{D} < \varepsilon_{11}^{W} < 0$.

As previously, $T > 0$ for all values of ε_{22} and ε_{11}. Now, if $\varepsilon_{22} < \varepsilon_{22}^{D}$, then $D' < 0$ in addition to $\varepsilon_{11}^{D} < 0$, which implies that $D < 0$ for $\varepsilon_{11} > \varepsilon_{11}^{D}$ and $D > 0$ for $\varepsilon_{11} < \varepsilon_{11}^{D}$. Moreover, $Z' > 0$ and $\varepsilon_{11}^{Z} < 0$ which entails that $Z > 0$ for $\varepsilon_{11} > \varepsilon_{11}^{Z}$ and $Z < 0$ for $\varepsilon_{11} < \varepsilon_{11}^{Z}$. Finally, $W' > 0$ and $\varepsilon_{11}^{W} < 0$, this gives that $W > 0$ for $\varepsilon_{11} > \varepsilon_{11}^{W}$ and $W < 0$ for $\varepsilon_{11} < \varepsilon_{11}^{W}$.

(i) For $\varepsilon_{11} < \varepsilon_{11}^{Z}$, we have $T > 0$, $D > 0$, $W < 0$ and $Z < 0$. So, three positive and one negative eigenvalues and the economy is stable.

(ii) For $\varepsilon_{11} \in \left(\varepsilon_{11}^{Z}, \varepsilon_{11}^{D}\right)$, we have $T > 0$, $D > 0$, $W < 0$ and $Z > 0$. So, three positive and one negative eigenvalues and the economy is stable.

(iii) For $\varepsilon_{11} \in \left(\varepsilon_{11}^{D}, \varepsilon_{11}^{W}\right)$, we have $T > 0$, $D < 0$, $W < 0$ and $Z > 0$. There is an endogenous fluctuation with three negative and one positive eigenvalues.

(iv) For $\varepsilon_{11} \in \left(\varepsilon_{11}^{W}, 0\right)$, we have $T > 0$, $D < 0$, $W > 0$ and $Z > 0$. The economy returns to be stable with three positive and one negative eigenvalues and the economy is stable.

- We consider the case where $\varepsilon_{11}^{D} < \varepsilon_{11}^{Z} < \varepsilon_{11}^{W} < 0$.

(i) For $\varepsilon_{11} < \varepsilon_{11}^{D}$, we have $T > 0$, $D > 0$, $W < 0$ and $Z < 0$. So, three positive and one negative eigenvalues and the economy is stable.

(ii) For $\varepsilon_{11} \in \left(\varepsilon_{11}^{D}, \varepsilon_{11}^{Z}\right)$, we have $T > 0$, $D < 0$, $W < 0$ and $Z < 0$. So, three positive and one negative eigenvalues and the economy is stable.

(iii) For $\varepsilon_{11} \in \left(\varepsilon_{11}^{Z}, \varepsilon_{11}^{W}\right)$, we have $T > 0$, $D < 0$, $W < 0$ and $Z > 0$. The economic system is unstable with three negative and one positive eigenvalues.

(iv) For $\varepsilon_{11} \in \left(\varepsilon_{11}^{W}, 0\right)$, we have $T > 0$, $D < 0$, $W > 0$ and $Z > 0$. The economy returns to be stable with three positive and one negative eigenvalues and the economy is stable.

Notice that regardless of the ranking of $\left(\varepsilon_{11}^{D}, \varepsilon_{11}^{Z}\right)$ the indeterminacy occurs in the two cases with sufficiently high ε_{11}.

(B) $\max\left\{\varepsilon_{11}^{Z}, \varepsilon_{11}^{D}\right\} < 0 < \varepsilon_{11}^{W}$.

- We consider the case where $\varepsilon_{11}^{D} < \varepsilon_{11}^{Z} < 0 < \varepsilon_{11}^{W}$.

As above, $T > 0$ for all values of ε_{22} and ε_{11} and $W < 0$ for all $\varepsilon_{11} < 0$. If $\varepsilon_{22} < \varepsilon_{22}^{D}$, then $D' < 0$ in addition to $\varepsilon_{11}^{D} < 0$, which implies that $D < 0$ for $\varepsilon_{11} > \varepsilon_{11}^{D}$ and $D > 0$ for $\varepsilon_{11} < \varepsilon_{11}^{D}$. In addition, $Z' > 0$ and $\varepsilon_{11}^{Z} < 0$ which indicates that $Z > 0$ for $\varepsilon_{11} > \varepsilon_{11}^{Z}$ and $Z < 0$ for $\varepsilon_{11} < \varepsilon_{11}^{Z}$.

(i) For $\varepsilon_{11} < \varepsilon_{11}^{D}$, we have $T > 0$, $D > 0$, $W < 0$ and $Z < 0$. The economy is stable with three positive and one negative eigenvalues.

(ii) For $\varepsilon_{11} \in \left(\varepsilon_{11}^{D}, \varepsilon_{11}^{Z}\right)$, we obtain $T > 0$, $D < 0$, $W < 0$ and $Z < 0$. The economy is stable with three positive and one negative eigenvalues.

(iii) For $\varepsilon_{11} \in \left(\varepsilon_{11}^{Z}, 0\right)$, we have $T > 0$, $D < 0$, $W < 0$ and $Z > 0$. The economy is unstable with three negative and one positive eigenvalues.

- We consider the second case where $\varepsilon_{11}^{Z} < \varepsilon_{11}^{D} < 0 < \varepsilon_{11}^{W}$.

(*i*) For $\varepsilon_{11} < \varepsilon_{11}^{Z}$, we have $T > 0$, $D > 0$, $W < 0$ and $Z < 0$. The economy is stable with three positive and one negative eigenvalues.

(*ii*) For $\varepsilon_{11} \in \left(\varepsilon_{11}^{Z}, \varepsilon_{11}^{D}\right)$, we have $T > 0$, $D > 0$, $W < 0$ and $Z > 0$. The economy is stable with three positive and one negative eigenvalues.

(*iii*) For $\varepsilon_{11} \in \left(\varepsilon_{11}^{D}, 0\right)$, we have $T > 0$, $D < 0$, $W < 0$ and $Z > 0$. The economy is unstable with three negative and one positive eigenvalues.

Notice that regardless of the ranking of $\left(\varepsilon_{11}^{Z}, \varepsilon_{11}^{D}\right)$, the economy is stable for low values of ε_{11} it changes it stability for high value of ε_{11}.

■

C- Representative agents problem.

Here, we analyze briefly the representative agents problem with capital cost. Agents choose their capital and consumption to maximize their utility

$$U = \int_0^\infty e^{-\rho t} u(c)\, dt$$

such that

$$\dot{k} = rk + wl - c - \xi(k)$$

After standard calculations, we obtain the dynamic system at equilibrium:

$$\begin{aligned} \dot{c} &= (1/\varsigma)\, c \left[f'(k) - \rho - \xi'(k)\right] \\ \dot{k} &= f(k) - c - \xi(k) \end{aligned}$$

From this system, it is easy to calculate the trace and the determinant:

$$T = \rho \tag{24}$$

$$D = f'' - \xi'' \tag{25}$$

Since there is one predetermined variable k, indeterminacy requires both eigenvalues to be negative. Given the SOC in Assumption (2), we deduce that the determinant is negative and so the steady state is always stable.

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