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The Role of Optimal Fiscal Policy in a Currency Union*

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Abstract

By constructing a dynamic stochastic general equilibrium model, this paper verifies the role of optimal fiscal policy in a currency union with nontradable goods. A single optimal monetary policy has a decisive role in enhancing welfare by stabilizing inflation and the output gap in each country simultaneously when all goods are tradable. However, because approximately half of all goods are nontradable in the Euro area, this result is not applicable. When nontradable goods exist, optimal monetary policy alone cannot maximize welfare because of a real exchange rate anomaly or the Balassa–Samuelson theorem, which holds that stabilizing inflation and the output gap in each country simultaneously cannot be achieved. In this case, an optimal monetary and cooperative optimal fiscal policy mix is essential to maximize welfare. Furthermore, we show that a self-oriented setting can bring about an optimal allocation that corresponds to one brought about by a cooperative setting.

Keywords: currency union, DSGE, real exchange rate anomaly, optimal monetary policy, monetary and fiscal policy mix

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1 Introduction

Currency unions, which were previously a matter of academic interest, became a reality when the European Monetary Union (EMU) was established. The creation of the EMU led to new challenges for policy makers. This study provides a tractable framework suitable for the analysis of fiscal and monetary policy in a currency union and studies its implications for the optimal design of such policies, not only within the context of a union-wide economy but also within the context of the individual countries that comprise the currency union.

Discussions of optimal monetary policy in a currency union have become vigorous following the establishment of the EMU.[1] Assuming that all goods are tradable, Benigno (2004) analyzes optimal monetary policy in simple situations including perfect risk sharing and a two-country model. He finds that a solitary central bank in a currency union can achieve welfare maximization not only union-wide but also in each country. In contrast, Gali and Monacelli (forthcoming) insist on a monetary and fiscal policy mix using a currency union model consisting of an infinite number of countries to maximize social welfare. Under this framework, the solitary central bank can maximize union-wide welfare whereas it needs strong support from the fiscal authorities to maximize welfare in each country. Ferrero (2007) and Beetsma and Jensen (2005) analyze optimal monetary and fiscal policy in a two-country currency union, and find that optimal fiscal policy is essential in a currency union to maximize social welfare.[2]

It is noteworthy that we refer to papers that do not use a dynamic stochas-

[1]Note that there are some papers that discuss the role of optimal monetary and fiscal policy. Schmitt-Grohe and Uribe (2004a, 2004b, forthcoming) find a relationship between fiscal policy, the output gap and inflation in a closed economy. Benigno and De Paoli (2006) find that it is optimal not to smooth taxes in an open economy.

[2]Gali and Monacelli (forthcoming), Ferrero (2007) and Beetsma and Jensen (2005) assume that all goods are tradable. Gali and Monacelli (forthcoming) does not consider a government budget constraint explicitly whereas Ferrero (2007) does so.

tic general equilibrium (DSGE) model. Focusing on fiscal theory of the price level, Canzoneri, Cumby and Diba (2001) discuss fiscal discipline and exchange rate systems. To maintain a common currency union, the discipline of a Ricardian regime is essential. They also point out that the Stability and Growth Pact, which is written into the Maastricht treaty, is sufficient for a Ricardian regime. This contrasts with policy implications derived by Gali and Monacelli (forthcoming) and Ferrero (2007). Furthermore, a conventional paper, such as McKinnon (1963), should be cited. McKinnon (1963) insists on the necessity of moving fiscal policy control from the local or national government to the central government. The existence of nontradable goods generates disparity among countries in a currency union. Thus, he advocates that fiscal transfers among countries under a centralized government are essential.

The policy implications of the papers we refer to above on the role of fiscal policy vary substantially. To stabilize immediately both inflation and the output gap under the assumption that all goods are tradable, Canzoneri, Cumby and Diba (2001) and Benigno (2004) suggest that additional fiscal policy is not essential, while Gali and Monacelli (forthcoming) and Ferrero (2007) discuss contradictory policy implications that support the necessity of fiscal policy. Assuming the existence of nontradable goods, McKinnon (1963) advocates fiscal transfers among countries via a centralized government. There is significant difference in the policy implications of the preceding studies, which discuss optimal policy in the currency union. The papers clearly leave some questions unanswered. Is fiscal policy needed in a currency union from the viewpoint of maximizing social welfare? What is the causation that creates differences in the policy implications of these papers? Can fiscal policy conducted by local government enhance social welfare, instead of income transfers by a centralized government? Can a decentralized setting for optimal fiscal policy be designed?

We should consider two key elements in addressing the questions raised by the preceding papers. Although many DSGE studies do not propose the form of the government budget constraint, we need to assume such a constraint to clarify the role of fiscal policy. Next, we analyze the role of fiscal policy in a currency union with nontradable goods, although few studies assume the existence of nontradable goods in recent DSGE studies.[3] Assuming the existence of nontradable goods may derive the policy implication that fiscal policy has an important role in a currency union because McKinnon (1963) advocates the necessity of fiscal policy assuming nontradable goods. Furthermore, assuming nontradable goods is plausible from the viewpoint of an actual currency union, namely, the Euro area. Following the definition that regards goods produced in the manufacturing industry, agriculture, forestry, fishery and mining as tradable goods and regards goods produced in other industries as nontradable goods, as used by Canzoneri, Cumby and Diba (1999), nontradable goods, in terms of both current and purchaser's prices, accounted for 50.3% of the sum of nontradable goods and tradable goods in major Euro area countries such as Belgium, Germany, France, Greece, Italy, the Netherlands, Portugal and Spain in 1999. It is obvious that the existence of nontradable goods should not be ignored in analyzing monetary policy.

This study constructs a DSGE model depicting a currency union consisting of two countries with nontradable goods and explicit government budget con-

[3]Neither papers on monetary policy in a currency union nor papers on monetary policy in an open economy, such as those of Benigno (2004), Benigno and Benigno (2008), Gali and Monacelli (forthcoming) and Okano (2007), consider the existence of nontradable goods, although these papers derive some important implications.

straints to obtain answers to the preceding questions. The model developed in this study has two distinctive features for analyzing an optimal policy design. Because it has two features, our model is a nested model of preceding papers such as those of Benigno (2004), Gali and Monacelli (forthcoming) and Ferrero (2007). First, because of the existence of nontradable goods, we focus on the Balassa–Samuelson theorem and real exchange rate anomaly, which explain deviations in the nominal exchange rate from purchasing power parity.[4] Needless to say, the effect of nontradable goods in an open economy is not widely considered in the DSGE literature. Whereas the nominal exchange rate does not appear in our model because the model is a closed currency union system, the Balassa–Samuelson theorem and real exchange rate anomaly explain the disparity in the consumer price indices (CPIs) between two countries comprising a currency union. Second, we allow implementation of fiscal policy by local governments to eliminate the effects of the nontradable goods. While the appropriateness of a centralized government is advocated by McKinnon (1963) for welfare maximization, this study verifies that a centralized government does not need to maximize social welfare even though all goods are not tradable. We prove that self-oriented local governments can create an optimal allocation brought about by cooperative local governments. Note that our contribution on developing the model is to mix nontradable goods with government budget constraints.[5]

[4]Some empirical analyses have focused on nontradable goods and the Balassa–Samuelson theorem. Analyzing exchange rate volatility rather than monetary policy, Stockman and Tesar (1995), Benigno and Thoenissen (2005) and Selaive and Tuesta (2006) focus on nontradable goods where the real exchange rate is volatile and tends to move in the opposite direction to relative consumption across countries, namely, the consumption–real exchange rate anomaly. These papers based on the Balassa–Samuelson theorem point out the relationship between the anomaly and the theorem. In this paper, another side effect relates to this anomaly.

[5]Under simple settings, monetary and fiscal interactions in an open economy have already been investigated by Lombardo and Sutherland (2004).

We show social welfare and macroeconomic volatility when all goods are tradable and when half of the goods are nontradable. In the case that all goods are tradable, which corresponds to the setting in Ferrero (2007), the role of optimal fiscal policy is unimportant although it supports the maximization of welfare. However, in the case that half of the goods are tradable, the role of optimal fiscal policy is very important. We can clarify the causation that creates disparities in the policy implications among the papers. Differences in the policy implications among these papers mainly depend on the price index used in each paper. In addition, we can prove that a self-oriented government can maximize social welfare using a game theory framework. Thus, neither income transfers nor a centralized government is needed to enhance social welfare. Details of these results are provided in latter sections.

The paper is organized as follows. Section 2 constructs the model. Section 3 log-linearizes the model. Section 4 analyzes a role of optimal fiscal policy under a cooperative setting. Section 5 considers the possibility of a decentralized setting to attain the allocation brought about by the cooperative solution. Section 6 concludes the paper.

2 The Model

We construct a closed-system currency union model belonging to the class of DSGE models with nominal rigidities and imperfect competition, following Obstfeld and Rogoff (2000) and Gali and Monacelli (2005).[6] The union-wide economy consists of two equally sized countries, countries H and F. Country H produces an array of differentiated goods indexed by the interval $h \in [0, 1]$,

[6]Appendices clarifying not only the derivation of the model but also details on other technical aspects are provided at the URL: http://www.cku.ac.jp/okano/papers_e.html.

while country F produces an array of differentiated goods indexed by $f \in [1, 2]$.

2.1 Households

The preferences of the representative household in country H are given by:

$$\mathcal{U} \equiv \mathrm{E}_t \sum_{t=0}^{\infty} \delta^t U_t, \tag{1}$$

where $U_t \equiv \ln C_t - \frac{1}{1+\varphi} N_t^{1+\varphi}$ denotes the period utility in country H, E_t denotes the expectation, conditional on the information set at period t, $\delta \in (0, 1)$ denotes the subjective discount factor, C_t denotes consumption in country H, $N_t \equiv N_{H,t} + N_{\mathcal{N},t}$ denotes hours of work in country H, $N_{H,t}$ and $N_{\mathcal{N},t}$ denote hours of work to produce tradable goods produced in country H and nontradable goods produced in country H, respectively, γ denotes the share of tradables in the CPI and φ denotes the inverse of the labor supply elasticity. $\mathcal{U}^*$, denoting the preference of the representative household in country F, is defined analogously. We note that quantities and prices peculiar to country F are denoted by asterisks while quantities and prices without asterisks are those in country H.

More precisely, private consumption is a composite index defined by:

$$C_t \equiv \left[\gamma^{\frac{1}{\eta}} C_{\mathcal{T},t}^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} C_{\mathcal{N},t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where $C_{\mathcal{T},t} \equiv 2 C_{H,t}^{\frac{1}{2}} C_{F,t}^{\frac{1}{2}}$ denotes the consumption index for tradables, $C_{H,t}$, $C_{F,t}$ and $C_{\mathcal{N},t}$ denote Dixit–Stiglitz-type indices of consumption across the tradables produced in country H and produced in country F, and nontradables produced in country H, respectively, $\theta > 1$ denotes the elasticity of substitution across goods produced within a country, and $\eta > 0$ denotes the elasticity of substitution between tradable and nontradable goods. Note that C_t^* is defined analogously to Eq.(2) whereas $C_{\mathcal{N},t}^*$, denoting the nontradables produced in country F, replaces

$C_{\mathcal{N},t}$.[7]

Total consumption expenditures by households in country H are given by $P_{H,t}C_{H,t}+P_{F,t}C_{F,t}+P_{\mathcal{N},t}C_{\mathcal{N},t}=P_tC_t$, with $P_{H,t}$ and $P_{F,t}$ being Dixit–Stiglitz-type indices of the price of tradable goods produced in countries H and F, respectively, and $P_{\mathcal{N},t}$ being Dixit–Stiglitz-type indices of the price of nontradable goods produced in country H. A sequence of budget constraints in country H is given by:

$$D_t^n + W_tN_t + S_t \geq P_tC_t + \mathrm{E}_tQ_{t,t+1}D_{t+1}^n, \tag{3}$$

where $Q_{t,t+1}$ denotes the stochastic discount factor, D_t^n denotes the nominal payoff of the bond portfolio purchased by households, W_t denotes the nominal wage and S_t denotes profits (net taxation) from ownership of the firms. The budget constraint in country F is defined analogously. Furthermore:

$$P_t \equiv \left[\gamma P_{\mathcal{T},t}^{1-\eta} + (1-\gamma)\, P_{\mathcal{N},t}^{1-\eta}\right]^{\frac{1}{1-\eta}}, \tag{4}$$

denotes the CPI, $P_{\mathcal{T},t} \equiv P_{H,t}^{\frac{1}{2}}P_{F,t}^{\frac{1}{2}}$ denotes the price index of tradables, $P_{H,t}$ and $P_{F,t}$ denote the price of tradables produced in countries H and F, respectively, and $P_{\mathcal{N},t}$ denotes the price of nontradables produced in country H. P_t^* is defined analogously to Eq.(4), whereas $P_{\mathcal{N},t}^*$, denoting the price of nontradables in country F, replaces $P_{\mathcal{N},t}$. We assume that the law of one price always holds, thus $P_{H,t} = P_{H,t}^*$ and $P_{F,t} = P_{F,t}^*$, implying that the prices of tradables are equal in both countries. However, $P_{\mathcal{N},t}$ and $P_{\mathcal{N},t}^*$ are not necessarily equal in both countries because these represent the prices of different goods. These facts imply that purchasing power parity (PPP) does not necessarily hold. When all goods are tradable, Eq.(4) implies $P_t = P_t^*$, namely, PPP always holds.

[7] Following Stockman and Tesar (1995), we assume that η is not necessarily unity, whereas Obstfeld and Rogoff (2000) implicitly assume that η is unity. Obstfeld and Rogoff (2000) assume $C_t \equiv \frac{C_{\mathcal{T},t}^{\gamma}C_{\mathcal{N},t}^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$. This implies $\eta = 1$ in our paper.

The optimal allocation of any given expenditure within each category of goods implies the demand functions as follows:

$$C_{H,t} = \frac{1}{2}\left(\frac{P_{H,t}}{P_{\mathcal{T},t}}\right)^{-1} C_{\mathcal{T},t} \quad ; \quad C_{F,t} = \frac{1}{2}\left(\frac{P_{F,t}}{P_{\mathcal{T},t}}\right)^{-1} C_{\mathcal{T},t},$$
$$C_{\mathcal{T},t} = \gamma\left(\frac{P_{\mathcal{T},t}}{P_t}\right)^{-\eta} C_t \quad ; \quad C_{\mathcal{N},t} = (1-\gamma)\left(\frac{P_{\mathcal{N},t}}{P_t}\right)^{-\eta} C_t, \tag{5}$$

where $P_{\mathcal{T},t} \equiv P_{H,t}^{\frac{1}{2}} P_{F,t}^{\frac{1}{2}}$ denotes the tradables price index.

The representative household maximizes Eq.(1) subject to Eq.(3). The optimality conditions are given by:

$$\delta \mathrm{E}_t\left(\frac{C_{t+1}^{-1} P_t}{C_t^{-1} P_{t+1}}\right) = \frac{1}{R_t}, \tag{6}$$
$$C_t N_t^{\varphi} = \frac{W_t}{P_t}, \tag{7}$$

where $R_t \equiv 1 + r_t$ satisfying $R_t^{-1} = Q_{t,t+1}$ denotes the gross nominal return on a riskless one-period discount bond paying off one unit of the common currency (for short, the gross nominal interest rate), and r_t denotes the net nominal interest rate. Eq.(6) is an intertemporal optimality condition, namely the Euler equation, and Eq.(7) is an intratemporal optimality condition. Optimality conditions in country F are given analogously to Eqs.(6) and (7).

Combining and iterating Eq.(6) with an initial condition, we have the following optimal risk-sharing condition:

$$C_t = \vartheta C_t^* \mathsf{Q}_t, \tag{8}$$

with $\mathsf{Q}_t \equiv \frac{P_t^*}{P_t}$ denoting the CPI differential between the two countries and ϑ denoting a constant depending on the initial value. When $C_{-1} = C_{-1}^* = P_{-1} = P_{-1}^* = 1$, we have $\vartheta = 1$.[8]

[8] See Chari, Kehoe and McGrattan (2002) for details.

2.2 Firms

Each producer can use a linear technology to produce a differentiated good as follows:

$$Y_{H,t}(h) = A_{H,t} N_{H,t}(h), \quad ; \quad Y_{\mathcal{N},t}(h) = A_{\mathcal{N},t} N_{\mathcal{N},t}(h), \tag{9}$$

where $Y_{H,t}(h)$ and $Y_{\mathcal{N},t}(h)$ denote the output of tradables h produced in country H, output of nontradables h produced in country H, respectively, and $A_{H,t}$ and $A_{\mathcal{N},t}$ denote stochastic productivity shifters associated with tradables produced in country H and nontradables produced in country H, respectively. Each producer in country F can use a technology similar to that in country H.

Each firm produces a single differentiated good and prices its good to reflect the elasticity of substitution across goods produced given the CPI. This is because each firm plays an active part in the monopolistically competitive market. We assume that Calvo–Yun-style price-setting behavior applies, and, therefore, that each firm resets its price with a probability of $1 - \alpha$ in each period, independently of the time elapsed since the last adjustment.

When setting a new price in period t, firms seek to maximize the expected discounted value of profits. The first-order necessary conditions (FONCs) are as follows:

$$\begin{aligned} \mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k (P_{t+k} C_{t+k})^{-1} \tilde{Y}_{H,t+k} \left(\tilde{P}_{H,t} - \zeta MC^n_{H,t+k} \right) \right] &= 0, \\ \mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k (P_{t+k} C_{t+k})^{-1} \tilde{Y}_{N,t+k} \left(\tilde{P}_{\mathcal{N},t} - \zeta MC^n_{N,t+k} \right) \right] &= 0, \end{aligned} \tag{10}$$

where $MC^n_{H,t} \equiv \frac{W_t}{(1-\tau)A_{H,t}}$ and $MC^n_{\mathcal{N},t} \equiv \frac{W_t}{(1-\tau)A_{\mathcal{N},t}}$ denote the nominal marginal costs associated with tradables produced in country H and nontradables produced in country H, respectively, $\tilde{Y}_{H,t}$ and $\tilde{Y}_{\mathcal{N},t}$ denotes the total demands following changes in the prices of tradables produced in country H and nontradables produced in country H, respectively, $\tilde{P}_{H,t}$ and $\tilde{P}_{\mathcal{N},t}$ denote the adjusted prices of tradables produced in country H and nontradables produced in country H, respectively, $\zeta \equiv \frac{\theta}{\theta-1}$ is a constant markup and τ denotes the tax rate.[9] Note that $(P_{t+k}C_{t+k})^{-1}$ is the marginal utility of nominal income.

Let $MC_{H,t} \equiv \frac{MC^n_{H,t}}{P_{P,t}}$ and $MC_{\mathcal{N},t} \equiv \frac{MC^n_{\mathcal{N},t}}{P_{P,t}}$ be the real marginal cost of tradables produced in country H and nontradables in country H, respectively, where $P_{P,t} \equiv \frac{P_{H,t}Y_{H,t}+P_{\mathcal{N},t}Y_{\mathcal{N},t}}{Y_{H,t}+Y_{\mathcal{N},t}}$ denotes the weighted average price of goods produced in country H. Combining Eq.(7) and the definition of real marginal cost, we have:

$$MC_{H,t} = \frac{C_t N_t^{\varphi} P_t}{(1-\tau) P_{P,t} A_{H,t}}, \quad ; \quad MC_{\mathcal{N},t} = \frac{C_t N_t^{\varphi} P_t}{(1-\tau) P_{P,t} A_{\mathcal{N},t}}. \tag{11}$$

Note that we call $P_{P,t}$ the producer price index (PPI) in country H, hereafter. We define the countrywide real marginal cost as $MC_t \equiv \frac{MC_{H,t}Y_{H,t}+MC_{\mathcal{N},t}Y_{\mathcal{N},t}}{Y_{H,t}+Y_{\mathcal{N},t}}$.

2.3 Local Government

Whereas monetary frictions are omitted and the limit of a "cashless economy" is considered following Woodford (2003) throughout this paper, monetary policy has important implications for fiscal decisions, as the level of the interest rate determines the debt burden and the inflation rate affects the real value of debt. Fiscal policy can choose one-period nominal risk-free debt to finance an exogenous process of public spending.[10] The flow government budget constraint

[9] Ferrero (2007) regards it as a value-added tax rate.

[10] We assume the tax rate is common in each country and constant over time for simplicity, whereas Ferrero (2007), Schmitt-Grohe and Uribe (2004a) and Canzoneri, Cumby and Diba (2001) assume it can vary in each country and over time. In contrast, Gali and Monacelli (forthcoming) assume a constant (negative) tax rate similar to our paper. A constant tax rate over time reflects actual regimes, not only in the Euro area but also in other countries, because adjustments of the tax rate are infrequent.

in country H is given by:

$$B_t^n = R_{t-1} B_{t-1}^n - \left[P_{P,t} \tau \left(Y_{H,t} + Y_{\mathcal{N},t} \right) - P_{G,t} \left(G_{H,t} + G_{\mathcal{N},t} \right) \right], \tag{12}$$

where $B_t^n \equiv P_t B_t$ denotes the nominal risk-free rate on bonds issued by the local government in country H, B_t denotes the real risk-free rate on bonds issued by local government in country H, and $P_{G,t} \equiv \frac{P_{H,t} G_{H,t} + P_{\mathcal{N},t} G_{\mathcal{N},t}}{G_{H,t} + G_{\mathcal{N},t}}$ denotes the average price of goods purchased by the government in country H. The local government in country F has a budget constraint similar to that shown in Eq.(12). For simplicity, we assume that government purchases are fully allocated to a domestically produced good and that the total amount of these is exogenous.[11] For any given level of public consumption, the government allocates expenditures across goods in order to minimize total cost.

Note that there is no borrowing or lending among households in equilibrium because all of them are identical. Thus, all interest-bearing asset holdings by households are in the form of government securities. That is:

$$D_t^n = R_{t-1} \left[B_{t-1}^n + \left(B_{t-1}^* \right)^n \right],$$

for all dates and all contingencies.[12]

Starting from Eq.(12) with an appropriate transversality condition, the resulting consolidated intertemporal budget constraint can be written as:

$$\frac{C_t^{-1}}{\Pi_t} R_{t-1} B_{t-1} = \mathrm{E}_t \left\{ \sum_{k=0}^{\infty} \delta^k C_{t+k}^{-1} \frac{\left[P_{P,t+k} \tau \left(Y_{H,t+k} + Y_{\mathcal{N},t+k} \right) - P_{G,t+k} \left(G_{H,t+k} + G_{\mathcal{N},t+k} \right) \right]}{P_{t+k}} \right\},$$

with $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ being the gross CPI inflation rate in country H, where we use the intertemporal optimality conditions Eq.(6). These equalities can be rewritten

[11] Thus, government purchases do not distort relative prices between tradables and nontradables.

[12] Thus, our households and government budget constraints are equivalent to those in Schmitt-Grohe and Uribe (2004a, 2004b), Canzoneri, Cumby and Diba (2001) and Woodford (1996).

as:

$$\frac{C_t^{-1}}{\Pi_t}R_{t-1}B_{t-1} = C_t^{-1}\frac{\left[P_{P,t}\tau\left(Y_{H,t}+Y_{\mathcal{N},t}\right)-P_{G,t}\left(G_{H,t}+G_{\mathcal{N},t}\right)\right]}{P_t} + \delta \mathrm{E}_t\left(\frac{C_{t+1}^{-1}}{\Pi_{t+1}}R_tB_t\right). \quad (13)$$

In contrast to the setting in Ferrero (2007), $\mathsf{Q}_t = 1$, which derives $C_t = C_t^*$, is no longer applied when all goods are not tradable. When $C_t = C_t^*$ is applied, Eq.(13) becomes similar to the one derived by Ferrero (2007). Note that because Eq.(13) is derived by combining the intertemporal government budget constraint and Euler equation, the equation becomes the new Keynesian IS (NKIS) curve by log-linearization.

2.4 Market Clearing

The market in country H for tradables clears when domestic demand equals domestic supply as follows:

$$Y_{H,t}(h) = C_{H,t}(h) + C_{H,t}^*(h) + G_{H,t}(h), \quad (14)$$

where $C_{H,t}^*(h)$ denotes country F's demand for generic tradables produced in country H. As for nontradables, equilibrium requires that:

$$Y_{\mathcal{N},t}(h) = C_{\mathcal{N},t}(h) + G_{\mathcal{N},t}(h). \quad (15)$$

As mentioned above, government purchases are fully allocated to a domestically produced good. The market clearing conditions in country F are analogous to Eqs.(14) and (15).

Let $Y_{H,t}$ denote a Dixit–Stiglitz-type index of the aggregate output of tradables produced in country H. Combining this definition and Eqs.(5) and (8), Eq.(14) can be rewritten as:

$$Y_{H,t} = \frac{\gamma}{2}\left(\frac{P_{H,t}}{P_{\mathcal{T},t}}\right)^{-1} C_t\left[\left(\frac{P_{\mathcal{T},t}}{P_t}\right)^{-\eta} + \left(\frac{P_{\mathcal{T},t}}{P_t^*}\right)^{-\eta}\mathsf{Q}_t^{-1}\right] + G_{H,t}. \quad (16)$$

Eq.(16) and its counterpart in country F imply that:

$$\frac{Y_{H,t} - G_{H,t}}{Y_{F,t} - G_{F,t}} = \mathsf{T}_t,$$

where $Y_{F,t}$ denotes the aggregate output of tradables produced in country F and $\mathsf{T}_t \equiv \frac{P_{F,t}}{P_{H,t}}$ denotes the terms of trade (TOT). Thus, the difference in the output of tradables between country H and country F is equal to the TOT.

Let $Y_{\mathcal{N},t}$ denote a Dixit–Stiglitz-type index of the aggregate output of nontradables produced in country H. Combining this definition and Eq.(5), Eq.(15) can be rewritten as follows:

$$Y_{\mathcal{N},t} = (1-\gamma)\left(\frac{P_{\mathcal{N},t}}{P_t}\right)^{-\eta} C_t + G_{\mathcal{N},t}. \tag{17}$$

Eq.(17) and its counterpart for country F imply that:

$$\frac{Y_{\mathcal{N},t} - G_{\mathcal{N},t}}{Y^*_{\mathcal{N},t} - G^*_{\mathcal{N},t}} = \mathsf{N}_t^{\eta} \mathsf{Q}_t^{-(\eta-1)},$$

where $\mathsf{N}_t \equiv \frac{P^*_{\mathcal{N},t}}{P_{\mathcal{N},t}}$ denotes the nontradables price difference between countries H and F (NPD). Analogous to the difference in output of tradables, the difference in output of nontradables between the two countries is equal to the price difference of nontradables between them.

Finally, we define countrywide output and government expenditure as follows:

$$Y_t \equiv \frac{P_{H,t}}{P_{P,t}} Y_{H,t} + \frac{P_{N,t}}{P_{P,t}} Y_{\mathcal{N},t} \quad ; \quad G_t \equiv \frac{P_{H,t}}{P_{G,t}} G_{H,t} + \frac{P_{N,t}}{P_{G,t}} G_{\mathcal{N},t}, \tag{18}$$

where Y_t and G_t denote output and government expenditure in country H, respectively.

3 Log-linearization of the Model

This section describes the stochastic equilibrium that arises from perturbations around the deterministic equilibrium. Lowercase letters denote percentage deviations of steady-state values for the respective uppercase letters when there is no note to the contrary; i.e., $v_t \equiv \frac{dV_t}{V}$, where V_t denotes the voluntary variable and V denotes the steady-state value of V_t. Lowercase letters accompanied by R as a superscript indicate the logarithmic difference between the two countries for the respective uppercase letters, i.e., $v_t^R \equiv v_t - v_t^*$, while lowercase letters accompanied by W as a superscript indicate the logarithmic weighted sum of the two countries for the respective uppercase letters; i.e., $v_t^W \equiv \frac{1}{2}(v_t + v_t^*)$. Finally, small letters accompanied by Δ indicate changes in the capital-letter variable; i.e., $\Delta v_t \equiv v_t - v_{t-1}$.

3.1 Aggregate Demand and Output

Log-linearizing Eq.(8), we obtain the following:

$$c_t^R = \mathsf{q}_t, \tag{19}$$

where q_t denotes the logarithmic CPI differential between the two countries. Eq.(19) implies that the logarithmic consumption differential between the two countries depends on the logarithmic CPI differential.

Log-linearizing and manipulating Eq.(4), we obtain:

$$\pi_t = \gamma \pi_{\mathcal{T},t} + (1-\gamma)\,\pi_{\mathcal{N},t}, \tag{20}$$

with $\pi_{\mathcal{T},t} = \frac{1}{2}\pi_{H,t} + \frac{1}{2}\pi_{F,t}$, where π_t denotes the CPI inflation rate in country H, $\pi_{\mathcal{T},t}$ denotes the tradable goods price inflation rate, $\pi_{H,t}$ and $\pi_{F,t}$ denote the inflation rates of tradables produced in countries H and F, respectively, and $\pi_{\mathcal{N},t}$ denotes the inflation rate of nontradables produced in country H.

Log-linearizing PPI, we have $p_{P,t} = \gamma p_{H,t} + (1-\gamma)\,p_{\mathcal{N},t}$, which implies that logarithmic PPI is the weighted sum of both the logarithmic price of tradables produced in country H and of nontradables produced in country H.[13] This equality implies that:

$$\pi_{P,t} = \gamma \pi_{H,t} + (1-\gamma)\, \pi_{\mathcal{N},t}, \tag{21}$$

where $\pi_{P,t}$ denotes the PPI inflation rate in country H. Note that $\pi_{P,t} = \pi_{H,t}$ is applied when all goods are tradable, i.e., $\gamma = 1$. Eq.(21) implies that the PPI inflation rate is the weighted sum of the inflation rates of both tradables and nontradables produced in country H.

Log-linearizing Eq.(18), we have:

$$\begin{aligned} y_t &= \gamma y_{H,t} + (1-\gamma)\, y_{\mathcal{N},t}, \\ g_t &= \gamma g_{H,t} + (1-\gamma)\, g_{\mathcal{N},t}. \end{aligned} \tag{22}$$

Log-linearizing Eqs.(16) and (17) and substituting these equalities into Eq.(22), we have:

$$y_t = (1-\sigma_G)\, c_t + \frac{(1-\sigma_G)\,\gamma}{2} \mathsf{t}_t + \frac{(1-\sigma_G)\,\psi}{2} \mathsf{n}_t + \sigma_G g_t, \tag{23}$$

with $\psi \equiv (1-\gamma)\,\gamma\,(\eta-1)$, where $\sigma_G \equiv \frac{G}{Y}$ denotes the steady-state ratio of government expenditure to output, t_t denotes the logarithm of TOT, n_t denotes the logarithm of NPD, and g_t denotes the percentage deviation of government spending from the steady-state output level in country H.

Subtracting the counterpart of Eq.(23) in country F from Eq.(23), we have:

$$y_t^R = \gamma\,(1-\sigma_G)\, \mathsf{t}_t + (1-\gamma)\,\varpi\,(1-\sigma_G)\, \mathsf{n}_t + \sigma_G g_t^R, \tag{24}$$

with $\varpi \equiv 1 + (\eta-1)\,\gamma$. Because of existing nontradables, the output differential between the two countries depends not only on the TOT but also on the NPD.

[13] Note that we have $p_{P,t} = p_{G,t}$ by log-linearizing the definition of $P_{G,t}$.

When $\gamma = 1$, implying that there are no nontradables, this equality reduces to $y_t^R = (1-\sigma_G)\,\mathsf{t}_t + (1-\sigma_G)\, g_t^R$, which is familiar in many new open economy macroeconomics (NOEM) studies. This equality shows that an increase in the price of domestic goods decreases domestic output when we ignore the effect of η.

Log-linearizing Eq.(13), we have:

$$\begin{aligned} b_t \quad = \quad & \mathrm{E}_t c_{t+1} - c_t - \frac{1}{\delta}\pi_t + \mathrm{E}_t \pi_{t+1} + \frac{1}{\delta}\hat{r}_{t-1} - \hat{r}_t + \frac{1}{\delta} b_{t-1} + \left(\frac{1-\delta}{\delta}\right)\frac{\gamma}{2}\mathsf{t}_t \\ & - \frac{\tau}{\sigma_B} y_t + \frac{\sigma_G}{\sigma_B} g_t, \end{aligned} \tag{25}$$

where $\hat{r}_t \equiv \frac{dR_t}{R}$ denotes the deviation of the nominal interest rate from its steady-state value and $\sigma_B \equiv \frac{B}{Y}$ denotes the steady-state ratio of the quantity of risk-free bonds to output.

Combining Eqs.(20), (22), (25) and the counterpart of Eq.(25), we have NKISs as follows:

$$\begin{aligned} y_t^W \quad = \quad & \frac{\beta_W}{1-\sigma_G}\mathrm{E}_t y_{t+1}^W + \beta_W \mathrm{E}_t \pi_{t+1}^W - \beta_W \hat{r}_t + \frac{\beta_W}{\delta}\hat{r}_{t-1} - \beta_W b_t^W + \frac{\beta_W}{\delta} b_{t-1}^W - \frac{\beta_W}{\delta}\pi_t^W \\ & + \sigma_G \nu_W g_t^W, \\ y_t^R \quad = \quad & -\beta_R \delta b_t^R + \beta_R (1-\gamma)\,\upsilon \mathsf{n}_t - \beta_R (1-\gamma)\,\mathsf{n}_{t-1} + \beta_R b_{t-1}^R + \sigma_G \nu_R g_t^R, \end{aligned} \tag{26}$$

with $\beta_W \equiv \frac{(1-\sigma_G)\sigma_B}{\sigma_B + (1-\sigma_G)\tau}$, $\beta_R \equiv \frac{\sigma_B(1-\sigma_G)}{(1-\sigma_G)\delta\tau - (1-\delta)\sigma_B}$, $\upsilon \equiv 1 - (1-\delta)\,\varpi$, $\nu_W \equiv \frac{[\sigma_B(1-\rho_G)+1-\sigma_G]}{\sigma_B+(1-\sigma_G)\tau}$, $\nu_R \equiv \frac{[(1-\sigma_G)\delta-(1-\delta)\sigma_B]}{(1-\sigma_G)\delta\tau-(1-\delta)\sigma_B}$, and $\rho_G < 1$ being the coefficient associated with exogenous government expenditure processes.[14] Note that the two equalities are not only demand curves but also government budget constraints. The first equality in Eq.(26) can be derived by summing Eq.(25) and its counterpart in country F. The model needs another demand curve such as

[14] We assume that the government expenditure and productivity shifters follow AR(1) processes. See subsection 4.2.

the second equality in Eq.(26). As mentioned by Ferrero (2007), two intertemporal government budget constraints are not necessarily satisfied, although the union-wide budget constraint is satisfied. Thus, the relative block of the NKIS, namely, the second equality in Eq.(26) is required. This can be explained another way. Although all goods are tradable, namely, $\gamma = 1$ is applied, $y_t^R = 0$ is not satisfied, namely, the relative block of the intertemporal government budget constraint affects the economy. This indicates that two intertemporal government budget constraints are not necessarily satisfied, although the union-wide budget constraint is satisfied.

Note that Eq.(26) accounts for the difference in policy implications between Benigno (2004) and Ferrero (2007). When all goods are tradable, i.e., $\gamma = 1$, the second equality of Eq.(26) reduces to:

$$y_t^R = -\beta_R \delta b_t^R + \beta_R b_{t-1}^R + \sigma_G \nu_R g_t^R.$$

Thus, only government bonds and government expenditure create a disparity between outputs in the two countries. This equality corresponds to the logarithmic relative block of the government budget constraint in Ferrero (2007).[15] In contrast, Benigno (2004) assumes a zero steady-state value of government expenditure, that the governments do not issue bonds, and that all goods are tradable. Thus, the above equality becomes $y_t^R = 0$, which implies there is no disparity in output between both countries, under the assumption of Benigno (2004). When changes in relative government expenditure occur, output disparity is amplified under Ferrero's (2007) settings. To eliminate the output disparity, issuing government bonds is essential because monetary policy can only cope with shifters on the union-wide or average block of the NKIS under

[15] Note that Ferrero (2007) does not show the logarithmic relative block of the government budget constraint explicitly.

his setting. However, $y_t^R = 0$ always holds, although relative shocks hit the economy under the assumption of Benigno (2004). Thus, optimal monetary policy alone can dissolve trade-offs in each country under the assumption of Benigno (2004). It is clear that the difference in policy implications between both papers stems from the steady-state value of government expenditure or the assumed form of the government budget constraint.

3.2 Aggregate Supply and Inflation

Log-linearizing Eq.(10) and rearranging, we can describe the dynamics of inflation in terms of marginal cost as follows:

$$\begin{aligned} \pi_{H,t} &= \delta \mathrm{E}_t \pi_{H,t+1} + \kappa (1-\gamma) p_{\mathcal{N},t} - \kappa (1-\gamma) p_{H,t} + \kappa mc_{H,t}, \\ \pi_{\mathcal{N},t} &= \delta \mathrm{E}_t \pi_{N,t+1} - \kappa \gamma p_{\mathcal{N},t} + \kappa \gamma p_{H,t} + \kappa mc_{\mathcal{N},t}, \end{aligned} \tag{27}$$

with $\kappa \equiv \frac{(1-\alpha)(1-\alpha\delta)}{\alpha}$.

Substituting Eq.(27) into Eq.(21), we have a PPI-based inflation dynamics equation, namely, a new Keynesian Philips curve (NKPC), as follows:

$$\pi_{P,t} = \delta \mathrm{E}_t \pi_{P,t+1} + \kappa mc_t, \tag{28}$$

where $mc_t = \gamma mc_{H,t} + (1-\gamma) mc_{\mathcal{N},t}$ which is derived by log-linearizing the definition of marginal cost at the country level. Combining the second equality of Eq.(27) and its counterpart for country F, the nontradables inflation differential is given by:

$$\pi_{\mathcal{N},t}^R = \delta \mathrm{E}_t \pi_{N,t+1}^R + \kappa \gamma \mathsf{n}_t - \kappa \gamma \mathsf{t}_t + \kappa mc_{\mathcal{N},t}^R, \tag{29}$$

with:

$$\pi_{\mathcal{N},t}^R \equiv -\Delta \mathsf{n}_t, \tag{30}$$

being relative nontradables inflation, which is a type of NKPC where, at first glance, Eq.(29) evolves into this version of the real exchange rate determiner and can be called the new Keynesian real exchange rate determiner (NKRD). Our model is a closed system while a two-country economy is assumed; however, as with the Balassa–Samuelson theorem, Eq.(29) explains the CPI disparity between the two countries, although the Balassa–Samuelson theorem addresses the problem of why the nominal exchange rate deviates from PPP in the international money and finance literature. Details on Eq.(29) are provided in a later section.

By log-linearizing the aggregated Eq.(9) and combining it with Eq.(22), we have:

$$y_t = \gamma a_{H,t} + (1-\gamma)\, a_{\mathcal{N},t} + n_t, \tag{31}$$

where we also use the log-linearized definition of hours of work, $n_t = \gamma n_{H,t} + (1-\gamma)\, n_{\mathcal{N},t}$.

Combining log-linearized Eq.(11), and Eqs.(23) and (31), we have:

$$\begin{aligned} mc_{H,t} &= \frac{\lambda}{1-\sigma_G} y_t - \frac{\psi}{2}\mathsf{n}_t - (1+\varphi\gamma)\, a_{H,t} - (1-\gamma)\,\varphi a_{\mathcal{N},t} - \frac{\sigma_G}{1-\sigma_G} g_t, \\ mc_{\mathcal{N},t} &= \frac{\lambda}{1-\sigma_G} y_t - \frac{\psi}{2}\mathsf{n}_t - \varphi\gamma a_{H,t} - [1+(1-\gamma)\,\varphi]\, a_{\mathcal{N},t} - \frac{\sigma_G}{1-\sigma_G} g_t, \end{aligned} \tag{32}$$

with $\lambda \equiv 1 + (1-\sigma_G)\,\varphi$, which implies that marginal cost depends not only on domestic output but also on the NPD.

Using the log-linearized definition of the marginal cost $mc_t = \gamma mc_{H,t} + (1-\gamma)\, mc_{\mathcal{N},t}$, Eq.(32) can be rewritten as follows:

$$mc_t = \frac{\lambda}{1-\sigma_G} y_t - \frac{\psi}{2}\mathsf{n}_t - (1+\varphi)\,\gamma a_{H,t} - (1+\varphi)(1-\gamma)\, a_{\mathcal{N},t} - \frac{\sigma_G}{1-\sigma_G} g_t. \tag{33}$$

When $\gamma = 1$, Eq.(33) reduces to:

$$mc_t = \frac{\lambda}{1-\sigma_G} y_t - (1+\varphi) a_{H,t} - \frac{\sigma_G}{1-\sigma_G} g_t,$$

because $\psi = 0$ when $\gamma = 1$. This equality is a familiar expression within DSGE models applied in NOEM studies.

Combining the second equality in Eq.(32) and its counterpart for country F, the logarithmic marginal cost differential associated with nontradables is given by:

$$\begin{aligned} mc_{\mathcal{N},t}^{R} &= \frac{\lambda}{1-\sigma_G} y_t^R - \psi \mathsf{n}_t - \varphi\gamma a_{H,t} + \varphi\gamma a_{F,t} - [1 + (1-\gamma)\varphi] a_{\mathcal{N},t}, \\ &\quad + [1 + (1-\gamma)\varphi] a_{\mathcal{N},t}^{*} - \frac{\sigma_G}{1-\sigma_G} g_t^R. \end{aligned} \tag{34}$$

3.3 Relative Price and Consumption

Log-linearizing Eq.(4) and rearranging yields:

$$\mathsf{q}_t = (1-\gamma)\,\mathsf{n}_t. \tag{35}$$

It is clear by paying attention to Eqs.(19) and (35) that the logarithmic consumption differential depends on both the logarithmic CPI differential and the logarithmic NPD. When $\gamma = 1$, Eq.(35) can be written as $\mathsf{q}_t = 0$, implying that the CPI is the same in both countries. In the international finance literature, this means that PPP holds.

Combining Eqs.(19) and (35), we have:

$$\mathsf{n}_t = \frac{1}{1-\gamma} c_t^R.$$

This equality and Eq.(35) imply that the existence of nontradables creates disparity in consumption between country H and country F when all goods are tradable, i.e., $\gamma = 1$, $\mathsf{q}_t = 0$ and $c_t^R = 0$. These imply that PPP holds and consumption is the same in both countries.

Benigno (2004) and Gali and Monacelli (forthcoming) assume that all goods are tradable and that the law of one price holds. However, PPP does not necessarily hold in Gali and Monacelli (forthcoming). They assume a currency union that consists of an infinite number of countries, whereas Benigno (2004) assumes a currency union consisting of two countries. The settings in Gali and Monacelli (2005) make a distinction in the CPI between one infinitesimal country- and union-wide economy because a price index in one infinitesimal country does not affect the union-wide CPI. This stems from the small open economy assumption. Thus, $\mathsf{q}_t = 0$ is not applied in Gali and Monacelli (forthcoming) although all goods are tradable. In a later section, we suggest that optimal fiscal policy is needed to stabilize both inflation and output gap simultaneously because of nontradables. The assumption of a small open economy does not permit applying $\mathsf{q}_t = 0$. Thus, the policy implications of Gali and Monacelli (forthcoming) and this paper are very similar, whereas the policy implications of Gali and Monacelli (forthcoming) and Benigno (2004) are contrary.

3.4 Marginal Cost and Output Gap

Following Gali and Monacelli (2005), we define the relationship between output, its natural level and the output gap as follows:

$$y_t \equiv \bar{y}_t + \tilde{y}_t,$$

where $\tilde{y}_t$ denotes the logarithmic output gap measured from its natural level, and $\bar{y}_t$ denotes the logarithmic natural output level. Under the long-run equilibrium, $\tilde{y}_t = 0$ must hold.[16]

When the fiscal authorities design their policies to reduce the distortion

[16] Following Gali and Monacelli (2005), nominal rigidities disappear in the long-run equilibrium.

generated by monopolistically competitive markets, real marginal costs under the long-run equilibrium are constant, and their logarithm is given by $mc_t = 0$. In addition, under the long-run equilibrium, PPP is applied.[17] Thus, the logarithmic NPD under the long-run equilibrium is given by $\mathsf{n}_t = 0$.

Combining these facts, Eq.(33) implies that:

$$\bar{y}_t = \bar{\beta}\gamma a_{H,t} + \bar{\beta}\left(1-\gamma\right) a_{\mathcal{N},t} + \frac{\sigma_G}{\lambda} g_t, \tag{36}$$

with $\bar{\beta} \equiv \frac{(1-\sigma_G)(1+\varphi)}{\lambda}$. Eq.(36) implies that the natural level of output consists of productivity and government spending, while it does not include any nominal variables reflecting the classical dichotomy. Our setting does not assume the level of investment or the capital stock, and the natural level of output is not determined by either of these variables.

Using Eq.(36), the log-linear approximated model can be rewritten in terms of the output gap. Eq.(26) can be rewritten as:

$$\begin{aligned}
\tilde{y}_t^W &= \frac{\beta_W}{1-\sigma_G}\mathrm{E}_t\tilde{y}_{t+1}^W - \beta_W\hat{r}_t + \beta_W\mathrm{E}_t\pi_{t+1}^W + \frac{\beta_W}{\delta}\hat{r}_{t-1} - \beta_W b_t^W + \frac{\beta_W}{\delta}b_{t-1}^W - \frac{\beta_W}{\delta}\pi_t^W, \\
&\quad - \frac{\gamma\bar{\beta}\beta_{\mathcal{T}}}{2}a_{H,t} - \frac{\left(1-\gamma\right)\bar{\beta}\beta_{\mathcal{N}}}{2}a_{\mathcal{N},t} - \frac{\gamma\bar{\beta}\beta_{\mathcal{T}}}{2}a_{F,t} - \frac{\left(1-\gamma\right)\bar{\beta}\beta_{\mathcal{N}}}{2}a_{\mathcal{N},t}^* + \sigma_G\varsigma_W g_t^W, \\
\tilde{y}_t^R &= -\beta_R\delta b_t^R + \beta_R\left(1-\gamma\right)\upsilon\mathsf{n}_t - \beta_R\left(1-\gamma\right)\mathsf{n}_{t-1} + \beta_R b_{t-1}^R - \bar{\beta}\gamma a_{H,t}, \\
&\quad + \bar{\beta}\gamma a_{F,t} - \bar{\beta}\left(1-\gamma\right)a_{\mathcal{N},t} + \bar{\beta}\left(1-\gamma\right)a_{\mathcal{N},t}^* + \varsigma_R\sigma_G g_t^R,
\end{aligned} \tag{37}$$

with $\beta_{\mathcal{T}} \equiv 1 - \frac{\beta_W\rho_{\mathcal{T}}}{1-\sigma_G}$, $\beta_{\mathcal{N}} \equiv 1 - \frac{\beta_W\rho_{\mathcal{N}}}{1-\sigma_G}$, $\varsigma_W \equiv \nu_W + \frac{\beta_W\rho_G}{(1-\sigma_G)\lambda} - \frac{1}{\lambda}$ and $\varsigma_R \equiv \nu_R - \frac{1}{\lambda}$ where $\rho_{\mathcal{T}} < 1$ and $\rho_{\mathcal{N}} < 1$ denote the coefficients associated with the exogenous processes on the productivity shifter of tradables and the productivity shifter of nontradables, respectively. Both equalities in Eq.(37) imply that the NKISs in the two countries are no longer homogeneous although risk sharing is perfect internationally. As mentioned when we derived the second equality of Eq.(26),

[17] Following Gali and Monacelli (2005), we assume a steady state where PPP is applied.

there are two major causes of disparity in the demand block between the countries: (1) a nonzero steady-state value of government bonds and expenditure, and (2) nontradable goods. When the steady-state value of government bonds and expenditure are zero, $\sigma_R = \sigma_G = 0$ is applied. When all goods are tradable, $1 - \gamma = 0$ is applied. Many open economy DSGE models, such as the model in Benigno (2004) and Benigno and Benigno (2008) adopt both $\sigma_R = \sigma_G = 0$ and $1 - \gamma = 0$. Thus, there is little or no output gap disparity between both countries. The model in Ferrero (2007) assumes a nonzero steady-state value of government bonds and expenditure because of an explicit government budget constraint. Thus, his model has a somewhat larger disparity. This paper includes both the earlier and latter cases. Our model has a larger disparity when nontradables exist.

The NKPCs in terms of the output gap are given by:

$$\pi_{P,t} = \delta \mathrm{E}_t \pi_{P,t+1} + \kappa \frac{\lambda}{1 - \sigma_G} \tilde{y}_t - \frac{\psi \kappa}{2} \mathsf{n}_t, \tag{38}$$

along with its counterpart in country F. These expressions become familiar when $\gamma = 1$. In this case, Eq.(38) can be rewritten as:

$$\pi_{P,t} = \delta \mathrm{E}_t \pi_{P,t+1} + \kappa \frac{\lambda}{1 - \sigma_G} \tilde{y}_t,$$

which corresponds with one derived by Gali and Monacelli (2005), who insist that inflation–output trade-offs can be dissolved simultaneously in a small open economy, under strong parameter restrictions, by inflation targeting. Indeed, when inflation targeting, such as $\pi_{P,t} = \pi^*_{P,t} = 0$ for all t, is introduced in our currency union with special restrictions, i.e., $\gamma = 1$ and $\sigma_B = \sigma_G = 0$, these equalities imply that $\tilde{y}_t = \tilde{y}^*_t = 0$ for all t and that the output gap is eliminated.

3.5 Balassa–Samuelson Theorem and Real Exchange Rate Anomaly

As mentioned in the previous subsection, we now turn to the relationship between the real exchange rate and the NKRD. Using Eq.(36), NKRD Eq.(29) can be rewritten as:

$$\begin{aligned}\pi^R_{\mathcal{N},t} &= \delta \mathrm{E}_t \pi^R_{N,t+1} + \kappa\varphi\tilde{y}^R_t + \kappa \mathsf{n}_t - \kappa\varphi\gamma\left(1-\bar{\beta}\right) a_{H,t} + \kappa\varphi\gamma\left(1-\bar{\beta}\right) a_{F,t}, \\ &\quad - \kappa\left[1+\varphi\left(1-\gamma\right)\left(1-\bar{\beta}\right)\right] a_{\mathcal{N},t} + \kappa\left[1+\varphi\left(1-\gamma\right)\left(1-\bar{\beta}\right)\right] a^*_{\mathcal{N},t}, \\ &\quad - \frac{\kappa\sigma_G}{1-\sigma_G}\left(1-\frac{\varphi}{\lambda}\right) g^R_t. \end{aligned} \tag{39}$$

Using Eqs.(30) and (35), Eq.(39) can be rewritten as follows:

$$\begin{aligned}\mathsf{q}_t &= (1-\gamma)\left\{\frac{1}{\kappa}\pi^R_{\mathcal{N},t} - \frac{\delta}{\kappa}\mathrm{E}_t\pi^R_{N,t+1} - \varphi\tilde{y}^R_t + \varphi\gamma\left(1-\bar{\beta}\right) a_{H,t} + \left[1+\varphi\left(1-\gamma\right)\left(1-\bar{\beta}\right)\right] a_{\mathcal{N},t}\right. \\ &\quad \left. -\varphi\gamma\left(1-\bar{\beta}\right) a_{F,t} + \kappa\left[1+\varphi\left(1-\gamma\right)\left(1-\bar{\beta}\right)\right] a^*_{\mathcal{N},t} - \frac{\varphi\sigma_G}{1+\varphi} g^R_t\right\},\end{aligned}$$

which implies that the CPI disparity disappears between the two countries, namely, $\mathsf{q}_t = 0$ holds when the currency union has no nontradables; i.e., as $\gamma = 1$. The problem with the CPI disparity is resolved, because each country has the same CPI. This implies that PPP holds in an ordinary open-economy model. Eq.(39) depicts the Balassa–Samuelson theorem in the international money and finance literature. However, it cannot be easily understood because Eq.(39) is different from the familiar equation of the Balassa–Samuelson theorem because it is a dynamic equation, as in the New Keynesian literature, which has correctly assumed nominal rigidities. To understand this characteristic easily, we inspect Eq.(39) without nominal rigidities. Under such a condition, Eq.(39) can be rewritten as:

$$\mathsf{q}_t = (1-\gamma)\left\{\varphi\gamma\left(1-\bar{\beta}\right) a_{H,t} + \left[1+\varphi\left(1-\gamma\right)\left(1-\bar{\beta}\right)\right] a_{\mathcal{N},t} - \varphi\gamma\left(1-\bar{\beta}\right) a_{F,t}\right.$$

$$-\left[1+\varphi\left(1-\gamma\right)\left(1-\bar{\beta}\right)\right]a^{*}_{\mathcal{N},t}-\frac{\varphi\sigma_{G}}{1+\varphi}g^{R}_{t}\Bigg\},\qquad(40)$$

because $\alpha=0$ and $\tilde{y}_t=\tilde{y}^{*}_t=0$ hold. Furthermore, note that when government expenditure is zero in the steady state, Eq.(40) can be rewritten as :

$$\mathsf{q}_t=\left(1-\gamma\right)\left(a_{\mathcal{N},t}-a^{*}_{\mathcal{N},t}-\frac{\varphi\sigma_{G}}{1+\varphi}g^{R}_{t}\right),\qquad(41)$$

because $\sigma_G=0$ implies $\bar{\beta}=1$. In Eq.(41), increasing the relative productivity of tradables produced in country H, i.e., decreasing the productivity of nontradables produced in country H, causes a decrease in the CPI disparity q_t. As the Balassa–Samuelson theorem explains, a rise in the productivity of the tradables sector in the home country causes a decrease (appreciation) in the real exchange rate through an increase in nontradables prices in the home country, which stems from an increase in wages in both the tradables and the nontradables sectors because of perfect labor mobility between each sector.

$\sigma_G=0$ is not consistent with our setting because we assume positive government expenditure in the steady state. Eq.(40) implies that an increase in the productivity of tradables produced in country H causes an increase (depreciation) in the real exchange rate. Because of an increase in the real marginal cost in the nontradables sector, the CPI increases. However, the CPI partially includes the PPI. Thus, the real marginal cost in both sectors decreases. This decreases the CPI via a decrease in the PPI. Finally, the real exchange rate increases (depreciates). Benigno and Thoenissen (2005) and Canzoneri, Cumby and Diba (1999) report a real exchange rate anomaly. They find that the actual direction of changes in the real exchange rate cannot be explained by the Balassa–Samuelson theorem. Eq.(40) well reflects their findings in a well-founded micro setting. Note that Eqs.(40) and (41) imply that the real exchange rate anomaly depends on steady-state share of government expenditure to output. When the share is zero, namely, government expenditure is zero at the steady state, changes in the real exchange rate follow the Balassa–Samuelson theorem. However, when the share is positive, namely, government expenditure is positive at the steady state, the effect of the Balassa–Samuelson theorem is offset by another side effect, the real exchange rate anomaly, which is pointed out by Benigno and Thoenissen (2005) and Canzoneri, Cumby and Diba (1999).

4 Optimal Cooperative Solution

In this section, we analyze the macroeconomic implications of an alternative policy regime for the Euro area: an optimal monetary policy without a fiscal policy regime and an optimal monetary and fiscal policy regime under a cooperative setting. Furthermore, we assume that each policy authority is responsible for minimizing social losses. Under an optimal monetary policy regime without fiscal policy, the central bank is the only policy authority whereas the central bank and local governments in the two countries are both authorities under an optimal monetary and fiscal policy regime. Policy authorities seek to minimize the social loss function subject to our structural model.[18] The period loss function is derived by a second-order Taylor expansion as approximated by the definition of period utility in Eq.(1), which is given by:

$$U_t^W = -L_t^W + \text{t.i.p.} + o\left(\|\xi\|^3\right),$$

with:

$$L_t^W = \frac{1}{1-\sigma_G}\left[\frac{\theta}{4\kappa}\pi_{P,t}^2 + \frac{\theta}{4\kappa}\left(\pi_{P,t}^*\right)^2 + \frac{1+\varphi}{2}\left(\tilde{y}_t^W\right)^2 + \frac{1+\varphi}{8}\left(\tilde{y}_t^R\right)^2\right], \quad (42)$$

[18]Our structural model consists of Eqs.(37), (38), (39) and a counterpart of Eq.(38) in country F.

where $U_t^W \equiv \frac{1}{2}\left(U_t + U_t^*\right)$ denotes the union-wide utility function, L_t^W denotes the union-wide period loss function, t.i.p. denotes the terms of the independent policy, and $o\left(\|\xi\|^3\right)$ denotes the terms that are higher than third order.[19]

We now consider the difference in welfare loss between our paper and the DSGE literature for an open economy. Using Eq.(36) we can obtain:

$$\tilde{y}_t^R = \gamma\left(1-\sigma_G\right)\mathsf{t}_t + \left(1-\sigma_G\right)\left(1-\gamma\right)\varpi\mathsf{n}_t + \sigma_G g_t^R - \bar{y}_t^R. \tag{43}$$

In the DSGE literature for an open economy, the welfare loss includes not only inflation and the output gap but also the TOT, namely t_t, while Eq.(42) includes only inflation and the output gap which may appear to be the loss function for a closed economy. Paying attention to Eq.(43), however, it is clear that the third term on the RHS of Eq.(42) includes the NPD n_t. When all goods are tradable, namely, $\gamma = 1$, Eq.(43) becomes $\tilde{y}_t^R = \left(1-\sigma_G\right)\mathsf{t}_t + \sigma_G g_t^R - \bar{y}_t^R$. Both this equality and Eq.(42) imply that the welfare loss does not include the square of the NPD but does include the square of the TOT, and the welfare loss boils down to one derived by most DSGE studies in an open economy in which all goods are tradable. Because our model allows for nontradables, our welfare loss does not necessarily correspond to the welfare loss in other DSGE studies for an open economy.

[19] Ferrero (2007) points out that the method of deriving the second-order approximated utility function by Rotemberg and Woodford (1997) cannot be applied to Ferrero's (2007) and our frameworks because of a nonnegative stock of debt and a nonnegative level of government expenditure in the steady state. Thus, Ferrero (2007) follows the method proposed by Sutherland (2004) or Benigno and Woodford (2003) and he derives the second-order approximated utility function without the presence of a linear term. In contrast to Rotemberg and Woodford (1997), our technique of deriving the second-order approximated utility function does not assume that the steady-state wedge between the marginal rate of substitution of consumption and leisure and the marginal product of labor is zero. Because of this, we can derive the second-order approximated utility function correctly, without the presence of a linear term. Note that Woodford (2003) discusses how the presence of linear terms generally leads to an incorrect evaluation of the welfare. A simple enlightening example of this result is proposed by Kim and Kim (2003).

4.1 Role of Optimal Fiscal Policy

In this subsection, we investigate the role of optimal fiscal policy by comparing FONCs, which clarify the relationship between PPI inflation and the output gap under the case of optimal monetary policy alone and the case of optimal monetary policy and fiscal policy. In both cases, policy authorities minimize the sum of the discounted value of social losses as follows:

$$\mathcal{L}^W = \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t L_t^W, \tag{44}$$

subject to the structural model with commitment. Hereafter, let us assume $\eta = 1$, implying that the elasticity of substitution between tradables and non-tradables is unity, which is assumed implicitly by Obstfeld and Rogoff (2000) for simplicity. In the case of optimal monetary policy alone, only the central bank minimizes Eq.(44) by choosing the sequence $\left\{\pi_{P,t}, \pi^*_{P,t}, \tilde{y}_t, \tilde{y}^*_t, \mathsf{n}_t, \hat{r}_t\right\}_{t=0}^{\infty}$, while both the central bank and two local governments cooperatively minimize Eq.(44) by choosing $\left\{\pi_{P,t}, \pi^*_{P,t}, \tilde{y}_t, \tilde{y}^*_t, \mathsf{n}_t, \hat{r}_t, b_t, b^*_t\right\}_{t=0}^{\infty}$ in the case of optimal monetary and fiscal policy.

4.1.1 Optimal Monetary Policy Alone

In the case of optimal monetary policy alone, the FONC for union-wide inflation and the output gap is given by:

$$\pi_t^W = -\frac{\bar{\beta}}{\theta}\left(\tilde{y}_t^W - \tilde{y}_{t-1}^W\right), \tag{45}$$

which is a familiar expression in papers discussing optimal monetary policy in an open economy.[20] This implies that local government does not need to dissolve union-wide inflation–output trade-offs. A solitary central bank can stabilize

[20] When $\sigma_G = 0$, Eq.(45) becomes $\pi_t^W = -\frac{1}{\theta}\left(\tilde{y}_t^W - \tilde{y}_{t-1}^W\right)$.

both inflation and the output gap simultaneously even though nontradables exist.

Next, we investigate the relative block of the FONC. We are interested in the effects of nontradables. Thus, we analyze the relative block of the FONC in both cases, namely, the case that all goods are tradable and the case that there are nontradables. When all goods are tradable, namely, $\gamma = 1$, the relative block FONC is given by:

$$\pi_{P,t}^{R} = -\frac{\bar{\beta}}{\theta}\left(\tilde{y}_t^R - \tilde{y}_{t-1}^R\right) - \frac{\bar{\beta}\varphi\left(1-\sigma_G\right)4}{\theta\left(1+\varphi\right)}\left(\mu_{2,t} - \mu_{2,t-1}\right), \tag{46}$$

where $\mu_{2,t}$ denotes the Lagrange multiplier associated with the second equality in Eq.(37), that is, the relative block of the NKIS. This case corresponds to the case assumed by Ferrero (2007). Eq.(46) implies that the inflation–output gap trade-offs no longer disappear simultaneously. Because of this, Ferrero (2007) insists that fiscal policy is needed to enhance social welfare. Next, we abandon the assumption that all goods are tradable. In this case, the relative block of FONC is given by:

$$\begin{aligned}
\pi_{P,t}^{R} &= -\frac{\bar{\beta}}{\theta}\left(\tilde{y}_t^R - \tilde{y}_{t-1}^R\right) + \frac{\bar{\beta}\varphi\left(1-\sigma_G\right)4}{\theta\left(1+\varphi\right)}\left(\mu_{2,t} - \mu_{2,t-1}\right), \\
&\quad + \frac{\bar{\beta}\left(1-\sigma_G\right)4\kappa\varphi}{\theta\left(1+\varphi\right)\left(1+\delta+\kappa\right)}\left(\mu_{5,t} - \mu_{5,t-1}\right), \\
\left(1-\gamma\right)\beta_R \upsilon \mu_{2,t} &= \mu_{5,t} - \frac{1}{1+\delta+\kappa}\mu_{5,t-1},
\end{aligned} \tag{47}$$

where $\mu_{5,t}$ denotes the Lagrange multiplier associated with Eq.(39), the NKRD. The two equalities in Eq.(47) imply not only that the inflation–output gap trade-offs do not disappear simultaneously, but also that the relationship between inflation and the output gap is weakened.

4.1.2 Optimal Monetary Policy and Fiscal Policy

In this optimal monetary and fiscal policy regime, the FONC for union-wide inflation and the output gap is given by Eq.(45). Thus, the union-wide inflation and the output gap are stabilized by optimal monetary policy and fiscal policy even though nontradables exist in a currency union.

The FONC for relative block inflation and the output gap is given by:

$$\pi_{P,t}^{R} = -\frac{\bar{\beta}}{\theta}\left(\tilde{y}_{t}^{R} - \tilde{y}_{t-1}^{R}\right). \tag{48}$$

This equality also implies that relative inflation and the output gap are stabilized by optimal monetary policy and fiscal policy. Both Eqs.(45) and (48) imply that inflation–output gap trade-offs disappear simultaneously, not only at the union-wide level but also in each country under the optimal monetary and fiscal policy regime. Note that we have Eq.(48), even though nontradables exist.

Why do we have Eq.(48) rather than Eq.(47)? Under the optimal monetary and fiscal policy regime, we have $\mu_{2,t} = 0$ as the optimality condition along with optimality conditions derived under the case of optimal monetary policy alone, because we not only have the nominal interest rate but also government bonds in the policy function. We obtain Eq.(48) by substituting $\mu_{2,t} = 0$ along with the initial condition $\mu_{5,-1} = 0$ into Eq.(47). Because of this, we have $\mu_{5,t} = 0$, although this equality is not obtained directly by implementation of fiscal policy. This fact implies that optimal fiscal policy removes the effects of the CPI disparity, which introduces the consumption disparity between both countries, by removing the disparity in the demand block.

4.2 Sensitivity Analysis

In this section, we illustrate the equilibrium behavior of the currency union under the alternative policy regime described above.

4.2.1 Parameterization

We run a series of dynamic simulations and adopt the following benchmark parameterization. We set the values of the inverse of the labor supply elasticity φ, the elasticity of substitution across goods θ, the elasticity of substitution between tradables and nontradables η, the subjective discount factor δ, the steady-state share of government bonds to output σ_B, the steady-state share of government expenditure to output σ_G and the tax rate τ equal to 3, 11, 0.75, 0.5, 1, 0.99, 2.4, 0.276 and 0.3, respectively, which is consistent with quarterly time periods in the model.[21] Except for γ, α, γ, φ and η, these parameterizations are used in Ferrero (2007).[22] As mentioned in the introduction, nontradables account for 50.3% of all goods in the Euro area; thus, we set the share of nontradables in the CPI as $\gamma = 0.5$. Following Obstfeld and Rogoff (2000), we set $\eta = 1$. We also assume that the government expenditure, productivity and preference shifters are described according to the following AR(1) processes:

$$a_{H,t} = \rho_{\mathcal{T}} a_{H,t-1} + \xi_{H,t} \quad ; \quad a_{F,t} = \rho_{\mathcal{T}} a_{F,t-1} + \xi_{F,t},$$

$$a_{\mathcal{N},t} = \rho_{\mathcal{N}} a_{\mathcal{N},t-1} + \xi_{\mathcal{N},t} \quad ; \quad a^*_{\mathcal{N},t} = \rho_{\mathcal{N}} a^*_{\mathcal{N},t-1} + \xi^*_{\mathcal{N},t},$$

$$g^W_t = \rho_G g^W_{t-1} + \xi^W_{G,t} \quad ; \quad g^R_t = \rho_G g^R_{t-1} + \xi^R_{G,t},$$

[21] $\sigma_B = 2.4$ implies that the steady-state debt–output annual ratio is 0.6.

[22] Many DSGE studies use the parameter values in Rotemberg and Woodford (1997). However, to compare our results with those derived by Ferrero (2007) and to analyze the Euro area, we mainly use his parameter values for the Euro area. More precisely, the parameter values of θ, δ, σ_B, σ_G are set as in Ferrero (2007). Ferrero (2007) sets a different degree of price rigidity in countries H and F, however. We set α equal to 0.75, which is assumed by Beetsma and Jensen (2005). Because φ does not appear in Ferrero (2007), we set it equal to 3, which is adopted by Gali and Monacelli (2005).

where $\xi_{H,t}$, $\xi_{F,t}$, $\xi_{\mathcal{N},t}$, $\xi^{*}_{\mathcal{N},t}$, $\xi^{W}_{G,t}$ and $\xi^{R}_{G,t}$ denote the i.i.d. shocks. We set $\rho_{\mathcal{T}}$, $\rho_{\mathcal{N}}$ and ρ_{G} equal to 0.705, 0.784 and 0.8 following Batini, Harrison and Millard (2001) and Ribeiro (2008).[23] Following Ferrero (2007), we set the standard deviation of the innovations equal to 0.01.

As an example of impulse responses, we focus not only on the one standard deviation changes in the productivity shifter of tradable goods in country H, $a_{H,t}$, and the productivity shifter of nontradable goods in country H, $a_{\mathcal{N},t}$, to investigate the effects of the existence of nontradables, but also on the unit standard deviation changes in the union-wide government expenditure shifter, g^{W}_{t}, and the relative government expenditure shifter, g^{R}_{t}, to compare our results with those of Ferrero (2007).

4.2.2 Optimal Monetary Policy Alone

Figure 1 depicts macroeconomic volatility under optimal monetary policy alone with commitment.[24] First, we consider the occurrence of an innovation in union-wide government expenditure. To secure funds for government expenditure, the nominal interest rate decreases and there is pressure for the output gap to increase (seventh panel in Figure 1). As shown in the first equality in Eq.(37), however, a decrease in the lagged nominal interest rate decreases the union-wide output gap. Thus, the union-wide output gap is stabilized (first panel

[23]There are few papers that estimate AR(1) parameters associated with the productivity of the tradables and nontradables sectors separately. Following Benigno and Thoenissen (2005), we adopt the result of Batini, Harrison and Millard (2001) who estimate AR(1) parameters associated with the productivity of tradables and nontradables sectors separately. Note that we recognize that their estimated parameter is smaller than those used in most RBC studies. Ribeiro (2008) is one of few papers that estimate autoregressive processes of government expenditure in Europe. We adopt his estimation result.

[24]See Table 1 for estimates of macroeconomic volatility. Under the benchmark parameterization, four eigenvalues are larger than 1 in value for four forward-looking variables. Thus, the Blanchard–Kahn conditions are met.

in Figure 1).[25] The union-wide inflation rate is also stabilized through the optimality condition in Eq.(45) (second panel in Figure 1). While nontradables account for half of all consumption goods, the output gap and inflation rate are stabilized simultaneously at both the union-wide and country levels. Note that Ferrero (2007) shows that the nominal interest rate increases to a unit innovation in union-wide government expenditure because the tax rate increases simultaneously to secure funds to finance government expenditure, whereas the tax rate is constant over time in our setting.

That the union-wide shock is absorbed by monetary policy alone can be understood intuitively even when nontradables exist. Next, we consider the occurrence of changes in the productivity shifter of tradables in country H. An increase in the productivity shifter of tradables in country H causes a decrease in the PPI inflation rate in country H through a decrease in the marginal cost of tradables in country H (fifth panel in Figure 1). An increase in real wages in country H stemming from an increase in the productivity of tradables produced in country H causes an increase in the marginal cost of nontradables produced in country H. This is the cause of a relative increase in the CPI in country H, because the price of nontradables produced in country F does not change. As shown in Eq.(35), a relative increase in the CPI in country H causes a decrease in the NPD. An increase in the price of nontradables in country H boosts demand for tradables in both countries. Thus, the output gap in country H decreases while it increases in country F (third and fourth panels in Figure 1). Because the logarithmic relative CPI or CPI disparity q_t departs from unity

[25] As mentioned in Section 2.3, all interest-bearing assets held by households are in the form of government bonds. Lowering the interest rate at the previous period reduces the nominal payoff of the bond portfolio purchased by households. Because of the budget constraint, this reduces output via a reduction in consumption.

through an increase in the CPI in country H, this can be called the (short-run) Balassa–Samuelson theorem effect. Note, however, that a decrease in the output gap in country H increases the CPI disparity by lowering the CPI in country H (eighth panel in Figure 1). However, this increase in the CPI disparity is inconsistent with the Balassa–Samuelson theorem, resulting in a real exchange rate depreciation corresponding to an increase in CPI disparity in our paper, which is confirmed by Benigno and Thoenissen (2005). The dynamics brought about by changes in the productivity shifter of tradables in country H can be confirmed by investigating the model, especially the relative block. The second equality in Eq.(37) shows that an increase in the productivity shifter in country H decreases the output gap disparity between both countries. Eqs.(30), (35) and (39) show that a decrease in the output gap disparity between both countries increases the relative CPI. Monetary policy alone can stabilize both the output gap and the inflation rate simultaneously at the union-wide level by increasing the nominal interest rate. However, it cannot be stabilized simultaneously at the country level because nontradables create the output gap disparity between countries.

The result of the occurrence of changes in the productivity shifter of nontradables in country H can be explained in the same manner as the occurrence of changes in the productivity shifter of tradables in country H. However, the coefficient of the productivity shifter of nontradables in the NKRD Eq.(39) is larger than the coefficient of the productivity shifter of tradables. Thus, the logarithmic relative CPI, i.e., real exchange rate q_t, increases and the volatility of other macroeconomic variables is higher in this case than in other cases (eighth panel in Figure 1 and Table 1). The Balassa–Samuelson theorem effect appears clearly in this case.

Changes in relative government expenditure mean that the output gap disparity between countries, and the output gap and inflation rate cannot be stabilized at the country level because of the existence of nontradables (third to sixth panels in Figure 1). Benigno (2004) implies that monetary policy alone can stabilize both the output gap and the inflation rate simultaneously even though asymmetric shocks hit the economy, assuming all goods are tradable. However, when nontradables exist in the currency union, monetary policy alone cannot eliminate the trade-offs between the output gap and inflation in each country.

4.2.3 Optimal Monetary Policy and Fiscal Policy Mix

Both the output gap and the PPI inflation rate are stabilized simultaneously at the union-wide level when any shocks occur in the case of optimal monetary policy alone. However, trade-offs between the output gap and the inflation rate cannot be eliminated in each country in this case. How do fiscal authorities cope with the fact that the trade-offs between the output gap and the inflation rate cannot be eliminated in each country?

Figure 2 depicts macroeconomic volatility under the optimal monetary policy and fiscal policy mix with commitment.[26] First, we consider changes in union-wide government expenditure. The central bank decreases the nominal interest rate to finance additional union-wide government expenditure (seventh panel in Figure 2). Both fiscal authorities decrease the issue of government bonds (ninth and 10th panels in Figure 2). However, decreasing both of them decreases

[26]See Table 2 for estimates of macroeconomic volatility. Under the benchmark parameterization, five eigenvalues are larger than 1 in value for five forward-looking variables. Thus, the Blanchard–Kahn conditions are met.

next period's union-wide output gap. This mechanism is basically the same as the case of optimal monetary policy alone. Because of a constant tax rate, our model implies somewhat different behavior of policy authorities from that which is implied by Ferrero (2007). While the effects of a union-wide shock, such as union-wide government expenditure, can be eliminated by monetary policy alone, not only the central bank but also the fiscal authorities cooperate against the union-wide shock. This result is the same as the one derived by Ferrero (2007).

To eliminate the effects of changes in the productivity shifter of tradables in country H, the fiscal authority in country H could increase government bonds on issue by more than the fiscal authority in country F (ninth and 10th panels in Figure 2). As mentioned above, an increase in the productivity shifter of tradables in country H decreases both the PPI inflation rate and output gap in country H while it increases both of them in country F. However, an increase in government bonds at the previous period increases the nominal payoff of the bond portfolio purchased by households. Thus, consumption in both countries increases. Because tradables produced in country H are cheaper than in country F, tradables produced in country H become more popular than tradables produced in country F in the goods market. This stabilizes the output gap and PPI inflation in both countries (third to sixth panels in Figure 2). This effect can be confirmed by paying attention to the second equality of Eq.(37) and Eq.(38) and their counterparts for country F.

To deal with the effects of changes in the productivity shifter of nontradables in country H, both fiscal authorities have similar behavior in the case of an increase in the productivity shifter of tradables in country H (ninth and 10th panels in Figure 2). An increase in government bonds at the previous period increases the output gap. This stabilizes the output gap in country H (third panel in Figure 2). Furthermore, an increase in government bonds at the previous period increases PPI inflation through an increase in the output gap. This stabilizes PPI inflation (fifth panel in Figure 2).[27] This is shown as the second equality of Eqs.(37) as well as (38) and its counterpart for country F.

When the relative government expenditure shifter shocks the economy, the fiscal authority in country F increases government bonds on issue while the fiscal authority in country H decreases government bonds on issue (ninth and 10th panels in Figure 2). As shown in the second equality in Eq.(37), an increase in relative government expenditure increases the output gap disparity. However, a decrease in relative government bonds in the previous period decreases the current output gap disparity. This reduces the effect of relative government expenditure on the output gap disparity. Stabilizing the output gap makes PPI inflation in both countries stabilize simultaneously (third to sixth panels in Figure 2).

It is noteworthy that the role of the fiscal authorities is larger than that of the central bank in stabilizing both inflation and the output gap. Volatility of the nominal interest rate decreases in the case of optimal monetary policy and fiscal policy (Tables 1 and 2). Furthermore, this tendency does not depend on the share of tradables and nontradables. The volatility of the CPI disparity in the case of optimal fiscal policy and monetary policy is smaller than it is in the case of optimal monetary policy alone (Tables 1 and 2).

[27] Changes in the productivity shifter of nontradables affects both the NKIS and NKRD, and, therefore, changes in the quantity of government bonds on issue are magnified. For the same reason that changes in the productivity shifter of nontradables affect both the NKIS and NKRD, the volatility of the CPI disparity is not zero.

Results in this sensitivity analysis prove the policy implications that are mentioned by international money and finance authors, such as McKinnon (1963). Cooperative fiscal authorities have a certain role in stabilizing both the output gap and inflation simultaneously when nontradable goods exist.

4.3 Welfare Analysis

In this section, we analyze the social welfare associated with both regimes, focusing on the share of nontradables. This paper finds that the role of optimal fiscal policy under the assumption that there are nontradables is more important than the one under the assumption that all goods are tradable. To clarify the role of optimal fiscal policy we compare our result to that of Ferrero (2007) who finds that such policy is essential for the minimization of social losses when all goods are tradable.

Now, we define the welfare criteria. Taking unconditional expectations on Eq.(44) and setting $\delta \to 1$, the expected welfare losses of any policy that deviates from the optimal cooperative solution can be written in terms of the variances of inflation and the output gap as follows:

$$\begin{aligned} \tilde{\mathcal{L}}^W \equiv\ & \frac{\theta}{4(1-\sigma_G)\kappa}\operatorname{var}(\pi_{P,t}) + \frac{\theta}{4(1-\sigma_G)\kappa}\operatorname{var}\left(\pi^*_{P,t}\right) + \frac{1+\varphi}{4(1-\sigma_G)}\operatorname{var}(\tilde{y}_t), \\ & + \frac{1+\varphi}{4(1-\sigma_G)}\operatorname{var}(\tilde{y}^*_t), \end{aligned} \tag{49}$$

with $\tilde{\mathcal{L}}^W$ being the expected welfare losses.

Table 3 depicts social losses associated with the two regimes analyzed in the previous section: optimal monetary policy alone and optimal monetary policy and fiscal policy. As noted above, both regimes are fully committed. When the share of nontradables increases, welfare losses increase under optimal monetary policy alone while optimal monetary policy and fiscal policy bring about zero welfare losses, independent of the share of nontradable goods. The necessity of optimal fiscal policy is clear from this analysis. Note that optimal monetary policy alone cannot result in a zero welfare loss when all goods are tradable, while the welfare losses are minimized among the losses brought about by the optimal monetary policy alone. As mentioned by Benigno (2004), optimal monetary policy alone can eliminate the inflation–output trade-offs simultaneously when all goods are tradable.[28] However, Ferrero (2007) insists that fiscal policy is needed even though all goods are tradable. This discrepancy stems from the assumption of the steady-state behavior of the fiscal authority. Using DSGE analysis, Benigno (2004) assumes a zero steady-state value of government expenditure and bonds. As in our setting, Ferrero (2007) does not assume a zero steady-state value of government expenditure and bonds. Government expenditure and bonds have nonzero values in the steady state. This results in additional elasticities, the steady-state share of government expenditure with respect to output, and the steady-state share of government bonds with respect to output, σ_G and σ_B, respectively. These elasticities change the format of the demand block of the economy, which inhibits perfect risk sharing. Thus, the study of Ferrero (2007) corresponds with our study in the case that all goods are tradable, namely $\gamma = 1$.

Our results insist that the existence of nontradables creates acute losses in the Euro economy, not only because of the assumption of a steady state but also because of the Balassa–Samuelson theorem and another side effect. In our benchmark setting where $\gamma = 0.5$, the welfare loss—the percentage deviation of consumption from its steady state, brought about by optimal monetary policy alone—is 8.50%, while it is 3.09% when all goods are tradable, $\gamma = 1$. As noted

[28]In addition, it is attained when price stickiness is the same in both countries.

above, approximately 50.3% of goods are nontradable; thus, the role of optimal fiscal policy is greater than that suggested by Ferrero (2007).

5 Implementing a Cooperative Solution by Self-oriented Fiscal Authorities

Some works, such as Benigno (2002), Obstfeld and Rogoff (2002) and Benigno and Benigno (2008), show that self-oriented monetary authorities can replicate the cooperative outcome in a decentralized framework so that there is no need for international monetary policy coordination. Following their context, we investigate whether it is possible that fiscal policies set in a noncooperative environment can implement the optimal cooperative solution in this section.

While the central bank commits to minimizing union-wide social loss $\mathcal{L}^W$ subject to the structural model, we assume that each fiscal authority commits to minimizing its respective losses as follows:

$$\mathcal{L}^{NC} \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t L_t^{NC} \ ; \ \mathcal{L}^{NC*} \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t L_t^{NC*},$$

subject to the structural model with:

$$L_t^{NC} \equiv \frac{1}{2\left(1-\sigma_G\right)} \left[\frac{\theta}{\kappa}\pi_{P,t}^2 + \omega\left(1+\varphi\right)\tilde{y}_t^2\right] \ ; \ L_t^{NC*} \equiv \frac{1}{2\left(1-\sigma_G\right)} \left[\frac{\theta}{\kappa}\left(\pi_{P,t}^*\right)^2 + \omega^*\left(1+\varphi\right)\left(\tilde{y}_t^*\right)^2\right], \quad (50)$$

where ω and ω^* are nonnegative parameters and L_t^{NC} denotes social losses assigned to the fiscal authority to replicate the cooperative outcome in country H.[29]

Next, we seek to find ω and ω^* that satisfy $\mathcal{L}^W = \mathcal{L}^{NCW}$, with $\mathcal{L}^{NCW} \equiv \frac{1}{2}\left[\mathcal{L}^{NC} + \mathcal{L}^{NC*}\right]$ being the union-wide social loss brought about by self-oriented

[29]Following Beetsma and Jensen (2005), we split the per period union-wide social loss function Eq.(42) as follows: $L_t^W = \frac{1}{2}\left(L_t + L_t^*\right)$ with $L_t \equiv \frac{1}{2(1-\sigma_G)}\left[\frac{\theta}{\kappa}\pi_{P,t}^2 + (1+\varphi)\tilde{y}_t^2\right]$ and $L_t^* \equiv \frac{1}{2(1-\sigma_G)}\left[\frac{\theta}{\kappa}\left(\pi_{P,t}^*\right)^2 + (1+\varphi)\left(\tilde{y}_t^*\right)^2\right]$. After this, we introduce ω and ω^*.

fiscal authorities in both countries. It can be said that $\mathcal{L}^{NCW}$ is the union-wide social loss in the Nash equilibrium under optimal monetary policy. When the noncooperative solution brings about the social loss that corresponds to the social loss brought about by the cooperative solution, $\mathcal{L}^{W} = \mathcal{L}^{NCW}$ is applied. Let us assume that there are cost push shocks that prevent the central bank from being able to stabilize both inflation and the output gap simultaneously and that have constant variance. These assumptions help us to calculate the social loss analytically.[30] Note that we still assume $\eta = 1$. With tedious calculations, we have ω and ω^* as follows:

$$\omega = \omega^* = 1.$$

This implies that a self-oriented fiscal authority can achieve the cooperative allocation in the Nash equilibrium without imposing a complicated loss function on fiscal authorities.

Benigno and Benigno (2008) investigate how ω and ω^* bring about the cooperative allocation in the Nash equilibrium. They show that when relative risk aversion multiplied by the elasticity of substitution between home and foreign goods is unity, $\omega = \omega^* = 1$ holds to achieve the cooperative allocation in the Nash equilibrium. Along with the assumptions of our model, namely, log utility and the unitary elasticity of substitution between home and foreign goods,$\eta = 1$ is assumed for simplicity. Thus, although the existence of nontradable goods is allowed, our result is the same as that derived by Benigno and Benigno (2008). However, it should be highlighted that the condition $\omega = \omega^* = 1$ always holds independently of the share of nontradables $1 - \gamma$.

Our result differs from the results in the literature such as McKinnon (1963),

[30] Following Walsh (2003) and Monacelli (2004), we calculate expected losses.

who insist on the necessity of moving fiscal policy control from the local or national government to the central government. Furthermore, as a New Keynesian author, Ferrero (2007) suggests the necessity of cooperation between fiscal authorities. Furthermore, our result is partially different from the result of Benigno and Benigno (2008), Benigno and Benigno (2006) and Beetsma and Jensen (2005) who analyze the necessity of policy coordination under a two-country DSGE model and investigate how self-oriented settings bring about the cooperative allocation in the Nash equilibrium. They show that an allocation derived in a centralized setting can be replicated in a self-oriented setting. However, they do not necessarily deny gains from a cooperative setting because of the TOT externality. Thus, to replicate allocation under a cooperative setting, they propose a somewhat rich and complicated individual or respective loss function whose sum do not equal the union-wide loss function. Our result implies that a rich and complicated respective loss function is not needed. There are contradictory policy implications between our paper and those papers. However, Benigno and Benigno (2008) and Benigno and Benigno (2006) show that there is no need to cooperate internationally under certain circumstances, even though gains from policy cooperation exist in all other situations.[31] Our utility function does not include any exogenous shifter. Because of this, the TOT do not appear in the second-order approximated utility function, which implies that there is no TOT externality. In contrast, the second-order approximated utility function derived by Benigno and Benigno (2008), Benigno and Benigno (2006) and Beetsma and Jensen (2005) imply a TOT externality that disappears only

[31] Benigno and Benigno (2006) show that there are no gains from cooperation when there are no mark-up shocks or government purchases. Benigno and Benigno (2008) show that the sum of the respective loss functions to replicate the cooperative allocation equal the union-wide loss function when both the degree of relative risk aversion and the elasticity of substitution between goods produced in both countries are unity.

under special circumstances. Our case corresponds to special cases of these papers. Thus, our policy implications are not necessarily inconsistent with those of Benigno and Benigno (2008) and Benigno and Benigno (2006).

Our game theoretical analysis following Benigno and Benigno (2008) implies that the national fiscal authority can eliminate welfare losses by having only one central bank when nontradables exist. In this case, the inflation–output trade-offs in the Euro area may not be an acute problem if the Stability and Growth Pact of the Maastricht Treaty is revised.[32]

6 Conclusion

This paper verified the role of optimal fiscal policy in a currency union. Assuming the existence of nontradables, we showed that the role of optimal fiscal policy is more essential than suggested by Ferrero (2007). Furthermore, in contrast to authors such as McKinnon (1963), we showed that there is no need to align fiscal authorities to eliminate social losses by eliminating inflation–output trade-offs. While we assume the existence of nontradables, Gali and Monacelli (forthcoming) assume a currency union that consists of small open economies. Considering the fact that relatively small economies will participate in the EMU hereafter, their assumption is plausible. Because the assumptions in our paper and in Gali and Monacelli (forthcoming) closely reflect the conditions in the Euro area, optimal fiscal policy is essential in the Euro area.

While this paper can reconcile some papers that have inconsistencies in their implications, this paper cannot reconcile the implications of Canzoneri, Cumby and Diba (2001), which support the Stability and Growth Pact of the Maastricht

[32]According to the Pact, national fiscal policies are bound to respect an upper threshold for the deficit to GDP and the debt to GDP ratios of 3% and 60%.

Treaty. While we can derive policy implications that suggest that fiscal policy is essential, the plausibility of the Stability and Growth Pact is not investigated explicitly. This is a future research agenda.

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Table 1: Macroeconomic Volatility: All Goods are Tradable ($\gamma = 1$)

Variable	Regime	Shocks			
		$a_{H,t}$	$a_{\mathcal{N},t}$	g_t^W	g_t^R
$\tilde{y}_t^W$	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0000	0.0000	0.0000	0.0000
π_t^W	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0000	0.0000	0.0000	0.0000
$\tilde{y}_t$	OMP	0.0066	0.0000	0.0000	0.0083
	OMFP	0.0000	0.0000	0.0000	0.0000
$\tilde{y}_t^*$	OMP	0.0066	0.0000	0.0000	0.0083
	OMFP	0.0000	0.0000	0.0000	0.0000
$\pi_{P,t}$	OMP	0.0082	0.0000	0.0000	0.0150
	OMFP	0.0000	0.0000	0.0000	0.0000
$\pi_{P,t}^*$	OMP	0.0082	0.0000	0.0000	0.0150
	OMFP	0.0000	0.0000	0.0000	0.0000
q_t	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0000	0.0000	0.0000	0.0000
$\hat{r}_t$	OMP	0.0115	0.0000	0.0123	0.0000
	OMFP	0.0008	0.0000	0.0067	0.0000
b_t	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0129	0.0000	0.0057	0.0044
b_t^*	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0081	0.0000	0.0057	0.0044

Notes:
OMP: Optimal solitary monetary policy
OMFP: Optimal monetary and fiscal policy

Table 2: Macroeconomic Volatility: Benchmark ($\gamma = 0.5$)

Variable	Regime	Shocks			
		$a_{H,t}$	$a_{\mathcal{N},t}$	g_t^W	g_t^R
$\tilde{y}_t^W$	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0000	0.0000	0.0000	0.0000
π_t^W	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0000	0.0000	0.0000	0.0000
$\tilde{y}_t$	OMP	0.0015	0.0023	0.0000	0.0042
	OMFP	0.0000	0.0000	0.0000	0.0000
$\tilde{y}_t^*$	OMP	0.0015	0.0023	0.0000	0.0042
	OMFP	0.0000	0.0000	0.0000	0.0000
$\pi_{P,t}$	OMP	0.0076	0.0153	0.0000	0.0236
	OMFP	0.0000	0.0000	0.0000	0.0000
$\pi_{P,t}^*$	OMP	0.0076	0.0153	0.0000	0.0236
	OMFP	0.0000	0.0000	0.0000	0.0000
q_t	OMP	0.0037	0.0084	0.0000	0.0101
	OMFP	0.0004	0.0038	0.0000	0.0001
$\hat{r}_t$	OMP	0.0058	0.0069	0.0130	0.0000
	OMFP	0.0004	0.0004	0.0065	0.0000
b_t	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0067	0.0088	0.0056	0.0046
b_t^*	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0041	0.0036	0.0056	0.0046

Notes:
OMP: Optimal solitary monetary policy
OMFP: Optimal monetary and fiscal policy

Table 3: Effect on Welfare of Varying Share of Tradable Goods

γ	$\mathcal{L}^W$ (Percentage)	
	OMP	OMFP
1	-3.0935	0.0000
0.9	-3.7844	0.0000
0.8	-5.9287	0.0000
0.7	-6.5082	0.0000
0.6	-7.5414	0.0000
0.5	-8.4962	0.0000
0.4	-13.0896	0.0000
0.3	-13.9473	0.0000
0.2	-15.6554	0.0000
0.1	-20.3470	0.0000
0	-21.4257	0.0000

Notes:
OMP: Optimal solitary monetary policy
OMFP: Optimal monetary and fiscal policy

Figure 1: Impulse Responses to Shocks under Optimal Monetary Policy Alone: Benchmark Case

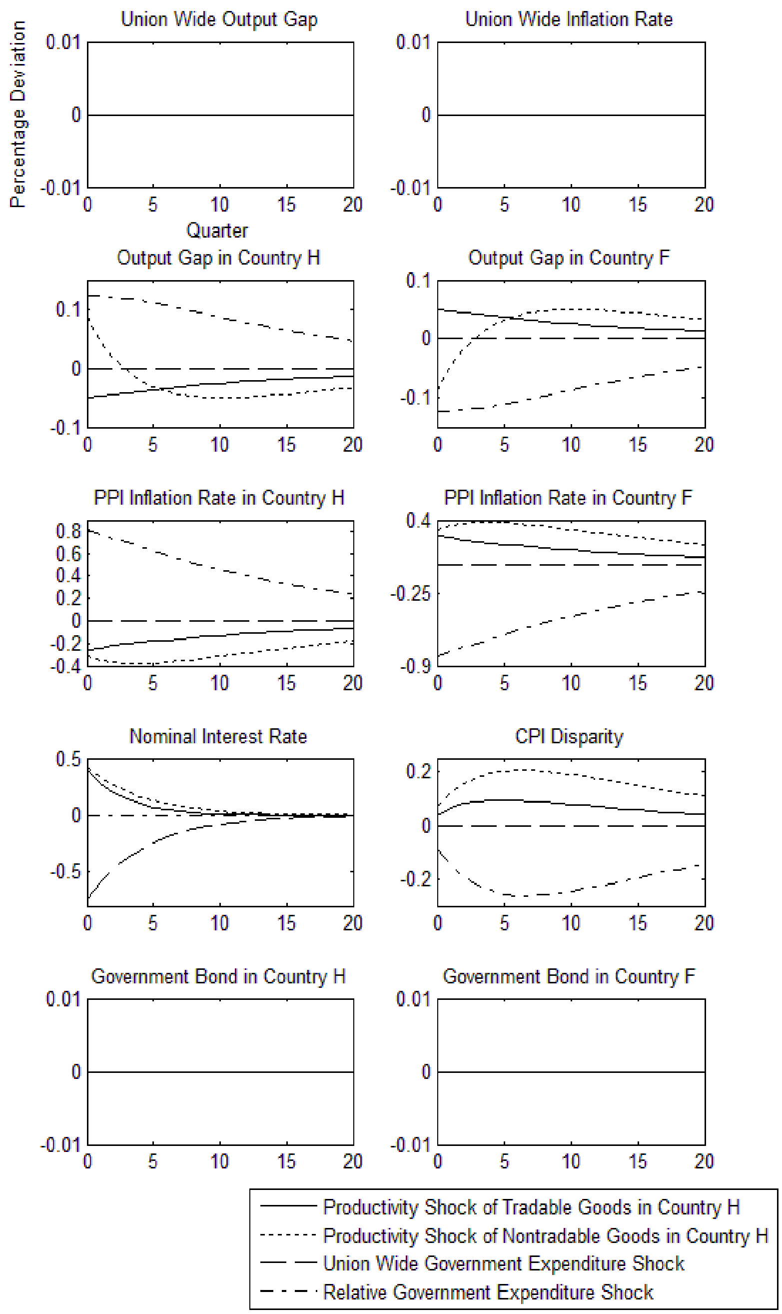

Figure 2: Impulse Responses to Shocks under Optimal Monetary Policy and Fiscal Policy: Benchmark Case