

Public External Debt, Informality and Production Efficiency in Developing Countries

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Abstract

This paper proposes an alternative approach to investigate the non-linear effect of external debt on growth. First, an endogenous growth model with formal and informal sectors is developed to analyse the effect of public external debt on production efficiency in developing countries. Second, using a stochastic frontier technique with unobserved heterogeneity, for a panel of 27 developing countries for the period of 1970-2005, we confirm that the turning point associated to the effect of the share external public debt is apparent at 84%.

Keywords: public external debt, production efficiency, informality.

JEL Classification: F34, O17, O40

1 Introduction

In recent decades the external debt in developing countries has steadily increased, making the analysis of the role of external debt in financing the development process particularly important. The debt cycle thesis proposed by Avramovic (1964) confirms that external debt is an important funding source for an economy characterized by low domestic savings. According to the debt cycle predictions, domestic savings in the long run should increase to finance a higher proportion of investment and to repay the external debt contracted in the first stage of development. However, many developing countries seem to be trapped in the first stage of this debt cycle as the stock of external debt is increasing and domestic saving is still low.

More recently, new studies by Krugman (1988), Sachs (1989) and Cohen (1992) gave rise to the theory of excessive debt (debt overhang). This theory establishes that beyond a certain threshold, external debt could discourage consumption and investment, and thus limit economic growth.

In an empirical study covering 29 sub-Saharan African LDCs over the period of 1970-80, Fosu (1996) confirms this finding as he identifies a non-linear effect of external debt on growth. Pattillo et al. (2002) confirm this finding and show that the average effect of debt on growth becomes negative when the ratio of debt to GDP exceeds a threshold between 35% and 40%. This non-linear effect is also confirmed by Clements et al. (2003) and Cordella and Arranz (2005). In addition, Imbs and Ranciere (2005) find that countries with good policies and good institutions have lower debt overhang.

This paper proposes an original theoretical model to explain the non-linear effect of external debt on growth. It is different from existing models of the debt overhang

theory in many aspects. First, it investigates the effect of the public external debt on a particular component of growth: the production efficiency. The focus on production efficiency is justified as few studies suggest that the negative effect of external debt on growth pass through total factor productivity decline rather than through a deceleration in the factors' accumulation dynamics (Fosu 1996, 1998, 2008, Pattillo, 2002, 2004). Indeed, Fosu (1996)¹ argues that both debt service payments and outstanding debt may affect GDP growth rate negatively even if these don't affect investment levels. He confirms that the debt has an adverse effect directly on growth by reducing productivity. Pattillo et al. (2004) analyse empirically the channels through which external debt affects economic growth. Their empirical study covers a sample of 61 developing countries from 1969 to 1998. Their results show that the negative impact of debt on growth is due to the adverse effects on the accumulation of physical capital (with an 1/3 contribution', on average) and the negative effects on total factor productivity (with 2/3 contribution, on average).

Moreover, production efficiency is the second component, in addition to technological progress, of the total factor productivity (Grosskopf 1993). While technological progress expands the set of production possibilities, an increase in production efficiency corresponds to a more efficient use of existing production factors: a convergence to the production frontier. In most developing countries, technological progress is weak and a feasible total factor productivity increase could take place through an improvement of production efficiency.

The second aspect that distinguishes our model is consideration of the informal economy which is an important aspect in the developing countries². We propose an endoge-

¹It is considered as the first to emphasize the productivity channel based on the 'Direct Effect of Debt Hypothesis (DEDH)'.
²Chickering and Salahdine (1991) argue that for the majority of developing countries, the informal sector contributes 35% to 65% to total employment and produces between 20% and 40% of GDP. According to Friedman et al. (2000) the size of the informal sector is approximately 68% in Egypt, 39% in Malaysia, 76% in Nigeria, 71% in Thailand, 45% in Tunisia, etc.

nous growth model with overlapping generations. Investors have to choose between formal projects and less efficient informal ones. We show that a misallocation of the external debt between the public sector and the private formal sector could reduce production efficiency and result in an enlarging of the informal sector. The excessive external public debt pushes the government to increase taxes to repay its debt service which in turn reduces the formal sector size. The fact that a large informal sector is associated with lower growth rates is widely accepted in the literature (Loayza1997, Johnson et al. 1999 and Schneider and Enste 2000). More recently, Ferreira-Tiryaki (2008) confirms that the need of informal firms to remain small in order to avoid detection implies that achieving economies of scale and, therefore, being productive is an unfeasible endeavor.

Numerous empirical studies examined the interrelationship between external debt and growth. The majority of these studies include external debt merely as an explanatory variable in the growth determinant function. In the empirical part of this paper we adopt a different and original approach. We use a stochastic frontier model to test the effect of external public debt on production efficiency. This enables us to determine if the external public debt reduces growth through a drop in production efficiency. Our econometric methodology is based on the random coefficient models recently proposed by Greene (2005) in order to remove the unmeasured heterogeneity from the efficiency measure . Indeed, in many estimations of the stochastic frontier, heterogeneity is confused with production (technical) efficiency. Any time-invariant unobserved heterogeneity is pushed into the efficiency component. However, as confirmed by Griliches (1957), unobserved heterogeneity if not accounted for, may cause biased estimates. Orea and Kumbhakar (2004) confirm that unobserved differences in technologies may be inappropriately labelled as efficiency if variation in technology is not taken into account. Our panel data, covering 27 developing countries over the period 1970 to 2005, include countries at different levels

of indebtedness and growth rates. The choice of the panel is justified by the fact that we need a heterogeneous panel to test the non-linear effect.

The rest of the paper is organized into four main sections. Section 2 presents the theoretical framework and discusses different channels through which external public debt affects productivity efficiency. Section 3 outlines the methodology and the data sources of the empirical investigation and discusses the results that emerge from the estimations. Section 4 concludes and gives some policy implications.

2 An endogenous growth model

2.1 Economic environment

We consider an economy with an infinite, discrete time horizon, $t = 0, 1, 2, \dots$. Date t corresponds to the beginning of period $t + 1$ and the end of period t . The economy is endowed with two production sectors with different technologies. The first sector produces a final (or consumption) good using capital and labour. The second sector produces an investment (or capital) good using the final good. At each date a new generation of two-period living agents of mass 1 is born. An initial generation of old agents coexists with young agents at date $t = 0$. The old of the first generation are endowed at $t = 0$ with a stock k_0 of capital good. All agents are endowed with one unit of labour which they supply during their first period of life inelastically at no disutility cost. In compensation for their work (when young) in the final good sector, they earn a wage which is invested during the second period in order to maximize the final wealth financing their consumption. Two investment opportunities are available for each young agent after receiving its wage: undertaking a formal or an informal investment project (producing the investment good). A formal project is eligible for a complementary external financing but is taxable. However, the informal project is self-financed, non-taxable and supports the cost of tax evasion.

2.1.1 Final good sector

The consumption good is tradable and produced instantaneously from the combination of two substitutable factors: capital (good) K and labour L . The technology which is assumed to be of Cobb-Douglas type exhibits constant factors' return but includes an aggregate level of "knowledge" A which enables the endogenous growth of the aggregate production: $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$. We associate A_t to the aggregate stock of capital $A_t = A \bar{k}_t^{1-\alpha} = A (K_t/L_t)^{1-\alpha}$. This choice ³ allows for a constant price of the final good which simplifies the model. Hence, the final good production is equal to the capital stock $Y_t = AK_t$ and the per capita output is given by

$$y_t = Ak_t \tag{1}$$

The output is entirely distributed to the workers (young agents) and to the investment good producers (the entrepreneurs among the old agents). Finally, the factors' prices are equal to their marginal productivity and capital depreciates fully after production:

$$\rho_t = \alpha A \tag{2}$$

$$w_t = A(1 - \alpha)k_t \tag{3}$$

2.1.2 Agents' investment decisions

The agent supplies, inelastically at no disutility cost, a unit of labour during its first-period of life. Hence, the total labour supply in each period is $L = 1$. In return, he earns a wage w_t which he invests during the second period in order to maximize its final consumption. Indeed, to simplify the model we assume consumption occurs only at the end of the second period. Under this assumption there is no trade-off between consumption and saving at the end of the first period. The only trade-off we consider in this model is between investing

³The choice of this technology is common in the literature (Bose and Cohtern 1996). For simplification we will set $A = 1$.

in a formal or in an informal project. Whatever the project's type is, investment good is produced using a linear technology transforming any quantity q of the final good in $(ag_t)q$ investment good with $a > 1$. The term g_t denotes the amount of public expenditures per capita which increases the productivity of the two types of projects⁴.

Undertaking a formal project When undertaking a formal project, an agent can obtain an external financing of d_t^f in terms of the final good. This amount is lent by international investors (through a domestic financial intermediary) in return of a gross interest rate denoted by r . Therefore, the total amount invested in the formal project is $w_t + d_t^f$ and the quantity of the investment good produced is

$$\kappa_{t+1}^f = ag_t (w_t + d_t^f) \quad (4)$$

This quantity is sold to the final sector at the price α which provides the agent an income $\alpha\kappa_{t+1}^f$ in terms of the final good. Hence, his gross profit after repaying his debt⁵ is $\pi_{t+1}^f = \alpha\kappa_{t+1}^f - rd_t^f$ and his net profit after paying the tax τ is $(1 - \tau_t) \pi_{t+1}^f$.

Undertaking an informal project An agent who undertakes an informal project has no access to the external financing and does not pay tax on the profit. He produces a quantity of the investment good given by

$$\kappa_{t+1}^j = ag_t w_t \quad (5)$$

His gross profit is $\pi_{t+1}^j = \alpha (\kappa_{t+1}^j - c_{t+1}^j)$ where c_{t+1}^j represents the cost of informality. This cost can be related to the masking of the activity through paying bribes or localities far from urban areas which exposes the agent to more risks and high transport costs. The agents are heterogenous relatively to this cost which is assumed to vary proportionally

⁴One can think about the quality of public services, infrastructure, etc.

⁵Note that the agent has no incentive to borrow if the cost of capital is more than its project return or equivalently $\alpha ag_t < r$.

to the production $c_{t+1}^j = (1 - \theta_j)\kappa_{t+1}^j$. The parameter θ_j is specific to each agent and is distributed uniformly on $[0, 1]$. This signifies that agents with very low cost of informality have a high value of θ_j (at the extreme no such cost if $\theta_j = 1$). Therefore, the profit derived from the informal project is given by $\pi_{t+1}^j = \alpha (\kappa_{t+1}^j - c_{t+1}^j) = \alpha\theta_j ag_t w_t$.

Agents' decisions Each agent chooses project by maximizing profit. Hence, at date t the informal projects are realized by the agents characterized by θ_j such that $\pi_{t+1}^j \geq (1 - \tau_t) \pi_{t+1}^f$. Using the above expression of the profits we obtain the set of informal entrepreneurs $\Theta = \{j \text{ such that } \theta_j \in [\theta_t, 1]\}$ where θ_t is defined by

$$\theta_t = \frac{(1 - \tau_t)\pi_{t+1}^f}{\alpha ag_t w_t} \quad (6)$$

$$= (1 - \tau_t) \left[1 + \left(1 - \frac{r}{\alpha ag_t} \right) \frac{d_t^f}{w_t} \right] \quad (7)$$

The set $\bar{\Theta}$ of formal entrepreneurs includes agents who support sufficiently high cost of informality and for whom it is more interesting to undertake formal projects $\bar{\Theta} = \{j \text{ such that } \theta_j \in [0, \theta_t]\}$. Hence, if the threshold θ_t is equal to one⁶, there is no informal projects in the economy during period $t + 1$. Note that we can interpret θ_t as the size of the formal sector and $1 - \theta_t$ as the size of the informal one.

2.1.3 Government

The stock of (inherited) external debt (in terms of the tradable good) at the beginning of period $t + 1$ is denoted by D_t and the economy raises a new line of external debt of an amount d_t assumed not to exceed the amount $(1 + \tau_t)w_t$. The government controls the allocation of the external debt in the economy. It allocates a proportion λ_t to finance its expenditures g_t and a proportion $1 - \lambda_t$ to the financing of the private sector (formal investment projects) so we have

⁶The case $\theta_t = 1$: there is no taxation $\tau_t = 0$ and the external financing is very costly so that $d_t^f = 0$.

$$g_t = d_t^g = \lambda_t d_t \quad (8)$$

$$d_t^f = (1 - \lambda_t) d_t$$

At date $t + 1$, the government uses its fiscal revenue to repay a proportion γ_{t+1} of the principal and service due on the stock of the inherited and new public debt.

$$T_t = \gamma_{t+1} r (D_t + d_t^g) \quad (9)$$

The fiscal revenue is the sum of taxes paid by the formal entrepreneurs of mass θ_t which gives $T_t = \theta_t \tau_t \pi_{t+1}^f$. Using (6) we obtain the proportion of the debt stock the government is willing to repay

$$\gamma_{t+1} = \frac{(\theta_t)^2}{\frac{r}{\alpha a w_t} \left(\frac{1}{\tau_t} - 1 \right) \left(1 + \frac{D_t}{d_t^g} \right)} \quad (10)$$

Note that this proportion increases if the size of the formal sector (θ_t) or the tax increases and if the interest rate decreases. It decreases if the inherited stock of public debt is high relative to new debt. Taking into account the repayment proportion we obtain the following dynamic of debt stock

$$D_{t+1} = r [(1 - \gamma_{t+1}) (D_t + d_t^g)] \quad (11)$$

Hence, a sufficient condition for the external debt to explode is $\gamma_{t+1} < \underline{\gamma} = 1 - 1/r$. Using (10) we obtain the minimal size $\underline{\theta}_t$ of the formal sector that corresponds to $\underline{\gamma}$

$$\underline{\theta}_t = \left[\frac{r-1}{\alpha a w_t} \left(\frac{1}{\tau_t} - 1 \right) \left(1 + \frac{D_t}{y_t} \frac{w_t}{(1-\alpha)\lambda_t d_t} \right) \right]^{\frac{1}{2}} \quad (12)$$

This expression signifies that the public external debt explodes if the size of the formal sector is inferior to $\underline{\theta}_t$. Figure 1 illustrates how the threshold $\underline{\theta}_t$ varies with the proportion λ_t of the public debt.

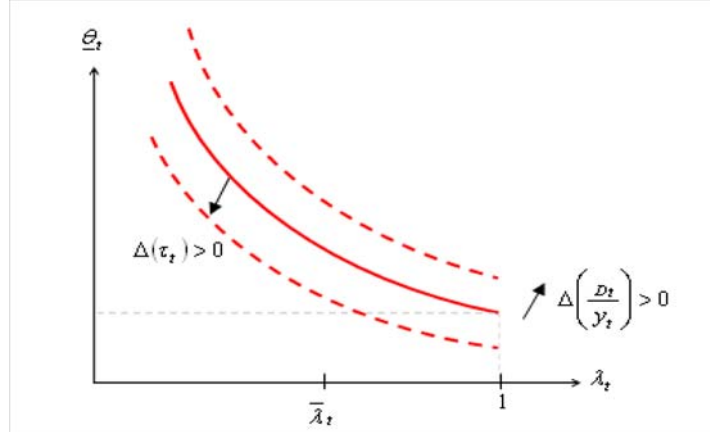


Figure 1

Note that the minimal size of the formal sector decreases when the proportion of the public debt increases. This is due to the increase in the formal project's net profit after the increase of public expenditure. Indeed, it is easy to show that $\frac{\partial \pi_{t+1}^f}{\partial \lambda} > 0$ means that increase of the positive externality induced by public expenditure more than compensates the negative effect resulting from the reduction of the private debt. Since the profit per formal project increases, the constraint on the size of the formal sector is relaxed. This figure shows also that for a given level of public debt proportion λ_t , an increase in taxation moves the frontier downward indicating a shrinking in the area of exploding external debt. This is because the government raises more taxes per project so that the total proportion of formal projects (which corresponds to the formal sector size) could be decreased in order to keep the tax revenues constant. Meanwhile, an increase of the ratio of external debt stock to production moves the frontier upward signifying that the debt burden is high and there is a need for a large formal sector to prevent it from exploding.

2.1.4 Formal sector's size

The size of the formal sector defined by (7) can be written equivalently using (8)

$$\theta_t = (1 - \tau_t) \left[1 + \left(1 - \frac{r}{\alpha a \lambda_t d} \right) \frac{(1 - \lambda_t) d_t}{w_t} \right] \quad (13)$$

This expression shows that an increase in the share of public sector external debt (λ_t) to the detriment of the private (formal) sector has an ambiguous effect on the share of the formal sector. On one hand, it increases public expenditures a fact which induces positive externality and increases project productivity. Through this channel, it affects positively the share of the formal sector, since more agents will have an incentive to quit the informal sector on observing that the return of formal project increases relatively the cost of external borrowing. This effect is captured through the term $\left(1 - \frac{r}{\alpha a \lambda_t d} \right)$. On the other hand, it reduces the amount of external financing to the formal projects, and this tends to diminish the size of the formal sector. This effect is captured through the term $(1 - \lambda_t)$.

Proposition 1

- i) *The size of the formal sector is maximal $\bar{\theta}_t$ for a share of the public external debt given by $\bar{\lambda}_t = \min(1, \tilde{\lambda}_t)$ where $\tilde{\lambda}_t = \sqrt{\frac{r}{\alpha a d_t}}$.*
- ii) *We have $\frac{\partial \theta_t}{\partial \lambda_t} (\lambda_t - \bar{\lambda}_t) \leq 0$.*

Proof. It is straightforward using (13) and differentiating θ_t relatively to λ_t obtaining

$$\frac{\partial \theta_t}{\partial \lambda_t} = \frac{(1 - \tau_t) d}{w_t} \left[\frac{\tilde{\lambda}_t^2}{\lambda_t^2} - 1 \right] \quad (14)$$

The following figure illustrates how the size of the formal sector θ_t varies when the share of the public external debt λ_t and the tax rate τ_t vary.

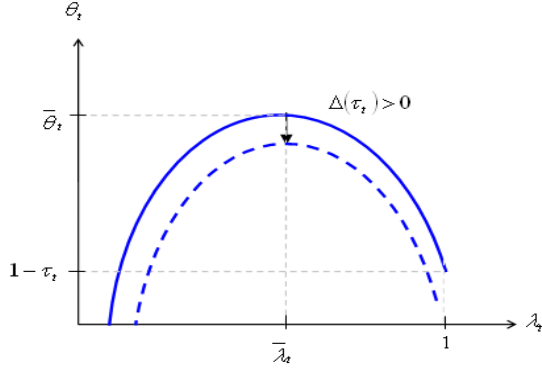


Figure 2

As it can be noted, the effect of public external debt share (λ_t) on the size of the formal sector is non linear and depends on the taxation rate. An increase in λ_t improves the size of the formal sector when the positive effect of an increase in government expenditure exceeds the negative effect of an increase in taxation. $\bar{\lambda}_t$ is the turning point after which any increase in the share of external public debt induces a smaller formal sector.

2.2 Production efficiency and external debt management

The purpose of this section is to investigate how the allocation of the external debt between the private sector and government expenditure affects production efficiency. Production efficiency is defined, at each period, as the ratio of the effective output per capita on the potential output per capita. For period $t + 1$ we have

$$\varphi_{t+1} = \frac{y_{t+1}}{\bar{y}_{t+1}}$$

Using (1) we obtain $\varphi_{t+1} = k_{t+1}/\bar{k}_{t+1}$ signifying that production efficiency is also the efficiency of the capital good production process. The quantity of the capital (investment) good available at $t + 1$ is the sum of the output of the formal projects $\theta_t \kappa_{t+1}^f$ and that of the informal projects $\int_{\theta_t}^1 \kappa_{t+1}^j d\theta_i$. Using (4) and (5) we obtain

$$\begin{aligned}
k_{t+1} &= \theta_t \left(ag_t \left(w_t + d_t^f \right) \right) + \int_{\theta_t}^1 (\theta_j ag_t w_t) d\theta_j \\
&= aw_t (\lambda_t d_t) h(\theta_t, \lambda_t)
\end{aligned} \tag{15}$$

where

$$h(\theta_t, \lambda_t) = \theta_t \left(1 + \frac{(1 - \lambda_t) d_t}{w_t} \right) + \frac{1 - (\theta_t)^2}{2} \tag{16}$$

The relationship between the external public debt share and production efficiency is summarized in the following proposition.

Proposition 2

- 1) For a given tax rate τ_t , the production efficiency is concave in λ_t and it is maximized in λ_t^* verifying

$$\begin{aligned}
\lambda_t^* &= 1 \quad \text{if} \quad \begin{cases} w_t \geq d \\ w_t < d < w_t(1 + \tau_t) \text{ and } \tau_t \geq \tau_t^* \end{cases} \quad \text{or} \\
\bar{\lambda}_t < \lambda_t^* < 1 & \quad \text{if} \quad \frac{r}{\alpha a} < w_t < d < w_t(1 + \tau_t) \text{ and } \tau_t < \tau_t^*
\end{aligned}$$

where τ_t^* depends only on $\frac{r}{\alpha a}$, d and w .

- 2) Production efficiency decreases with the tax rate.

Proof. See the appendix.

The following figure illustrates two possible configurations of production efficiency satisfying the above proposition.

Case $w_t \geq d_t$ or

$$\frac{r}{\alpha\lambda} < w_t < d_t < w_t(1 + \tau_t) \text{ and } \tau_t > \tau_t^*$$

Case $\frac{r}{\alpha\lambda} < w_t < d_t < w_t(1 + \tau_t)$

$$\text{and } \tau_t < \tau_t^*$$

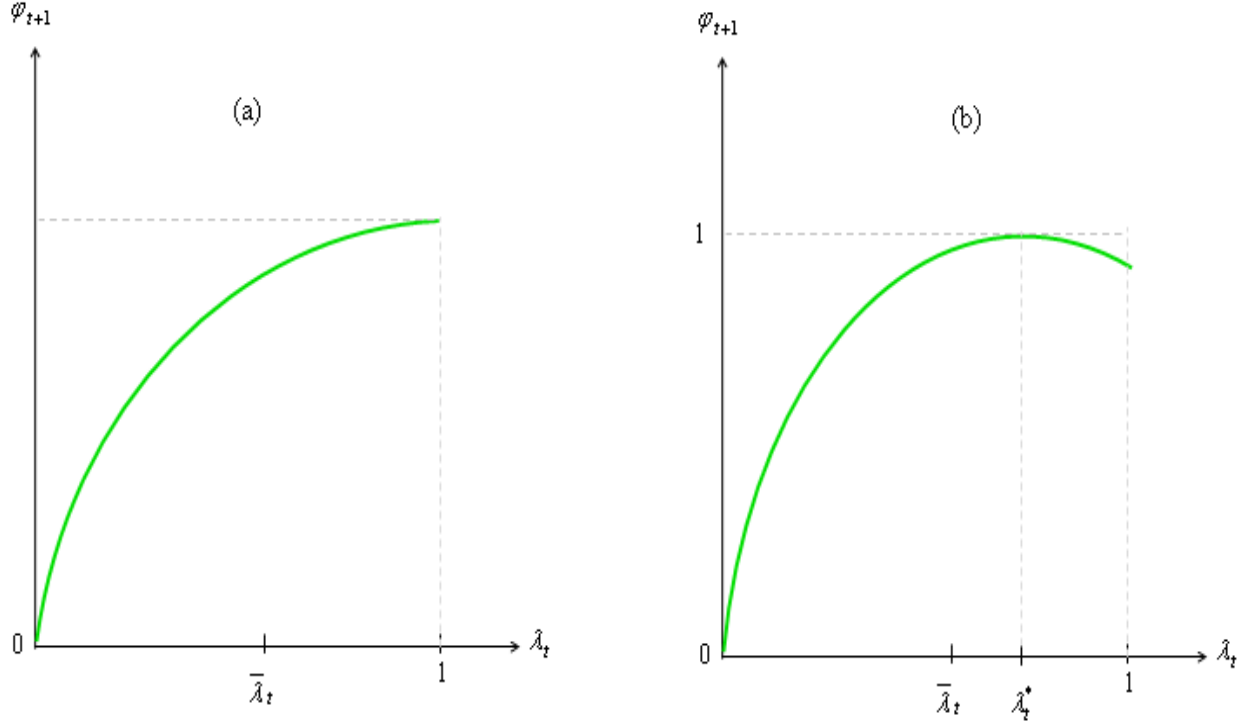
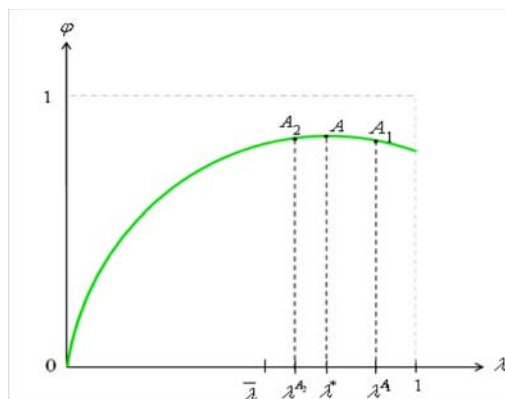


Figure 3

As showed in proposition 1, when $\lambda_t > \bar{\lambda}_t$, an increase in the external public debt share λ_t diminishes the size of the formal sector. However, as long as the share λ_t remains in $[\bar{\lambda}_t, \lambda_t^*]$, this negative effect is dominated by the positive effect of higher externalities generated by the increase of public expenditures. Therefore, the resulting effect is positive and there is an increase in growth although the informal sector expands. In the following we will focus on configuration (a). Configuration (b) is a particular case of (a) since it corresponds to $\lambda_t^* = 1$.

2.2.1 Direct effect of the external debt management

The direct effect is defined relative to the variation of the share of public external debt relative to the optimal share λ_t^* . The following figure illustrates two situations.



The first situation corresponds to the negative direct effect of a decrease of the share in the public external debt. This is the case when the initial share is less than λ_t^* for example λ^{A_2} . The second situation corresponds to the positive direct effect of a decrease of this share. This is the case when it is initially superior to λ_t^* for example λ^{A_1} . In many developing economies, the structural adjustment programmes resulted in a diminished share of the public external debt. For the countries we considered it decreased from 80% in 1985-89 to almost 70% in 2000-2004. It is interesting to analyze empirically the type of direct effect this reduction induced.

2.2.2 Indirect effect of the external debt management

We showed that a government that repays a fraction $\gamma < \underline{\gamma}$ of its debt stock at each period will put the economy on the path of an explosive external debt stock relative to output. As showed in the section 1.4 the explosive dynamic of the ratio $\frac{D}{y}$ cannot be stopped if the minimum size of the formal sector has not been reached. However, this minimal size itself is increasing with the ratio $\frac{D}{y}$. This situation is illustrated in the following figure (b) showing an increase of the minimum size of the formal sector from $\underline{\theta}_t$ to $\underline{\theta}_{t+1}$. This may

generate a sufficiently high stock to force the government to raise the tax rate in order to relax this constraint. This corresponds to the downward movement from $\underline{\theta}_{t+1}$ à $\underline{\theta}_{t+2}$. However, raising the tax has a negative (indirect) effect on growth as illustrated in the figure 5. This is due to the reduction of the formal sector size.

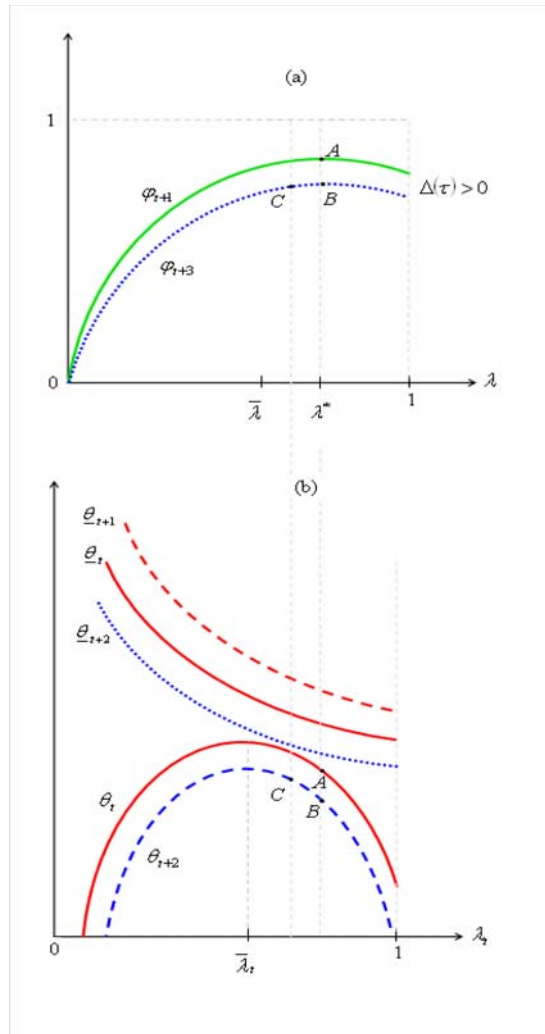


Figure 5

Note that the government may accompany the tax rate increase with a reduction of its external debt share in order to sterilize partially the negative effect on the size of the formal sector. Indeed, as figure (5-b) shows reducing this share to λ^C moves the size of the formal sector from B to C which is closer to the new frontier defined by $\underline{\theta}_{t+2}$. Bear in mind that reducing the distance to the frontier lowers the explosion dynamic of the external

debt stock. In this case, Figure (5-a) shows that we have a combination of a negative direct effect and a negative indirect effect which sets the final production efficiency at φ^C . However, note that the accompanied direct effect may be positive if the initial share is superior to λ_t^* .

3 The empirical analysis

In this paper we use recent advances made in the econometric of stochastic frontier methods to examine the effect of external debt on economic efficiency. Our econometric methodology rests upon the recent development techniques proposed by Greene (2005).

In many estimations of the stochastic frontier, the heterogeneity is confused with technical efficiency. Any time-invariant unobserved heterogeneity is pushed into the inefficiency component. However, as confirmed by Griliches (1957), unobserved heterogeneity if not accounted for, may cause biased estimates. Orea and Kumbhakar (2004) confirm that unobserved differences in technologies may be inappropriately labeled as inefficiency if variation in technology is not taken into account.

Many techniques are offered in the literature for differentiating heterogeneity from inefficiency. Huang (2004) proposes a stochastic frontier model with random coefficients to distinguish technical efficiency from technological differences across individuals. Kumbhakar (1991) uses a panel data model to estimate production function and technical efficiency. He distinguishes technical inefficiency from individual and time specific effects by allowing the mean of the technical inefficiency to be a function of exogenous variable. Both random and fixed effects are considered in the estimation. Stochastic frontiers with fixed effect (Heshmati et al. 1995, Greene 2005) and random coefficient models (Tsionas 2002, Huang 2004; Alvarez, Arias and Greene 2004) and Latent Class models (Orea and Kumbhakar 2004).

In line with Greene (2005), we estimate Random Coefficient models to deal with

unobserved heterogeneity. The stochastic frontier function can be specified as

$$\ln(y_{it}) = \beta_0 + w_i + \beta \ln(k_{it}) + \gamma \ln(h_{it}) + \mu \ln(l_{it}) + v_{it} + u_{it} \quad (17)$$

where w is a time-invariant, individual-specific random term assumed to capture specific heterogeneity. This model is fit by maximum simulated likelihood method.

The subscripts i and t denote individual countries and years, respectively; y represents output; k is physical capital, l is employment and h human capital.

3.1 Data definitions

Our empirical analysis consists of the first step of an estimate of a stochastic frontier model. The efficiency series will be used in the second stage to test the nonlinear effect of the share of public debt. In order to confirm our theoretical prediction, we proceed to an additional regression to estimate the relationship between the share of public debt and informal sector size. The analysis uses data from a balanced panel of 27 developing countries observed over the period 1970-2005. The panel data include countries with different levels of indebtedness and growth rates. The choice of the panel is justified by the fact that we need a heterogeneous panel to test the nonlinear effect. The output variable is defined as the GDP at constant price. We follow Bosworth and Collins (2003) and we define the human capital indicators as $H_{it} = h_{it}L_{it}$ where $h_{it} = [1.07]^{s_{it}}$, s is the average number of schooling years (Barro-Lee 2000). We use the perpetual inventory method to calculate the stock of physical stock series; the depreciation rates (di) are assumed to be 4 per cent.

We define informal activity as market-based illegal production of goods and services that escapes detection in the official estimates of GDP. It includes unreported income from the production of legal goods and services either from monetary or barter transac-

tions. The physical input (electricity consumption) method is used (Johnson et al. 1999) to measure the size of the informal sector. The input method is developed under the assumption that electricity consumption is the single best physical indicator of economic activity. If this is the case, the growth rate in total electricity consumption is a proxy for the growth in overall economic activity, including both informal and formal sectors. With this method, the electricity consumption to GDP for a base period is calculated and then extrapolated to the present. Assuming that the short-run electricity consumption/GDP elasticity is close to one and assuming a relative constant ratio of electricity consumption to GDP, it is possible to calculate the expected (or overall) GDP for each year following the base period. As officially measured GDP by definition captures only the official part of the economy, the difference between an overall (or expected) and officially measured GDP gives an estimate of the size of the shadow economy. Data for GDP, investment, labour, debt, and electricity consumption are extracted from the World Bank Data base. The initial stock of capital is from Nehru and Dhareshwar (1993).

Table1. Descriptive statistics of the data

Variables	Mean	Minimum	Maximum	StdDev.
GDP(billion)	34591.56	0.03	1750657	192478
Capital(billion)	190441.5	0.47	14391562	1421144
Labour (million)	25.21	0.46	230.60	60.99
Human Capital	4.07	0.32	8.74	1.79
Public Debt Share	0.88	0.25	1	0.14
Debt to GDP	52.65	6.15	612.04	42.51
Informal sector share	30.89	7.52	57.80	15.59

Source : Authors' calculations .

All variables are in real terms

As shown by the descriptive statistics, both output and inputs display a significant amount of variation. The standard deviation indicates a high degree of heterogeneity

among production decision of countries. The findings justify the use of heterogeneous panel data techniques.

3.2 Econometric results

The estimation of the frontier model is presented in Table 2. The Limdep 8 package was used to estimate the frontier model. The t-values show that all coefficients are significant at the 1 per cent level. All the elasticities are positive as expected and are all significantly different from zero. The results indicate that output is most responsive to labor, with an elasticity of 0.61. The elasticity of output with respect to capital is several times less than that of labour, suggesting that production is labour intensive. The coefficient associated to human capital is positive but relatively low, suggesting a weak contribution to output.

Table2. Estimated stochastic frontier model

Variables	Coefficients	t-stat
Labor	0.61***	51.46
Physical Capital	0.11***	40.92
Human Capital	0.01***	5.90
Constant	0.06***	3.30
Efficiency(average)	0.789	
σ	0.20***	21.56
σ_u	0.15	
σ_v	0.13	
λ	1.12***	5.52
Likelihood	400.5	

*** Significant at 1 per cent level.

The efficiency is based on the average efficiency for each country over the simple period.

In addition to the parameter estimates, technical efficiency was also estimated for each country⁷. The estimated efficiency measures of all countries range between a minimum level of 0.396 to a maximum level of 0.977 with an average level of 0.789. The next

⁷Efficiency mesures over time for each country are presented in the appendix.

step will be to test the non-linear effect of public external debt on economic efficiency. We introduce an indicator of financial development (*FinDev*) as well as an indicator of institutional quality (*InvestProfil*) as additional determinants of efficiency. We use the ratio of total private credit to *GDP* as a measure of the quality of the financial system. The quality of the institutional environment is measured by an indicator of the investments profile and an increase implies better conditions to invest⁸. The estimated model is :

$$\begin{aligned}
 Efficiency_{it} = & - \underset{(3.06)}{3.09} FinDev_{it} + \underset{(0.2)}{0.42} Invest\ Profil_{it} \\
 & + \underset{(0.09)}{0.62} PublicDebt_{it} - \underset{(0.06)}{0.37} PublicDebt_{it}^2 + u_{it}
 \end{aligned}$$

As we can note, financial development has no significant effect on efficiency. An improvement in the investment environment enhances efficiency; coefficient associated is significantly different from zero at the 5 per cent level. We find a non linear effect of public debt on efficiency as coefficient of the linear term is positive and that of the quadratic term is negative and the two are significant at the one per cent level. This result is coherent with our intuition presented in the theoretical part about the plausible (configuration (b) in figure 3) relationship between the share of external public debt and economic efficiency. The threshold λ^* beyond which an increase of the share of external public debt affects negatively production efficiency is 84 per cent of the total external debt.

Finally, in order to confirm that the configuration (b) of figure 3 is the econometrically verified, we should prove that there is a non linear relationship between external public debt and the size of the informal sector. In addition, we need to verify that the turning threshold denoted by $\bar{\lambda}$ in the theoretical model is smaller than 84 per cent. Therefore,

⁸The data is extracted from the International Country Risk (ICR) Guide (2005). Standard errors in parentheses.

we test the relationship between public debt and informal sector size (*Informal*). We assume that corruption (*Corrup*)⁹ and unemployment (*Unemp*)¹⁰ encourage unofficial activities. Indeed, for countries suffering from high level of unemployment, informality or informal economic activity is a best alternative for combatting unemployment and poverty. Informal activities are also the best option to avoid the costs associated with bureaucracy and corruption. The size of the informal economy is expressed as a fraction of a country's gross domestic product (*GDP*). The estimated model is¹¹:

$$\begin{aligned}
 Informal_{it} = & \frac{1.68}{(0.303)} Corrup_{it} + \frac{0.23}{(0.111)} Unemp_{it} \\
 & - \frac{0.75}{(0.04)} PublicDebt_{it} + \frac{0.6}{(0.03)} PublicDebt_{it}^2 + v_{it}
 \end{aligned}$$

All coefficients are significant at the conventional significance level. Unemployment and corruption have a positive effect on the informal sector. The non linear effect is confirmed and the turning point is approximately at 62 per cent. In addition, as predicted by the model, the turning point associated with the effect of the share of external public debt on the formal sector size ($\bar{\lambda}_t$) is less than the turning point associated with the effect of the share of external public debt on production efficiency (λ_t^*). Indeed, even if the size of the formal sector diminishes, the remaining formal projects becomes more efficient (due to an increase in government expenditure) which tends to increase production efficiency. Hence, it may be optimal, from a production efficiency perspective, to have a smaller formal sector with highly efficient projects. As mentioned previously, there is a threshold below which any increase in the share of external public debt improves production efficiency (the positive direct effect). However, beyond this threshold, an increase in external

⁹This variable proxies the degree which government agents use their political power for private gain. Data are extracted from International Country Risk (ICR) Guide (2005).

¹⁰It is measured by the ratio of unemployment and data are extracted from the World Bank database.

¹¹Data available from 1980 to 2003 and standard errors in parentheses.

public debt negatively affects production efficiency (the negative indirect effect). This result means that for a relatively high share of public external debt, the size of the formal private sector begins to decrease. The negative indirect effect of external public debt is linked to the increasing level of taxation. Indeed, debt service increases when external public debt increases, and given the constant levels of public spending and taxation, the government needs to increase its share of external debt to repay the increasingly high interest. If such a financing option is not possible, the government raises taxes (investors can anticipate the increase in taxation) which increases the size of the informal sector and reduces production efficiency.

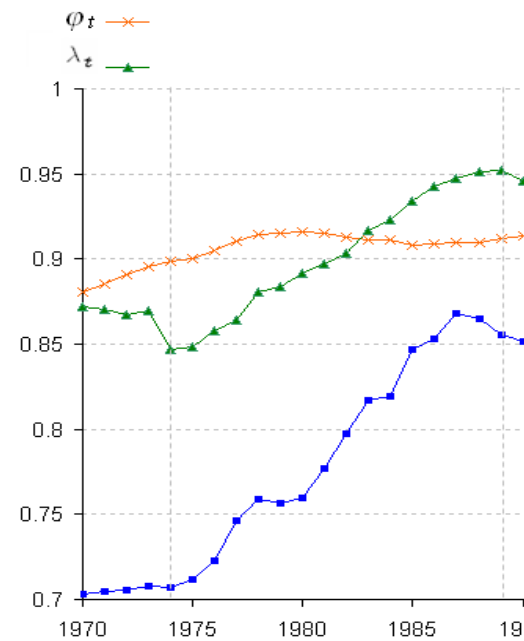


Figure 6

In Figure 6 , we can note that during the period 1974-89, the share of external debt was over the 84 per cent threshold. We can therefore confirm that the indirect negative effect of external public debt on production efficiency, had dominated during this period. The increasing share of the debt has been accompanied with an increasing stock of public debt from about 22% of GDP in 1974 to 61% of GDP in 1989. This could have led to a negative indirect effect if government had increased their taxation in addition to increasing

their shares of foreign debt. However, production efficiency for the same period rose from 0.90 in 1974 to about 0.92 in 1989. During the 1990-2005 period, the share of public external debt has declined from about 95% in 1990 to about 84% in 2005. This decline of the external debt has been associated with an increase in efficiency, as the positive effect dominates the negative effect.

4 Conclusion

Production efficiency is the second component of total factor productivity, in addition to technology. Production efficiency improves when the economy approaches the level of its potential production frontier through better use of production factors. Given the importance of external public debt in developing countries, we tried to answer the following question: Does the public external debt constitute a source of production inefficiency in developing countries?

We proposed an endogenous growth model with nested generations taking into account the informal sector, an important essential characteristic of developing countries. We have shown that the external public debt can impact on production efficiency through a direct effect (positive or negative) and a negative indirect effect. The direct effect is related to the fact that increasing the share of external public debt improves production efficiency through a positive externality effect. The crowding-out effect associated with an increase in the share of the external public debt reduces the size formal sector in favour of an informal sector. This direct effect of external public debt on efficiency becomes negative beyond an optimal level. On the other hand, accumulated stock of public external debt pushes the government to increase taxes, in order to repay debt servicing, causing a reduction in the size of the formal sector. As informal activities are considered less productive than formal ones, an increase in external public debt reduces production

efficiency.

Using a stochastic frontier production model to test the effect of external debt on the production efficiency of a sample of 27 developing countries between 1970 and 2005, we confirm the non-linear effect of external public debt on the production efficiency with a turning point of about 84 per cent. Econometric results confirm also a non-linear effect of the external public debt on the formal sector.

We show that the share of public external debt was sub-optimal throughout the studied period and particularly in the late 1980s. Reducing the share of public external debt from the 1990s has contributed to the improvement of production efficiency.

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5 Appendix

Proof of proposition 2

From (15) the maximal production \bar{y}_{t+1} is obtained for λ_t^* maximizing $f(\lambda) = \lambda h(\theta(\lambda), \lambda)$ which is not necessarily $\bar{\lambda}$. The production efficiency is $\varphi_{t+1} = \frac{f(\lambda_t)}{f(\lambda_t^*)}$ and we have $\frac{d^2\varphi_{t+1}}{d\lambda_t^2} = \frac{1}{f(\lambda_t^*)} \frac{d^2f(\lambda_t)}{d\lambda_t^2}$. We have

$$\frac{df}{d\lambda} = h + \lambda \frac{\partial h}{\partial \theta} \frac{\partial \theta}{\partial \lambda} + \lambda \frac{\partial h}{\partial \lambda}$$

$$\begin{aligned} \frac{d^2f}{d\lambda^2} &= \left(\frac{\partial h}{\partial \theta} \frac{\partial \theta}{\partial \lambda} + \frac{\partial h}{\partial \lambda} \right) + \left(\frac{\partial h}{\partial \theta} \frac{\partial \theta}{\partial \lambda} + \lambda \frac{\partial^2 h}{\partial \theta^2} \left(\frac{\partial \theta}{\partial \lambda} \right)^2 + \lambda \frac{\partial^2 h}{\partial \theta \partial \lambda} \left(\frac{\partial \theta}{\partial \lambda} \right) + \lambda \frac{\partial h}{\partial \theta} \frac{\partial^2 \theta}{\partial \lambda^2} \right) \\ &\quad + \left(\frac{\partial h}{\partial \lambda} + \lambda \frac{\partial^2 h}{\partial \lambda \partial \theta} \left(\frac{\partial \theta}{\partial \lambda} \right) \right) \\ &= 2 \left(\frac{\partial h}{\partial \lambda} + \left(\frac{\partial h}{\partial \theta} + \lambda \frac{\partial^2 h}{\partial \theta \partial \lambda} \right) \left(\frac{\partial \theta}{\partial \lambda} \right) \right) + \lambda \frac{\partial^2 h}{\partial \theta^2} \left(\frac{\partial \theta}{\partial \lambda} \right)^2 + \lambda \frac{\partial h}{\partial \theta} \frac{\partial^2 \theta}{\partial \lambda^2} \\ &= 2 \left[-\frac{\theta d}{w} + \left(\frac{\partial h}{\partial \theta} - \frac{d\lambda}{w} \right) \left(\frac{\partial \theta}{\partial \lambda} \right) \right] - \lambda \left(\frac{\partial \theta}{\partial \lambda} \right)^2 + \lambda \frac{\partial h}{\partial \theta} \frac{\partial^2 \theta}{\partial \lambda^2} \end{aligned}$$

Since,

$$\frac{\partial^2 \theta}{\partial \lambda^2} = -\frac{2}{\lambda} \frac{\partial \theta}{\partial \lambda} - \frac{2(1-\tau)d}{\lambda w}$$

we obtain

$$\frac{d^2f}{d\lambda^2} = \frac{-2d}{w} \left[\theta + \lambda \frac{\partial \theta}{\partial \lambda} \right] - \frac{2(1-\tau)d}{w} \frac{\partial h}{\partial \theta} - \lambda \left(\frac{\partial \theta}{\partial \lambda} \right)^2 \quad (18)$$

In addition, we have (14) and (16):

$$\begin{aligned} \frac{\partial h}{\partial \theta} &= 1 - \theta + \frac{(1-\lambda)d}{w} \\ \theta + \lambda \frac{\partial \theta}{\partial \lambda} &= (1-\tau) \left[1 + \frac{d}{w} \left(1 - 2\lambda + \tilde{\lambda}^2 \right) \right] \end{aligned}$$

which substituted in (18) enables us to write

$$\frac{d^2f}{d\lambda^2} = \frac{-2d}{w}(1-\tau) \left[2 - \theta + \frac{d}{w} \left(2 - 3\lambda + \tilde{\lambda}^2 \right) \right] - \underbrace{\lambda \left(\frac{\partial\theta}{\partial\lambda} \right)^2}_{>0} \quad (19)$$

and we can easily verify that $2 - \theta + \frac{d}{w} \left(2 - 3\lambda + \tilde{\lambda}^2 \right) > 0$ under the assumption that $d < w(1 + \tau)$. Therefore, $\frac{d^2f}{d\lambda^2} < 0$ which implies that the function $\frac{df}{d\lambda}$ is strictly decreasing.

* Case $d \leq w$

We have

$$\begin{aligned} \left. \frac{df}{d\lambda} \right|_{\lambda=1} &= h(\theta(1), 1) + \left. \frac{\partial h}{\partial\theta} \right|_{\lambda=1} \left. \frac{\partial\theta}{\partial\lambda} \right|_{\lambda=1} + \left. \frac{\partial h}{\partial\lambda} \right|_{\lambda=1} \\ &= \theta(1) \left(1 - \frac{d}{w} \right) + \frac{1 - (\theta(1))^2}{2} + (\theta(1) - (\theta(1))^2) \left(\frac{d}{w} \left[\frac{r}{\alpha ad} - 1 \right] \right) \\ &= \theta(1) \left(1 + \left[\frac{r}{\alpha ad} - 2 \right] \frac{d}{w} \right) - \frac{1}{2} (\theta(1))^2 \left[1 + \frac{d}{w} \left[\frac{2r}{\alpha ad} - 2 \right] \right] + \frac{1}{2} \end{aligned} \quad (20)$$

since $\theta(1) = 1 - \tau$. Let's set

$$g(\tau) = (1 - \tau) \left(1 + \left[\frac{r}{\alpha ad} - 2 \right] \frac{d}{w} \right) - \frac{1}{2} (1 - \tau)^2 \left[1 + \frac{d}{w} \left[\frac{2r}{\alpha ad} - 2 \right] \right] + \frac{1}{2} \quad (21)$$

We have $g(1) = \frac{1}{2}$ and $g(0) = 1 - \frac{d}{w} \geq 0$. Varying τ in $[0, 1]$ it is easy to show that $g(\tau) = \left. \frac{df}{d\lambda} \right|_{\lambda=1} > 0$. Hence, $\frac{df}{d\lambda} > \left. \frac{df}{d\lambda} \right|_{\lambda=1} > 0$ and we obtain $f(\lambda) < f(1)$ for every λ . We conclude that the production efficiency is equal to 1 for $\lambda^* = 1 > \bar{\lambda}$.

** Case $w < d < w(1 + \tau)$

We show using (21) that it exists $\tau^* \in]0, 1[$ depending only on $\frac{r}{\alpha a}$, d and w verifying $g(\tau^*) = 0$ such that

$$\left. \frac{df}{d\lambda} \right)_{\lambda=1} \begin{cases} \geq 0 & \text{if } \tau \geq \tau^* \\ < 0 & \text{if } \tau < \tau^* \end{cases} \quad (22)$$

We have $\frac{df}{d\lambda} = h + \frac{\partial \theta}{\partial \lambda} \frac{\partial h}{\partial \theta} + \lambda \frac{\partial h}{\partial \lambda}$ and since $\frac{\partial \theta}{\partial \lambda} \Big|_{\lambda=\bar{\lambda}} \geq 0$ we obtain

$$\begin{aligned} \left. \frac{df}{d\lambda} \right)_{\lambda=\bar{\lambda}} &\geq h(\theta(\bar{\lambda}), \bar{\lambda}) + \bar{\lambda} \left. \frac{\partial h}{\partial \lambda} \right)_{\lambda=\bar{\lambda}} \\ &\geq \theta(\bar{\lambda}) \left(1 + (1 - \bar{\lambda}) \frac{d}{w} \right) + \frac{1 - (\theta(\bar{\lambda}))^2}{2} - \bar{\lambda} \frac{d}{w} \theta(\bar{\lambda}) \\ &\geq \theta(\bar{\lambda}) \left(1 + (1 - 2\bar{\lambda}) \frac{d}{w} \right) + \frac{1 - (\theta(\bar{\lambda}))^2}{2} \end{aligned}$$

At this step it is easy to show that a sufficient condition to have $\left. \frac{df}{d\lambda} \right)_{\lambda=\bar{\lambda}} > 0$ is $w > \frac{r}{\alpha a}$. In this case, and given (22) enables us to conclude that it exists $\lambda^* \in]\bar{\lambda}, 1]$ which maximises the value of f and therefore the efficiency.

Finally, let's show that the production efficiency is decreasing with the tax rate. From (16) we have $\frac{\partial h}{\partial \theta_t}(\theta_t, \lambda_t) > 0$ and from (13) we have $\frac{\partial \theta_t}{\partial \tau} < 0$. Hence, $\frac{\partial h}{\partial \tau} < 0$ which is all we need to obtain the result.

Efficiency by country

