# Tradable Permits Under Environmental and Cost-reducing R\&D 

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#### Abstract

This paper models the simultaneous investments in cost-reducing and environmental R\&D by asymmetric firms competing à la Cournot. Pollution rights are allocated by the regulator, and firms can trade pollution permits. Both R\&D competition and R\&D cooperation are considered; in the latter case, firms fully share information about technologies. In a 3 -stage game, firms first invest in $R \& D$, then trade permits, and then compete in output. The strategic interaction between different types of R\&D investments is analyzed. It is found that the permit price depends on total permits only, not on their initial allocations. The optimal allocation of pollution rights by the social planner is also considered; the allocation of permits between firms matters for social welfare in the presence of environmental R\&D under noncooperative R\&D, but is irrelevant under cooperative $R \& D$. Moreover, it is optimal to give firms less permits when spillovers are higher. In addition, grandfathering permits (proportion to prepermit output) is studied under R\&D noncooperation. Compared with social optiaml allocation, grandfathering allocates too many permits to the large firm and too many permits to the small firm. Furthermore, an R\&D budget constraint is introduced. When the constraint is binding, firms underinvest more in standard $R \& D$ than in environmental R\&D.


## 1 Introduction

Permit trading is one of the cost-effective regulatory instruments to reduce pollution. The global market for tradable permits started from 1997 with the Kyoto Protocol, and so far the most effective cap-and-trade (governments set emission targets, then firms can trade permits) trading system is the European Union Emissions Trading Systems. The Canadian government also proposes to apply emission trading as part of environmental regulation.

Baumol and Oate (1971) are the first to argue that charges on waste emissions can help to achieve a certain pollution reduction target with minimum cost. Reminiscent of Coase (1960), Montgomery (1972) argues that tradable permits can help to achieve cost minimization, irrelevant of initial permit allocation. However, cost effectiveness may not be realized when firms face market imperfections. Hahn (1984) argues that if the dominant firm acts as a monopolist (monoposonist) in the permit market, efficiency would be distorted if the permits are not allocated exactly as it needs. While Sartzetakis and McFetridge (1999) study price-taking behavior in the permit market with oligopoly in the product market, as well as the case where firms have market power in both markets, concluding that with positioning strategies (raising rival's cost), overall efficiency may be increased even though industry output decreases. Sartzetakis (1997) shows welfare improving with Cournot oligopolists engaging in competitive permit trading, compared with regualtion of command-and-control, and this welfare enhancement is independent of permit allocations.

So far, there has been a number of extensions of Hahn (1984). Some authors assume a perfectly competitive permit market with an imperfect product market, such as Sanin and Zanaj (2007), studying both the incentive of innovation and welfare. Some consider imperfections in both permit and output markets, such as Montero (2000a, 2000b), ranking the incentive of innovation under different policy tools (tax, permits, subsidy, etc). Others focus on environmental $\mathrm{R} \& \mathrm{D}$ only, which can reduce emissions but not production costs, such as Poyago-Theotoky (2007), ranking R\&D and social welfare levels under R\&D noncooperation and cartel. Petrakis and Poyago-Theotoky (2002) also analyze environmental regulation with
both standard (cost-reducing) and environmental $R \& D$, comparing welfare under either $R \& D$ cooperation or R\&D subsidization.

Environmental policy affects not only pollution levels, but also firms' incentives to invest in innovation aiming developing cleaner technologies. The paper focuses on this dimension, in addition to incorporating imperfections in the permit trading market. This paper is the first to study an asymmetric duopoly with permit trading, where firms invest in both standard and environmental $R \& D$ simultaneously. In a three-stage game, firms first invest in $R \& D$, then trade permits, and then compete in output. Both R\&D cooperation and noncooperation are considered. First of all, we find that the permit price depends on total permits only, not on initial allocations. Then, we compare R\&D levels between cooperation and noncooperation, and find that cooperation always increases standard $R \& D$ by the small firm; the effect of cooperation on the other R\&D investments depends on spillover and other parameters.

The social optimum is also considered. We derive second best results with the government controlling only initial permit allocations. Both fixed and non-fixed total permits are discussed.

Some other extensions are also studied. One is quantity-ratio based grandfathering permits. We find that grandfathering allocates too few permits to the large firm and too many permits to the small firm, which reduces social welfare compared with the second best. The other extension we consider is an R\&D budget constraint. For this extension, we assume a symmetric duopoly to obtain tractable results. When firms face a binding budget constraint, they underinvest in both types of $R \& D$, but more so for standard $R \& D$.

The paper is organized as follows. Section 2 introduces the basic framework. Sections 3 and 4 present the basic $\mathrm{R} \& D$ noncooperation and cooperation models, and provide some comparative statics results. Section 5 compares R\&D levels under cooperation and noncooperation. Section 6 studies the second best with fixed and non fixed total permits. Section 7 analyzes quantity-ratio based grandfathering and the R\&D budget constraint. Section 8 concludes.

## 2 The basic framework

There are two asymmetric firms, 1 and 2 , with marginal costs $A_{1}$ and $A_{2}$, producing a homogeneous good in quantities $q_{1}$ and $q_{2}$. The inverse market demand is $P=a-q_{1}-q_{2}$. Each firm invests in two types of R\&D: standard (cost-reducing) R\&D and environmental R\&D. Environmental R\&D reduces pollution for a given level of output. Firms also face an R\&D spillover $\beta \in[0,1]$, which allows firms to benefit from each other's technology without payment. Here, spillovers reduce not only production costs, but also emissions. For simplicity, we assume that a unique spillover rate applies to both firms and to both types of $\mathrm{R} \& \mathrm{D}$ investments.

With standard $\mathrm{R} \& \mathrm{D} x_{i}$, the production cost is reduced from $A_{i}$ to $A_{i}-x_{i}-\beta x_{j}$, where $x_{i}$ represents the $\mathrm{R} \& \mathrm{D}$ output of firm $i, i \neq j, i, j=1,2$. The cost of standard $\mathrm{R} \& \mathrm{D}$ is $\frac{\gamma}{2} x_{i}^{2}$.

Production causes pollution, and firms need permits to pollute. Initially, there is a one-to-one relationship between quantity $q_{i}$ and emission $M_{i}$, where $q_{i}=M_{i}$. With environmental $\mathrm{R} \& \mathrm{D} w_{i}$, the emission level is reduced to $f_{i}=q_{i}-w_{i}-\beta w_{j}$. The cost of environmental $\mathrm{R} \& \mathrm{D}$ is $\frac{\delta}{2} w_{i}^{2}$.

In order to regulate pollution, the regulator allocates some emission permits, namely $e_{1}$ and $e_{2}$, to each firm freely. However, the initial allocations of permits may not be exactly what firms need, thus there will in general be trading between them at unit price $\sigma$. Each firm determines how many permits it wants to buy or sell, taking the permit price as given. After trading, firm $i$ uses $f_{i}$ permits to produce, where $f_{1}+f_{2}=e_{1}+e_{2}$. In other words, firms are not allowed to bank permits. However, because of the duopolistic structure of the industry, each firm's decision to buy or sell permits will have a significant impact on the price of permits and the number of traded permits.

## 3 R\&D noncooperation model

Firms play a three-stage game: first of all, they invest in both types of $R \& D$; then, they trade permits; finally, they compete in output. We analyze the game by backward induction.

### 3.1 Third stage: quantity competition

The total cost of production and $\mathrm{R} \& \mathrm{D}$ investments for firm $i$ is:

$$
\begin{equation*}
C_{i}=\left(A_{i}-x_{i}-\beta x_{j}\right) q_{i}+\frac{\gamma}{2} x_{i}^{2}+\frac{\delta}{2} w_{i}^{2}, \quad i \neq j, i, j=1,2 \tag{1}
\end{equation*}
$$

If the firm is a permit buyer (seller), it pays (receives) $\sigma\left(q_{i}-w_{i}-\beta w_{j}-e_{i}\right)$ in trading permits. Then, the profit function for firm $i$ is:

$$
\begin{equation*}
\pi_{i}=\left(a-q_{i}-q_{j}\right) q_{i}-C_{i}-\sigma\left(q_{i}-w_{i}-\beta w_{j}-e_{i}\right) \tag{2}
\end{equation*}
$$

Firm $i$ 's profit will be maximized by choosing the optimal quantity $\widehat{q}_{i}$. The first-order condition with respect to $q_{i}$ is:

$$
\begin{equation*}
\frac{\partial \pi_{i}}{\partial q_{i}}=\left[\left(a-q_{i}-q_{j}\right)-q_{j}\right]-\left(A_{i}-x_{i}-\beta x_{j}\right)-\sigma=0 \tag{3}
\end{equation*}
$$

where $\left[\left(a-q_{i}-q_{j}\right)-q_{j}\right]$ is marginal revenue and $\left(A_{i}-x_{i}-\beta x_{j}\right)$ is marginal cost of production. Rewriting (3), we get:

$$
\begin{equation*}
\left[\left(a-q_{i}-q_{j}\right)-q_{j}\right]-\left(A_{i}-x_{i}-\beta x_{j}\right)=\sigma \tag{4}
\end{equation*}
$$

The left hand side is marginal revenue minus marginal cost, which can be seen as marginal abatement cost through reducing quantity $q_{i}$; the right hand side is the unit price of permits. This implies that firms set marginal abatement cost equal to the unit permit price when maximizing profits: the permit price is just the forgone net profit (Mansur, 2007). However, Sartzetakis and McFetridge (1999:49) point out that "Equalization of marginal abatement
cost across firms yields the efficient distribution of abatement effort, but due to the oligopolistic product market structure, it cannot achieve the efficient production allocation.... Trading of permits does not necessarily yield the first-best allocation of resources when product markets are imperfectly competitive."

By solving the two first-order conditions with respect to $q_{i}$ and $q_{j}$, we get $\widehat{q}_{i}$ :

$$
\begin{equation*}
\widehat{q}_{i}=\frac{1}{3}\left[a-\sigma-2 A_{i}+A_{j}+(2-\beta) x_{i}-(1-2 \beta) x_{j}\right] \tag{5}
\end{equation*}
$$

### 3.2 Second stage: permit trading ${ }^{1}$

Firms are price takers in the permit market, and the equilibrium price is such that the demand for permits equals the the supply. Moreover, firms are not allowed to bank permits. Rewriting the equation $f_{1}+f_{2}=e_{1}+e_{2}$, we have $f_{1}-e_{1}=-\left(f_{2}-e_{2}\right)$. With $f_{i}=q_{i}-w_{i}-\beta w_{j}$, we get:

$$
\begin{equation*}
q_{1}-w_{1}-\beta w_{2}-e_{1}=-\left(q_{2}-w_{2}-\beta w_{1}-e_{2}\right) \tag{6}
\end{equation*}
$$

Then, we can get the equilibrium permit price by solving for $\sigma$ :

$$
\begin{equation*}
\widehat{\sigma}=\frac{1}{2}\left[2 a-A_{1}-A_{2}-3\left(e_{1}+e_{2}\right)-3(1+\beta)\left(w_{1}+w_{2}\right)+(1+\beta)\left(x_{1}+x_{2}\right)\right] \tag{7}
\end{equation*}
$$

From equation (7), it is clear that the effect of environmental $R \& D$ on the permit price is negative. Environmental R\&D reduces the demand for permits by a permit buyer, and increases the supply of permits by a seller. This is consistent with Montero (2002a), who uses a permits-Cournot game. Substituting $\widehat{\sigma}$ into (5), we get:

$$
\begin{equation*}
\widehat{q}_{i}=\frac{1}{2}\left[-A_{i}+A_{j}+e_{i}+e_{j}+(1+\beta)\left(w_{i}+w_{j}\right)+(1-\beta)\left(x_{i}-x_{j}\right)\right] \tag{8}
\end{equation*}
$$

The profit function $\pi_{i}=\pi_{i}\left(x_{1}, x_{2}, w_{1}, w_{2}, \beta, A_{1}, A_{2}, e_{1}, e_{2}, \gamma, \delta\right)$ is obtained by substituting $\widehat{q}_{i}$ into (2). See Appendix 1 for the full expressions of both profit functions.

[^0]We first analyze the strategic interactions between $R \& D$ investments.

Proposition 1 The standard $R \mathcal{G} D$ levels are strategic substitutes, and so are the environmental $R \mathcal{G} D$ levels. The own standard and environmental $R \mathcal{G} D$ levels are strategic complements, and so are the cross standard and environmental $R \& D$ levels.

Proof. From the two profit functions (A1) and (A2) in Appendix A1, we get:

$$
\begin{array}{rlrl}
\frac{\partial^{2} \pi_{i}}{\partial x_{i} \partial x_{j}} & =-\frac{1}{2}(-1+\beta)^{2} \leq 0 & \frac{\partial^{2} \pi_{i}}{\partial w_{i} \partial w_{j}}=-(1+\beta)^{2}<0 \\
\frac{\partial^{2} \pi_{i}}{\partial x_{i} \partial w_{i}}=\frac{1}{2}\left(2+\beta-\beta^{2}\right)>0 & \frac{\partial^{2} \pi_{i}}{\partial x_{i} \partial w_{j}}=\frac{1}{2}(1+\beta)>0
\end{array}
$$

The first inequality holds strictly when $\beta<1$.
When firm $i$ increases its standard $\mathrm{R} \& \mathrm{D}$, firm $j$ responds by investing less in its standard R\&D. Firm $i$ needs more permits to produce, which increases (decreases) its demand (supply) for permits, thus firm $j$ can hold less permits in hands, which leads to less production by firm $j$. This induces firm $j$ to invest even less in standard $\mathrm{R} \& \mathrm{D}$, the strategic substitutability between $x_{i}$ and $x_{j}$. Similarly, when firm $i$ invests more in environmental R\&D, firm $j$ responds by investing less in its environmental R\&D. Firm $i$ needs less permits, which decreases (increases) its demand (supply) for permits, then firm $j$ can hold more permits in hand. This induces firm $j$ to invest less in environmental $R \& D$, whence the strategic substitutability between $w_{i}$ and $w_{j}$.

Furthermore, when firm $i$ does more standard $\mathrm{R} \& \mathrm{D}$, it has to invest more in its own environmental $\mathrm{R} \& \mathrm{D}$ to produce more, whence the strategic complementarity between $x_{i}$ and $w_{i}$. Meanwhile, the need for more pollution rights increases (decreases) its demand (supply) for permits, thus firm $j$ has to do more environmental $\mathrm{R} \& \mathrm{D}$, whence the strategic complementarity between $x_{i}$ and $w_{j}$.

These results are different from the model by d'Aspremont and Jacquemin (1988), where there is only standard $\mathrm{R} \& \mathrm{D}$. In that model, the strategic interactions depend on $\beta: \mathrm{R} \& \mathrm{D}$
levels are strategic substitutes when $\beta<\frac{1}{2}$, and strategic complements when $\beta>\frac{1}{2}$. ${ }^{2}$ This is because "when firms independently decide the $\mathrm{R} \& \mathrm{D}$ levels, their decision will inflict a positive (negative) externality upon the other firm when $\beta>\frac{1}{2}\left(\beta<\frac{1}{2}\right)^{\prime \prime}$ (Steurs, 1994:13).

### 3.3 First stage: R\&D competition

Under $\mathrm{R} \& \mathrm{D}$ competition, in the first stage, both firms choose their own standard ( $\widehat{x}_{1}, \widehat{x}_{2}$ ) and environmental $\mathrm{R} \& \mathrm{D}\left(\widehat{w}_{1}, \widehat{w}_{2}\right)$ to maximize own profits:

$$
\begin{equation*}
\left(\widehat{x}_{i}, \widehat{w}_{i}\right)=\arg \max _{x_{i}, w_{i}} \pi_{i} \tag{9}
\end{equation*}
$$

Substituting $\left(\widehat{x}_{1}, \widehat{x}_{2}\right)$ and $\left(\widehat{w}_{1}, \widehat{w}_{2}\right)$ into (7), we get the permit price $\widehat{\sigma}$ :

$$
\begin{equation*}
\widehat{\sigma}=-\frac{\left(A_{1}+A_{2}-2 a \gamma\right)\left[(1+\beta)^{2}+2 \delta\right]+\delta[-3+(-2+\beta) \beta+6 \gamma]\left(e_{1}+e_{2}\right)}{2(1+\beta)[-3+7 \gamma+\beta(-2+\beta+\gamma)]+4 \gamma \delta} \tag{10}
\end{equation*}
$$

See Appendix A2 for details.

Proposition 2 The equilibrium permit price is independent from the initial permit allocations $\left(e_{1}, e_{2}\right)$ if the amount of total permits is fixed.

Proof. From equation (10) we see that $\widehat{\sigma}$ depends on $\left(e_{1}+e_{2}\right)$ as a whole if $\left(e_{1}+e_{2}\right)$ is fixed.

As shown in Figure 1, when the seller is given less permits (from $e_{0}$ to $e_{0}^{\prime}$ ), it will sell less, which drives down the supply (supply curve moves in from $S$ to $S^{\prime}$ ); meanwhile, the buyer gets more permits and will buy less, which drives down the demand also (demand curve moves in from $D$ to $D^{\prime}$ ). Then, with less demand and supply, the price remains constant. Similarly, if the seller is given more permits (from $e_{0}$ to $e_{0}^{\prime \prime}$ ), it will sell more, which drives up the supply (supply curve moves out from $S$ to $S^{\prime \prime}$ ); meanwhile, the buyer gets less permits

[^1]and has to buy more, which increases the demand also (demand curve moves out from $D$ to $\left.D^{\prime \prime}\right)$. Then, with higher demand and supply, the price remains constant. This result is the same as Sanin and Zanaj (2007), who also incorporate a symmetric Cournot duopoly in permit trading.


Figure 1: Supply and demand of permits

### 3.3.1 Comparative statics

We now present comparative statics. As the solutions are algebraically complicated, we focus on extreme spillover values: $\beta=0$ and $\beta=1$. See Appendix A3 for all the comparative statics results.

With $\gamma$ large enough, the higher the marginal costs of firm $i$, the lower the production $\left(\frac{\partial \widehat{q}_{i}}{\partial A_{i}}<0\right)$, then there is less need to invest in environmental $\mathrm{R} \& \mathrm{D}\left(\frac{\partial \widehat{w}_{i}}{\partial A_{i}}<0\right)$; meanwhile, the opponent will produce more $\left(\frac{\partial \widehat{q}_{j}}{\partial A_{i}}>0\right)$ and thus needs to do more environmental $\mathrm{R} \& \mathrm{D}$ $\left(\frac{\partial \widehat{w}_{j}}{\partial A_{i}}>0\right)$.

Paradoxically, reducing the output of a polluting industry will also result in a reduction in investment in environmental $\mathrm{R} \& \mathrm{D}$, reducing the net social benefit of reducing pollution. This is in addition to the negative effect of production reduction on cost-reducing R\&D.

Proposition 3 When the amount of total permits is not fixed, $\frac{\partial \widehat{\sigma}}{\partial A_{i}}<0, \frac{\partial \widehat{\sigma}}{\partial A_{j}}<0$ : the higher the marginal costs of firm $i$ or $j$, the lower the permit price. Also, $\frac{\partial \widehat{\sigma}}{\partial e_{i}}<0, \frac{\partial \widehat{\sigma}}{\partial e_{j}}<0$ : the more initial permits firm $i$ or $j$ has, the lower the permit price.

The increase in firm $i$ 's marginal costs leads to a decrease in its production and pollution, so there is less (more) demand (supply) for permits. When the amount of permits is not fixed, this drives the permit price down.

Also, without a fixed number of permits, the more permits firm $i$ gets, the more (less) it can sell (buy) to firm $j$, which drives up (down) the supply (demand) of permits and leads to a lower price.

Proposition $4 \frac{\partial \widehat{q}_{i}}{\partial e_{i}}>0, \frac{\partial \widehat{q}_{i}}{\partial e_{j}}>0$ : with a non-fixed amount of permits, when a firm gets more initial permits, both firms produce more.

Firms need pollution rights to produce. Thus, the more permits firm $i$ gets initially, the more it can produce. When firm $j$ gets more initial permits, it would need to buy (sell) less (more), which leads firm $i$ to have more permits in hand, and produce more.

Proposition $5 \frac{\partial \widehat{f_{i}}}{\partial A_{i}}<0, \frac{\partial \widehat{f}_{j}}{\partial A_{i}}>0$ : the higher the marginal costs of firm $i$, the less permits firm $i$ holds finally, and the more permits firm $j$ holds finally. Furthermore, $\frac{\partial \widehat{f}_{i}}{\partial e_{i}}>0, \frac{\partial \widehat{f}_{j}}{\partial e_{i}}>0$ : the more permits a firm gets initially, the more permits it, as well as the other firm will hold finally.

The higher firm $i$ 's marginal cost, the lower its own production, thus it needs less final permits to produce; meanwhile, firm $i$ 's higher marginal cost leads firm $j$ to produce more,
thus firm $j$ needs more final permits. When firm $i$ is given more permits initially, it can produce more; meanwhile, it can offer (demand) more (less) permits to (for) firm $j$, which will also hold more permits.

Proposition $6 \frac{\partial \widehat{x}_{i}}{\partial e_{i}}>0$, the more initial permits firm i gets, the more it invests in standard $R \xi D$.

The more initial permits a firm gets, the more it can produce for any given investment in environmental R\&D. This increases the value of cost reduction, which leads the firm to invest more in cost-reducing R\&D.

Proposition $7 \frac{\partial \widehat{w}_{i}}{\partial e_{i}}<0$, $\frac{\partial \widehat{w}_{j}}{\partial e_{i}}<0$ : with a non-fixed amount of permtis, the more permits firm $i$ gets, the less both firms invest in environmental $R \mathcal{B} D$.

When firm $i$ gets more permits, it can produce and pollute more with doing less environmental R\&D. Furthermore, now it can sell more or buy less permits from firm $j$. Without fixed amount of permits, this leads firm $j$ to hold more permits in hand and invest less in environmental R\&D as well.

Proposition 8 When $\beta=0, \frac{\partial \widehat{x}_{i}}{\partial A_{j}}>0$, the higher firm $j$ 's marginal costs, the higher firm $i$ 's standard $R \mathcal{E} D ; \frac{\partial \widehat{x}_{i}}{\partial e_{j}}>0$, the higher firm $j$ 's initial permits, the higher firm $i$ 's standard $R \xi D$. When $\beta=1, \frac{\partial \widehat{x_{i}}}{\partial A_{j}}<0$, the higher firm $j$ 's marginal costs, the lower firm $i$ 's standard $R \varepsilon D ; \frac{\partial \widehat{x}_{i}}{\partial e_{j}}<0$, the higher firm $j$ 's initial permits, the lower firm $i$ 's standard $R \mathcal{G} D$. However, $\frac{\partial \widehat{w}_{i}}{\partial A_{i}}<0$ and $\frac{\partial \widehat{w}_{i}}{\partial A_{j}}>0$ hold independent of $\beta$.

When $\beta=0$, firms do not benefit from each other's R\&D. Thus, when the marginal cost of firm $j$ increases (especially that $x_{j}$ declines with non-negative effect on firm $i$ because $\beta=0$ ), this leads to an increase in the market share of firm $i$, which induces it to increase its standard R\&D. When firm $j$ gets more permits, the output of firm $i$ increases, given that
it buys some of those additional permits (see Proposition 4). This leads firm $i$ to increase its standard $\mathrm{R} \& \mathrm{D}$, especially that with $\beta=0$, the net benefit of this increase is very high.

When $\beta=1$, the increase in the marginal cost of firm $j$ leads to a reduction in $x_{j}$. This increases the marginal cost of firm $i$, reducing the value of cost reduction for firm $i$, inducing it to reduce its standard R\&D. Even though firm $i$ now produces more, it is content with the high spillover which allows it to benefit from $x_{j}$.

Finally, when $\beta=1$, when firm $j$ gets more permits, the output of firm $i$ increases (through permit purchases). But again, the high spillover rate reduces the net benefit of any increase in $x_{i}$, inducing instead firm $i$ to reduce its standard $\mathrm{R} \& \mathrm{D}$. Here, firm $i$ is content with benefiting from the higher standard $\mathrm{R} \& \mathrm{D}$ of $j$ (remember that $\frac{\partial \widehat{x}_{j}}{\partial e_{j}}>0$ ), especially with the high spillover rate.

When firms are symmetric $\left(A_{i}=A_{j}=A\right)$, most of the comparative statics results above continue to hold. However, in that case, $\frac{\partial \widehat{x}_{i}}{\partial A}<0, \frac{\partial \widehat{w}_{i}}{\partial A}<0$, and $\frac{\partial \widehat{q}_{i}}{\partial A}<0$, irrespective of $\beta$. In this symmetric case, the own-cost effects always dominate the cross-cost effects. Furthermore, $\frac{\partial \widehat{f}_{i}}{\partial A}=0$.

For example, a unit tax on the polluting output - whether paid for by producers or consumers, will have the beneficial effect of reducing pollution; however, it will also have the unintended effect of reducing cost-reducing and environmental innovation. This may cause the optimal pollution tax to be lower.

## $4 \quad \mathrm{R} \& \mathrm{D}$ cooperation model

Everything is the same as above except that firms cooperate in both types of R\&D. Cooperation entails full information sharing $(\beta=1)$, as well as joint profit maximization with respect to $\mathrm{R} \& \mathrm{D}$. By backward induction, we get the same expressions of quantities and permit price as before. By substituting them into the individual profit functions, we get the total profit
function:

$$
\begin{equation*}
\pi_{c}=\pi_{1}\left(x_{1}, x_{2}, w_{1}, w_{2}, A_{1}, A_{2}, e_{1}, e_{2}, \gamma, \delta, \beta\right)+\pi_{2}\left(x_{1}, x_{2}, w_{1}, w_{2}, A_{1}, A_{2}, e_{1}, e_{2}, \gamma, \delta, \beta\right) \tag{11}
\end{equation*}
$$

The full expression is given in Appendix A4.
Leahy and Neary (2005) are the first to define cooperative substitutes and complements. The main difference between cooperative substitutes / complements and the standard definition of strategic substitutes / complements is that the former is "the effect of one firm's R\&D on the marginal contribution of another firm's R\&D to industry profits" while the latter "refers to the cross-effect of one firm's R\&D on the marginal profits of another firm" (Leahy and Neary 2005:383).

Proposition 9 In the model of d'Aspremont and Jacquemin (1988), with only standard $R \xi D$, the $R \xi D$ levels are cooperative substitutes when $\beta<\frac{1}{2}$ and cooperative complements when $\beta>\frac{1}{2}$.

Proof. See Appendix A5.
The intuition is the same as for a strategic interaction. When $\beta$ is low, when its opponent increases its R\&D level, the firm's benefit is limited, thus it reduces its $\mathrm{R} \& \mathrm{D}$. When $\beta$ is high, when its opponent increases it R\&D investment, the firm benefits a lot, which induces it to increase its $R \& D$.

Proposition 10 Under R $\mathcal{B} D$ cooperation, the standard $R \mathcal{G} D$ levels are cooperative substitutes, and so are environmental REBD levels; furthermore, both of them are cooperatively independent under full information sharing or perfect spillovers. The own standard and environmental R $\mathcal{B} D$ levels are always cooperative complements, and so are the cross standard and environmental $R \mathcal{G} D$ levels.

Proof. From equation (A18), we have:

$$
\begin{aligned}
\frac{\partial^{2} \pi_{c}}{\partial x_{i} \partial x_{j}} & =-(-1+\beta)^{2} \leq 0 & \frac{\partial^{2} \pi_{c}}{\partial w_{i} \partial w_{j}} & =-2(1+\beta)^{2}<0 \\
\frac{\partial^{2} \pi_{c}}{\partial x_{i} \partial w_{i}} & =\frac{1}{2}(1+\beta)^{2}>0 & \frac{\partial^{2} \pi_{c}}{\partial x_{i} \partial w_{j}} & =\frac{1}{2}(1+\beta)^{2}>0
\end{aligned}
$$

Cooperative interaction is different in our model compared with the basic model (see Proposition 8). When the opponent invests more in standard R\&D, the substitution effect between $\mathrm{R} \& \mathrm{D}$ expenditures dominates, and the firm reduces its standard $\mathrm{R} \& \mathrm{D}$. At the same time, it increases its environmental $\mathrm{R} \& \mathrm{D}$ to be able to produce more (due to lower marginal costs). With perfect information sharing, the total $\mathrm{R} \& \mathrm{D}$ level is fixed to maximize industry profits, and there is no cooperative strategic interaction between standard $R \& D$ investments.

Independently of $\beta$, environmental $\mathrm{R} \& \mathrm{D}$ levels are cooperative substitutes. With permit trading, the more firm $i$ invests in environmental $\mathrm{R} \& \mathrm{D}$, the less permits it needs, then consequently, firm $j$ will hold more permits and need to do less environmental $\mathrm{R} \& \mathrm{D}$.

The firm's own standard and environmental R\&D levels are cooperative complements. The more standard $\mathrm{R} \& \mathrm{D}$ a firm does, the more it produces and pollutes, and then the more it should invest in environmental R\&D.

The cross standard and environmental R\&D levels are also cooperative complements. When $\beta=0$, the more standard $\mathrm{R} \& \mathrm{D}$ firm $i$ does, the more permits it needs to produce, which leads firm $j$ to use less permits and do more environmental $R \& D$. When $\beta>0$, there is an additional effect: firm $j$ benefits from firm $i$ 's standard $\mathrm{R} \& \mathrm{D}$, produces more, which drives up firm $j$ 's environmental $\mathrm{R} \& \mathrm{D}$ level.

Comparing Propositions 1 and 9, we see that the results for cooperative substitutability / complementarity (Proposition 9) are (qualitatively) similar to the strategic interactions in Proposition 1 under noncooperation.

By solving the four first-order conditions with respect to $x_{1}, x_{2}, w_{1}$ and $w_{2}$, we obtain the equilibrium cooperative $\mathrm{R} \& \mathrm{D}$ levels $\left(\widetilde{x}_{1}, \widetilde{x}_{2}, \widetilde{w}_{1}, \widetilde{w}_{2}\right)$ (refer to Appendix A6 for details).

## 5 Comparison of cooperative and noncooperative R\&D

## 6 The optimal allocation of permits

When firms pursue profit maximization, they do not consider pollution damage. For the social planner, this negative externality must be incorporated when maximizing social welfare. We define the second best as a situation where the social planner controls initial permits allocations.

To incorporate that dimension, the game is extended to four stages. In the first stage, the social planner maximizes social welfare by choosing the allocations of permits. The other three stages are as before.

Social welfare ( $S W$ ) is total surplus ( $T S$ ) net of total damage ( $T D$ ), where total surplus is total benefit $(T B)$ net of total cost $(T C)$. Total damage $D\left(e_{1}+e_{2}\right)$ is a function of total permits, where $D^{\prime}\left(e_{1}+e_{2}\right)>0, D^{\prime \prime}\left(e_{1}+e_{2}\right)>0$ (i.e. damages increase with pollution at an increasing rate).

Total benefit is given by

$$
\begin{equation*}
T B=\int_{0}^{Q}(a-x) d x \tag{12}
\end{equation*}
$$

Total cost is composed of firms' production costs and R\&D costs. Furthermore, trading permits is only a transfer. Therefore, total cost is

$$
\begin{equation*}
T C=\sum_{j \neq i, i, j=1}^{2}\left\{\left(A_{i}-x_{i}-\beta x_{j}\right) q_{i}+\frac{\gamma}{2} x_{i}^{2}+\frac{\delta}{2} w_{i}^{2}\right\} \tag{13}
\end{equation*}
$$

Also, we define the total damage as

$$
\begin{equation*}
T D=D\left(e_{1}+e_{2}\right)=\theta\left(e_{1}+e_{2}\right)^{2} \tag{14}
\end{equation*}
$$

Two scenarios are considered. In the first scenario, the total amount of permits is fixed
at $L ;^{3}$ in this case, the social planner's choice reduces to allocating $L$ between the two firms. In the second scenario, the social planner is not constrained by a permit ceiling.

### 6.1 Fixed amount of permits

### 6.1.1 R\&D noncooperation

The social planner maximizes social welfare by choosing the initial permit allocations $e_{1}$ and $e_{2}$ :

$$
\begin{align*}
& \max _{e_{1}, e_{2}} S W=T S-T D  \tag{15}\\
& \text { s.t. } e_{1}+e_{2}=L
\end{align*}
$$

Under R\&D noncooperation, we plug ( $\widehat{x}_{1}, \widehat{x}_{2}, \widehat{w}_{1}, \widehat{w}_{2}$ ) into the social welfare function and substitute $e_{2}=L-e_{1}$. We can now maximize social welfare with respect to $e_{1}$ and obtain the second best results $\left(x_{1 s}^{n}, x_{2 s}^{n}, w_{1 s}^{n}, w_{2 s}^{n}\right) .{ }^{4}$ Due to the algebraically complicated solutions, we use the same numerical simulations as before; in addition, we set $\theta=1$ and $L=100$. The optimal permit allocations are:

$$
\begin{align*}
& e_{1 s}^{n}=\frac{50\left(4983+5137 \beta+282 \beta^{2}-118 \beta^{3}-5 \beta^{4}+\beta^{5}\right)}{(1+\beta)\left(3459+358 \beta-112 \beta^{2}-6 \beta^{3}+\beta^{4}\right)}  \tag{16}\\
& e_{2 s}^{n}=\frac{50\left(1935+2497 \beta+210 \beta^{2}-118 \beta^{3}-5 \beta^{4}+\beta^{5}\right)}{(1+\beta)\left(3459+358 \beta-112 \beta^{2}-6 \beta^{3}+\beta^{4}\right)} \tag{17}
\end{align*}
$$

Obviously, the optimal allocations depend on spillovers. As $\beta$ increases, the social value of R\&D increases, and the social planner would prefer to give both firms less permits to induce them to increase their environmental R\&D. But because of the fixed permit constraint, social planner can't reduce permit allocations to both firms. In addition, fixed total permits has no effect on total environmental R\&D levels because it leads to $\frac{\partial \widehat{w}_{1}}{\partial e_{1}}=\frac{\partial \widehat{w}_{2}}{\partial e_{2}}$ and $\frac{\partial \widehat{w}_{1}}{\partial e_{2}}=\frac{\partial \widehat{w}_{2}}{\partial e_{1}}$. Furthermore, we have $\frac{\partial e_{1 s}^{n}}{\partial \beta}<0, \frac{\partial e_{2 s}^{n}}{\partial \beta}>0$ : as spillover increases, the social planner allocates

[^2]less permits to the large firm and more permits to the small firm. From Table 1 in Appendix A8.1.2, even though the large firm produces more when spillover is low due to large benefit from cost-reducing R\&D, with increasing spillover rate, it reduces production. Thus, $\frac{\partial x_{i}}{\partial e_{i}}>0$ leads to a lower market share and profit of the large firm and a higher production (it always benefit from large firm's cost-reducing $R \& D$ ), market share and profit of the small firm, because the large firm has a higher marginal cost while the small firm has a lower marginal cost. ${ }^{56}$ Thus, even though there is loss in large firm's profit, the gain of the consumer surplus ${ }^{7}$ and the small firm's profit are so significant that these effects dominate the loss and induce a higher social welfare. Thus, with higher spillover, the social planner is willing to allocate more permits to the small firm. Note that due to the fixed amount of permits, the effect of permit allocation on environmental R\&D cancelled out each other.

### 6.1.2 R\&D cooperation

Under R\&D cooperation, we plug $\left(\widetilde{x}_{1}, \widetilde{x}_{2}, \widetilde{w}_{1}, \widetilde{w}_{2}\right)$ into the social welfare function and substitute $e_{2}=L-e_{1}$, and maximize social welfare function with respect to $e_{1}$ to obtain the second best results $\left(x_{1 s}^{c}, x_{2 s}^{c}, w_{1 s}^{c}, w_{2 s}^{c}\right)$. However, equation (A40) in Appendix A8.1.3 shows that social welfare dependents on $L$ only, and not on the distribution of permits between firms. This is because with full information sharing, total R\&D is fixed, and, social welfare is maximized for any initial distribution of permits. Also, from equations (A26) and (A27) in Appendix A6, we can see that the equilibrium cooperative R\&D levels do not depend on either $e_{1}$ or $e_{2}$ but only on $L$.

[^3]
### 6.2 Non-fixed amount of permits

### 6.2.1 R\&D noncooperation

Now, the social planner maximizes social welfare by choosing the initial permit allocations $e_{1}$ and $e_{2}$ without any constraint:

$$
\begin{equation*}
\max _{e_{1}, e_{2}} \quad S W=T S-T D \tag{18}
\end{equation*}
$$

We plug ( $\widehat{x}_{1}, \widehat{x}_{2}, \widehat{w}_{1}, \widehat{w}_{2}$ ) into the social welfare function. By maximizing social welfare with respect to $e_{1}$ and $e_{2}$, we get the optimal permit allocations:

$$
\begin{align*}
& e_{1 s}^{n n}=\left[-200\left(-3.4 * 10^{9}-4.4 * 10^{9} \beta-10.3 * 10^{9} \beta^{2}-2 * 10^{8} \beta^{3}-10^{7} \beta^{4}+8.6 * 10^{5} \beta^{5}-1.7 * 10^{5} \beta^{6}\right.\right. \\
& \left.\left.-2.9 * 10^{4} \beta^{7}+762 \beta^{8}\right)\right] /\left(1.2 * 10^{10}+1.8 * 10^{10} \beta+7.6 * 10^{9} \beta^{2}+1.6 * 10^{9} \beta^{3}+1.3 * 10^{8} \beta^{4}-2 * 10^{7} \beta^{5}\right. \\
& \left.-5.2 * 10^{6} \beta^{6}-3.4 * 10^{5} \beta^{7}+1.4 * 10^{4} \beta^{8}+3291 \beta^{9}+146 \beta^{10}+2 \beta^{11}\right)  \tag{19}\\
& e_{2 s}^{n n}=\left[-200\left(-8.8 * 10^{8}-1.1 * 10^{9} \beta+1.8 * 10^{7} \beta^{2}+1.2 * 10^{8} \beta^{3}+3.6 * 10^{7} \beta^{4}+4.8 * 10^{6} \beta^{5}+9885 \beta^{6}\right.\right. \\
& \left.\left.-2.5 * 10^{4} \beta^{7}+798 \beta^{8}\right)\right] /\left(1.2 * 10^{10}+1.8 * 10^{10} \beta+7.6 * 10^{9} \beta^{2}+1.6 * 10^{9} \beta^{3}+1.3 * 10^{8} \beta^{4}-2 * 10^{7} \beta^{5}\right. \\
& \left.-5.2 * 10^{6} \beta^{6}-3.4 * 10^{5} \beta^{7}+1.4 * 10^{4} \beta^{8}+3291 \beta^{9}+146 \beta^{10}+2 \beta^{11}\right) \tag{20}
\end{align*}
$$

Obviously, the optimal allocations depend on the spillover rate. Furthermore, in contrast to Section 6.1.1, where the permit constraint plays an important role, here we have $\frac{\partial e_{1}}{\partial \beta}<0$, $\frac{\partial e_{2}}{\partial \beta}<0$. As $\beta$ increases, the value of environmental $R \& D$ to society increases. We want firms to do more environmental $R \& D$, hence we give them less permits. And because here we are not constrained by a fixed number of total permits, we do not need to choose which firm gets more permits.


Figure 2: Socially optimal permit allocations with fixed number of total permits

Figures 2 and 3 illustrate optimal permit allocations, by using equations (16), (17), (19) and (20). With both fixed and non-fixed amount of permits, the large firm is given more permits than the small firm. This makes sense as the large firm produces more and pollutes more.

With these allocations, we get the optimal R\&D levels $\left(x_{1 s}^{n n}, x_{2 s}^{n n}, w_{1 s}^{n n}, w_{2 s}^{n n}\right)$. See Appendix A8.2.1.

### 6.2.2 R\&D cooperation

Similarly, by substituting ( $\left.\widetilde{x}_{1}, \widetilde{x}_{2}, \widetilde{w}_{1}, \widetilde{w}_{2}\right)$ into the social welfare function, we can maximize social welfare by choosing $e_{1}$ and $e_{2}$. We get:

$$
\begin{equation*}
e_{1 s}^{c n}+e_{2 s}^{c n}=\frac{\gamma \delta[-16+\gamma(24+\delta)]\left(2 a-A_{1}-A_{2}\right)}{2\left[512(-1+\gamma)^{2} \theta+\gamma \delta^{2}(-2+\gamma+2 \gamma \theta)+32(-1+\gamma) \delta(-1+\gamma+2 \gamma \theta)\right]} \tag{21}
\end{equation*}
$$

Under R\&D cooperation, the allocation of permits between firms is irrelevant, only the total amount of permits, $e_{1 s}^{c n}+e_{2 s}^{c n}$, matters, and it is chosen to maximize social welfare. In Section 6.1.2, the fixed amount of permits is exogenous; here, it is endogenous, chosen by the
social planner. See Appendix A8.2.2 for $\left(x_{1 s}^{c n}, x_{2 s}^{c n}, w_{1 s}^{c n}, w_{2 s}^{c n}\right)$.
We consider two extensions of the model. First, we consider permit allocations based on grandfathering, and compare them with the socially optimal allocations derived in Section 6. Second, we consider how a budget constraint distorts firms' investments in standard and environmental R\&D.

## 7 Grandfathering permits

One common way of allocating permits is in proportion to (pre-permit) output. ${ }^{8}$ As explained above, permit allocations play no role under $R \& D$ cooperation, thus here we only consider noncooperation with a fixed total amount of permits.

The equilibrium quantities $\left(q_{1}, q_{2}\right)$ are obtained without permit trading. See Appendix A9 for details. Then, we get the grandfathering allocations:

$$
\begin{align*}
& \frac{e_{1}^{g}}{L}=\frac{q_{1}}{q_{1}+q_{2}}=\frac{-2042-75 \beta+23 \beta^{2}}{52\left(-58-3 \beta+\beta^{2}\right)}  \tag{22}\\
& \frac{e_{2}^{g}}{L}=\frac{q_{2}}{q_{1}+q_{2}}=\frac{-974-81 \beta+29 \beta^{2}}{52\left(-58-3 \beta+\beta^{2}\right)} \tag{23}
\end{align*}
$$

Rewriting these two equations, we get:

$$
\begin{align*}
& e_{1}^{g}=\frac{-2042-75 \beta+23 \beta^{2}}{52\left(-58-3 \beta+\beta^{2}\right)} L  \tag{24}\\
& e_{2}^{g}=\frac{-974-81 \beta+29 \beta^{2}}{52\left(-58-3 \beta+\beta^{2}\right)} L \tag{25}
\end{align*}
$$

These are the allocations of permits consistent with grandfathering based on output ratios. Figures 4 and 5 compare the grandfathering allocations ((24) and (25)) with the second best

[^4]((16) and (17)), where permit allocations maximize social welfare.


Figure 4: Permits allocation comparison for the large firm

Figure 5: Permit allocation comparison for the small firm

Proposition 11 (Pre-permit quantity-based) grandfathering permits allocates too few permits to the large firm and too many permits to the small firm.

Proof. It is easily observed from Figures 4 and 5.
We see that grandfathering gives firm 1 too few permits, and firm 2 too many permits. Thus, somewhat surprisingly, grandfathering gives too few permits to the large firm and too many permits to the the small firm. Giving more permits to the large firm induces it to increase its standard $\mathrm{R} \& \mathrm{D}$, as it can now pollute more and produce more. $x_{1}$ is more socially valuable than $x_{2}$, because $x_{1}$ is applied to a larger output. Moreover, giving more permits to the large firm increases its market share ${ }^{9}$. This is socially desirable, because a larger share of output is now produced at a lower marginal cost. In a sence, the regulator attempts to accentuate the initial cost asymmetry, to the benefit of consumers (and of the large firm).

In addition, due to the convex $R \& D$ cost, the last unit of $R \& D$ of the large firm costs more, because it invests more in $\mathrm{R} \& \mathrm{D}$. The social planner needs to equalize $\mathrm{R} \& \mathrm{D}$ marginal costs to achieve maximum social welfare. Thus, to reduce the gap between $\mathrm{R} \& \mathrm{D}$ marginal costs, the social planner reduces the $R \& D$ of the large firm, by giving it more permits.

[^5]Mackenzie et al. (2008) conclude that allocating permits based on historical output in a dynamic market setting is never optimal. This is because a particular firm's increasing/decreasing output would affect the aggregate output, and then the permits the other firm may receive; in other words, firms' choices are strategically interdependent. This leads firms' behaviors to be typically different from when face an absolute amount of permits (under second best).

## 8 R\&D budget constraint

Until now, we have not taken into account the fact that firms are financially constrained in how much they can invest in innovation. Such a constraint will distort firms' investments in innovation - both cost-reducing and environmental innovation - from their profit-maximizing levels. This issue becomes particularly relevant in an economic downturn, where firms face declining profits, reduced demand, lower employment and reduced growth prospect. This section incorporates such a constraint. The goal is to see how financially constrained firms allocate their limited resources to cost-reducing and environmental $R \& D$. Given the difficulty of incorporating this constraint into the model with asymmetric firms, we limit the analysis here to symmetric firms.

### 8.0.3 R\&D noncooperation

The game is the same as above except for the constraint in the first stage ( $R \& D$ competition). Let $B>0$ be the total amount that can be invested in $\mathrm{R} \& \mathrm{D}$ by a firm. After substituting from equations (7) and (8) into (2), firm $i$ now solves

$$
\begin{align*}
& \max _{x_{i}, w_{i}} \pi_{i}=\left(a-q_{i}-q_{j}\right) q_{i}-C_{i}-\sigma\left(q_{i}-w_{i}-\beta w_{j}-e_{i}\right) \\
& \text { s.t } \frac{\gamma}{2} x_{i}^{2}+\frac{\delta}{2} w_{i}^{2} \leq B \tag{26}
\end{align*}
$$

By solving the binding constraint, we have $w_{i}=\frac{\sqrt{2 B_{i}-\gamma x_{i}^{2}}}{\sqrt{\delta}}$. Substituting $w_{i}$ and $w_{j}$ into
equation (25) and with symmetry, we can solve for $x_{i}^{n b}$, and $w_{i}^{n b}$. We use the following configuration: $a=300, A_{1}=A_{2}=10, \gamma=40, \delta=10, \beta=0.5$. With the graph below, we see how the budget constraint affects innovation. Figure 7 illustrates the ratio of constrained to non-constrained $\mathrm{R} \& \mathrm{D}$ investments.


Figure 6: Noncooperative R\&D level comparisons
with and without financial constraint

When there is no constraint, firm $i$ invests a total of $\$ 297.73$ in both types of $\mathrm{R} \& \mathrm{D}$. If the budget is beyond this amount, firm $i$ will invest the same amount as in the unconstrained game. Figure 15 shows that when the constraint is binding, investments in both types of $R \& D$ are reduced, but more so (proportionally) for cost-reducing $R \& D^{10}$.

While distorting either standard or environmental $R \& D$ from their unconstrained levels reduces profits, reducing environmental $\mathrm{R} \& \mathrm{D}$ is particularly costly, since it forces the firm to buy more permits. Hence, the higher the permit price, the lower severe will be the negative effect of a financial constraint on investments in environmental innovation.

[^6]
### 8.0.4 R\&D cooperation

We now analyze the effect of the constraint under R\&D cooperation. Firms solve

$$
\begin{align*}
& \max _{x_{1}, x_{2}, w_{1}, w_{2}} \quad \pi_{c}=\pi_{1}+\pi_{2}  \tag{27}\\
& \text { s.t } \frac{\gamma}{2} x_{1}^{2}+\frac{\delta}{2} w_{1}^{2} \leq B, \frac{\gamma}{2} x_{2}^{2}+\frac{\delta}{2} w_{2}^{2} \leq B
\end{align*}
$$

By using the same numerical parametrization as in section 7.2.1, we obtain (qualitatively) similar results to those obtained with noncooperation.


Figure 7: Cooperative R\&D level comparisons
with and without financial cosntraint

## 9 Conclusion

In this paper, we address permit trading with both cost-reducing and environmental $\mathrm{R} \& \mathrm{D}$ invested by two asymmetric Cournot duopolists. First of all, the permit price depends on total permits only, but not on initial allocations. Then, we consider the second best, where the social planner chooses the optimal permit allocations. Both fixed and non-fixed amounts of permits are analyzed. The allocation of permits between firms matters for social welfare in the presence of environmental $\mathrm{R} \& \mathrm{D}$ under noncooperative $\mathrm{R} \& \mathrm{D}$, but is irrelevant under
cooperative R\&D.
We consider two extensions of the model. One is grandfathering permits. Surprisingly, we find that grandfathering gives too few permits to the large firm, and too many permits to the small firm. The other extension incorporates an $R \& D$ budget constraint. It is shown that the constraint induces firms to underinvest in both types of $\mathrm{R} \& \mathrm{D}$, but more so for cost-reducing R\&D.

Future research could incorporate $\mathrm{R} \& \mathrm{D}$ subsidies into the model. Moreover, bankable permits, which are allowed in many countries, will affect the trading and investment behavior of firms. Also, we may consider the pruchase of permits from third parties, which means that permits come from outsiders. Furthermore, it would be interesting to incorporate permit trading between vertically related markets.

## Appendix

## Appendix A1: Profits under R\&D noncooperation

By substituting $\widehat{q}_{1}$ and $\widehat{q}_{2}$ into the profit functions, we get:

$$
\begin{align*}
& \pi_{1}=\frac{1}{4}\left\{A_{1}^{2}+A_{2}^{2}+4 a e_{1}-5 e_{1}^{2}-4 e_{1} e_{2}+e_{2}^{2}+4 a w_{1}-2(5+2 \beta) e_{1} w_{1}+2(-2+\beta) e_{2} w_{1}-\right. \\
& \left(5+4 \beta-\beta^{2}+2 \delta\right) w_{1}^{2}+4 a \beta w_{2}-(2+14 \beta) e_{1} w_{2}-\left(4+8 \beta+4 \beta^{2}\right) w_{1} w_{2} \\
& +\left(1-4 \beta+5 \beta^{2}\right) w_{2}^{2}+4 e_{1} x_{1}-2(-1+\beta) e_{2} x_{1}+\left(4+2 \beta-2 \beta^{2}\right) w_{1} x_{1}+(2+2 \beta) w_{2} x_{1} \\
& -\left(2 \beta-\beta^{2}+2 \gamma\right) x_{1}^{2}+2 A_{2}\left[e_{2}+\beta w_{1}+w_{2}-(-1+\beta)\left(x_{1}-x_{2}\right)\right]-2 A_{1}\left[A_{2}+2 e_{1}+e_{2}\right. \\
& \left.+(2+\beta) w_{1}+(1+2 \beta) w_{2}-(-1+\beta)\left(x_{1}-x_{2}\right)\right]+2\left[2 \beta e_{1}+(-1+\beta) e_{2}+\beta(1+\beta) w_{1}\right. \\
& \left.\left.+\left(-1+\beta+2 \beta^{2}\right) w_{2}-(-1+\beta)^{2} x_{1}\right] x_{2}+(-1+\beta)^{2} x_{2}^{2}\right\} \tag{A1}
\end{align*}
$$

$$
\begin{align*}
& \pi_{2}=\frac{1}{4}\left\{A_{1}^{2}+A_{2}^{2}+4 a e_{2}-5 e_{2}^{2}-4 e_{1} e_{2}+e_{2}^{2}+4 a \beta w_{1}-2(1-2 \beta) e_{1} w_{1}+2(2+5 \beta) e_{2} w_{1}\right. \\
& -\left(1-4 \beta-5 \beta^{2}\right) w_{1}^{2}+4 a w_{2}-(14-6 \beta) e_{1} w_{2}-\left(4+8 \beta+4 \beta^{2}\right) w_{1} w_{2}-\left(5+4 \beta-\beta^{2}+2 \delta\right) w_{2}^{2} \\
& -(-2+2 \beta) e_{1} x_{1}+4 \beta e_{2} x_{1}-\left(2-2 \beta-4 \beta^{2}\right) w_{1} x_{1}+\left(2+2 \beta^{2}\right) w_{2} x_{1}-(\beta-1)^{2} x_{1}^{2} \\
& +2 A_{1}\left[-A_{2}+e_{1}+w_{1}+\beta w_{2}+(-1+\beta)\left(x_{1}-x_{2}\right)\right]-2 A_{2}\left[e_{1}+2 e_{2}+(1+2 \beta) w_{1}+(2+\beta) w_{2}\right. \\
& \left.+(-1+\beta)\left(x_{1}-x_{2}\right)\right]+2\left[2 e_{2}-(-1+\beta) e_{1}+(1+\beta) w_{1}+\left(2+\beta-\beta^{2}\right) w_{2}\right. \\
& \left.\left.-(-1+\beta)^{2} x_{1}\right] x_{2}+\left[(-1+\beta)^{2}-2 \gamma\right] x_{2}^{2}\right\} \tag{A2}
\end{align*}
$$

## Appendix A2: R\&D equilibria under noncooperation

The first and second order conditions with respect to $x_{1}$ are:

$$
\begin{gather*}
\frac{\partial \pi_{1}}{\partial x_{1}}=\frac{1}{2}\left\{(-1+\beta)\left(A_{1}-A_{2}-e_{2}\right)+2 e_{1}+(1+\beta)\left[(2-\beta) w_{1}+w_{2}\right]+(1-\beta)^{2}\left(x_{1}-x_{2}\right)-2 \gamma x_{1}\right\}=0  \tag{A3}\\
\frac{\partial^{2} \pi_{1}}{\partial x_{1}^{2}}=\frac{1}{2}(-1+\beta)^{2}-\gamma<0 \text { iff } 2 \gamma>(1-\beta)^{2} \tag{A4}
\end{gather*}
$$

The first and second order conditions with respect to $w_{1}$ are:

$$
\begin{align*}
\frac{\partial \pi_{1}}{\partial w_{1}}=\frac{1}{2}\left[2 a-(2+\beta) A_{1}+\beta A_{2}\right. & -(5+2 \beta) e_{1}+(-2+\beta) e_{2}-\left(5+4 \beta-\beta^{2}+2 \delta\right) w_{1}-2(1+\beta)^{2} w_{2} \\
\left.+\left(2+\beta-\beta^{2}\right) x_{1}+\left(\beta+\beta^{2}\right) x_{2}\right] & =0  \tag{A5}\\
\frac{\partial^{2} \pi_{1}}{\partial w_{1}^{2}} & =-\frac{1}{2}\left(5+4 \beta-\beta^{2}+2 \delta\right)<0
\end{align*}
$$

The first and second order conditions with respect to $x_{2}$ are:

$$
\begin{gather*}
\left.\frac{\partial \pi_{2}}{\partial x_{2}}=\frac{1}{2}\left\{(-1+\beta)\left(A_{2}-A_{1}-e_{1}\right)+2 e_{2}+(1+\beta)\left[(2-\beta) w_{2}+w_{1}\right]-(1-\beta)^{2}\left(x_{1}-x_{2}\right)-2 \gamma x_{2}\right)\right\}=0  \tag{A7}\\
\frac{\partial^{2} \pi_{2}}{\partial x_{1}^{2}}=\frac{1}{2}(1-\beta)^{2}-\gamma<0 \text { iff } 2 \gamma>(1-\beta)^{2} \tag{A8}
\end{gather*}
$$

The first and second order conditions with respect to $w_{2}$ are:

$$
\begin{align*}
& \frac{\partial \pi_{2}}{\partial w_{2}}=\frac{1}{2}\left[2 a-(2+\beta) A_{2}+\beta A_{1}-(5+2 \beta) e_{2}+(-2+\beta) e_{1}-\left(5+4 \beta-\beta^{2}+2 \delta\right) w_{2}-2(1+\beta)^{2} w_{1}\right. \\
&\left.+\left(2+\beta-\beta^{2}\right) x_{2}+\left(\beta+\beta^{2}\right) x_{1}\right]=0  \tag{A9}\\
& \frac{\partial^{2} \pi_{2}}{\partial w_{1}^{2}}=-\frac{1}{2}\left(5+4 \beta-\beta^{2}+2 \delta\right)<0 \tag{A10}
\end{align*}
$$

Solving the 4 first-order conditions simultaneously, we get:

$$
\begin{aligned}
& \widehat{x}_{1}=\left\{2 a(-3+\beta)(1+\beta)\left[\left(1-\beta^{2}\right)(4+2(-3+\beta) \beta-3 \gamma)+2\left((-1+\beta)^{2}-\gamma\right) \delta\right]-\left\{(-1+\beta)(1+\beta)^{2}\right.\right. \\
& *[24-37 \gamma+\beta(-20+13 \gamma+2 \beta(2+\gamma))]+2(1+\beta)[-6+\beta(8+\beta(-2+\gamma)-13 \gamma)+14 \gamma] \delta \\
& \left.-4(-1+\beta) \gamma \delta^{2}\right\} A_{1}+\left\{(-1+\beta)(1+\beta)^{2}[4(-3+\beta)(-2+\beta) \beta+(-19+\beta(7+2 \beta))] \gamma-2(1+\beta)\right.
\end{aligned}
$$

$$
\left.*[2(-3+\beta)(-1+\beta) \beta-(8+(-11+\beta) \beta) \gamma] \delta-4(-1+\beta) \gamma \delta^{2}\right\} A_{2}+\delta\{-15+16 \gamma+\beta[\beta(-6
$$

$$
\left.+\beta(\beta(13-2 \beta)+4(-5+\gamma)))+2(7+6 \gamma)]+2\left[(-3+\beta)(1-+\beta)^{2}+4 \gamma\right] \delta\right\} e_{1}+\delta\{[-(1+\beta)(9-2 \gamma
$$

$$
\left.+\beta(-39+\beta(-33+\beta(-13+2 \beta)-2 \gamma)+20 \gamma))+2(-1+\beta)(3+(-4+\beta) \beta-2 \gamma) \delta]\} e_{2}\right\} /\left\{2 \left[\left(-1+\beta^{2}\right)\right.\right.
$$

$$
\left.\left.*(4+2(-3+\beta) \beta-3 \gamma)-2\left((-1+\beta)^{2}-\gamma\right) \delta\right][(1+\beta)(-3+7 \gamma+\beta(-2+\beta+\gamma))+2 \gamma \delta]\right\}
$$

$$
\widehat{x}_{2}=\left\{2 a(-3+\beta)(1+\beta)\left[\left(1-\beta^{2}\right)(4+2(-3+\beta) \beta-3 \gamma)+2\left((-1+\beta)^{2}-\gamma\right) \delta\right]+\left\{(-1+\beta)(1+\beta)^{2}\right.\right.
$$

$$
*[4(-3+\beta)(-2+\beta) \beta+(-19+\beta(7+2 \beta)) \gamma]-2(1+\beta)[2(-3+\beta)(-1+\beta) \beta-(8+(-11+\beta) \beta) \gamma] \delta
$$

$$
\left.-4(-1+\beta) \gamma \delta^{2}\right\} A_{1}+\left\{(-1+\beta)(1+\beta)^{2}[24-37 \gamma+\beta(-20+13 \gamma+2 \beta(2+\gamma))]+2(1+\beta)\right.
$$

$$
\left.*(-6+\beta(8+\beta(-2+\gamma)-13 \gamma)+14 \gamma) \delta-4(-1+\beta) \gamma \delta^{2}\right\} A_{2}+\delta\{-(1+\beta)[9-2 \gamma+\beta(-39+\beta(33
$$

$$
+\beta(-13+2 \beta)-2 \gamma)+20 \gamma)]+2(-1+\beta)[3+(-4+\beta) \beta-2 \gamma] \delta\} e_{1}+\delta\{-15+16 \gamma+\beta[\beta(-6+\beta((13
$$

$$
\left.\left.-2 \beta) \beta+4(-5+\gamma)))+2(7+6 \gamma)]+2\left[(-3+\beta)(-1+\beta)^{2}+4 \gamma\right] \delta\right\} e_{2}\right\} /\left\{2 \left[\left(-1+\beta^{2}\right)(4+2(-3+\beta) \beta\right.\right.
$$

$$
\begin{equation*}
\left.-3 \gamma)-2\left((-1+\beta)^{2}-\gamma\right) \delta\right][(1+\beta)(-3+7 \gamma+\beta(-2+\beta+\gamma))+2 \gamma \delta] \tag{A12}
\end{equation*}
$$

$$
\begin{align*}
& \widehat{w}_{1}=\left\{2 a \gamma\left[\left(-1+\beta^{2}\right)(4+2(-3+\beta) \beta-3 \gamma)+2\left(-(1-\beta)^{2}+\gamma\right) \delta\right]-\{\gamma[(1+\beta)(-7+10 \gamma+\beta(5(1+\gamma)\right. \\
& \left.\left.+\beta(-9+3 \beta+\gamma)))-2\left((1-\beta)^{2}-(2+\beta) \gamma\right) \delta\right]\right\} A_{1}+\gamma\left\{-(1+\beta)\left[-1-4 \gamma+\beta\left(15-7 \beta+\beta^{2}\right.\right.\right. \\
& -(11+\beta) \gamma)]+2[1+\beta(-2+\beta+\gamma)] \delta\} A_{2}+\{(1+\beta)[4+2(-3+\beta) \beta-3 \gamma][-3+7 \gamma+\beta(-2+\beta \\
& \left.+\gamma)+\left[(-3+\beta)(-1+\beta)^{2}(1+\beta)+\left(14-13 \beta+3 \beta^{3}\right) \gamma-2(5+2 \beta) \gamma^{2}\right] \delta\right\} e_{1}-\{\beta(1+\beta)[4+2(-3 \\
& +\beta) \beta-3 \gamma][-3+7 \gamma+\beta(-2+\beta+\gamma)]-\left[(-3+\beta)(-1+\beta)^{2}(1+\beta)-(-6+\beta(9+(-8+\beta) \beta)) \gamma\right. \\
& \left.\left.\left.+2(-2+\beta) \gamma^{2}\right] \delta\right\} e_{2}\right\} /\left\{\left[\left(-1+\beta^{2}\right)(4+2(-3+\beta) \beta-3 \gamma)-2\left((-1+\beta)^{2}-\gamma\right) \delta\right][(1+\beta)(-3+7 \gamma\right. \\
& +\beta(-2+\beta+\gamma))+2 \gamma \delta]\}  \tag{A13}\\
& \widehat{w}_{2}=\left\{2 a \gamma\left[\left(-1+\beta^{2}\right)(4+2(-3+\beta) \beta-3 \gamma)+2\left(-(1-\beta)^{2}+\gamma\right) \delta\right]+\gamma\left\{-(1+\beta)\left(-1-4 \gamma+\beta\left(15+\beta^{2}\right.\right.\right.\right. \\
& -11 \gamma-\beta(7+\gamma)))+2(1+\beta(-2+\beta+\gamma)) \delta\} A_{1}-\gamma\{(1+\beta)[-7+10 \gamma+\beta(5(1+\gamma)+\beta(-9+3 \beta \\
& \left.+\gamma))]-2\left[(-1+\beta)^{2}-(2+\beta) \gamma\right] \delta\right\} A_{2}-\{\beta(1+\beta)[4+2(-3+\beta) \beta-3 \gamma][-3+7 \gamma+\beta(-2+\beta+\gamma)] \\
& \left.-\left[(-3+\beta)(1-\beta)^{2}(1+\beta)-(-6+\beta(9+(-8+\beta) \beta)) \gamma+2(-2+\beta) \gamma^{2}\right] \delta\right\} e_{1}-12 e_{2}+\left\{2 \beta^{5}-3 \delta\right. \\
& +\beta^{4}(-8+2 \gamma+\delta)+\gamma(37-21 \gamma+14 \delta-10 \gamma \delta)+\beta^{3}(7 \gamma-4 \delta+3 \gamma \delta)+\beta^{2}[-3 \gamma(9+\gamma)+2(10+\delta)] \\
& \left.-\beta[2-4 \delta+\gamma(-+13 \delta+4 \gamma(6+\delta))]\} e_{2}\right\} /\left\{\left[\left(-1+\beta^{2}(4+2(-3+\beta) \beta-3 \gamma)-2\left((-1+\beta)^{2}-\gamma\right) \delta\right]\right.\right. \\
& *((1+\beta)(-3+7 \gamma+\beta(-2+\beta+\gamma))+2 \gamma \delta)\} \tag{A14}
\end{align*}
$$

By substituting (A11) (A12) (A13) and (A14) into (7) and (8), we get

$$
\begin{equation*}
\widehat{\sigma}=-\frac{\left(A_{1}+A_{2}-2 a \gamma\right)\left[(1+\beta)^{2}+2 \delta\right]+\delta[-3+(-2+\beta) \beta+6 \gamma]\left(e_{1}+e_{2}\right)}{2(1+\beta)[-3+7 \gamma+\beta(-2+\beta+\gamma)]+4 \gamma \delta} \tag{A15}
\end{equation*}
$$

$$
\begin{align*}
& \widehat{q}_{1}=\left\{4 a(1+\beta) \gamma\left[\left(-1+\beta^{2}\right)(4+2(-3+\beta) \beta-3 \gamma)+2\left(-(1-\beta)^{2}+\gamma\right) \delta\right]+\gamma\left\{-(-1+\beta)(1+\beta)^{2}[17\right.\right. \\
& \left.\left.-6 \beta+\beta^{2}-3(9+\beta) \gamma\right]+2(1+\beta)\left[5-2 \beta+\beta^{2}+2(-6+\beta) \gamma\right] \delta-4 \gamma \delta^{2}\right\} A_{1}-\gamma\left\{(-1+\beta)(1+\beta)^{2}\right. \\
& \left.*\left[-1+7 \beta^{2}+3 \beta(-6+\gamma)+15 \gamma\right]-2(1+\beta)\left[-1+3 \beta^{2}+8 \gamma-2 \beta(3+\gamma)\right] \delta-4 \gamma \delta^{2}\right\} A_{2}-\delta\left\{\left(-1+\beta^{2}\right)\right. \\
& \left.*\left[(-3+\beta)(1+\beta)^{2}+(-1+(20-3 \beta) \beta) \gamma+6 \gamma^{2}\right]+2[1+\beta(-4+3 \beta)-2 \gamma] \gamma \delta\right\} e_{1}+\delta\left\{\left(-1+\beta^{2}\right)\right. \\
& \left.\left.*\left[(-3+\beta)(1+\beta)^{2}+(15+\beta(-4+5 \beta)) \gamma-6 \gamma^{2}\right]+2 \gamma[-3-(-4+\beta) \beta+2 \gamma] \delta\right\} e_{2}\right\} / \\
& \left\{2[(1+\beta)(-3+7 \gamma+\beta(-2+\beta+\gamma))+2 \gamma \delta]\left[\left(-1+\beta^{2}\right)(4+2(-3+\beta) \beta-3 \gamma)+2\left(-(-1+\beta)^{2}+\gamma\right)\right] \delta\right\}  \tag{A16}\\
& \text { (A16) } \\
& \widehat{q}_{2}=-\left\{4 a(1+\beta) \gamma\left[\left(-1+\beta^{2}\right)(-4-2(-3+\beta) \beta+3 \gamma)+2\left((1-\beta)^{2}-\gamma\right) \delta\right]+\gamma\left\{(-1+\beta)(1+\beta)^{2}[-1\right.\right. \\
& \left.\left.+7 \beta^{2}+3 \beta(-6+\gamma)+15 \gamma\right]-2(1+\beta)\left[-1+3 \beta^{2}+8 \gamma-2 \beta(3+\gamma)\right] \delta-4 \gamma \delta^{2}\right\} A_{1}-\gamma\{-(-1+\beta) \\
& \left.*(1+\beta)^{2}\left[17-6 \beta+\beta^{2}-3(9+\beta) \gamma\right]+2(1+\beta)\left[5-2 \beta+\beta^{2}+2(-6+\beta) \gamma\right] \delta-4 \gamma \delta^{2}\right\} A_{2} \\
& +\delta\left\{\left(-1+\beta^{2}\right)\left[(-3+\beta)(1+\beta)^{2}+(15+\beta(-4+5 \beta)) \gamma-6 \gamma^{2}\right]-2 \gamma[-3-(-4+\beta) \beta+2 \gamma] \delta\right\} e_{1} \\
& \left.+\delta\left\{\left(-1+\beta^{2}\right)\left[(-3+\beta)(1+\beta)^{2}-(1+\beta(-20+3 \beta)) \gamma+6 \gamma^{2}\right]-2 \gamma[-1+(4-3 \beta) \beta+2 \gamma] \delta\right\} e_{2}\right\} / \\
& \left\{2\left[\left(-1+\beta^{2}\right)(4+2(-3+\beta) \beta-3 \gamma)-2\left((-1+\beta)^{2}-\gamma\right)\right] \delta[(1+\beta)(-3+7 \gamma+\beta(-2+\beta+\gamma))+2 \gamma \delta]\right\}
\end{align*}
$$

## Appendix A3: Comparative statics under noncooperation

When $\beta=0$, we have:

$$
\begin{aligned}
& \frac{\partial x_{i}}{\partial A_{i}}<0 ; \quad \frac{\partial x_{i}}{\partial A_{j}}>0 ; \quad \frac{\partial x_{i}}{\partial e_{i}}>0 ; \quad \frac{\partial x_{i}}{\partial e_{j}}>0 \\
& \frac{\partial w_{i}}{\partial A_{i}}<0 ; \quad \frac{\partial w_{i}}{\partial A_{j}}>0 ; \quad \frac{\partial w_{i}}{\partial e_{i}}<0 ; \quad \frac{\partial w_{i}}{\partial e_{j}}<0 \\
& \frac{\partial \sigma}{\partial A_{i}}<0 ; \quad \frac{\partial \sigma}{\partial e_{i}}<0 ; \quad \frac{\partial q_{i}}{\partial A_{i}}<0 ; \quad \frac{\partial q_{i}}{\partial A_{j}}>0 \\
& \frac{\partial f_{i}}{\partial A_{i}}<0 ; \quad \frac{\partial f_{i}}{\partial A_{j}}>0 ; \quad \frac{\partial f_{i}}{\partial e_{i}}>0 ; \quad \frac{\partial f_{i}}{\partial e_{j}}>0
\end{aligned}
$$

When $\beta=1$, we have:

$$
\begin{aligned}
& \frac{\partial x_{i}}{\partial A_{i}}<0 ; \quad \frac{\partial x_{i}}{\partial A_{j}}<0 ; \quad \frac{\partial x_{i}}{\partial e_{i}}>0 ; \quad \frac{\partial x_{i}}{\partial e_{j}}<0 \\
& \frac{\partial w_{i}}{\partial A_{i}}<0 ; \quad \frac{\partial w_{i}}{\partial A_{j}}>0 ; \quad \frac{\partial w_{i}}{\partial e_{i}}<0 ; \quad \frac{\partial w_{i}}{\partial e_{j}}<0 \\
& \frac{\partial \sigma}{\partial A_{i}}<0 ; \quad \frac{\partial \sigma}{\partial e_{i}}<0 ; \quad \frac{\partial q_{i}}{\partial A_{i}}<0 ; \quad \frac{\partial q_{i}}{\partial A_{j}}>0 \\
& \frac{\partial f_{i}}{\partial A_{i}}<0 ; \quad \frac{\partial f_{i}}{\partial A_{j}}>0 ; \quad \frac{\partial f_{i}}{\partial e_{i}}>0 ; \quad \frac{\partial f_{i}}{\partial e_{j}}>0
\end{aligned}
$$

## Appendix A4: Joint profit under cooperation

By summing firm 1's profit function and firm 2's profit function, we get the joint profit
function:

$$
\begin{align*}
& \pi_{c}=\frac{1}{2}\left\{A_{1}^{2}+A_{2}^{2}-2\left(e_{1}+e_{2}\right)^{2}-4(1+\beta)\left(e_{1} w_{1}+e_{2} w_{1}+e_{1} w_{2}+e_{2} w_{2}\right)-\left(2+4 \beta+2 \beta^{2}+\delta\right)\left(w_{1}^{2}+w_{2}^{2}\right)\right. \\
& +2 a \beta w_{2}-4(1+\beta)^{2} w_{1} w_{2}+2 a\left[e_{1}+e_{2}+(1+\beta) w_{1}+w_{2}\right]+(1+\beta)\left(e_{1} x_{1}+e_{2} x_{1}\right) \\
& +(1+\beta)^{2}\left(w_{1} x_{1}+w_{2} x_{1}\right)+\left[(1-\beta)^{2}-\gamma\right] x_{1}^{2}-A_{2}\left[e_{1}+e_{2}+(1+\beta)\left(w_{1}+w_{2}\right)+2(-1+\beta)\left(x_{1}-x_{2}\right)\right] \\
& +\left[(1+\beta)\left(e_{1}+e_{2}+(1+\beta)\left(w_{1}+w_{2}\right)\right)-2(-1+\beta)^{2} x_{1}\right] x_{2}+\left[(-1+\beta)^{2}-\gamma\right] x_{2}^{2}-A_{1}\left[2 A_{2}+e_{1}+e_{2}\right. \\
& \left.\left.+(1+\beta)\left(w_{1}+w_{2}\right)+2(1-\beta)\left(x_{1}-x_{2}\right)\right]\right\} \tag{A18}
\end{align*}
$$

## Appendix A5: R\&D strategic / cooperative interactions in d'Aspremont and Jacquemin (1988)

In d'Aspremont and Jacquemin (1988), there are two symmetric firms which invest in standard R\&D $x$ only. The inverse demand function is $P=a-b Q$, where $Q=q_{1}+q_{2}$. By assuming no fixed cost, the marginal production cost is $c_{i}=\left(A-x_{i}-\beta x_{j}\right)$. R\&D costs are $\frac{\gamma}{2} x_{i}^{2}$. The profit function for firm $i$ is $\pi_{i}=\left(P-c_{i}\right) q_{i}-\frac{\gamma}{2} x_{i}^{2}$. There are two scenarios: R\&D noncooperation and $\mathrm{R} \& \mathrm{D}$ cooperation.

Under R\&D noncooperation, there are two stages: first, they compete in R\&D investments; second, they compete à la Cournot. By backward induction, we maximize profits with respect to quantity, which yields:

$$
\begin{equation*}
q_{i}=\frac{A-a+(1-2 \beta) x_{j}-(2-\beta) x_{i}}{3 b}, i \neq j, i, j=1,2 \tag{A19}
\end{equation*}
$$

By substituting $q_{i}$ and $q_{j}$ into the profit functions, we get:

$$
\begin{equation*}
\pi_{i}=\frac{2\left[a-A+(-1+2 \beta) x_{j}\right]^{2}-4(-2+\beta)\left[a-A+(-1+2 \beta) x_{j}\right] x_{i}+\left[2(-2+\beta)^{2}-9 b \gamma\right] x_{i}^{2}}{18 b} \tag{A20}
\end{equation*}
$$

We can analyze the strategic interaction between the $\mathrm{R} \& \mathrm{D}$ levels:

$$
\begin{equation*}
\frac{\partial^{2} \pi_{i}}{\partial x_{i} \partial x_{j}}=-\frac{2[2+\beta(-5+2 \beta)]}{9 b} \tag{A21}
\end{equation*}
$$

which depends on $\beta$ : when $\beta<\frac{1}{2}$, $\frac{\partial^{2} \pi_{i}}{\partial x_{i} \partial x_{j}}<0$, the $\mathrm{R} \& \mathrm{D}$ levels are strategic substitutes; otherwise, they are strategic complements

Under R\&D cooperation, firms cooperate in $R \& D$ investments but still compete à la Cournot. In the first stage, firms maximize total profits with respect to $x_{1}$ and $x_{2}$. The total profit function is:

$$
\begin{align*}
& \pi_{c}=\left\{4(a-A)^{2}+[10+2 \beta(-8+5 \beta)-9 b \gamma]\left(x_{1}^{2}+x_{2}^{2}\right)+4(a-A)(1+\beta)\left(x_{1}+x_{2}\right)\right. \\
& \left.-8[2+\beta(-5+2 \beta)] x_{1} x_{2}\right\} / 18 b \tag{A22}
\end{align*}
$$

Then, we can analyze the cooperative interaction between the $R \& D$ levels:

$$
\begin{equation*}
\frac{\partial^{2} \pi_{c}}{\partial x_{1} \partial x_{2}}=-\frac{4(-2+\beta)(-1+2 \beta)}{9 b} \tag{A23}
\end{equation*}
$$

which depends on $\beta$ : if $\beta<\frac{1}{2}$, the $\mathrm{R} \& \mathrm{D}$ levels are cooperative substitutes, otherwise they are cooperative complements.

## Appendix A6: R\&D equilibria under cooperation

The 4 first-order conditions with respect to $x_{1}, x_{2}, w_{1}, w_{2}$ are:

$$
\begin{gather*}
\frac{\partial \pi_{c}}{\partial x_{1}}=\frac{\partial \pi_{c}}{\partial x_{2}}=2\left(e_{1}+e_{2}\right)+4\left(w_{1}+w_{2}\right)-2 \gamma x_{1}=0  \tag{A24}\\
\frac{\partial \pi_{c}}{\partial w_{1}}=\frac{\partial \pi_{c}}{\partial w_{2}}=4 a-2\left(A_{1}+A_{2}\right)-8\left(e_{1}+e_{2}\right)-16\left(w_{1}+w_{2}\right)-2 \delta w_{1}+4\left(x_{1}+x_{2}\right)=0 \tag{A25}
\end{gather*}
$$

Solving the four equations, we obtain $\left(\widetilde{x_{1}}, \widetilde{x_{2}}, \widetilde{w_{1}}, \widetilde{w_{2}}, \widetilde{q_{1}}, \widetilde{q_{2}}\right)$ :

$$
\begin{gather*}
\widetilde{x_{1}}=\widetilde{x_{2}}=-\frac{-8 a+4\left(A_{1}+A_{2}\right)-\delta\left(e_{1}+e_{2}\right)}{16(\gamma-1)+\gamma \delta}  \tag{A26}\\
\widetilde{w_{1}}=\widetilde{w_{2}}=-\frac{-2 a \gamma+\gamma\left(A_{1}+A_{2}\right)+4(\gamma-1)\left(e_{1}+e_{2}\right)}{-16+16 \gamma+\gamma \delta} \tag{A27}
\end{gather*}
$$

By substituting (A26) and (A27) into $f_{i}$ and equation (7), we also get:

$$
\begin{gather*}
\widetilde{f}_{1}=\frac{-A_{1}+A_{2}+e_{1}+e_{2}}{2}  \tag{A28}\\
\widetilde{f}_{2}=\frac{A_{1}-A_{2}+e_{1}+e_{2}}{2}  \tag{A29}\\
\widetilde{\sigma}=-\frac{\left(-2 a+A_{1}+A_{2}\right) \gamma(4+\delta)+(-4+3 \gamma) \delta\left(e_{1}+e_{2}\right)}{-32+2 \gamma(16+\delta)} \tag{A30}
\end{gather*}
$$

## Appendix A7: Numerical simulations for equilibria under noncooperation and cooperation

Let $a=300, A_{1}=10, A_{2}=70, \delta=10, \gamma=40, e_{1}=75, e_{2}=25$, which guarantee that all quantities, prices and profits are positive. Then, we have:

$$
\begin{gather*}
\widetilde{x_{1}}=\widetilde{x_{2}}=\frac{385}{128}, \widetilde{w_{1}}=\widetilde{w_{2}}=\frac{325}{64}  \tag{A31}\\
\widehat{x_{1}}=-\frac{5\left(-308757+1322 \beta+76724 \beta^{2}+1660 \beta^{3}-5413 \beta^{4}-382 \beta^{5}+46 \beta^{6}\right)}{\left(1077+315 \beta+39^{2}+\beta^{3}\right)\left(448+23 \beta-69 \beta^{2}-3 \beta^{3}+\beta^{4}\right)}  \tag{A32}\\
\widehat{x_{2}}=-\frac{5\left(-99819-25882 \beta+32612 \beta^{2}+4020 \beta^{3}-2651 \beta^{4}+62 \beta^{5}+58 \beta^{6}\right)}{\left(1077+315 \beta+39^{2}+\beta^{3}\right)\left(448+23 \beta-69 \beta^{2}-3 \beta^{3}+\beta^{4}\right)}  \tag{A33}\\
\widehat{w_{1}}=-\frac{25\left(-113774+44805 \beta+28465 \beta^{2}-4294 \beta^{3}-868 \beta^{4}+33 \beta^{5}+\beta^{6}\right)}{\left(1077+315 \beta+39^{2}+\beta^{3}\right)\left(448+23 \beta-69 \beta^{2}-3 \beta^{3}+\beta^{4}\right)}  \tag{A34}\\
\widehat{w_{2}}=-\frac{25\left(-135314+10503 \beta+15187 \beta^{2}-4434 \beta^{3}-420 \beta^{4}+107 \beta^{5}+3 \beta^{6}\right)}{\left(1077+315 \beta+39^{2}+\beta^{3}\right)\left(448+23 \beta-69 \beta^{2}-3 \beta^{3}+\beta^{4}\right)} \tag{A35}
\end{gather*}
$$

## Appendix A8: Numerical simulation for second best

## A8.1 Fixed amount of permits

A8.1.1 R\&D noncooperation By substituting $e_{1 s}^{n}$ and $e_{2 s}^{n}$ into equations (A11), (A12), (A13) and (A14), we have the second best R\&D levels:

$$
\begin{equation*}
x_{1 s}^{n}=\frac{30\{121+\beta[-57+(-5+\beta) \beta]\}}{3459+\beta[358+(-14+\beta) \beta(8+\beta)]}+\frac{20\left(114+\beta-13 \beta^{2}\right)}{1077+\beta[315+\beta(39+\beta)]} \tag{A36}
\end{equation*}
$$

$$
\begin{align*}
& x_{2 s}^{n}=\frac{30\{121+\beta[-57+(-5+\beta) \beta]\}}{3459+\beta[358+(-14+\beta) \beta(8+\beta)]}+\frac{20\left(114+\beta-13 \beta^{2}\right)}{1077+\beta[315+\beta(39+\beta)]}  \tag{A37}\\
& w_{1 s}^{n}=\frac{600(-1+2 \beta)}{3459+\beta[358+(-14+\beta) \beta(8+\beta)]}-\frac{50[-139+\beta(38+\beta)]}{1077+\beta[315+\beta(39+\beta)]}  \tag{A38}\\
& w_{2 s}^{n}=\frac{600(1-2 \beta)}{3459+\beta[358+(-14+\beta) \beta(8+\beta)]}-\frac{50[-139+\beta(38+\beta)]}{1077+\beta[315+\beta(39+\beta)]} \tag{A39}
\end{align*}
$$

A8.1.2 Effect of permit allocations on the large firm's market share The market size of the large firm is defined as $s_{1}=\frac{q_{1}}{q_{1}+q_{2}}$ and that of the small firm is $s_{2}=\frac{q_{2}}{q_{1}+q_{2}}$. Figures A1 and A2 show the spillover effect on market sizes:


Figure A1: Spillover effect on $s_{1}$


Figure A2: Spillover effect on $s_{2}$

The spillover effect on other social welfare function components are:

|  | $\beta=0.2$ | $\beta=0.3$ |  | $\beta=0.5$ |  | $\beta=0.7$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $e_{1 s}^{n}=71.145$ | $e_{1}=70.801$ | $\Delta_{1}$ | $e_{1 s}^{n}=70.253$ | $\Delta_{2}$ | $e_{1 s}^{n}=69.852$ | $\Delta_{3}$ |
| $S W$ | 14752 | 14844.1 | 92.1 | 14975.7 | 131.6 | 15044.9 | 69.2 |
| $C S$ | 6476.17 | 6510.21 | 34.04 | 6547.68 | 37.47 | 6548.94 | 1.26 |
| $\pi_{2 s}^{n}=28.855$ | $e_{2}=29.199$ |  | $e_{2 s}^{n}=29.747$ |  | $e_{2 s}^{n}=30.148$ |  |  |
| $\pi_{2}$ | 3776.17 | 3841.88 | 65.71 | 3948.21 | 106.33 | 4026.92 | 78.71 |
| $T D$ | 10000 | 10000 | - | 10000 | - | 10000 | - |
| $q_{1}$ | 87.649 | 87.665 | 0.016 | 87.60 | -0.065 | 87.418 | -0.182 |
| $q_{2}$ | 26.160 | 26.443 | 0.283 | 26.838 | 0.395 | 27.028 | 0.19 |
| $Q$ | 113.808 | 114.107 | 0.299 | 114.435 | 0.328 | 114.446 | 0.011 |
| $s_{1}$ | 0.770 | 0.768 | -0.002 | 0.765 | -0.003 | 0.764 | -0.001 |
| $s_{2}$ | 0.230 | 0.232 | 0.002 | 0.235 | 0.003 | 0.236 | 0.001 |

Table 1: Spillover effect on social welfare function components
$\Delta_{1}$ : changes of $S W, C S, \pi_{1}, \pi_{2}, T D, q_{1}, q_{2}, Q, s_{1}$ and $s_{2}$ when $\beta$ changes from 0.2 to 0.3 , $e_{1 s}^{n}$ from 71.145 to 70.801 and $e_{2 s}^{n}$ from 28.855 to 29.199.
$\Delta_{2}$ : changes of $S W, C S, \pi_{1}, \pi_{2}, T D, q_{1}, q_{2}, Q, s_{1}$ and $T D$ when $\beta$ changes from 0.3 to 0.5, $e_{1 s}^{n}$ from 70.801 to 70.253 and $e_{2 s}^{n}$ from 29.199 to 29.747.
$\Delta_{3}$ : changes of $S W, C S, \pi_{1}, \pi_{2}, T D, q_{1}, q_{2}, Q, s_{1}$ and $T D$ when $\beta$ changes from 0.5 to 0.7, $e_{1 s}^{n}$ from 70.253 to 69.852 and $e_{2 s}^{n}$ from 29.747 to 30.148 .

Obvisouly, with higher spillover, the large firm gets less permits and makes less profit, while the small firm gets more permits and makes more profits. Moreover, consumer surplus decreases with increases in $\beta$. However, as a whole, social welfare increases with $\beta$.

A8.1.3 R\&D cooperation By substituting $e_{2}=L-e_{1}$ into equation (13), we get the social welfare function:

$$
\begin{align*}
& S W=\left\{-32 L^{2} \delta+[-16+\gamma(24+\delta)]\left(8 a^{2} \gamma+2 a L \gamma \delta\right)+L^{2}[-(-2+\gamma) \gamma \delta(32+\delta)\right. \\
& \left.-2(-16+\gamma(16+\delta))^{2} \theta\right]+\left\{256-32 \gamma(17+\delta)+\gamma^{2}[304+\delta(34+\delta)]\right\} A_{1}^{2} \\
& -\gamma(8 a+L \delta)[-16+\gamma(24+\delta)] A_{2}+\left\{256-32 \gamma(17+\delta)+\gamma^{2}[304+\delta(34+\delta)]\right\} A_{2}^{2} \\
& -A_{1}\{\gamma(8 a+L \delta)[-16+\gamma(24+\delta)] \tag{A40}
\end{align*}
$$

Obviously, social welfare depends on $L$ only.

## A8.2 Non-fixed amount of permits

A8.2.1 R\&D noncooperation By substituting $e_{1 s}^{n n}$ and $e_{2 s}^{n n}$ into (A11), (A12), (A13) and (A14), we get $\left(x_{1 s}^{n n}, x_{2 s}^{n n}, w_{1 s}^{n n}, w_{2 s}^{n n}\right)$ :

$$
\begin{align*}
& x_{1 s}^{n n}=\frac{30\{121+\beta[-57+(-5+\beta) \beta]\}}{3459+\beta[358+(-14+\beta) \beta(8+\beta)]} \\
& -\frac{260(-3+\beta)\{7539+\beta[2974+\beta(713+2 \beta(40+\beta))]\}}{3337503+\beta\{1434280+\beta[386452+\beta(53828+\beta(4307+2 \beta(78+\beta)))]\}}  \tag{A41}\\
& x_{2 s}^{n n}=\frac{30\{121+\beta[-57+(-5+\beta) \beta]\}}{3459+\beta[358+(-14+\beta) \beta(8+\beta)]} \\
& -\frac{260(-3+\beta)\{7539+\beta[2974+\beta(713+2 \beta(40+\beta))]\}}{3337503+\beta\{1434280+\beta[386452+\beta(53828+\beta(4307+2 \beta(78+\beta)))]\}} \tag{A42}
\end{align*}
$$

$$
\begin{align*}
& w_{1 s}^{n n}=200\left(\frac{-3+6 \beta}{3459+\beta[358+(-14+\beta) \beta(8+\beta)]}\right. \\
& \left.+\frac{52\{3123+\beta[810+\beta(171+4 \beta)]\}}{3337503+\beta\{1434280+\beta[386452+\beta(53828+\beta(4307+2 \beta(78+\beta)))]\}}\right) \tag{A43}
\end{align*}
$$

$$
\begin{align*}
& w_{1 s}^{n n}=200\left(\frac{3-6 \beta}{3459+\beta[358+(-14+\beta) \beta(8+\beta)]}\right. \\
& \left.+\frac{52\{3123+\beta[810+\beta(171+4 \beta)]\}}{3337503+\beta\{1434280+\beta[386452+\beta(53828+\beta(4307+2 \beta(78+\beta)))]\}}\right) \tag{A44}
\end{align*}
$$

A8.2.2 R\&D cooperation By substituting $e_{1 s}^{c n}$ and $e_{2 s}^{c n}$ into (A26) and (A27), we get $\left(x_{1 s}^{c n}, x_{2 s}^{c n}, w_{1 s}^{c n}, w_{2 s}^{c n}\right):$

$$
\begin{align*}
x_{1 s}^{c n} & =\frac{[-16+\gamma(16+\delta)](\delta+16 \theta)\left(2 a-A_{1}-A_{2}\right)}{2\left[512(-1+\gamma)^{2} \theta+\gamma \delta^{2}(-2+\gamma+2 \gamma \theta)+32(-1+\gamma) \delta(-1+\gamma+2 \gamma \theta)\right]}  \tag{A45}\\
x_{2 s}^{c n} & =\frac{[-16+\gamma(16+\delta)](\delta+16 \theta)\left(2 a-A_{1}-A_{2}\right)}{2\left[512(-1+\gamma)^{2} \theta+\gamma \delta^{2}(-2+\gamma+2 \gamma \theta)+32(-1+\gamma) \delta(-1+\gamma+2 \gamma \theta)\right]}  \tag{A46}\\
w_{1 s}^{c n} & =\frac{\gamma[-\gamma \delta-32 \theta+2 \gamma(16+\delta) \theta]\left(2 a-A_{1}-A_{2}\right)}{512(-1+\gamma)^{2} \theta+\gamma \delta^{2}(-2+\gamma+2 \gamma \theta)+32(-1+\gamma) \delta(-1+\gamma+2 \gamma \theta)}  \tag{A47}\\
w_{2 s}^{c n} & =\frac{\gamma[-\gamma \delta-32 \theta+2 \gamma(16+\delta) \theta]\left(2 a-A_{1}-A_{2}\right)}{512(-1+\gamma)^{2} \theta+\gamma \delta^{2}(-2+\gamma+2 \gamma \theta)+32(-1+\gamma) \delta(-1+\gamma+2 \gamma \theta)} \tag{A48}
\end{align*}
$$

## Appendix A9: Pre-permit trading quantities under R\&D noncoop-

 erationCompared with the game under $\mathrm{R} \& \mathrm{D}$ noncooperation, the only difference is that there is no permit trading. We still solve the game by backward induction. The profit function becomes

$$
\begin{equation*}
\pi_{i}=\left(a-q_{i}-q_{j}\right) q_{i}-\left(A_{i}-x_{i}-\beta x_{j}\right) q_{i}-\frac{\gamma}{2} x_{i}^{2}-\frac{\delta}{2} w_{i}^{2} \tag{A49}
\end{equation*}
$$

The first order condition is

$$
\begin{equation*}
\frac{\partial \pi_{i}}{\partial q_{i}}=a-A_{i}-2 q_{i}-q_{j}+x_{i}+\beta x_{j}=0 \tag{A50}
\end{equation*}
$$

Solving for $q_{i}$, we get

$$
\begin{equation*}
q_{i}=\frac{1}{3}\left(a-2 A_{i}+A_{j}+2 x_{i}-\beta x_{i}-x_{j}+2 \beta x_{j}\right) \tag{A51}
\end{equation*}
$$

Substituting (A51) into (A49), and then taking derivatives with respect to ( $x_{i}, x_{j}, w_{i}, w_{j}$ ), we obtain

$$
\begin{gather*}
x_{1}=\frac{-2(-2+\beta)\left\{a[4+2(-3+\beta) \beta-3 \gamma]+2(-2+\beta+3 \gamma) A_{1}+[-2(-2+\beta) \beta-3 \gamma] A_{2}\right\}}{[4+2(-3+\beta) \beta-3 \gamma][-4+2(-1+\beta) \beta+9 \gamma]} \\
x_{2}=\frac{-2(-2+\beta)\left\{a[4+2(-3+\beta) \beta-3 \gamma]+2(-2+\beta+3 \gamma) A_{2}+[-2(-2+\beta) \beta-3 \gamma] A_{1}\right\}}{[4+2(-3+\beta) \beta-3 \gamma][-4+2(-1+\beta) \beta+9 \gamma]}  \tag{A52}\\
w_{1}=w_{2}=0 \tag{A54}
\end{gather*}
$$

Substituting (A52), (A53) and (A54) into (A51), we get ( $q_{1}, q_{2}$ ). Using the same numerical simulations, we get

$$
\begin{align*}
\frac{q_{1}}{q_{1}+q_{2}} & =\frac{-2042-75 \beta+23 \beta^{2}}{52\left(-58-3 \beta+\beta^{2}\right)}  \tag{A55}\\
\frac{q_{2}}{q_{1}+q_{2}} & =\frac{-974-81 \beta+29 \beta^{2}}{52\left(-58-3 \beta+\beta^{2}\right)} \tag{A56}
\end{align*}
$$

## Appendix A10: Spillover effect on social optimal permit allocation

The large firm's market share without permit trading is defined as

$$
\begin{equation*}
s_{1}=\frac{q_{1}}{q_{1}+q_{2}} \tag{A57}
\end{equation*}
$$

Thus, if we substitute grandfathering permit allocation $\left(e_{1}^{g}, e_{1}^{g}\right)$ (equations (25) and (26)) and social optimal allocation $\left(e_{1 s}^{n}, e_{2 s}^{n}\right)$ (equations (17) and (18)) into (A57) respevtively and use
the same numerical simulations, we get


Figure A3: Comparison of large firm's market share between grandfathering and social optimal permit allocations

From Figure A1, it is clear that giving the large firm more permits (from $e_{1}^{g}$ to $e_{1 s}^{n}$ ) increases its market share.

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[^0]:    ${ }^{1}$ Firms are forced to use all permits.

[^1]:    ${ }^{2}$ Details are provided in Appendix A5.

[^2]:    ${ }^{3}$ Through an international agreement, for expamle.
    ${ }^{4}$ See Appendix 8.1.1.

[^3]:    ${ }^{5}$ See Appendix A8.1.2.
    ${ }^{6}$ Even though in Appendix A3 we derive that the sign of $\frac{\partial x_{i}}{\partial e_{j}}$ depends on spillover, the effect of $\frac{\partial x_{i}}{\partial e_{i}}$ always dominates $\frac{\partial x_{i}}{\partial e_{j}}$. This is an even more important result with fixed amount of permits.
    ${ }^{7}$ From Table 1, total production increases in spillover rate.

[^4]:    ${ }^{8}$ In our model, allocating permits in proportion to output or to pollution is equivalent. This is because of the one-to-one relationship between production and pollution.

[^5]:    ${ }^{9}$ See Appendix A10 for details.

[^6]:    ${ }^{10}$ The results are robust for changes in the parameters.

