How Much Greener is Really Green? -
Carbon Taxation Design and Resource Extraction

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- preliminary version -

Abstract

Recently, there have been increasing doubts that further increases in carbon taxes, which enjoy an ever increasing public support, are a proper instrument to slow down Global Warming. Indeed, our analysis confirms that, under some assumptions, an acceleration of "green" policies leads to the opposite effect since resource owners try to escape their misery by pushing today's extraction even more. However, it is not true in general, but depends on the long-run expectations of the resource owners and on the policy design itself. In any case, we are able to suggest a carbon taxation design that has all desired properties, namely being climate friendly and politically feasible.

Keywords: Global Warming, Carbon taxation, Optimal taxation

JEL Classification: Q38, Q54, H21

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1 Introduction

One of the biggest challenges humanity is facing in the twenty-first century is Global Warming. Scientists with very different backgrounds are involved in the policy debate on how to cope with this problem. Obviously, the vast majority will argue that the industrialized nations have to reduce their demand for oil and other hydrocarbons in order to emit less of the climate-damaging carbon dioxide gas. But however logical this requirement seems it does not reflect reality: Even though 175 countries have ratified the Kyoto protocol so far, carbon dioxide emissions are still on a rising scale.

In general, a market for a good is characterized by a demand and a supply side. In case of the hydrocarbon market, the supply side has been neglected to a high degree, by both economists and politicians. However, since it is only possible to consume fossil fuels that have been extracted beforehand, supply or extraction of resources, respectively, is crucial for the accumulation of carbon dioxide in the atmosphere. Hence, the market equilibrium is determined by the demand for fossil fuels and by the supply, so that an analysis of both components is necessary. Thus, the question arises whether today's carbon taxation design can actually induce resource owners to extract on a lower scale, given the demand function for fossil fuels. In this context, [Sinn 2007] shows that a gradually increasing tax on hydrocarbon may lead to more extraction in the short-run as it changes the optimal time-path of the resource owner, i.e. it makes extraction in the distant future more costly, and hence, extraction today more favourable. Essentially, this kind of analysis goes back to the theory of [Hotelling 1931].

The purpose of this article is to examine different carbon taxation designs in a comprehensive dynamic-optimization model for a supplier of a non-renewable resource. Thereby, we allow for any symmetric structure of the supply side, since our analysis for oligopolistic producers includes the competitive market and the monopoly as border cases. In general, we think of two phases in the maximization problem of the resource owner: The first stage covers the period, when current carbon taxation policies are already settled (until time T), and the second stage represents everything that comes afterwards, i.e. when no climate targets are specified yet. The resource owner knows about the tax policy that will be applied up to T, and forms expectations about the long-run policies after T. Obviously, as T is in the distant future, it is reasonable to assume that the resource owner will assess the remaining stock of his resource at a certain value per unit. This value may actually be interpreted as the price of a backstop-technology that becomes available in T, so that the carbon taxation policy points towards this backstop-technology.[3] An interesting question to ask in this context is how the extraction path of the resource owner is affected when the announced tax policy up to T is changed unexpectedly. The impact on the climate is obviously positive if the policy ensures that extraction is postponed to future periods, whereas it is negative if extraction is brought forward.

[3]See e.g. [Davison 1978], [Kamien/Schwartz 1978] and [Heal 1976] for a model with backstop-technology.

We show that the climate impact of a further greening of government policies, i.e. an additional increase in the carbon taxes, highly depends on the carbon taxation design itself, but also on the long-run expectations of the resource owner and the internal rate of return that is demanded.[4] The term carbon taxation design means on the one hand the choice of a unit-tax or an ad-valorem tax, and on the other hand the various implementation options by choosing a specific functional form and thus a specific time path for the development of the tax.

In general, a further tax increase tends to be beneficial if the resource owner's long-run expectation regarding the tax is not affected by the present policy change. In contrast, if he incorporates the increase in the tax trend entirely, then this policy change will typically lead to more extraction in the short-run. In the latter case, a reduction of the tax growth might be a climate friendly option, however, as Global Warming advances, a tax-cut with respect to an already defined policy framework will obviously not be an acceptable solution in a political context.

However, we are able to specify for any given carbon tax time-path a transformation term with three desirable properties: (1) it increases the carbon tax rate at any point in time compared to the current time-path (2) it postpones extraction to the interval after the period for which a reduction is intended (3) it is independent of the long-run expectations of the resource owners. Thus, a further tax increase can be clearly beneficial from a Global Warming perspective if the carbon taxation design is appropriately chosen. A remarkable result of our analysis is that the conditions for a tax increase to be positive or negative for Global Warming are independent of the market structure, i.e. regardless of the supply market being controlled by a cartel like the OPEC or being perfectly competitive.

The remaining chapters of the paper is organized as follows: In Section 2 we use the maximum principle to solve a resource extraction problem in a dynamic optimization model. Thereby, we derive an optimal extraction path and an optimal price path under the assumption of differing profit functions for the short-run and a long-run planning horizon of the resource owner. We then analyze the effects of changes in the carbon taxation design for the extraction path and prove for which functional form a further tax increase is beneficial in the sense, that resource extraction is postponed to later points in time. We also analyze the effects of differing expectations of the resource owner about the persistence of a carbon tax increase. Complementary to our theoretical analysis, we present in Section 3 data on current carbon taxation trends. In Section 4, we summarize our results and propose policy measures.

2 Model: Dynamic Optimization

The purpose of our analysis is to get a deeper understanding of how a change in the tax trend of a carbon tax affects extraction behaviour. Due to the dynamic

[4][Farzin 1984] was first to describe the relationship between the discount factor and resource extraction.

features of resource extraction, a dynamic optimization model is used. We assume that the market consists of N identical resource owners, each of them faces the objective function stated in (1), where p_t is the price of the resource, R_t the quantity extracted, c_t the underlying cost function and v_t the tax function. As the resource owner has a time preference we introduce ρ as required internal rate of return, being the discount rate. First we define the problem with a tax that is independent of the price, however, we show afterwards that a similar result can be derived for an ad-valorem tax as well.

$$\max_{R_t} \int_0^\infty e^{-\rho t} \left[p_t(R_t) \cdot R_t - c_t \cdot R_t - v_t(\theta) \cdot R_t\right] dt \tag{1}$$

$$\dot{S} = -R_t \tag{2}$$

$$S(0) = S_0 \tag{3}$$

$$\lim_{t \to \infty} S(t) \geq 0 \tag{4}$$

The equation (2) states the law of motion that determines how much stock is still in situ at time t. Moreover, equation (3) constitutes the initial condition and equation (4) the terminal condition. Moreover, we divide the time horizon of the resource owner into a "well informed" short-run and into an "uninformed" long-run planning interval. The first planning interval is effective for $t < T$, and we assume the resource owner to know the market structure (i.e. the demand function) and the government announced tax policy. Thus, the inverse demand and the tax remain functions of their original variables.

$$p_t = p_t\left(R_t\right) \tag{5}$$

$$v_t = v_t(\theta) \tag{6}$$

For the long-term $t \geq T$, i.e. when no climate targets are specified yet, the resource owner has simple expectations regarding the tax policy and the market price or his personal valuation of the remaining stock in situ, respectively, and equations (7) and (8) become valid.[5]

$$p_t = P \tag{7}$$

$$v_t = W + \beta \cdot v_T(\theta) \tag{8}$$

The parameter $\beta \geq 0$ indicates the resource owner's expectation about how persistent the tax level at the end of the announced tax period is in the long-run. For $\beta < 1$ he assumes that the tax level after time T will underproportionally depend on the short-run tax level (however, the total tax level can still be higher than in the short-run if $W > 0$), for $\beta > 1$ he expects the tax level to depend

[5] For simplicity we assume the "uninformed" long-run time horizon to begin at the same point in time for the tax policy and for the demand structure. This assumption could be easily relaxed, and we leave it to the reader to show the results with differing switching points.

overproportionally on the short-run level. In general, the resource owner may also assume that the tax is going to be reduced to a minimum level W if e.g. the climate goals are achieved by then or Global Warming has shown not to be that dramatic as many expected it to be. Thus the constant W describes the part of the tax expectation that is independent of the announced developments, while the latter part describes influence of the current tax policy on the expectations. The current value Hamiltonian takes the form shown in equation (9) and we can apply the maximum principle to solve for the optimal extraction path.

$$H^c = p_t(R_t) \cdot R_t - c_t \cdot R_t - v_t(\theta) \cdot R_t - \lambda_t \cdot R_t \tag{9}$$

$$\frac{\partial H^c}{\partial R_t} = \frac{\partial p_t(R_t) \cdot R_t}{\partial R_t} - c_t - v_t(\theta) - \lambda_t \overset{!}{=} 0 \tag{10}$$

$$\dot{\lambda}_t \overset{!}{=} \rho \cdot \lambda_t - \frac{\partial H^c}{\partial S_t} \tag{11}$$

We assume for the remainder of our analysis that the resource stock is big enough so that it will not be exhausted before T. The complementary analysis of a resource owner who actually depletes the resource completely by T is presented in the Appendix. From (10), (11) and the conditions (7) and (8) we can derive the shadow-price of a unit more stock at the beginning of the maximization problem, λ_0.

$$\lambda_0 = e^{-\rho T} \cdot [P - c_T - W - \beta \cdot v_T(\theta)] \tag{12}$$

Since we aim to analyze the effect of different tax policies on the extraction path explicitly, we assume a specific inverse demand function. In a very general formulation let the inverse demand function satisfy equation (13), where Q_t is the total quantity supplied by all resource owners and A, B and α represent parameters such that price is falling in total quantity supplied[6]. Additionally, it seems reasonable to make the assumption that price must be nonpositive when supply approaches infinity. This gives us the following condition on the parameters A and α: If $\alpha < 0$, then $A \leq 0$.

$$\widetilde{p}_t\left(Q_t\right) = A - B \cdot Q_t^{\alpha} \tag{13}$$

For a Cournot competition and with η being the market share of total supply of the representative resource owner, the individual inverse demand function and marginal revenue have the following form:

$$p_t = A - B \cdot \left[[1-\eta] \cdot Q_t + \underbrace{\eta \cdot Q_t}_{R_t} \right]^{\alpha} \tag{14}$$

$$\frac{\partial p_t\left(R_t\right) \cdot R_t}{\partial R_t} = \left[A - B \cdot [[1-\eta] \cdot Q_t + R_t]^{\alpha}\right] - B \cdot \left[\alpha \cdot [[1-\eta] \cdot Q_t + R_t]^{\alpha - 1}\right] \cdot R_t \tag{15}$$

[6]Since the inverse demand function is monotonically decreasing we know that $\partial p_t / \partial Q_t = -\alpha \cdot B \cdot Q_t^{\alpha-1} < 0$ and thus $\alpha \cdot B > 0$.

Since resource owners are all identical, we can substitute $R_t = \eta \cdot Q_t$ to include the reaction of the other resource owners and thereby are able to derive the effects in the total market in terms of a positive analysis. Therefore, we will use for the remainder the following result[7]:

$$\frac{\partial p_t(R_t) \cdot R_t}{\partial R_t} = A - B \cdot \underbrace{\frac{[1 + \eta \cdot \alpha]}{\eta^\alpha}}_{m} R_t^\alpha \tag{16}$$

$$\varepsilon_{R_t,p_t} = \frac{1}{\eta \cdot \alpha} \cdot \left[1 - \frac{A}{B \cdot Q_t^\alpha}\right] = \frac{1}{\eta} \cdot \varepsilon_{Q_t,p_t} < -1 \tag{17}$$

It is easy to see that in equation (17) the price-elasticity of supply of the particular resource owner is equal to the total price-elasticity if his market share η is one. On the other hand, if his market share is very small as it is the case in a competitive market, his price-elasticity approaches infinity. Thus, we are able to study this model for any symmetric market structure between a monopoly with only one resource owner and a competitive market with an infinite number of resource owners. For the solution not to be trivial, the individually observed price-elasticity has to be smaller than unity since it would otherwise always be optimal to further reduce extraction for positive extraction quantities.

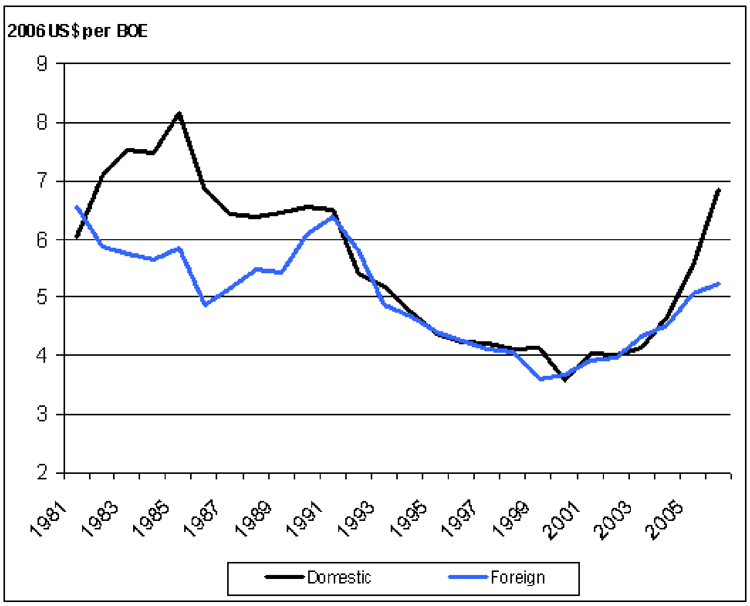
2.1 Cost function

Having discussed the revenue side of the objective function, we can now turn our attention to the cost function of the resource owner. We draw our functional form of the cost function from real world data since we provide a positive analysis with our model. It is rather difficult to get the correct figures on the cost of extraction, however, before simply making a false assumption, it may be reasonable to have a look at some data that is available. In Figure 1 we see the direct costs of extracting oil and gas for the 28 largest private companies for the period 1981-2006. Obviously, the costs per barrel of oil-equivalent peaked at points in time when the price of oil was high. In these periods, these companies found it worthy to extract much, and therefore also exploited sources that are more expensive. Another plausible explanation might be that the companies decided to employ more capital and labour in order to extract more from a given resource, but there exist decreasing returns to scale for these production factors. In any case, the costs went down as the price got back to its expected path. As a result, the extraction costs per barrel are observed to be rather slightly falling over the last 25 years with a recent increase as the oil prices rose. For our model - being supposed to serve actually for deriving policy advices about changes in the tax rate - we assume the cost function to be linear in the amount of extracted resource and time-constant while the results could be easily

[7] This result includes the solutions for a monopoly with $A - 2 \cdot B \cdot R_t^\alpha$ and $A - B \cdot Q_t^\alpha$ for the perfect competitive case

generalized to a more complex cost function. Hence, we let $c_t = C$, where C is the constant marginal cost.

Figure 1: Direct costs of extracting oil and gas for the 28 largest private companies for the period 1981-2006



Source: EIA

2.2 Results, unit tax

The optimal extraction path of the resource owner for a unit tax is described by equation (18) where C is the marginal cost as described in Section 2.1.

$$R_t = \left[\frac{A - c_t - v_t(\theta) - e^{\rho[t-T]} \cdot [P - C - W - \beta \cdot v_T(\theta)]}{m \cdot B} \right]^{\frac{1}{\alpha}} \text{ for } t < T \quad (18)$$

At first glance an interpretation seems difficult, however, we can simplify the equation to get the intertemporal optimality condition in equation (19). It states that the marginal profit of any period t must equal the present value of the prespecified net price in the long-term. Moreover, at this point we get back to one assumption we made regarding the inverse demand function, namely that $A \leq 0$ if $\alpha < 0$. We must exclude the case that the marginal revenue is negative in any period. The variable b can only be negative if $\alpha < -1$. However, then by assumption A would have to be negative which must not be the case. To get a reasonable result for our extraction path we therefore have to require $\alpha > -1$ such that the variable m is positive in any case.

$$\underbrace{\underbrace{A - m \cdot B \cdot R_t^{\alpha}}_{\text{marginal revenue}} - \underbrace{C}_{\text{marginal cost}} - \underbrace{v_t(\theta)}_{\text{marginal tax}}}_{\text{marginal profit}}$$

$$= \underbrace{e^{\rho[t-T]} \cdot [P - C - W - \beta \cdot v_T(\theta)]}_{\text{PV of marginal profit of extraction in T}} \quad \text{for } t < T \tag{19}$$

It should be noted that this result holds only if the stock S_0 is large enough so that the terminal condition is not violated. To allow for the resource to be exhausted before T the approach has to be generalized as shown in appendix I. An interesting observation is that as the planning horizon approaches infinity, the intertemporal optimality condition reduces to the standard static optimality condition of marginal revenue equalling marginal cost plus marginal tax.

Moreover, we are able to derive a rule that describes the relationship between the internal rate of return and the price path (equation (20)). It is the Hotelling equivalent rule for our problem.

$$\rho = \frac{\dot{p}_t - \frac{\partial v_t(\theta)}{\partial t}}{p_t - A + \frac{A - C - v_t(\theta)}{[1+\eta \cdot \alpha]}} \tag{20}$$

It is easy to see the familiar results for this rule by setting $\eta = 1$ for the monopoly case or $\eta = 0$ for perfect competition.

2.2.1 Supply elasticities of a change in carbon taxation

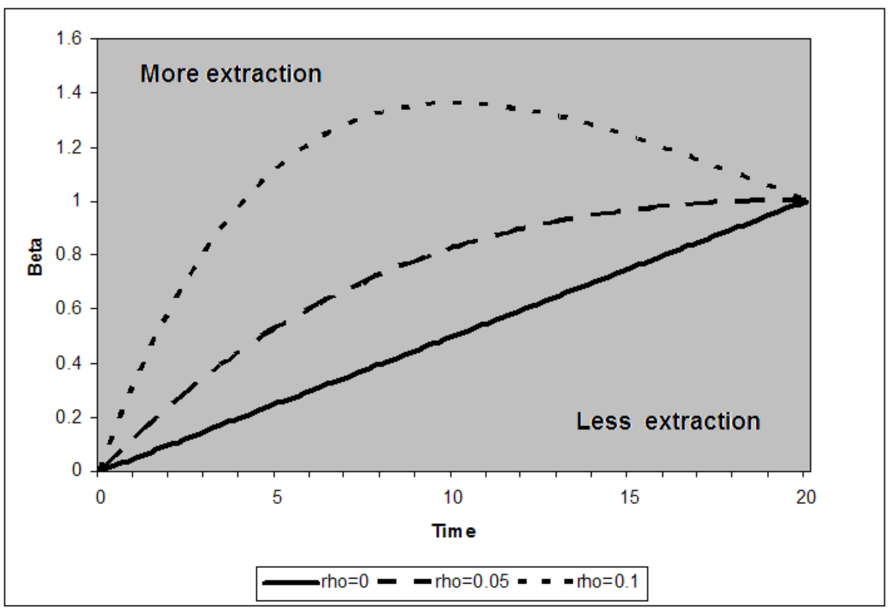
We can now analyse what happens if green taxes increase as the fear of global warming continues to rise. We present in Section 3, Figure 6, data on how green taxes have developed in the past. But will a further increase in the tax growth really lead to less extraction? For a carbon tax policy to mitigate global warming it is necessary that this policy induces resource owners to postpone extraction. Let us analyze the effect of a change in the tax trend θ on the supply[8]. The supply elasticity of a change in this tax trend of the unit tax is shown in equation (21).

$$\varepsilon_{R_t,\theta} = \frac{\partial R_t}{\partial \theta}\frac{\theta}{R_t} = \underbrace{\frac{\theta}{\alpha \cdot m \cdot B}[R_t]^{-\alpha}}_{\oplus} \cdot \left(\underbrace{\beta \cdot e^{\rho[t-T]} \cdot \frac{\partial v_T(\theta)}{\partial \theta}}_{\oplus \text{ or } 0} \underbrace{- \frac{\partial v_t(\theta)}{\partial \theta}}_{\ominus} \right) \tag{21}$$

[8] We define the tax to be an increasing function of $\theta \rightarrow \partial v_t(\theta)/\partial \theta > 0$

We can see that the tax elasticity of supply is indepentent of the market structure and only depends on the term in the last parenthesis. The tax-policy is neutral for any given period if the term in this last parenthesis is equal to zero, since the product of all other variables is strictly positive. If the term is negative the policy change will reduce extraction in that period, and it will increase extraction if the term is positive. Let us recall that β measures the resource owners expectations of the effect of a change in the current tax-policy on the taxes after the period for which the tax-policy is announced. If $\beta = 0$, then the policy change does not affect the expectation with regard to the long-run taxes. If $\beta = 1$ then the resource owner expects the policy change to persist in the long-term on a one to one basis. For a given tax function we can now determine the "policy-neutral" level of β in any period by setting the term in the parenthesis equal to zero.

Figure 2: Climate neutral expectations



In equation (22) we show the condition for a tax change to be neutral at time t. If a tax increase is not optimal in the way as we specify it in Section 2.3, the tax increase will lead to more extraction at the beginning of the maximization problem. More extraction will persist until the neutrality-condition becomes valid for the first time. Thereafter, less extraction takes place either until the condition becomes valid once more or time T is reached.

$$\beta(t) = e^{\rho[T-t]} \cdot \frac{\partial v_t(\theta)/\partial\theta}{\partial v_T(\theta)/\partial\theta} \tag{22}$$

If we now observe this condition exemplarily for a the tax being an arbitrary linear function of time then $\beta(t) = e^{\rho[T-t]} \cdot t/T$. Figure 2 shows the effect of different discount rates for the neutrality of a tax increase. The repective fuction that describe the values of $\beta(t)$ for which the tax increase would be extraction neutral becomes more concave as the discount rate rises, and thus the area under the function in which the tax increase is beneficial for the environment gets larger.

2.3 Optimal tax policy

Having a close look at equation (21) again we can see that the sufficient condition for a tax increase to be strictly beneficial is $\beta \cdot e^{\rho[t-T]} \cdot \partial v_T(\theta)/\partial\theta = 0$ since than the elasticity will be stricly negative. This is either the case if $\beta = 0$ and thus the tax increase does not affect long run expectations, or if the tax rate in the long run stays unchanged. Figure 3 displays a stylized optimal tax policy according to that condition. Any policy that changes the tax path such that it is higher than before for $t < T$ but not for $t \geq T$ is optimal in a way that it will postpone extraction to the long-run.

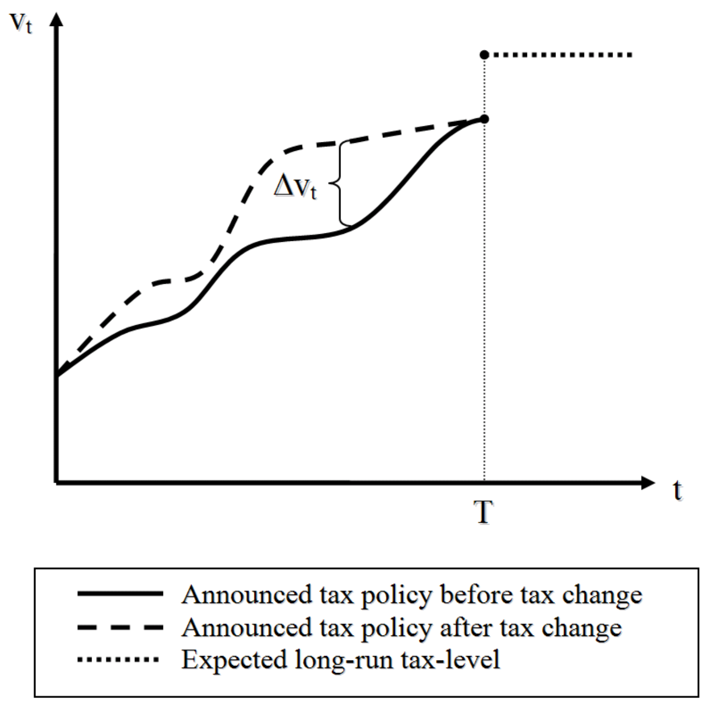
Thus an optimal policy crucially depends on its credibility. Its focus should be not to rise the expectations about long-run taxes (i.e. in model terms: an increase in β) and policy makers should then strictly stick to the aimed long-run taxation level. Both of these advices seem to be interconnected in reality and become more difficult to achieve as the end of the period for which the tax policy is initially announced comes closer. Additionally the time scope for environmental policy is then limited to the period for wich a tax policy is currently known (and believed). Any announcment which is higher than the current policy for T or the current expectations for $t > T$ will accelerate todays extraction. Increasing taxes in the short-run will postpone extraction if it is credible that the expected tax levels in T and beyond will not be affected. However, it seems questionable that this policy might be credible. The basic question is whether those tax increases implicitly change the expectations, or whether they can be evaluated separately.

2.4 Results, ad-valorem tax

If we assume that there exists an ad-valorem tax instead of a unit tax, the orginally stated maximization problem remains unchanged. However, the revenue function is now $\varphi_t(\theta) \cdot R_t \cdot p_t = \varphi_t(\theta) \cdot R_t \cdot (A - B \cdot [[1-\eta] \cdot Q_t + R_t]^{\alpha})$ and the term previously describing the unit-tax disappears.[9]The new optimal extraction path is described by equation (23).

[9]The tax function has now the following property: $\frac{\partial\varphi_t(\theta)}{\partial\theta} < 0$.

Figure 3: optimal tax



$$R_t = \left[\frac{\varphi_t(\theta) \cdot A - C - e^{\rho[t-T]} \cdot [\varphi_T(\theta) \cdot P - C]}{\varphi_t(\theta) \cdot m \cdot B} \right]^{\frac{1}{\alpha}} \text{ for } t < T \tag{23}$$

Again we can simplify the equation to get the intertemporal optimality condition in equation (24). It states that the marginal profit of any period t must equal the present value of the marginal profit in T.

$$\underbrace{\underbrace{\varphi_t(\theta) \cdot (A - m \cdot B \cdot R_t^{\alpha})}_{\text{marginal revenue net of tax}} - \underbrace{C}_{\text{marginal cost}}}_{\text{marginal profit}} = \underbrace{e^{\rho[t-T]} \cdot [\varphi_T(\theta) \cdot P - C]}_{\text{PV of marginal profit in T}} \tag{24}$$

2.4.1 Supply elasticities of change in carbon taxation, ad-valorem

We can analyse the effect on the supply of a change in the tax growth. Since it is not very convincing that green taxes will remain constant forever as the fear of global warming will continue to rise, this is the interesting case. We show the elasticity of supply for a change in tax growth in equation (25).

$$\varepsilon_{R_t,\theta} = \underbrace{\frac{\theta}{\alpha \cdot m \cdot B}\left[A - \frac{C + e^{\rho[t-T]} \cdot [\varphi_T(\theta) \cdot P - C]}{\varphi_t(\theta)}\right]^{-1} \cdot \frac{1}{\varphi_t(\theta)^2}}_{\oplus}$$

$$\cdot \left(\underbrace{\frac{\partial \varphi_t(\theta)}{\partial \theta} \cdot \left[C + e^{\rho(t-T)}\left(P - \beta \cdot P + \beta \cdot P \cdot \varphi_T(\theta) - C\right)\right]}_{\ominus} \underbrace{- \beta \cdot e^{\rho(t-T)} \cdot P \cdot \varphi_t(\theta) \cdot \frac{\partial \varphi_T(\theta)}{\partial \theta}}_{\oplus \text{ or } 0} \right) \tag{25}$$

As for the unit-tax, the sign of the elasticity is determined by only one term, this time being the last factor of the product. Once again, the elasticity is clearly negative if β is zero or the change in taxation has no effect on the long-run taxation. Therefore, our qualitative results for the unit-tax hold also for an ad-valorem tax and in principle there is no advantage or disadvantage in choosing an ad-valorem tax instead of a unit-tax.

2.5 Short-run and long-run planning horizon

For our analysis we assume the overall planning horizon of the resource owner to be devided into a short-run period and a long-run period. We define the short-run period as the time, for which the resource owner is informed about governments' climate targets and about the demand function. In practice, you might think of the short-run being associated with the period for which the Kyoto agreement is already defined. Those announcements are included in the maximization problem on a one to one basis. Moreover, the resource owner forms expectations about the tax regime after current climate target dates will be reached. We leave these expectations very general as we define the long-run carbon tax as the sum of some constant and a share of the announced tax rate at the end of the short-run, where the share also may exceed unity. In general, it is also reasonable to assume that a resource owner has only vague knowledge about long-run demand. It is uncertain whether a backstop-technology will arise at some point, and limit the price of the resource. Therefore, a resource owner will include a long-run price in his today's maximization problem, which reflects his valuation of a unit of the resource that is not extracted at the end of the short-run. If he assumes a backstop-technology to be in place by the end of his short-run planning horizon, then his valuation is limited by the price of the backstop-technology. Certainly the resource owner's information on the demand and on the government tax policies may have different end points, however, this factum does not change the results, so we consider only the case in which both coincide.

2.6 Two-period interpretation of the results

Our previous analysis can easily be illustrated in a two period diagram, where we use our definitions for the short-run and the long-run as before. Obviously, the two-periods illustration is less powerful than the previously derived extraction path, since it does not incorporate the adjustments within the short-run, but only the overall extracted quantity. However, it is still powerful enough to demonstrate the main effects. Using equation (18) to substitute R_t in the solution to the differential equation in (2) and defining the present value of the short run marginal profit in t as $\overline{MP} \equiv e^{-\rho t} \cdot [A - m \cdot B \cdot R_t^{\alpha} - C - v_t(\theta)]$ in equation (19) we can determine $\widetilde{S}$, the extraction until T, as:

$$\widetilde{S} = \int_0^T \left[\frac{A - C - v_t(\theta) - e^{\rho t} \cdot \overline{MP}}{m \cdot B} \right]^{\frac{1}{\alpha}} dt \qquad \text{for } t < T \tag{26}$$

Assuming for simplicity a linear demand function ($\alpha = 1$) it is trival to see that $\partial\widetilde{S}/\partial\overline{MP} < 0$. Knowing that the present value of the marginal profit is equal in all periods it holds for the long run marginal profit that $MP_T = \overline{MP}$ for $t \geq T$. Now we can take the inverse of both functions to display the relation as shown in Figure 4. The figure allows an easy interpretation on how the long-run carbon tax expectation of the resource owner influences the extraction decision. In particular, the example explains that no increase in carbon taxation itself is necessary for a shift of extraction to the short-run to occur. If the resource owner begins to belief that the short-run carbon taxes will persist to a higher degree in the long-run (e.g. $\beta \uparrow$ or $v_T(\theta) \uparrow$), he will consequently bring extraction forward to the short-run period. In contrast, if governments could convince resource owners that the carbon taxation policies will be relaxed in the long-run, e.g. after climate targets are achieved, the long-run marginal profit curve would shift upwards. Then extraction would be postponed into the long-run.

Figure 5 describes a particular situation, in which carbon taxes actually do change. However, as the long-run marginal profit curve remains unaffected, this tax policy is conform to our optimal tax policy. Therefore, we observe that less is extracted in the short-run and more is postponed to the second period. This conclusion, however, must not be generalized to all tax increases, as a specification that does not match our optimality conditions also shifts the long-run marginal profit curve. For this case, the two-period presentation is not a suitable method since the extraction behaviour within the short-run period is not clear-cut.

Figure 4: Change in expectations

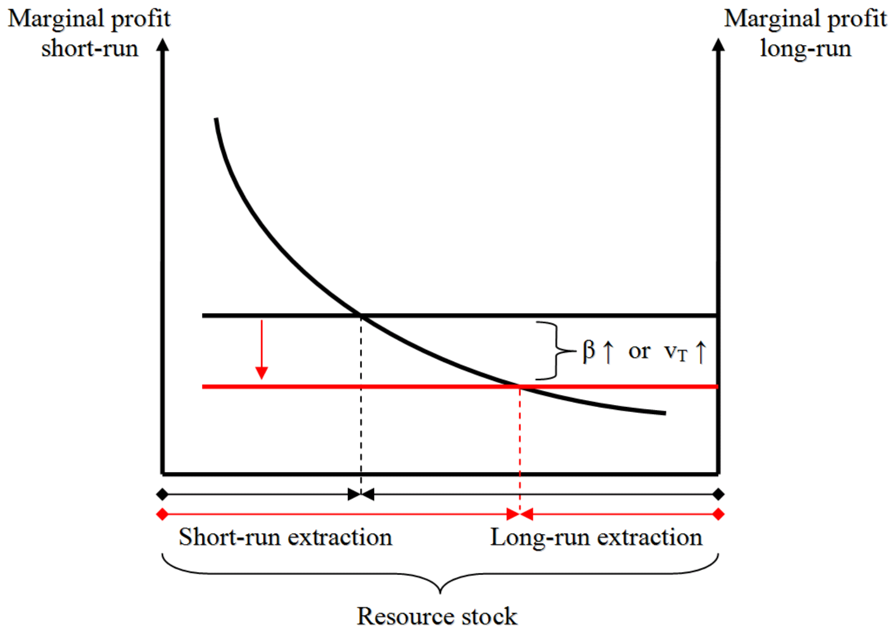
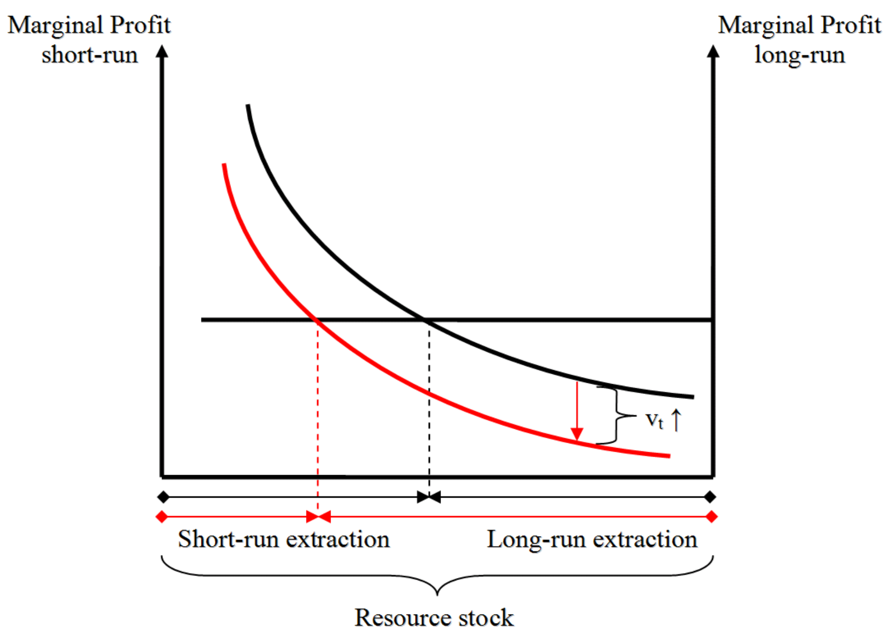


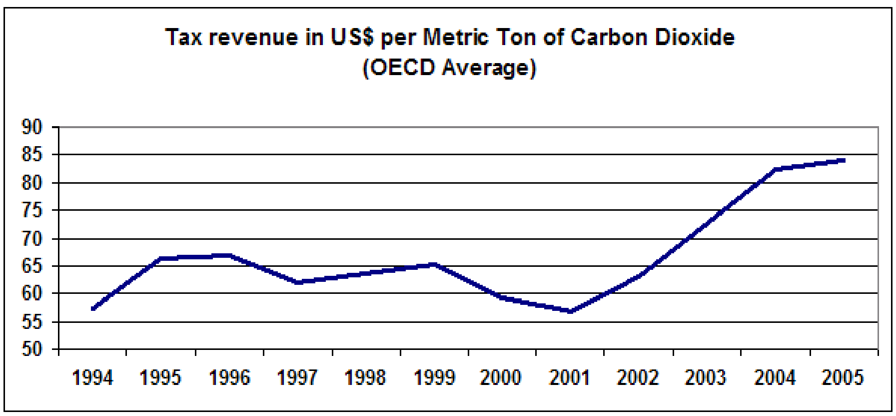
Figure 5: Change in the tax-policy



3 Current trends in taxation

Complementary to our theoretical analysis, we present in this section data on current carbon taxation trends. In principle, this article is an in-depth discussion of policies becoming greener over time with the aim of mitigating global warming. We have left it open so far whether the common feeling of carbon taxes increasing over time can be confirmed by data. However, Figure 6 verifies this hypothesis quite strongly. In the period from 1994-2005 the tax revenue per metric ton of carbon dioxide emitted increased from US$ 57 to US$ 84, in real terms.[10] Especially the increase after 2001 is remarkable; there is no doubt that OECD policies are recently becoming greener.

Figure 6: Tax revenues development

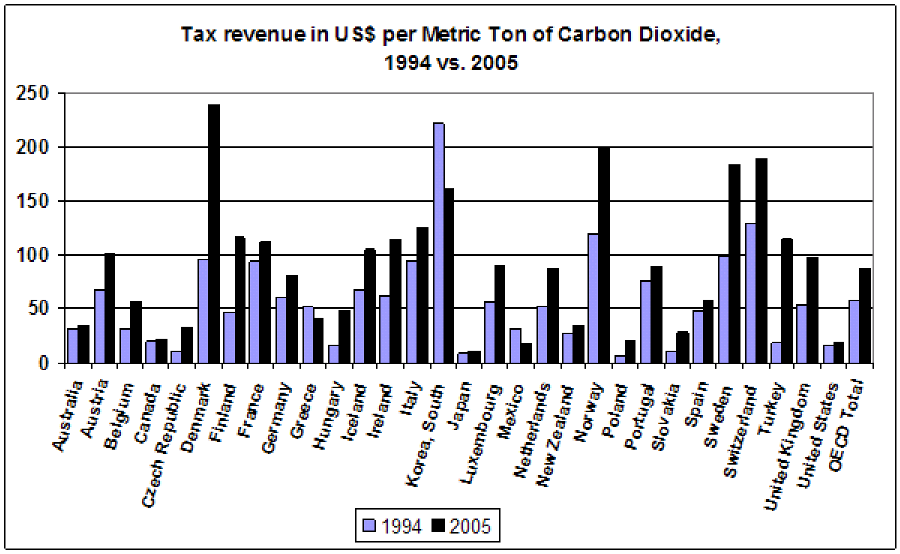


Source: own calculations, EEA, IMF, EIA

The Scandinavian countries (Denmark, Norway, and Sweden) and Switzerland have the highest carbon taxes within the OECD in 2005 (Figure 7). Moreover, the two by far biggest polluters, the United States and Japan, maintain a very low tax level compared to almost all other member states. Although, carbon taxes are increasing by almost 4 percent on average in the OECD, the developments in the single member countries are rather different (see Figure 8). Turkey reports the highest growth rates, however, as we have seen previously it started from quite a low level in 1994. It is again the Scandinavian countries, and here also the some Eastern European countries that have the fastest greening policies. On the other hand, only Greece, South Korea and Mexico have lowered their taxes on carbon dioxide emission in the period 1994-2005.-

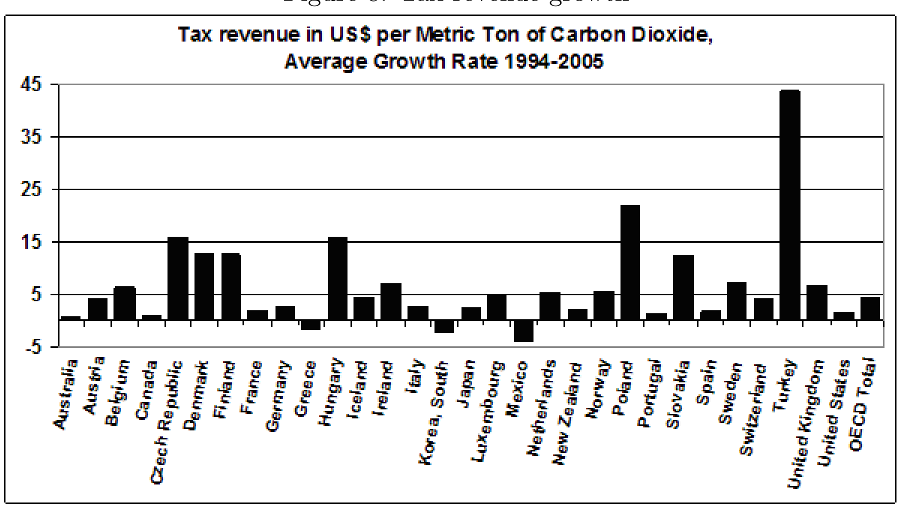
[10]Figure 6 displays total OECD tax revenue in US$ devided by total OECD carbon dioxide emissions.

Figure 7: Tax revenues 1995 vs. 2005



Source: own calculations, EEA, IMF, EIA

Figure 8: Tax revenue growth



Source: own calculations, EEA, IMF, EIA

In general, we find broad confirmation of the hypothesis that carbon taxes are rising over time. Thus our analysis is all the more useful, as we have shown which effects may occur when governments increase taxes in a non-optimal way.

4 Conclusion

One of the biggest challenges humanity is facing in the twenty-first century is Global Warming. Recently, there have been increasing doubts that further increases in carbon taxes, which enjoy an ever increasing public support, are a proper instrument to slow down Global Warming. Indeed, our analysis confirms recent findings that, typically, an accelaration of green policies leads to the opposite effect since resource owners try to escape their misery by pushing today's extraction even more. A highly important result of our model is that the long-run expectations of resource owners play a key role for the climate impact of increases in carbon taxation. We have shown that an increase in taxation is clearly beneficial whenever long-run tax expectations remain unchanged. Also, it is important that tax increases in the short-run do not spillover to the long-run tax level. If a tax is designed in such a way that it rises taxation level up to some point in time, but does not change the long-run taxes, then we show that extraction will be postponed.

Nevertheless, it seems questionable whether a tax increase in the short-run can be accomplished that does not alter long-run expectations. Today's tax increases and long-run tax levels seem naturally connected, however, it does not need to be the case if announcements are made credibly. Climate policies should be settled up to some point in time, and should be announced not to increase anymore afterwards. Especially, if a backstop-technology will be available eventually, it may not be necessary to increase taxes anymore so that this policy is also conform to current climate goals. As other climate options, e.g. directly limiting the global demand are politically not feasible at the moment (as negotiations about the post-Kyoto agreement show), it is advisable to design a carbon tax regime that will slow down global warming. Based on our analysis, we therefore suggest to increase carbon taxes, however, simultaneously to specify an end point for this policy change, which must be communicated credibly to resource owners.

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5 Appendix

Short-run depletion in the simple case:

As we have analysed in the main part of this article a situation in which the resource owner does not deplete the resource completely in the short-run, we can restudy our results for the situation when the resource owner exhausts the stock by time T. Then:

$$\lambda_0 > e^{-\rho T} \cdot [P - C - W - \beta \cdot v_T(\theta)] \tag{27}$$

$$R_t = \left[\frac{A - C - v_t(\theta) - e^{\rho t} \cdot \lambda_0}{m \cdot B}\right]^{\frac{1}{\alpha}} \tag{28}$$

$$S_0 = \int_0^T \left[\frac{A - C - v_t(\theta) - e^{\rho t} \cdot \lambda_0}{m \cdot B}\right]^{\frac{1}{\alpha}} dt \tag{29}$$

For the case of a linear inverse demand function ($\alpha = 1$):

$$\lambda_0 = T \cdot \frac{\rho}{e^{\rho T} - 1} \cdot \left[A - \frac{S_0}{T} \cdot m \cdot B - C - \frac{\int_0^T v_t(\theta) dt}{T}\right] \tag{30}$$

$$R_t = \frac{1}{m \cdot B} \cdot \left[A - C - v_t(\theta) - \frac{e^{\rho t}}{e^{\rho T} - 1} \cdot \rho \cdot \left[A \cdot T - S_0 \cdot m \cdot B - T \cdot C - \int_0^T v_t(\theta) dt\right]\right] \tag{31}$$

$$\underbrace{\underbrace{A - m \cdot B \cdot R_t}_{\text{marginal revenue}} - \underbrace{c_t}_{\text{marginal cost}} - \underbrace{v_t(\theta)}_{\text{marginal tax}}}_{\text{marginal profit}}$$

$$= \underbrace{e^{\rho t} \cdot T \cdot \frac{\rho}{e^{\rho T} - 1} \cdot \left[A - \frac{S_0}{T} \cdot m \cdot B - C - \frac{\int_0^T v_t(\theta) dt}{T} \right]}_{\text{PV of marginal profit of extraction in T}} \tag{32}$$

Thus the implications for the sufficient condition for a tax increase to be envi-romentally beneficial are the same as before.

$$\varepsilon_{R_t,\theta} = \frac{\partial R_t}{\partial \theta} \frac{\theta}{R_t} = \underbrace{\frac{1}{m \cdot B} \cdot \frac{\theta}{R_t}}_{\oplus} \cdot \left(\underbrace{\frac{e^{\rho t} \cdot \rho}{e^{\rho T} - 1} \cdot \int_0^T \frac{\partial v_t(\theta)}{\partial \theta} dt}_{\oplus \text{ or } 0} - \underbrace{\frac{\partial v_t(\theta)}{\partial \theta}}_{\ominus} \right) \tag{33}$$

Special case $\rho \to 0$:

$$R_t = \frac{S_0}{T} + \frac{1}{m \cdot B} \cdot \left[\frac{1}{T} \cdot \int_0^T v_t(\theta) dt - v_t(\theta) \right] \tag{34}$$

Under constant marginal extraction costs, extraction is equally distributed over time if the current marginal tax is equal to the average marginal tax at any point in time. If this not true, then there exists an adjustment which depends on the difference between the current marginal tax and the average marginal tax. Note that $[e^{\rho t} \cdot T \cdot \rho] / [e^{\rho T} - 1] \to 1/T$ as $e^{\rho T} = 1$ for $\rho = 0$ and for a marginal variation $\rho = 0 + \varepsilon$ the first order approximation (which is equal to the true value for a marginal variation around a known value) of the exponential function is $e^{\rho T} = 1 + \rho T$.

Special case $T \to \infty$:

$$\underbrace{A - m \cdot B \cdot R_t}_{\text{marginal revenue}} = \underbrace{C}_{\text{marginal cost}} + \underbrace{v_t(\theta)}_{\text{marginal tax}} \tag{35}$$

This can easily be seen from equation (32) where the term in brackets must be non-negative as otherwise extraction cannot be profitable. Also, it is true that $T/ [e^{\rho T} - 1] \to 0$ for $T \to \infty$.

A general approach to the maximization problem if the ressource is not depleted in the short-run:

We can formulate the problem in a more general way since we know that in the short-run the elasticity of marginal revenue $MR(R_t)$ for a change in the tax development θ has the opposite sign of the elasticity of supply R_t for a change in the tax develpment θ. From equation (9) and equation (11) we can derive the general optimality condition (36).

$$\underbrace{\frac{\partial p_t(R_t^\star)\cdot R_t^\star}{\partial R_t^\star}}_{MR(R_t^\star)} = C + v_t(\theta) + e^{\rho[t-T]}\cdot\lambda_T^\star \tag{36}$$

Which is the optimal solution to the following generalized dynamic maximization problem in which not all remaining stock is liquidated in T.

$$\max_{R_t}\int_0^T e^{-\rho t}\left[p_t(R_t)\cdot R_t - C\cdot R_t - v_t(\theta)\cdot R_t\right]dt$$

$$+\int_T^\infty e^{-\rho t}\left[p_t(R_t)\cdot R_t - C\cdot R_t - W + \beta\cdot v_T(\theta)\cdot R_t\right]dt \tag{37}$$

$$s.t.\qquad p_t(R_t)\leq P\qquad \text{for } t\geq T \tag{38}$$

Where the asterix indicates that these are the optimal values as they follow from the solution to the dynamic optimization problem. Additionally, since $\lambda_T^\star = \lambda_T\left(S_T\left(R_t^\star(\theta)\right), W+\beta\cdot v_T(\theta), C\right)$, the marginal profit in T is the reference point for any current point in time. This specifiction is more general than the one before since we allow $\lambda_T^\star$ to decline in the optimal stock $S_T\left(R_t^\star\right)$ that is left at time T.

Totally differentiating equation (36) while keeping c_t constant, we derive:

$$\frac{\partial MR\left(R_t^\star\right)}{\partial R_t^\star}\cdot dR_t^\star + \frac{\partial MR\left(R_t^\star\right)}{\partial R_t^\star}\cdot\frac{\partial R_t^\star}{\partial\theta}\cdot d\theta =$$

$$\frac{\partial v_t(\theta)}{\partial\theta}\cdot d\theta + e^{\rho[t-T]}\frac{\partial\lambda_T\left(S_T\left(R_t^\star\right), W+\beta\cdot v_T(\theta), C\right)}{\partial S_T\left(R_t^\star\right)}\cdot\frac{\partial S_T\left(R_t^\star\right)}{\partial R_t^\star}\cdot dR_t^\star$$

$$+e^{\rho[t-T]}\cdot\left[\frac{\partial\lambda_T\left(S_T\left(R_t^\star\right), W+\beta\cdot v_T(\theta), C\right)}{\partial v_T(\theta)}\cdot\frac{\partial v_T(\theta)}{\partial\theta}\cdot d\theta\right.$$
$$\left.+\frac{\partial\lambda_T\left(S_T\left(R_t^\star\right), W+\beta\cdot v_T(\theta), c_T\right)}{\partial S_T\left(R_t^\star\right)}\cdot\frac{\partial S_T\left(R_t^\star\right)}{\partial R_t^\star}\cdot\frac{\partial R_t^\star}{\partial\theta}\cdot d\theta\right] \tag{39}$$

This can used to define the elasticity of supply for a marginal change in θ given the current optimal extraction path:

$$\varepsilon_{R_t^\star,\theta} = \frac{\frac{\partial v_t(\theta)}{\partial\theta} + e^{\rho[t-T]}\cdot\frac{\partial\lambda_T\left(S_T\left(R_t^\star\right), W+\beta\cdot v_T(\theta), C\right)}{\partial v_T(\theta)}\cdot\frac{\partial v_T(\theta)}{\partial\theta}}{\frac{\partial MR\left(R_t^\star\right)}{\partial R_t^\star} - e^{\rho[t-T]}\cdot\frac{\partial\lambda_T\left(S_T\left(R_t^\star\right), W+\beta\cdot v_T(\theta), C\right)}{\partial S_T\left(R_t^\star\right)}\cdot\frac{\partial S_T\left(R_t^\star\right)}{\partial R_t^\star}}\cdot\frac{\theta}{R_t^\star}$$

$$-\frac{\partial R_t^\star}{\partial \theta} \cdot \frac{\theta}{R_t^\star} \cdot \frac{\frac{\partial MR\left(R_t^\star\right)}{\partial R_t^\star} - e^{\rho[t-T]} \cdot \frac{\partial \lambda_T\left(S_T\left(R_t^\star\right), W+\beta \cdot v_T(\theta), C\right)}{\partial S_T\left(R_t^\star\right)} \cdot \frac{\partial S_T\left(R_t^\star\right)}{\partial R_t^\star}}{\frac{\partial MR\left(R_t^\star\right)}{\partial R_t^\star} - e^{\rho[t-T]} \cdot \frac{\partial \lambda_T\left(S_T\left(R_t^\star\right), W+\beta \cdot v_T(\theta), C\right)}{\partial S_T\left(R_t^\star\right)} \cdot \frac{\partial S_T\left(R_t^\star\right)}{\partial R_t^\star}} \tag{40}$$

$$\varepsilon_{R_t^\star,\theta} = \frac{\frac{\partial v_t(\theta)}{\partial \theta} + e^{\rho[t-T]} \cdot \frac{\partial \lambda_T\left(S_T\left(R_t^\star\right), W+\beta \cdot v_T(\theta), C\right)}{\partial v_T(\theta)} \cdot \frac{\partial v_T(\theta)}{\partial \theta}}{\frac{\partial MR\left(R_t^\star\right)}{\partial R_t^\star} - e^{\rho[t-T]} \cdot \frac{\partial \lambda_T\left(S_T\left(R_t^\star\right), W+\beta \cdot v_T(\theta), C\right)}{\partial S_T\left(R_t^\star\right)} \cdot \frac{\partial S_T\left(R_t^\star\right)}{\partial R_t^\star}} \cdot \frac{\theta}{R_t^\star} - \varepsilon_{R_t^\star,\theta} \tag{41}$$

$$\varepsilon_{R_t^\star,\theta} = \frac{1}{2} \cdot \frac{\frac{\partial v_t(\theta)}{\partial \theta} + e^{\rho[t-T]} \cdot \frac{\partial \lambda_T\left(S_T\left(R_t^\star\right), W+\beta \cdot v_T(\theta), C\right)}{\partial v_T(\theta)} \cdot \frac{\partial v_T(\theta)}{\partial \theta}}{\frac{\partial MR\left(R_t^\star\right)}{\partial R_t^\star} - e^{\rho[t-T]} \cdot \frac{\partial \lambda_T\left(S_T\left(R_t^\star\right), W+\beta \cdot v_T(\theta), C\right)}{\partial S_T\left(R_t^\star\right)} \cdot \frac{\partial S_T\left(R_t^\star\right)}{\partial R_t^\star}} \cdot \frac{\theta}{R_t^\star} \tag{42}$$

The denominator is negative since the first order effect of a rise in $R_t^\star$ on the marginal revenue is negative and dominates the second order effect on $\lambda_T^\star$. Thus as seen for the special cases, the sign of the elasticity depends only on the nominator, since the denominator is clearly negative and the ratio $\theta/R_t^\star$ is positive. Therefore, the sufficient condition for a tax increase to be beneficial remains $\partial v_T(\theta)/\partial\theta = 0$. However, more precisely, the necessary condition is:

$$\frac{\partial v_t(\theta)}{\partial \theta} + e^{\rho[t-T]} \cdot \frac{\partial \lambda_T\left(S_T\left(R_t^\star\right), W + \beta \cdot v_T(\theta), C\right)}{\partial v_T(\theta)} \cdot \frac{\partial v_T(\theta)}{\partial \theta} > 0 \tag{43}$$

Setting $t = T$ in the above equation and using condition (8) we can define a constant:

$$\frac{\partial \lambda_T\left(S_T\left(R_t^\star\right), W + \beta \cdot v_T(\theta), C\right)}{\partial v_T(\theta)} \cdot \frac{\partial v_T(\theta)}{\partial \theta} = -\frac{\partial W + \beta \cdot v_T(\theta)}{\partial \theta} \tag{44}$$

Substituting equation (44) in equation (43) we get the condition for a tax change to be extraction neutral:

$$\frac{\partial v_t(\theta)}{\partial \theta} - e^{\rho[t-T]} \cdot \frac{\partial W + \beta \cdot v_T(\theta)}{\partial \theta} = 0 \tag{45}$$

or

$$\frac{\partial v_t(\theta)}{\partial \theta} = e^{\rho[t-T]} \cdot \frac{\partial W + \beta \cdot v_T(\theta)}{\partial \theta} \tag{46}$$

Thus, for a tax increase that also raises the tax in T to be environmentally beneficial, the tax increase in any other period $t < T$ must be be larger than the net present value of the expected tax change in T.

$$\frac{\partial v_t(\theta)}{\partial \theta} > e^{\rho[t-T]} \cdot \frac{\partial W + \beta \cdot v_T(\theta)}{\partial \theta} \tag{47}$$

Derivation of the price-elasticity of supply:

$$p_t = A - B \cdot \left[[1-\eta] \cdot Q_t + \underbrace{\eta \cdot Q_t}_{R_t} \right]^{\alpha} \tag{48}$$

$$\varepsilon_{p_t,R_t} = \alpha \cdot \frac{B \cdot [[1-\eta] \cdot Q_t + R_t]^{\alpha-1} R_t}{A + B \cdot [[1-\eta] \cdot Q_t + R_t]^{\alpha}} \tag{49}$$

$$\varepsilon_{p_t,R_t} = \alpha \cdot \frac{B \cdot [[1-\eta] \cdot Q_t + \eta \cdot Q_t]^{\alpha-1} \eta \cdot Q_t}{A + B \cdot [[1-\eta] \cdot Q_t + \eta \cdot Q_t]^{\alpha}} \tag{50}$$

$$\varepsilon_{R_t,p_t} = \frac{1}{\eta \cdot \alpha} \cdot \frac{A + B \cdot [[1-\eta] \cdot Q_t + \eta \cdot Q_t]^{\alpha}}{B \cdot [[1-\eta] \cdot Q_t + \eta \cdot Q_t]^{\alpha-1} \cdot Q_t} \tag{51}$$

$$\varepsilon_{R_t,p_t} = \frac{1}{\eta \cdot \alpha} \cdot \left[\frac{[1-\eta] \cdot Q_t + \eta \cdot Q_t}{Q_t} + \frac{A}{B \cdot [[1-\eta] \cdot Q_t + \eta \cdot Q_t]^{\alpha-1} \cdot Q_t} \right] \tag{52}$$

$$\varepsilon_{R_t,p_t} = \frac{1}{\eta \cdot \alpha} \cdot \left[1 + \frac{A}{B \cdot Q_t^{\alpha}} \right] = \frac{1}{\eta} \cdot \varepsilon_{Q_t,p_t} \tag{53}$$