

Jobless Recoveries, Idle Productivity, and the Role of Capital

There is, however, another characteristic of what we call the trade cycle which our explanation must cover; namely, the phenomenon of the crisis—the fact that the substitution of a downward for an upward tendency often takes place suddenly and violently, whereas there is as a rule, no such sharp turning points when an upward is substituted for a downward tendency.

Keynes (1936)

Abstract

The U.S. economy is in a deepening recession in the moment. An important question is whether a “jobless recovery” will hit the economy once again. To answer the question, one first needs to resolve the puzzle of “jobless recoveries”.

This paper incorporates a hiring game, a simplified version of the seminal job matching model of Kelso and Crawford (1982), into a framework of an efficiency wage model to resolve this puzzle. Our Nash equilibrium shows that there is a spike in job destruction when an economy is hit by a recession. The economy may not create new jobs in the immediate recovery of the recession. After a substantial duration of an economic recovery, the economy will eventually create new jobs together with new addition of capital. Unlike the job destruction, the job creation is a gradual process.

Beyond the jobless recoveries, we also show how technological improvements may lead to an increase in unemployment rate and a reduction in the investment of capital goods in the short-run, as observed in a number of empirical studies, and demonstrate why inter-industrial wage and unemployment rate differentials co-persist across time and spaces, all at equilibrium with flexible real wage.

Journal of Economic Literature Classification Numbers: E24, E32, D0, D24, J41

Running Head: Jobless Recoveries

1 Introduction

An economic model that aims at resolving the “jobless recovery” puzzle must answer at least two important questions. Why does an economy create new jobs after a substantial duration of an economic expansion but not create new jobs for the immediate recovery of a recession? Since not all economies in the world and not all recessions of the U.S. economy have “jobless recoveries”, why do some economies start to create new jobs immediately after a recession is over while others do not? Beyond these two questions, it must answer why job destruction during a recession is much more concentrated¹ than job creation during an expansion, as observed in the empirical studies of Davis and Haltiwanger (1990) and Hall (1999). In this paper we provide such a model.

According to the Bureau of Economic Analysis, real gross domestic product (GDP)-the output of goods and services produced within the U.S. grew at an annual rate of 8.2 percent in the third quarter of 2003. The Labor Department reported that in the same quarter nonfarm productivity of American workers-output per hour increased at an annual rate of 9.4 percent. The strong growth in GDP and productivity in the quarter lead economists and market analysts to expect the labor market to create 100,000 to 150,000 new nonfarm jobs in December 2003. In contrast, the Labor Department reported on January 9 of 2004 that only 1,000 nonfarm jobs were created in the month. The revised number reported on February 8 of 2004 was 16,000, majority of them were the labor-intensive construction and retail trade service jobs. The capital-intensive manufacturing labor market remained quite weak.

The weak job market situation in December 2003 was by no means alone. It had been a characteristic of the U.S. economy during the latest two recessions and their immediate recoveries. The payroll survey from the Bureau of Labor Statistics showed that there were more than one and half million job losses during the 2001 recession from March 2001 to November 2001, largely due to layoffs exercised simultaneously by the U.S. firms, even though real GDP fell just mildly. The 1990-1991 recession had a similar pattern in job creation during its recovery, though less severe than the 2001 recession. Now economists call such a characteristic of a weak job market during a recovery the “jobless recovery”.

Bernanke (2003) and Schreft and Singh (2003) had provided some detailed discussions about the causes of the 2001 “jobless recovery”. Bernanke (2003) showed that there were three factors that might contribute to the jobless recovery: an increase in “the long run productivity growth of the U.S. economy”, “the unmeasured increases in the work effort of employees”, and “the delayed result of firms’ heavy investment in high-technology equipment in the later part of the 1990s”. Schreft and Singh (2003) observed that “productivity gains in the current recovery [after the 2001

¹Large corporations announce layoffs in concentration one after another during a recession. But majority of them just have one or two waves of layoffs in a typical recession. For example, Sony and Siemens so far announced one and only one major layoffs.

recession were] determined to a relatively greater extent by firms' efforts to lower costs by shrinking payrolls and working existing employees more intensively." Thus, factors such as effort of work, productivity growth, technological progress, and firms' investment in capital before the recession appear to be important in explaining a "jobless recovery". A question that is left unanswered is why it is a recession that triggers these factors to work together against the creation of new jobs. This question is critical as the U.S. economy did not have problems in creation of new jobs during economic expansions right before the two recessions. In fact, economists worried about the tightness of the labor market that might lead to high inflation right before the two recessions. Why did the labor market change so drastically in job creation before and after a recession? Indeed, why?²

Our paper is built on the insights provided in Bernanke (2003) and Schreft and Singh (2003). It contributes to the literature by integrating those factors discussed in Bernanke (2003) and Schreft and Singh (2003) into a formal hiring game in a framework of an efficiency wage model. Our hiring game is a simplified version of the noted job matching model of Kelso and Crawford (1982). Our efficiency wage model is based on Solow (1979). However, our effort function $e(w, u)$ is taken from Akerlof (1984), with an exception that the unemployment rate u is endogenously determined at equilibrium, an idea critical to the shirking model of Shapiro and Stiglitz (1984). The unemployment rate in our model, as in Shapiro and Stiglitz (1984), is better to be considered as a local unemployment rate of an island economy (Phelps (1970)). The whole economy can be considered as a network of islands (Dickens and Katz (1987), Krueger and Summers (1988), Davis and Haltiwanger (1990), and Blanchflower and Oswald (1994)).

Akerlof (2002), Akerlof and Yellen (1986), and Stiglitz (2002) have provided a detailed account of the literature of the existing efficiency wage models. It should be aware that the real wage in an efficiency wage model should be seen as a flexible real wage in nature. Otherwise, a firm will lose its freedom in controlling its wage and thus workers' effort. For example, as the labor demand curve shifts, the real wage in Shapiro and Stiglitz (1984) must adjust accordingly so that there always exists a minimal number of unemployed workers to be presence in order to prevent the hired workers from shirking. Somehow ironically, efficiency wage models are often used as a key base for the hypothesis of rigidity real wage. One reason is that the real wage at equilibrium in the efficiency wage model of Solow (1979) is rigid. It depends on the effort function alone. Out of the equilibrium, the real wage in Solow (1979) must be flexible so that the firm can adjust it until the equilibrium is eventually achieved. Another reason, which is more critical, is that economists require the rigidity real wage to explain why there is involuntary unemployment in the labor market. It is a uniform belief among economists that a flexible real wage must result

²Structural changes and offshore outsourcing may slow job creation during a recovery after a recession. But they are unlikely to be the major causes for jobless recoveries since they are typically a gradual and time-consuming process. Either of them can answer the posed question.

in an equilibrium that clears the labor market so that involuntary unemployment is not possible without job match frictions. But there always exists a spike in involuntary unemployment during a recession. A fundamental question is why there is such a spike. The Keynesian theory sees that a recession is a short-run event and answers the question with rigidity real wage, as long as it is rigidity enough to be above the market-clearing wage. But the rigidity real wage is often objected by others since real wage is observed to be procyclical³, not so rigid as expected in theory. The neoclassical theory, after abandoning the hypothesis of rigidity real wage, answers the question with search and match. Mismatch frictions are needed to show why involuntary unemployment exists with either posted or bargaining wage. But the answer to the question why there is a spike in the unemployment rate in a recession has yet been satisfactorily achieved in this growing search and match literature. Our model, while built on the existing efficiency wage models, allows procyclical real wage and shows why such a spike happens at equilibrium in the absence of mismatch frictions.

A novel feature of our model is the introduction of an upper bound on the effort function. Such an upper bound represents a worker's productivity potential. It mainly depends on the capital per hired worker and the existing production technology. There are two phases that our island economy may operate during a business cycle: Either each hired worker works up to her potential (i.e., the upper bound of the effort function is reached) or each hired worker does not work up to her potential (i.e., the upper bound of the effort function is not reached)⁴. In the later case, we say that the hired workers have idle productivity since they are not working up to their potential. Clearly, an economy likely operates under the former phase after a substantial duration of an economic expansion. It may well operate under the later phase during a recession and its immediate recovery (Corrado (2002), Fay and Medoff (1985)).

How does our economic model perform under the two phases? Once the economy operates under the former phase where the upper bound is reached, the Nash equilibrium of our model shows that the unemployment rate will gradually lower when the economy expands to satisfy additional increase in demand. An increase in output requires creation of new jobs and addition of new capital (Theorem 4).

Things change dramatically when the economy operates under the later phase where the upper

³It is known that real wage is procyclical for post-War era. Blanchflower and Oswald (1994) studied how real wage is related to the local unemployment rates and found the elasticity of the real wage with respect to the local unemployment rate is about negative 10 percent. Swanson (2007) studied the PSID data and also found that the elasticity of the real wage with respect to the national unemployment rate is about negative 10 percent. The real wage at equilibrium in our model may fall during a recession, depending on how deep the recession may be. Nevertheless, the fall in the real wage is tempered by the sharp reduction in employment levels.

⁴The unbounded effort function of work in the existing efficiency wage models can happen under different scenarios. First, firms may have the ability to work the existing workers more and more intensively for a given level of capital stock and technology or they may enhance the productivity potential through internal improvements in organization or reallocation of the existing resources (Greenspan (2002)). Second, a firm can grow individual worker's productivity potential (i.e., the upper bound) with new investment in capital or technology for its long run growth. Third, such a bound, though existing, may not be reached during a recession phase in a business cycle. In a recession, it is known that the utilization of capacity in the U.S. economy dropped substantially (Corrado (2002)).

bound is not reached. The Nash equilibrium of our model shows that the unemployment rate will become sticky while wage remains flexible (Theorem 2). The unemployment rate is sticky because an increase in demand is first satisfied by the idle productivity of the existing hired workers, without creating new jobs. Only after the idle productivity is worked out, the economy will start to create new jobs. Our idle productivity is similar to the labor hoarding empirically found in Fay and Medoff (1985). It is noteworthy that even though there is idle productivity for each hired worker, there are no idle workers. Every hired worker must be hired at full time to produce goods at equilibrium with full time effort. There are no clean-up or maintenance tasks available for hired workers and there are no rooms for hired workers to shirk or rest on duty. The idle productivity is just the gap between a worker's productivity potential and her current productivity on duty.

The idea delivered by our model for a jobless recovery may be summarized as follows: Suppose the economy expands in an equilibrium path such that both labor and capital operate at their productivity potential. Under such an environment, any additional increase in demand must be met by adding new labor and capital as shown in Theorem 4. Such a process continues up to a point where the economy is hit by a recession. Since investment in capital is typically irrevocable, at least in the short run, it is in the best interest of firms to layoff workers but not to liquidate capital stock. This exercise raises the ratio of capital per existing worker. Due to capital deepening and the likely cleansing effect of recessions (Caballero and Hammour (1994)), the productivity potential of the existing workers may well be higher (Bernanke (2003), Schreft and Singh (2003), and Greenspan (2002)). With higher productivity potential and a weaker demand, it is plausible to expect that workers and capital operate under their potential productivity (Fay and Medoff (1985)). Under such a circumstance, our model also shows that the economy will jump into an equilibrium path with a sticky unemployment rate (Theorem 2). The economy will experience a job destruction spike during a recession because the sticky unemployment rate is much higher than the unemployment rate before the recession. Since there is some idle productivity at equilibrium, any increase in demand during the immediate recovery of the recession will be met by the idle productivity of the existing workers. There is no creation of new jobs. After the idle productivity potential has been worked out, the economy will enter another equilibrium path with new hiring and new addition of capital, in a gradual process. Along the new equilibrium path of another expansion, the level of productivity is higher than the previous expansion while the unemployment rate is gradually lower again.

It is not always the case that our economy experiences a jobless recovery when a recession hits. If the economy jumps from an equilibrium path to another where the upper bounds of two effort functions are reached in both cases, the economy will still start to create new jobs during an immediate recovery of a recession. This may explain why not all economies had the jobless recoveries after a recession and why not all recessions of the U.S. economy had the jobless recoveries. It may

also explain why jobless recoveries happen more often to capital-intensive sectors like manufacturing than to labor-intensive sectors like construction. The U.S. data observed from the 2001 recession also supports our model that jobless recovery is less likely in an economy that is labor-intensive than an economy that is capital-intensive.

The equilibrium behavior of our model is consistent with a great number of empirical findings in the literature. Empirical studies by Basu, Fernald and Kimball (2006) and Gali (1999) showed that technological improvements destroy existing jobs and reduce non-residential investment sharply. A sticky price model is used by these authors to explain their findings. Our model with flexible wage can provide an alternative. A technological improvement shifts the upper bound of the effort function so that the upper bound will not be reached at the current equilibrium. Since the upper bound is now not reached, our equilibrium will jump from the current to a new position with higher unemployment rate. Firms will stop to add new capital because there is idle productivity for each existing hired worker after the improvement. Firms meet the additional increase in demand with the existing hired workers who now have higher productivity. To recognize the higher productivity, workers need to have higher effort of work. Workers do not work for free. Their higher efforts are paid with higher real wage. After the idle productivity is worked out, firms start to create new jobs and add new capital, with higher productivity and higher real wage for each hired worker. Thus, technological improvements can pull the economy back in the short-run by destroying jobs and reducing other input demands like non-residential investment. But it is quite beneficial for the long run growth of an economy.

In the long run, real wage and productivity in the U.S. economy have formed an upward trend without a bound in sight while unemployment rates have been fluctuated in a range. Blanchard and Katz (1997) pointed out that any model on productivity and the average rate of unemployment should satisfy the condition that there is no long run effect of the level of productivity on the average rate of unemployment. Stiglitz (1997) and Trehan (2001) both insisted that the tradeoff between level or rate of change of productivity and the unemployment rate, if any, must be transient not permanent. Summer (1988) made a similar remark on the tradeoff between real wages and average unemployment rates in the long run. He wrote that “[i]t is striking that real wages have doubled several times over the last century without having a large impact on average unemployment rates”. These authors have demonstrated why it is often a challenge task to build a model that is consistent with the observed relationships between productivity, the real wage, and the unemployment rate in the long-run and short-run. Our unbounded effort function of work captures the idea that the growth in productivity or real wage is in a trend in the long run without a bound. But the bounded effort function captures the idea that the productivity potential of an individual worker for time being can be constrained by the current technology, the existing accumulated capital stock, and

the current economic status⁵. The real wage or productivity does affect the unemployment rate in the short-run in our model. But there is no long run tradeoff between the real wage, productivity, or technological improvements, and the unemployment rate.

Krueger and Summers (1988) found that inter-industry wage differentials for equally skilled workers are substantial and stable across time and spaces. They concluded that their findings support the efficiency wage theory where the high pay industries pay workers with supra-competitive wage while low pay industrials pay workers with market-clearing wage. Firms in the industrials that pay supra-competitive wage often possess certain monopoly power and earn more than normal profits.⁶

While wage differentials across industries are extensively discussed in the literature, it is surprising that the persistence nature of inter-industrial unemployment rate differentials across time and spaces (Table 4) is not investigated well in the same literature. One challenge issue in the explanation of the inter-industrial wage differentials by an efficiency wage model that has a rigidity wage is that the unemployment rates for the industries paying supra-competitive wages should have been higher than the industries paying competitive wages. This is because labor market should always be clear for industries paying competitive wages while involuntary unemployment exists in industries paying supra-competitive wages. The empirical data show that the opposite is often true. There is another controversy for the existing efficiency wage models to deal with wage and unemployment rate differentials in a recession. If the efficiency wage is rigidity, the unemployment rates for the industries paying supra-competitive wages during a recession should increase more than the unemployment rates for the industries paying competitive wages. Empirical data also show that this is not the case (Tables 2, 3, and 4). For example, Textile and Apparel are the two low pay industries while Mining and Chemicals are the two high pay industrials. But the unemployment rates in Textile and Apparel are often much higher than Mining and Chemicals across time and spaces during both recessions and expansions.

Our game theoretical model resolves these two controversies. We will show how the wage and unemployment rate differentials co-persist across time and spaces. In particular, we will demonstrate how low pay industrials like Textile and Apparel may have higher unemployment rates than high pay industrials like Mining and Chemicals.

The rest paper is organized as follows. Section 2 provides a general model of a hiring game. Sections 3 and 4 present the general results derived from our hiring game. Section 5 provides a

⁵No body believes an ordinary person can fly to the moon at present time. No body can say either this will never be possible.

⁶An efficiency wage model should better apply to an economy where firms have certain monopoly power either in the commodity market or in the factor market. Otherwise, firms can only pay competitive wages in order to operate with nonnegative profits because a firm in a perfect competitive industry paying competitive wage earns no more than zero normal profit. That is, without certain monopoly power, a firm that is willing to pay supra-competitive wage will not be able to afford to pay such a wage.

detailed discussion how our model may resolve the “jobless recovery” puzzle. Section 6 studies the inter-industrial wage and unemployment rate differentials. We also discuss the technical change and income inequality and show how the unemployment insurance or labor institutions may affect unemployment. Section 7 concludes the paper.

2 A Hiring Game

There are a finite number of identical workers, $i = 1, 2, \dots, N$, and a finite number of identical firms, $j = 1, 2, \dots, m$. A wage offer \tilde{w}_j by firm j is defined by $0 \leq \tilde{w}_j = \{\tilde{w}_{ij}\}_{i=1,2,\dots,N}$. A worker i who is hired by firm j and paid with wage offer \tilde{w}_{ij} chooses an effort e_{ij} of work, which depends on not only his wage offer \tilde{w}_{ij} but also many other factors such as the fair-wage, the norms of work and the unemployment rates (Akerlof(1984)).

A firm j , $j = 1, 2, \dots, m$, has a production function $F_j : 2^{\{e_{1j}, e_{2j}, \dots, e_{Nj}\}} \rightarrow R_+$, where e_{ij} is the effort of work of worker i in firm j if worker i is hired by firm j . Since a worker’s effort of work depends on the unemployment rate u , formally defined later, and the other factors, firm j ’s profit depends on not only those hired by firm j but also those hired by other firms. So the choices of what wage offers may be provided and who will be hired become a strategic game.

Let $\tilde{S}_j \subset \{1, 2, \dots, N\}$ denote the group of workers that firm j is willing to hire at \tilde{w}_j , $j = 1, 2, \dots, m$. Let $\tilde{W} = (\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_m)$ denote a strategy profile of wage offers and $\tilde{S} = (\tilde{S}_1, \tilde{S}_2, \dots, \tilde{S}_m)$ denote a strategy profile of workers firms are willing to hire under \tilde{W} . If a worker i receives more than one offer, he rejects all but the one he likes the most. He may use any lottery to break ties, if any. Let $S(\tilde{W}, \tilde{S}) = (S_1(\tilde{W}, \tilde{S}), S_2(\tilde{W}, \tilde{S}), \dots, S_m(\tilde{W}, \tilde{S}))$ denote the actual hired profile of workers.

Given a strategy profile (\tilde{W}, \tilde{S}) , a firm j ’s profit function is defined by

$$\pi_j(\tilde{W}; \tilde{S}) = F_j(\{e_{ij} : i \in S_j(\tilde{W}, \tilde{S})\}) - \sum_{i \in S_j(\tilde{W}, \tilde{S})} \tilde{w}_{ij}.$$

A strategy profile (\tilde{W}, \tilde{S}) is a (pure) Nash equilibrium if for every j , $j = 1, 2, \dots, m$,

$$\pi_j((\tilde{w}_j, \tilde{w}_{-j}); (\tilde{S}_j, \tilde{S}_{-j})) \geq \pi_j((\tilde{w}'_j, \tilde{w}_{-j}); (\tilde{S}'_j, \tilde{S}_{-j}))$$

for all $\tilde{w}'_j \geq 0$ and all $\tilde{S}'_j \subset \{1, 2, \dots, N\}$.

3 General Results: No Upper Bound

Section 2 has established a general hiring game whose main structure is a mixture of the “partial-gift exchange” model initiated by Akerlof (1982) and the job matching market of Kelso and Crawford (1982). In this section we present an application of it to study the issue of involuntary unemployment in a labor market with identical firms and identical workers, a framework used by Shapiro

and Stiglitz (1984). Since we have identical workers and firms, the wage offer at equilibrium is the same for all workers. Moreover, the norms of effort e_n equals the effort of work e (Akerlof (1982)). So the effort of work for a worker⁷

$$e = e(w, u)$$

is a function of his wage w and the unemployment rate u such that derivatives $e'_w(w, u)$ and $e'_u(w, u)$ are strictly positive (the exogenous variables are left out in the expression which can be embodied without changing the results in this paper). Let $F(el)$ be the production function for a firm, assuming that the neoclassical conditions are satisfied: $F(0) = 0$, $F' > 0$, $F'' < 0$, and $F'(0) = \infty$.

Let w_j denote firm j 's wage offer for each hired worker and l_j denote its employment level. Feasibility implies that $w_j \geq 0$, $j = 1, 2, \dots, m$, and

$$l_1 + l_2 + \dots + l_m \leq N.$$

The unemployment rate u is defined by

$$u = \frac{N - \sum_{j=1}^m l_j}{N}.$$

Firm j 's maximization problem is to choose a wage offer w_j and an employment level l_j to maximize its profit, given (w_k, l_k) , $k = 1, 2, \dots, m$, $k \neq j$,

$$F(e(w_j, u)l_j) - w_j l_j,$$

subject to the feasibility constraint that each worker is hired by at most one firm.

Since all firms and all workers are identical, it is natural to consider a symmetric Nash equilibrium (in pure strategy) under which all firms choose the same wage offer w and the same employment level $0 \leq l \leq N$ at equilibrium. Thus, the unemployment rate u at a symmetry Nash equilibrium is defined by $u = 1 - \frac{m}{N}l$ or equivalently $\frac{m}{N}l = 1 - u$.

At a symmetry Nash equilibrium, the first order conditions for firm j are respectively given by

$$(1) \quad F'(e(w, u)(1 - u)\frac{N}{m})e'_w(w, u) - 1 = 0,$$

$$(2) \quad F'(e(w, u)(1 - u)\frac{N}{m})(-\frac{1}{m}(1 - u)e'_u(w, u) + e(w, u)) - w = 0,$$

assuming that there is an interior solution at equilibrium.

Thus, one can solve equations (1) and (2) to obtain the optimal wage offer w^Δ and the unemployment rate u^Δ at a symmetric Nash equilibrium. The optimal employment level is defined by $l^\Delta = (1 - u^\Delta)\frac{N}{m}$.

⁷It is known that private universities in U.S. pay better in salary than public universities. Do you believe that professors in private universities on average are more productive? Consider an economy that has zero or 25% unemployment rates. Under what scenario do you think you may work harder?

It should be aware that the price level p for each unit output has been normalized to be unity. As a result, w is the real wage. Eq.(2) is the demand curve for labor which should be nonlinear in general. It says that at a symmetric Nash equilibrium, the marginal product of an efficiency labor equals the real wage w . Note that real wage at equilibrium when unemployment matters is lower than that when it does not matter. That is, our demand curve for labor shifts down when unemployment matters.

Substitute (1) into (2) by eliminating the term $F'(e(w, u)l)$, we have

$$(3) \quad \lambda_{e,w} = \frac{e'_w(w, u)}{-\frac{1}{m}(1-u)e'_u(w, u) + e(w, u)}w = 1.$$

In the Solow (1979) model, a representative neoclassical firm with production function $F(e(w)l)$ chooses an efficiency wage w and the level of labor l such that its profit $F(e(w)l) - wl$ is maximized. The equilibrium condition is that the elasticity of effort with respect to the real wage is unity⁸:

$$\frac{e'(w)}{e(w)}w = 1.$$

The Nash equilibrium condition in (3) is different from the Solow equilibrium condition, even in the limit of $m \rightarrow \infty$ since the unemployment rate u and the real wage w are interacting with each other. The Solow equilibrium condition implies that the efficient wage at equilibrium is sticky, perhaps above the competitive equilibrium wage. Our equilibrium condition shows that there is a case where the real wage at equilibrium is flexible while the unemployment rate is sticky.

We now provide a necessary condition how an economy achieves a full employment.

Theorem 1. *Full employment at a symmetric Nash equilibrium is achieved only if the following holds:*

$$e(w, 0) = e'_w(w, u)w + \frac{1}{m}e'_u(w, u) \Big|_{u=0}.$$

Conversely, if $e'_u(w, u) \neq 0$ and the following holds for all w, u :

$$(4) \quad e(w, u) = e'_w(w, u)w + \frac{1}{m}e'_u(w, u),$$

then there is a full employment at every symmetric Nash equilibrium.

Proof. A full employment at a symmetric Nash equilibrium means that $u = 0$. It follows from Eq.(3) that

$$e(w, 0) = e'_w(w, u)w + \frac{1}{m}e'_u(w, u) \Big|_{u=0}.$$

⁸Here we follow the reformulated version of the Solow (1979) model well presented in Yellen (1984) and Akerlof and Yellen (1986).

If Eq.(4) holds, then Eq.(3) implies that $ue'_u(w, u) = 0$. Thus, we have $u = 0$. □

An interesting observation about Theorem 1 is that even if the demand curve for labor depends on the production function, the sufficient condition to achieve full employment at a symmetric Nash equilibrium does not depend on it. One can tell from effort function of work alone if there is nonzero involuntary unemployment at equilibrium, no matter what the production function may be. The necessary condition for full employment at a symmetric Nash equilibrium shows that nonzero involuntary unemployment is by no means a specific phenomenon but a general one. Theorem 1 shows that it is possible to find a family of effort functions of work to achieve full employment at equilibrium. But any economy with a general form of effort function that does not satisfy the necessary condition in Theorem 1 has nonzero involuntary unemployment at equilibrium, no matter what a production function the economy may have. Because of the restrictive manner in the necessary condition for full employment, full employment at equilibrium is quite special in nature.

The natural rate of unemployment, defined as “the average level around which the unemployment rate fluctuates” (Mankiw (1997, p.124)), is important for both theory and economic policy. The conventional view is that the natural rate of unemployment generally consists of frictional and structural unemployment rates but not the involuntary unemployment rate (Rogerson (1997)). Our result above shows that this is possible only for a particular family of effort functions. In general cases, nonzero involuntary unemployment persists in the average level of unemployment rates.

Next we consider a class of effort functions with nonzero involuntary unemployment at equilibrium such that the underlying production function has impacts on the individual worker’s productivity and equilibrium wage but not on the involuntary unemployment rate. This result makes our study useful to analyze involuntary unemployment in the long run and the “jobless recovery”. It provides an additional evidence that involuntary unemployment may present in the average or natural rate of unemployment.

Theorem 2. *Assume the effort function of work is as follows:*

$$e(w, u) = aw + g(u),$$

where $g(u)$ is nonnegative and bounded on $[0, 1]$, satisfying $g'(0) = \infty$.⁹ Then there exists a unique symmetric Nash equilibrium that is also feasible. Moreover, the involuntary unemployment rate $0 < u^\Delta < 1$ at equilibrium satisfies the following:

$$(5) \quad (1 - u)g'(u) = mg(u).$$

⁹This condition means that, with or without the probability to be unemployed, it matters very much for workers in their productivity. Our economists are not lack of the same experience: It matters very much whether an academic position is tenured or not tenured.

Furthermore, there exists a constant $c > 0$ such that the effort of work e^Δ at equilibrium is given by

$$(6) \quad e^\Delta = \frac{c}{1 - u^\Delta}$$

and the efficiency wage w^Δ at equilibrium is given by

$$(7) \quad w^\Delta = \frac{c}{a(1 - u^\Delta)} - \frac{1}{a}g(u^\Delta).$$

Proof. Eq.(5) follows from Eq.(3). Note that the function $f(u) = \frac{1}{m}(1 - u)g'(u) - g(u)$ is monotone and continuous on $(0, 1)$. Moreover, $f(0) \rightarrow \infty$ and $f(1) \leq 0$. Thus, the existence and uniqueness follow. It follows from Eqs.(2) and (5) that

$$(8) \quad aF'(e^\Delta(1 - u^\Delta)\frac{N}{m}) = 1.$$

Thus, there is a constant $c > 0$ such that

$$e^\Delta(1 - u^\Delta) = c.$$

This shows eq.(6). Note that the efficiency wage w^Δ at equilibrium is given by

$$w^\Delta = \frac{e^\Delta}{a} - \frac{1}{a}g(u^\Delta).$$

Thus, eq.(7) follows. □

Henceforth, the effort function of work is assumed to have the separable form given in Theorem 2. Moreover, we assume that the function $g(u)$, if without further specification, is concave.

Theorem 2 may be used to analyze involuntary unemployment and the efficiency wage or productivity at equilibrium in the long run. For example, consider a sequence of economies with effort functions $e_t(w, u) = a_t w_t + b_t g(u)$ and the production functions $F_t(e_t l)$ with N_t number of workers and m_t number of firms. Then such a sequence of economies has the same involuntary unemployment rates at equilibrium. But equilibrium wage and productivity may well be different for different time periods. In particular, it is possible that real wage and productivity form an upward trend in the long run while involuntary unemployment rate at equilibrium remains steady, a theoretic result that is consistent with the view expressed by Blanchard and Katz (1997), Stiglitz (1997), and Trehan (2001), among others. Therefore, Theorems 1 and 2 provide some interesting insights on the involuntary unemployment rate at equilibrium and its relationships with the equilibrium effort of work and the equilibrium efficiency wage.

4 General Results: Upper Bound

A key feature in the effort function of work given in Theorem 2 is that it does not have an upper bound. An unbounded effort function of work is consistent with the upward trend in the level of productivity in the U.S. economy in the long run. But it can be a short-run phenomenon. For example, Corrado (2002) reported that, during the 1990-1991 recession, utilization of capacity in the manufacturing industries had dropped from its peak 84.1 percent in 1988 to the trough 78.3 percent in 1991; During the 2001 recession, utilization of capacity had fallen from 81.4 percent in 2000 to 73.8 percent in 2002. The total industrial capacity utilization during the two recessions had dropped in a similar pattern. These data provided strong evidence to support our view that workers and capital have not operated up to their potential during a recession or its immediate recovery. That is, the upper bound of the effort function may not be reached in the short run when the economy is in recession.

When an expansion or recovery lasts for a substantial period, an upper bounded effort function of work is more realistic in the economic analysis, at least in the short run. Such a bound mainly depends on the capital stock per worker and the technical progress. Reorganization and reallocation of resources during a recession may also be a factor (Benanke (2003), Greenspan (2002), Schreft and Singh (2003)).

Let K denote the total capital stock in a firm and l denote the number of hired workers. Thus, the productivity potential of an individual worker $\bar{e}(A, k)$ is defined as a function of the multifactor productivity or technical progress A and the ratio $k = \frac{K}{l}$ of capital stock per worker. Assume that $\bar{e}'_2(A, k) > 0$. We first consider the case where k is constant. Then we determine k endogenously.

When k is constant, the upper bound $\bar{e}(A, k)$ is constant for a given production technology A .¹⁰ Note that managerial skills, organization forms in production and job market status may also affect \bar{e} through A .¹¹

Let $\bar{e} = \bar{e}(A, k)$ be the fixed upper bound. Then firm j 's maximization problem is to maximize its profit, with respect to l_j ,

$$F(\bar{e}l_j) - \frac{1}{a}(\bar{e} - g(u))l_j,$$

subject to the feasibility condition.

Theorem 3 (Constant Upper Bound). *There exists a unique symmetric Nash equilibrium such*

¹⁰ A is often called multifactor productivity. In empirical studies, A is identified as the Solow residual. This may not be quite accurate. The Solow residual measures changes in three things: a shift in production function due to change in the underlying production technology, a movement along the production function due to change in demand for the product, or a change in the utilization of production factors.

¹¹Greenspan (2002) explicitly stated that the multifactor productivity A "includes technical progress, organizational improvements, cyclical factors, and myriad other influences on output per hour."

that the equilibrium unemployment rate u^Γ satisfies that

$$0 < u^\Gamma \leq u^\Delta.$$

Proof. Recall that

$$u = \frac{N - \sum_{k=1}^m l_k}{N}.$$

Thus, at a symmetric Nash equilibrium under which all firms employ the same level l and $l = (1 - u)\frac{N}{m}$, the first order condition is given by

$$F'(\bar{e}l)\bar{e} - \frac{1}{a}[g'(u)\frac{1}{N}l + \bar{e} - g(u)] = 0.$$

That is,

$$F'(\bar{e}(1 - u)\frac{N}{m})\bar{e} - \frac{1}{a}[\frac{1}{m}g'(u)(1 - u) + \bar{e} - g(u)] = 0.$$

Let

$$\begin{aligned} L(u) &= F'(\bar{e}(1 - u)\frac{N}{m})\bar{e} - \frac{1}{a}[\frac{1}{m}g'(u)(1 - u) + \bar{e} - g(u)] \\ &= \frac{\bar{e}}{a}[aF'(\bar{e}(1 - u)\frac{N}{m}) - 1] - \frac{1}{a}[\frac{1}{m}(1 - u)g'(u) - g(u)]. \end{aligned}$$

By the assumption that the bound \bar{e} is reached at equilibrium, it follows that $e^\Delta \geq \bar{e}$. Since, by eq. (8),

$$aF'(e^\Delta(1 - u^\Delta)\frac{N}{m}) = 1,$$

it follows that

$$aF'(\bar{e}(1 - u^\Delta)\frac{N}{m}) \geq 1$$

since $F'' < 0$. This means that $L(u^\Delta) \geq 0$ since, by eq. (5),

$$\frac{1}{m}(1 - u^\Delta)g'(u^\Delta) - g(u^\Delta) = 0.$$

To complete the proof, note that $g'(0) = \infty$ (Theorem 2). Thus, we have that $L(0) = -\infty$. Since $L(u)$ is continuous in $(0, u^\Delta]$, there exists $u^\Gamma \in (0, u^\Delta]$ such that $L(u^\Gamma) = 0$. Note that $L'(u) > 0$. Thus, uniqueness follows from the fact that $L(u)$ is monotonically increasing. \square

The equilibrium wage w^Γ is given by

$$w^\Gamma = \frac{1}{a}(\bar{e} - g(u^\Gamma)),$$

which is the equilibrium locus that has a negative slope.

Next we study what may happen when capital K is private and also costly to firms. Suppose that all firms on the island purchase capital goods from an economy-wide capital market with competitive (real) price r . Thus, firm j 's problem is to maximize its profit,

$$F(\bar{e}(A, \frac{K_j}{l_j})l_j) - \frac{1}{a}(\bar{e}(A, \frac{K_j}{l_j}) - g(u))l_j - rK_j,$$

with respect to K_j and l_j , subject to the feasibility condition.

Let

$$\lambda_{\bar{e},k} = \frac{\bar{e}'_2(A, k)}{\bar{e}(A, k)}k$$

be the elasticity of the upper bound with respect to the capital per worker. This elasticity $\lambda_{\bar{e},k}$ plays a key role in determining how the unemployment rate at equilibrium is related to u^Δ and u^Γ .

Theorem 4 (Role of Capital). *There exists a unique symmetric Nash equilibrium such that the equilibrium unemployment rate u^γ satisfies that (a)*

$$u^\gamma = u^\Delta$$

for $\lambda_{\bar{e},k} = 1$; (b)

$$0 < u^\Gamma < u^\gamma < u^\Delta < 1$$

for $\lambda_{\bar{e},k} < 1$; and (c)

$$0 < u^\Gamma < u^\Delta < u^\gamma < 1,$$

for $\lambda_{\bar{e},k} > 1$.

Proof. The first order conditions at a symmetric Nash equilibrium are given by

$$(9) \quad F'(\bar{e}(A, k)(1 - u)\frac{N}{m}) - \frac{1}{a} = \frac{r}{\bar{e}'_2(A, k)},$$

and

$$(10) \quad L(u) - rk = 0,$$

where $k = \frac{K}{l}$, $\bar{e}'_2(A, k) = \frac{d\bar{e}(A, k)}{dk}$, and $L(u)$ is given by

$$L(u) = F'(\bar{e}(A, k)(1 - u)\frac{N}{m})\bar{e}(A, k) - \frac{1}{a}[\frac{1}{m}g'(u)(1 - u) + \bar{e}(A, k) - g(u)].$$

By the proof of Theorem 3, $L(u)$ is monotonically increasing. It shifts down by a positive term rk . Thus, $u^\Gamma < u^\gamma$. To complete the proof of the first part, note that, by (9) and (10),

$$L(u) = \frac{r\bar{e}(A, k)}{\bar{e}'_2(A, k)} - \frac{1}{a}[\frac{1}{m}g'(u)(1 - u) - g(u)].$$

Since $\frac{1}{a}[\frac{1}{m}g'(u)(1-u) - g(u)] = 0$ when $u = u^\Delta$ by (5), we have that

$$L(u^\Delta) - rk = \frac{r\bar{e}(A, k)}{\bar{e}'_2(A, k)} - rk = rk\left(\frac{1}{\lambda_{\bar{e}, k}} - 1\right),$$

which is zero when $\lambda_{\bar{e}, k} = 1$, strictly positive when $\lambda_{\bar{e}, k} < 1$, and strictly negative when $\lambda_{\bar{e}, k} > 1$.

Thus there is a unique u^γ such that $u^\gamma = u^\Delta$ when $\bar{e} = k\bar{e}'_2$, and

$$0 < u^\gamma < u^\Delta < 1$$

when $\bar{e} > k\bar{e}'_2$. To complete the proof of the last part, note that $L(u)$ is positive as $u \rightarrow 1$. So there exists a unique u^γ in $(0, 1)$ such that $L(u^\gamma) - rk = 0$ and

$$0 < u^\Gamma < u^\Delta < u^\gamma < 1.$$

□

Theorem 4 shows that a linear upper bound $\bar{e}(A, k)$ in k implies that $u^\gamma = u^\Delta$. The condition $\bar{e}(A, k) > k\bar{e}'_2(A, k)$ in Theorem 4 is satisfied with a great number of upper bound functions such as $\bar{e}(A, k) = z(A)k^{\frac{1}{1+\varphi}}$ for $\varphi \in (0, 1)$.

Theorem 4 shows that higher costs in capital and capital deepening raise unemployment rate u^γ at equilibrium. As a result, automation of a production process that involves heavy investment in capital may indeed slow creation of new jobs. In particular, the unemployment rate u^Γ with cost-free capital in Theorem 3 remains lower than that with costly capital in Theorem 4. During the immediate recovery of a recession, the capital that was purchased before recession and not liquidated can be seen as free capital. Theorems 3 and 4 will show that job creation during an expansion with cost-free capital will be faster than job creation during an expansion with costly capital. Moreover, investment in social infrastructure by the governments will be helpful in creation of new jobs across all sectors in the economy, not just those sectors that are directly involved. Thus, Theorems 3 and 4 provide a number of interesting results about how capital investment and job creation are related.

The optimal level k^γ of capital per worker and the unemployment rate u^γ at equilibrium are given by

$$(11) \quad F'(\bar{e}(A, k)(1-u)\frac{N}{m}) - \frac{1}{a} = \frac{r}{\bar{e}'_2(A, k)},$$

and

$$(12) \quad \left(\frac{1}{\lambda_{\bar{e}, k}} - 1\right)k = \frac{1}{ar}\left[\frac{1}{m}g'(u)(1-u) - g(u)\right].$$

Unemployment rate u^γ and the optimal level k^γ of capital per worker are negatively related. The equilibrium wage w^γ is given by $\bar{e}(A, k^\gamma) = aw^\gamma + g(u^\gamma)$, which is the equilibrium locus in the space (u, w) for various optimal levels k^γ of capital per worker. It has a steeper slope than the equilibrium locus $\bar{e}(A, k) = aw^\Gamma + g(u^\Gamma)$ for a constant level k of capital per worker.

5 “Jobless Recovery” Puzzle

Theorems 2 and 4 provide necessary tools to resolve the “jobless recovery” puzzle. During a business cycle, an economy can operate under two different phases: either it operates such that each hired worker works up to her potential or it operates such that each hired worker does not work up to her potential. The former case is where the upper bound of effort function is always reached. So firms’ decision in hiring is governed by the result in Theorem 4. The later case is where the upper bound of effort function is not reached. Under such a scenario, firms’ decision in hiring is governed by the result in Theorem 2.

As the economy expands for a substantial period of time, there is a good reason to expect that each hired worker will work up to her potential so that the upper bound of the effort function is always reached. However, when an economy is hit by a recession, it is more likely that each hired worker will not work up to her potential so that the upper bound of effort function is not reached. The upper bound may not be reached even during an immediate recovery of a recession.

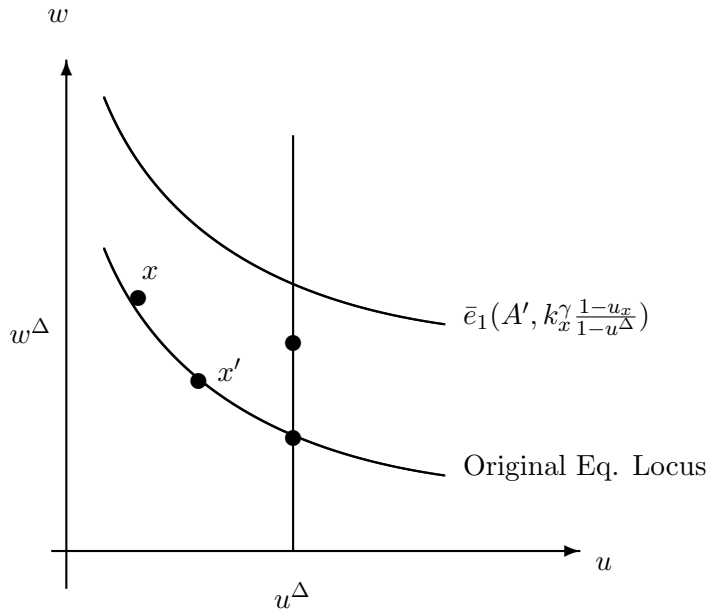


Fig.1. The gap between (u^Δ, w^Δ) and $\bar{e}_1(A', k_x^\gamma \frac{1-u_x}{1-u^\Delta})$ contributes to the “jobless recovery”.

Now consider the scenario where the economy expands for a substantial period of time such that the upper bound of effort function is reached. Theorem 4 shows that the economy will expand such that an additional increase in demand will be met with new hiring and new addition of capital, in a gradual manner, along the original equilibrium locus in Fig.1 described by

$$\bar{e}(A, k^\gamma) = aw^\gamma + g(u^\gamma),$$

together with equations (11) and (12). Suppose the expansion of the economy continues up to

an equilibrium point x , say, where level of capital per worker is k_x^γ . After the equilibrium x , the economy is hit into a recession by an adverse demand shock. In a perfect case, the economy should have adjusted along the original equilibrium locus to a point x' , say, with gradual job cuts and gradual liquidity in capital. But capital investment is typically irreversible or at least it is very costly to liquidate capital in the short-run. So firms are only left to lay off workers, with limited or no liquidity in capital. Such an exercise raises the ratio of capital per worker for the existing workers. On the other hand, reorganization and reallocation of resources in the recession also enhances the productivity potential of the existing hired workers. So the upper bound of the effort function after the economy is hit by a recession may well shift upwards as shown in Fig.1. Since there is a weak demand during the recession, it is expected that the higher upper bound of effort function may not be reached for the existing workers. Under such a situation, Theorem 2 shows that the economy will jump from equilibrium x to the equilibrium (u^Δ, w^Δ) . A massive layoff of workers will be observed since the economy jumps from an equilibrium to another with much higher unemployment rate.

Now observe that there is a gap between equilibrium (u^Δ, w^Δ) and the new upper bound $\bar{e}_1(A', k_x^\gamma \frac{1-u^\gamma}{1-u^\Delta})$. This means that idle productivity exists for each existing hired worker. Note that the equilibrium locus drawn in Fig.1 is given by

$$\bar{e}_1(A', k_x^\gamma \frac{1-u^\gamma}{1-u^\Delta}) = aw + g(u)$$

for constant $k_x^\gamma \frac{1-u^\gamma}{1-u^\Delta}$, where A' , increased from A , includes a possible gain in productivity potential from the cleansing effect of the recession (Caballero and Hammour (1994)).¹² The reorganization and reallocation of resources during a recession by the firms may also lead to an increase in A (Greenspan (2002)).

Suppose there is a recovery after the recession. Theorem 2 will show that such a recovery will not create jobs as long as there is idle productivity. This provides a possibility for a jobless recovery. During the immediate recovery of a recession, liquidation in capital by firms can close the gap faster and thus make jobless recovery shorter in time. Once the gap is closed, the economy will start to create new jobs again, together with new investment in capital.

It should be aware that each hired worker must be hired at equilibrium. That is, there is idle productivity for each hired worker but there are no idle workers. There are no concerns in transaction costs like job training or searching in hiring.

Capital plays an important role in the above discussion. If the new upper bound shifts in a small amount, the economy may jump from equilibrium x to a new equilibrium where the new upper bound is still reached. Under such a scenario, a recovery after a recession will still create new jobs, together with new addition in capital, by Theorem 4. This provides an answer why some

¹² A' is the potential productivity which may not be achieved when the economy is in the immediate recovery of a recession.

economies have jobless recovery while others do not and why some recessions have a very short jobless recovery while others have a very long jobless recovery.

Theorems 2 and 4 also show why job creation is a gradual process while job destruction during a recession is a spike. Job creation along the equilibrium locus of the economy with a substantial period of expansion must accompany with new addition in capital. Such a new addition in capital slows the creation of new jobs. When a recession hits the economy, job destruction can be a spike since the economy jumps to an equilibrium with much higher unemployment rate. This is consistent with empirical data. For example, it took the California State seven years to lower its unemployment rates from 10 percent in 1983 to 5 percent in 1990. After the 1990-1991 recession, in less than three years, the CA unemployment rates increased from 5 percent in 1990 to 9.6 percent in 1993. The creation and destruction of jobs before and after the 2001 recession in CA had the same pattern.

One important implication from our analysis is that, everything else constant, the recovery after a recession in labor-intensive sectors should create new jobs faster than the capital-intensive sectors. This is because the shift of the new upper bound for the labor-intensive sectors should be smaller than that for the capital-intensive sectors. A narrower gap implies a shorter or no jobless recovery. Since capital is expected to play more and more important roles in the U.S. economy in the future, it is quite safe to expect that “jobless recovery” may well happen once again.

Empirical studies by Basu, Fernald and Kimball (2006) and Gali (1999) showed that technological improvements raise unemployment rates and reduce non-residential investment sharply. They showed that a sticky price model can explain why this may be the case in the absence of monetary policy intervention. Our model with flexible wage can provide an alternative. A technological improvement shifts the upper bound of the effort function so that the upper bound will not be reached at the current equilibrium. With unreached upper bound, the equilibrium will jump from the current position to a new one with higher unemployment rate. Investment in capital falls because firms stop to add new capital. Any increase in demand is met by the existing hired workers who now have higher productivity. To recognize the higher productivity, workers are working with higher effort of work. Their higher efforts are paid with higher real wage. Thus, a technological improvement can pull back the economy in the short-run by destroying jobs and reducing other input demands like capital. But there is no long run trade-off between the two.

The above discussion does not imply that a technological regress will symmetrically increase creation of new jobs or demands for other inputs like capital. This is due to a beauty of asymmetry of the market system. For example, a severe technological regress may push firms to shut down their business if there is no profit to make for the regress technology. We believe that the nature of asymmetry in the adjustment of the market system from one equilibrium to another may be the main cause for the irregularity of business cycles. The jump of our Nash equilibrium during a

recession is due to the asymmetry in adjustment of capital investment.

6 Inter-industry Wage and Unemployment rate Differentials

Krueger and Summers (1988), among others, found that inter-industry wage differentials for equally skilled workers are substantial and stable across time and spaces. They showed that these wage differences are hardly explained by the unobserved quality of workers, the union threats, and various working conditions. They concluded that their findings support the efficiency wage theory in the sense that the high pay industries paid supra-competitive wage. Their studies suggested that each industry should be examined in detail for its wage determination. This implies that our stylized island economy is probably the right one in the study of wage determination.

While wage differentials across industries are extensively discussed in the literature, it is surprising that the persistence nature of inter-industrial unemployment rate differentials across time and spaces (Table 4) is hardly discussed in the same literature. One challenge issue of the efficiency wage theory in the explanation of the inter-industrial wage differentials is that the unemployment rates for the industries with high pay supra-competitive wage should be higher than the industries with low pay competitive wage. This is because labor market should clear for industries paying competitive wage while involuntary unemployment exists in industries paying supra-competitive wage. The empirical data show otherwise. For example, Textile and Apparel are the two low pay industries while Mining and Chemicals are the two high pay industrials. Table 4 shows that the unemployment rates in Textile and Apparel are much higher than Mining and Chemicals across time.

It is of great interest to deliver a theory to show why the persistence nature of inter-industrial wage and unemployment rate differentials across time and spaces co-exists. Such a study provides a more complete understanding of the very issue of the inter-industry wage differentials posed in Krueger and Summers (1988) and the related literature.¹³

6.1 An Example

Assume that the effort function of work is given by

$$e(w, u) = aw + bu^\beta, \quad a, b, \beta > 0$$

and the production function is given by $F(el) = A(el)^\alpha$, $\alpha \in (0, 1)$. The Arrow-Pratt coefficient of absolute risk aversion is given by

$$r_e(u) = \frac{1 - \beta}{u}.$$

¹³We have observed the similarity of this issue to those studied in the new trade theory and the geography economics. A detailed discussion of this with a network of island economies is left for a future study.

By Theorem 2, the equilibrium unemployment rate u^Δ satisfies equ. (5)

$$(1 - u)g'(u) = mg(u).$$

Therefore,

$$u^\Delta = \frac{\beta}{m + \beta}.$$

Unemployment rate at equilibrium depends on how much risk aversion a worker has towards unemployment and how many firms compete for workers. The higher the risk aversion the lower the unemployment rate. The larger the number of firms the lower the unemployment rate.

From equ. (8), we obtain that

$$e^\Delta = \frac{m + \beta}{N} (Aa\alpha)^{\frac{1}{1-\alpha}}.$$

Let $d = \frac{b}{a} \left(\frac{\beta}{m+\beta}\right)^\beta$. The efficiency wage at equilibrium is given by

$$w^\Delta = \frac{m + \beta}{aN} (Aa\alpha)^{\frac{1}{1-\alpha}} - d.$$

The total output Y^Δ and profit π are respectively as follows:

$$Y^\Delta = F(e^\Delta l^\Delta) = A^{\frac{1}{1-\alpha}} (a\alpha)^{\frac{\alpha}{1-\alpha}},$$

and

$$\pi = A^{\frac{1}{1-\alpha}} (a\alpha)^{\frac{\alpha}{1-\alpha}} (1 - \alpha) + \frac{N}{(m + \beta)} d = Y^\Delta (1 - \alpha) + \frac{N}{(m + \beta)} d.$$

Risk averse workers towards unemployment receive less than α share in the total output. Precisely,

$$w^\Delta l^\Delta = \alpha Y^\Delta - \frac{N}{(m + \beta)} d.$$

That is, firms get higher profits when workers are more risk-averse towards unemployment. Globalization, outsourcing, and unions likely affect d through the three parameters a , b and β . If d forms an increasing trend along time, the labor share of the national income will form a declining trend.¹⁴

As the number of firms approaches infinite, our model shows that the labor market approaches full employment with zero unemployment rate. Workers receive α share of outputs while firms receive $1 - \alpha$ share. The efficiency wage model with a finite number of firms in the Keynesian school collapses into a market-clearing model with an infinite number of firms in the Neoclassic school.

Note that the total output Y^Δ has nothing to do with the size of the labor force N . That is, an economy with a smaller size in population can have higher output than an economy with a

¹⁴In the last two to three decades, the U.S. economy experienced a declining trend in the number of union members and the labor share. Our model showed that such a possibility does exist.

larger size in population, consistent with what has been observed in the world economy. However, this does not mean that an economy with a large size in population will not matter. Consider the real wage at equilibrium. It becomes lower with a larger size of population. This means that an economy with a larger size of population has an advantage in wages in production. So it will be attractive to multinational corporations seeking lower labor costs. Globalization and offshore outsources clearly benefit economies with a larger size in population like China and India. But size alone is not the only reason for a multinational corporation to go global. It is the production technology A of the underlying economy that matters the most.

The above discussion is based on the condition that the upper bound $\bar{e}(A, k)$ of the effort function is not reached. The minimum capital-labor ratio k^Δ satisfying this condition is given by $\bar{e}(A, k^\Delta) = e^\Delta$, where the growth in N can play an important role.

As in the Solow growth model, the growth of output Y^Δ in our model depends on technical progress in A with steady α and a . The output elasticity of the technical progress A equals $\frac{1}{1-\alpha}$. For an economy that has 1% annual growth rate in A , the output can grow at annual rate 3% for $\alpha = \frac{2}{3}$. Thus, our model may be useful to forecast the long run growth in output of an economy.

6.2 Wage Differentials: No Upper Bound

To see how inter-industrial wage differentials may persist over time and spaces, we use the example given above. Consider two island economies i and j with different coefficients A and β . All else are equal. Suppose that island i 's $A(i)$ is greater than island j 's $A(j)$. Moreover, β_i is smaller than β_j . At equilibrium, unemployment rate u_i^Δ on island i is lower than unemployment rate u_j^Δ on island j .

But wage on island i can be higher than wage on island j . Precisely, the wage premium at equilibrium is given by

$$\frac{w^\Delta(i) + d(i)}{w^\Delta(j) + d(j)} \approx \left(\frac{A(i)}{A(j)}\right)^{\frac{1}{1-\alpha}},$$

which is larger than 1.

It is noteworthy that the efforts of work at equilibrium on the two islands also satisfy that

$$\frac{e^\Delta(i)}{e^\Delta(j)} \approx \left(\frac{A(i)}{A(j)}\right)^{\frac{1}{1-\alpha}}.$$

Therefore, the island that is initially endowed with more “profitable” production technology pays higher equilibrium wage than the island endowed with less “profitable” production technology. However, nothing is for free. Higher wage accompanies with higher effort of work at equilibrium.

The next question is how it is possible such a wage differential can sustain across time, with a possible mobility of labors between the two islands. Consider two periods 0 and 1 island economies. Let x_i and y_j denote the growth rates of $A(i)$ and $A(j)$ respectively. Let z denote the net mobility rate of labors from island j to island i . For simplicity, assume that the mobility of labors is done

right by the end of period zero and the beginning of period one. Assume there are no mobility in labors in the middle of each period. Thus, the ratio of number of firms over number of workers in island i becomes $\frac{m}{N}(1-z)$ by the beginning of period one. It is $\frac{m}{N}(1+z)$ in island j by the beginning of period one. Then the wage premium at equilibrium in period one is given by

$$\frac{w_1^\Delta(i) + d}{w_1^\Delta(j) + d} = \frac{w^\Delta(i) + d}{w^\Delta(j) + d} \frac{1-z}{1+z} \left(\frac{1+x}{1+y}\right)^{\frac{1}{1-\alpha}} \approx \frac{w^\Delta(i) + d}{w^\Delta(j) + d} > 1,$$

as long as x, y and z are small and $x = y + 2(1-\alpha)z$. The mobility rate z can be small due to mobility barriers or costs. Once again, higher wage at equilibrium in period one in island i is not for free. The higher equilibrium wage in island i in period one accompanies with higher effort of work, since the efforts of work in period one also satisfy

$$\frac{e_1^\Delta(i)}{e_1^\Delta(j)} = \frac{e^\Delta(i)}{e^\Delta(j)} \frac{1-z}{1+z} \left(\frac{1+x}{1+y}\right)^{\frac{1}{1-\alpha}} \approx \frac{e^\Delta(i)}{e^\Delta(j)}.$$

It should be aware that the unemployment rates at equilibrium for the two islands do not change in period one even though there is a mobility of labors between the two islands. Therefore, both inter-industry wage and inter-industry unemployment rate differentials co-persist across time and spaces. Higher pay island has lower unemployment rate while lower pay island has higher unemployment rate.

There is another reason why inter-industry wage unemployment rate differentials or income inequality persist across time and spaces. Suppose that islands i and j have workers with different norms of work

$$e_i(w, u) = a(i)w + b(i)u^\beta$$

and

$$e_j(w, u) = a(j)w + b(j)u^\beta.$$

Assume that $a(i) > a(j)$ and $b(i) > b(j)$, with constant $\frac{a(i)}{b(i)}$ and $\frac{a(j)}{b(j)}$, everything else being equal. Then both islands have the same unemployment rates at equilibrium. But workers in island i enjoy higher wage with higher effort of work at equilibrium than workers in island j . Such a wage differential can persist across time and spaces, as shown in Fig.2. The two curves in Fig.2 are the two indifference curves of utility for the two islands. The line follows from the wage equation

$$w^\Delta = \frac{e^\Delta}{a} - \frac{b}{a}(u^\Delta)^\beta.$$

Since the elasticity of equilibrium wage and effort of work with respect to a is given respectively by

$$\lambda_{w,a} = \frac{\alpha}{1-\alpha} \left(1 + \frac{d}{w^\Delta}\right)$$

and

$$\lambda_{e,a} = \frac{1}{1-\alpha},$$

it follows that with higher a , the equilibrium wage and effort of work are both higher.

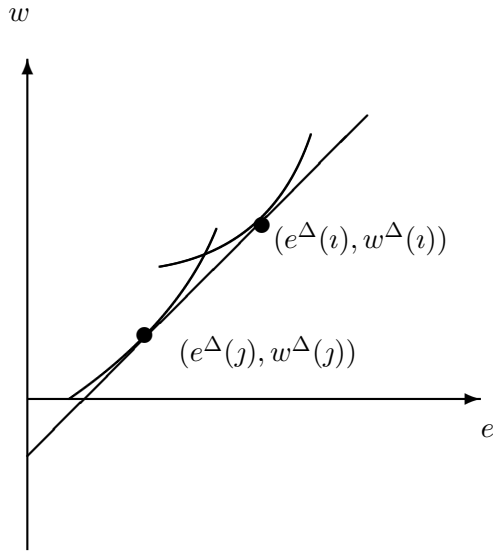


Fig.2. Persistent Wage Differentials

6.3 Technical Change and Income Inequality

A related question to the wage differential is the income inequality and technical change. Acemoglu (2002) provided a survey and a detailed discussion on technical change and income inequality of the U.S. labor market during the last several decades. He concluded that “technical change has been skill-biased during the past sixty years, and probably for most of the twentieth century”. Such a skill-biased technical change has been recognized to be the main cause for the sharp rising since 1970s in the income inequality between high-skill and low-skill workers. In fact, real income for high-skill workers rises sharply during 1980s while real wages for low-skill workers have fallen since 1970. These facts together with the pattern of unemployment invite an interesting question: How is it possible that a steady pattern of unemployment coexists with deteriorating income inequality?

We now provide an answer to the question with our example above. Imagine that island i is for the high-skill workers and island j is for the low-skill workers. Assume that high-skill workers as a work group are more risk averse than low-skill workers. Then island i has lower unemployment rate than island j at equilibrium. The unemployment rates at equilibrium remain steady unless workers have changes in their attitudes toward unemployment risk.

Now, the skill (wage) premium (Acemoglu (2002)) at equilibrium is given by

$$\frac{w^\Delta(i) + d(i)}{w^\Delta(j) + d(j)} \approx \left(\frac{A(i)}{A(j)}\right)^{\frac{1}{1-\alpha}}.$$

The income inequality can be widened at period one as long as $x - y > 2(1 - \alpha)z$, because the ratio of equilibrium wages at period one becomes

$$\frac{w^\Delta(i) + d(i)}{w^\Delta(j) + d(j)} \frac{1 - z}{1 + z} \left(\frac{1 + x}{1 + y}\right)^{\frac{1}{1-\alpha}} > \frac{w^\Delta(i) + d(i)}{w^\Delta(j) + d(j)}.$$

Technical change that is skill-biased means that $x > y$. If the net mobility from low-skill labor market to high-skill one is small, then the condition that $x - y > 2(1 - \alpha)z$ can be easily satisfied.

If technical change generates an “erosion effect” (i.e., $y < 0$) on the productivity of low-skill workers (Acemoglu (2002)), then equilibrium wage for low-skill workers can decline in trend if $(1 - \alpha)z + y < 0$. Acemoglu (2002) pointed out that the decline in real wages for the low-skill workers since 1970 is one of the puzzles associated with technical progress. Our model can have a reasonable explanation of that fact. It does one more thing because it also shows why unemployment rate can remain high for low-skill workers, while income inequality deteriorates.

6.4 Inter-industrial Wage Differential: Role of Capital

The analyses in the last section apply to an economy in the long run where productivity forms an upward trend without a bound in sight. However, empirical studies are often based on data observed in the short-run where the level of capital plays an important role in the wage and unemployment rate determination. We now use the results in Theorem 4 to see how wage and unemployment rate differentials co-persist across time and spaces.

Once again, the effort function of work is given by

$$e(w, u) = aw + bu^\beta, \quad a, b, \beta > 0$$

and the production function is given by $F(el) = A(el)^\alpha$, $\alpha \in (0, 1)$, where A is the total factor productivity beyond capital and labor. The upper bound of the effort function $\bar{e}(A, k) = z(A)k$. By Theorem 4, the unemployment rate at equilibrium

$$u^\gamma = u^\Delta = \frac{\beta}{m + \beta}.$$

We need to compute the level k^γ of capital per worker and the wage w^γ at equilibrium.

By equ. (9), k^γ is given by

$$k^\gamma = \frac{m + \beta}{z(A)N} \left(\frac{a\alpha Az(A)}{ar + z(A)}\right)^{\frac{1}{1-\alpha}}.$$

Thus, we obtain the equilibrium wage w^γ as follows

$$w^\gamma = \frac{Ak^\gamma}{a} - \frac{d}{a}.$$

Note that $\frac{dw^\gamma}{dA} > 0$ and $d'(\beta) > 0$ as long as $z' > 0$. Consider the two island economies i and j given above again. We conclude that the equilibrium wage on island i is higher than the equilibrium wage on island j while the unemployment rate on island i is lower than on island j . With the same technique introduced above, such wage and unemployment rate differentials co-persist across time. The higher pay jobs on island i are not free lunch. They are accompanied with higher level of capital per worker and higher level of effort in work.

6.5 Unemployment Insurance

The economy presented in the last two subsections may be used to study unemployment insurance or effects of labor market institutions. An increase in the parameter β implies a decrease in risk aversion. This can be accomplished by unemployment insurance which helps reduce the fear of a worker for being unemployed. A consequence of such a policy is an increase in the unemployment rate at equilibrium. The reduction in employment level at equilibrium has been compensated by an increase in the effort of work. Everything else constant, an increase in risk aversion decreases real wage at equilibrium.

The impact of unemployment benefits and job protection or regulation can be analyzed with respect to their risk aspects. Longer duration and better unemployment benefits are expected to reduce risk aversion. Such a policy, if implemented permanently, helps the unemployed but has potential to increase involuntary unemployment rate at equilibrium in the long run. Thus, unemployment insurance with better benefits makes unemployed better off in the short run but worse off in the long run. Thus, unemployment insurance and the related policy may increase the overall average unemployment rate.

One key difference between the labor markets in the U.S. and OECD Europe is that the existing workers in the U.S. labor market are less secure in their jobs. It is well known that it is harder for a firm to fire a worker in the OECD. Therefore, it is plausible to expect that workers in OECD Europe are less risk aversion than in the United States. Our model then shows that the labor market in OECD Europe will have higher unemployment rates at equilibrium than in the U.S.

7 Conclusions

The U.S. economy is in a deepening recession in the moment. A great number of jobs have been lost in a very short period of time. Our theory showed why firms behave in such a way by destroying jobs sharply during such a recession. Our model also showed that a “jobless recovery” will likely hit the economy once again as long as the demand during the recovery remains weak. How to reduce the duration of such a jobless recovery? Theorems 3 and 4 can provide some useful suggestions. They show that investment in the public capital like social infrastructure by the government is much more effective in creating new jobs than the private investment in capital goods. A stimulus

plan focusing on social infrastructure can be effective to reduce the jobless recovery duration during the current recession.

The rest we learned from the study of this paper is that involuntary unemployment should be a general phenomenon in an economy even if the wage is flexible. Our study reveals why it is difficult to eliminate the involuntary unemployment. An economy with zero involuntary unemployment rate for a substantial period of time likely reduces workers' risk aversion for being unemployed. Such a reduction in the risk aversion means higher involuntary unemployment rate at equilibrium in the future. On the other hand, involuntary unemployment rate cannot sustain with a high level. A high level of involuntary unemployment rate makes workers more risk averse for being unemployed. Higher risk aversion implies lower involuntary unemployment at equilibrium in the future. It should be aware that this conclusion is based on the assumption that a high level of unemployment rate does not result in any change in the policy about labor market such as the job protection and unemployment compensation. Any irrevocable change in the policy in favor of the unemployed due to the current high level in unemployment rate likely makes the current high unemployment rate a permanent phenomenon in the future, creating a kind of hysteresis of unemployment. The lesson we learned is this: Help but do not spoil unemployed!

Appendix

Table 1 Nonfarm Business Sector: Productivity, output, hourly compensation, seasonally adjusted. Percentage change from previous quarter at annual rate.

Year and quarter	Output per hour of all persons	Output	Hours of all persons	Nominal compensation per hour	Real compensation per hour
2001Q1	-1.4	-0.9	0.5	2.8	-0.9
Q2	-0.1	-2.7	-2.6	0.1	-3.0
Q3	2.1	-0.8	-2.9	1.0	0.1
Q4	7.2	2.9	-4.0	1.5	2.1
Annual	1.1	-0.1	-1.2	2.7	-0.1
2002Q1	8.6	6.2	-2.3	2.9	1.6
Q2	1.7	0.9	-0.7	4.0	0.4
Q3	5.5	5.2	-0.2	1.8	-0.3
Q4	0.7	1.7	0.9	3.9	1.9
Annual	4.8	2.7	-2.0	2.4	0.8
2003Q1	1.9	1.8	-0.1	3.4	-0.4

Source: U.S. BLS Current Population Survey Table 2 (June 3 2003).

Table 2 Unemployment rates by education^a, 25 yrs. & over

	93	94	95	96	97	98	99	00	01	02	03 ^b
Less Than a High School Diploma	10.8	9.8	9.0	8.7	8.1	7.1	6.7	6.3	7.2	8.5	8.8
High School, No College	6.3	5.3	4.7	4.7	4.2	4.0	3.5	3.5	4.2	5.3	5.5
Some College or Associate Degree	5.2	4.4	4.0	3.8	3.3	3.0	2.8	2.7	3.3	4.5	4.8
Bachelor's Degree and Higher	3.0	2.6	2.5	2.3	2.0	2.2	1.8	1.7	2.3	2.9	3.1

a). Source. U.S. BLS.

b). From January to July.

Table 3 Unemployment and skill

Unemployment Rates by Occupations	April 1987 ^a	2001 ^b	2002 ^b
Managerial and Professional Specialty	2.1	2.3	3.1
Technical, Sales, and Administrative Support	4.3	4.1	5.2
Service Occupations	7.6	5.8	6.7
Precision Production, craft, and repair	6.5	4.6	6.1
Operators, fabricators, and laborers	9.8	7.7	8.9

a). Source. Akerlof and Yellen (1990).

b). Source. U.S. BLS.

Table 4 Unemployment rates by industry and sex (Source: U.S.BLS Household data annual average Table 26 Unemployed persons by industry and sex).

Industry			Men		Women	
	2001	2002	2001	2002	2001	2002
Total, 16 years and over	4.7	5.8	4.8	5.9	4.7	5.6
Nonagricultural private wage and salary workers	5.0	6.2	5.0	6.2	4.9	6.1
Mining	4.7	6.2	4.7	6.1	4.2	6.8
Construction	7.3	9.2	7.5	9.5	5.1	7.3
Manufacturing	5.2	6.7	4.7	6.0	6.4	8.2
Durable goods	5.3	7.0	4.8	6.4	6.5	8.8
Lumber and wood products	6.4	6.9	6.6	7.0	5.2	6.8
Furniture and fixtures	5.0	7.2	4.5	6.1	6.0	9.8
Stone, clay, and glass products	5.4	5.6	5.3	4.8	5.7	8.8
Primary metal industries	5.3	7.8	4.3	7.7	9.5	8.2
Fabricated metal products	4.9	6.7	4.4	6.7	6.6	6.8
Machinery, except electrical	5.0	7.8	4.9	7.2	5.3	9.5
Electrical machinery, equipment, and supplies	6.0	8.4	4.8	7.2	7.8	10.4
Transportation equipment	4.5	5.1	4.1	4.7	5.8	6.4
Automobiles	5.1	5.4	4.7	4.8	6.5	7.4
Other transportation equipment	3.6	4.6	3.3	4.6	4.6	4.6
Professional and photographic equipment	3.9	6.1	3.3	5.4	4.9	7.5
Other durable goods industries	8.3	9.2	8.6	7.0	7.7	12.6
Nondurable goods	5.2	6.1	4.4	5.2	6.3	7.6
Food and kindred products	5.2	6.2	4.7	5.0	6.2	8.5
Textile mill products	8.3	9.3	7.4	7.9	9.3	11.0
Apparel and other textile products	9.7	10.6	7.6	7.9	10.9	12.3
Paper and allied products	4.0	3.5	3.2	3.1	6.5	4.8
Printing and publishing	4.1	5.5	4.1	5.2	4.1	6.0
Chemicals and allied products	3.9	4.9	3.6	4.7	4.4	5.2
Rubber and miscellaneous plastics products	4.8	6.1	4.1	5.7	6.2	6.9
Other nondurable goods industries	4.6	6.1	4.2	6.3	5.5	5.5
Transportation and public utilities	4.1	5.5	3.9	5.0	4.5	6.6
Transportation	4.5	5.4	4.4	5.1	5.0	6.3
Communications and other public utilities	3.4	5.7	3.1	4.9	3.9	7.0
Wholesale and retail trade	5.5	6.8	5.1	6.2	6.0	7.5
Wholesale trade	3.9	5.0	3.5	4.2	4.8	6.8
Retail trade	5.9	7.2	5.6	6.8	6.2	7.6
Finance, insurance, and real estate	2.7	3.3	2.7	2.9	2.8	3.5
Service industries	4.6	5.5	4.9	6.1	4.4	5.1
Professional services	2.9	3.5	2.7	3.3	2.9	3.6
Other service industries	7.0	8.6	6.6	8.5	7.5	8.7
Agricultural wage and salary workers	9.5	9.1	9.4	9.0	9.7	9.4
Government, self-employed and unpaid family workers	2.1	2.5	2.0	2.6	2.1	2.4

References

- [1] Acemoglu, Daron (2002), "Technical Change, Inequality, and the Labor Market," *Journal of Economic Literature*, **XL (1)**, pp.7-72.
- [2] Akerlof, George A. (1982), "Labor Contracts As Partial Gift Exchange," *Quarterly Journal of Economics*, **97**, pp.543-69.
- [3] Akerlof, George A. (1984), "Gift Exchange and Efficiency-Wage Theory: Four Views," *American Economic Review*, AER Papers and Proceeding, **74(2)**, pp.79-83.
- [4] Akerlof, George A. (2002), "Behavioral Macroeconomics and Macroeconomic behavior," *The American Economic Review*, **92(3)**, pp.411-433.
- [5] Akerlof, George A., and Yellen, Janet L. (1986), "Introduction," in George A. Akerlof and Janet L. Yellen, eds., *Efficient Wage Models of the Labor Market*. Cambridge University Press.
- [6] Akerlof, George A., and Yellen, Janet L. (1990), "The Fair Wage-Effort Hypothesis and Unemployment," *Quarterly Journal of Economics*, **105(2)**, pp.255-83.
- [7] Basu, Susanto, John Fernald, and Kimball, Miles (2006), "Are Technology Improvements Contractionary?" *American Economic Review*, **96**, pp.1418-1448.
- [8] Blanchard, Olivier, and Katz, Lawrence F. (1997), "What We Know and Do Not Know about the Natural Rate of Unemployment," *Journal of Economic Perspectives*, **11(1)**, p.51-72.
- [9] Blanchflower, David G., and Oswald, Andrew J. (1994). *The Wage Curve*. Cambridge, Massachusetts and London: MIT Press.
- [10] Bernanke, Ben S. (2003), "The Jobless Recovery," At the Global Economic and Investment Outlook Conference, Carnegie Mellon University, Pittsburgh, Pennsylvania, November 6, 2003.
- [11] Caballero, Richard J., and Hammour, Mohamad L. (1994), "The Cleansing Effect of Recessions," *American Economic Review*, **84(5)**, pp.1350-1368.
- [12] Corrado, Carol (2002), "Industrial Production and Capacity Utilization: The 2002 Historical and Annual Revision," the Board Division of Research and Statistics, Federal Reserve Board.
- [13] Davis, Steven J., and Haltiwanger, John (1990), "Gross Job Creation and Destruction: Microeconomics Evidence and Macroeconomic Implications," NBER Macroeconomics Annual 5, pp.123-168.

- [14] Dickens, William T. and Katz, Lawrence F. (1987), "Inter-industry Wage Differences and Industry Characteristics," in Kevein Lang and Jonathan S. Leonard, eds., *Unemployment and the Structure of Labor Markets*. New York: Blackwell, pp. 48-89.
- [15] Fay, Jon and Medoff, James (1985), "Labor and Output over the Business Cycle," *American Economic Review*, September, pp.638-655.
- [16] Gali, Jordi (1999), "Technology, Employment, and the Business Cycle: Do Technology shocks explain aggregate fluctuations?" *American Economic Review* **89(1)**, pp.249-271.
- [17] Greenspan, Alan (2002), "Productivity," at the U.S. Department of Labor & American Enterprise Institute Conference, Washington, D.C., October 23, 2002.
- [18] Gould, Elise (2003), "Measuring Employment Since the Recovery: a comparison of the household and payroll surveys," Briefing Paper, Economic Policy Institute, Washington, D.C.
- [19] Hall, Robert (1999), "The Concentration of Job Destruction," NBER Working Paper 7025.
- [20] Katz, Lawrence F., and Summer, Lawrence H. (1989), "Industry Rents: Evidence and Implications," *Brookings Papers Econ. Activity: Microeconomics*, pp.209-75.
- [21] Kelso, Jr., A.S., and Crawford, V.P.(1982), "Job Matching, Coalition Formation, and Gross Substitutes," *Econometrica*, **50**, pp.1483-1504.
- [22] Krueger, Alan B., and Summers, Lawrence H. (1988), "Efficiency Wages and the Inter-industry Wage Structure," *Econometrica*, **56(2)**, pp.259-93.
- [23] Mankiw, N. Gregory (1997). *Macroeconomics*. Third Edition. Worth Publishers. New York.
- [24] Phelps, Edmund S. (1970), "Introduction: The New Microeconomics in Employment and Inflation Theory," in *Microeconomic Foundations of Employment and Inflation Theory* by Edmund S. Phelps et al, Norton, New York.
- [25] Rogerson, Richard (1997), "Theory Ahead of Language in the Economics of Unemployment," *Journal of Economic Perspectives*, **11(1)**, pp.73-92.
- [26] Schreft, Stacey L., and Singh, Aarti (2003), "A Closer Look at Jobless Recoveries," *Economic Review*, Second Quarter 2003, Federal Reserve Bank of Kansas City. Published in website at www.kc.frb.org.
- [27] Shapiro, Carl, and Stiglitz, Joseph E. (1984), "Equilibrium Unemployment as a Worker Discipline Device," *American Economic Review*, **74 (3)**, pp.433-444.

- [28] Solow, Robert, M. (1979), "Another Possible Source of Wage Stickiness," *Journal of Macroeconomics*, 1, pp.79-82.
- [29] Stiglitz, Joseph, E. (1997), "Reflections on the Natural Rate Hypothesis," *Journal of Economic Perspectives*. 11(1), pp.3-10.
- [30] Stiglitz, Joseph, E. (2002), "Information and the Change in the Paradigm in Economics," *American Economic Review*, 92(3), pp.460-501.
- [31] Summers, Lawrence H. (1988), "Relative Wages, Efficiency Wages, and Keynesian Unemployment," *American Economic Review*, **78(2)**, pp.383-388.
- [32] Swanson, Eric T. (2007), "Real Wage Cyclicalities in the PSID," Working Paper 2007-15, Federal Reserve Bank of San Francisco.
- [33] Trehan, Bharat (2001), "Unemployment and Productivity," *FRBSF Economic Letter*, Number 2001-28, Research Department, Federal Reserve Bank of San Francisco.
- [34] Yellen, Janet L. (1984), "Efficiency Wage Models of Unemployment," *American Economic Review*, AER Papers and Proceedings, **74(2)**, pp.200-205.