Comparisons of linear item pricing methods for iterative multi-unit reverse combinatorial auctions[1]

M. S. Iftekhar, A. Hailu and R. K. Lindner
School of Agricultural and Resource Economics, University of Western Australia, Australia

Abstract

In multi-round reverse combinatorial auctions bidders submit bids to supply single items or bundles of items in a sequence of rounds. At the end of each round the auctioneer computes the provisional allocations and provides feedbacks on implied item prices to facilitate bidding process. Ask prices could be either on bundles or on individual items. Individual prices may be same for all bidders (anonymous) or vary from bidder to bidder (non-anonymous). For a bundle, sum of its items prices may be equal (linear) or unequal (non-linear) to its price. Auction designs based on linear anonymous prices have been successfully tested and applied in different experimental and practical contexts. It has been observed that they can offer flexibility and achieve substantial efficiency for combinatorial auctions. However, linear pricing schemes have rarely been used for cases where the items or services auctioned can be offered at different levels (e.g. areas conserved or species protected) rather than being distinct items (e.g. airport landing slots). Theoretically, the former resemble multi-unit reverse combinatorial auctions and are more appropriate for natural resource conservation problems. This paper focuses on testing pricing schemes for combinatorial biodiversity auctions where farmers are contracted to conserve packages of target species populations. We report performance measures for linear pricing schemes based on the Resource Allocation Design (RAD; DeMartini et al., 1998; Kwasnica et al., 2005) and nucleolus algorithms (Dunford et al., 2007). An agent-based computational model is used to thoroughly compare the pricing schemes in terms of allocative and budgetary efficiency outcomes for different levels of competition and heterogeneity in the bidder resource and cost structures.

Key words

Agent based model, Biodiversity conservation, Combinatorial auction, Resource Allocation Design, Nucleolus based algorithms

1. Introduction

Combinatorial auction allows simultaneous trading of multiple items as participants can bid on combinations or packages of items (Pekeč and Rothkopf, 2003). The bidders have flexibility in expressing their preferences as these auctions allow them to nominate prices for individual or combination of items (Cramton et al., 2006). This design is helpful in the trading of complementary goods, as the bidders can express their precise valuations for any collection of items (de Vries and Vohra, 2003). Combinatorial auctions are frequently used in trading a variety of related items such as office equipments, bus routes, radio spectrum licences and take-off and landing slots at airports (Jehiel and Moldovanu, 2003). Stoneham et al. (2005) have proposed that combinatorial auctions can be used to manage additive natural resources, such as allocation of areas of native timber or the allocation of aquaculture sites.

[1] Part of on-going PhD research of MSI. Correspondence: M. S. Iftekhar, School of Agricultural and Resource Economics, University of Western Australia, 35 Stirling Highway, Crawley, Perth, WA 6009, Australia, E-mail - mdsayediftekhar@yahoo.com

In terms of timing, combinatorial auctions could be either continuous or round based (for a review of the design space of combinatorial auction see Parkes, 2006). In continuous auction, a timer is started and bids are submitted. The bids that fit within the logistic and feasibility constraints of the auction are posted. New bids can be placed at any time. The auction ends if no allocation-changing bids are submitted during a fixed period of time (Porter et al., 2003). The round-based auction could be composed of single or multiple rounds. In multi-round auctions, bids are submitted in a sequence. After each round a winner determination problem (WDP) is solved to determine a set of provisional allocations. Information related to the state of the market (such as the winners and prices) is posted and a new round is then started. Often activity rules for bid submission are imposed to prompt faster convergence of auction outcomes. The auction may end after a set number of rounds or when there are no new winners or no new bids (Porter et al., 2003).

A key feature of combinatorial auctions is that the auctioneer facilitates bidding by providing information on item prices. The prices are based on provisional allocations for the current round and reflect the prices implied by these provisional allocations. Ideally, one would want these feedback prices to be such that the value of winning packages are equal to the actual bids at these prices, while the losing packages should be deemed less valuable at these prices relative to the actual bids (see Xia et al., 2004 for details). If these prices are per-item prices (also known as *linear prices*) the feedback would be intuitive and easy to understand for bidders for several reasons (Pikovsky, 2008). Firstly, only a limited number of prices have to be communicated in each round. This reduces the cognitive, computational, and communication burden placed on the bidders and on the auctioneer (Xia et al., 2004). Secondly, it gives guidance to bid formation and evaluation. A bidder can easily estimate values for different bundles, even if no bid was submitted for this bundle in previous rounds. So, such auctions tend to be simpler to bid on, run faster, and require less communication and computation and thus are feasible for a larger number of items (Kwon et al., 2005).

Auction designs based on linear prices have been successfully tested and applied in different experimental and practical contexts. It has been observed that they can offer flexibility and achieve substantial efficiency for combinatorial auctions. There are couple of algorithms for calculating linear prices (for a review see Pikovsky, 2008). These include the Resource Allocation Design (RAD) which was first proposed by DeMartini et al. (1999) and the nucleolus algorithm (proposed by Dunford et al., 2007). RAD has been extensively tested in laboratory and simulation experiments and used in different practical contexts (Goeree et al., 2007). The concept of a nucleolus is a key concept in coalitional game theory, which has been recently applied in combinatorial auctions (Dunford et al., 2007). Both algorithms are interesting and offer advantages over earlier algorithms, so we have chosen them for study. However, the focus in the combinatorial auction literature so far has been on auctions for distinct items[2], i.e. items that do not come in multiple quantities. As a result, these schemes have not yet been tested for cases where the items or services auctioned can be offered at different levels. In natural resource management, the interest is likely to be in designing auctions for cases where bidders can offer different services with each service coming at different levels. For example, a farmer is capable of undertaking conservation activities to benefit individual or multiple species but the level of benefit can also be varied for each line of benefit.

This paper focuses on testing linear pricing schemes for combinatorial biodiversity auctions where farmers are contracted to conserve packages of target species populations. We construct an agent-based model to examine the performance of the designs. The simulated auctions are procurement auctions where a government agent has a target conservation level and is running a combinatorial auction to allocate contracts. The bidders resemble a

[2] Aparicio et al. (2008) have developed a Data Envelopment Analysis (DEA) based linear pricing scheme that focuses on multi-unit forward combinatorial auction. In this paper we study multi-unit reverse combinatorial auction

population of farmers with different conservation capacities and cost structures. Bidders submit packages indicating the type and level of species conservation they are willing to undertake and the prices they would like to be paid. Auctions run for a fixed number of rounds and, in response to price feedback from the government agent, bidders use simple bid updating rules to revise their bids with the objective of winning the contracts. The performance of each auction format is evaluated for different levels of competition. The comparative analysis is also undertaken for different levels of heterogeneity in terms of capacity and cost structures of the bidder populations.

The remainder of the paper is organized as follows. In Section 2, we discuss combinatorial allocation problem for biodiversity auction. We introduce linear pricing concept in Section 3. Several candidate linear pricing schemes are reviewed in Section 4. We discuss the modifications we have done to the algorithms to accommodate the multi-unit nature of our combinatorial auctions. In Section 5, we provide the framework for the simulation experiments. Results are presented and discussed in Section 6. In Section 7, we conclude the paper.

2. Combinatorial biodiversity auction

In conservation auctions, contracts are allocated to undertake environmental conservation activities based on project proposals and prices from the participating farmers. The amount of private information that farmers have extends beyond cost on individual items to the nature of cost complementarity among different levels or types of projects. For example, the presence of ecological complementarity and inter-dependence among the species as well as the presence of jointness in conservation technology might mean that the cost of undertaking projects with multiple objectives is lower from what one can deduce by looking at individual projects. Different farmers may have cost and benefit advantages for different types of activities. Combinatorial auction, which allows bidding on single as well as packages of items, can make it possible to exploit these potential advantages (Parkes, 2006). Farmers would have the opportunity to present more information on the nature of projects they can undertake and the procuring agency would have more flexibility in meeting its target under these auctions.

Iftekhar et al. (2009) have espoused the idea of combinatorial biodiversity auctions that can be used for allocation of contracts for conservation of multiple species on farmlands. Here, the agency specifies the ‘goods’ in terms of items (species) and units (population size of species). Following invitation for participation in an auction, the farmers place bids showing their willingness to maintain a certain set of species and their respective populations at what cost (Table 1). The auctioneer then selects the bids according to pre-defined criteria. The normal criterion would be the minimization of the total cost while fulfilling the target.

Table – 1: Examples of bids in a hypothetical combinatorial biodiversity auction

Bid / Package	Bidder’s ID	Item (Species and population size in the tenth year)			Ask price ($)
		Malleefowl	Carpet python	Red-tailed phascogale	
1	01	10	10	2	97,542
2	02	40	40	6	374,538
3	01	60	60	6	580,962
4	02	60	70	4	580,180
5	01	60	80	4	607,742

Source: Iftekhar et al. (2009)

To clarify the concept, let's assume that our agency needs to maintain a set of species, $G = \{1, 2,\ldots, g\}$. It specifies number of animals (or units) of each species it wants, $U = \{u_1, u_2,\ldots, u_g\}$, $u_i \in \Re^+$. There are N sellers $\{1, 2,\ldots, n\}$. Each bidder submit a set of asks, $A_i = \{A_{i1}, A_{i2}, \ldots, A_{im}\}$. An ask is a tuple $A_{ij} = \left\langle \left(\lambda_{ij}^1, \lambda_{ij}^2, \ldots, \lambda_{ij}^g\right), p_{ij} \right\rangle$, where $\lambda_j^k \geq 0$ is the number of units of species k offered in the bid j submitted by bidder i. The ask price is $p_{ij} \geq 0$. The winner determination problem (WDP) is to find the least expensive set of bids under the constraint that the agency receives all the target units of the species (Sandholm, 2006):

$$\begin{aligned} &\min \sum_{i=1}^{N} \sum_{j=1}^{n_i} p_{ij} x_{ij} \\ &s.t. \sum_{i,j} \lambda_{ij}^k x_{ij} \geq U \\ &\sum_{i} x_{ij} \leq 1 \\ &x_{ij} \in \{0,1\} \end{aligned} \tag{1}$$

Here, x_{ij} is a binary variable, indicating whether contract for bundle j is awarded to bidder i. The first constraint is the resource requirement constraint and the second constraint reflects the condition that at most one bundle is selected from each bidder.

What makes combinatorial auctions complex is not the winner determination problem. It is the fact that the options or strategies that bidders have are so complex that the outcomes from a single round would not be sufficient. The bidding process needs to be facilitated through the provision of price feedbacks that would help the bidders to formulate their bids in subsequent rounds.

3. Linear pricing for combinatorial auctions

Linear pricing schemes provide price feedback in the form of item prices that rationalizes the results of the winner determination problem. The ideal set of feedback prices would be compatible with the given allocation and the given bids, i.e., computed value for packages in winning (loosing) bids are not lower (higher) than the respective bids. Compatible prices provide indications to the winners, why they have won, and to the losers, why they have lost.

However, such linear competitive equilibrium (CE) prices may not exist in the presence of strong sub-additive bidder valuations for multiple bundles (Pikovsky, 2008). For example, consider a two items (a and b) and two bidders (1 and 2), procurement auction. Bidder 1 is asking \$3 for {a, b} and bidder 2 is asking \$1, \$1 and \$4 for {a}, {b} and {a, b} respectively. At most one bid is selected from a bidder. Bidder 1 wins for {a, b} and total cost is \$3. To be compatible with this allocation, anonymous item prices, $p_{ask}(a)$ and $p_{ask}(b)$, have to be less than or equal to \$1. This implies that the auctioneer could spend only \$2 by purchasing the items separately, which is lower than the cost from the optimal allocation. So, no anonymous linear prices exist in this case supporting the efficient allocation identified by the WDP.

In such situations, the use of an approximate linear price system has been advised. This approach was used for the first time by Rassenti et al. (1982). This system tries to find a set of linear prices which satisfies the following conditions of the dual price system[3] as much as possible:

[3] Dual or shadow price is the change in the objective value of the constrained optimization problem due to a change in the binding constraint. The constraint is relaxed by moving them into the objective function with a penalty term in proportion to the

- Primal feasibility: It ensures that no unique item is traded / allocated for more than once.
- Primal complementary slackness: This condition states that the winners are asked to pay exactly what they have bid. In other words, computed value for bundles in the winning bids should be equal to their respective bids.
- Dual complementary slackness: The items not in optimal allocation receive zero prices. So, the price of an unsold item is zero.
- Dual feasibility: The item prices should be as such that the non-winning bids are priced out. That is the computed value for the bundle is less than the submitted bids.

However, often dual feasibility constraint is relaxed, which may result in estimation of prices for some bundles in loosing bids higher than their submitted bids in a procurement auction (Drexl and Jørnsten, 2007). That's why these prices are often called pseudo-dual linear prices (Rassenti et al., 1982).

Let's consider the following example to clarify the concept (Table 2). There are 3 bidders and 3 items. In a cost minimizing multi-unit combinatorial auction each of them has submitted bids on 3 packages in any round; however, a maximum of one package is provisionally selected from each bidder. The auctioneer's target is to achieve 6 units of X, 4 units of Y and 3 units of Z. She solves model (1) to achieve the target at minimum cost. The winners are marked with asterisks.

Table - 2: Example of bids ($) in a 3 bidders and 3 items multi-unit auction. Bids with *s are in winning combination

Bidder	Bid ID	Bid ($)	Units of different items		
			X	Y	Z
AA	1	150	1	3	3
	2	420	1	7	3
	3	500	2	8	3
BB	1	480	1	8	1
	2	370	3	6	1
	3*	375	3	6	2
CC	1	250	2	4	1
	2*	260	3	1	1
	3	360	4	2	3

Now, the auctioneer has to provide feedback on item prices. These prices are obtained by forcing the computed value (sum of the item price multiplied by units of the item) of the package comprising a provisionally winning bid to equal its respective bid amount but allowing the computed value of packages comprising non-winning bids to be less than the bid for the respective package. The target of the auctioneer is to reduce the amount by which the computed value for the loosing bids fluctuate to keep the pseudo dual prices as close to the dual prices. Let, δ_j be the slack variable that represents the difference between the bid amount of non-winning bid j and the computed value of the bundles contained in non-winning bid j. However, the auctioneer has considerable flexibility in choosing an objective function that will help in selecting among multiple solutions while still ensuring that the set of pseudo-dual prices yields the minimum cost of the round. The auctioneer could either minimize maximum slack variable or minimize total amount of slack. Let's assume that the auctioneer has tried options 2.1-2.3 while solving 2.

amount of infeasibility (de Vries and Vohra, 2000). For example, if a farm is already operating at its maximum area, the shadow price would be the price of adding an extra unit of land.

$$
\begin{aligned}
&\min_{\gamma, h, \delta} \quad h \\
&\textit{subject to} \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} = p^j \ \forall j \in W_t \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} - \delta^j \le p^j \ \forall j \in L_t \\
&\quad \delta^j \ge 0 \qquad \forall j \in L_t \\
&\quad \gamma_t^k \ge 0
\end{aligned} \tag{2}
$$

$$h \ge \sum\nolimits_j \left(\delta^j\right) \quad \forall j \in L_t \tag{2.1}$$

$$h \ge \sum\nolimits_j \left(\delta^j\right)^2 \quad \forall j \in L_t \tag{2.2}$$

$$h \ge \ \delta^j \quad \forall j \in L_t \tag{2.3}$$

here $\lambda_j^k \ge 0$ is the number of units of item *k* offered by the bid *j* submitted by bidder *i*. The ask price is $p_{ij} \ge 0$. x_{ij} is a binary variable, indicating whether contract for bundle *j* is awarded to bidder *i*. Per-unit prices for each item is γ_t^k . The results from the optimisations are presented in Table - 3.

Table - 3: Pseudo dual prices for the items and the slack variables for different objective functions solved for Table 2

Bidder	Bid	2.1	2.2	2.3
Slack variables				
AA	1	0	0	1.941
	2	0	0	0
	3	0	0	0
BB	1	0	0	0
	2	5.00	4.423	3.816
	3*			
CC	1	0	0	0
	2*			
	3	2.00	2.885	3.816
Total slack		7.00	7.308	9.573
Per unit price				
X		79.00	78.846	78.684
Y		23.00	22.885	22.763
Z		0.00	0.577	1.184

We can see that different objective functions produce different set of prices and slack variables. Model 2.1 has minimum total slack but has the maximum slack for any bid among the schemes. Also it has a zero price for item Z, which may confuse the bidders in formulating their next round of bids. Model 2.2 lowered the maximum slack even though total slack is higher than model 2.2. Model 2.3 works on individual slacks and produce lower maximum slack than the other two models.

Therefore, pricing schemes should be chosen carefully as many sets of pseudo dual prices can satisfy the constraint set (Dunford et al., 2007). In a procurement auction, if the feedback price of some bundle is approximated too low, this can keep a bidder from submitting a potentially winning bid. If the price is approximated too high, a new bid might have little chances of winning. An unfavourable price selection can exacerbate the threshold problem. By threshold problem we mean the situation where small bidders with combined least cost fail to coordinate and outbid a large bid and loose ultimately. All these factors can reduce the efficiency of the auction (Pikovsky, 2008). So, the prices should provide enough indication for new bids to be competitive in the next round (Kwasnica et al., 2005). Linear (LP) or nonlinear (NLP) constrained optimization schemes are solved sequentially to get a unique set of prices.

4. Competing linear pricing rules for combinatorial biodiversity auctions

There are couple of linear pricing schemes that have been used for iterative single-unit combinatorial auctions (for a review see Parkes, 2006). In this paper, we study two of these schemes, namely, the Resource Allocation Design (RAD) and the nucleolus based mechanism. RAD is one of the pioneer linear pricing designs. The RAD was developed by incorporating features from two competing designs – package bidding from the Adaptive User Selection Mechanism (AUSM) and an iterative format from the Simultaneous Multiple Round (SMR) auctions. It added a feature of providing feedback on prices (Kwasnica et al., 2005). RAD lets the bidders submit bids using OR bidding language[4]. It calculates the prices based on the LP relaxation of the WDP by solving an optimization problem. Then another optimization is solved to reduce the threshold problem, by fine-tuning and balancing the prices across the items. Feedback on prices and winning bids are provided. In the following round bidders can revise their loosing bids, although winning bids remain active. The auction stops when there is no new bid or new allocation is made for consecutive number of rounds. Cognitive simplicity and its dynamic ask price computation algorithm have made RAD design attractive to auctioneers (Pikovsky, 2008).

Nucleolus based algorithms have retained the basic properties of RAD, such as, package bidding, iterative format, use of OR bidding language, stopping rules and a linear price feedback system. It differs in the way it calculates the feedback prices. It uses the nucleolus concept to find an optimal set of prices (Dunford et al., 2007). Here, instead of bidders, items are treated as agents. Every package is considered as a coalition of the items. The price set tries to distribute the total cost among the items fairly. Depending on the wining and losing condition the maximum dissatisfaction / excess for any coalition (i.e., bid) is sequentially minimized that could occur regarding a specific price estimate of a given item. Here, 'excess' refers to the differences between the computed values for the bundles and respective bids.

Let's consider a single-unit procurement combinatorial auction. There are N items where the cost savings, $v(P)$, to each package (P) is $v(P) = \sum_{i \in P} c_i - c(P), \ \forall P \subseteq N$. Here, c_i is the production cost of the item i individually and $c(P)$ is the production cost of the package P. It will be beneficial to bundle the items only if $v(P) < 0$. Any price allocation x among the items requires the distribution of the total cost among the items. In order to find the core, the nucleolus concept relies on minimizing the dissatisfaction or 'excess', which is the difference between the cost savings the items enjoys at the allocation x and the cost savings it could obtain by acting alone: $e(x, P) = \sum_{i \in P} (c_i - x_i) - c(P)$.

[4] OR bidding language allows a bidder to submit and win multiple bundles (Xia et al., 2004). Another language is XoR, which allows a bidder to win, at most, one bundle even though multiple bids could be submitted by the bidder (Parkes, 2006).

An allocation *y* is more favourable to package *P* than an allocation *x* whenever: $e(y,P) < e(x,P)$. A package has an 'objection' to *x* if there is another allocation *y* that is more favourable to *P*. Their objection has a 'counter objection' if there exists another coalition *T* that is worse off at *y* and furthermore, whose dissatisfaction (excess) with *y* is greater than coalition *P*'s dissatisfaction with *x*, that is, *T* has a counter objection to *y* if $e(y,T) > e(x,T)$ and $e(y,T) \geq e(x,P)$. The nucleolus is the set of all allocation *x* with the property that for every objection (*y*, *P*) there is a counter objection (Carter and Walker, 1996). Table 4 presents some basic features of the tested linear pricing schemes.

Table - 4: Basic features of some linear pricing schemes, which have been tested in this paper

Schemes	Optimization	Sign of slack variable	Source
RAD – LP	1. Minimizing the maximum of slack variables 2. Maximizing the minimum price*	Positive	Kwasnica et al. (2005)
RAD-NLP	1. Minimizing the sum of the squared values of the slack variables 2. Maximizing the minimum price*	Positive	Kwasnica et al. (2005)
Nucleolus	1. Winning bids are lumped into a single bid. Minimizing the maximum of slack variables	Free	Dunford et al. (2007) Drexl and Jørnsten (2007)
Constrained Nucleolus	1. Minimizing the maximum of slack variables	Free	Dunford et al. (2007)

* We have modified this to minimizing the maximum price

Theoretically, combinatorial biodiversity auctions resemble multi–unit reverse combinatorial auctions, where bidders submit packages of multiple units of items and the auctioneer selects least cost combination of bids to fulfil its target. We are not aware of any study, which has tested these linear pricing schemes for multi–unit reverse combinatorial auctions. In absence of proper theoretical analysis, it is difficult to predict the behaviour of these pricing schemes. In order to test the pricing schemes we have accommodated the features of a multi-unit reverse combinatorial auction.

All schemes are adapted to reverse auctions, where the auctioneer tries to reduce the procurement cost instead of maximizing the revenue. This has been done to allow the conservation agency to allocate the contracts to the least cost farmers while fulfilling the targets. We have retained the free disposal property of the winner determination problem. This means that while fulfilling the target the auctioneer would not mind to have some extra unit of targets if the total cost is reduced.

The schemes have been adapted to accept XoR bids. This means that at most one bid is accepted from a bidder even though she is allowed to submit as many bids as feasible. This feature will help the bidders to precisely express their valuations. We consider it reasonable as it has been observed that conservation cost of different population sizes of target species may not be additive. There could be substantial economies of scale in the conservation of endangered species. Consider an example where the valuations of a farmer for two species {A} & {B} and their package {AB} are \$10, \$20 and \$25 respectively. If the auctioneer targets multiple units of the items and all three bids are in least cost combination, it is possible that all three bids from the farmer are selected, which clearly exceeds the capacity of the farmer. In this case, the farmer shall either have to submit bids on individual items or only on the package. Either way, it restricts the flexibility of the farmers in submitting bids and of the auctioneer in selecting least cost combination. XoR bidding solves this problem by selecting any one bid from a farmer. The bidders can submit bids on any suitable combination of items and auctioneer could use the information.

In auctions for unique items, each item is procured only once. So, two bids on the same item can never be winners. However, in multi-unit combinatorial auctions, multiple bids on same item can be winners. This makes it hard to satisfy the primary complementary slackness condition, which is supposed to ensure that the computed values for the bundles in winning bids are equal to their respective bids. Let's consider an example of an auction where the target is to obtain 30 units of item X. There are three bids: bid1 (10 units, \$10), bid2 (20 units, \$15) and bid3 (30 units, \$50). The provisional winners are bid1 and bid2. Per unit price for bid1 and bid2 is \$1 and \$0.75 respectively. This is a contradiction and there is no linear anonymous price for item X in this case. So, in our modifications, we have relaxed the primary complementary slackness condition and have allowed the computed values for packages in winning bids to be equal to or greater than their bids. Thus, in this example, per unit price would be \$1. In the following sub-sections we present the details of our modifications to the existing designs.

4.1 Resource Allocation Design (RAD)

The Resource Allocation Design (RAD) was first proposed by DeMartini et al. (1999), which calculates pseudo-dual prices based on the LP relaxation of the combinatorial allocation problem. Since there is only a limited number of cases where a set of ideal prices may exist, in the RAD some slacks are allowed in calculating the feedback prices. These slacks are a measure of the perturbation of the calculated prices from the 'ideal' prices. The degree of distortion is defined as the sum of squared slacks or as the largest slack. RAD chooses these feedback prices so that they constitute the least distortion over the 'ideal' prices. Several alternative objective functions have been suggested to derive item prices in RAD:

- Minimization of the sum of the squares of the slack variables for all loosing bids, followed by minimization of the maximum item prices (we refer to it as RAD NLP version)
- Minimization of the maximum of the slack variable for all loosing bids, followed by minimization of the maximum item prices (we refer to it as RAD LP version)

The description of the two versions provided below follows the presentation in Kwasnica et al. (2005).

4.1.1 RAD NLP version

In RAD NLP we try to get a feasible set of prices and slacks that ensures that the computed values for bundles in the winning bids are equal to or more than the respective bids and for bundles in losing bids the computed values are somewhat less than their bids by solving RAD NLP 01. The variable δ^j is the amount of slack or deviation from the ideal price for each losing package. We minimize the sum of the squares of the δ_j's. Let $L_t = B_t \setminus W_t$ be the loosing bid at round *t*. We solve the following problem –

$$\begin{aligned}
&\min_{\gamma,Z,\delta} \quad Z \\
&\textit{subject to} \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} \geq p^j \ \forall j \in W_t \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} - \delta^j \leq p^j \ \forall j \in L_t \qquad \text{(RAD NLP 01)} \\
&\quad Z \geq \sum\nolimits_j \left(\delta^j\right)^2 \quad \forall j \in L_t \\
&\quad \delta^j \geq 0 \qquad \forall j \in L_t \\
&\quad \gamma_t^k \geq 0
\end{aligned}$$

Here, we minimize the total size of the slack with greater emphasis on larger slacks. Let $\delta^*, \gamma^* \ and \ z^*$ be a solution to RAD NLP 01. It should be noted that the positive slack values indicate the amount by which the computed value is greater than the bid amount. In other words, positive slacks provide wrong signals to the losing bidders: the computed value for the package in losing bid is greater than the ask price. For this reason, these prices are considered approximate instead of exact prices.

Further, the prices we obtain from RAD NLP 01 may not be 'balanced' across the items and would not be able to provide sufficient guideline to the new bidders. For example, consider the following procurement auction version of an example provided in Kwasnica et al. (2005). There are two items (a and b) and three bidders (1, 2 and 3). Bidder 1 is asking \$6 for {a, b} and bidder 2 is asking \$6 for {a}. Bidder 3 has not bid yet but its lower bound is \$2 for {b}. Bidder 1 is the provisional winner. Any prices such that $\gamma_A + \gamma_B = 6$ and $\gamma_A \leq 6$ will satisfy RAD NLP 01. If we select $\gamma_A = 6$ and $\gamma_B = 0$ then bidder 3 has no incentive to submit any new bid in the following round since it cannot bid below \$2. This means that bids from small bidders could not be combined. The more natural decision would be to divide the prices equally among the items.

In order to balance the prices, we run another optimization where we work on the prices while keeping the slack variables fixed that we have obtained from RAD NLP 01. Let, $\overline{\delta}^J = \delta^{*J}$. In the original design the minimum price has been maximized for forward auction. For our reverse auction we minimize the maximum item price sequentially. Let *K* is the set of item prices and $\hat{K} = K$.

$$\begin{aligned}
&\min_{Y,\gamma} \quad Y \\
&\textit{subject to} \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} \geq p^j \qquad \forall j \in W_t \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} - \overline{\delta}^j \leq p^j \qquad \forall j \in L_t \qquad \text{(RAD NLP 02)} \\
&\quad \gamma_t^k \leq Y \qquad \forall k \in \widetilde{K} \\
&\quad \gamma_t^k = \gamma_t^{*k} \qquad \forall k \in K \setminus \widetilde{K}
\end{aligned}$$

Let, $Y^* and \ \gamma^*$ solve RAD NLP 02. We identify the set of prices (s) which have been minimized and are equal to Y^*, i.e., $\gamma_k^t = Y^*$. We separate them into a set *K** and fix them, i.e., $\gamma_t^k = \gamma_t^{k*}$. We define another set with the remaining prices, i.e., $\widetilde{K} = \widetilde{K} \setminus K*$. In the following iteration we minimize remaining prices while keeping the minimized prices obtained in earlier iteration. When we have sequentially minimized all the prices, in other words $\widetilde{K}$ is empty, we are done. The prices obtained from the final iteration are our prices for the following round.

4.1.2 RAD LP version

RAD LP focuses on the maximum slacks and work on them iteratively, unlike RAD NLP which focuses on total slack. At first, we try to get a feasible set of prices and slacks that fulfils the constraints by solving RAD LP 01.

$$
\begin{aligned}
&\min_{\gamma, Z, \delta} \quad z \\
&\textit{subject to} \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} \geq p^j && \forall j \in W_t \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} - \delta^j \leq p^j && \forall j \in L_t \\
&\quad 0 \leq \delta^j \leq z && \forall j \in L_t \\
&\quad \delta^j \geq 0 && \forall j \in L_t \\
&\quad \gamma_t^k \geq 0
\end{aligned}
\qquad \text{(RAD LP 01)}
$$

Let δ^*, γ^* *and* z^* be a solution to RAD LP 01. If there is no slack in any of the losing bids (i.e., $z^* = 0$) or if all the slacks have a value equal to z^* (i.e., $\delta^* = z^*$) we are done with the minimization of the slacks. Then we go to the price minimization optimization. On the other hand, if the values of slacks are different then we try to reduce the maximum of the slacks sequentially. We work on the initial set of slacks that we have obtained in RAD LP 01. We separate the bids which have slacks equal to z^* into a separate set J^* (i.e., $J^* = \{ j \in L_t | z^* = \delta^{*j} \}$) and permanently fix $\delta^j = \delta^*, \forall j \in J^*$. Then we solve RAD LP 02 –

$$
\begin{aligned}
&\min_{\gamma, Z, \delta} \quad z \\
&\textit{subject to} \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} \geq p^j && \forall j \in W_t \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} - \delta^{*j} \leq p^j && \forall j \in J_t^* \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} - \delta^j \leq p^j && \forall j \in L_t \setminus J_t^* \\
&\quad 0 \leq \delta^j \leq z && \forall j \in L \setminus J_t^* \\
&\quad \delta^j \geq 0 && \forall j \in L_t \\
&\quad \gamma_t^k \geq 0
\end{aligned}
\qquad \text{(RAD LP 02)}
$$

Let $\hat{\delta}, \hat{\gamma}$ *and* $\hat{z}$ be the solution. Again, if there is no slack (i.e., $\hat{z} = 0$) or all the slacks have a value equal to $\hat{z}$ (i.e., $\hat{\delta} = \hat{z}$) we go to the price minimization optimization. Otherwise, we separate the bids with $\hat{\delta} = \hat{z}$ into a set $\hat{J}$, let $J^* = J^* \cup \hat{J}$ and go to RAD LP 02 again. This way we reduce the amount of distortions by finding the smallest values for slacks across all losing packages.

It should be noted that in the original design for single unit auctions there is strong equality condition in the second constraint of RAD LP 02. However, contrary to the original design, in our design we maintain a weak equality condition for the second constraint. After solving RAD LP 01 we separate the bids with minimized optimal slack into the set of bids (J^*). Since, selection of bids with optimal slack does not ensure that the computed values minus the slack will be equal to the bids in J^* so it may be difficult to satisfy the strong equality condition.

When the iteration on RAD LP 02 is complete we have the set of minimized slacks. Now in order to balance the prices across the items we run a price minimization optimization as described in RAD NLP 02. We keep the slack variables fixed that we have obtained from the last iteration. Let $\hat{\delta}$ be the solution from the last iteration of RAD LP 02. Let $\bar{\delta}^J = \hat{\delta}^J$ and solve RAD NLP 02.

Intuitively, it means that RAD LP starts with an optimization to find out the initial set of slacks. It sets aside the loosing bids with maximum slacks. It then iteratively works on the slack variables of other loosing bids to reduce them if possible. However, the prices obtained may not be unique and may not help in overcoming the threshold problem. So, we run another optimization to minimize the maximum prices in order to find a unique set of price, which is balanced across the items as far as possible.

4.2 Nucleolus based algorithms

Similar to RAD, nucleolus based algorithms also try to find the set of pseudo-dual prices by minimizing the slacks or infeasibility. Here, the largest slacks are iteratively minimized and the slacks can take any sign (positive or negative). Dunford et al. (2007) have suggested two different ways to use nucleolus concept:

- Nucleolus algorithm: Winning bids are lumped into a single bid. The maximum of the slack variables for the loosing bids is minimized sequentially (Drexl and Jørnsten, 2007).
- Constrained Nucleolus: Sum of the prices of items in a winning bid is required to be equal to or more than the winning bid amount. The maximum of slack variables is minimized sequentially.

Below we describe the approaches following Dunford et al. (2007).

4.2.1 Nucleolus algorithm

In the nucleolus algorithm (hereafter Nuc), the winning bids are lumped together into one single bid. Computed value for the aggregate winning bid is forced to be equal to the provisional procurement cost of the auctioneer that is obtained from the winner determination problem. Computed values for packages in individual winning bids may be greater than, less than or equal to their respective bids. Computed values for loosing bids are forced to be less than or equal to their respective bids (Drexl and Jørnsten, 2007). The first iteration of the nucleolus algorithm is:

$$\begin{aligned}
&\min \quad z \\
&\textit{subject to} \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} = MinCost \qquad \forall j \in W_t \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} - \delta^j \le p^j \qquad \forall j \in L_t \\
&\quad \delta^j \le z \\
&\quad \delta^j \quad \textit{is unrestricted in sign}
\end{aligned} \tag{Nuc 01}$$

Let $z^* \textit{ and } \delta_1^*$ be the solution. If there is no slack (i.e., z* = 0) the iteration is complete. Otherwise, we separate the set of bids (*J**), for which the computed value minus the optimal slack (*z**) is equal to the respective bids, $J_1^* = \left\{ j \middle| \sum_{i \in I^j} \gamma_t^k \lambda_{ij}^k x_{ij} - z_1^* = p_j, j \in L_t \right\}$. Let $J^* = J_1^*$.

If $J^* = B$ the iteration is complete. Otherwise –

$$\begin{aligned}
&\min \quad z \\
&\textit{subject to} \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} = MinCost \qquad \forall j \in W_t \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} - \delta^{*j} = p^j \qquad \forall j \in J^* \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} - \delta^j \le p^j \qquad \forall j \in L_t \setminus J^* \\
&\quad \delta^j \le z \\
&\quad \delta^j \quad \textit{is unrestricted in sign}
\end{aligned} \tag{Nuc 02}$$

Let $z^* \textit{ and } \delta_k^*$ be the solution. At the end of iteration *k*, we set $J_k^* = \left\{ j \middle| \sum_{i \in I^j} \gamma_t^k \lambda_{ij}^k x_{ij} - \delta_k^* = p_j, j \in L_t \setminus J^* \right\}$. Set $J^* = J^* \cup J_k^*$. The algorithm terminates either when z* = 0 or $J^* = B$. The values γ^k after termination of the algorithm, is our desired set of item prices.

4.2.2 Constrained nucleolus algorithm

In the constrained nucleolus algorithm (hereafter ConsNuc), we allow the computed values (sum of the prices of the items multiplied by the number of units) for packages in the winning bids to be equal or greater than the winning bid amount. For packages in loosing bids we allow the computed values to be less than or equal to the bid. The first iteration of the constrained nucleolus algorithm is:

$$
\begin{aligned}
&\min \quad z \\
&\textit{subject to} \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} \ge p^j \qquad \forall j \in W_t \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} - \delta^j \le p^j \qquad \forall j \in L_t \quad \ldots \text{(ConsNuc 01)} \\
&\quad \delta^j \le z \\
&\quad \delta^j \quad \textit{is unrestricted in sign}
\end{aligned}
$$

Let δ_1^* be the solution. Create the set of bids where computed value minus the optimal slack is equal to the respective bid, $J_1^* = \left\{ j \middle| \sum_{i \in I^j} \gamma_t^k \lambda_{ij}^k x_{ij} - \delta_1^* = p_j,\ j \in L_t \right\}$. Let $J^* = J_1^*$. We separate the bids in J^* and reduce maximum slack in the remaining losing bids iteratively, where at iteration k, ConsNuc 01 becomes –

$$
\begin{aligned}
&\min \quad z \\
&\textit{subject to} \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} \ge p^j \qquad \forall j \in W_t \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} - \delta^j = p^j \qquad \forall j \in L_t \setminus J^* \quad \ldots \text{(ConsNuc 02)} \\
&\quad \sum_j \gamma_t^k \lambda_{ij}^k x_{ij} - \delta^{*j} \le p^j \qquad \forall j \in J^* \\
&\quad \delta^j \le z \\
&\quad \delta^j \quad \textit{is unrestricted in sign}
\end{aligned}
$$

Let δ_k^* be the solution. At the end of iteration *k*, we set $J_k^* = \left\{ j \middle| \sum_{i \in I^j} \gamma_t^k \lambda_{ij}^k x_{ij} - \delta_k^* = p_j,\ j \in L_t \setminus J^* \right\}$. Set $J^* = J^* \cup J_k^*$. The algorithm terminates when $J^* = B \setminus W$. We get the prices γ^k after termination of the algorithm.

In the previous sub-sections, we have presented our modifications to the existing designs of four linear pricing schemes based on Resource Allocation Design (RAD) and nucleolus algorithm. The RAD NLP pricing scheme emphasizes on the total deviation from the ideal prices and stops when the total sum of squares of the slacks is minimized. Then it runs an optimization on the price set to try to balance the prices across the items by minimizing the maximum price. RAD LP focuses on the minimization of the maximum of the slacks. This is then followed by the price minimization optimization.

Nucleolus based algorithms reduce the maximum slack iteratively like RAD LP. However, instead of separating the bids with optimal slack in the sequence of iterations like RAD LP, they separate the set of bids for which computed value minus the optimal slack is equal to the respective bids. They do not involve explicit procedures for balancing the prices across the items like RAD LP and RAD NLP. They allow the slack variables to take any (positive or negative) sign. So, they may have more flexibility in selecting the slacks compared to RAD based algorithms. It is expected that such differences in the construction of the algorithms should affect the performances of the auctions.

5. Experimental setting

The experimental framework used to test the designs in the context of a combinatorial auction for biodiversity conservation is described below.

5.1 Farmer types

We have used a bioeconomic model developed by Iftekhar et al. (2009) for three native endangered species (red-tailed phascogale, carpet python and malleefowl) found in the wheatbelt of Western Australia. This model generates optimal costs for conserving different population sizes of target animals for different types of farmers. We have assumed two categories of farmers differing in intervention costs: high and low cost farmers. Further, under each category, we have assumed three types of farmers with different sizes of remnant vegetations (<30, <50 and <70 ha) that they are able to put under conservation scheme. So, in total, we get cost profiles for six farmer types.

For each type of farmer we ran the bio-economic model to generate a cost functions for different population target which is feasible for the given remnant size. Then we conducted regression analysis to get conservation cost functions for different farmers (Table - 5). We can see that with increases in remnant size, the cost for conservation of phascogale and python goes down. In the case of malleefowl, per unit cost first declines and then increases again. Positive coefficients for the intra-species interactions indicate that it is more costly to conserve larger population of any species. However, negative co-efficient for inter-species interactions indicate that it is more cost-effective to conserve different species together.

Table – 5: Cost functions for different types of farmers

	Low cost			High cost		
Land	<30 ha	<50 ha	<70 ha	<30 ha	<50 ha	<70 ha
Farmer ID	Farmer 1 (AA)	Farmer 2 (BB)	Farmer 3 (CC)	Farmer 4 (DD)	Farmer 5 (EE)	Farmer 6 (FF)
Intercept	-18,874	-12,487	-836	-7,369	7,253	20,585
Malleefowl (M)	5,255	4,539	5,064	7,247	5,129	6,050
Phascogale (Ph)	4,763	4,130	3,976	3,735	4,875	4,387
Python (Py)	32,815	27,303	21,873	29,989	22,735	11,979
M_sq	163	103	63	177	116	67
Ph_sq	61	39	28	104	42	30
Py_sq	2,328	1,113	855	2021	1,166	489
M_Ph	-226	-126	-89	-273	-140	-96
M_Py	-1,470	-683	-440	-1478	-564	-218
Ph_Py	-1,218	-540	-417	-933	-510	-220
M_Ph_Py	42	11	7	38	10	4
Adjusted R Square	0.98	0.99	0.99	0.99	0.99	0.99
F	1327.56***	976.58***	1307.26***	1192.12***	1647.04***	1955.41***

It should be noted that the maximum size of population the farmer could conserve depends on the maximum size of the remnant the farmer can put under conservation scheme. So, we can see that a farmer with less than 30 ha remnant vegetation (AA and DD) can conserve a package consisting of up to 40 malleefowl, 40 phascogale and 4 pythons. Similarly, a farmer with less than 50 ha of remnant vegetation (BB and EE) could conserve a package of up to 60 malleefowl, 60 phascogale and 6 pythons. On the other hand, farmers with the largest remnant size (< 70 ha; CC and FF) can conserve a package of up to 80 malleefowl, 80 phascogale and 6 pythons. This means that farmers with bigger remnants can offer packages of larger population sizes. Another trend is that, with bigger remnants, costs for conserving medium size packages are comparatively lower than in the case for smaller size remnants.

5.2 Packages used in the simulation experiment

In order to select the packages we have considered two factors: maximum size of the package that could be conserved with given remnant size and relative cost disadvantage in high population sizes. Within each cost category, for smaller sized packages conservation cost is same for each remnant size. But it gets costlier for small landholders to conserve bigger sized packages. So, it would be prudent for the small landholders to bid on smaller sized packages. On the other hand, for big landholders it may be suitable to select packages of larger sizes to exploit cost advantage. Following this argument, we have considered three levels of population sizes for each species, which is different for each type of remnant size (Table - 6). Thus, for each farmer we compute cost estimates for 27 packages from the bio-economic model.

Table – 6: Different levels of target population size for each type of farmer

Remnant size	Farmer ID	Population level		
		Malleefowl	Phascogale	Python
<30 ha	AA, DD	10	20	2
		20	30	3
		30	40	4
<50 ha	BB, EE	20	20	2
		40	40	4
		60	60	6
<70 ha	CC, FF	40	40	2
		60	60	4
		80	80	6

5.3 Simulation framework

We have developed the simulation framework following Hailu and Schillizi (2004). The model incorporates two types of agents representing the actual players in a real auction. These are:

- Farmers bidding for the conservation contract. Each farmer has several packages of target species and an opportunity cost associated with each package.
- Auctioneer (conservation agency), which selects winning farmers and awards contracts based on the pre-determined criteria. Auctioneer has a fixed target, which may or may not be pre-declared.

Each auction round incorporates the following three major steps or activities.

Step 1: Farmers construct their bids. The bids farmers make depend on their respective opportunity costs, cost categories, their previous bid prices, and price information provided by the auctioneer as well as their success or failure in the previous round. In the first round, farmers do not have any prior experience and start by bidding more of their true opportunity costs depending on their position in cost categories, which ultimately provides the upper bound of the government expenditure. We have assumed that low cost farmers are more aggressive and their bids depend on the random number drawn from a uniform distribution of [2, 3]. Similarly, the bids of high cost farmer depend on the random number drawn from a uniform distribution of [1, 2]. The random draws are mark-up factors that are used to multiply or scale up the package costs in the construction of the bids.

In the subsequent rounds, the farmers use a very simple learning algorithm to revise their bids (Table - 7). Let's assume that there are two bidders (1 and 2) and each of them has submitted two packages (A and B). Due to XoR regulation only one bid from each bidder may be selected. After the very first round, the Package B of Bidder 1 is provisionally selected. In the subsequent round:

1) Bidder 1 will maintain the same bid for winning package.
2) For the loosing bids, Bidder 1 will revise the bids in such a way that total tentative profit is not reduced should any of them be selected in the next round.
3) For the loosing bids with computed value higher than the production cost, the loosing bidder (Bidder 2) will submit bid between computed value and cost.
4) For the loosing bids with computed value lower than the production cost, the loosing bidder (Bidder 2) will submit bid between previous bid and cost.

Table – 7: Bid revision rules used in the experiment

				Bid revision
Winner (Bidder 1)	Winning bid (B)			Current bid (B) = Previous bid (B)
	Loosing bid (A)	Price (A) > Cost (A)	Price (A) - Cost (A) > Profit (B)	Current bid (A) = Price (A)
			Price (A) - Cost (A) < Profit (B)	Current bid (A) = Cost (A) + Profit (B)
		Price (A) < Cost (A)		Current bid (A) = Cost (A) + Profit (B)
Looser (Bidder 2)	Loosing bid (A, B)	Price (A) > Cost (A)		Current bid (A) = Price (A) – (Price (A) – Cost (A)) X random_2*
		Price (A) < Cost (A)		Current bid (A) = Previous bid (A) – (Previous bid (A) – Cost (A)) X random_3

*Low cost farmer: random_2 = random_22 [0, .2] - random_22 [0, .2] / round no; High cost farmer: random_2 = random_22 [0, .3] - random_22 [0, .3] / round no; Low cost farmer: random_3 = random_33 [0, .1] - random_33 [0, .1] / round no; and High cost farmer: random_3 = random_33 [0, .2] - random_33 [0, .2] / round no.

The values of the random numbers are different in each round and different for each bidder. We have assumed that the high cost bidders are more eager to win and reduce bids for their loosing bids more quickly than the low cost bidders. In the later rounds, all bidders reduce their bids at a larger proportion compared to the earlier rounds. This is commonly observed in the real world and experiments that in a discriminatory auction overbidding is highest for the lowest cost bidders, whereas the highest-cost bidders bid closest to their true costs (Latacz-Lohmann and Schilizzi, 2005).

Step 2: The auctioneer takes the submitted bids and computes an expenditure-minimizing allocation in which each bidder receives at most one package, while fulfilling the conservation target. Thus, a multidimensional multiple choice knapsack problem is solved to provisionally select the winners (Lehmann et al., 2006). Then the auctioneer runs a pricing algorithm to determine the per unit price of each item. After finishing the calculations it informs each bidder whether their bids have been successful or not and tentative per animal price for each species.

Step 3: Farmers calculate the total price of their packages based on the item prices provided by the auctioneer. Farmers update their contract status based on the message from the auctioneer and their learning experience. The auction stops after a fixed number of rounds, which depends on the complexity in auction environment.

Since the experiments involve the use of random or stochastic elements, the auctions are replicated 20 times to average over these stochastic elements in the simulation.

5.4 Simulation settings

We have tested the pricing schemes for different levels of competition and heterogeneity in the size and cost structure of the bidder population. We use two sets of test cases to analyse performances.

5.4.1First set of tests

The first set consists of four small case studies with simpler bidders' population and cost structures, which would allow us to understand the behaviour of the pricing schemes for different levels of complexity in bidder populations and competition. For case studies, we increase the number of rounds in the auctions with increasing complexity in bidder population. We replicate each auction for 20 times and report average performances over these replications. The case studies are described in more detail below.

<u>Case study 1:</u> In the first test we consider a homogenous population of six farmers (farmer type AA) each with one identical package to submit (Table – 8). We set the target of the auction as 30 malleefowl, 30 phascogale and 3 python. This target is equal to one-sixth of the aggregate capacity of the bidders and thus represents a fair degree competition or rationing among the bidders. According to the bio-economic model, the optimal cost of achieving or 'producing' this target is $277,255. Homogeneity in the population will help us to understand the effect of pricing on the bidding behaviour and the performance of the auctions. Each auction comprises of 250 rounds.

Table -8: Packages of farmer AA used in case study 1

	Malleefowl	Phascogale	Python	Cost ($)
Bidder 1-6 (AA)	30	30	3	277,255

<u>Case study 2:</u> In the second case study, we consider a semi heterogeneous population of three bidders of farmer type AA (small bidder with low cost, <30 ha remnant) and three bidders of farmer type FF (large bidder with high cost, <70 ha remnant), each submitting a single package (Table – 9). The target of the auctioneer is to achieve 60 malleefowl, 80 phascogale and 6 python, which is one third of the aggregate supply/capacity of the bidders. It should be noted that any two small farmers or a big farmer could supply the whole target. It is expected that the provisional cost will reach the second lowest cost $688,946 pretty quickly due to the competition between the two types of farmers. After that, provisional cost will keep going down albeit at much slower pace due to competition among the small bidders until they reach the lowest production cost of $573,218. Each auction comprises of 500 rounds as the competition pressure is less compared to case study 1.

Table –9: Packages of the farmers used in case study 2

	Malleefowl	Phascogale	Python	Cost ($)
Bidder 1-3 (AA)	30	40	3	286,609
Bidder 4-6 (FF)	60	80	6	688,946

Case study 3: In the third case study, we consider a heterogeneous population of six bidders from each farmer type. Each farmer submits a single package (Table – 10). The target of the auctioneer is to achieve 80 malleefowl, 80 phascogale and 6 python, which is about one third of the aggregate supply/capacity of the bidders. Individual bidders (FF, $887,694) or coalition of bidders ((CC & EE, $811,956) or (AA, DD & EE, $891,732) or (AA, DD & CC, $815,802) can supply the target at different cost. Depending on the performance of the pricing schemes, saturation point would be somewhere in between the optimal cost $811,956 (CC and EE) and second lowest cost $815,802. Each auction comprises of 500 rounds.

Table - 10: Packages of the farmers used in case study 3

	Malleefowl	Phascogale	Python	Cost ($)
Bidder 1 (AA)	10	20	2	124,968
Bidder 2 (BB)	20	20	2	184,567
Bidder 3 (CC)	40	40	2	368,013
Bidder 4 (DD)	30	20	2	322,821
Bidder 5 (EE)	40	40	4	443,943
Bidder 6 (FF)	80	80	6	887,694

Case study 4: In the fourth case study, we accept multiple bids from a heterogeneous bidder's population. Each farmer submits two packages (Table – 11), from which any one may be selected. The target of the auctioneer is to achieve 80 malleefowl, 80 phascogale and 6 python. There are many combinations of bids, which can supply the target. If the auction is run for sufficient number of rounds, the procurement cost would be somewhere in between the optimal cost $738,269 and second lowest cost $761,624 irrespective of the pricing schemes adopted. Each auction comprises of 1000 rounds.

Table - 11: Packages of the farmers used in case study 4

Bidder ID	Package ID	Malleefowl	Phascogale	Python	Cost ($)
Bidder 1 (AA)	P – 1	20	20	4	195,628
	P – 2	20	20	2	184,567
Bidder 2 (BB)	P – 1	60	40	2	550,393
	P – 2	60	60	2	565,996
Bidder 3 (CC)	P – 1	40	40	2	368,013
	P – 2	60	60	4	553,702
Bidder 4 (DD)	P – 1	20	20	2	221,481
	P – 2	20	40	4	300,628
Bidder 5 (EE)	P – 1	40	40	2	441,859
	P – 2	60	60	4	686,464
Bidder 6 (FF)	P – 1	40	40	4	444,127
	P – 2	80	80	6	887,694

For the case studies in the first test, we examine the behaviour of the pricing schemes in detail. We study the amount of bids submitted by the bidders, profit made by each type of farmer, per unit prices of the animals, level of rent extraction and allocative efficiency.

5.4.2 Second set of tests

In the second set of tests, we test the sensitivity of the performance of the pricing schemes in presence of variation in auction environments. It has been observed that asymmetry in bidding capacities and auction environment can have a more powerful impact on the performance of the auction than the auction type itself (Sade et al., 2006). We test the effects of some important factors, such as, bidders' heterogeneity, number of packages, number of bidders, level of competition and level of complementarity. We describe them below:

Bidder's heterogeneity: Since in combinatorial auctions bids from different bidders could be part of a coalition, participation of heterogeneous bidders in the same auction means that the bidders only compete when the bidders belong to the same category and complement bids from other categories. So, bidders' homogeneity should induce more competition. We test the schemes for 20%, 60% and 100% bidders' heterogeneity. In order to get multiple data points we test the schemes for different number of packages (Table 12). To maintain a uniform level of competition across the tests, we set the target in such a way that for any level of heterogeneity and any number of packages any two bidders could supply the target. We run each auction for 250 rounds and replicate for 20 times.

Table - 12: Test schemes and their codes in terms of level of bidder's homogeneity and number of packages submitted by each bidder

		Number of packages [NP]			
Bidders' homogeneity [BH] (%)	Composition of bidders' population	1	2	3	4
100	5 AA	BH100NP1	BH100NP2	BH100NP3	BH100NP4
60	3 AA, 1 BB, 1 CC	BH60NP1	BH60NP2	BH60NP3	BH60NP4
20	1 AA, 1 BB, 1 CC, 1 DD, 1 EE	BH20NP1	BH20NP2	BH20NP3	BH20NP4

Number of packages: With increasing number of packages the auctioneer should enjoy more flexibility in selecting least costly combinations of bids to fulfil the target. However, with increasing number of packages, bidders face the cognitive burden to evaluate all packages. For example, Chen and Takeuchi (2005), who experimented with 4 items in a combinatorial VCG auction, have observed that the bidders have consistently failed to bid on all combination of items (maximum of 15 packages). So, in our experiments we have tested the schemes for up to 4 packages. In order to get multiple data points, we test the schemes for three different bidders' homogeneity levels (Table 12). To maintain a uniform level of competition across the tests we set the target in such a way that for any level of heterogeneity and any number of packages any two bidders could supply the target.

Number of bidders: With respect to the number of bidders, there may be a trade-off between the interests of the auctioneer (more participation is preferred) and those of the bidders (less participation is preferred). From bidders' point of view, less number of bidders means less competition. From auctioneer's perspective it is better to have more participants. It has been observed that subject to some restrictions on the seller's choice of mechanism, an auction with N + 1 bidders beats any standard mechanism for selling to N bidders (Bulow and Klemperer, 1996), which means that additional competition generated through an extra bidder outweighs the benefits from any other mechanism (including auction) with N bidders. We have tested the schemes for 5, 15 and 40 bidders and allowed each bidder to submit 4 packages. In order to get multiple data points we test the schemes for three different competition levels (Table 12).

Level of competition: Level of competition is a large determinant of auction efficiency. The higher is the competition the greater the benefits of auction mechanisms. However, a balance is required to maintain an optimum level of competition. Although a huge proportion of successful bidders mean less competition and less cost savings for the agency, a huge proportion of unsuccessful bidders mean negative political and psychological impact and reduced chance for the unsuccessful bidders in participating in the next round of auction. In our experiment, we have defined the level of competition as the percentage of bidders who could supply the target and tested the schemes for 20%, 40% and 60% competition (Table 13). Each bidder submits 4 packages. We run each auction for 1000 rounds and replicate for 20 times.

Table - 13: Test schemes and their codes in terms of bidder's number and level of competition

		Level of competition (% of bidders could supply the target) [CM]		
Bidders number [BN]	Composition of bidders' population	20%	40%	60%
5 bidders	1 AA, 1 BB, 1 CC, 1 DD, 1 EE	BN5CM20	BN5CM40	BN5CM60
15 bidders	3 AA, 3 BB, 3 CC, 3 DD, 3 EE	BN15CM20	BN15CM40	BN15CM60
40 bidders	8 AA, 8 BB, 8 CC, 8 DD, 8 EE	BN40CM20	BN40CM40	BN40CM60

Level of complementarity in conservation of multiple species: For conservation of multiple species we consider the presence and absence of complementary in conservation cost. In our case, absence of complementarity means that the farmer is planning to conserve the species in separate patches of remnants, which have no connection. So, cost for conservation of multiple species is the summation of their individual costs. On the other hand, by presence of complementarity, we mean that the farmer plans to conserve multiple species in the same remnant and there are some cost savings. To get multiple data points we have run the tests for two different numbers of packages (Table 14). In order to maintain a uniform level of competition across the tests we set the target in such a way that for any level of complementary and for any number of packages any two bidders could supply the target. All bidders are medium size low cost bidders (type BB). Each test has been run for 250 rounds and repeated for 20 times. Then we have averaged the data for different pricing rules and level of complementarity.

Table – 14: Test schemes and their codes in terms of level of complementarity and number of packages submitted by each bidder

Level of complementary (%) [CP]		Number of packages [NP]	
		1	2
0	All (5) bidders have additive cost	CP00NP1	CP00NP2
20	4 bidders have additive cost	CP20NP1	CP20NP2
40	3 bidders have additive cost	CP40NP1	CP40NP2
60	2 bidders have additive cost	CP60NP1	CP60NP2
80	1 bidder has additive cost	CP80NP1	CP80NP2
100	All (5) bidders have complementary cost	CP100NP1	CP100NP2

5.5 Performance measures

There are several criteria for judging the performance of an auction. Allocative efficiency and the degree of rent extraction are two basic criteria that are applicable to all auctions. For combinatorial auctions, additional criteria can be defined. Following Goeree et al. (2007), our set of performance measures include the following three.

Allocative efficiency (AE): Allocative efficiency is a desirable property of an auction. It is achieved when one minimizes the total cost to the winners of the items being auctioned (Pekec and Rothkopf, 2003). It can be measured as the ratio of the total cost of the resulting allocation X to the total cost of an efficient allocation X* (Kwasnica et al., 2005). If AE =1, allocative efficiency is maximized and the lower the AE, the lower the allocative efficiency of the auction. We have used the following formula to calculate AE:

Allocative efficiency = minimum cost to meet the target / total cost of the winning projects

Degree of rent extraction (RE): Degree of rent extraction shows the degree of overpayment or rent to the bidders. Given the resulting allocation X, the degree of rent extraction is measured as the ratio of the auctioneer's expenditure to the minimum cost of an efficient allocation X*:

Rent Extraction = minimum cost to meet the target / auctioneer's total payment

If RE =1, profit to the bidders is minimized. The lower the RE, the higher is the winners' profit.

Price monotonicity (PM): In procurement auction, gradual reduction in item prices in the course of the auction is necessary to reflect the competitive situation. However, often linear prices fluctuate as demand for different items varies. This may confuse bidders. Following Pikovsky (2008), we measure the price non-monotonicity as the sum of the ask price increases $\Delta^{+}\gamma_t^k$ divided by the sum of the ask price decreases $\Delta^{-}\gamma_t^k$ for all species *k* in all rounds *t*. This results in the price non-monotonicity measure, PM, where PM = 0 describes fully monotonic ask prices.

$$PM = \frac{\sum_k \sum_t \Delta^{+}\gamma_t^k}{\sum_k \sum_t \Delta^{-}\gamma_t^k}$$

We also report per unit prices, bidders' profit, and submitted bids to show auction dynamics. In suitable cases, we have conducted univariate analysis for variance to test between subject differences. This allows us to test significance of differences in performance estimates of the pricing rules. Then we conduct multiple comparisons using least significant difference (LSD) t test to find out the sources of differences. For analysis we have used SPSS Version 17.

6. Results and discussions

The results from the simulation experiments are presented and discussed in this section.

6.1 Results from first set

In this section we present results from the first set of tests. We use these case studies to demonstrate the behaviour of the algorithms.

6.1.1 Per unit prices

The individual price estimates across the pricing schemes are quite different. In the first case study, all pricing schemes have allocated the entire package price to python. Nuc has produced lower prices for python than other schemes in the initial rounds. Then, at around round number 90, all schemes have started to produce identical prices. The ask prices have been fully monotonic for all pricing schemes, i.e., have decreased consistently. Similarly, in the second case study, all pricing schemes have allocated entire package price to python. Nuc has produced lower prices for python than other schemes in most of the rounds, even though at the end all schemes have produced almost identical prices. The prices are fully monotonic for all pricing schemes.

In the third case study, initially, all pricing schemes have divided the package prices among the species. For python, Nuc has produced lower average per unit price for most of the rounds followed by RAD LP and ConsNuc. However, at the end Nuc has produced zero prices for python. For phascogale and malleefowl, RAD LP and RAD NLP respectively have produced lower per unit price at the end. Overall, RAD LP has produced least fluctuated per unit price (PM = 0.38) followed by Nuc (PM = 0.78), ConsNuc (PM = 0.95) and RAD NLP (PM = 1.97).

In the fourth case study, RAD LP has allocated the entire package price to malleefowl and subsequently produced highest per unit average price for malleefowl. ConsNuc and Nuc have produced lower per unit prices for malleefowl and python respectively. For phascogale, RAD NLP has produced lower prices followed by Nuc and ConsNuc (Figure 1). Overall, RAD LP has produced least fluctuated per unit price (PM = 0.10) followed by ConsNuc (PM = 0.78), RAD NLP (PM = 0.95) and Nuc (PM = 0.96).

We can see that for simpler cases (case 1 and 2) pricing schemes tend to allocate price for whole package to any one species. They also produce fully monotonic prices. For complex cases (case 3 and 4) the schemes estimate prices for multiple items (Figure 1). RAD LP tends to produce monotonic prices. We shall see later that derivation of lower per unit prices have influence on the bidding dynamics and overall performance of the pricing schemes.

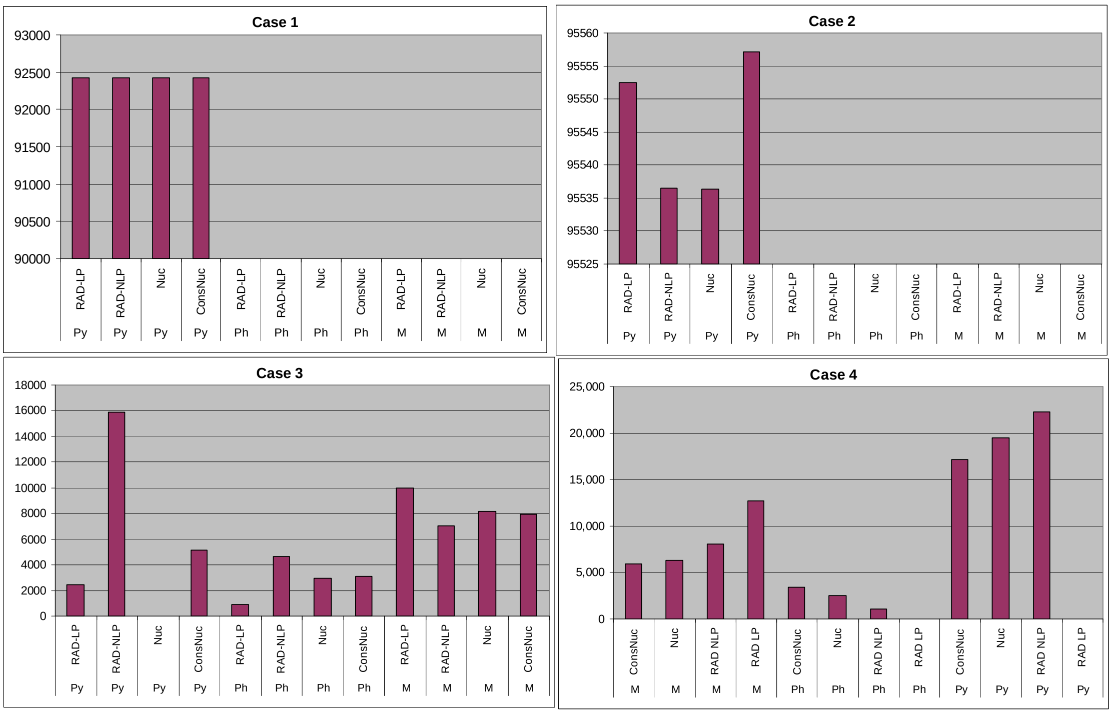

Figure - 1: Average per unit price ($) of different species (on Y axis) in the terminal round under different pricing schemes (on X axis) for the case studies. Here, Py = Python, Ph = Phascogale and M = Malleefowl

6.1.2 Bidding dynamics

In the first case study, bidders started to bid their production costs earliest in Nuc (in round # 69) followed by RAD NLP (in round # 80), RAD LP (in round # 89) and ConsNuc (in round # 96). In the second case study, in Nuc algorithm small bidders have started bidding their production costs earlier (at round #359); this trend was followed by RAD NLP (round # 469). In case of ConsNuc and RAD LP, small bidders never bid their true cost within the given number of rounds. However, they were very close to their production costs at the end of the terminal round. On the other hand, in none of the schemes, large bidders had to bid their production cost ($688,946) within the given number of rounds (500). On average, their bidding was closest to the production costs in RAD LP ($688,948) and furthest in RAD NLP ($689,031). The statistics for ConsNuc and Nuc are the same ($688,954).

In the third case study, we can see that within the given number of rounds (500) the bidders have never bid their production costs. In RAD LP and Nuc algorithms, bidders have consistently bid lower than RAD NLP and ConsNuc. In RAD LP all bidders (except medium sized bidders BB and EE) have consistently bid closer to their production cost, whereas, in Nuc algorithm medium sized bidders BB and EE have consistently bid closer to their production costs. In RAD NLP all bidders have bid higher than in other pricing schemes. This trend was followed in ConsNuc. In the fourth case study each bidder has submitted multiple (two) packages. In Nuc algorithm, all bidders have bid consistently closer to their production costs for all packages. This trend has been followed by ConsNuc, RAD NLP and RAD LP (Figure 2). In summary, we can observe that in Nuc algorithm the bidders have bid closer to their production costs in most of the cases.

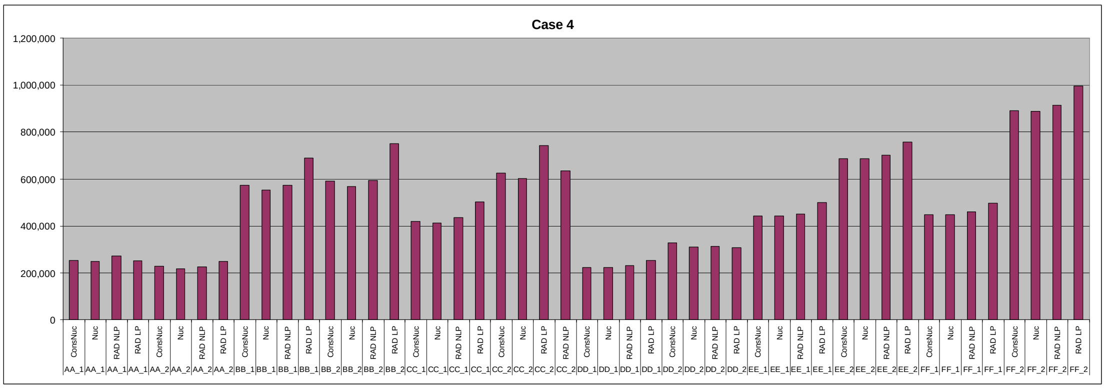

Figure - 2: Average bids ($, on Y axis) submitted by the bidders in the terminal round for different pricing schemes (on X axis) in the case study 4.

Another motivation behind bid revision is the tentative profit from the bid should it be selected. In our framework, the bidder will maintain the same bid as long as it is winning. For the loosing bids, it will use the computed value information to reduce the bid until it reaches its conservation / production cost. We consider the average profit made by the bidders at the terminal round as a measure of performance of the schemes. We analyse bidders profit only for case study 4. It has been observed that even though bids from high cost medium and large bidders (EE and FF) were initially selected they were ultimately priced out in all pricing schemes. Low cost small farmer (AA) has made less profit in RAD LP. Others (bidder BB, CC and DD) have made less profit in Nucleolus based algorithms (Table 15).

Table - 15: Relative position of pricing schemes in terms of bidders' tentative profit. Scheme with lowest average profit has been ranked one

Bidder ID	ConsNuc	Nuc	RAD NLP	RAD LP
Bidder 1 (AA)	4	2	3	1
Bidder 2 (BB)	2	1	3	4
Bidder 3 (CC)	1	3	4	2
Bidder 4 (DD)	2	1	4	3
Bidder 5 (EE)	N/A	N/A	N/A	N/A
Bidder 6 (FF)	N/A	N/A	N/A	N/A

It should be noted that in case study 4 no bidder had to bid their production costs in any pricing schemes. Nuc and ConsNuc have performed better as after the initial rounds they were able to select bidders in optimal allocation. However, when the bidders in optimal allocation are selected, competition in the auction ceases, as the winning bidders do not have to reduce their bids anymore. So, bidders' profit never reaches to zero in this case study.

6.1.3 Rent extraction

Degree of rent extraction depends on how quickly the pricing scheme can select the bids in optimal allocation and how well per unit prices can guide the bidding. We have considered the value at which RE measures stabilize as an estimate of convergence for each pricing scheme. In the first case study, we can see that Nuc reaches to convergence most quickly (round # 47) followed by RAD NLP (round # 53), RAD LP (round # 59) and ConsNuc (round # 71). For case study 2, we can see that Nuc reaches convergence most quickly (round # 253) followed by RAD NLP (round # 313). In case of RAD LP and ConsNuc, rent extraction never reaches to one even though it reaches very close (0.999) at the end of terminal round. In the third case study, all pricing schemes reach plateau before the terminal round. RAD LP algorithm performed best with 0.921 followed by Nuc (0.918), ConsNuc (0.912) and RAD NLP (0.861). In the fourth case study, Nuc algorithm performed best (0.903) followed by ConsNuc (0.875), RAD NLP (0.870) and RAD LP (0.794). We can observe that the performances of all linear pricing schemes have gradually declined with increasing complexity in bidder's heterogeneity and number of packages (Table 16). The pricing schemes producing lower per unit prices for the species have performed better.

6.1.4 Allocative efficiency

Capacity of auction designs to achieve allocative efficiency is a desirable property. In the first case study, allocative efficiency is always one as the bidders' population is homogeneous (Table 16). In the second case study, since any two small bidders can supply the target at least cost, when the large bidders are priced out of the competition the allocative efficiency reaches to one. Optimal allocation was made consistently at the earliest (in other words, AE reaches one) in case of Nuc (round # 21) followed by RAD NLP (round # 22), ConsNuc (round # 24) and RAD LP (round # 26). However, selecting least cost farmers does not mean the end of competition as small farmers continue to compete with each other to win the contracts, and so RE continues to fall. In the third case study, Nuc algorithm performed best (AE = 1.00 at round # 102) followed by RAD NLP (round # 135) and RAD LP (round # 246). In ConsNuc AE never could reach 1 within the given number of rounds. In fourth case study, the pricing schemes have failed to select bids in optimal allocation. Nuc algorithm has performed best (AE = 0.996 at convergence) followed by RAD NLP, ConsNuc and RAD LP. We can see that similar to rent extraction estimates average allocative efficiency has declined with increases in bidders' population complexity.

Table - 16: Average rent extraction and allocative efficiency estimates at convergence for the case studies under different pricing schemes

	RE				AE			
	RAD LP	RAD NLP	Nuc	ConsNuc	RAD LP	RAD NLP	Nuc	ConsNuc
Case study 1 (Homogeneous population)	1	1	1	1	1	1	1	1
Case study 2 (Semi-homogeneous population)	0.999	1	1	0.999	1	1	1	1
Case study 3 (Heterogeneous population with single package)	0.920	0.861	0.918	0.911	1	1	1	0.995
Case study 4 (Heterogeneous population with 2 packages)	0.794	0.870	0.903	0.875	0.941	0.990	0.996	0.980

In the first set of tests, we study the amount of bids submitted by the bidders, profits made by each type of farmer, per unit prices of the animals, level of rent extraction and allocative efficiency under each pricing schemes. From the very limited number of testing we have conducted, it has been observed that all linear pricing schemes have achieved allocative efficiency higher than 0.90 for all the bidder population structures (Table – 16). However, their performances in terms of rent extraction (revenue distribution) have been affected by bidder population complexity. For homogeneous and semi homogeneous populations, all pricing schemes have performed equally well. In case of heterogeneous population, nucleolus-based algorithms have performed better than other schemes. They have been able to produce lower per unit prices for the species for most of the rounds than other schemes, which suggest that they may have been able to guide the bidding better. Even though the nucleolus based algorithms have performed relatively well, RAD LP has produced least fluctuated prices for the species which may help bidders in actual auctions. Further study is needed to see if the same result holds true for much larger and complex auctions. In the second set of tests, we examine the pricing schemes thoroughly in more complex environment with varying degree of bidder population composition in terms of cost and capacity while the bidders are facing different levels of competition.

6.2 Results from second set

After exploring the behaviour of the pricing schemes in the first set of tests, in the second set we examine the performances of the schemes with changes in the auction environment. We test the effect of bidder's heterogeneity, bidder's population size, level of competition, level of complementarity in conservation costs for multiple species and the number of packages each bidder is allowed to submit. We discuss the results in the following sub-sections.

6.2.1 Effect of number of packages submitted by the bidders

In most of the conservation auctions, the farmers are allowed to submit a single project. The project selection is of an 'all or nothing' nature where the farmer is either contracted to carry out the whole project or nothing. However, such project formulation and selection precludes contracting the farmers to perform a number of interventions that are relatively cost-effective (Chan et al., 2003). Moreover, lumpy bids conceal variations in the marginal cost over different combinations of conservation outcomes. For example, if the marginal cost increases with the population size of the conserved species, then landowners will avoid bidding for larger population size. On the other hand, if the marginal cost decreases with the increase in the number of species conserved, the farmer will be inclined to bid on mix or package of species. So, when, a single package is allowed, the auction could exploit only differences in the average costs of individual farmers, but not the differences in marginal costs of conservation of different size and mix of populations for any farmer. This phenomenon may lead to lower participation, higher conservation cost and lower efficiency (Hailu and Thoyer, 2006). So it may be beneficial to allow the farmers to submit multiple packages.

However, preparation of conservation projects requires substantial resources and technical expertise. There is considerable uncertainty and effort involved in developing the management plans, which forms the basis of the bid assessment process. Different landholder groups may view these transaction costs differently. For example, in the Desert Uplands Project, it was observed that several landholders have submitted a lumpy bid covering the whole farm area instead of estimating a detailed bid for a particular section of their property (Whitten et al., 2007).

Therefore, in this test, we have allowed the bidders to submit a maximum of four packages. In order to keep the intensity of competition similar we have set the target as such that for any number of packages any two bidders could supply the target. We can see from the allocative efficiency and rent extraction estimates that the performances of all pricing schemes have enhanced with increasing number of packages (Figure 3). In terms of allocative efficiency, Nuc algorithm has performed best for NP1, NP3 and NP4 and ConsNuc for NP2. RAD LP and RAD NLP have performed better than ConsNuc in all other cases.

In terms of rent extraction estimates, univariate analysis indicates that there is significant difference among different number of packages (Type III sum of squares 61.655, d.f. 3, F 4805.572**). RE estimates have been highest for the cases with four packages (NP4) and have significant differences with other package sizes (NP1: 0.228*, NP2: 0.334* and NP3: 0.120*). This is followed by cases for three packages, NP3 (NP1: 0.107 and NP2: 0.214*). RE estimates for cases with NP1 is significantly higher than cases with NP2 (0.107*). One explanation for this could be that the presence of additional packages has provided more opportunities to the auctioneer to select suitable bids.

The performances of the pricing schemes (in terms of RE) are also significantly different (Type III sum of squares 1.920, d.f. 3, F 149.650**). Among the pricing rules, Nuc algorithm has obtained higher efficiencies and have significant differences with other schemes (ConsNuc: 0.033*, RAD LP: 0.055* and RAD NLP: 0.052*). ConsNuc has obtained second highest average rent extraction estimates, which is significantly higher than RAD LP (0.022*) and RAD NLP (0.019*). This is followed by RAD NLP, which has significant differences with RAD LP (0.002*). In terms of price monotonicity, all schemes have produced lower fluctuating prices for medium number of packages (NP2 and NP3). Nuc algorithm has produced least fluctuating prices for NP1 and NP2. For higher package sizes RAD LP has produced least fluctuating prices (Figure 4).

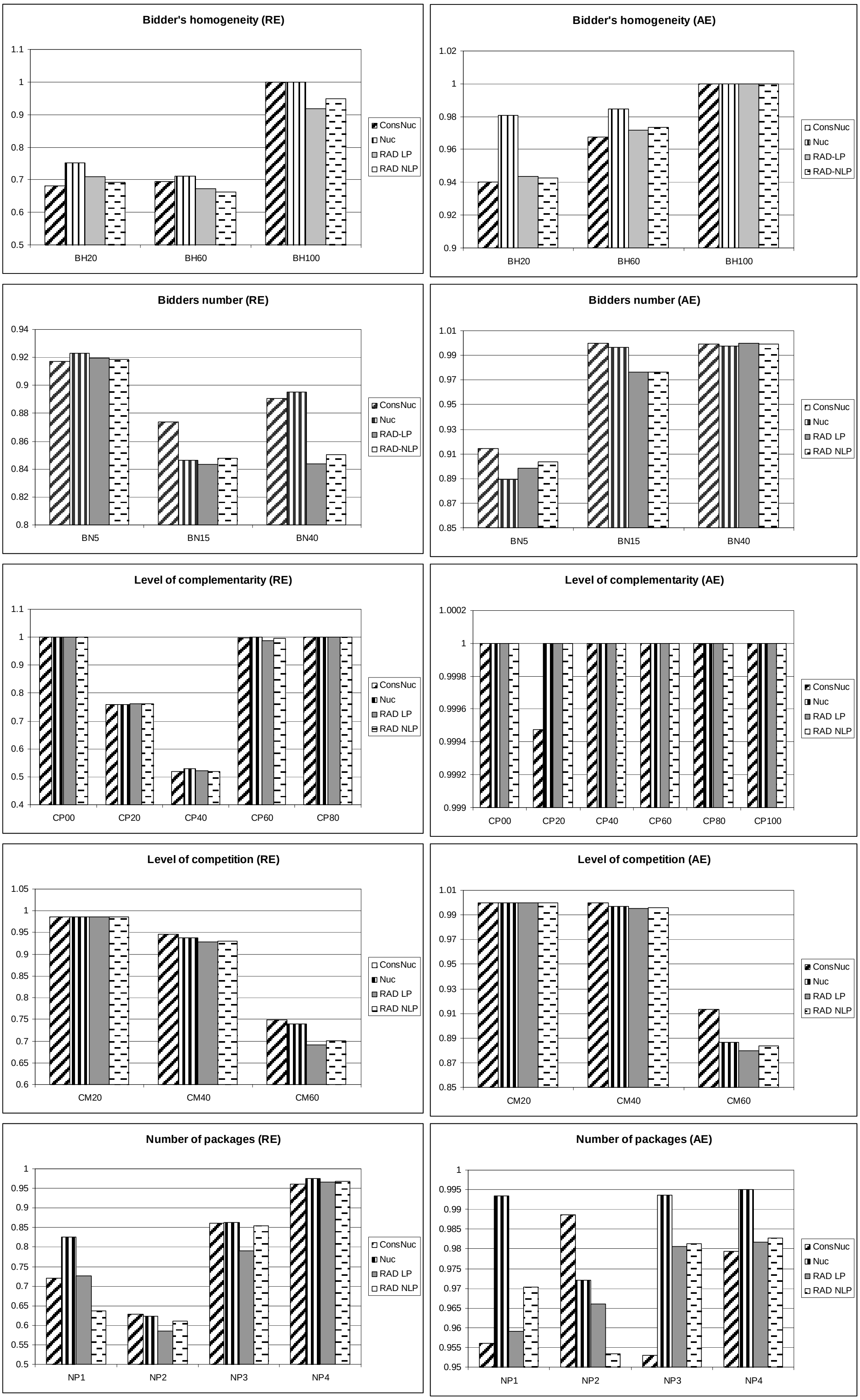

Figure - 3: Average allocative efficiency and rent extraction estimates (on Y axis) at convergence for different pricing schemes under different test conditions (on X axis)

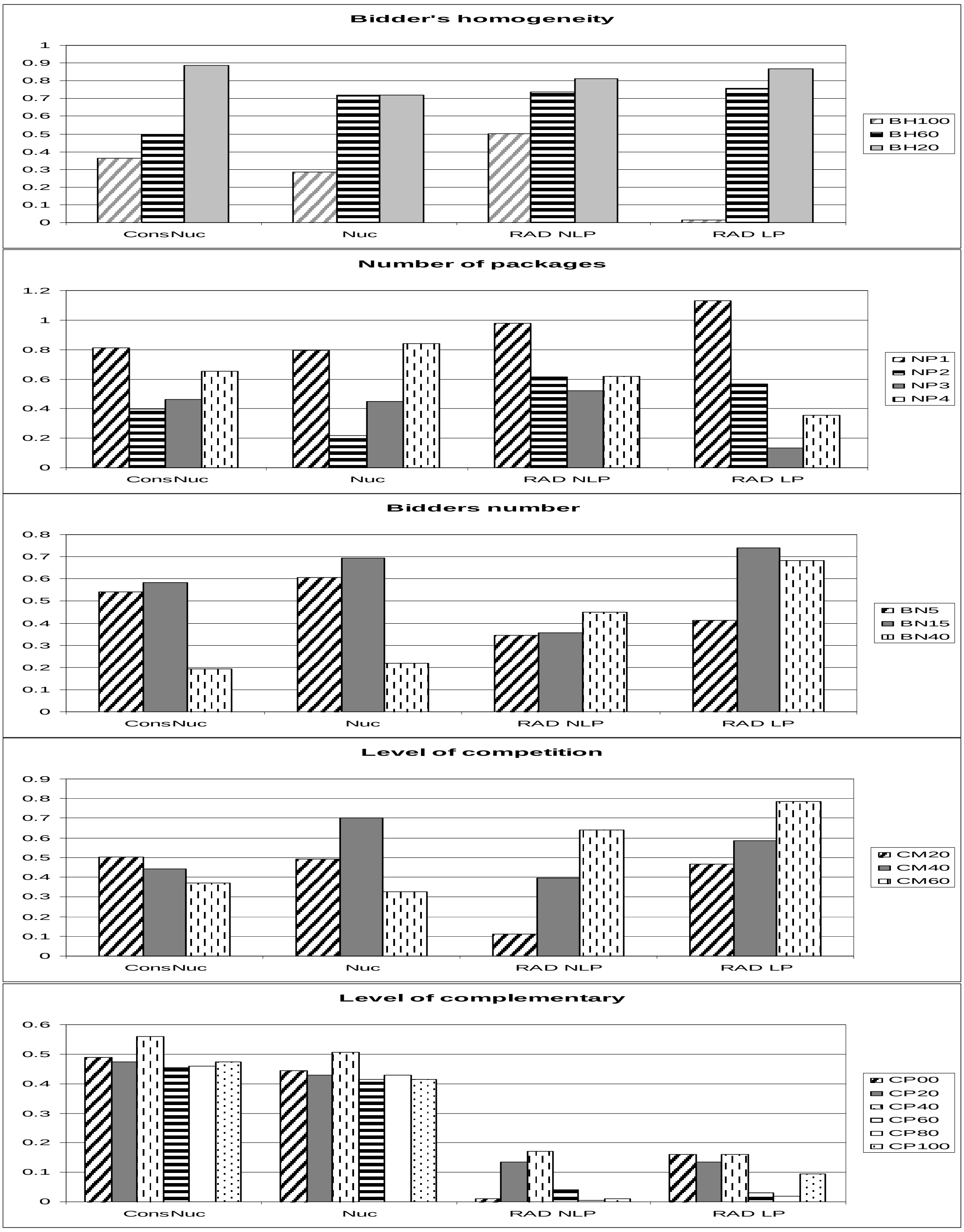

Figure - 4: Average price monotonicity estimates (PM on Y axis) for different pricing schemes (on X axis) under different test conditions

6.2.2 Effect of bidders' population size

Market participation is often defined as the number of bidders who participate in the bidding process. The design and target of the auction affects the number of bidders drawn to it (Klemperer, 2004). Conservation auctions often have specific conservation goal (e.g., pollution reduction, native vegetation restoration), confined to specific geographic regions and target a certain group of farmers (Rolfe et al., 2008). All these factors limit participation. The costs of participation and strategic motives may also be deterrents (Chan et al., 2003). If the costs of participation are high bidders may be discouraged from participating. Some bidders may use tactics to discourage other bidders from participating to reduce competition and enhance chances of winning (Bryan et al., 2005).

Auctions work by promoting low cost bidders, so not only the number of bidders but also the type of bidders is important. If only high cost bidders participate there will be no benefits in terms of cost effectiveness. So, widespread participation of bidders may be beneficial from cost saving perspective (Whitten et al., 2007). However, due to practical and political implications often auctioneer tries to optimize the number of bidders instead of maximizing them (Arsenault, 2007). High participation rates may result in large proportions of unsuccessful bidders which may be undesirable.

There is relatively little research on what constitutes a minimum level of participation to ensure competitive efficiency. It has been observed that efficiency losses can be substantial with only two or three bidders, but are negligible with seven bidders or more (Goeree and Offerman, 2003). In many situations four agents are sufficient to imply a competitive outcome (Holt et al., 2007). As a guideline for a conservation auction in Queensland, Windle and Rolfe (2005) have suggested that there should be at least eight active bidders in a tender and ideally more than 15.

In this test we have examined the schemes for three distinct population sizes (5, 15 and 40) of bidders. We can observe that the allocative efficiency estimates for the schemes have reduced with increasing number of bidders (Figure 3). For lower number of bidders all schemes have performed equally well and achieved complete allocative efficiency. For medium population size (BN15) allocative efficiency has been reduced, which has again increased for larger population (BN40). For rent extraction estimates the trend is similar.

Univariate analysis indicates that there is significant difference in rent extraction estimates among different population sizes (Type III sum of squares 9.415, d.f. 2, F 4354.716**). RE estimates have been highest for the cases with smallest bidder population (BN5), which have significant differences with other bidder population sizes (BN15: 0.064* and BN40: 0.052*). This is followed by cases with largest population, BN40 (BN15: 0.012*).

The performances of the pricing schemes (in terms of RE) are also significantly different (Type III sum of squares 1.495, d.f. 3, F 460.875**). Among the pricing rules, ConsNuc algorithm has obtained higher efficiencies, even though it does not have significant difference with Nuc. The average rent extraction estimates for ConsNuc is significantly higher than RAD LP (0.02419*) and RAD NLP (0.02138*). Nuc has significant differences with RAD LP (0.023*) and RAD NLP (0.02*). RAD NLP has produced significantly higher average RE estimates than RAD LP (0.003*). In terms of price monotonicity, all schemes have produced higher fluctuating prices for medium population size of bidders (BN15) compared to other population sizes. RAD NLP has produced least fluctuating prices for BN5 and BN15. Whereas, for higher population size ConsNuc has produced least fluctuating prices (Figure 4).

6.2.3 Effect of level of competition

Conservation auctions vary in terms of level of competition. In general, the higher is the level of competition, the greater the competitive efficiency, and so the greater the benefits of auction mechanism. Competition depends on the degree of heterogeneity among landholders in their conservation costs. If costs are highly variable, the advantages of increased competition from increased bidders' population are plausible. Auctions should be open to anyone willing and able to meet financial prequalification, as they will enhance competition and limit opportunities for collusion. Along with the number of active bidders, level of targets defines the extent of competition (Whitten et al., 2007).

There is no specific rule on what should be the appropriate level of competition. It depends on the objectives of the auction, approximate number of bids, cost of assessing the bids to the agency, likely impact on unsuccessful bidders (Whitten et al., 2007). The agency may add some conditions to control the level of competition (Lehmann et al., 2006). Commercial firms may want to maintain diversity in contract allocation in terms of suppliers' capacity and location so that changes in circumstances do not disrupt essential supplies. But too many winners increase overhead (Abrache et al., 2007). In government auctions, there may be a ceiling on what fraction of the contracts may go to select bidders. If the allocation is too small it may discourage the bidders and if it is too large the government may become dependent on particular suppliers. For example, Holt et al. (2007) have suggested restricting any entity from purchasing (or taking a beneficial role) more than 33% of the allowances for sale in the auction for CO_2 emission.

In this test, we have defined competition in terms of percentage of bidders that could fulfil the target in the optimal allocation. For example, CM20 means that there are twenty percent bidders in the optimal allocation. We can observe that the allocative efficiency estimates for the schemes have decreased with reduced level of competition. For high levels of competition (CM20 and CM40) all schemes have performed equally well and achieved complete allocative efficiency. For cases of reduced competition (CM60), ConsNuc has achieved higher allocative efficiency (Figure 3).

We can see from the RE lines that the performances of all pricing schemes have enhanced with increasing competition and they have performed best with highest level (20%) of competition. Univariate analysis indicates that there is significant difference in rent extraction estimates among different levels of competition (Type III sum of squares 155.556, d.f. 2, F 70743.840**). In cases for intense competition (CM20), all pricing schemes have achieved high rent extraction estimates. With gradual reduction in intensity of competition (CM40 and CM60) the amount of rent extraction has gone down.

The performances of the pricing schemes (in terms of RE) are also significantly different (Type III sum of squares 1.495, d.f. 3, F 453.176**). Among the pricing rules ConsNuc algorithm has obtained higher efficiencies in all cases, even though it does not have significant difference with Nuc. The average rent extraction estimates for ConsNuc is significantly higher from RAD LP (0.024*) and RAD NLP (0.021*). This is followed by Nuc, which has significant differences with RAD LP (0.023*) and RAD NLP (0.020*). Performances of RAD LP and RAD NLP are also significantly different (0.003*) with RAD NLP producing higher average RE estimates.

In terms of price monotonicity, RAD NLP has produced least fluctuating prices for CM20 and CM40. For CM60, Nuc has produced least fluctuating prices. For RAD LP and RAD NLP, price fluctuations have increased with increasing level of competition. For Nuc and ConsNuc price monotonicity shows reverse trend (Figure 4).

6.2.4 Effect of bidders' homogeneity

Bidders' homogeneity in terms of cost structure and bidding strategy has significant effect on the performances of the auctions (Sade et al., 2006). If the bidders are homogenous, they will compete with each other and reduce the procurement costs. But, if all bidders are high cost then bidders homogeneity will help little in reducing overall procurement costs. On the other hand, if the bidders are heterogeneous the high cost bidders will bid closest to their production costs. The low costs bidders will bid slightly lower than the high cost bidders in order to be selected. Thus they will be competing directly only with those offering the same quality mix. This will reduce competition (Latacz-Lohmann and Schilizzi, 2005). This may be case for conservation auctions, since there is high heterogeneity in landholder's opportunity costs. For example, it has been observed that the average bid per hectare in BushTender was $274/ha but the standard deviation of bids was $349/ha, even though the auction was confined to a relatively homogeneous agricultural production system (Box Iron Bark vegetation classification) (Eigenraam et al., 2006).

In this test, we have defined bidder's homogeneity in terms of percentage of bidders having identical packages and similar bidding strategy. For example, BH60 means that sixty percent of bidder's population have identical packages and have similar bidding strategy. We can observe that the allocative efficiency estimates for the schemes have decreased with increased level of heterogeneity. This may be the case due to the fact that with increased heterogeneity the pricing schemes not only have to separate high-value bidders from low-value bidders but also reflect the demands of bidders across commodities (Ledyard et al., 1997). For heterogeneous populations (BH60 and BH20) ConsNuc has achieved higher allocative efficiency (Figure 3).

In terms of rent extraction estimates, univariate analysis indicates that there is significant difference among different levels of bidders homogeneity (Type III sum of squares 42.454, d.f. 2, F 4730.573**). In cases for homogenous population, all pricing schemes have achieved high revenue efficiencies. But when the bidder population is semi heterogeneous (BH60), the schemes produce lower rent extraction compared to heterogeneous population (BH20). This means that in case of semi heterogeneous population the pricing schemes have not been able to guide the bidders sufficiently to make the auction more competitive. This may also be the case when bidders are competing only with others in the same group and so the level of competition has reduced.

The performances of the pricing schemes in terms of RE are also significantly different (Type III sum of squares 1.44, d.f. 3, F 106.971**). Among the pricing rules Nuc algorithm has obtained higher efficiencies in all cases. The average rent extraction estimates for Nuc is significantly higher from other pricing rules (ConsNuc: 0.033*, RAD LP: 0.055* and RAD NLP: 0.052*). This is followed by ConsNuc, which has significant differences with RAD LP (0.055) and RAD NLP (0.052). However, performances of the RAD LP and RAD NLP are not significantly different. Price fluctuations have increased with increasing level of bidder's heterogeneity. For homogeneous population (BH100) RAD LP has produced least fluctuating prices. For BH60 and BH20 ConsNuc and Nuc has produced least fluctuating prices respectively (Figure 4).

6.2.5 Effect of level of complementarity

Presence of complementarity / synergy in production costs of the items is one of the main reasons for using combinatorial auctions. Performances of combinatorial auctions have been tested for different settings of synergy. For example, Ledyard et al. (1997) have reviewed 130 auction experiments conducted for allocating Personal Communications licenses by the Federal Communications Commission of USA. They have observed that over a very wide range of complementarities, combinatorial auction (weakly) dominates simultaneous auction, which in turn (weakly) dominates sequential auction. Lunander and Nilsson (2004), who have compared the designs in sealed bid first price format, also observed higher efficiency of combinatorial auction in allocating complementary construction contracts. Cramton et al. (2006) noted that where complementarities are both strong and varied across bidders, package bids could improve the efficiency. Goeree et al. (2007) have made similar observations. But in case of un-related goods, sequential and simultaneous auctions generate almost similar revenue (Menezes and Monteiro, 2003).

In these experiments, for a procurement auction, level of complementarity is defined as the amount by which procurement / production costs of two items are reduced when they are produced together. For example, if A and B are two items, and c (.) denotes the farm's production cost, A and B are said to be complementary if c ({A, B}) = c ({A}) + c ({B}) – α (AB), α (AB) > 0. Here α (AB) is the estimate / level of synergy (Abrache et al., 2007). Different authors have varied this estimate in their experiments to test the performances of CAs. For example, Leufkens et al. (2006) have tested the bidding in a sequential private value auction under three treatments: a baseline with no synergies, one with mild synergies (s = 1.5) and one with strong synergies (s = 2.0). Chernomaz and Levin (2007) investigated bidding in a first-price sealed-bid multi-unit demand auction with and without package bidding. There are two local bidders competing against a global. The global bidder draws a single value from the same uniform distribution as local bidders. The value to the global bidder for obtaining both items is vg = 2βsg where β represents the synergy value and sg is the global bidder's value.

However, in this paper, we have defined the level of synergy differently. We have considered that the bidders could either have synergy or not. Absence of synergy means that the farmers are conserving the species in different patches and conservation costs are unrelated. So, the cost of the project is the sum of the costs for different patches. Presence of synergy means that the farmers are conserving the species in the same patch and the costs are complementary. We have varied the number of farmers with synergistic packages. For example, CP20 means that twenty percent bidders have synergistic packages while the rest has additive packages. We can observe that all the pricing schemes have obtained complete allocative efficiency, although the rent extraction estimates have varied with the changes in level of complementarity (Figure 3).

Univariate analysis indicates that there is significant difference in rent extraction estimates among different levels of complementarity (Type III sum of squares 156.95, d.f. 5, F 3159.249**). Cases where all or majority of the farmers have either additive (CP00) or complementary costs for (CP100 and CP80) packages, all pricing schemes have achieved complete revenue efficiency. But when these two groups are mixed the schemes produce lower rent extraction estimates (CP40 and CP20) even though they can achieve complete allocative efficiency (CP40). This means that the bidders are competing only with bidders in the same group and the level of competition has reduced.

The performances of the pricing schemes (in terms of RE) are also significantly different (Type III sum of squares 0.454, d.f. 3, F 15.218**). Among the pricing rules, Nuc algorithm has obtained highest efficiencies in all cases. The average rent extraction estimates for Nuc is significantly higher than other pricing rules (ConsNuc: 0.0178*, RAD LP: 0.0227* and RAD NLP: 0.0183*). However, the performances of other pricing schemes are not significantly different among each other.

Price fluctuations have varied with changes in the level of complementarity. Price fluctuations have increased for medium level of complementarities (CP20 and CP40). RAD LP and RAD NLP have produced less fluctuating prices compared to Nuc and ConsNuc for all cases (Figure 4).

7. Concluding remarks

Combinatorial auctions offer advantages over traditional or simpler conservation auctions that have been attracting policy interest especially in the last decade. However, the flexibility offered by combinatorial auctions requires the design of price feedback schemes or algorithms that help bidders formulate and revise their bids in these auctions. There has been several price feedback algorithms suggested in the literature. However, they have been proposed and applied in auctions for distinct items rather than services that can be offered at different levels (e.g. area conserved). This paper focuses on applying and evaluating the price feedback algorithms for auctions where the bidders can offer not only different services but also different levels of the individual services, separately or in combination.

We test some linear pricing schemes for multi-unit reverse (procurement) combinatorial auction. A bio-economic model is used to generate costs for conservation of different packages of target species on lands owned by farmers. These farmers compete to win contracts to conserve sets of target species. In order to accommodate the multi-unit features of our auctions, we have modified the existing designs of four linear pricing schemes based on Resource Allocation Design (RAD) and nucleolus algorithms. Bidding and auction outcomes are simulated for bidder populations with different levels of heterogeneity and complexities in the auction environment.

In order to test the schemes in wide range of environment we have studied the performances with variation in some major factors of auction environment, such as, bidders' heterogeneity, number of packages, number of bidders, level of competition and level of complementarity. We summarize the main findings –

- All pricing schemes have obtained very high allocative efficiency (AE > 0.95) and reasonably well rent extraction estimates (RE > 0.50).
- Performance of the schemes have enhanced with the increase in number of packages
- The schemes have performed better in low and high bidder's population sizes
- With enhanced competition the schemes have obtained higher allocative efficiency and rent extraction estimates
- The allocative efficiency estimates decrease with increasing bidders' heterogeneity. In terms of rent extraction, the schemes were more efficient for homogenous and highly heterogeneous bidders' population.
- Similarly in presence of high proportion of farmers with additive or complementary costs, the schemes were more efficient

Overall, RAD based algorithms have produced least fluctuating prices. This may be due to the presence of price balancing optimization across the items. However, nucleolus based algorithms have performed consistently better than other pricing schemes in terms of allocative efficiency and rent extraction estimates. We postulate two reasons. Firstly, in the nucleolus based algorithms the sign of the slack variable is free, so the algorithms have more freedom in selecting both positive and negative slacks and positive prices compared to RAD based schemes. Secondly, in the Nuc algorithm all the winning bids are merged into an aggregated bid. While, computed value for the aggregated bid is forced to be equal to the minimum cost, individual winning bids may have lower, higher or equal computed values. So, the algorithm has more freedom in selecting prices compared to other schemes.

However, there is scope for further study. Although, nucleolus based algorithms have achieved better efficiency, per unit prices produced by them fluctuate between the rounds. In the simulation experiments, it may not hamper the performances of the bidders. But in experiments with human subjects and in real life applications highly fluctuating / volatile prices may confuse the bidders. In the literature, price anchoring or smoothing techniques have been applied for single unit combinatorial auctions. This can be adapted for multi-unit auction. There are also alternative algorithms to compute linear prices. For example, Aparicio et al. (2008) have developed a DEA based pricing schemes for multi-unit forward auction. This can be adapted to reverse auction to test its performances. Lastly, the learning rules used for the bidders are very simplistic. More complex bidder's learning rules could be used to test the schemes. We have plans to test some of these aspects in future experiments.

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