Can Central Bank Interventions Affect the Exchange Rate Volatility? Multiperiod GARCH Approach Using Constrained Nonlinear Programming

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Abstract

This study examines the impact of foreign currency market interventions of the Central Bank of the Republic of Turkey (CBRT) in a multivariate GARCH framework. CBRT has switched to the floating exchange rate regime since 2001 crisis and announced that the interventions in the foreign exchange markets are aimed at reducing the expected volatility of the USD/YTL and EUR/YTL. However the literature documents that foreign exchange interventions leads to an increase in exchange rate volatility. In an attempt to calculate the expected volatility, we employ a bivariate GARCH estimation with non-linear constrained optimization (NLP) [19] and BEKK [1] on the USD/YTL and EUR/YTL. Our results shed some doubt about the efficiency of these interventions in stabilising the Turkish Lira market.

Keywords. Time series econometrics, Constrained Nonlinear programming, Multivariate GARCH, FOREX Interventions.

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1 Introduction

After the collapse of Bretton Woods system central banks used foreign exchange market interventions to stabilise the market. During the last 30 years we are witnessing an ongoing debate on the ability of these interventions in influencing both the level and volatility of the exchange rate. The main motive of the interventions is keeping the volatility of the exchange rate market at reasonable levels. Over the last 15 years Turkey as an emerging market has been experiencing sudden high volatility periods as result of the instable economic conditions. After the crisis in 2001 CBRT announced a floating rate regime and intervened in the foreign exchange market since then to decrease expected volatility. This paper studies the impact of these direct interventions on the volatility of the USD/YTL and EUR/YTL between 2002-2005.

Many academic studies tried to test the impact of these interventions on the level and volatility of the exchange rates mostly on the currencies of developed economies. For example Dominguez [8] investigates the effect of interventions on USD-DM and USD-JPY between 1977-1994 by using univariate GARCH models. He documents that the interventions are positively correlated with the level of volatility. Ballie and Osterberg [2] show that interventions between 1985-1990 had no significant effect on the level and volatility of the USD-DM exchange rate. Kim et.al [17] examines foreign exchange interventions of the Reserve Bank of Australia between 1983-1997. They conclude that large interventions have a stabilising effect in terms of the level and volatility of the exchange rate. Frenkel et.al [11] studies the interventions of the Bank of Japan using the official data between 1993-2000 and find that interventions increases the USD-JPY volatility. Sarno and Taylor [20] provides a detailed survey of this literature. Recently Beine et.al. [3] examines the impact of interventions carried out by Federal Reserve, European Central Bank and Bank of Japan over the period 1989-2003. Their results confirm the increased volatility hypothesis caused by interventions particularly for interventions carried out unilaterally.

After the floating rate regime in 2001 CBRT carried out interventions in four different ways. 1) Direct interventions-CBRT buys or sells USD without an official a priori announcement, 2) Auctions of foreign currency announced earlier according to publicly available rules and regulations, 3) Foreign currency borrowings of the banks from the CBRT, 4) announcements and press releases of the CBRT. Starting from 2002 CBRT publicly announced that there are two motives for direct interventions. First one is to control or rather decrease expected volatility. As CBRT was very determined about the flexible exchange rate regime interventions are not directed towards a possible change in the level of USD/YTL or EUR/YTL. The second one is to accumulate enough reserves for the foreign debt payments of the treasury and other foreign currency denominated obligations of the CBRT.

CBRT is aiming to reduce the volatility of the foreign exchange market yet the literature mentioned earlier document that direct interventions either increase or does not affect volatility. Two recent studies by Beine et.al. [4, 3] using Realized moments method and Bayesian framework try to examine the effects of interventions on the currency components of the exchange rates. Although many of the studies involving GARCH framework are conducted using univariate models, it is also necessary to observe the dynamics of variances and covariances of exchange rates during a central bank intervention in a multivariate setting. However Beine et.al. [4] states that due to the technical difficulties in the optimization of the multivariate model GARCH many studies choose to use the univariate representation. The major difficulty in the multivariate case stems from the highly nonlinear and non-convex nature of the resulting optimization problem.

This paper will overcome that difficulty with the nonlinear programming technique proposed in Salih et.al [19]. In the NLP multivariate GARCH approach there is no need to impose artificial restrictions for tractability. GARCH models are usually presented as unconstrained optimization models in econometrics,(see e.g., Hamilton [15], Gourieroux [14]) with recursive terms whereas they actually fall into the domain of non-convex nonlinearly constrained nonlinear programming. They can be solved by extensions of Newton or quasi-Newton methods that take into account the recursive nature of terms defining the objective function. We solve the problem with nonlinearly constrained non-convex programs using the AMPL modeling language (Fourer et al. [12]), and the state-of-the-art optimization packages available through the recently developed NEOS[1] interface at the Argonne National Laboratory.

Main contribution of this study is to analyze the impact of direct foreign exchange market interventions carried by CBRT during 2002-2005 through NLP multivariate GARCH and BEKK multivariate GARCH representations. The paper is organized as follows. Next section will give a brief review about the direct interventions by CBRT. Section 3 presents the NLP multivariate GARCH and compare NLP with BEKK and VECH models. Section 4 will discuss the empirical results. Section 5 will conclude.

2 Direct Purchase (Sale) Interventions

The method most commonly used by the monetary authorities for foreign currency interventions is the unannounced direct sales and purchases of US dollar. As mentioned earlier the scope of these interventions is to lower the predicted or actual volatility in foreign currency markets by unannounced interventions. If the Lira is depreciating against the US dollar the Central Bank intervenes with US dollar sales, if YTL is appreciating, Central Bank purchases US dollars.

Real growth in the Turkish economy picked up after the 2001 crisis - reforms encompassing, Central Bank Independence, restructuring of the banking sector, single party government, progress made on the EU front lead to a fall in the country risk premium. Moreover the speed of privatizations of public institutions initiated long - term capital inflows in Turkey. With achieved economic stability, domestic investors too began to hold more of YTL reserves than US dollars in their investment portfolios. The immediate impact was on the appreciation of YTL against the US dollar. In this context, volatility increased in the USD/YTL parity due to accelerated appreciation of the YTL. To counteract the excess volatility Central Bank actively entered into US dollar purchase inveteventions. Table 1 presents the details of the purchase and sale interventions in the 2002 - 2005 period. Numbers indicated with red represent direct sale interventions. In this period, Central Bank purchased 16.9 billion US dollars with an equivalent increase in Central Bank reserves.

3 Model Specifications

The interventions of CBRT are aimed at reducing predicted volatility and GARCH family of models have been a major tool in predicting volatility since their introduction in 1982 by Engle [9]. To describe the multivariate GARCH representations that are employed in this study we will adopt the following notation. Following autoregressive process for USD/YTL and EUR/YTL exchange rate returns are assumed, which explains the behavior of the random variable in terms of its past values as

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + \phi_m Y_{t-m} + \varepsilon_t$$

where $\varepsilon = (\varepsilon_t)$ is a weak white noise satisfying the martingale difference sequence condition:

$$E(\varepsilon_t | \underline{\varepsilon_{t-1}}) = 0$$

where the notation $E(.)$ denotes mathematical expectation and $\underline{\varepsilon_{t-1}} = \{\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots\}$ represents the vector of past values. When the error term ε_t is a multivariate process of dimension n, for all $t = 1, \ldots, T$ we have $Y_t \in \Re^n$ and $\varepsilon_t \in \Re^n$ with components Y_{lt} and $\varepsilon_{lt}, l = 1, \ldots, n$, respectively. We denote the components of the $n \times n$ conditional variance-covariance matrix $H_t = E(\varepsilon_t \varepsilon_t^T | \underline{\varepsilon_{t-1}})$ by h_{klt}.

[1] `http://www-neos.mcs.anl.gov`; see Cyzik et al. [7]

Date	Amount
$11.July.2002$	3
$02.December.2002$	16
$24.December.2002$	9
$12.May.2003$	62
$21.May.2003$	517
$09.June.2003$	566
$18.July.2003$	938
$10.September.2003$	704
$25.September.2003$	$1,442$
$16.February.2004$	$1,283$
$11.May.2004$	9
$27.January.2005$	$1,347$
$09.March.2005$	$2,361$
$03.June.2005$	$2,056$
$22.July.2005$	$2,366$
$04.October.2005$	$3,271$

Table 1: Turkish YTL Purchase and Sale Interventions (million US dollars)

3.1 Vech and Diagonal Vech Model

Following Kraft and Engle [18] and Bollerslev, Engle, and Wooldridge [5], Vech model can be formulated as follows:

$$vech(H_t) = vech(C) + \sum_{i=1}^{q} A_i vech(\varepsilon_{t-i}\varepsilon'_{t-i}) + \sum_{j=1}^{p} B_j vech(H_{t-j})$$

where vech(.) is the operator that stacks the lower triangle and diagonal elements of an $N \times N$ matrix to a $N(N+1)/2 \times 1$ vector. The Vech model is pretty intuitive and easy to understand, and estimates the covariances as a geometrically declining weighted average of past cross products of the error terms. The major weakness of this model is the number of parameters to be estimated.For example for the simplest Vech(1,1) model $N(N+1)(N(N+1)+1)/2$ number of parameters must be estimated , which can be difficult in practice. Moreover, there is no guarantee for a positive definite covariance matrix without imposing additional restrictions.

Bollerslev, Engle, and Wooldridge [5] proposed the Diagonal Vech model where A_i and B_i are assumed to be diagonal matrices. For GARCH(1,1) process the entries of the H_t can be written as

$$h_{ijt} = c_{ij} + a_{ij}\varepsilon_{i,t-1}\varepsilon_{j,t-1} + b_{ij}h_{ij,t-1}$$

and in matrix notation it can be characterized as follows:

$$H_t = C + A \odot (\varepsilon_{t-1}\varepsilon'_{t-1}) + B \odot H_{t-1}$$

where $\odot$ represents the Hadamard products. In D-Vech specification the number of parameters to be estimated reduces to $N(N+5)/2$. Despite the decreased number of parameters, restrictions on semi-definiteness on C, A, B and initial matrix H_0 still remain.

3.2 BEKK Model

Engle and Kroner [10] suggested a new model to eliminate the hard restrictions imposed by Vech model on positive definiteness of H_t. BEKK with an exogenous dummy variable in the conditional variance-covariance matrix can be characterized by the following equation as implemented by S-Plus:

$$H_t = C'C + B'H_{t-1}B + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + DZ_tD'$$

where A, B and C are $N \times N$matrices with C symmetric and positive definite while Z_t is a diagonal matrix with exogenous dummy on the diagonal and D is the coefficient matrix. While the model makes progress on restrictions of H_t, it increases the number of parameters to be estimated. From a numerical optimization point of view, the BEKK model also increases the nonlinearity of the constraints by utilizing a higher-order polynomial representation.

3.3 Constrained Nonlinear Programming Model

For multivariate GARCH optimization problem, Salih et al. [19] proposed a new representation, which transforms the general multivariate GARCH problem into a nonlinearly constrained nonconvex program as follows:

$$\max \; -\frac{TN}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}(\log|H_t| + \varepsilon'_t H_t^{-1}\varepsilon_t)$$

subject to

$$vech(H_t) = vech(C) + \sum_{i=1}^{q} A_i vech(\varepsilon_{t-i}\varepsilon'_{t-i}) + \sum_{j=1}^{p} B_j vech(H_{t-j}).$$

The above mathematical program is the most general multivariate GARCH specification model, from which simplified specifications were obtained by imposing certain restrictions on matrices A_i and B_j.

In the particular NLP approach we parametrize the H_t as $L_t D_t L'_t$, *t=1,..T* where L_t is a unit lower triangular matrix and D_t is a diagonal matrix. Therefore,H_t yields:

$$\begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} = \begin{pmatrix} d_{1t} & d_{1t}l_{21t} \\ d_{1t}l_{21t} & d_{1t}l_{21t}^2 + d_{2t} \end{pmatrix}$$

As we will employ an intervention dummy in the variance equation for our estimation purposes a GARCH(1,1) process with dummy variables in the conditional variance, NLP representation of vech(H_t) takes the following form:

$$\begin{bmatrix} d_{1t} \\ d_{1t}l_{21t} \\ d_{1t}l_{21t}^2 + d_{2t} \end{bmatrix} = \begin{bmatrix} c_{11} + a_{11}\varepsilon_{1.t-1}^2 + a_{12}\varepsilon_{1.t-1}\varepsilon_{2.t-1} + a_{13}\varepsilon_{2.t-1}^2 + b_{11}d_{1,t-1} \\ +b_{12}d_{1,t-1}l_{21,t-1} + b_{13}(d_{1,t-1}l_{21,t-1}^2 + d_{2,t-1}) + \gamma_1 z_{1t} \\ \\ c_{12} + a_{12}\varepsilon_{1.t-1}^2 + a_{22}\varepsilon_{1.t-1}\varepsilon_{2.t-1} + a_{23}\varepsilon_{2.t-1}^2 + b_{12}d_{1,t-1} \\ +b_{22}d_{1,t-1}l_{21,t-1} + b_{23}(d_{1,t-1}l_{21,t-1}^2 + d_{2,t-1}) + \gamma_2 z_{1t} \\ \\ c_{22} + a_{13}\varepsilon_{1.t-1}^2 + a_{23}\varepsilon_{1.t-1}\varepsilon_{2.t-1} + a_{33}\varepsilon_{2.t-1}^2 + b_{13}d_{1,t-1} \\ +b_{23}d_{1,t-1}l_{21,t-1} + b_{33}(d_{1,t-1}l_{21,t-1}^2 + d_{2,t-1}) + \gamma_3 z_{1t}. \end{bmatrix}$$

In this model, positive definiteness of H_t can be satisfied with $D_{ii,t} \geq 0, t = 1, .., T$.

The log-likelihood function to be maximized in the multivariate NLP case is given as:

$$L(\theta) = -\frac{TN}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}(\log|H_t| + \varepsilon_t' H_t^{-1}\varepsilon_t).$$

Gaussian-maximum likelihood estimation is used in the estimation process for two reasons. First, it is easy to implement and second, following Weiss [21], Bollerslev and Wooldridge [6], when the normality assumption is violated but the first two conditional moments are specified, under suitable regularity conditions QMLE estimates of L(θ) will be asymptotically normal and consistent. Robust standard errors of Bollerslev and Wooldridge [6] for the MLEs that use Gaussian maximum likelihood are calculated. Following Kawakatsu [16], robust BW asymptotic covariance matrix for the MLEs can be written as:

$$V(\theta) = \frac{1}{n}(\frac{1}{n}\sum_{i=1}^{n}\Im_t)^{-1}(\frac{1}{n}\sum_{i=1}^{n}\frac{\partial l_t}{\partial\theta}\frac{\partial l_t}{\partial\theta'})\frac{1}{n}(\frac{1}{n}\sum_{i=1}^{n}\Im_t)$$

where Fischer information matrix is:

$$\Im_t = (\nabla\varepsilon_t)' H_t^{-1}(\nabla\varepsilon_t) + \frac{1}{2}(\nabla H_t)'(H_t^{-1} \otimes H_t^{-1})(\nabla H_t).$$

4 Estimation and Empirical Results

In this section we test the impact of direct CBRT interventions on the volatility and the correlation of the USD/YTL and EUR/YTL exchange rates using the popular bivariate GARCH(1,1) BEKK estimation and the NLP GARCH(1,1) with intervention dummy in the variance-covariance matrix. Data used in the estimations is the daily log returns of the two exchange rates calculated from the exchange rate levels supplied by the CBRT recorded at 15:30 local time.[2] The data set covers the period from 4.4.2002 to 6.10.2005. GARCH (1,1) BEKK estimation is performed using S-PLUS GARCH module implementing the BHHH algorithm (see the text [15] for a discussion of the BHHH algorithm), and NLP GARCH(1,1) is estimated with SNOPT software [13].

[2]For GARCH diagnosis, autocorrelation functions and Ljung-Box statistics have been checked. The data can be supplied upon request.

Table 2 reports the coefficients, standard errors, and the log-likelihood values for the NLP and BEKK GARCH(1,1) specifications with intervention dummy included in the variance covariance matrix. Standard errors are the robust standard errors of Bollerslev and Wooldridge [6].

It is important to realize that the coefficients do not have an intuitive interpretation for both the Constrained NLP and the BEKK. However, log-likelihood values show that Constrained NLP brings a substantial improvement over the BEKK representation in the solution of the multivariate GARCH formulation. For completeness we also report the AIC and SIC statistics

Although both of the models provide a solution to the same multivariate GARCH estimation problem, one can say that Constrained NLP estimation is superior to BEKK model based on the AIC and SIC tests.

An examination of coefficients reveals that for bivariate GARCH(1,1) BEKK with intervention dummy in the variance-covariance matrix all the coefficients except a_{11},a_{21},a_{22} and b_{21} are statistically significant at the 5 percent level. Moreover the dummy variables of the volatility equations of USD/YTL and EUR/YTL are positive and statistically significant at the 5 percent level. This means if the CBRT is using BEKK model to predict volatility then direct interventions have a tendency to increase the volatility rather than decreasing it. The coefficients of the GARCH(1,1) NLP with dummy variable in the variance-covariance matrix are statistically significant at the 5 percent level with the exception of theb_{33} and the coefficients of the dummy variables. Therefore from this exercise one can infer that interventions have no effect on neither the volatility nor the correlation of these two exchange rates.

In Figures 1, 2 and 3 below we plot the conditional annualized volatility estimates obtained from GARCH specifications, for the USD/YTL, EUR/YTL and the conditional correlation estimates of the two exchange rates obtained from NLP and BEKK representations. Visual inspection of these figures also confirm our statistical findings that CBRT direct interventions do not decrease the volatility of the exchange rates. In summary, the present paper documents that CBRT interventions do not serve the purpose of decreasing volatility in the foreign exchange market.

5 Conclusions

This study analyzes the effect of direct foreign exchange market interventions carried by CBRT during 2002-2005 by employing the NLP multivariate GARCH formulation of Salih et.al. [19] and BEKK multivariate GARCH representation of Baba et.al [1]. First we conclude that the direct interventions do not serve the purpose of decreasing predicted volatility in the foreign exchange market as announced by the CBRT.Second NLP brings a substantial improvement over the BEKK representation in the solution of the bivariate GARCH(1,1) with dummy variables in the variance-covariance matrix, in terms of log-likelihood, AIC and SIC criteria.

Therefore, as documented in Salih et.al [19] this paper also confirms that the simplifications in the estimation of the multivariate GARCH model in the interest of solvability are unnecessary from an optimization point of view with the current state-of-the-art in optimization technology. However, there is certainly need for further research to ascertain the comparative advantage of the Constrained NLP approach, especially from a forecasting accuracy point of view.

Coefficients	Constrained NLP	BEKK
c_{11}	-0.076	0.052
	(0.020)	(0.031)
c_{12}	0.583	0.075
	(0.032)	(0.049)
c_{22}	0.071	0.055
	(0.016)	(0.043)
a_{11}	0.741	0.315
	(0.049)	(0.051)
a_{12}	-1.169	0.160115
	(0.073)	(0.044)
a_{21}		0.198
		(0.056)
a_{13}	0.556	
	(0.039)	
a_{22}	2.197	0.263
	(0.119)	(0.048)
a_{23}	-0.955	
	(0.057)	
a_{33}	0.490	
	(0.043)	
b_{11}	-0.500	0.960
	(0.119)	(0.052)
b_{12}	2.524	-0.092
	(0.155)	(0.049)
b_{21}		0.005
		(0.057)
b_{13}	-0.385	
	(0.109)	
b_{22}	-8.115	0.879
	(0.481)	(0.053)
b_{23}	0.897	
	(0.074)	
b_{33}	-0.044	
	(0.085)	
γ_1	-0.006	0.223
	(0.065)	(0.063)
γ_2	0.010	
	(0.065)	
γ_3	-0.003	0.308
	(0.104)	(0.066)
Log-likelihood	599.843	-322.479
AIC	-1155.686	678.958
SIC	901.807	760.255

Table 2: Results with the Bivariate Model on the USD/YTL and EUR/YTL Data (Numbers in parentheses are robust standard errors).

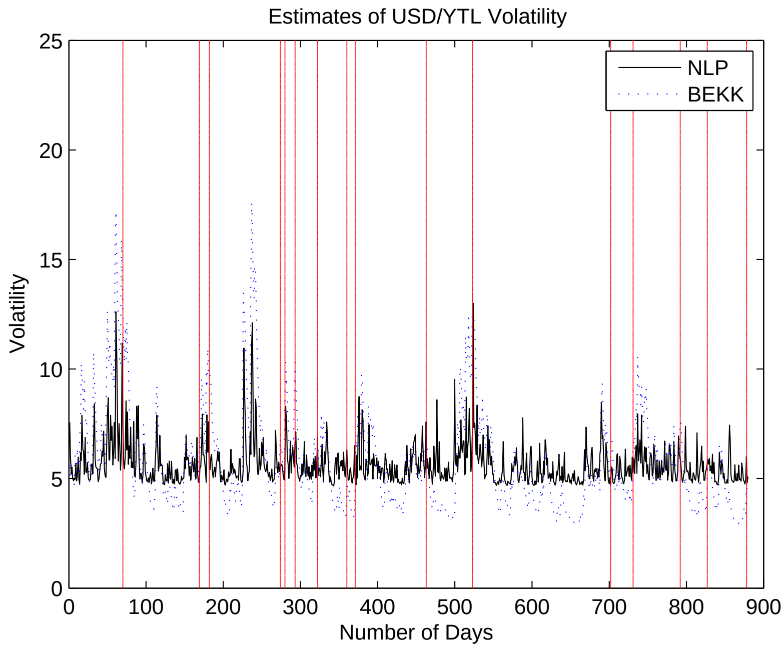

Figure 1: Daily Volatility Estimates for USD/YTL

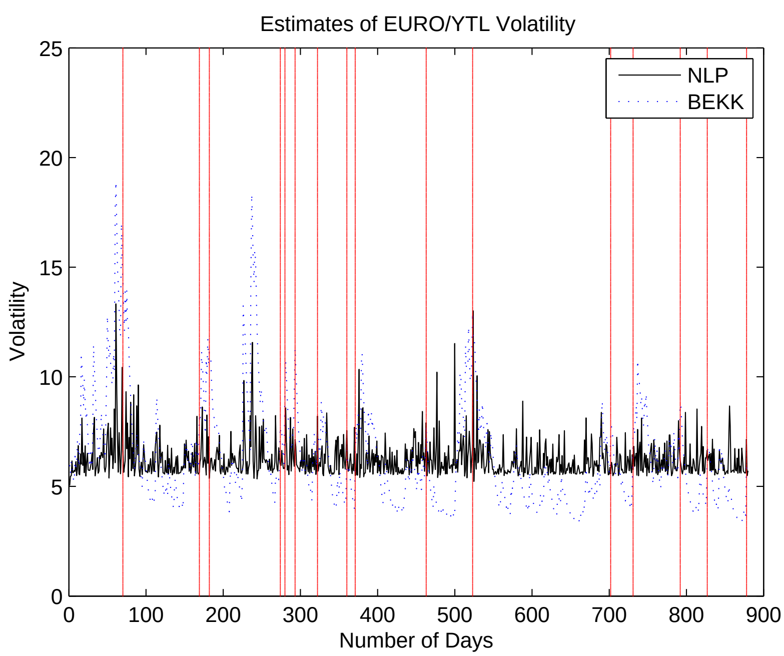

Figure 2: Daily Volatility Estimates for EUR/YTL

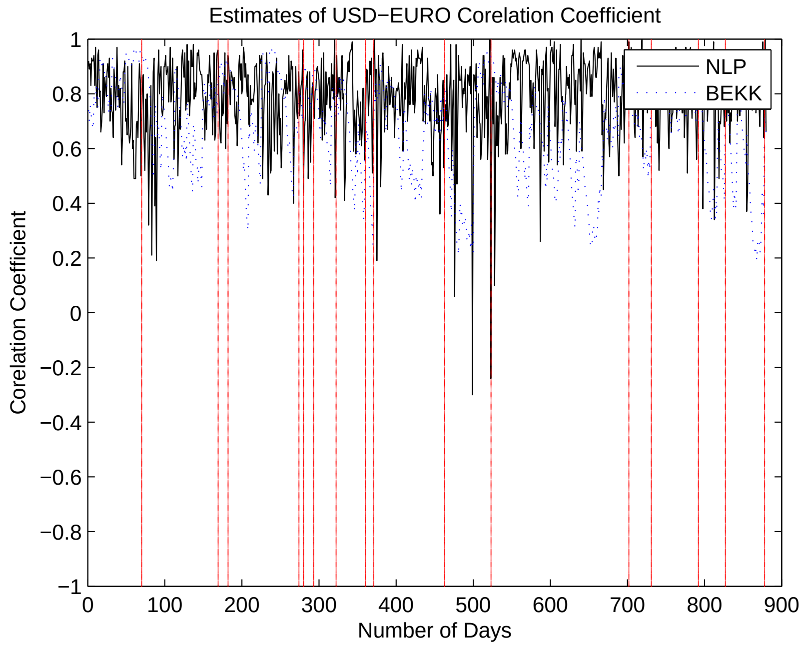

Figure 3: Daily Conditional Correlations of USD/YTL and EUR/YTL

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