An OLG-CGE Modeling Framework for Bilateral Migration

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Abstract

To accommodate possible negative impacts of ageing and compensate for potential labour shortages, many countries are adopting more open immigration policy toward the developing countries. Meanwhile, the demand from many emerging developing economies for foreign intelligence is increasing with a deeper degree of globalization and the outspread of multinational enterprises and outsourcing. Based on a simple two-country, three-input and two-generation OLG model, this paper introduces the concept of bilateral migration into a general equilibrium framework. In the model, two countries are assumed to be perfect symmetric in every aspect except for their population growth. Labour stock in each country is differentiated by both their skill levels and countries of origin. In this dynamic general equilibrium modeling, we introduce a series of CET (constant elasticity of transformation) equations to capture the supply of each kinds of labour. Simulation results from our model suggest some stylized facts which are in accord with the new era of globalization. For example, it is shown that more people migrate from the younger developing countries to the older developed countries. There will also be wage differentials between native and immigrant skilled or unskilled workers. Finally, as the degrees of ageing diverge across two countries, there will be significant tendency for workers to move from the developed countries to the developing countries with the booming of bilateral trade between these them.

1. INTRODUCTION

Over the next several decades, countries around the world will experience different degrees of population ageing. Demographic projections show that all developed countries will be in the process of fast ageing after 2010 when the baby-boom generation (those born between 1946 and 1966) begins to retire (Mérette 2002).

Population ageing is widely viewed as a threat to living standard and economic growth, especially for the ageing developed countries, as shown by related researches by World Bank (1994) and OCED (1998). Pessimistic views urged these countries to find solutions to accommodate possible negative effects of ageing. Besides some internal reform scenarios, such as debt stabilization and delayed retirement (OECD 1998), adopting more immigrants is considered by many countries as an option to relax the pressure of ageing on labour market by opening door to international labour market so as to improve the age structure of an ageing economy. International migration had already become a hot issue in almost all industrial countries.

Different from the European mass emigration to the new continents before 1914, the post-WWII wave of world migration is accompanied with a deeper integration of national markets (globalization) and an accelerating process of population ageing. On one hand, developed countries have incentives for attracting immigrants to deal with the shortage of labour supply and alleviate possible fiscal impacts of ageing. With the development of information and telecommunication (ICT) industry, many developed countries have also put emphasis on attracting more high-skilled foreign labour from the developing world. This is a new characteristic of the post-WWII wave of international migration. On the other hand, deeper integration of national markets provides incentives for a large scale of industry out-sourcing, from the developed world to the developing world, and more and more multinational enterprises (MNEs) are operating overseas. Consequently, many emerging developing economies (the East-Asian dragons, for example) are demanding more and more foreign intelligence, especially those from advanced countries, as a means of upgrading local industries and filling the technology gaps between them and the developed world.

Overall, over the past decade, we have observed a large scale of migration of labour, at all kinds of skill levels, from the developing countries to the developed countries. In addition, more and more people from the developed world have moved to emerging economies. Thus, it is highly relevant to study the so-called bilateral migration to capture the new trend of double-way international migration both from the developing world to the developed world and from the developed world to the developing world.

This model uses a simple 2*3*2*2 type overlapping-generation computable general equilibrium (OLG-CGE) model to provide a basic modeling framework to study the issue of double-way migration between two regions. Two countries in the model are assumed to be perfectly symmetric at the initial steady states. Working individuals in the two countries are allowed to make choices on their time allocation into either working in his or her homeland or working abroad. In the dynamic, population growth rates of two countries are assumed to be different so that one country grows into an older economy. Changes of age structures differentiate these two countries' labour markets, and the resulting wage differentials create incentives for people to move. A series of CET functions are used to model the supply side of labour so that working individuals in each country could make migration decision based on both their preferences in working locations and changes of their expected earnings.

2. MODEL DESCRIPTIONS

In the 2*3*2*2 type OLG-GE model, the world economy has been divided into two countries (*J1* and *J2*). In each of the them, two homogenous[i] goods (*S1*, *S2*) are produced and two major inputs (capital and labour, labelled as *Kd* and *Ld*) are used in the production of each good. Labour input used in the production of each good is a constant elasticity of substitution (CES) aggregation of two kinds of labour differentiated by their skill levels (*Skilled and unskilled*).

In the model, each country is populated with two generations (the working generation *G1*, or *gj*, and the retired generation *G2*, or *gm*) of population, and *G1* is assumed to be the only fertile generation. The first generation, *G1*, is also the only working generation in each country as the labor input in the production processes. At the initial steady state, we assume population of each of the two generations equals to 1/2 of total population, and the shares of skilled and unskilled workers in each generation are equal across countries. In the dynamic as the degrees of ageing diverge across countries, the resulting wage differentials between the two countries' labor markets generates incentives for both the skilled and the unskilled workers to move in both directions (from the younger country to the older country and from the older country to the younger country). Consequently, the shares of skilled and unskilled workers in each country' labor force vary overtime, as a result of bilateral migration.

To simplify our model and put emphasis on the decision making of different types of labor, government behaviour is not taken into account. We also ignore the existence of bequest and heritance in the optimization process for a typical individual.

2.1 Producer behavior

In each country, there are two sectors producing each one of the two goods *s* (*S1* or *S2*) using Cobb-Douglas (CD) production technologies. A representative firm's problem is to minimize the production cost subject to the embedded constraint that characterizes the firm's technology:

$$\underset{Kd_{j,s,t},Ld_{j,s,t}}{Maximize}\, Pq_{j,s,t}X_{j,s,t} - \mathrm{R}_{j,t}\, Kd_{j,s,t} + W_{j,t}Ld_{j,s,t} \quad (1)$$

$$s.t.\ X_{j,s,t} = SP_s Kd_{j,s,t}^{\alpha_s^K} Ld_{j,s,t}^{(1-\alpha_s^K)} \quad (2)$$

In equation (1), $Pq_{j,s,t}$ is the production price of good *s* in country *j* at time *t*. $R_{j,t}$ and $W_{j,t}$ represent the rental price and wage rate in country *j* at time *t* respectively. $Kd_{j,s,t}$ is the capital demanded by production sector *s* in country *j* at time *t*. $Ld_{j,s,t}$ is the aggregate labour demanded.

In equation (2), SP_s[1] is a scaling constant parameter and α^K_s is the expenditure share measuring the intensity of use of capital in production. *S1* is capital-intensive good and *S2* is labour-intensive good. The technology is represented by a CD production function in which the sum of expenditure shares for capital and labour inputs is equal to one.

1 In this paper, $SP_{s,t}$ are assumed to be one in all sectors.

In this model, we assume that the aggregate labour supply, $Ld_{j,s,t}$, is a CES mixture of two kinds of labour supply (*qual*) in the economy: the skilled labour and the unskilled labour (equation 3).

$$Ld_{j,s,t} = (\delta^{Q}_{s,skill} LdQ_{j,s,skill,t}^{\frac{\sigma_s^{LD}-1}{\sigma_s^{LD}}} + \delta^{Q}_{s,unskill} LdQ_{j,s,unskill,t}^{\frac{\sigma_s^{LD}-1}{\sigma_s^{LD}}})^{\frac{\sigma_s^{LD}}{\sigma_s^{LD}-1}} \qquad (3)$$

Skilled and unskilled labour is differentiated by the quantity of labour supply they could provide per unit of time. In equation (3), $\delta^{Q}_{s,qual}$ is the share of labour at skill level *qual* demanded by the production of good *s* and the sum of $\delta^{Q}_{s,skill}$ and $\delta^{Q}_{s,unskill}$ is equal to one. σ_s^{Ld} is the elasticity of substitution between different kinds of labour in sector *s*. The value of $\delta^{Q}_{s,qual}$ is calibrated to be different both across sectors and across countries.

Differentiating equation (1) with respect to equation (2), we get the following equations (4) and (5) as the first order conditions for a producer:

$$\mathrm{R}_{j,t} = \alpha_s^K \frac{Pq_{j,s,t} X_{j,s,t}}{Kd_{j,s,t}} \qquad (4)$$

$$W_{j,t} = (1-\alpha_s^K) \frac{Pq_{j,s,t} X_{j,s,t}}{Ld_{j,s,t}} \qquad (5)$$

After the rental price and the aggregate wage rate are determined, a firm's optimization problem turns to be minimizing its expenditure on labour input. That is, it minimizes the following equation (6) subject to the above equation (3):

$$W_{j,t} Ld_{j,s,t} = WdQ_{j,skill,t} LdQ_{j,s,skill,t} + WdQ_{j,unskill,t} LdQ_{j,s,unskill,t} \qquad (6)$$

where $WdQ_{j,qual,t}$ is the wage rate paid for labour input at skill level *qual* in country *j* at time *t*. The following equation (7) and equation (8) are the first order conditions:

$$LdQ_{j,s,qual,t} = \delta_{s,qual}^{Q^{\sigma_s^{DQ}}} \left[\frac{W_{j,t}}{WdQ_{j,qual,t}} \right]^{\sigma_s^{DQ}} Ld_{j,s,t} \tag{7}$$

$$W_{j,t}^{1-\sigma^{DQ}} = \sum_s \delta_{s,qual}^{Q^{\sigma_s^{DQ}}} WdQ_{j,qual,t}^{1-\sigma_s^{DQ}} \tag{8}$$

Equation (7) shows the demand for labour input at skill level *qual*, $LdQ_{j,s,qual,t}$, increases with the aggregate labour demanded, the share of this kind of labour in the technology and decreases with the wage rate paid for this kind of labour. Equation (8) shows the aggregate wage in region *j* is a weighted sum of wage rate paid for all kinds of labour.

In this model, migration is introduced so that the aggregate labour demand at skill level *qual* by sector *s*, $LdQ_{j,s,qual,t}$, could be supplied by workers from both the native labour market and foreign labour market. In other words, this labour supply is a CES aggregation of labour supply differentiated by their countries of origin ($LdM_{j,j,s,qual,t}$ and $LdM_{i,j,s,qual,t}$), as shown by the following equation (9):

$$LdQ_{j,s,qual,t} = (\delta_{j,j,s,qual}^{Ldm} LdM_{j,j,s,qual,t}^{\frac{\sigma_s^{DM}-1}{\sigma_s^{DM}}} + \delta_{i,j,s,qual}^{Ldm} LdM_{i,j,s,qual,t}^{\frac{\sigma_s^{DM}-1}{\sigma_s^{DM}}})^{\frac{\sigma_s^{DM}}{\sigma_s^{DM}-1}} \tag{9}$$

where $\delta_{j,j,s,qual}{}^{Ldm}$ is the share of labour supplied by workers born in country *j* to sector s in country j and $\delta_{i,j,qual,s}{}^{Ldm}$ is the share of labour supplied by workers born in country *i* to sector *s* in country *j*. $LdM_{j,j,qual,t}$ ($LdM_{i,j,qual,t}$) is the quantity of labour supplied by individuals born in country *j* (*i*) and work in country *j*. In other words, $LdM_{j,j,s,qual,t}$ and $LdM_{i,j,s,qual,t}$ are labour supplied by native and foreign workers respectively. $\sigma_s{}^{DM}$ is the elasticity between native labour demand and foreign labour demand in sector *s*.

At time t, the problem for a producer in sector s in country j is to minimize its aggregate wage expenditure on native and foreign workers subject to the above equation (9). The aggregate wage expenditure is determined by the following equation (10):

$$Wdq_{j,qual,t} LdQ_{j,s,qual,t} = WdM_{j,j,skill,t} LdM_{j,j,s,qual,t} + WdM_{i,j,qual,t} LdM_{i,j,s,qual,t} \qquad (10)$$

where $WdM_{j,j,s,qual,t}$ and $WdM_{i,j,s,qual,t}$ are demand wage rates paid by producers in country j to native and immigrant workers working in sector s in country j.

The first order conditions for this stage of optimization are the following equations (11) and (12):

$$LdM_{i,j,s,qual,t} = \delta_{i,j,s,qual}^{Ldm^{\sigma_s^{DM}}} \left[\frac{WdQ_{j,qual,t}}{WdM_{i,j,qual,t}} \right]^{\sigma_s^{DM}} LdQ_{j,s,qual,t} \qquad (11)$$

$$WdQ_{j,qual,t}^{1-\sigma_s^{DM}} = \sum_i \sum_s \delta_{i,j,s,qual}^{Ldm^{\sigma_s^{DM}}} WdM_{i,j,qual,t}^{1-\sigma_s^{DM}} \qquad (12)$$

Equation (11) shows the demand for native (immigrant) workers increases with the ratio of aggregate wage rate for workers at skill level *qual* to the wage rate for native (immigrant) workers, the share parameter of native (immigrant) workers in total labour force and the aggregate demand for workers at skill level *qual* in sector s. Equation (12) shows the aggregate demand wage rate is a weighted sum of demand wage rates for native and immigrant workers working in country j.

2.2 Household behavior

An Allais-Samuelson overlapping generation framework characterizes households in the economy, so that this model is based on the life-cycle theory of savings. In period t, each economy is populated by two generations g ($G1$ and $G2$) of population. A representative cohort born in period t works in the first period of its life (from 16 to 50 years of age), retires in period $t+2$ and lives another period of time (from 51 to 85 years of age). Thus each period t approximately represents 30 years in our model.

The lifetime utility for a representative cohort is represented by a time-separable nested CES function with the elasticity different from one. The optimization problem for a cohort at skill level *qual* can be divided into three steps in an open economy with international trade. First, immediately after birth, a cohort decides on the allocation of its labour income on either current or future consumptions. On the second step of optimization, the aggregate consumption expenditure is allocated among different kinds of consumption goods. Finally, on the third step of optimization, an individual decides on the composition of each consumption good s in terms of the country of origin. In other words, consumption of good s is an aggregation of both locally- produced products and imported products following the Armington assumption.

The intertemporal optimization problem for a representative cohort at the skill level *qual* in country j is to maximize its lifetime utility subject to the budget constraint:

$$\underset{C_{j,g,t}}{Maximize}\, U(C_{j,qual,g,t})_{j,qual,t} = \frac{\sigma^{IT}}{\sigma^{IT}-1}\sum_{g=1}^{7}\left(\frac{1}{1+\rho}\right)^{g} C_{j,qual,g,t+g-1}^{\frac{\sigma^{IT}-1}{\sigma^{IT}}} \tag{9}$$

$$s.t. \quad LW_{j,qual,t} = \sum_{g=1}^{7} PCon_{j,g,t+g-1}(1+\tau^{C})(C_{j,qual,g.t+g-1}) \tag{10}$$

The CES-form utility function is shown by equation (9), and the lifetime utility is an aggregation of the present values of current consumption and future consumptions. In each country j, $U(C_{j,qual,g,t})_{j,qual,t}$ is the lifetime utility for a representative cohort at skill level *qual* and $C_{j,qual,g,t}$ is the quantity of aggregate consumption. σ^{IT} is the intertemporal elasticity of substitution, and ρ is the pure rate of time preference. The larger ρ, the more of its lifetime resources a cohort would spend in the first stage of life and the less it saves. As shown in equation (10), the lifetime wealth for an individual at the skill level *qual*, $LW_{j,qual,t}$, is allocated into current consumption and future consumptions. In this equation, $PCon_{j,g,t}$ is the aggregate consumption price for an individual of age g at time t. $LW_{j,qual,t}$ is represented by the following equation (11):

$$LW_{j,qual,t} = \sum_{g=1}^{7} \left\{ \frac{1}{1 + Rin\mathrm{t}_{j,t+g-1}} \right\}^{t} [LInc_{j,qual,g,t+g-1}(1 - CtR_t) + Pens_{j,qual,g,t+g-1}] \qquad (11)$$

where $LInc_{j,\ qual,g,t}$ is the labour income for an individual at skill level *qual* of age g at time t. $Rint_{j,t}$ is the interest rate in region j at time t. CtR_t represents the contribution rate of pension out of total labour income; $Pens_{j,qual,g,t}$ is the pension income received by the last two generations.

In both countries, an individual's labour income depends both on the individual's age-dependent productivity and the skill level. Thus, the labour income for an individual in the working generation gj at time t can be represented by the following equations (12) and (13):

$$LInc_{j,qual,gj,t} = wage_{j,qual,t} EP_{j,g,qual} \qquad (12)$$

$$EP_{j,g,qual} = \alpha_{qual}^{Q} (\gamma + \lambda g - \psi g^2), \quad \gamma, \lambda, \psi, \geq 0 \qquad (13)$$

The intertemporal maximization problem is as follows. Households in all countries are forward looking and have perfect foresight. A representative cohort is born in the first period and works in this period as the labour force in the economy. Part of its labour income is saved as investment in capital stock and the rest is consumed. In the last two periods, this cohort spends all its wealth, including investment in the first period and interest income, on consumption. Immediately after birth, an individual decides on the allocation of his or her expected labour income into current consumption and future consumptions. Differentiating the above equation (9) with respect to the budget constraint (10) yields the following first order condition for the aggregate consumption, $C_{j,qual,g,t}$, consumed by an individual at skill level *qual* of age *g* (Equation 14):

$$C_{j,qual,g+1,t+g} = \left[\frac{(1+Rint_{t+g}(1-\tau^K))(1+\tau^C)PCon_{g,t+g-1}}{(1+\rho)(1+\tau^C)PCon_{g+1,t+g}} \right]^{\sigma^{IT}} C_{j,qual,g,t+g-1} \qquad (14)$$

Equation (14) shows the ratio of aggregate consumption by individual of age *g* at skill level *qual* at time *t+1* to it is at time *t* increases with the ratio of aggregate consumption price at time *t* to the aggregate consumption price at time *t+1*. In other words, higher are future consumption prices, higher is current consumption and lower is future consumption.

On the second step of optimization, an individual of age *g* allocates the expenditure on the optimal aggregate consumption, $C_{j,qual,g,t}$, across sectors. The problem is to maximize the following CES utility function subject to budget constraint:

$$\underset{CS_{j,S1,qual,g,t},CS_{j,S2,qual,g,t}}{Maximize} C_{j,qual,g,t} = (\delta_{S1}^{CS} CS_{j,S1,qual,g,t}^{\frac{\sigma^{CS}-1}{\sigma^{CS}}} + \delta_{S2}^{CS} CS_{j,S2,qual,g,t}^{\frac{\sigma^{CS}-1}{\sigma^{CS}}})^{\frac{\sigma^{CS}}{\sigma^{CS}-1}} \qquad (15)$$

$$s.t.\ PCon_{j,g,t} C_{j,qual,g,t} = PCons_{j,S1,t} CS_{j,S1,qual,g,t} + PCons_{j,S2,t} CS_{j,S2,qual,g,t} \qquad (16)$$

In equation (15), δ_s^{CS} as the parameter representing the preference of a consumer of age g on consumption goods across sectors. σ^{CS} is the elasticity of substitution for consumptions across sectors. $CS_{j,s,qual,g,t}$ is the quantity of consumption of good s by an individual of age g at skill level *qual* living in country j at time t. In equation (16), $PCons_{j,s,t}$ is consumption price of good s. The following equations (17) and (18) are the first order conditions for the second step of optimization:

$$CS_{j,qual,s,g,t} = \delta_S^{CS^{\sigma^{CS}}} \left[\frac{PCon_{j,g,t}}{PCons_{j,s,t}} \right]^{\sigma^{CS}} C_{j,qual,g,t} \quad (17)$$

$$PCon_{j,g,t}^{1-\sigma^{CS}} = \sum_s \delta_S^{CS^{\sigma^{CS}}} PCons_{j,s,t}^{1-\sigma^{CS}} \quad (18)$$

Equation (17) shows the consumption of good s by individual of age g at skill level *qual* at time t increases with the aggregate consumption but decreases with the consumption price of good s on the market. Equation (18) shows the aggregate consumption price is a weighted sum of the consumption prices for different good s.

In our Armington-type trade model, an individual in country j allocates the consumption expenditure on good s by country of origin, which is the third step of optimization. In other words, a decision is made on how much to spend on either locally-produced good s or imported good s. The following equations show the third step of consumer's optimization:

$$\underset{CSJ_{j,j,s,qual,g,t},CSJ_{i,j,s,qual,g,t}}{Maximize} CS_{j,s,qual,g,t} = (\delta_{j,j,s}^{CE} CSJ_{j,j,s,qual,g,t}^{\frac{\sigma^{CE}-1}{\sigma^{CE}}} + \delta_{i,j,s}^{CE} CSJ_{i,j,s,qual,g,t}^{\frac{\sigma^{CE}-1}{\sigma^{CE}}})^{\frac{\sigma^{CE}}{\sigma^{CE}-1}} \quad (19)$$

$$s.t.\ PCons_{j,s,t} CS_{j,s,qual,g,t} = Pq_{j,s,t} CSJ_{j,j,s,qual,g,t} + Pq_{i,s,t} CSJ_{i,j,s,qual,g,t} \quad (20)$$

In equation (19), $\delta_{i,j,s}{}^{CE}$ as the parameter representing the preference of a consumer on goods s produced in country i, and σ^{CE} is the elasticity of substitution for locally-produced and imported consumption good s. $CSJ_{i,j,qual,s,g,t}$ is the quantity of consumption of good s produced in country i but consumed by country j's consumer. In other words, set i shows the country of origin of a consumed good and set j shows where this good is consumed. Accordingly, $CSJ_{j,j,qual,s,g,t}$ represents locally-produced consumption good s consumed by generation g in country j at time t. The following equation (21) and equation (22) are the first order conditions for the third step of optimization:

$$CSJ_{i,j,s,qual,g,t} = \delta_{i,j,s}^{CE^{\sigma^{CE}}} \left[\frac{PCons_{j,s,t}}{Pq_{i,s,t}} \right]^{\sigma^{CE}} CS_{j,s,qual,g,t} \tag{21}$$

$$PCons_{j,s,t}^{1-\sigma^{CE}} = \sum_i \delta_{i,j,s}^{CE^{\sigma^{CE}}} Pq_{i,s,t}^{1-\sigma^{CE}} \tag{22}$$

2.3 Labour supply, migration and demographics

In this sub-section of model description, we put emphasis on our contribution as the modeling of the supply side of labor market in a dynamic CGE model. Since the evolution of each country's population is no longer exogenously determined and becomes more dependent on the results of bilateral migration, we will also introduce how the post-migration population of each country evolutes.

In each country, the aggregate labour supply is provided by workers either born in this country or born abroad. Thus, there are four types of labour on the market: native skilled workers, immigrant skilled worker, native unskilled workers and immigrant unskilled workers. The decision making of labour supply is closely correlated to household behavior. At time t, each individual in country j is endowed with a certain quantity of labour. Since there is no constraint on the movement of labour across borders, so this individual has the freedom to allocate his or her time into either working in the home country or working abroad.

In the model, the aggregate labour supply at skill level *qual* in country *j* at time *t*, $LsQ_{j,qual,t}$, is provided by a number of people at skill level *qual* ($Pop_{j,qual,t}$), who are working and living in this country:

$$LsQ_{j,qual,t} = Pop_{j,qual,t} EP_{j,gj,qual} \tag{27}$$

where $LsQ_{j,qual,t}$ is a CET aggregation of labour supplied by native workers and foreign workers ($LsM_{j,j,qual,t}$ and $LsM_{j,i,qual,t}$), as shown by the following equation (28):

$$LsQ_{j,qual,t} = (\delta^{Lsm}_{j,j,qual} LsM_{j,j,qual,t}^{\frac{\sigma^{SM}+1}{\sigma^{DM}}} + \delta^{Lsm}_{j,i,qual} LsM_{j,i,qual,t}^{\frac{\sigma^{SM}+1}{\sigma^{SM}}})^{\frac{\sigma^{SM}}{\sigma^{SM}+1}} \tag{28}$$

In equation (28), $\delta_{j,j,qual,,s}{}^{Lsm}$ is the share of time allocated by an individual born in country *j* into working in country *j*, and $\delta_{j,i,qual,s}{}^{Lsm}$ is the share of time allocated by an individual, born in country *j*, into working in country *i*. $LsM_{j,j,qual,t}$ ($LsM_{j,i,qual,t}$) is the quantity of labour supplied by an individual born in country *j* and work in country *j*(*i*). σ^{SM} is the elasticity of transformation between labour supplied to production in country *j* by workers born in country *j* and labour supplied to production in country *i* by workers born in country *i*. This elasticity value shows the responsiveness of native and foreign-born labor to the wage rates paid for them by producers in country *j*.

In the numerical model, both countries are assumed to be perfectly symmetric at the initial steady state, so all share parameters and elasticity values are assumed to be equal across countries. The values of related parameters are presented on the following Table 1.

As mentioned above, an individual born in country *i* allocates the time into either working in his or her home country or abroad. The optimization problem for this individual is to maximize its labour income gained from both working in his or her home country and abroad. In the model, the problem is to maximize the following equation (29) subject to equation (28):

$$WsQ_{j,qual,t} LsQ_{j,qual,t} = WsM_{j,j,qual,t} LsM_{j,j,qual,t} + WsM_{j,i,qual,t} LsM_{j,i,qual,t} \qquad (29)$$

where $WsQ_{j,qual,t}$ is the aggregate wage rate paid for an individual, at skill level *qual*, born in country *j* at time *t*. $WsM_{j,j,qual,t}$ ($WsM_{j,i,qual,t}$) is the wage rate paid by producers in country *j* (*i*) for native (immigrant) workers born in country *j*.

The following two equations (30) and (31) are the first order conditions:

$$LsM_{j,i,qual,t} = \delta_{j,i,qual}^{Lsm^{\sigma^{SM}}} \left[\frac{WsM_{j,i,qual,t}}{WsQ_{j,qual,t}} \right]^{\sigma^{SM}} LsQ_{j,qual,t} \qquad (30)$$

$$WsQ_{j,qual,t}^{1+\sigma^{SM}} = \sum_{i} \delta_{j,i,qual}^{Lsm^{\sigma^{SM}}} WsM_{j,i,qual}^{1+\sigma^{SM}} \qquad (31)$$

Equation (30) suggests the quantity of labour provided by native (immigrant) workers born in country *j* increases with the ratio of wage rate for native (immigrant) workers in home (destination) country to the aggregate supply wage rate for workers, at skill level *qual*, born in country *j*. It also augments with the share of labour provided by native or immigrant workers in the aggregate labour supply and the aggregate quantity of labour supply provided by workers at skill level *qual* and born in country *j*. Equation (31) suggested the aggregate supply wage rate for an individual born in country *j* is the weighted sum of supply wage rate paid by producers in country *j* to native workers and supply wage rate paid by producers in country *i* to immigrant workers born in country *j*.

In the first chapter of this dissertation, we introduce the demographic shock as given the projected fertility rates, we calculate each country's population and effective labor supply for each period. The changes of all economic variables in the dynamic are based on this demographic shock. In chapter 2, we introduced a one-period exogenous shock of unilateral migration so that the population of the source country and the destination country will be recalculated for all periods after the timing of migration, as shown by the following equations (32) and (33):

$$POP_{i,qual,t+1} - MigCoe_{i,j,t+1,g} Pop_{i,qual,gi,t+1} \quad (32)$$

$$POP_{j,qual,t+1} + MigCoe_{i,j,t+1,g} Pop_{i,qual,gi,t+1} \quad (33)$$

where $MigCoe_{i,j,qual,t,g}$ is the migration coefficient calculated based on wage differentials across countries.

The economic implication of this migration coefficient is the percentage of out-migrating people out of the population of the source countries, so we only have to adjust the population for all countries for one period and allow their population to regenerate based on the adjusted population in the following periods. However, with this setting, we are unable to model the bilateral migration as this ratio is exogenously given.

In this paper, modeling of the supply side of the labor market enables us to differentiate native and immigrant workers. In addition, we no longer rely on the migration coefficient to exogenously determine the number of migrating people. In other words, for the fist time in the dynamic CGE modeling, we treat labor stock in each country as commodities on the good markets based on the rationale of Armington assumption. How much time an individual allocates into working in the home country and working abroad are determined by both his or her preferences at the initial steady state and the endogenously determined changes of wage rates for native and immigrant workers.

Consequently, the population of both countries is also exogenously determined in each period. This raises a problem of the population of both countries at each time period could be either the number of people before migration or the number of people after migration. Another obstacle is the timing for us to impose the exogenous fertility rates.

To solve these problems, we first assume that both native and immigrant workers follow the fertility rates in the destination countries. Then, we introduce two types of population (the native population $PopM_{j,j,qual,t,g}$, and the immigrant population $PopM_{i,j,qual,t,g}$) into the model.

The following four equations are used to relate the two variables of $Pop_{j,qual,t,g}$ and $PopM_{i,j,qual,t,g}$ and to capture the evolution of population for country j:

$$PopM_{j,i,qual,t,G1} = LsM_{j,i,qual,t} / (LuM_{j,i,qual,G1} EPM_{j,i,qual,G1}) \quad (34)$$

$$Pop_{j,qual,t+1,G1} = \sum_i PopM_{i,j,qual,t,G1} FR_{j,t} \quad (35)$$

$$PopM_{j,i,qual,t+1,G2} = PopM_{j,i,qual,t,G1} \quad (36)$$

$$Pop_{j,qual,t+1,g+1} = Pop_{j,qual,t,g} \quad (37)$$

where $PopM_{i,j,qual,t,g}$ is the population of people, at skill level *qual*, born in country j but working in country i at time t. $LuM_{j,i,qual,g}$ is the unit labor supplied by this individual and $EPM_{j,i,qual,g}$ is the efficiency coefficient.

Equation (34) is used to calculate the number of native and immigrant populations based on the equilibrium levels of labor supply. Equation (35) introduces the exogenous fertility rate and suggests that the total population in country j at time *t+1* equals to the sum of native and immigrant workers at time t augments with the fertility rate in country *j*. Equations (36) and (37) show that the population of the next generations of native, immigrant workers and the total population of country *j* at time *t+1* equals to their population of previous generations at time *t*.

2.4 Determinants of investment demand

In each period *t*, savings of the younger generations finance investment equipped by firms to increase and maintain the stock of physical capital in period *t+1*. The usual law of motion of capital stock can be represented by the following equation (38):

$$KS_{j,t+1} = I_{j,t} + (1 - DR)KS_{j,t} \tag{38}$$

In equation (38), $I_{j,t}$ is the aggregate investment in country *j* at time *t*. The capital stock in the economy at time *t+1*, $KS_{j,t+1}$, augments with investment in the previous period *t* and declines with depreciation rate (*DR*). This equation shows the aggregate investment level is determined by the gap between demand of capital stock in the next period and current capital stock net of depreciation.

The capital stock is a composite good composed of the available goods in the economy. Consequently, the capital stock in each economy is built using an investment technology with both the capital-intensive good and the labour-intensive good. After the aggregate investment level is determined, there are also two steps of optimization for the investors in the economy, which are depicted by the following four equations:

$$\underset{ISJ_{i,j,s,t},ISJ_{j,j,s,t}}{Maximize}\ I_{j,t} = (\delta_{S1}^{IS} IS_{j,S1,t}^{1-\frac{1}{\sigma^{IS}}} + \delta_{S2}^{IS} IS_{j,S2,t}^{1-\frac{1}{\sigma^{IS}}})^{\frac{\sigma^{IS}}{\sigma^{IS}-1}} \tag{39}$$

$$s.t. \qquad IS_{j,s,t} = (\delta_{j,j,s}^{IE} ISJ_{j,j,s,t}^{1-\frac{1}{\sigma^{IE}}} + \delta_{i,j,s}^{IE} ISJ_{i,j,s,t}^{1-\frac{1}{\sigma^{IE}}})^{\frac{\sigma^{IE}}{\sigma^{IE}-1}} \tag{40}$$

$$and \quad PI_{j,t} I_{j,t} = PIS_{j,S1,t} IS_{j,S1,t} + PIS_{j,S2,t} IS_{j,S2,t} \tag{41}$$

$$and \quad PIS_{j,s,t} IS_{j,s,t} = Pq_{j,s,t} ISJ_{j,j,s,t} + Pq_{i,s,t} ISJ_{i,j,s,t} \tag{42}$$

In equation (39), δ_s^{IS} is the given share of good s in aggregate investment, and σ^{IS} is the elasticity of substitution for investment goods. $IS_{j,s,t}$ is the investment good s used in country j at time t. In equation (40), $\delta_{i,j,s}^{IE}$ is the given share of investment good s produced in country i but used in country j. $ISJ_{i,j,s,t}$ is the investment good s used in country j but produced in country i at time t. Accordingly, $ISJ_{j,j,s,t}$ is the locally-produced investment good s used in country j at time t. σ^{IE} represents the elasticity of substitution for locally-produced and imported investment good s. In equations (41) and (42), $PI_{j,t}$ represents the aggregate price of all investment goods in country j at time t and $PIS_{j,s,t}$ is the price of investment good s in country j at time t.

On the second step of optimization, an investor's problem is to maximize equation (39) subject to equation (41). The following equations (43) and (44) are the first order conditions:

$$IS_{j,s,t} = \delta_s^{IS^{\sigma^{IS}}} \left[\frac{PI_{j,t}}{PIS_{j,s,t}} \right]^{\sigma^{IS}} I_{j,t} \tag{43}$$

$$PI_{j,t}^{1-\sigma^{IS}} = \sum_s \delta_s^{IS^{\sigma^{IS}}} PIS_{j,s,t}^{1-\sigma^{IS}} \tag{44}$$

After the $IS_{j,s,t}$ is determined, on the third step of optimization, an investor allocates its investment expenditure on investment good s across countries. The optimization problem is to maximize equation (40) subject to equations (42). The following equations (45) and equation (46) are the first order conditions:

$$ISJ_{i,j,s,t} = \delta_{i,j,s}^{IE^{\sigma^{IE}}} \left[\frac{PI_{j,s,t}}{Pq_{i,s,t}} \right]^{\sigma^{IE}} IS_{j,s,t} \qquad (45)$$

$$PIS_{j,s,t}^{1-\sigma^{IE}} = \sum_i \delta_{i,j,s}^{IE^{\sigma^{IE}}} Pq_{i,s,t}^{1-\sigma^{IE}} \qquad (46)$$

2.5 Foreign trade with the rest of the world

As explained in the above sections, we allocate demand in country j for goods produced in country i based on the Armington assumption. In other words, even though individual producers are microscopic price takers, good s are assumed to be differentiated in demand by country of origin. At the initial steady state where all countries are assumed to be perfectly symmetric, each country imports and exports the same quantity of *S1* and *S2*. Trade exists when countries are perfectly symmetric because each of them demands both locally-made and foreign-made goods. During the processes of population ageing, changes in relative factor abundances bring changes to two countries' comparative advantages. Accordingly, demands for the four goods change overtime.

The aggregate level of one country's import or export of good s (denoted as $IMP_{j,s,t}$ and $EXP_{j,s,t}$) is determined by the above optimization steps for agents (consumer, investor and government), which can be shown by the following equation (47) and equation (48):

$$IMP_{j,s,t} = \sum_{i,qual,g} POP_{j,qual,g,t} CSJ_{i,j,qual,s,g,t} + ISJ_{i,j,s,t} \quad for\, i \neq j \tag{47}$$

$$EXP_{j,s,t} = \sum_{i,qual,g} POP_{i,qual,g,t} CSJ_{j,i,qual,s,g,t} + ISJ_{j,i,s,t} \quad for\, i \neq j \tag{48}$$

2.6 Equilibrium conditions

A general equilibrium solution is one in which all economic behavior are consistent with both current prices and future prices and all markets clear. In this model, we impost the following four equilibrium conditions for good market, labour market, capital market and the market for financial asset respectively:

$$X_{j,s,t} = \sum_{i,g} POP_{i,g,t} CSJ_{j,i,s,g,t} + ISJ_{j,i,s,t} + GovSJ_{j,i,s,t} \tag{49}$$

$$\sum_{gj} POP_{j,qual,gj,t} LS_{j,qual,gj,t} = \sum_{s} Lsq_{j,qual,s,t} \tag{50}$$

$$KS_{j,t} = \sum_{s} Kd_{j,s,t} \tag{51}$$

$$\sum_{g,qual} POP_{j,qual,gm,t+1} LD_{j,qual,gm,t+1} = PI_{j,t} KS_{j,t+1} + PGov_{j,t} Bond_{j,t+1} \tag{52}$$

In equation (49), $LS_{qual,g,t}$ is the unit labour supply provided by an individual at skill level *qual* in the working generation *gj* in country *j*. In equation (50), $LD_{j,qual,g,t}$ is the financial asset owned by the old generations *gm* living in country *j* at time *t*.

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i Homogenous in this paper means that the technologies employed in the production of each good are identical across countries.