Coopetition in a Mixed Duopoly Market

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Abstract

This study aims to investigate the impact of privatization on the degree of cooperation and competition in a mixed duopoly market. We consider a duopoly market that comprises one semipublic firm and one private firm. Each firm is assumed to determine the level of two types of effort: the cooperative effort made to enlarge the total market size and the competitive effort made to increase market share. In a contest framework, our results show that the competitive effort level of the semipublic firm is smaller than that of the private firm. The more the semipublic firm is concerned for the social welfare, the less it competes. On the basis of average costs, we then identify the conditions under which only the semipublic firm undertakes cooperative effort while the private firm behaves as a free rider. Furthermore, we find that the semipublic firm expends more cooperative effort than does the private firm.

Keywords: Coopetition, Mixed oligopoly, Contests, Free rider.
JEL classification: C72, L13, L33.

1 Introduction

In the business world, many firms choose to undertake not only competitive activities but also cooperative activities with each other. Studies such as Brandenburger and Nalebuff (1996), Dagnino and Padula (2002) describe situations that contain both cooperative and competitive activities simultaneously in terms of coopetition. Literatures on coopetition have developed

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rapidly, particularly in recent years, and the concept has been used to explain many economic and social phenomena in various industries and in different countries. However, to the best of our knowledge, previous works on coopetition have focused either on the activities of private firms or those of public firms. In other words, they have not addressed the market where there are both private and public firms. Seminal works such as Merrill and Schneider (1966), Harris and Wiens (1980), Bös (1986, 1991), Vickers and Yarrow (1988) and De Fraja and Delbono (1990) describe this market as a "mixed oligopoly market".

With privatization and deregulation waves in both developed and developing countries, we can find easily many examples of coopetition in mixed oligopoly markets. For instance, the three mobile phone operators in the French market, Orange, SFR and Bouygues Telecom, initiated a cooperation project. Orange is a semipublic firm in the sense that 18.17% of its holding company, France Telecom, is controlled by the French government. Motivated to enhance the telecommunications services offered to their clients and thus increase their profits in the "saturated" French market, these rivals decided to cooperate to offer the best homogenous public wifi service covering the whole country. To do so, in October 2003, they created the Wireless Link Association and have since attracted nine members of varying size and some start-ups. Because establishing a high quality nationwide wifi network is a complex task and requires enormous investment in infrastructure and technology research, no operator can meet this challenge alone. Hence, cooperation between these rival operators unifies the complementary innovative techniques and the different experiences that each member has in different processes, toward providing a public wifi service for the common good.

Another example is the case of the Japanese life insurance market[1]. In this market, there are two types of life insurance providers, private and semipublic. Japan Post was originally a completely public entity, but in 2003 it reformed as a semipublic organization to improve its efficiency. Both insurers cooperate to develop a healthier and more disciplined life insurance market. In October 2007 Japan Post was admitted to the Life Insurance Association of Japan which coordinates services for policyholders, evaluates moral hazard and so on. Japan Post and private life insurers are expected to adopt cooperative strategies to develop their common interests. However, they also compete on premiums and quantities to expand their market shares. Thus, the Japanese life insurance market can also be considered as a coopetitive mixed oligopoly market.

The purpose of this study is to investigate the coopetition in such a

[1]A more detailed explanation of this is given by Okura and Kasuga (2007)

situation. For a given set of market conditions determined, for example, by the number of firms and the timing of decisions, the activities chosen by semipublic and private firms may differ because of differences in their respective objective functions. Specifically, if one firm is semipublic rather than private, how does this affect market equilibrium?

To answer this question, we consider the simplest duopoly market that contains one semipublic firm and one private firm. Each firm chooses two types of effort level: cooperative effort made to enlarge the market size and competitive effort made to increase market share. In other words, we develop a coopetition model of effort levels in a mixed duopoly market.

The remainder of this study is organized as follows. In section 2, we review the related literature in order to shed light on the contribution made by our research. In section 3, we develop a model of cooperative and competitive effort in a mixed oligopoly market that contains both private and semipublic firms. The model is used to derive several interesting results. Concluding remarks are given in section 4.

2 Related literature

In general, economists model the objectives of public firms in two ways. One way is to specify that a public firm's objective is pure social welfare maximization (see, for example, De Fraja and Delbono (1990), and more recently, White (1996), Fjell and Pal (1996), Mujumdar and Pal (1998) and Pal (1998)). The second (Bös 1991 and more recently, Matsumura 1998, Barcena-Ruiz and Garzon 2003 and Matsumura and Kanda 2005) is to specify that the public firm is only a partial social welfare maximizer. In this paper, we adopt the second approach which treat the first as a special case. In many cases, the state has held, and sometimes still holds, a nonnegligible proportion of shares in privatized firms, and there are a number of firms that have a mixture of private and public ownership. Because privatized firms with mixed ownership must also respect the interests of private shareholders, they cannot behave as pure welfare maximizers.

Moreover, our paper is closely related to the interesting work of Chung (1996) on the endogenization of prizes in contests. Chung (1996) analyzes the effort levels expended by players and focuses on social waste in rent-seeking contests, an important issue in the rent-seeking contest literature, in which the prize increases with the efforts of all players.

Our research is also inspired by Krishnamurthy (1999) and Dearden and Lilien (2001), who model coopetition in a contest framework. Both study firm behavior under the assumption that firms collaborate on advertising to

increase the total market, so representing the cooperation aspect, at the same time as attempting to increase their shares of that demand, representing the competition aspect.

Our work contributes to this literature in two ways. First, while previous work assumes that payoff functions of players are similar, we consider the general case in which players pursue different objectives and thus have distinct payoff functions. Second, following Ngo (2006) we employ a two-stage model. As pointed out by Dumez and Jeunemaître (2006), there are two types of coopetition. In the one-stage game, cooperation and competition occur simultaneously in a multi-dimension framework, and in a two-stage or multi-stage game, cooperation and competition take place sequentially.

The merit of modeling coopetition in a contest framework is that it enables one to take into account agents' competitive efforts, which are neglected in the traditional Cournot and Bertrand models. Ngo (2006) argues that many economic and social phenomena can be viewed as coopetition contests in which agents spend resources in order to win one or more prizes. Many examples of coopetition contests can be found in real life: employees compete with each other for promotion in organizational hierarchies but also work collectively to develop their firms; domestic firms compete for market share but also join together against foreign firms; athletes compete for prizes but are mutually responsible for attracting a crowd.

3 The model

For simplicity, we consider a model of two firms. Suppose that these firms play a two-stage game. Firm A is semipublic, while firm B is private.

In the first stage, both firms choose their cooperative effort levels simultaneously. The cooperative effort level is denoted by y_i for $i \in \{A, B\}$, which increases the total market. The overall market demand function is:

$$y = a + y_A + y_B$$

where a represents initial demand without any cooperative effort.

In the second stage, both firms simultaneously choose their competitive effort levels, which are denoted by x_i. Competitive effort levels can enhance individual firms' competitive power and market shares. That is, if there is no competitive effort, the market is divided equally to two firms. Otherwise, the pie share of firm i is determined by the ratio $x_i/(x_A + x_B)$.

Let the inverse demand function be $p(\cdot)$. Assume that $p'(\cdot) < 0$. Average cost denoted by c is assumed to be constant and the same for both firms.

Then, the objective function of firm B is[2]:

$$U_B = \Pi_B = (p - c)\, y \frac{x_B}{x_A + x_B} - k_x x_B - k_y y_B^2 \tag{1}$$

where $p \equiv p(y)$, $k_x x_B$ and $k_y y_B^2$ represent the costs of expending competitive and cooperative efforts respectively[3].

Because firm A is a semipublic firm, its objective function is:

$$U_A = \alpha W + (1 - \alpha)\, \Pi_A \tag{2}$$

where W, which represents the social surplus, is the sum of the producer's profit and the consumer's surplus; the parameter $\alpha \in [0, 1]$ can be interpreted at two levels. At one level, it represents the weight of the government's participation in the firm A. At the other level, it can be regarded as the importance level attributed to the government's objective, i.e. the social welfare, in contrast with the profit objective. $\alpha = 0$ signifies that the firm A is solely concerned about its profit. $\alpha = 1$ means that the firm A aims to maximize the social welfare regardless of its profit.

It follows that:

$$\Pi_A = (p - c)\, y \frac{x_A}{x_A + x_B} - k_x x_A - k_y y_A^2 \tag{3}$$

and the social surplus can be written as:

$$W = \int_0^y p(q)\, dq - cy - k_x (x_A + x_B) - k_y \left(y_A^2 + y_B^2\right) \tag{4}$$

To derive the extensive form game, we solve the game by backward induction. That is, the equilibrium in the second stage is derived on the basis of the first stage before the first stage has been played. Once the equilibrium in the second stage is determined, the equilibrium in the first stage is derived by using the results from the second stage.

The second stage is described below.

[2]Here we consider a deterministic outcome competition in which each party receives a share of what is under dispute. The equivalent results under a probabilistic competition, i.e. a winner-take-all competition, can be derived under the assumption of risk neutrality.

[3]The ratio $x_i/(x_A + x_B)$ is concave in x_i while the overall market demand function is linear in y_i. Thus, in order to guarantee the existence of the optimal y_i and x_i, we assume that the cost functions of y_i and of x_i are, respectively, quadratic and linear.

The first-order conditions with respect to x_i are:

$$\frac{\partial U_A}{\partial x_A} = \frac{(1-\alpha)(p-c)x_B y - k_x (x_A + x_B)^2}{(x_A + x_B)^2} = 0$$
$$\frac{\partial U_B}{\partial x_B} = \frac{(p-c)x_A y - k_x (x_A + x_B)^2}{(x_A + x_B)^2} = 0$$

Given $\alpha \in [0, 1)$, the equilibrium competitive effort levels are:

$$x_A^* = \frac{(1-\alpha)^2 (p-c) y}{(2-\alpha)^2 k_x} \; ; \; x_B^* = \frac{(1-\alpha)(p-c) y}{(2-\alpha)^2 k_x} \tag{5}$$

and the market shares of public and private firm are:

$$s_A^* = \frac{1-\alpha}{2-\alpha} \; ; \; s_B^* = \frac{1}{2-\alpha}$$

When $\alpha \to 1$, it is reasonable to assume that $x_A^* \to 0, x_B^* = \varepsilon$ (where ε represents a very small positive number) and the market shares are $s_A^* \to 0, s_B^* \to 1$.

These results are used to state the following lemma.

Lemma 1 (relationship between competitive effort levels)**.**
Both competitive effort levels satisfy: $x_A^* = (1-\alpha) x_B^*$ *Furthermore, the relationship between market shares of firm A and B:* $s_A^* = (1-\alpha) s_B^*$.

This implies that the competitive effort level of the semipublic firm is below that of the private firm. The more concerned is the semipublic firm for social welfare (the closer is α to 1), the less it competes. As a result of it, the market share of the public firm is never bigger than that of the private firm.

Several comments on the equilibrium cooperative effort levels shown in (5) are warranted. In this context, consider the case in which only one variable changes.

First, $\partial x_i^* / \partial k_x < 0$ and $\partial x_j^* / \partial k_x < 0$ have the simple and intuitive implication that the higher is the cost level, the lower is competitive effort.

Second, consider the relationship between competitive and cooperative effort levels. From (5), the following derivatives are obtained:

$$\frac{\partial x_A^*}{\partial y_A} = \frac{\partial x_A^*}{\partial y_B} = \frac{(1-\alpha)^2 \{p' + p - c\}}{(2-\alpha)^2 k_x} = \frac{(1-\alpha)^2 \left\{p\left(1 - \frac{1}{e}\right) - c\right\}}{(2-\alpha)^2 k_x}$$
$$\frac{\partial x_B^*}{\partial y_B} = \frac{\partial x_B^*}{\partial y_A} = \frac{(1-\alpha) \{p' + p - c\}}{(2-\alpha)^2 k_x} = \frac{(1-\alpha) \left\{p\left(1 - \frac{1}{e}\right) - c\right\}}{(2-\alpha)^2 k_x}$$

where $e \equiv -\frac{p}{p'y}$ is the price elasticity of demand.

Thus, if the demand function is sufficiently price elastic, i.e. $e > p/(p-c)$ then we have the following results: (i) $\partial x_i^*/\partial y_i > 0$ implying that both types of effort spent by a firm are complements; (ii) $\partial x_i^*/\partial y_j > 0$ implying that both types of effort spent by two different firms are also complements. By contrast, if the demand is sufficiently price inelastic, i.e. $e < p/(p-c)$ then we have the following results: (i) $\partial x_i^*/\partial y_i < 0$ implying that both types of effort spent by a firm are substitutes; (ii) $\partial x_i^*/\partial y_j < 0$ implying that both types of effort spent by two different firms are also substitutes.

In general, competition and cooperation are considered as two polar opposites, that is, a higher level of cooperation naturally leads to a lower level of competition and *vice versa*. On the contrary, in the coopetitive game, the relation between competition and cooperation can be positive or negative depending on the price elasticity level of the demand. The following lemma summarizes these results.

Lemma 2 (relationship between competitive and cooperative efforts)**.**
If e is sufficiently large ($e > p/(p-c)$), then both types of effort are complements. In contrast, if e is sufficiently small ($e < p/(p-c)$), then both types of effort are substitutes.

At present, we analyze the first stage of the game. Substituting x_A^*, x_B^* into equations (1) and (2) yields:

$$U_A = \frac{1}{(2-\alpha)^2}(p-c)\,y - \alpha p y - k_y\left(y_A^2 + \alpha y_B^2\right) + \alpha \int_0^y p\,(q)\,dq$$

$$U_B = \frac{1}{(2-\alpha)^2}(p-c)\,y - k_y y_B^2$$

The first-order conditions with respect to y_A and y_B are:

$$\frac{\partial U_A}{\partial y_A} = \frac{1}{(2-\alpha)^2}p'y + \frac{1}{(2-\alpha)^2}(p-c) - \alpha p'y - 2k_y y_A = 0 \tag{6}$$

$$\frac{\partial U_B}{\partial y_B} = \frac{1}{(2-\alpha)^2}p'y + \frac{1}{(2-\alpha)^2}(p-c) - 2k_y y_B = 0 \tag{7}$$

To obtain interior solutions from equations (6) and (7), one can remark that the following condition must be satisfied: $p'y + p - c = p\left(1 - \frac{1}{e}\right) - c \geq 0$

From equations (6) and (7), the following proposition can be derived.

Proposition 1 (degree of cooperation)**.**
Define that $\bar{c} \equiv p\left[1 - \frac{1}{e}\left(1 - \alpha\,(2-\alpha)^2\right)\right]$ and $\underline{c} \equiv p\left(1 - \frac{1}{e}\right)$. Then, there are

three outcomes for cooperative effort levels corresponding to three different average cost levels.

1. *If average cost is high, i.e.* $c > \bar{c}$ *, both firms expend no cooperative effort. Moreover, equilibrium competitive effort levels are* $(x_A^*, x_B^*) = \left(\frac{(1-\alpha)^2(p(a)-c)a}{(2-\alpha)^2 k_x}, \frac{(1-\alpha)(p(a)-c)a}{(2-\alpha)^2 k_x}\right)$.

2. *If average cost is moderate, i.e.* $\underline{c} < c < \bar{c}$ *, only the semipublic firm expends cooperative effort. The private firm free rides.*

3. *If average cost is low, i.e.* $c < \underline{c}$ *then, both firms expend cooperative effort.*

Proof.
From equation (6), the following condition is necessary for $x_A^* > 0$: $c < \bar{c}$.
From equation (7), the following condition is necessary for $x_B^* > 0$: $c < \underline{c}$.
It is easy to verify the following inequality because $\alpha(2-\alpha)^2 \geq 0$: $\underline{c} < \bar{c}$.

From these above inequalities, all three cases in relation to average costs can be derived. If the inequality $c < \bar{c}$ is not satisfied, then the best strategy for both firms is to produce no output ($y_A = 0$ and $y_B = 0$). The equilibrium competitive effort levels can be derived by substituting $y_A = 0$ and $y_B = 0$ into equations (6) and (7). □

Proposition 1 has interesting implications. When α rises, while the semipublic firm behaves more like a public firm, it has ambiguous effect on private firm's behaviour as free rider. In fact, when $\alpha < 2/3$, the case 2 is more likely to arise because $\partial\bar{c}/\partial\alpha = p/e.(2-\alpha)(2-3\alpha) > 0$. This property implies that the semipublic firm behaves more like a public firm and the private firm is more likely to free ride. However, when $\alpha > 2/3$, $\partial\bar{c}/\partial\alpha < 0$, surprisingly, rising α will lead the private firm more likely to free ride. In the context of a quantity-setting oligopoly, De Fraja and Delbono (1990) show that the welfare may be higher when a public firm maximizes profits rather than the welfare. Their results suggest that, in some cases, a public firm should be privatized. In our coopetitive framework, privatizing public firms does not always mitigate the free-rider problem.

Moreover, the smaller is e, the more likely is the case 2 to arise because $\frac{\partial\bar{c}}{\partial e} = \frac{p\left(1-\alpha(2-\alpha)^2\right)}{e^2} < \frac{p}{e^2} = \frac{\partial\underline{c}}{\partial e}$. This implies that the lower is the price elasticity of demand, the more likely is the private firm to free ride.

Next, we try to make a general comparison of cooperative effort levels yields the following proposition.

Proposition 2 (cooperative effort levels)**.**
The private firm has no strategic incentive to spend more cooperative effort

than the semipublic firm. That is, $y_B^* \leq y_A^*$ with strict equality when $c > \bar{c}$ or $\alpha = 0$.

Proof.
In case 1, i.e. $c > \bar{c}$, both firms do not spend cooperative effort ($y_A^* = y_B^* = 0$). In case 2 and 3, just by subscribing (7) from (6), we get

$$\alpha p' y = 2k_y \left(y_B^* - y_A^*\right)$$
$$\Rightarrow y_B^* - y_A^* = \frac{\alpha p' y}{2k_y} \leq 0,$$

equality holds if and only if $\alpha = 0$. □

4 Concluding remarks

In this study, we developed a coopetition model of a mixed duopoly market. We built the model to describe a situation in which both public and private firms determine their levels of competitive effort to expand their market shares having chosen their cooperative effort levels to maximize the total market size.

Our results shown that the competitive effort level of the semipublic firm is below that of the private firm. The more concerned is the semipublic firm about social welfare, the less it competes. On the basis of average cost, we then identified the conditions under which only the semipublic firm expends cooperative effort while the private firm behaves as a free rider. Furthermore, we found that the semipublic firm expends more cooperative effort than does the private firm. Our analysis generates many insights of interest to the government, public and private firms. Regarding the government, it can use the privatization level as a mean to regulate the competitiveness of public firms and to mitigate free-rider problem. Our model offers an explanation of why public firms almost always contribute the most in cooperative associations with private firms but the former often have difficulties in competing with the latter.

However, our research is incomplete in several respects. Following two out of such several aspects are the most interesting and important. First, we assumed that both firms choose their effort levels simultaneously. However, in reality, semipublic firms may choose their cooperative effort levels before private firms do. For example, to increase social benefits, Japan Post developed and enlarged the potential demand for ordinary civilians who had little money. To represent this, a Stackelberg model may be more appropriate than a Nash one. Second, the extent to which the semipublic firm cares

about social welfare is implicitly assumed to be common knowledge. Thus, the private firm knows the extent to which the semipublic firm considers the social surplus. If there is some uncertainty about this and the private firm is risk averse, the private firm may change its cooperative and competitive effort levels.

These issues remain open to discussion. Much additional work is required by future researchers to improve our model. However, our results have important implications for coopetition in mixed oligopoly markets.

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