LABOUR SHARE DYNAMICS IN EUROPE: A TIME-HORIZON APPROACH

A. Arpaia and E. Pérez**

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Abstract

This paper seeks to understand labour share dynamics in Europe. An important point to clarify when discussing labour share movements is the time horizon over which these movements are observed. We consider three different time scales: the long run, the medium run and the short run. We start by documenting some basic empirical regularities of the labour share at the various time horizons. Although in the long run the share of national income accruing to labour is roughly constant, there is supportive evidence of large medium-term swings and significant movements at business cycle frequencies. We present a shift-share decomposition which illustrates the contribution of changes in the sectoral and employment composition of the economy to observed medium-term variations in the labour share. The findings from the shift-share analysis being on the descriptive side, we subsequently proceed to identify the fundamental factors underlying labour share movements through a model-based approach. Building on Bentolila and Saint Paul (2003), we present a solidly micro-founded expression to account for medium- and short-term movements of the labour share. As the sources of labour share movements can be expected to differ depending on the time horizon under consideration, matching labour share movements with the relevant time horizon in which they occur can be regarded as one of the main assets of the theoretical model. From an econometric perspective, the specification of the labour share can be regarded as a general model from which nested versions can be obtained by imposing various economically-meaningful restrictions. We estimate the general equation and various nested versions using EU KLEMS annual panel data for a sample of OECD countries over the period 1970-2004.

1. Introduction

The functional distribution concerns the distribution of income between production factors. The distribution of increases in output between the proprietors of the two main production factors, labour and capital, has occupied the attention of the profession for decades. It has also been the object of concern among policy makers and the public opinion.

The interplay between increases in output and factor shares can be regarded both from a long-run and a short-run perspective. From a long-run perspective, the relevant framework of analysis is provided by the theory of economic growth. From a short-run perspective, the proper analytical framework is provided by the theory of business cycles. The predominant view in the theoretical and empirical literature that focuses on these two extreme time horizons seems to be that movements in factor shares, if any, are of second-order importance. As a way of illustration take the Solow (1958) quotation: "Even if it is sometimes observed that the pattern of distributive shares shows long-run shifts and short-run fluctuations, the former can be explained away and the latter neglected in principle".

This paper looks at the functional distribution from the labour share perspective. On a secular basis, the widespread belief that the share of national income going to labour is nearly constant is deeply anchored in economists' minds. In the context of the theory of growth and capital accumulation, the constancy of the labour share is associated with models that possess a steady state. As is well known, the convergence property that characterizes the neoclassical growth model relies on the Cobb-Douglas production function. Alternatively, one may adopt the more general Constant-Elasticity-of Substitution (CES) technology coupled with the assumption that all technical progress is labour augmenting. Empirically, the status of "stylised fact" attributed to the labour share of income is confirmed by the few countries for which this data are available on a secular basis, namely, France, the UK and the US.

The conventional wisdom that oscillations in the labour share at business-cycle frequencies are irrelevant is more arguable. The increasing body of literature focussing on labour share movements in the short run proves that there is probably something to it. Empirical work has sought to identify the regularities affecting the cyclical behaviour the labour share, which are informative enough to suggest that one should cautious not to neglect short-run fluctuations in this variable.

In between the long and the short run there is the medium run, which is undoubtedly the most relevant period for policy makers and the public opinion, yet the most difficult to deal with from a theoretical perspective. To begin with, labour share movements over few decades may be rationalised in terms of the transitional dynamics of a neoclassical growth model, which is governed by, *inter alia,* the degree of substitution between production factors, the process of capital accumulation and the effect of technological progress, all of them operating at a time. On top of that, it is usually the case that product and labour markets work in an imperfectly competitive fashion over the medium run, which may provide additional explanatory power for labour share movements. One should finally bear in mind that worldwide institutional changes, such as the globalisation process, also matter in the medium term.

This paper seeks to understand labour share dynamics in Continental Europe. An important point to clarify when discussing labour share movements is the time horizon over which these movements are observed. Thus, Section 2 starts by considering three different time scales -the long run, the medium run and the short run- on the basis of which we document some basic empirical regularities of the labour share. We conclude that, although in the long run the share of national income accruing to labour is roughly constant, there is supportive evidence of large medium-term swings and significant movements at business-cycle frequencies. This leads us to compute a shift-share decomposition of the labour share in Section 3, which illustrates the contribution of changes in the sectoral and employment composition of the economy to observed medium-term variations in this variable. The findings from the shift-share analysis being on the descriptive side, in Section 4 we proceed to identify the fundamental factors underlying labour share movements at the various time horizons through a model-based approach. From an econometric perspective, one may see our specification of the labour share as a general model from which nested versions can be obtained by imposing various economically-meaningful restrictions. This endeavour is pursued in Section 5, which presents the estimates of the general equation and several of its nested versions using EU KLEMS annual panel data for EU15 countries over the period 1970-2004. Concluding remarks are presented in Section 6.

2. Empirical regularities

An important point to clarify when discussing labour share movements is the time horizon over which these movements are observed. As conventional in macroeconomics, one may consider three different time scales: the long run, the medium run and the short run.

Although any definition of time horizons on the basis of how variable the labour share is expected to be is too subjective, we may define the long run as a situation where factor shares in national income are roughly constant. The relative stability of the labour share of income has acquired the condition of a "stylized fact". Empirically, constant shares of value added accruing to production factors seem to materialise over various decades. However, due to lack of long data series, supporting evidence of constant labour shares over the long run is limited to few countries. This conventional wisdom is not too far from the pattern for France, UK and the US, as documented in Gollin (2002), Gomme and Rupert (2004), Gordon (2005), Piketty (2007), Piketty and Saez (2007), and Zuleta and Young (2007). The medium run may be defined as a situation where there are marked movements in the labour share, i.e., variations up to around 15%, usually taking place over periods as long as 10 or 20 years. There is a vast empirical literature that reports substantial medium-term swings of the labour share. Two such studies focussing on a large number of countries include Harrison (2003) and Jones (2003). The short run may be defined as a situation where changes in the labour share are of a business-cycle nature, with fluctuations no higher than a 2-3% ensuing from cyclical upturns/downturns.

This section first presents our preferred measure of the labour share. Then we proceed to document the medium-run stylised facts and cyclical properties of the labour share while taking for granted the constancy of the share of labour in the long run.

From the income perspective, the gross value added (GVA) of an economy at current basic prices is equal to the sum of compensation of employees, corporate profits, rental income, net interest income, the proprietors' income, and the capital depreciation. Of these income sources, compensation of employees is unambiguously labour income. In principle, computing the labour income share simply entails dividing compensation of employees by GVA at current basic prices, as in:

$$(1)\quad LS_t^{aggregate\ data} = \frac{CE_t}{GVA_t}$$

The main drawback of (1) is that it ignores the labour income of proprietors. National accounts do not identify separately the labour income of the self-employed, which is typically a mix of capital and labour. The consensus in the literature[1] is that this ambiguous income should be allocated to labour and capital in the same proportions they represent in the remainder of the economy. This simplifying assumption leaves us with the so-called "adjusted labour share":

$$(2)\quad ALS_t^{aggregate\ data} = \frac{CE_t}{GVA_t} * \frac{TE_t}{E_t}$$

where CE_t, GVA_t, TE_t, E_t respectively stand for compensation of employees, GVA at current basic prices, total employment and the employees of the economy. Expression (2) attributes to proprietors' income the average compensation of wage earners as remuneration of their labour[2]. Scaling up the average compensation of wage earners for the entire workforce will be a good adjustment to the extent that the self-employed command essentially the same wages as people

[1] See Gollin (2002).

[2] The correction of the labour share by attributing a certain proportion of the proprietors' income to labour was first discussed by Kravis (1962), who pointed out that entrepreneurial income as a share of GDP was shrinking over time as a result of long-term shifts in the structure of employment—away from agriculture and self-employment and into industrial wage labour. More recently, Gollin (2002) has argued that when labour shares are corrected to impute the labour income of the self-employed, the large differences in labour shares between rich and poor countries become much smaller.

who work as employees. On the contrary, it will be a poor assumption if there are systematic differences in earnings between employees and the self-employed. Askenazy (2003) has underlined that imputing the national average compensation to the self-employed distorts the measure of the labour share: as it stands, equation (2) can be expected to overestimate the income of the self-employed in the 1970s, when these non-employee workers were mainly farmers with low earnings. Symmetrically, this method can be expected to underestimate their income today, as a large part of these workers (doctors, lawyers…) earn more than the average employee. Therefore, a better estimate may easily be obtained by attributing to these workers the compensation of the average employee of their own activity branch (instead of the national average compensation). This methodological improvement leads to the following expression for the adjusted labour share:

$$(3)\quad \mathrm{A}LS_t^{\sec toral\ data} = \frac{\sum_{i=1}^{k} CE_{i,t} * TE_{i,t}}{\sum_{i=1}^{k} va_{i,t} * E_{i,t}} = \sum_{i=1}^{k} \frac{va_{i,t}}{GVA_t} * \frac{CE_{i,t}}{va_{i,t}} * \frac{TE_{i,t}}{E_{i,t}} = \sum_{i=1}^{k} \omega_{i,t} * aws_{i,t}$$

where for any economic sector *i*,, $CE_{i,t,} va_{i,t}, TE_{i,t}, E_{i,t}, aws_{i,t}, \omega_{i,t}$, respectively denote compensation of employees, gross value added at current basic prices, total employment, the employees*,* the adjusted labour share and the weight of the sector's value added in the value added of the whole economy. Employment is measured in headcounts, with no adjustment for hours worked. According to (3), the adjusted labour share is calculated as a weighted average of the adjusted labour share for each sector *i* in the economy, with sector shares in total value-added as weights.

We now proceed to explore empirical evidence on labour share patterns across EU15 countries according to the various measures discussed above. We use EU KLEMS data covering the period 1970-2004. The sectoral breakdown used in the analysis includes 24 sectors grouped into 9 broadly-defined industries (NACE code in brackets), namely, Agriculture, Hunting, Forestry and Fishing (A-B), Mining and Quarrying (C), Total Manufacturing (D), Electricity, Gas and Water Supply (E), Construction (F), Wholesale and Retail Trade (G), Hotels and Restaurants (H), Transport and Storage and Communication (I), Finance, Insurance, Real Estate and Business Services (J-K). Note that Community Social and Personal Services (L-Q) are excluded, as value added generated by these sectors is merely wage and salary income, so there is no genuine concept of labour share involved. In practical terms, including NACE categories L-Q in the analysis would result in an upward bias of labour's income.

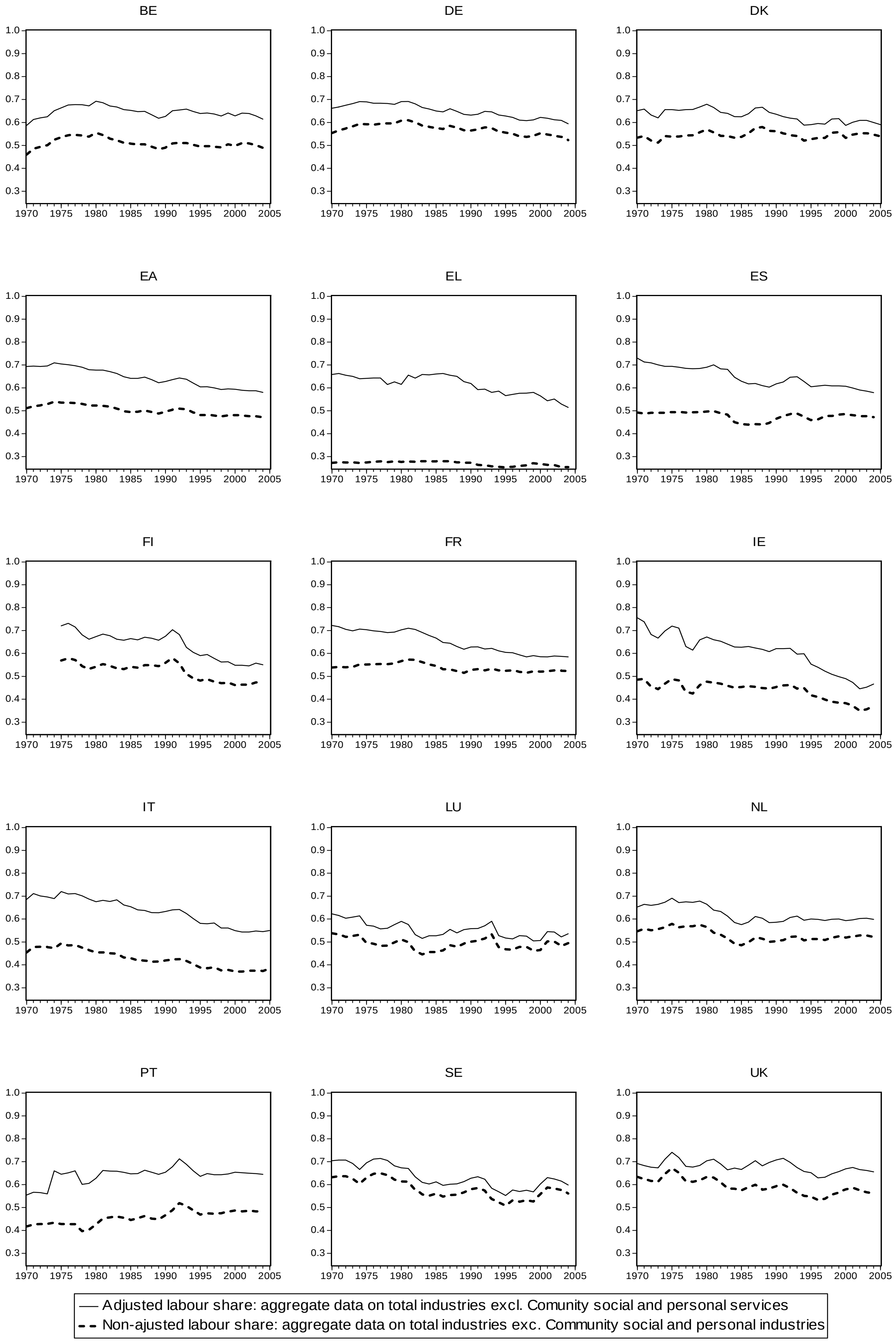
To see the effect induced by the imputation of labour income to the self-employed, Graph 1 compares non-adjusted and adjusted labour shares calculated on the basis of aggregate data on total industries excluding Community social and personal services. These series correspond to expressions (1) and (2) in the main text. The dashed line is the "naive" measure, constructed as compensation of employees over gross value added. The solid line incorporates the correction for the self-employment. Inspection of Graph 1 reveals that computing the labour share according to (2) results in an augmentation in the labour share. This obviously stems from the fact that there is always a certain amount of self-employed workers who provide labour services in the economy. We also learn from the data that such adjustment generally preserves the dynamic patterns in labour shares[3]. Self-employment as a proportion of employees has decreased markedly in Greece, Ireland, France and Spain. The UK stands out as the only country where the number of employees as a proportion of total workforce has actually shrunk, as illustrated by the increasing gap over time between non-adjusted and adjusted labour shares. In the remaining EU15 countries, the structure of employment in the whole economy has remained broadly the same. Conversely, the two series converge for countries experiencing a reduction of the share of self-employeed in agriculture.

Following Askenazy (2003), we subsequently compute labour shares by attributing to the self-employed the compensation of the average employee of their own activity branch, instead of the national average compensation. Graph 2 compares expressions (2) and (3) in the main text. The dashed line plots the adjusted labour share calculated on the basis of aggregate data whereas the solid line is the preferred measure, which incorporates the correction for the self-employment using sectoral data. Although the refinement does not seem to change the broad picture in several EU15 members, in various others Askenazy's alternative results in a downward revision of the labour share. Revisions are remarkable in Greece, quite sizeable in Spain, Italy and Portugal while more modest in France and Ireland. It is apparent that adjusting the labour share on the basis of aggregate data tends to largely overestimate the income of the self-employed in the 1970s in Greece, Spain and Italy. This is due to the fact that the agricultural population remained pretty large in 1970 in these countries, i.e., self-employed workers were mainly farmers with low

[3] Readers should be aware of the fact that Austria has been excluded from the analysis. This is because the imputation of labour income to the self-employed as implied by (2) results in an adjusted labour share exceeding one. This is due to the fact that the correction implied by (2) is not very reliable when the wages for the two types of employment largely differ, which is the case at stake. Specifically, in the case of Austria, equation (2) largely overestimates the income of the self-employed in the 1970s, when these non-employee workers were mainly farmers with low earnings. In this country, the share of employees in total employment in the Agriculture sector in 1970 was barely 6%, i.e., atypically low as compared with European standards. This measurement problem tends to be less troublesome when calculating the adjusted labour share on the basis of sectoral data, i.e. following expression (3) in the main text.

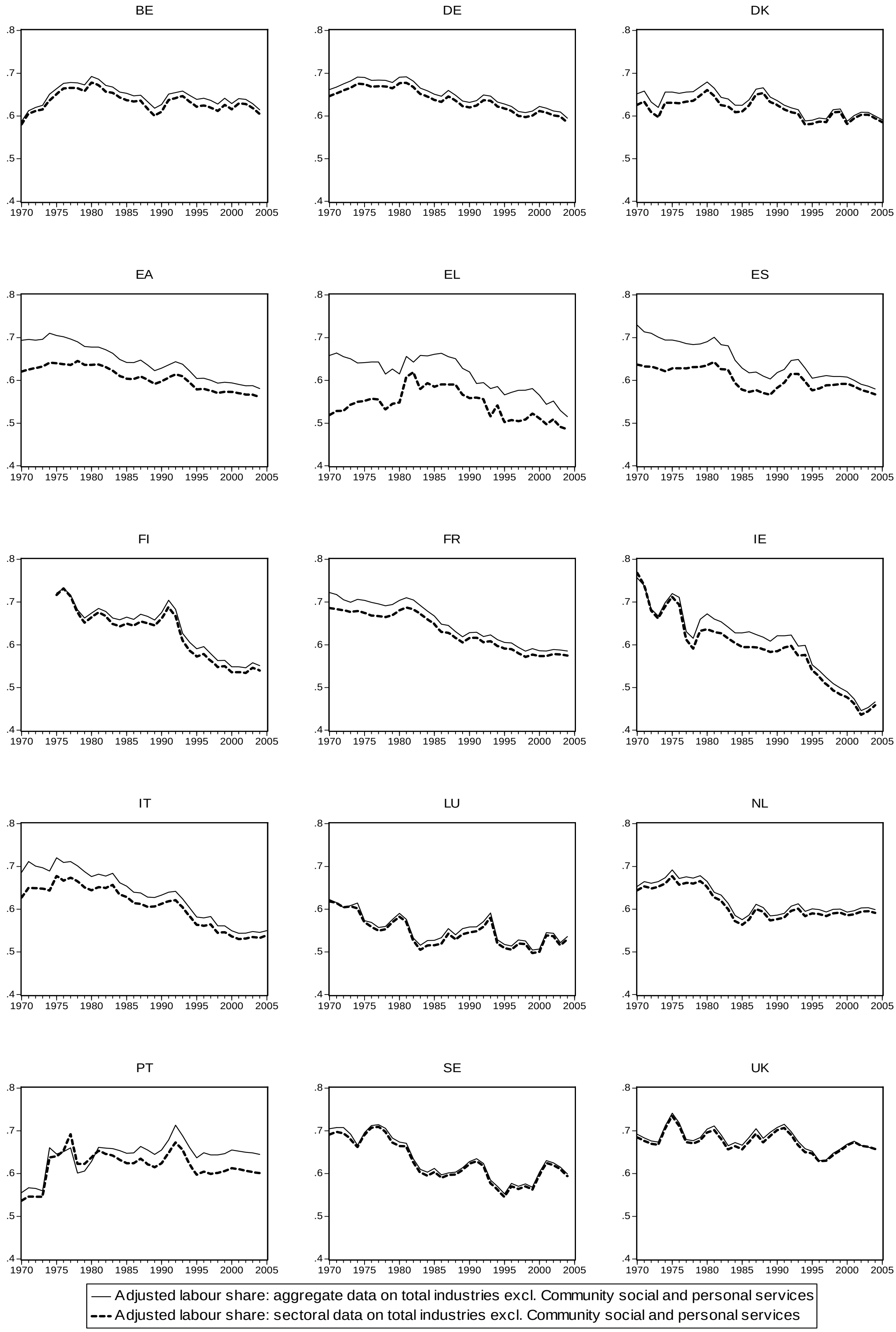
earnings. We interpret these results as a confirmation that imputing to the self-employed the national average compensation is a poor approximation when there are systematic and substantial differences in the earnings ability between employees and the self-employed.

Graph 1 - Non-adjusted versus adjusted labour share on the basis of aggregate data, EU15 Member States excl. Austria
Comparison of expressions (1) and (2) in the main text fed with EU KLEMS data, 1970-2004



Source: Commission services.

Graph 2 - Adjusted labour share on the basis of both aggregate and sectoral data, EU15 Member States excl. Austria
Comparison of expressions (2) and (3) in the main text fed with EU KLEMS data, 1970-2004



Source: Commission services.

3. Stylised Facts

We now document a few stylised facts present in our preferred measure of the labour share, as given by (3). Table 1 reports averages, the maximum and the minimum values and the coefficient of variation by country and by industry. It also displays the pp. variation of the labour share by country during the periods 1970-1985, 1986-1995 and 1996-2004.

In most countries the labour share reaches a peak in the early 1970s and a low in the late 1990s and early 2000s. Only in Belgium and Portugal was the labour share lower in 1970 than in the recent past. The coefficient of variation is highest in Ireland, where the adjusted labour share reached a high of 0,76 in 1970 and a low of 0,45 in 2002, followed by a considerable distance by Finland, Italy, Sweden, France and Greece[4]. The adjusted labour share was most stable in Belgium and the United Kingdom. In Spain, Ireland, Luxembourg, Netherlands, Austria and Sweden, the pp. fall in the labour share was most pronounced between 1970 and 1985. In Denmark, Greece and Italy the largest pp. decline in the labour share is registered during the period 1986-1995 whereas in Belgium and Germany the pp. reduction in the labour share has been highest over the last decade. The adjusted labour share varies more widely across industries than across countries, reflecting the importance of technological differences across industries: the range for the country's average adjusted labour share goes from 0,39 in Electricity, gas and water supply, to 0,77 in Agriculture, hunting, forestry and fishing.

[4] The coefficient of variation in Austria is biased upwards. In this country the imputation to the self-employed of the average compensation of wage earners in Agriculture, forestry, hunting and fishing results in an adjusted labour share exceeding one in this industry all over the sample. Given the relatively high share of the Agriculture in the value added of the whole economy in the early 1970s, the adjusted labour share calculated on the basis of (3) is close to one at the beginning of the sample. Although this measurement problem due to the imputation of labour income to the self-employed persists till the end of the period under consideration, it becomes of second order importance at the end of the sample, because of the decreasing economic weight of Agriculture in total value added. This explains the high value of the coefficient of variation in this country.

Table 1 - Medium-term stylised facts of the adjusted labour share in EU15 countries

Standard descriptive statistics on the adjusted labour share by country and by industry, EU KLEMS data, 1970-2004 (Finland 75-04)

Descriptive statistics of the labour share by country

Country	pp. change 70-85	pp. change 86-95	pp.change 96-04	Mean	Maximum	(year)	Minimum	(year)	Coefficient of variation (levels)
EU15	-0,03	-0,04	-0,01	0,65	0,70	1975	0,59	2004	5,48
Belgium	0,07	-0,01	-0,03	0,65	0,69	1980	0,59	1970	3,60
Germany	-0,01	-0,02	-0,03	0,65	0,69	1981	0,59	2004	4,53
Denmark	-0,03	-0,05	0,00	0,63	0,68	1980	0,59	2000	4,19
Greece	0,00	-0,10	-0,06	0,61	0,66	1971	0,51	2004	7,07
Spain	-0,10	-0,01	-0,03	0,65	0,73	1970	0,58	2004	6,78
Finland	-0,06	-0,07	-0,04	0,64	0,73	1976	0,55	2002	9,21
France	-0,05	-0,04	-0,02	0,65	0,72	1970	0,59	1998	7,56
Ireland	-0,13	-0,08	-0,07	0,61	0,76	1970	0,45	2002	13,65
Italy	-0,03	-0,06	-0,03	0,64	0,72	1975	0,54	2001	9,11
Luxembourg	-0,10	-0,02	0,02	0,55	0,62	1970	0,50	1999	5,99
Netherlands	-0,08	0,01	0,00	0,62	0,69	1975	0,58	1985	5,68
Austria	-0,23	-0,05	-0,07	0,77	0,99	1970	0,63	2004	11,92
Portugal	0,09	-0,01	0,00	0,64	0,71	1992	0,56	1970	5,42
Sweden	-0,09	-0,04	0,02	0,63	0,71	1977	0,55	1995	7,85
United Kingdom	-0,03	-0,03	0,03	0,68	0,74	1975	0,63	1996	3,63

Descriptive statistics of the labour share by industry

Industry	Mean	Maximum	(country)	Minimum	(country)	Coefficient of variation
Agriculture, hunting, forestry and fishing	0,77	0,97	Germany	0,50	Spain	45,52
Mining and quarrying	0,40	0,88	Germany	0,07	Netherlands	41,37
Total manufacturing	0,71	0,76	Sweden/UK	0,51	Ireland	9,99
Electricity, gas and water supply	0,39	0,56	Ireland	0,21	Sweden	22,69
Construction	0,74	0,92	Denmark	0,41	Greece	18,69
Wholesale and retail trade	0,75	0,84	France	0,55	Greece	12,84
Hotels and restaurants	0,76	0,97	Germany	0,46	Greece	20,36
Transport and storage and communication	0,70	0,80	nited Kingdon	0,55	Finland	9,05
Finance, insurance, real estate and business services	0,41	0,59	nited Kingdon	0,25	Greece	21,68

Source: Commission services.

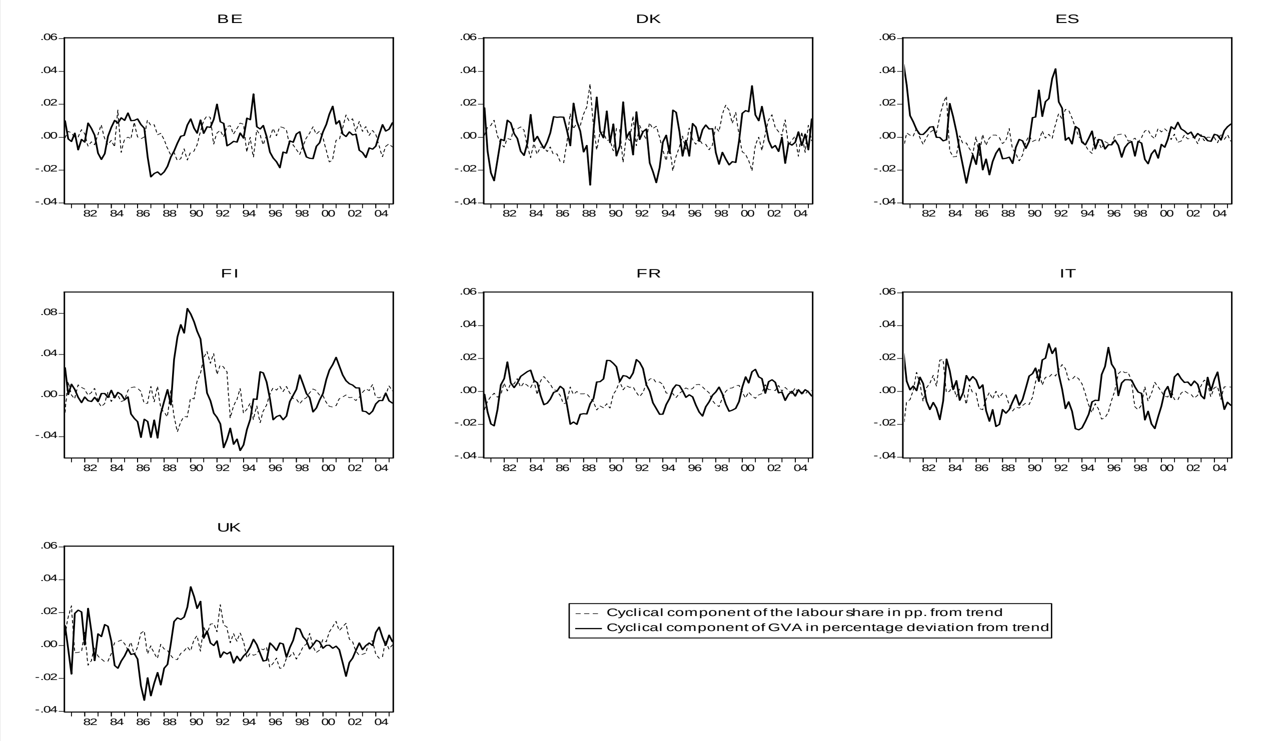
Maximum/minimum: maximum/minimum value of the adjusted labour share in pp.; coefficient of variation: standard deviation of labour share divided by mean, reported as a percentage.

Readers should be aware of the fact that descriptive statistics by industry exclude the observations of the labour share that exceed 1. This is the case of Agriculture, hunting , forestry and fishing in Austria and Portugal, Construction in Ireland and Hotels and restaurants in Belgium. This is due to the fact that the correction implied by (2) is not very reliable when the wages for the self-employed and the employees largely differ, which is the case at stake.

From a short-run perspective, Graph 3 plots the cyclical components of the labour share and gross value added. Table 2 displays some standard business-cycle statistics calculated on the basis of the HP-filtered GVA and labour share series. The data have been taken from the TRIMECO database and cover the period 1980Q3-2005Q2. Compensation of employees and GVA are seasonally and working day adjusted whereas the series of total employment and employees are not. Unlike the annual data used to describe the medium term movements of the labour share, the quarterly data used here are limited to a few countries. More fundamentally, lack of data on the public sector on a quarterly basis, we obtain the labour share corresponding to all industries in the economy. The statistics we look at are the maximum and minimum oscillation of the cyclical component of the labour share in the first two columns, the contemporaneous correlation of the cyclical component of the labour share with GVA in the third column, the standard deviation of the cyclical component of the labour share relative to the standard deviation of the cyclical component of the GVA in the fourth column, and the first autocorrelation of the cyclical component of the labour share in the fifth column.

Over the period 1980Q3-2005Q2, the share of gross value added accruing to labour has registered sizeable high frequency movements, especially in the 1980s. Moreover, the labour share is counter-cyclical, which reflects pro-cyclical productivity and nominal wages rigidity. As suggested by the fourth column, the standard deviation of the labour share is more than half of that of output in most countries. The labour share is quite persistent: the auto-correlation coefficient is above 50% in all cases. Perhaps more important is the phase-shift of these variables reported in Table 3. Before the peak of an expansion, the labour share is below average, with the negative correlation being largest two to one quarter before the peak of output. Subsequently, the labour share starts to increase quite above its mean, with its maximum peaking one year after output did, implying that the labour share lags output by one year or so.

Graph 3 - Cyclical components of the labour share and GVA in selected EU15 countries
Cyclical components calculated on the basis of HP-filtered GVA and labour share series, 1980Q3-2005Q2



Source: Commission services.
Readers should be aware of the fact that the scales of the graphs are uniform for all countries but Finland

Table 2 - Cyclical properties of the labour share in selected EU15 countries
Standard business-cycle statistics calculated on the basis of HP-filtered GVA and labour share series, 1980Q3-2005Q2

	Maximum	Minimum	Synchronization	Volatility	Persistence
Belgium	0,02	-0,01	-0,32	0,68	0,52
Denmark	0,03	-0,02	-0,68	0,78	0,54
Spain	0,02	-0,01	0,07	0,51	0,60
France	0,01	-0,01	-0,16	0,46	0,59
Italy	0,02	-0,02	-0,10	0,67	0,68
Finland	0,04	-0,04	-0,31	0,49	0,69
UK	0,02	-0,01	-0,31	0,64	0,64
Average			*-0,26*	*0,60*	*0,61*

Source: Commission services.
Maximum/minimum: maximum/minimum value of the cyclical component of the labour share in pp.; synchronization: contemporaneous correlation between the cyclical components of the labour share and gross value added; volatility: standard deviation of the cyclical component of the labour share relative to standard deviation of the cyclical component of the GVA; persistence: auto-correlation coefficient of the cyclical component of the labour share.

Table 3 - Phase-shift of the labour share in selected EU15 countries
Cross-correlations calculated on the basis of HP-filtered GVA and labour share series, 1980Q3-2005Q2

Cross-correlation of the cyclical component of contemporaneous GVA with the cyclical component of the labour share at different leads and lags											
	LS_{t-5}	LS_{t-4}	LS_{t-3}	LS_{t-2}	LS_{t-1}	LS_{t}	LS_{t+1}	LS_{t+2}	LS_{t+3}	LS_{t+4}	LS_{t+5}
Belgium	-0,11	-0,08	-0,05	-0,17	-0,23	-0,32	-0,05	0,12	0,33	0,45	0,56
Denmark	0,07	-0,11	-0,20	-0,24	-0,33	-0,68	-0,21	-0,05	0,14	0,30	0,31
Spain	0,03	0,16	0,24	0,21	0,19	0,07	0,27	0,34	0,42	0,42	0,33
France	-0,05	-0,02	-0,03	-0,06	-0,09	-0,16	0,02	0,20	0,39	0,49	0,55
Italy	-0,22	-0,11	-0,05	0,03	0,02	-0,10	0,10	0,26	0,36	0,47	0,51
Finland	-0,52	-0,54	-0,51	-0,47	-0,38	-0,31	-0,03	0,18	0,38	0,53	0,65
UK	-0,19	-0,18	-0,13	-0,11	-0,14	-0,31	0,00	0,15	0,19	0,25	0,30

Source: Commission services.

4. A shift-share decomposition of medium-term movements in the labour share

The current framework of wage moderation has been accompanied by declining labour share patterns, therefore giving rise to distributional concerns. However, declining labour share patterns do not stem exclusively from wage moderation. An obvious explanation for the decline in the labour share is that there may be changes at work in the sectoral composition and the employment structure of the economy. To illustrate this argument, this section pursues a shift-share analysis to decompose movements in the labour share.

Starting from the definition of adjusted labour share, its change can be split into 3 components: i) the "sectoral composition effect", ii) the "employment structure effect", and iii) the "industrial labour share effect", which measures changes in the adjusted labour share of the economy coming respectively from changes in: i) the sectoral composition of the economy, ii) the employment structure of the economy, and iii) the ratio of compensation of employees to value added at the industry level. According to the first effect, a shift from high-labour-share sectors to low-labour-share sectors will translate into an aggregate decline in the labour share, all other things being equal. According to the second effect, widespread reductions in the ratio of total employment to the number of employees across the various economic sectors will translate, all other things being equal, into a lower aggregate labour share, because of a lower level of compensation per employee being imputed to a higher level of self-employed. According to the third effect, generalised reductions in the ratio of compensation of employees to value added across the various economic sectors will translate into a lower labour share for the economy as a whole, all other things being equal. In symbols,

$$(4)\quad \Delta ALS_t^{\text{sectoral data}} = \sum_{i=1}^{k}\left[\underbrace{\frac{CE_{i,t}}{va_{i,t}} * \frac{TE_{i,t}}{E_{i,t}} * \Delta\omega_{i,t}}_{\text{Sectoral composition effect}} + \underbrace{\omega_{i,0} * \frac{TE_{i,0}}{E_{i,0}} * \Delta\frac{CE_{i,t}}{va_{i,t}}}_{\text{Industrial labour share effect}} - \underbrace{\frac{CE_{i,t}}{va_{i,t}} * \frac{1}{q_{i,t}} * \omega_{i,t} * \frac{\Delta q_{i,t}}{q_{i,0}}}_{\text{Employment structure effect}}\right]$$

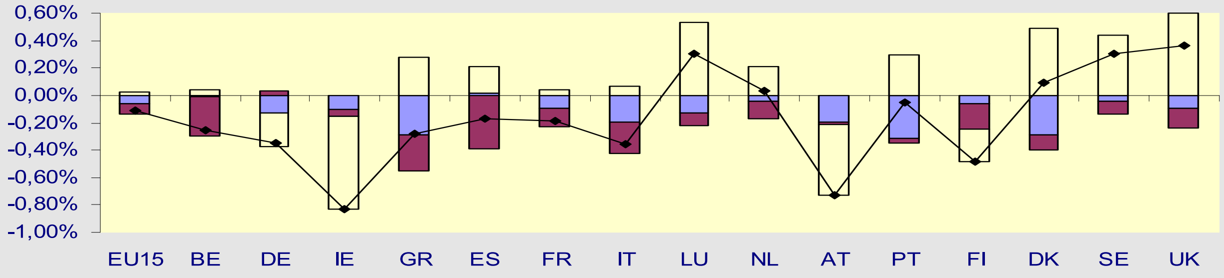
with $q_{i.t} = \dfrac{E_{i,t}}{TE_{i,t}}$.

It therefore becomes as obvious that changes in the employment structure (reductions in the ratio of total employment to the number of employees) and in the sectoral composition of the economy, which materialise over the medium-term, influence the trend in the labour share.

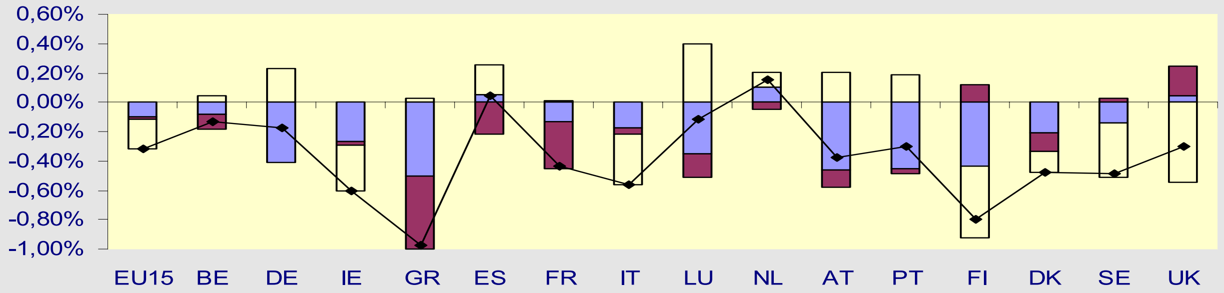
This decomposition is performed for three selected sub-periods, namely 1970-1985, 1986-1995 and 1996-2004 (Graph 4). Notwithstanding the complexity and heterogeneity of labour share movements across countries, one may identify some common patterns in the data: i) Over the period 1970-2004, the sectoral and the employment composition effects have both contributed to a reduction in the aggregate labour share, and ii) the effect of the industrial labour share effect has been most sizeable during the sub-periods 1970-1985 and 1996-2004. Whether this latter effect has contributed to a downward rather than an upward movement in the aggregate labour share strongly depends on the country under consideration.

The shift-share analysis reveals the importance of structural forces in driving aggregate labour share movements. To illustrate this more clearly, we construct a counterfactual adjusted labour share (expression 3) calculated for sectoral and employment composition at 1970 levels. This allows disentangling the industrial labour share component from the other two structural sources of labour share movements. The main conclusion is that, when the sectoral and the employment composition of the economy are kept constant, the labour share takes a higher value and its declining pattern is notably less so. For the EU15, for instance, the observed decline in the labour share was 5.95% over this period. Our calculations show that if there had been no change in the sectoral and employment structure of the economy, this decline would have been 2.56%.

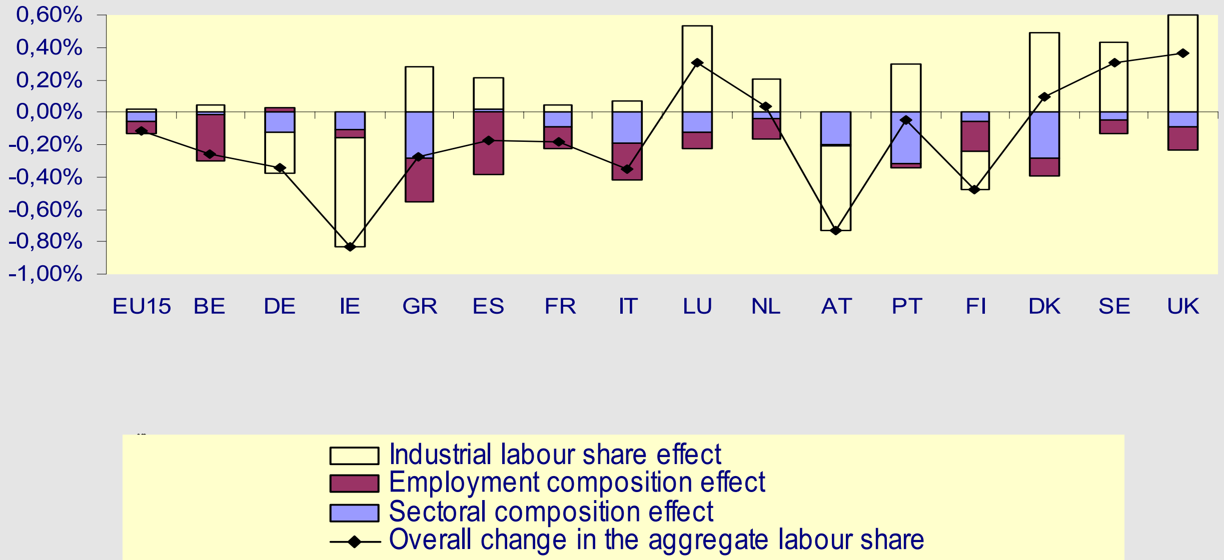
Graph 4 – Sources of changes in the labour share, EU15 countries
Average annual percentage change 1970-1985



Graph 5 – Sources of changes in the labour share, EU15 countries
Average annual percentage change 1986-1995

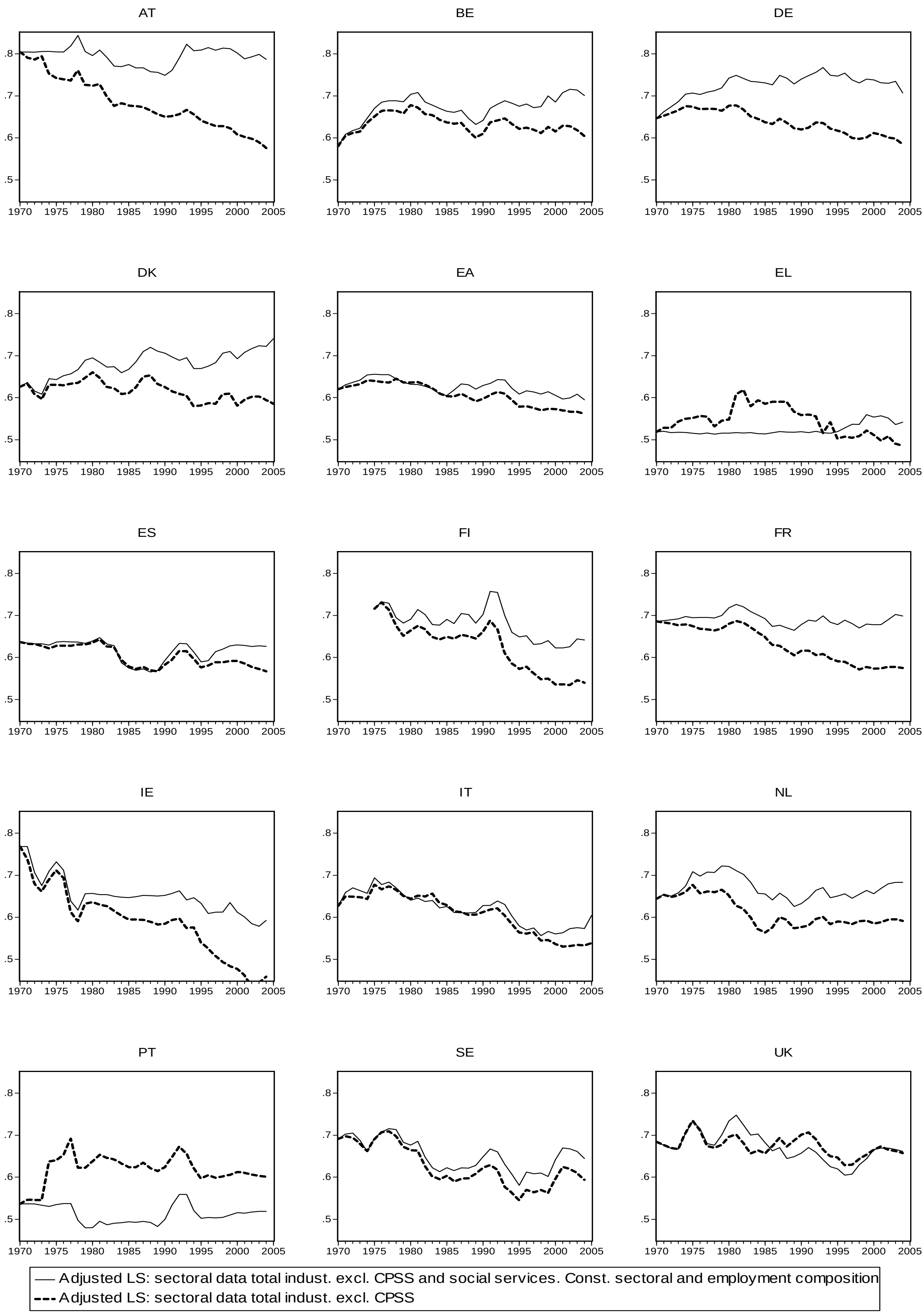


Graph 6 - Sources of changes in the labour share, eu15 countries
Average annual percentage change 1996-2004



Graph 7 – Adjusted labour share (dashed line) **versus alternative Adjusted labour share measure for given sectoral and employment composition at 1970 levels** (solid line)**, EU15 Member States excl. Luxembourg**

Comparison of expression (3) in the main text (dashed line) with an alternative measure of the adjusted labour share where sectoral and employment composition are kept constant at their prevailing levels in 1970 (solid line)



Source: Commission services.

5. Theoretical model

5.1. Methodological approach

The shift-share analysis is just a description of the interplay within different components. In this section, building on Bentolila saint Paul (2003) we review the ultimate factors underlying labour share movements. This will be addressed in the following section. The identification of such factors requires a model-based approach.

The aim of our methodological approach is twofold. First, we come up with an expression for the labour share which is solidly micro-founded. Second, from an econometric perspective, we wish the specification of the labour share to be the most general possible, so that nested versions can be obtained by imposing various economically-meaningful restrictions. These two objectives require matching labour share movements with the relevant time horizon in which they occur, as the sources of labour share movements can be expected to differ depending on the time horizon under consideration. An important point to clarify when discussing labour share movements is, therefore, the time horizon over which these movements are observed. As is conventional in macroeconomics, one may consider three different time scales: the long run, the medium run and the short run. The modelling strategy developed in this section is an attempt to identify the sources of labour share movements operating at different time scales.

5.2. The labour share in the long run

A first important observation is that shares of value added accruing to labour show no secular trend. In the context of the theory of growth and capital accumulation, constant labour shares are associated with models that possess a steady state. In turn, there are two possibilities for the neoclassical growth model to deliver a steady-state solution: either the production function is Cobb-Douglas or one may adopt a Constant-Elasticity-of- Substitution (CES) production function coupled with the assumption that all technical progress is purely labour augmenting[56].

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[5] The assumption of labour-augmenting technical progress implies that technical progress only increases the efficiency of labour and does not affect the efficiency of capital: overtime a constant amount of output can be produced with a constant amount of capital and a decreasing amount of labour. This implying that the labour output ratio decreases over time.

[6] See Barro and Sala-i-Martin, 2003, pp. 78-80. The intuition behind the proof is that there are two ways of getting a steady state, either the neoclassical production function takes a CES form and all technical progress is labour augmenting, or the production function takes the Cobb-Douglas form. Recall that if the production function is

The competition between these two alternatives has become more obvious in two recent papers. Lack of evidence for a fading away of capital-augmenting technical change, Jones (2003) contends that the long-term production function is Cobb-Douglas. Conversely, in a recent paper Klump *et al.* (2004) have found that the elasticity of substitution is significantly below unity and that the growth rates of technical progress are biased towards labour.

While estimates of the elasticity of substitution between labour and capital range widely, the weight of the evidence seems to support a value of the elasticity in the range of 0.4 and 0.6 (Chirinko, 2008).This result therefore supports Acemoglu (2003) view that technical progress is purely labour augmenting in the long run, thus advocating for the use of CES production function.

Unlike the Cobb-Douglas technology, the CES production function can deliver fluctuations in the labour share over the medium term, i.e., along the transitional dynamics. Thus, in what follows, the CES production function with labour-augmenting technical progress will be adopted, as it is consistent not only with the long-run stability of factor shares[7], but also with medium-term swings. In symbols, technological possibilities are given by:

$$(1)\ Y = \left[\alpha K^{(\sigma-1)/\sigma} + (1-\alpha)(BL)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}$$

where *Y, K, B,* and *L* are value added, capital services, labour-augmenting technical progress, and labour services. For this production function it holds that $0 < \sigma < \infty$ [8] and $0 < \alpha < 1$ where

Cobb-Douglas, we can always express technological change –whatever its nature as capital augmenting, labour augmenting or total factor productivity- as purely labour augmenting.

[7] It should be noted that labour-augmenting technical progress is a necessary, though not sufficient condition, for the CES production function to generate the standard neoclassical convergence property. To be more specific, it can be shown that if there is a high degree of substitution between capital and labour, i.e., if σ > 1, the convergence property requires the saving rate to be sufficiently low. If the saving rate does not satisfy the key condition, the CES model will generate endogenous, steady-state growth. Dynamics of this model will be similar to the *AK* model, not the standard neoclassical growth model. Conversely, it can be shown that if there is a low degree of substitution between capital and labour, i.e., if σ < 1, the convergence property requires the saving rate to be sufficiently high. If the saving rate does not satisfy the key condition, the capital stock will decline continuously until a trivial equilibrium at $(K/BL)^* = 0$ is obtained (see Barro and Sala-i-Martin, 2003, pp. 68-71 for a formal proof). In what follows, we will assume that, whatever the degree of substitution between capital and labour, the key condition is satisfied, so that the CES production function delivers the convergence result that characterises the standard neoclassical growth model.

[8] Note that in a two-factor economy, the possibility of σ < 0 is naturally excluded. This means that, if, for instance, there is an increase in the relative price of labour, the capital-labour ratio will, at most, remain constant for any given level of output. Put differently, this movement in relative prices cannot possibly cause a reduction in the capital-labour ratio for any given level of output.

σ denotes the elasticity of substitution between labour and capital, i.e., how the factors' demand change with their relative price. The CES technology encompasses several well-known production functions, depending on the value of the parameter σ[9]*:* i) The Leontieff production function (*σ = 0),* illustrates the case where there is no substitution between labour and capital*;* ii) The Cobb-Douglas production function (*σ = 1),* which illustrates the case where the capital-labour ratio responds positively and proportionally to an increase in the relative price of labour*;* iii) The linear production function (*$\sigma = \infty$)*, which illustrates the case where capital and labour are perfect substitutes. The main focus of the paper is on the two dense regimes in between these extreme cases: iv) $0 < \sigma < 1$, which illustrates the case where the capital-labour ratio responds positively and less than proportionally to an increase in the relative price of labour, implying a low degree of substitution between capital and labour (or complementarity between capital and labour); and v) $1 < \sigma < \infty$, which illustrates the case where the capital-labour ratio responds positively and more than proportionally to an increase in the relative price of labour, implying a high degree of substitution.

We will refer to α as "the constant attached to capital", instead of sticking to the conventional expression "the distribution parameter". The latter term reflects the fact that, if the production function is Cobb-Douglas (*σ = 1*), labour and capital factor shares are constant and respectively equal to α and ($1-\alpha$), either along the transitional dynamics or the steady-state. However, in the more general CES case adopted here, not only α, but also σ, are distribution parameters: σ matters for the dynamics off the steady state, i.e., during the period over which capital accumulation is at work, whereas the two of them, α and *σ,* jointly determine the steady-state level of factor shares. To see this more clearly, it suffices to derive the expression of the labour share consistent with the production function described in (1).

If labour market is perfectly competitive, profits' maximising firms equate the real wage to the marginal productivity of labour, i.e. $w^{PC} = MPL$. Thus, the labour share is

$$(3) \quad LS^{PC} = \frac{L * MPL}{Y}$$

And with a labour augmenting CES production function, it takes the form:

$$(4) \quad LS_{LATP}^{PC} = 1 - \alpha \left(\frac{K}{Y} \right)^{(\sigma-1)/\sigma}$$

[9] See Varian (1992) pp. 19-20 for a formal proof.

When $\sigma=1$ *we get* a Cobb-Douglas[10], and the labour share is a constant given by ($1-\alpha$). Thus, the theoretical constancy of factor shares at all frequencies results from assuming a Cobb-Douglas technology with constant coefficient α and maintaining the connection between factor prices and their respective marginal productivity. When $\sigma \neq 1$, provided that all technical progress is labour-augmenting, there exists a steady-state solution for the labour share, LS^*_{LATP}, whose value depends on the steady-state level of the capital-output ratio and the values of the parameters α and σ. Furthermore, we will show in Section 2.1.1 below that, along the transitional dynamics, the average productivity of capital will decrease (K/Y will increase) and the labour share will raise (decline), if there is a low (high) degree of substitution between capital and labour, i.e., if $0 < \sigma < 1$ ($1 < \sigma < \infty$). To get an intuition, consider that when the elasticity of substitution is high it is possible to change greatly the factor proportions in response to a change in their relative price. Thus, in response to an increase in the price of labour relative to that of capital, it is "easier" to change the relative capital-labour ratio when the elasticity of substitution is high and still produce the same amount of output. Due to the concavity of the production function the wage share falls. A symmetric argument is valid when $\sigma>1$.

Beyond accounting for labour share movements in the medium term, the adoption of a CES specification is further justified by the fact that the elasticity of substitution may be expected to vary across sectors to reflect specific technical and institutional features. De La Grandville (1989) regards the elasticity of substitution σ as "a measure of the efficiency of the productive system". As pointed out by Hicks (1963), in a multi-sectoral setting, technical substitution between factors of production can take place through inter- and intra-sectoral factor reallocations, and the application of new methods of production in one sector. On the other hand, the elasticity of substitution is also influenced by the institutional framework. Possible institutional determinants are, according to Klump and Preissler (2000), competition on good and labour markets, openness to trade, and institutions promoting knowledge spillovers. For instance, the absence of public regulations preventing intra- and inter-sectoral reallocations can be conjectured to be associated with high elasticities of substitution. Openness is thoroughly discussed in Ventura (1997), who has shown that a small country open to international trade can be modelled as possessing a linear aggregate production function ($\sigma = \infty$). More generally, globalisation is claimed to increase the elasticity of labour demand with respect to the real wage (see OECD, 2007), with the value of this elasticity obviously depending on σ. Finally, Weder and Grubel

[10] For a formal proof, see Sala-i-Martin (2003), pp. 80-81, or Varian (1992) pp. 20.

(1993) claim that industry-wide research associations can also cause high elasticities of substitution, as they favour knowledge spillovers which result in new methods of production.

Overall, in order to account for secular trendless labour shares, it will be assumed that the production function is given by a CES with labour-augmenting technical progress. This specification is consistent with the long-run constancy of factor shares, with $1-\alpha\left(K^*/Y^*\right)^{(\sigma-1)/\sigma}$ standing for the share of value added accruing to labour.

5.3. The labour share in the medium run

A second important observation is that there are large fluctuations in the shares of value added accruing to labour in Continental Europe over the past few decades. The subsequent analysis will explain to what extent technology, market structure in the products and the labour market, the institutional framework and globalisation forces contribute to explain medium- term variations in the labour share. All these aspects are addressed in separate sections, except globalisation, which is treated in several sections at a time. Details on algebra are provided in Appendix 1.

The starting point of the modelling approach we adopt is the theorem of XX, which reads as follows: "If technology is Cobb-Douglas and factor prices are competitive, then factor shares are constant". In order to account for medium-term labour share movements (i.e., along the transitional dynamics) one may therefore propose models that change technology and/or break competitive factor markets. Following Bentolila and Saint-Paul (2003), we first show that the assumption of a CES technology with labour-augmenting technical progress results in a stable relationship between the labour share and the capital-output ratio. This setting can deliver either increasing or decreasing labour shares along the transitional dynamics depending on the interaction between capital deepening and labour-augmenting technical progress. We then consider three factors that shift this stable relationship: capital-augmenting technical progress, labour heterogeneity and the introduction of intermediate inputs in the production function. Finally, we abandon the perfect competition assumption in the products and the labour market. Breaking the connection between factor prices and their respective marginal productivity is shown to have an additional explanatory power to account for medium-term labour share movements.

4.3.1. Technology

4.3.1.1. The CES production function with labour-augmenting technical progress

Consider that at any time *t,* for each industry *i,* technological possibilities are given by a production function like (1). Then the behaviour of the labour share off the steady state implied by the neoclassical growth model with labour-augmenting technical progress satisfies the following two expressions (for the sake of simplicity, we drop the time and industry indexes *t* and *i*):

$$(4) \quad LS^{PC}_{LATP} = \frac{L*w^{PC}}{Y} = \frac{L*MPL}{Y} = 1-\alpha\left(\frac{K}{Y}\right)^{(\sigma-1)/\sigma}$$

Alternatively

$$(5) \quad LS^{PC}_{LATP} = \frac{L*w^{PC}}{Y} = \frac{L*MPL}{Y} = \frac{(1-\alpha)}{\alpha\left(\frac{K}{BL}\right)^{(\sigma-1)/\sigma}+(1-\alpha)}$$

Equation (4), already presented in the previous section, and equations (5) are essentially the same relationship.[11] They represent two different ways of looking at the labour share, either through changes in the capital-labour ratio measured in efficiency units, or through changes in the capital-output ratio. Indeed, there is a monotonic relationship between these two variables (Appendix 1):

$$(6) \ \frac{K}{Y} = \left[\alpha+(1-\alpha)\left(\frac{K}{BL}\right)^{(1-\sigma)/\sigma}\right]^{\sigma/(1-\sigma)}$$

Thus, changes in the capital-output ratio reflect changes in the capital-labour ratio triggered by variations in factor endowments, in the relative factor prices and/or by changes in the labour-augmenting technical progress. These changes do not affect the stability of the relationship between the labour share and the capital-output ratio.

The impact of the capital-output ratio on the labour share depends on the elasticity of substitution between capital and labour. We show in Appendix 2 that increases in the capital output ration (i.e. reductions in the average productivity of capital) come along with increasing (decreasing) labour shares if $\sigma < 1$ ($\sigma > 1$, i.e. if there is a low (high) degree of substitution between capital and labour. In symbols, $\partial LS / \partial(K/Y) < (>) 0$ if $\sigma > (<) 1$

[11] Although equations (4) and (5) are essentially the same relationship, equation (5) is not easy to estimate, as it requires computing *B,* i.e., labour-augmenting technical progress. By contrast, from an empirical point of view, the main virtue of equation (4) is that it expresses the labour share as a function of the observable capital-output ratio.

In equation (5) the labour share is expressed as a function of the capital-labour ratio, with labour measured in efficiency units.. We show in Appendix 2 that $\partial LS / \partial (K/L) > 0$ if $\sigma < 1$, $\partial LS / \partial (K/L) < 0$ if $\sigma > 1$, i.e., all other things being equal, capital deepening along the transitional dynamics comes along with increasing (decreasing) labour shares if there is a low (high) degree of substitution between capital and labour. We also show that $\partial LS / \partial B > 0$ if $\sigma > 1$ and $\partial LS / \partial B < 0$ if $\sigma < 1$, i.e., all other things being equal, labour-augmenting technical progress comes along with increasing (decreasing) labour shares if there is a high (low) degree of substitution between capital and labour.

However, in the real world capital deepening and labour-augmenting technical progress take place simultaneously so that the *ceteris paribus* clause does not apply. In detail, with $\sigma < 1$ ($\sigma > 1$) the labour share increases over time if the capital-labour ratio grows faster (slower) than labour-augmenting technical progress. Intuitively, when capital grows faster than labour measured in efficiency units, the smaller the elasticity of substitution between capital and efficient labour, the higher the increase in the relative price of labour following capital accumulation. As such, the price effect –i.e., an increase in the relative price of labour-, will dominate the quantity effect –i.e., an increase in the capital-labour ratio measured in efficiency units- if the substitution elasticity is below one, so that the labour income share increases. Conversely, in the case of an elasticity of substitution larger than one, the quantity effect will be stronger than the price effect and the labour income share will decrease when the capital-to-labour ratio measured in efficiency units increases.

In a nutshell, we learn from equation (5) that the neoclassical growth model can deliver either increasing or decreasing labour shares along the transitional dynamics. It all depends on the interaction between capital deepening and labour-augmenting technical progress. In turn, this interaction is governed by the elasticity of substitution between capital and labour. Following Bentolila and Saint Paul, there is a stable one to one relationship between the wage share and the capital-output ratio: $LS_{LATP}^{PC} = g(K/Y)$.

4.3.1.2. Capital-augmenting technical progress

The incorporation of capital-augmenting technical progress to the CES technology displaces the relationship between the labour share and the capital-output ratio. It also causes shifts in the stable relationship between the capital-output ratio and the capital-labour ratio measured in efficiency units. To this aim, let us assume that the production function is now given by:

$$(7)\ Y = \left[\alpha (AK)^{(\sigma-1)/\sigma} + (1-\alpha)(BL)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}$$

where capital-augmenting technical progress A also enters the CES production function. In this case, the labour share is equal to:

$$(8)\ LS_{LCATP}^{PC} = 1 - \alpha \left(\frac{AK}{Y}\right)^{(\sigma-1)/\sigma}$$

where LS_{LCATP}^{PC} is the labour share calculated under the assumption of perfect competition and a CES production function with labour- and capital-augmenting technical progress like. The comparison of equations (4) and (8) illustrates that, unlike the case where all technical progress is labour-augmenting, capital-augmenting technical progress causes shifts in the relationship between the labour share and the capital-output ratio. In detail, capital-augmenting technical progress has a direct impact on the labour share, as reflected by the term $A^{(\sigma-1)/\sigma}$. One can conclude further from equation (8) that capital-augmenting technical progress has an indirect impact on the labour share through its influence on the capital-output ratio, which itself depends on *A,* as indicated by:

$$(9)\quad \frac{K}{Y} = \left[\alpha A^{(\sigma-1)/\sigma} + (1-\alpha)\left(\frac{K}{BL}\right)^{(1-\sigma)/\sigma}\right]^{\sigma/(1-\sigma)}$$

Similarly, unlike the case where all technical progress is labour-augmenting (eq. 5), capital-augmenting technical progress alters the relationship between the labour share and the capital-labour ratio in efficiency units. Indeed, substituting the capital-output ratio according to (9) into (8) yields:

$$(10)\quad LS_{LCATP}^{PC} = \frac{(1-\alpha)}{\alpha\left(\frac{AK}{BL}\right)^{(\sigma-1)/\sigma} + (1-\alpha)}$$

Thus for a given capital labour ratio, capital-augmenting technical progress will decrease the labour share as long as there is a high degree of substitution between capital and labour, i.e

$\partial LS_{LCATP}^{PC} \big/ \partial A < 0$ if $\sigma > 1$; conversely with capital-labour complementarity, the labour share rises in response to capital augmenting technological progress, i.e. $\partial LS_{LCATP}^{PC} \big/ \partial A > 0$ if $\sigma < 1$[12]. In addition, assuming *A* constant, capital deepening will decrease (increase) the labour share as long as there is a high (low) degree of substitution between capital and labour, i.e. $\partial LS_{LCATP}^{PC} \big/ \partial K/BL < (<) 0$ if $\sigma > (<) 1$.

We therefore learn from comparative statics that the effects of capital-augmenting-technical progress and the capital-output ratio measured in efficiency units on the labour share have the same sign.

As shown in this section, the assumption of labour-augmenting technical progress results in a monotonic relationship between the labour share and capital deepening. The incorporation of capital-augmenting technical progress to the production function alters this stable relationship and provides additional explanatory power for medium-term labour share movements. Although the assumption of labour-augmenting technical progress has been more common in macroeconomics, insofar it is compatible with a balanced-growth path and thus, consistent with trendless factors shares in the long run, the possibility of capital-augmenting technical progress needs to be considered in the medium run. Moreover, as shown by Acemoglu (2003), it is possible to reconcile capital-augmenting technical progress as a medium run phenomenon with purely labour-augmenting technical as a long run economic growth factor.

4.3.1.3. Labour heterogeneity

It has been assumed so far that the workforce is homogeneous. It is often argued, though, that both skilled and unskilled labour enter the production function in a way such that there is less substitution between skilled labour and capital than between unskilled labour and capital. Indeed, a related empirical literature has demonstrated that physical capital and skilled labour have been relatively complements in the past two centuries and are still so today. Goldin and Katz (1996) show that economy-wide capital and skilled labour complementarity emerged as a result of the

[12] see Appendix 2 for derivation.

adoption of several crucial technological advances, including the shift from the factory to continuous-process or batch methods, with electrification and the adoption of unit-drive machines reinforcing the change through the automation of hauling and conveying operations. Moreover, the capital-skilled labour complementarity is believed to be in full blossom today with ICT developments having a skill-biased component. Caselli and Coleman (2001), present robust findings that high levels of educational attainment are important determinants of computer-technology adoption. Krusell *et al.* (2000) show that capital-skill complementarity can be the source behind the increased of the US skilled premium. The growth in the stock of capital equipment combined with different degree of substitution with skilled and unskilled labour services raises the marginal product of skilled relative to high skilled people jointly with an increase in their relative labour supply. Briefly, empirical research indicates that new technologies tend to substitute for unskilled labour in the performance of routine tasks, while assisting skilled workers in executing qualified work.

Following Krussel et al., labour heterogeneity is introduced assuming that output is produced with unskilled labour and a composite capital made of imperfectly substitutable physical capital and skilled labour. Such array of production possibilities is ensured by the following "two-level CES production technology" (Sato 1967).[13] :

$$Y=\left[\alpha X^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(B_u L_u)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

(11) where

$$X=\left((AK)^{\frac{\eta-1}{\eta}}+(B_s L_s)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

In our notation, L_u and L_s stand for unskilled and skilled labour and B_u and B_s for their relative efficiencies. Two pair-wise elasticities are present in this technology. η is the elasticity of substitution between the two capital goods; σ is the elasticity of substitution between the composite capital and the unskilled labour services which is always positive ($0<\sigma<\infty$). Thus, an increase in the relative price of skilled labour (w_s/w_u) will trigger some substitution between the composite capital input and the skilled labour. This production function has the desirable properties that the (Allen partial) elasticity of substitution between skilled and unskilled is the

[13] Papageorgiou and Saam (2005) discuss the sufficient conditions for the existence of a steady state solution with such a production function embedded in the neoclassical growth model. In this work, we will take for granted that (11) enables a long-run steady state solution characterized by constant factor shares.

same as the elasticity between capital and unskilled (Sato, 1967). Conversely, the (Allen partial) elasticity of substitution between capital and skilled labour depends on the substitution effect between the two capital inputs and between unskilled labour and the composite capital. As in Krussel et al. (2000), we assume complementarity between capital and skilled labour, meaning that elasticity of substitution between capital and unskilled labour is higher than between capital and skilled labour (i.e. η<σ).[14]

In the appendix it is shown that the labour share equals:

$$(12)\ LS^{PC}_{LCATP,LH} = \frac{w_u^{PC} L_u + w_s^{PC} L_s}{Y} = 1 - (Ak)^{\rho} \left\{ \alpha^{\varepsilon} + (1-\alpha)^{\varepsilon} l^{\frac{\varepsilon-\sigma}{\sigma}} \omega^{\varepsilon-1} \right\}^{\frac{\sigma\rho}{\varepsilon(\sigma-1)}}$$

where $k = \frac{AK}{Y}$ is the capital-ouput ratio in efficency units; $\omega = \frac{w_s^{PC}}{B_s} / \frac{w_u^{PC}}{B_u}$ is the wage premium expressed in efficiency units; $l = \frac{B_s L_S}{B_U L_U}$ relative supply of labour services; $\rho = \frac{\eta - 1}{\eta}$; $\varepsilon = \frac{\sigma\rho}{\sigma(\rho-1)+1}$ a parameter depending on the technical parameters of the production function. Similarly to the case of homogeneous labour, the labour share move along a stable non-linear relationship with the capital-output ratio. Labour heterogeneity introduces a shift factor, which depends on the relative supply of labour services and the wage premium.

In appendix 2 it is shown that when the quantities of the two types of labour inputs and the capital labour ratio are fixed, an increase in the wage premium is accompanied by a fall in the wage share (i.e. $\frac{\partial LS^{PC}_{LATP,LH}}{\partial w} < 0$). Similarly, the wage share responds negatively to an increase in the supply of skilled , i.e. $\frac{\partial LS^{PC}_{LATP,LH}}{\partial l} < 0$. Finally, If the substitution between capital and skilled labour is high ($\eta > 1$) an increase in the capital output ratio is accompanied by a fall in the wage share, i.e. $\frac{\partial LS^{PC}_{LATP,LH}}{\partial k} < 0 \text{ if } \eta > 1$; The opposite is valid in the case of capital akill complementarities.

[14] For the two level production function considered, the Allen partial elasticity of substitution between skilled and capital is $\sigma_{K,S} = \sigma + \frac{\eta - \sigma}{\theta_\eta}$ where θ_η is the relative share of the composite capital in total output. (Sato 1967). Imposing physical capital to be less substitutable with skilled than unskilled labour (i.e. σ≥η) implies an Allen elasticity of substitution between the inputs of the composite capital lower than between unskilled labour and the skilled labour - or capital because of the property of asymmetry - (i.e. $\sigma_{K,S} \leq \sigma$). Thus our restrictions are $\sigma \geq \sigma_{K,S}$ and σ≥η.

Thus, all other things being equal, a technology characterised by imperfect substitution between capital and skilled labour and between these and unskilled labour input can account for episodes, where declining labour shares are accompanied by increases in the skill premium and in the labour supply of highly-qualified workers.

4.3.1.4. Intermediate inputs

The previously described technology links the production factors with value added. As the labour share is defined in terms of value added one may be tempted to think that, whatever the demand for intermediate inputs, the fraction of domestic income accruing to labour will be unaffected. This section show that changes in the relative price of intermediate goods shift the stable relationship between the wage share and the capital output ratio, as the fraction of value added absorbed by labour is not independent of the firm's optimisation behaviour as regards intermediate goods. To see this more formally, it is convenient to define the production function in terms of gross output, instead of value added. Let us assume that we adopt the following CES specification for gross output:

$$(13)\ \tilde{Y} = \left\{ \gamma \left[\left(\alpha X^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(B_u L_u)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \right]^{\frac{\lambda-1}{\lambda}} + (1-\gamma) I^{\frac{\theta-1}{\theta}} \right\}^{\frac{\lambda}{\lambda-1}} \quad \text{where } X = \left((AK)^{\frac{\eta-1}{\eta}} + (B_s L_s)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

where $\tilde{Y}$, and I respectively stand for gross output and intermediate input. As in expression (11), it is assumed that capital and (skilled and unskilled) labour are combined by means of a two level CES aggregator. For the parameters of this production function it holds that $0 < \theta < \infty$ and $0 < \theta < 1$. Very broadly, intermediate inputs can be of two kinds, depending on their degree of substitution with the CES composite input of capital and labour: whereas intermediate energy inputs exhibit a low degree of substitution with the capital-labour composite ($0 < \omega < 1$), the opposite applies to intermediate material and services inputs ($1 < \omega < \infty$). Note that the specification above is rather general, in that intermediate inputs can be produced in the domestic economy or imported from abroad. As such, I could represent, for instance, imported raw materials, which are low substitutes to the capital-labour composite. Another possibility would be to feed I with a measure of off-shoring, which is the outsourcing of intermediate production to companies in locations outside the country. This practice allows firms to respond more flexibly to shocks via changes in the mix of production at home and abroad. As such, off-shoring can be regarded as high substitute to the capital-labour composite.

On an accounting basis, value added can be defined as:

$$(14) \quad Y = \widetilde{Y} - \frac{p_I}{p} I$$

where p and p_I respectively denote the price deflator of gross output and intermediate inputs, so $\frac{p_I}{p}$ represents the real price in terms of gross output of intermediate inputs. In this case, it can be shown (Appendix 1) that the labour share in value added is given by:

$$(15) \quad LS^{PC}_{LCATP,LH,I} = \frac{w_u^{PC} L_u + w_s^{PC} L_s}{Y} = 1 - (Ak)^{\rho} \frac{\gamma^{\frac{\theta}{\theta-1}}}{\left[1 - \frac{(1-\gamma)^{\theta}}{(p_I/p)^{\theta-1}}\right]^{\frac{1}{\theta-1}}} \left\{ \alpha^{\varepsilon} + (1-\alpha)^{\varepsilon} l^{\frac{\varepsilon-\sigma}{\sigma}} \omega^{\varepsilon-1} \right\}^{\frac{\sigma\rho}{\varepsilon(\sigma-1)}}$$

where $LS^{PC}_{LCATP,LH,I}$ is the labour share in value added calculated under the assumption of perfect competition and a CES specification for gross output like (13), i.e., with labour- and capital-augmenting technical progress, labour heterogeneity and intermediate inputs. The sign of the partial derivative $\partial LS^{PC}_{LCATP,LH,I} \big/ \partial\, p_I/p$ is ambiguous.

4.3.2. Market conditions

The discussion in the preceding section has assumed that the products and the labour market work in a competitive fashion. In this section we allow firms to have some product market power. We also extend the model by assuming a bargaining framework in the labour market. In both cases, the connection between real wages and the marginal productivity of labour is broken, which will be shown to provide additional explanatory power to account for medium-term labour share movements.

4.3.2.1. Imperfect competition in the products market

To illustrate how imperfect competition in the goods sector affects the behaviour of the labour share, let us recall the basic definition of the labour share given by (2), which we reproduce here for the sake of clarity:

$$(2)\ LS = \frac{L * w}{Y}$$

Under the assumption of perfect competition discussed so far, the real wage equates the marginal productivity of labour, and the labour share is equal to the marginal productivity of labour times the inverse of the average productivity of labour. This is reflected in expression (3) presented in Section 4.2:

$$(3)\quad LS^{PC} = \frac{L * MPL}{Y}$$

In other words, the labour share matches the concept of the employment elasticity of output, this implying that the share of value added accruing to labour is technologically given. Imperfect competition in the products market generates a gap between the marginal product of labour and the real wage. We will show below that breaking the connection between real wages and the marginal productivity of labour therefore provides additional explanatory power to account for medium-term labour share movements.

To proceed further, let us reconsider the specification for the labour share reflected in (15) once we adopt the (more realistic) assumption that firms operate in a non-competitive setting in the medium run. Imperfect competition may stem, for instance, from regulations and barriers to competition. If firms enjoy some market power they will not behave as price-takers but they will instead set prices over marginal costs in the following way:

$$(16)\ p = (1+\mu)MC = (1+\mu)\frac{W}{MPL}$$

where *p, W* and μ respectively denote the price deflator of gross output, the nominal wage and the markup of prices over marginal costs. One may proceed to work out the real wage as a function of the markup:

$$(17)\ \left(\frac{W}{P}\right)^{IC} = w^{IC} = \frac{MPL}{(1+\mu)}$$

where w^{IC} denotes the equilibrium real wage under imperfect competition in the products market. Equation (17) suggests that, in an imperfectly-competitive framework, the real wage does not equate the marginal productivity of labour, but rather the marginal productivity of labour corrected for the markup. One may now substitute the real wage according to (17) into the definition of the labour share given by (2), which yields:

$$(18)\ LS^{IC} = \frac{1}{(1+\mu)}\frac{L * MPL}{Y}$$

Because a positive markup requires $\mu > 0$, it can be seen by comparing equations (3) and (18) that, under imperfect competition in the products market, the labour share will be lower than under Walrasian conditions. Intuitively, the imperfectly-competitive equilibrium entails a lower level of employment and the real wage than the Walrasian one[15], which explains the reduction in the labour share. Note that, if the production function for gross output is given by (13), then expression (17) applies separately to skilled and unskilled labour. Combining (17) with (15) yields the following expression for the labour share:

(19) $$LS^{IC}_{LCATP,LH,I} = \frac{w_u^{IC} L_u + w_s^{IC} L_s}{Y} = \frac{1}{(1+\mu)}\left(1 - A\left(\frac{K}{Y}\right)\gamma^{\omega/(\omega-1)}\left[1 - \frac{(1-\gamma)^{\omega}}{(p_I/p)^{\omega-1}}\right]^{-1/(\omega-1)}\left[\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right]^{-1}(1-\alpha)^{\sigma/(\sigma-1)}\left\{1 + \left(\frac{\alpha}{1-\alpha}\right)^{\sigma}\left[\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right]^{\sigma-1}\right\}^{1/(\sigma-1)}\right)$$

where $LS^{IC}_{LCATP,LH,I}$ is the labour share in value added calculated under imperfect competition in the products market and a CES specification for gross output like (13), with labour- and capital-augmenting technical progress, labour heterogeneity and intermediate inputs. Because a positive markup requires $\mu > 0$, by comparing expressions (15) and (19) one can easily see that, all other things being equal, the labour share will be reduced in an imperfectly-competitive setting as compared with a competitive one. Equation (19) also indicates that the labour share is a decreasing function of the markup, and thus of the monopoly power of firms. This is so because the higher the firms' market power, the lower the levels of employment and the real wage in the new imperfectly-competitive equilibrium position.

4.3.2.2.Bargaining in the labour market

It has been assumed so far that the labour market works in a Walrasian fashion. We now consider how the labour share is affected by the introduction of regulations and institutions that prevent competitive forces from playing fully in the labour market. We consider one particular way in which the labour market deviates from spot competitive markets. Specifically, we will further assume that unions, or more generally, employed workers, may have some bargaining power that leads to a different pattern of real wages and employment than would be observed under perfect competition.

To analyse the implications of bargaining for the labour share one may develop a framework in which the bargaining parties are represented by a union and a firm (see, for instance, Blanchard and Fischer, 1989, ch. 9 and Booth, 1995, ch. 5). Alternatively, one may frame bargaining with reference to the more recent labour-market-search paradigm, where the bargaining parties are represented by one single worker and the firm (see, for instance, Trigari, 2004). The difference between these two approaches is merely methodological, as it does not affect the results concerning the equilibrium levels of employment, the real wage and the labour share. In what follows, we will consider that negotiating parties are represented by the union and the firm.

We examine the implications of two alternative structures of bargaining for labour share movements, one in which the union and the firm bargain over the wage and the firm then chooses employment, and one in which the union and the firm bargain simultaneously over employment and the wage. Put differently, bargaining can take place along two dimensions, according to whether the firm retains the right to manage employment. If it does, there is sequence by which the firm and the union first bargain over the real wage, and then employment is chosen by the firm unilaterally so as to maximize profit (i.e., the firm chooses a point on the labour demand curve). This is referred as to as the "right-to-manage" approach. But, as first pointed out by

[15] As indicated by equation (17), with market power in the products market, firms are willing to pay a lower level of real wage for any given level of employment, i.e., the labour demand shifts leftwards and crosses the labour supply for lower levels of employment and the real wage.

Leontief (1946), such contracts are not efficient: the union and/or the firm (or both) could be made better off by bargaining over employment as well as wages. This alternative assumption in which the union and the firm simultaneously bargain over wages and employment is referred to as the "efficient bargaining" model.

As we will show in this section, the two bargaining models have different implications for the way employment is determined and the allocative role played by real wages. Obviously, whether employment, together with wages, is the object of bargaining is an empirical matter. In principle, the right-to-manage model is widely seen as a good description of how bargaining actually takes place in many countries, insofar the labour input, both in the dimension of employment and hours of work per employee, is rarely the object of bargaining agreement (see, for instance, Layard, 2005). This has not precluded most academics from adopting the efficient bargaining setup to account for labour share movements, not only because of its efficiency properties, but most importantly because, unlike the right-to-manage framework, efficient bargaining delivers higher wages with no detriment for employment following an increase in the workers' bargaining power (see, most notably, Blanchard and Giavazzi, 2003) [16]. This is regarded as a desirable property of efficient bargaining, at least in the short run.

In the paragraphs that follow we will adopt a comprehensive approach and therefore discuss how the labour share equation is affected under each of these two bargaining paradigms.

A. Right-to-manage bargaining

The traditional right-to-manage model assumes that first the firm and the union bargain over the wage, and then the employment is freely chosen by the firm to maximize the profit.

According to this approach, wages are determined by maximisation of the product of each agent's gains from reaching a bargain, weighted by their respective bargaining strengths. We can write the Nash bargaining problem for wages (the product of the weighted net gains to each party) as:

$$(20) \quad \max_{w} B = \left\{ \frac{L}{T}\left[U(w) - U(RW)\right] \right\}^{\beta} \{Y - wL\}^{1-\beta}$$

where *L, T, U(.), w, RW, Y,* and β respectively stand for employed union members, total union members, the union's representative worker utility, the real bargained wage, the reservation wage, the firm's value added and the union's bargaining power. It will be assumed that $U'(w) > 0; U''(w) < 0$, i.e., the representative union member is risk-averse. Whereas the firm's net gain is its profits function, the net gain of the union is given by that of one representative union member. Since the union raises wages above the competitive level, each member T will face a probability of being unemployed $\left(1 - \frac{L}{T}\right)$. If unemployed, a worker receives the reservation level of utility $U(RW)$. But if there is bargain, the expected utility of a member is $\frac{L}{T}U(w) + \left(1 - \frac{L}{T}\right)U(RW)$. The net gain to the union is

[16] We quote Blanchard and Giavazzi (2003) at length: "Why assume efficient bargaining? First, it seems like a natural assumption in this context. But also, we want to capture the possibility that firms may not be operating on their demand for labour. In more informal terms, we want to allow for the fact that, when there are rents, stronger workers (a higher β) may be able to obtain a higher wage without suffering a decrease in employment, at least in the short run. Efficient bargaining naturally delivers that implication".

thus $\frac{L}{T}U(w)+\left(1-\frac{L}{T}\right)U(RW)-U(RW)$, which yields $\frac{L}{T}[U(w)-U(RW)]$. The reservation wage, which is be taken as exogenous, is the lowest wage rate at which a worker would be willing to accept a job.

The solution to this bargaining problem is as follows:

$$(21)\quad \frac{\beta w^{RM}U'\left(w^{RM}\right)}{U\left(w^{RM}\right)-U(RW)}=\beta\xi+(1-\beta)\frac{w^{RM}L}{Y-w^{RM}L}$$

where w^{RM} is the solution for the real wage under right-to-manage bargaining and $\xi=-\frac{-L'\left(w^{RM}\right)w^{RM}}{L}$ is the wage elasticity of labour demand evaluated at the equilibrium position. Equation (21) suggests that the wage solution under right-to-manage approach is such that the proportional marginal benefit to the union from a unit increase in wages is exactly equal to the proportional marginal cost to each party, weighted by each party's bargaining power. The left-hand side of (21) represents the benefit from a wage increase, which is felt only by the union, and thus is weighted by the union's bargaining power *β*. The first term on the right-hand side is the union's proportional marginal cost (the percentage reduction in employment due to the proportional wage increase[17]) weighted by the parameter representing union power, *β*. The second term on the right-hand-side represents the firm's proportional marginal cost weighted by the firm's power *(1-β).*

Once wages are determined as a result of the bargain, the firm continues to choose the number of workers it wishes to employ. This means that firms operate on their demand for labour, i.e., employees will be hired up to the point where the marginal labour productivity (corrected for the markup) is equal the real wage, that is:

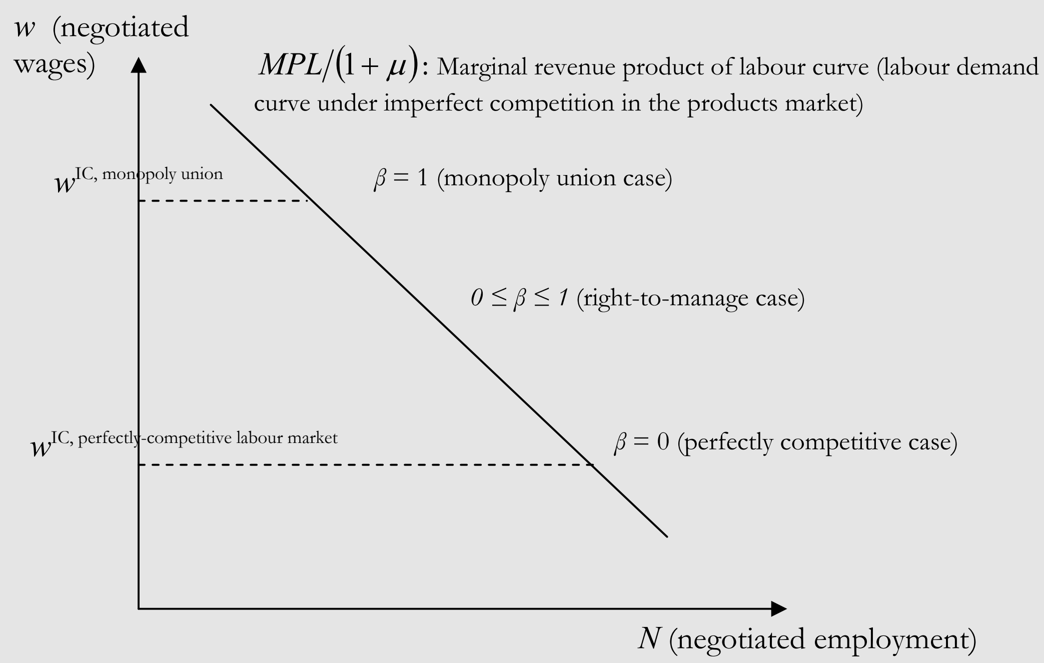
$$(22)\quad \left(\frac{W}{P}\right)^{IC,RM}=w^{IC,RM}=\frac{MPL}{(1+\mu)}$$

where $w^{IC,RM}$ denotes the equilibrium value for the real wage under imperfect competition in the products market and right-to-manage bargaining in the labour market. Now, let us compare equations (17) and (22), which share the imperfectly competitive setting as regards the products market while assuming two different approaches for the labour market, i.e., a competitive labour market in equation (17) and a right-to-manage framework in equation (22). It can be easily seen that, in both cases, the solution pair for employment and the real wage lies on the labour demand (i.e., the marginal product of labour curve corrected for the markup), though there is only one value of *β,* i.e. *β=0,* for which the two solutions coincide. This means that the competitive labour market outcome given by (17) is a special case of the right-to-manage outcome given by (22) under the assumption that the bargaining power of workers is equal to 0. For any $\beta \neq 0$, the union will use its bargaining power to obtain higher wages, and, by consequence, lower employment outcomes than in the case of a competitive labour market. If the union has all the power, i.e. if

[17] Note that, given the negative slope of the labour demand, the union's marginal cost is represented by the reduction in employment due to the marginal wage increase.

β=1, the union is able to extract the entire surplus and the right-to-manage model collapses to the monopoly union model. We illustrate these results in Graph 8.

Graph 8 - Labour market outcomes under the right-to-manage bargaining model



The graph illustrates the right-to-manage labour market outcome. Note that the monopoly union model is a special case of the right-to-manage bargaining model where $\beta = 1$, while the perfectly competitive model is a special case where $\beta = 0$.

Note that, as the marginal product equation (22) remains valid, modified versions of equations (18) and (19) can be obtained, where the real wage now represents the right-to-manage bargained wage. In detail, one may substitute the real wage according to (22) into the definition of the labour share given by (2), which yields a modified version of (18):

$$(18')\ LS^{IC,RM} = \frac{1}{(1+\mu)} \frac{L * MPL}{Y}$$

The difference between (18) and (18') is that, as generally, *β > 0,* then $w^{IC,RM} > w^{IC}$, so, $L^{IC,RM} < L^{IC}$. In words, when workers have some bargaining power, the equilibrium values for the real wage (and the marginal productivity) is higher, and the level of employment is lower as compared to a situation in which *β=0.*

If the production function for gross output is given by (13), then expression (22) applies separately to skilled and unskilled labour. Combining (22) with (15) yields the modified version of (19) for the labour share:

$$(19')\ LS^{IC,RM}_{LCATP,LH,I} = \frac{w_u^{IC,RM} L_u + w_s^{IC,RM} L_s}{Y} = \frac{1}{(1+\mu)} \left(1 - A\left(\frac{K}{Y}\right) \gamma^{\omega/(\omega-1)} \left[1 - \frac{(1-\gamma)^{\omega}}{(p_I/p)^{\omega-1}} \right]^{-1/(\omega-1)} \left[\frac{B_u}{B_s} \frac{MPL_s}{MPL_u} \right]^{-1} (1-\alpha)^{\sigma/(\sigma-1)} \left\{ 1 + \left(\frac{\alpha}{1-\alpha} \right)^{\sigma} \left[\frac{B_u}{B_s} \frac{MPL_s}{MPL_u} \right]^{\sigma-1} \right\}^{1/(\sigma-1)} \right)$$

The only difference between equations (19) and (19') is the level of real wages, which will be generally higher under right-to-manage as long as $\beta > 0$. Apart from that, (19') is undistinguishable from (19). This is because, like in the competitive labour market case, the equilibrium position for real wages and employment under the right-to-manage approach still lies on the labour demand, so, in that regard, equation (19) is unaffected. Put differently, changes in the bargaining power of workers affect the labour share thorough variations in the {real wage, employment} equilibrium along the labour demand curve.

B. Efficient Nash bargaining

As is well-known, the allocation of resources within the right-to-manage model is inefficient in that at least one of the parties could be better off by bargaining over employment as well as wages. In the efficient bargaining model, the union and the firm simultaneously determine wages and employment. We can now write the Nash bargaining problem modified to allow for bargaining over both the real wage and employment as[18]:

$$(23)\quad \max_{w,L} \widetilde{B} = \left\{ \frac{L}{T}\left[U(w) - U(RW)\right] \right\}^{\beta} \left\{ P(Y)Y(L) - WL \right\}^{1-\beta}$$

where *L, T, U(.), W, RW, P(Y), Y,* and β respectively stand for employed union members, total union members, the union's representative worker utility, the nominal bargained wage, the reservation wage, the inverse of the demand curve faced by the imperfectly competitive firm, the firm's value added and the union's bargaining power. The solution for this maximization problem is as follows: the union and the firm will set the real wage and employment such that the wage is equal to the sum of the average and marginal products of labour, weighted respectively by the union's bargaining strength β, and the firm's bargaining strength $(1-\beta)$. This will lie on the contract curve. The {real wage, employment} equilibrium is characterised by the following two equations:

$$(24)\quad \frac{MPL}{(1+\mu)} - w^{IC,EB} = -\frac{\left[U\left(w^{IC,EB}\right) - U(RW)\right]}{U'\left(w^{IC,EB}\right)}$$

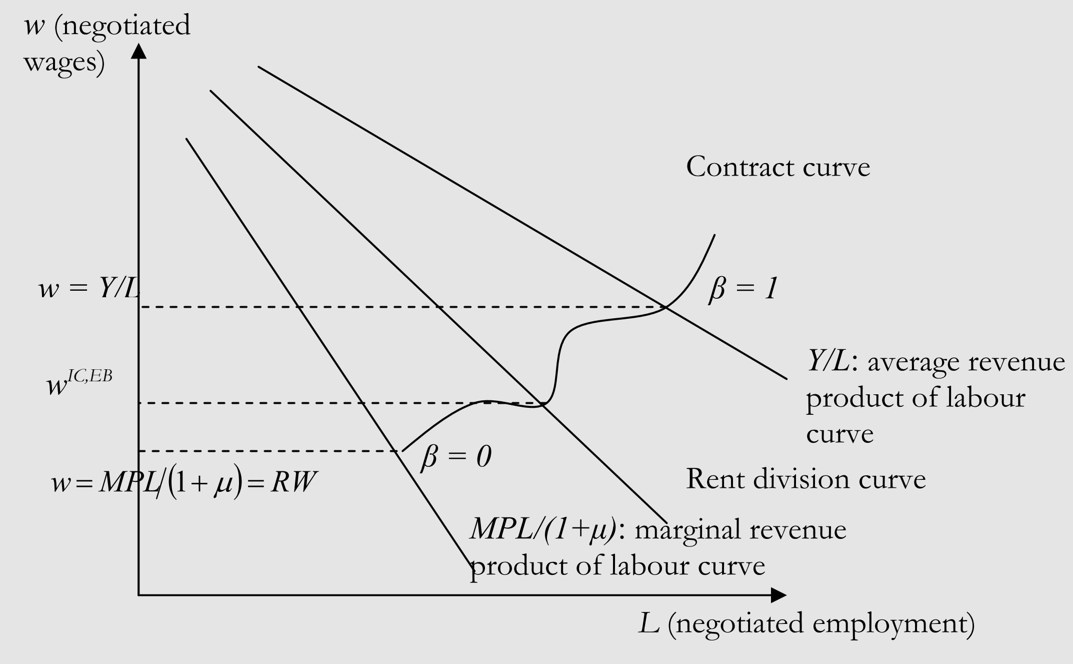
$$(25)\quad \left(\frac{W}{P}\right)^{IC,EB} = w^{IC,EB} = \beta\left(\frac{Y}{L}\right) + (1-\beta)\frac{MPL}{(1+\mu)}$$

where $w^{IC,EB}$ is the solution for the real wage under imperfect competition in the products market and efficient bargaining in the labour market. Equation (24) is referred as the "contract curve". It states that an efficient wage and employment outcome is one where the slopes of an isoprofit curve and an indifference curve are the same. Efficiency therefore means that the marginal rates of substitution of employment for wages, for both the union and the firm, are equal. Note that, as

[18] It is worthwhile drawing the attention to the fact that the net gain to the firm from reaching the bargain is defined in (23) in terms of nominal output and nominal wages, as opposed to real output and real wages as we did in (20). This is because in (23) one derives not only with respect to wages, but also with respect to employment. As an imperfectly-competitive firm faces a downward-sloping curve, one is then obliged to consider the reduction in prices arising from a marginal increase in employment. However, the fact that prices are a function of output (i.e., $P(Y)$) under in an imperfect competitive environment does not affect the derivation in (20) with respect to wages, so we preferred to simplify the notation in (20) by expressing variables in real terms.

the value of the marginal product of labour (corrected by the markup) is less than the real wage (by an amount which is equal to the union marginal rate of substitution of employment for wages), the contract curve lies to the right of the labour demand curve for any $w > RW$. If $w = RW$, then one obtains the result under the competitive labour market. Thus the contract curve starts at the competitive equilibrium. It can also be shown that the contract curve is upward sloping[19]. Intuitively, as wages are increased above the competitive level ($\beta > 0$), any members who are laid off have an increasing opportunity cost of being unemployed. The union therefore insures members against this risk by bargaining for increased employment[20]. Thus, if the contract is efficient, the union and the firm choose a point on the contract curve. Which point is chosen depends on the relative bargaining power of the firm and of the union, as indicated by equation (25). Equation (25) is referred as the "rent division curve", which is negatively sloped in the {real wage, employment} space[21]. If the union has no power ($\beta=0$), then the rent division curve collapses to the marginal product of labour (corrected for the markup), i.e., $\left(\frac{W}{P}\right)^{IC,EB} = \frac{MPL}{(1+\mu)}$, the outcome under a perfectly- competitive labour market. If the firm has no power ($\beta=1$), the rent division curve becomes the average product of labour, i.e., $\left(\frac{W}{P}\right)^{IC,EB} = \frac{Y}{L}$. The efficient Nash bargaining labour market equilibrium is illustrated in Graph 9. The equilibrium wage and employment levels are given by the intersection of the rent division curve and the contract curve.

Graph 9 - Labour market outcomes under the efficient Nash bargaining model



The graph illustrates the efficient Nash bargaining outcome where bargaining occurs over wages and employment.

[19] For a formal proof see, for instance, Booth (1995), pp. 130.

[20] This reasoning assumes that union's members are risk-averse, that is $U'(w) > 0; U''(w) < 0$. If members were risk-neutral, the contract curve would be vertical; members are not offered insurance against the risk of being unemployed. If members were risk-loving, the contract curve would be negatively sloped.

[21] This result comes from the concavity of the production function.

Now, substituting the real wage according to (25) into the definition of the labour share given by (2), one gets:

$$(26)\ \ LS^{IC,EB}=\frac{L*w^{IC,EB}}{Y}=\beta+(1-\beta)\frac{MPL}{(1+\mu)}*\frac{L}{Y}=\beta+(1-\beta)\frac{1}{(1+\mu)}\frac{L*MPL}{Y}$$

It can be seen by comparing equations (18) and (26) that, under imperfect competition in the products market and efficient bargaining in the labour market, the labour share will be generally higher than in the case where there is imperfect competition in the products market without any bargaining power allocated to workers in the labour market. The labour share given by (26) ranges between its value provided by equation (18) $\frac{1}{(1+\mu)}\frac{L*MPL}{Y}$ (if β =0, i.e., all bargaining power is allocated to firms) and 1 (if β =1, i.e., all bargaining power rests with workers).

If the production function for gross output is given by (13), then expression (25) applies to skilled and unskilled labour separately. Assuming that $\beta_u = \beta_s$ and combining (25) with (15) yields the following expression for the labour share:

$$(27)\ \ LS^{IC,EB}_{LCATP,LH,I}=\frac{w_uL_u+w_sL_s}{Y}=2\beta+\frac{(1-\beta)}{(1+\mu)}\left(1-A\left(\frac{K}{Y}\right)\gamma^{\omega/(\omega-1)}\left[1-\frac{(1-\gamma)^{\omega}}{(p_I/p)^{\omega-1}}\right]^{-1/(\omega-1)}\left[\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right]^{-1}(1-\alpha)^{\sigma/(\sigma-1)}\left\{1+\left(\frac{\alpha}{1-\alpha}\right)^{\sigma}\left[\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right]^{\sigma-1}\right\}^{1/(\sigma-1)}\right)$$

where $LS^{IC,EB}_{LCATP,LH,I}$ is the labour share in value added calculated under imperfect competition in the goods sector, efficient bargaining in the labour market, and a CES specification for gross output like (13), with labour- and capital-augmenting technical progress, labour heterogeneity and intermediate inputs.

Let us consider now the impact of changes in β, μ and the reservation wage on the labour share. For the sake of clarity, we will calculate partial derivatives on the basis of (26) (instead of the more complicated version (27)). We show in Appendix 2 that $\partial LS^{IC,EB}/\partial\beta>0 \text{ if } \sigma<1$ whereas $\partial LS^{IC,EB}/\partial\beta<0 \text{ if } \sigma>1$, $\partial LS^{IC,EB}/\partial RW>0 \text{ if } \sigma<1$ whereas $\partial LS^{IC,EB}/\partial RW<0 \text{ if } \sigma>1$, and $\partial LS^{IC,EB}/\partial\mu>0 \text{ if } \sigma<1$ whereas $\partial LS^{IC,EB}/\partial\mu<0 \text{ if } \sigma>1$. The detailed calculations and some intuition of the economics behind these results can be found in Appendix 2.

The sign of the derivatives above therefore depends on the degree of input substitutability between capital and labour. In the remaining of this section we pursue to show that this is no longer case when we refine (26) by incorporating the no-entry condition. Let us denote by κ the cost of entry faced by imperfectly-competitive firms. For the sake of algebraic simplicity we will assume that κ is proportional to output[22]. The no-entry condition states that, in equilibrium, rents must cover entry costs, which translates into the condition that profit per worker must cover entry costs. In detail:

[22] With κ proportional to output, the profit per unit of output in equilibrium must be equal to κ, and, in the limit, the equilibrium converges to the competitive equilibrium as κ goes to zero.

$$(28)\quad \frac{Y - w^{IC,EB} L}{L} = \kappa$$

Let us substitute the real wage according to (25) into (28), thus obtaining:

$$(29)\quad (1+\mu) = \frac{MPL}{(Y/L) - [\kappa/(1-\beta)]}$$

Equation (29) can be regarded as the no-entry condition under imperfect competitive conditions. It tells us that the markup is no longer an exogenous parameter, but is determined in equilibrium by κ and β. An increase in κ and/or β both lead to exit of firms, thus lower elasticity of demand and a higher equilibrium value of μ.

One may now substitute (29) in (26) so as to get a new version of the labour share that satisfies the no-entry condition:

$$(30)\quad LS^{IC,EB} = \frac{L * w^{IC,EB}}{Y} = 1 - \frac{\kappa}{Y/L}$$

On the basis of (30), we show in Appendix 2 that $\partial LS^{IC,EB} / \partial \beta < 0$, $\partial LS^{IC,EB} / \partial RW > 0$ and $\partial LS^{IC,EB} / \partial \mu > 0$. Note that these signs coincide with the ones described above (before considering the no-entry condition) for the case where $\sigma < 1$.

In empirical applications, one may think of μ as a time-varying series determined by κ and β, instead of taking it as exogenous parameter[23]. In turn, one may conceive κ as coming from product market regulations. Two relevant dimensions affecting product market regulation in the EU context are the completion of the single market and the policy reforms undertaken in the context of the Lisbon Strategy. Through the elimination of tariff barriers, or standardization measures making it easier to sell domestic products in other EU countries, the single market increases the elasticity of demand facing monopolistic firms, thereby reducing their market power. Product market deregulations implemented at the national level as outlined in the National Reform Programs may reflect measures undertaken to remove the entry costs faced by firms or the elimination of state monopolies. To the extent that globalisation has led to an intensification of product market competition, this may have further reinforced the movement towards liberalisation in product markets.

In an equal manner, there is no need to take β as an exogenous parameter, but one would rather think of β as a time-varying series. Traditionally, the bargaining power of workers has been made

[23] As documented in the literature, many empirical difficulties arise when measuring markups in presence of bargaining in the labour market. From the perspective of our paper, the main problem is that, under efficient bargaining, the conventional method used to construct the labour share on the basis of equation (18) is no longer valid. As illustrated by equation (26), under efficient bargaining, the labour share now depends also on β. In practical terms this implies that what is interpreted as an increase in the markup may in fact reflect lower bargaining power of workers in the labour market.

a function of several aspects of labour market regulation, such as coverage rates, the rules on the right to strike etc… Interestingly enough, Hornstein *et al.* (2002) argue that faster rate of innovation in the form of capital-augmenting technological progress has resulted in an increase in the job turnover in the economy, which has risen the bargaining power of firms, allowing them to push workers closer to their outside option (i.e., their reservation wage)[24]. A more novel set of hypothesis points to globalisation as one force behind the reduction in the workers' bargaining power. Increased labour demand elasticity and intensified foreign competition could be reducing the workers' bargaining power. There is some evidence that the rents previously accruing to workers are squeezed after trade liberalisation exposes their employers to increased import competition (see Boulhol *et al.,* 2006, and Kramarz, 2006).

4.4. The labour share in the short run

According to the evidence presented in Section 3, in the short run, the labour share fluctuates counter-cyclically. It also tends to lag output by around one year while it seems to be more volatile in those countries characterized by more flexible labour markets. In order to account for such cyclical properties of the labour share, this section will examine the role of labour market institutions that result in labour hoarding.

4.4.1. Labour hoarding

Labour hoarding is to a large extent determined by adjustment costs, such as firing and hiring restrictions, search and training costs. Adjustment costs affect the behaviour of the labour share in two ways:

i. The fluctuations of labour demand are dampened as compared to a situation in which these institutional aspects are absent. This is because adjustment costs induce less hiring when demand is strong, but also less firing when business conditions are less favourable. This means that, the overall change in employment across the economic cycle is lower than in a situation where such adjustment costs are absent.

ii. If convex in the change in employment, adjustment costs result in a gradual distribution over time of any given magnitude of the change in employment, which rationalizes a lagged response of employment to changes in output.

This means that, even in the unitary elasticity framework (i.e., a Cobb-Douglas technology), the labour share will fluctuate along the business cycle once adjustment costs are considered (see Kessing, 2001). In the more general framework of the CES production function, this section will show that adjustment costs tend to dampen labour share fluctuations across the economic cycle.

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[24] To be more clear, Hornstein *et al.* (2002, 2003) show that a faster rate of innovation and obsolescence of putty-clay capital can raise the profit share if there are search frictions in the labour market. The faster rate of innovation makes new capital goods more attractive to firms relative to their existing capital, so they want to change their capital and production processes more often than before. But with putty-clay capital, this means more frequent changes in their employment levels to make best use of the new technology. Therefore, there is more employment churn [funny expression I didn't know, a churn is originally a vessel used to make butter!] *ex ante*, which would reduce the rate of matching between firms and workers. Workers are therefore more likely to lose their jobs and experience during a period of unemployment. This increases firms' bargaining power *endogenously*, so they can reap a larger share of the rents that result from the interaction of search frictions and the nature of technological progress. [is this argument also true for qualified labour?]

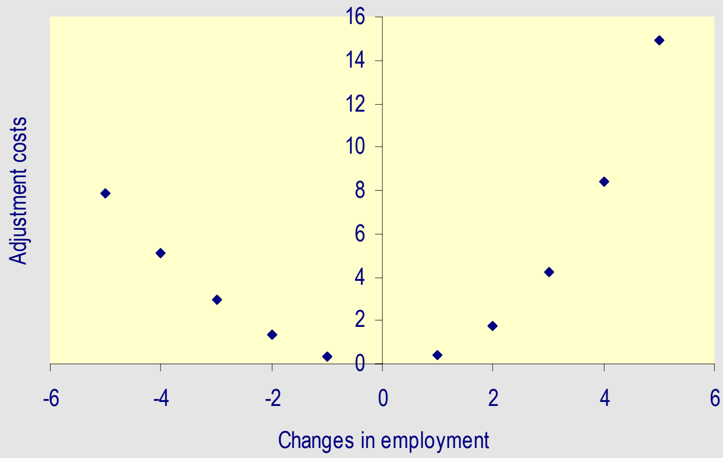
To do so, one may first of all adopt a specification for the labour adjustment costs function, and then characterize the new equilibrium in the labour market. For ease of analysis, adjustment costs have most often been represented using a convex symmetric function. But this way of specifying them does not allow us to explain asymmetric adjustments in employment. For this reason, one may postulate asymmetric adjustment costs, as in Pfann and Palm (1993) that assume a relation of the form:

$$(31)\quad AC(\Delta L) = -1 + e^{(\phi \Delta L)} - \phi \Delta L + \frac{\chi}{2}(\Delta L)^2 \qquad \phi > 0, \chi > 0$$

where *AC' (ΔL)>0, AC'' (ΔL)>0*. This specification implies an asymmetry between positive and negative variations in employment. We return to a symmetric formulation when *a = 0*. Conversely, when *a > 0* (or *a < 0*), the marginal cost of an increase in employment is greater (or less) than that of a reduction (see Graph 10).

Graph 10 - The costs of employment adjustment

Parameter values are a = b = 0,5



Adjustment costs are an asymmetric convex function of the quantity of labour adjusted. Convexity means that the unitary cost of adjusting ΔL_2 is higher than the unitary cost of adjusting ΔL_1, where $\Delta L_2 > \Delta L_1$. Asymmetry means that the marginal cost of an increase in employment is greater than that of a reduction.

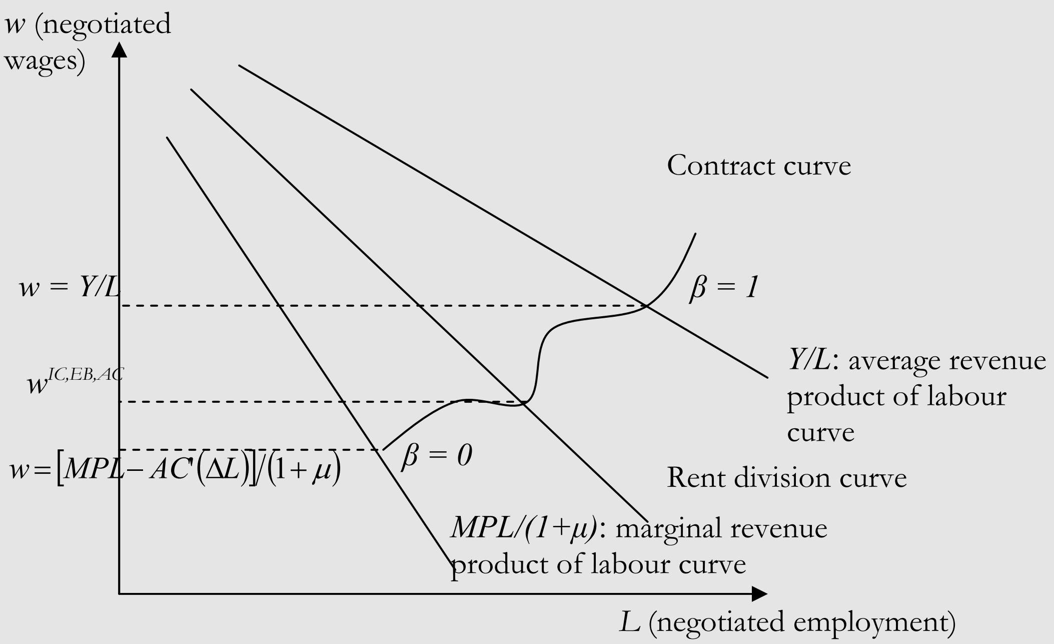
In presence of labour adjustment costs, the equilibrium in the labour market is characterized by the following expressions:

$$(32)\quad \frac{MPL - AC'(\Delta L)}{(1+\mu)} - w^{IC,EB,AC} = -\frac{\left[U\left(w^{IC,EB,AC}\right) - U(RW)\right]}{U'\left(w^{IC,EB,AC}\right)}$$

$$(33)\quad \left(\frac{W}{P}\right)^{IC,EB,AC} = w^{IC,EB,AC} = \beta\left(\frac{Y}{L}\right) + (1-\beta)\frac{MPL - AC'(\Delta L)}{(1+\mu)}$$

where $w^{IC,EB,AC}$ is the solution for the real wage under imperfect competition in the products market, efficient bargaining in the labour market and the presence of labour-consuming adjustment costs. According to the modified version of the contract curve (32), efficiency requires the marginal rates of substitution of employment for wages, for both the union and the firm, be equal. For the firm, this is given by the marginal product of labour minus its marginal adjustment cost (corrected by the markup), this implying that, during an expansion, the firm will be more reluctant to hire new workers in the presence of adjustment costs. The modified version of the real wage solution (33) shows that the firm's threat point is given by the marginal product of labour minus its marginal adjustment cost (corrected by the markup), this implying that, if the union has no power (*β=0),* during an expansion, the firm will be willing to pay a lower level of wages for any given level of employment in the presence of adjustment costs. The equilibrium wage and employment levels in presence of adjustment costs are given by the intersection of the rent division curve and the contract curve, as illustrated in Graph 11.

Graph 11 - Labour market outcomes under the efficient Nash bargaining model with labour adjustment costs



The graph illustrates the efficient Nash bargaining outcome in presence of adjustment costs.

To proceed further, it is worthwhile noting that, from a technical point of view, one may distinguish a situation where adjustment costs are payments from the firm to the worker –as is the case for severance payments– or a resource cost that uses labour –for example if new hires have to be recruited by an employment agency, or if they have to be trained by the firm's existing workforce, thus diverting it from direct productive activity–, from a situation where they are not. In any event, the labour market outcome will vary as compared to a situation with no adjustment costs, and so will do the labour share. However, the real resources consumed by adjustment costs will be included in the definition of the labour share only if adjustment costs are resource costs that use labour. In what follows, we discuss how the labour share is affected by adjustment costs under each of these two hypotheses.

Consider first the case where adjustment costs are resource-consuming from the firm's perspective, though not in terms of the labour input. In such circumstances, the labour share under imperfect competition in the products market and efficient bargaining in the labour market is given by:

$$(34) \quad LS^{IC,EB,AC} = \frac{L * w^{IC,EB,AC}}{Y} = \beta + (1-\beta)\frac{[MPL - AC(\Delta L)]}{(1+\mu)} * \frac{L}{Y} = \beta + (1-\beta)\frac{1}{(1+\mu)}\frac{L * [MPL - AC(\Delta L)]}{Y}$$

On the basis of (34) and a function for adjustment costs like (31), we show in Appendix 2 that the labour share is a decreasing function of the change in employment, implying that labour share fluctuations are dampened in presence of convex adjustment costs as compared to a situation in which adjustment costs may be linear or may not exist at all.

Consider now the case where adjustment costs use labour. Accordingly, the labour share will include the remuneration to the labour services that facilitate the incorporation to the firm of additional employment:

$$(35) \quad LS^{IC,EB,AC} = \frac{L * w^{IC,EB,AC}}{Y} = \beta + (1-\beta)\frac{[MPL - AC(\Delta L)]}{(1+\mu)} * \frac{L}{Y} + \frac{AC(\Delta L)}{Y}$$

On the basis of (35) and a function for adjustment costs like (31), we show in Appendix 2 that the labour share is also a decreasing function of the change in employment.

Before we finish, let us reconsider expression (27) in presence of labour-consuming adjustment costs. If the production function for gross output is given by (13), then expression (33) applies to skilled and unskilled labour separately. Assuming for simplicity that $\beta_u = \beta_s$, and that adjustment costs affecting skilled and unskilled workers are the same, then expression (27) becomes:

$$(35) \quad LS^{IC,EB,AC}_{LCATP,LH,I} = \frac{w_u L_u + w_s L_s}{Y} + \frac{AC(\Delta L)}{Y} = 2\beta + \frac{(1-\beta)}{(1+\mu)}\left(1 - A\left(\frac{K}{Y}\right)\gamma^{\omega/(\omega-1)}\left[1 - \frac{(1-\gamma)^{\omega}}{(p_I/p)^{\omega-1}}\right]^{-1/(\omega-1)}\left[\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right]^{-1}(1-\alpha)^{\sigma/(\sigma-1)}\left\{1 + \left(\frac{\alpha}{1-\alpha}\right)^{\sigma}\left[\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right]^{\sigma-1}\right\}^{1/(\sigma-1)}\right) - \left(\frac{AC'(\Delta L)}{(1+\mu)} * \frac{L}{Y}\right) + \frac{AC(\Delta L)}{Y}$$

where $LS^{IC,\,EB,\,AC}_{LCATP,LH,I}$ is the labour share in value added calculated under imperfect competition in the goods sector, efficient bargaining in the labour market, adjustment costs in labour changes, and a CES specification for gross output like (13), with labour- and capital-augmenting technical progress, labour heterogeneity and intermediate inputs.

Finally, note that in the discussion above we have implicitly assumed that firms are perfectly informed as regards the nature of the shocks that hit the economy, i.e., whether these shocks are of a temporary or a permanent nature. One could also rightly argue that in a context characterised by some degree of uncertainty, the counter-cyclical behaviour of the labour share could either be reinforced or dampened. In terms of the model sketched above, uncertainty would modify further modify the marginal product of labour, as the marginal adjustment costs now consist of two terms: the current marginal adjustment cost generated by an extra unit of labour, as already captured by *AC' (ΔL),* and the shadow expected future marginal adjustment costs generated by that unit, which we will denote as *AC'(θ)*, with *θ* a measure of firms' uncertainty about the nature of future shocks. In this context, the impact of higher uncertainty on the labour share is ambiguous. On the one hand, higher uncertainty might be expected to increase the likelihood a worker be fired, thus increasing the shadow cost of labour and inducing firms to be more prudent regarding their hiring behaviour, as more hiring today might mean more necessity to fire tomorrow if demand turns weak. This would reinforce the counter-cyclical behaviour or the labour share. On the other hand, as in the case of investment (see Nickell 1977), an increase in uncertainty may well increase incentives to hire. Empirically, the preceding argument indicates that taking future marginal adjustment costs into account should lead to adding a function of perceived uncertainty to capture the *shadow* expected future marginal adjustment cost, as in expression (36) below:

(36)

$$LS^{IC,EB,AC}_{LCATP,LH,I} = \frac{w_u L_u + w_s L_s}{Y} + \frac{AC(\Delta L)}{Y} = 2\beta + \frac{(1-\beta)}{(1+\mu)}\left(1 - A\left(\frac{K}{Y}\right)\gamma^{\omega/(\omega-1)}\left[1 - \frac{(1-\gamma)^{\omega}}{(p_I/p)^{\omega-1}}\right]^{-1/(\omega-1)}\left[\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right]^{-1}(1-\alpha)^{\sigma/(\sigma-1)}\left\{1 + \left(\frac{\alpha}{1-\alpha}\right)^{\sigma}\left[\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right]^{\sigma-1}\right\}^{1/(\sigma-1)}\right) - \left(\frac{[AC'(\Delta L) - AC'(\theta)]}{(1+\mu)} * \frac{L}{Y}\right) + \frac{AC(\Delta L)}{Y}$$

Now, if following an increase in uncertainty, fear of future firing predominates over willingness to hiring in the firm behaviour, then $\partial LS^{IC,\,EB,\,AC}_{LCATP,LH,I} \big/ \partial(\Delta\theta) < 0$. If vice-versa, then $\partial LS^{IC,\,EB,\,AC}_{LCATP,LH,I} \big/ \partial(\Delta\theta) > 0$.

4.4.2. Interaction of labour hoarding with the cyclical behaviour of markups

Bearing in mind expression (34), one may forcefully argue that the counter-cyclical behaviour of the labour share caused by labour hoarding might be tempered by the counter-cyclical behaviour of the mark-up in the products market, if strong demand conditions allowed firms operating in an imperfectly-competitive framework to raise margins. In the framework developed above, the markup is fixed across the different states of business conditions. However, there are both theoretical and empirical reasons to conceive counter-cyclical markups[25]. If this is the case, movements in the markup μ, which also enter expressions (26) and (34), will tend to counter-balance the fluctuations in the labour share outlined above. It is therefore an empirical issue to determine the outcome of these opposing effects.

[25] For a thorough review of the topic of markups' cyclical behaviour see Rotemberg and Woodford (1999).

6. Empirical Evidence

The implication of the theory is that the wage share is related to the capital-output ratio (in efficiency units) by a stable relationship, which is negatively or positively sloped depending upon the elasticity of substitution between factors of productions. Movements in the relative price of labour or in factor augmenting technological progress do not modify this relationship. As shown in the analysis by Bentolila-Saint Paul (2003) reviewed in section 4, firm's profits maximisation qualifies uniquely this relationship, which survives to alternative ways of completing the model. Thus, changes in the relative supply of skilled labour, in the wage premium and in the price of imported materials shift the curve linking the wage share to the capital output ratio upward or downward.

Our aim is that of establishing the sign of the relationship between the wage share and the capital output ratio, controlling for possible shifters of this relationship. Following, Bentolila and Saint Paul we explore this relationship at the industry level. We aim at exploiting as far as possible both the time series and the cross-section properties of the data. This has the major advantage of improving the statistical properties of estimates when the number of observations over time is limited. We use yearly observations from 1970 to 2004 for the 18 OECD countries disaggregated by 9 main market industries. Data are taken from the KLEMS database (Appendix 3).

Visual inspection of the data reveals that the wage share varies with the capital-output ratio, which is consistent with an elasticity of substitution between capital and labour different from one (**Error! Reference source not found.** to **Error! Reference source not found.**). In addition, the sign of the relationship varies across different country and industry combinations, which might be the outcome of the interaction between technological constraints and institutional set-up. The relationship is also hump and/or U-shaped, suggestive either of shifts in or of movements off the relationship between the wage share and the capital labour ratio. A similar pattern is observed when differences between countries are offset through averaging.

6.1. Econometric estimation

Following Bentolila and Saint Paul, we estimate the following relationship

$$\ln ws_{ijt} = \lambda_{ij} + \mu_t + \beta k_{ijt} + \gamma l_{ijt} + \delta wp_{ijt} + \varepsilon_{ijt}$$

Where

λ_{ij} : country/industry fixed effects

μ_t : period fixed effect

l_{ijt} : relative supply of skilled labour

wp_{ijt} : wage premium of skilled over unskilled labour

We start estimating our model with OLS. It is well known that the OLS estimates are unbiased and consistent only if the error term is uncorrelated with the explanatory variables. However, these estimates are inconsistent if the error term contains temporal and/or cross- section common components, which may reflecting unobserved factors correlated with the explanatory variables. Thus, we report OLS estimates to verify how the estimated coefficients change when we allow for unobserved heterogeneity across time and space. The OLS estimator uses both the cross-sectional and time dimension. Running OLS regression on average values of the variables over time yields the Between estimator, which gives consistent estimates when the correlation between the regressors and the unobserved individual effects is zero. Conversely, fixed effects models allow unobserved heterogeneity potentially correlated with the observed regressors to be taken into account.

The first 6 columns of Table 4 report the estimates of the wage share equation without any shifter, while columns 8 to 12 introduce as controls the ratio between skilled and unskilled labour and the relative wage premium. The reported t-statistics are based on standard errors robust to heteroskedasticity. Panel a) to c) reports estimates for the full sample, for the EMU and the non-EMU sub-samples.

When we assume that the capital output ratio is independent of any country/industry specific effects (cols. 1 and 2), the coefficient of the capital-output ratio turns out to be negative, which is consistent with an elasticity of substitution between capital and labour larger than one (see eq. 4). This finding is robust to the inclusion of the relative supply of skilled labour and wage premium (cols 7 and 8) or of period dummies capturing common unobserved trend components (col. 11). The estimated coefficients of the wage premium are both positive, which is inconsistent with what expected from equation 12. Accounting with the possible correlation between the explanatory factors and unobserved country and industry specific component (col. 3), switches the sign of the coefficient of the capital output ratio from negative to positive - i.e. capital and labour are complement and an increase in the capital intensity of production is accompanied by an increase in the wage share. It is also worth mentioning that the elasticity of the wage share to the capital output ratio is larger when the regression is done on the subsample of non-EMU countries but its coefficient non statistically significant, an indication that for these countries σ is not different from 1. These coefficients are robust to the inclusion of the labour supply and the wage premium in col. 9. In this case, an increase in the relative supply of skilled labour is accompanied by an increase in the wage share. Conversely, the share in labour income falls with the skill premium. Finally in columns 6 and 12 we run the regression controlling for unobserved country/industry specific and time components. The estimated coefficients are not different from what found when unobserved common shocks are discarded (compare 6 with 3 and 12 with 9).

So far we have imposed that the coefficient of the explanatory variables are the same across industries. In Table 5 we allow the coefficient to vary across industries through interaction of the explanatory variables with industries dummies (cols 1 to 3). The equations are estimated with industry- country specific fixed effects; period dummies are included to capture shocks to the wage share common across countries and industries. We next allow the effects of the relative supply of skilled labour and the skilled wage premium to play a role (cols. 4 to 6). The estimated coefficient of the relative supply of skilled labour is positive, implying that, everything constant, the labour share rises with the supply of skilled labour. Conversely an increase in the wage premium is accompanied by a reduction of the labour share in gross value added. These findings reflect the estimates obtained restricting the sample to the group of non-emu countries. For countries members of the monetary union, the coefficients are insignificant. Looking at equation 12, the wage share is unrelated to both the ratio of skilled over unskilled labour and the wage premium if $\sigma=0$ or $\rho=0$. In the first case, there is no substitution between the composite capital and the unskilled labour input (i.e. the composite production function in 11 is of the Leontief type); the second case implies that the capital-skilled labour ratio responds positively and proportionally to an increase in the relative price of labour (i.e. the X production function is a Cobb-Douglass). Alternatively, the wage share is independent of factor prices and factor supplies when $\sigma=1$, i.e. when the relative demand of unskilled labour relative to the composite capital change proportionally and negatively with its relative price.

Consistently with the results of Bentolila and Saint Paul, our estimates suggest that the effect of the capital output ratio on the wage share is industry specific. The coefficient is negative in some industries suggesting that capital and labour are substitute and positive in others, which implies capital and labour complementarities. For the sample of all countries, our estimates suggest that capital and labour complementarities prevail in a Manufacturing, Mining, Construction (only for non-emu countries), Hotels (only for non-EMU countries), Wholesale (but only in EMU countries). In Agriculture, Electricity and Finance, the estimated coefficients points toward a substitution between capital and labour. However, in the second industry the coefficient is statistically different from zero only when we control for the supply of skilled labour and the wage premium. It is also worth mentioning that the coefficient of the capita labour ratio change sign or become insignificant when we include the skilled variables (compare columns 1 and 2 with 4 and 5). Only in the case of non-EMU countries, the coefficient suggests that capital and labour are substitute in this sector.

Table 4 –Wage share equation

	OLS	Between	FE	RE	FE	FE	OLS	Between	FE	RE	FE	FE
a) all countries	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Capital-output	-0.22	-0.25	0.06	0.05	-0.22	0.07	-0.22	-0.25	0.08	0.05	-0.22	0.08
ratio	(-31.8)	(-7.2)	(4.0)	(4.4)	(-33.6)	(4.3)	(-21.3)	(-6.2)	(2.89)	(4.5)	(-20.2)	(3.0)
Relative supply							0.02	0.01	0.08	0.07	0.03	0.08
of skilled labour							(5.4)	(1.2)	(6.1)	(10.9)	(5.3)	(4.6)
Wage premium							0.01	0.04	-0.03	-0.03	0.01	-0.04
							(2.0)	(1.6)	(-4.3)	(-6.7)	(1.1)	(-2.2)
Country and Industry FE	No	:	Yes	:	No	Yes	No	:	Yes	Yes	No	Yes
Period FE	No	:	No	:	Yes	Yes	No	:	No	No	Yes	Yes
Obs	4523	4523	4523	4523	4523	4523	3123	2043	3123	3123	3123	3123
R^2	0.18	0.18	0.18	0.18	0.18	0.17	0.19	0.18	0.04	0.009	0.18	0.005
b) EMU countries												
Capital-output	-0.18	-0.22	0.06	-0.22	-0.18	0.06	-0.23	-0.26	0.10	0.09	-0.23	0.10
ratio	(-21.6)	(-4.4)	(4.9)	(-4.4)	(-21.4)	(5.1)	(-17.2)	(-5.0)	(6.11)	(5.0)	(-16.8)	(6.0)
Relative supply							0.04	0.05	0.06	0.05	0.05	0.01
of skilled labour							(6.2)	(2.3)	(5.7)	(5.8)	(6.22)	(1.0)
Wage premium							0.03	0.05	-0.03	-0.03	0.03	0.26
							(2.9)	(1.4)	(-4.3)	(-4.3)	(2.7)	(1.4)
Country and Industry FE	No	:	Yes	:	No	Yes	No	:	Yes	Yes	No	Yes
Period FE	No	:	No	:	Yes	Yes	No	:	No	No	Yes	Yes
Obs	2797	2797	2797	2797	2797	2797	2043	2043	2043	2043	2043	2043
R^2	0.12	0.12	0.12	0.12	0.12	0.10	0.18	0.18	0.007	0.007	0.19	0.04
b) no-EMU countries												
Capital-output	-0.28	-0.27	0.10	-0.27	-0.28	0.09	-0.26	-0.30	0.03	-0.04	-0.25	0.02
ratio	(-21.5)	(-5.0)	(0.99)	(-5.0)	(-21.5)	(-21.5)	(-16.3)	(-4.1)	(0.28)	(-0.44)	(-14.1)	(0.2)
Relative supply							0.01	-0.02	0.18	0.10	0.02	0.21
of skilled labour							(2.38)	(-0.09)	(4.12)	(3.6)	(3.0)	(4.2)
Wage premium							0.01	0.05	-0.02	-0.03	-0.01	-0.04
							(1.5)	(1.01)	(-2.18)	(-2.75)	(-0.7)	(-1.1)
Country and Industry FE	No	:	Yes	:	No	Yes	No	:	Yes	Yes	No	Yes
Period FE	No	:	No	:	Yes	Yes	No	:	No	No	Yes	Yes
Obs	918	918	918	918	918	918	828	828	828	828	828	828
R^2	0.29	0.29	0.29	0.29	0.29	0.29	0.26	0.25	0.05	0.09	0.27	0.06

Source: Commission services.

Estimates are robust to heteroschedasticity

Table 5 –Wage share equation: industry specific slopes

	FE All countries (1)	FE EMU countries (2)	FE No EMU (3)	FE All countries (4)	FE EMU countries (5)	FE No EMU countries (6)
Capital-output ratio:	-0.26	-0.26	-0.50	-0.38	-0.25	-0.59
Agriculture	(-6.1)	(-2.7)	(-6.5)	(-6.0)	(-1.9)	(-6.2)
Capital-output ratio	0.13	0.11	0.27	0.14	0.16	0.03
Mining	(4.7)	(6.1)	(1.4)	(3.4)	(8.1)	(0.2)
Capital-output ratio:	0.13	0.12	0.42	0.31	0.29	0.31
Manufacturing	(3.9)	(3.4)	(3.9)	(8.9)	(8.0)	(3.3)
Capital-output	-0.08	-0.04	0.04	-0.15	-0.20	0.09
Electricity	(-0.3)	(-0.12)	(0.4)	(-2.48)	(-2.76)	(0.6)
Capital-output	0.11	0.023	0.17	0.098	0.04	0.19
Construction	(4.1)	(0.9)	(1.7)	(3.2)	(1.2)	(1.78)
Capital-output	-0.01	0.03	0.05	0.12	0.14	-0.027
Wholesale	(-0.5)	(1.58)	(0.4)	(4.8)	(5.2)	(-0.31)
Capital-output	0.12	0.013	1.11	0.17	0.008	0.90
Hotels	(4.1)	(0.3)	(7.6)	(3.8)	(0.26)	(5.8)
Capital-output	0.17	0.17	0.22	0.25	0.27	-0.039
Transport	(7.0)	(6.6)	(1.98)	(4.4)	(3.9)	(-0.37)
Capital-output	-0.098	-0.23	-0.90	0.14	-0.13	-0.38
Finance	(-3.1)	(-9.0)	(-8.2)	(3.6)	(-1.54)	(-2.45)
Relative supply of				0.09	0.01	0.19
skilled labour				(4.7)	(0.76)	(3.0)
Wage premium				-0.05	0.23	-0.02
				(-2.7)	(1.2)	(-0.54)
Country and Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Period FE	Yes	Yes	Yes	Yes	Yes	Yes
Obs	4523	2797	2043	4523	2043	828
R^2	0.23	0.27	0.29	0.23	0.29	0.13

Source: Commission services.
Estimates are robust to heteroschedasticity

7. Conclusions

This paper seeks to understand labour share dynamics in Europe. Although in the long run the share of national income accruing to labour is roughly constant, there is supportive evidence of large medium-term swings and significant movements at business cycle frequencies. We present a shift-share decomposition which illustrates the contribution of changes in the sectoral and employment composition of the economy to observed medium-term variations in the labour share. We subsequently proceed to identify the fundamental factors underlying labour share movements through a model-based approach building on Bentolila and Saint Paul (2003). We show that with a CES technology there is a stable relationship between the wage share and the capital output ratio. The introduction of deviation from the homogeneity of labour and or the inclusion of raw material makes this relationship to shift upward or downward. We show that an increase in the supply of skilled labour and in the wage premium is associated to a fall of the wage share. The preliminary econometric evidence suggests that substitution between capital and labour characterises some industries while complementarity prevails in others.

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Appendix 1: Derivation of the labour share under different theoretical assumptions

This appendix shows basic analytical results regarding the specification of the labour share under the various theoretical assumptions adopted in the main text.

Expression (4)

If labour market is perfectly competitive, firms optimization imposes firms to equate marginal product of labour wit the real wage: $w^{PC} = MPL$. With imperfect substitutions between capital and labour as in (1), the marginal productivity of labour is given by:

$$MPL = \left[\alpha K^{(\sigma-1)/\sigma} + (1-\alpha)(BL)^{(\sigma-1)/\sigma}\right]^{1/(\sigma-1)}\left[(1-\alpha)(BL)^{-1/\sigma}B\right]$$

Substituting the marginal productivity of labour into the definition of the labour share we obtain, after some simplification:

$$LS_{LATP}^{PC} = \frac{L*w}{Y} = \frac{(1-\alpha)(BL)^{(\sigma-1)/\sigma}}{\alpha K^{(\sigma-1)/\sigma} + (1-\alpha)(BL)^{(\sigma-1)/\sigma}} = \frac{(1-\alpha)}{Y^{(\sigma-1)/\sigma} + (BL)^{(\sigma-1)/\sigma}}$$

From the production function:

$$(BL)^{(\sigma-1)/\sigma} = \frac{Y^{(\sigma-1)/\sigma} - \alpha K^{(\sigma-1)/\sigma}}{(1-\alpha)}$$

Then:

$$LS_{LATP}^{PC} = \frac{(1-\alpha)}{Y^{(\sigma-1)/\sigma}}\frac{Y^{(\sigma-1)/\sigma} - \alpha K^{(\sigma-1)/\sigma}}{(1-\alpha)} = 1-\alpha\left(\frac{K}{Y}\right)^{(\sigma-1)/\sigma}$$

Following the same approach, one can derive equation 8.

Expression (5)

If labour market is perfectly competitive, optimizing behaviour by firms implies that $w^{PC} = MPL$. If the production function is CES with labour-augmenting technical progress as in (1), then the marginal productivity of labour is given by:

$$MPL = \frac{\sigma}{\sigma-1}\left[\alpha K^{(\sigma-1)/\sigma} + (1-\alpha)(BL)^{(\sigma-1)/\sigma}\right]^{1/(\sigma-1)}\left[(1-\alpha)\frac{\sigma-1}{\sigma}(BL)^{-1/\sigma}B\right]$$

Substituting the marginal productivity of labour into the definition of the labour share we obtain after some simplification:

$$LS_{LATP}^{PC} = \frac{Lw}{Y} = \frac{(1-\alpha)(BL)^{(\sigma-1)/\sigma}}{\alpha K^{(\sigma-1)/\sigma} + (1-\alpha)(BL)^{(\sigma-1)/\sigma}} = \frac{(1-\alpha)}{\alpha\left(\frac{K}{BL}\right)^{(\sigma-1)/\sigma} + (1-\alpha)}$$

Expression (6)

Starting from the production function given by (1), one may compute the value added per unit of efficient labour as:

$$\frac{Y}{BL}=\left[\alpha\left(\frac{K}{BL}\right)^{(\sigma-1)/\sigma}+(1-\alpha)\right]^{\sigma/(\sigma-1)}$$

And the average productivity of capital as:

$$\frac{Y\big/BL}{K\big/BL}=\left[\alpha+(1-\alpha)\left(\frac{K}{BL}\right)^{(1-\sigma)/\sigma}\right]^{\sigma/(\sigma-1)}$$

Which one may invert to obtain the capital-output ratio presented in equation (5):

$$\frac{K}{Y}=\left[\alpha+(1-\alpha)\left(\frac{K}{BL}\right)^{(1-\sigma)/\sigma}\right]^{\sigma/(1-\sigma)}$$

In the case of labour and capital augmenting technological progress we get

$$\frac{K}{Y}=\left[\alpha A^{(\sigma-1)/\sigma}+(1-\alpha)\left(\frac{K}{BL}\right)^{(1-\sigma)/\sigma}\right]^{\sigma/(1-\sigma)}$$

Expression (10)

It is straightforward to obtain expression by plugging the capital-output ratio according to equation (9) into the expression for the labour share given by (8).

Expression (12)

Consider a specification for the production function with labour heterogeneity like (11). The marginal productivity of skilled and unskilled labour is respectively given by:

$$MPL_S=\alpha B_S(B_S L_S)^{\rho-1}Y^{1/\sigma}\left[(AK)^{\rho}+(B_S L_S)^{\rho}\right]^{-\frac{1}{\varepsilon}}$$
$$MPL_U=(1-\alpha)B_U(B_U L_U)^{-1/\sigma}Y^{1/\sigma}$$

Under perfect competition the skill premium is:

$$\frac{w_S^{PC}}{w_U^{PC}}=\frac{\alpha}{1-\alpha}\frac{B_S}{B_U}l^{-\frac{1}{\sigma}}\left[1+\left(\frac{AK}{B_s L_s}\right)^{\rho}\right]^{\frac{1}{\varepsilon}}=\frac{\alpha}{1-\alpha}\frac{B_S}{B_U}l^{-\frac{1}{\sigma}}\left[(AK)^{\rho}+(B_s L_s)^{\rho}\right]^{\frac{1}{\varepsilon}}$$

Where $\varepsilon=\dfrac{\sigma\rho}{\sigma(\rho-1)+1}$. Thus, when the labour inputs are unchanged, an increase in the capital output is associated with an increase in the wage premium when $\sigma>\eta$. Under the assumption of perfect competition in the labour market, substitution of the MPL for wages in the expression for the labour share gives

$$LS^{PC}_{LCATP,LH} = \frac{\alpha B_u L_u \left(AK + B_u L_u\right)^{-1/\sigma} + \left(B_s L_s\right)^{1-(1/\sigma)}(1-\alpha)}{\alpha\left(AK + B_u L_u\right)^{(\sigma-1)/\sigma} + (1-\alpha)\left(B_s L_s\right)^{(\sigma-1)/\sigma}}$$

From the skill premium

$$\left[(AK)^{\rho} + \left(B_s L_s\right)^{\rho}\right]^{\varepsilon} = \left(\frac{\alpha}{1-\alpha}\right)^{\varepsilon}\left(B_S L_S\right)^{(\rho-1)\varepsilon}\left(B_U L_U\right)^{\frac{\varepsilon}{\sigma}}\omega^{-\varepsilon}$$

where ω is the wage premium in efficiency units. The capital-output ratio is:

$$\left(A\frac{K}{Y}\right)^{(\sigma-1)/\sigma} = \frac{(AK)^{(\sigma-1)/\sigma}}{\alpha\left[(AK)^{\rho} + \left(B_S L_S\right)^{\rho}\right]^{(\sigma-1)/\sigma} + (1-\alpha)\left(B_U L_U\right)^{(\sigma-1)/\sigma}}$$

Substituting $(AK)^{\rho} + \left(B_S L_S\right)^{\rho}$ in the previous equation according to its value into the preceding one and working out *AK* we find:

$$AK = Ak(1-\alpha)^{-\frac{\varepsilon}{\rho}}\left(B_U L_U\right)\left\{\alpha^{\varepsilon} l^{\frac{\sigma-\varepsilon}{\sigma}}\omega^{1-\varepsilon} + (1-\alpha)\right\}^{\frac{\sigma}{\sigma-1}}$$

Substituting *AK* into the expression $(AK)^{\rho} + \left(B_S L_S\right)^{\rho}$, one gets:

$$B_S L_S = \left(\frac{\alpha}{1-\alpha}\right)^{\varepsilon}\left(B_S L_S\right)^{(\rho-1)\varepsilon}\left(B_U L_U\right)^{\frac{\varepsilon}{\sigma}}\omega^{-\varepsilon} - \left(\frac{1}{1-\alpha}\right)^{\varepsilon}(Ak)^{\rho}\left(B_U L_U\right)^{\rho}\left\{\alpha^{\varepsilon} l^{\frac{\sigma-\varepsilon}{\varepsilon}}\omega^{1-\varepsilon} + (1-\alpha)^{\varepsilon}\right\}^{\frac{\sigma\rho}{\sigma-1}}$$

which one may substitute together with the expression above for $(AK)^{\rho} + \left(B_S L_S\right)^{\rho}$ into the labour share equation to obtain, after tedious calculations, equation (12):

$$LS^{PC}_{LCATP,LH} = 1 - (Ak)^{\rho}\left\{\alpha^{\varepsilon} + (1-\alpha)^{\varepsilon} l^{\frac{\varepsilon-\sigma}{\sigma}}\omega^{\varepsilon-1}\right\}^{\frac{\sigma\rho}{\varepsilon(\sigma-1)}}$$

<u>Expression (15)</u>

Consider a specification for gross output like (13). The first-order condition for profit maximization with respect to intermediate inputs *I* is given by:

$$(1-\gamma)\left(\frac{Y}{I}\right)^{\frac{1}{\theta}} = \frac{p_I}{p}$$

This condition can be solved for *I,* which yields: $I = (1-\gamma)^{\theta}\left(\frac{p_I}{p}\right)^{-\theta} Y$. Substituting the definition of Y and working out I one gets

$$I = \gamma^{\frac{\theta}{\theta-1}}(1-\gamma)^{\theta}\left(\frac{p_I}{p}\right)^{-\theta} \frac{\left(\alpha X^{(\sigma-1)/\sigma} + (1-\alpha)(B_U L_U)^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}}{\left[1-(1-\gamma)^{\theta}\left(\frac{p_I}{P}\right)^{\theta}\right]^{\frac{\theta}{\theta-1}}}$$

Substituting in the definition of value added given by (14) worked out from the first-order condition above one can get:

$$Y = \frac{\gamma^{\frac{\theta}{\theta-1}}}{\left[1-(1-\gamma)^{\theta}\left(\frac{p_I}{P}\right)^{\theta}\right]^{\frac{\theta}{\theta-1}}}\left(\alpha X^{(\sigma-1)/\sigma} + (1-\alpha)(B_U L_U)^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}$$

Define the labour share in value added as:

$$LS = \frac{w_u L_u + w_s L_s}{Y}$$

Following the same steps to find expression 12 one gets

$$LS^{PC}_{LCATP,LH,I} = \frac{w^{PC}_{u} L_u + w^{PC}_{s} L_s}{Y} = 1-(Ak)^{\rho}\frac{\gamma^{\frac{\theta}{\theta-1}}}{\left[1-\frac{(1-\gamma)^{\theta}}{(p_I/p)^{\theta-1}}\right]^{\frac{1}{\theta-1}}}\left\{\alpha^{\varepsilon}+(1-\alpha)^{\varepsilon} l^{\frac{\varepsilon-\sigma}{\sigma}}\omega^{\varepsilon-1}\right\}^{\frac{\sigma\rho}{\varepsilon(\sigma-1)}}$$

<u>Expression (19)</u>

Much in the same manner as we did with expression (15), consider a specification for gross output like (13). The first-order condition for profit maximization with respect to intermediate inputs I is given by:

$$\left[\gamma Y^{(\omega-1)/\omega} + (1-\gamma)I^{(\omega-1)/\omega}\right]^{\omega/(\omega-1)]-1}(1-\gamma)I^{[(\omega-1)/\omega]-1} = \frac{p_I}{p}$$

This condition can be solved for I, which yields:

$$I = \frac{\left(\alpha(AK + B_u L_u)^{(\sigma-1)/\sigma} + (1-\alpha)(B_s L_s)^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}}{\left(\frac{1}{\gamma}\right)^{\omega/(\omega-1)}\left[\left(\frac{p_I/p}{1-\gamma}\right)^{(\omega-1)} - (1-\gamma)\right]^{\omega/(\omega-1)}}$$

Substituting in the definition of value added given by (14) $\left[\gamma Y^{(\omega-1)/\omega} + (1-\gamma)I^{(\omega-1)/\omega}\right]^{\omega/(\omega-1)}$ worked out from the first-order condition above one can get:

$$Y = \frac{p_I}{p}\frac{\gamma}{(1-\gamma)}\left[\left(\alpha\left(AK + B_u L_u\right)^{(\sigma-1)/\sigma} + (1-\alpha)\left(B_s L_s\right)^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}\right]^{(\omega-1)/\omega} I^{1-[(\omega-1)/\omega]}$$

Now substitute I by its value according to the first-order condition above so as to get the following expression for value added:

$$Y = \gamma^{\omega/(\omega-1)}\left[1 - \frac{(1-\gamma)^{\omega}}{(p_I/p)^{\omega-1}}\right]^{-1/(\omega-1)}\left(\alpha\left(AK + B_u L_u\right)^{(\sigma-1)/\sigma} + (1-\alpha)\left(B_s L_s\right)^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}$$

The marginal productivity of unskilled and skilled labour are respectively given by:

$$MPL_u = \gamma^{\omega/(\omega-1)}\left[1 - \frac{(1-\gamma)^{\omega}}{(p_I/p)^{\omega-1}}\right]^{-1/(\omega-1)}\left[\alpha\left(AK + B_u L_u\right)^{(\sigma-1)/\sigma} + (1-\alpha)\left(B_s L_s\right)^{(\sigma-1)/\sigma}\right]^{1/(\sigma-1)}\alpha\left(AK + B_u L_u\right)^{-1/\sigma} B_u$$

$$MPL_s = \gamma^{\omega/(\omega-1)}\left[1 - \frac{(1-\gamma)^{\omega}}{(p_I/p)^{\omega-1}}\right]^{-1/(\omega-1)}\left[\alpha\left(AK + B_u L_u\right)^{(\sigma-1)/\sigma} + (1-\alpha)\left(B_s L_s\right)^{(\sigma-1)/\sigma}\right]^{1/(\sigma-1)}(1-\alpha)\left(B_s\right)^{1-(1/\sigma)}\left(L_s\right)^{-1/\sigma} B_u$$

In an imperfectly-competitive setting, profit maximisation by firms implies:

$$p = (1+\mu)MC = (1+\mu)\frac{W_u}{MPL_u} = (1+\mu)\frac{W_s}{MPL_s}$$

where W_u and W_s respectively denote nominal wages for skilled and unskilled workers. Rearranging terms:

$$\left(\frac{W_u}{p}\right)^{IC} = w_u^{IC} = \frac{MPL_u}{(1+\mu)}$$
$$\left(\frac{W_s}{p}\right)^{IC} = w_s^{IC} = \frac{MPL_s}{(1+\mu)}$$

Define the labour share in value added as:

$$LS_{LCATP,LH,I}^{IC} = \frac{w_u^{IC} L_u + w_s^{IC} L_s}{Y}$$

where w_u^{IC} and w_s^{IC} respectively denote real wages for skilled and unskilled workers under imperfect competition in the products market. Then substitute real wages for skilled and unskilled workers by the respective marginal productivity of skilled and unskilled labour corrected by the markup, thereby getting:

$$LS_{LCATP,LH,I}^{IC} = \left(\frac{1}{1+\mu}\right)\frac{\alpha B_u L_u\left(AK + B_u L_u\right)^{-1/\sigma} + \left(B_s L_s\right)^{1-(1/\sigma)}(1-\alpha)}{\alpha\left(AK + B_u L_u\right)^{(\sigma-1)/\sigma} + (1-\alpha)\left(B_s L_s\right)^{(\sigma-1)/\sigma}}$$

Define $\dfrac{MPL_s}{MPL_u}$ as:

$$\frac{MPL_d}{MPL_u}=\frac{(1-\alpha)B_s\left(B_sL_s\right)^{-1/\sigma}}{\alpha B_u\left(AK+B_uL_u\right)^{-1/\sigma}}$$

This equation can be rearranged as:

$$\left(AK+B_uL_u\right)=\left(\frac{\alpha}{1-\alpha}\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right)^{\sigma}B_sL_s$$

Compute the capital-output ratio as:

$$A\frac{K}{Y}=\frac{AK}{\gamma^{\omega/(\omega-1)}\left[1-\frac{(1-\gamma)^{\omega}}{\left(p_I/p\right)^{\omega-1}}\right]^{-1/(\omega-1)}\left[\alpha\left(AK+B_uL_u\right)^{(\sigma-1)/\sigma}+(1-\alpha)\left(B_sL_s\right)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}}$$

Substituting $\left(AK+B_uL_u\right)$ in the previous equation according to its value into the preceding one and working out AK we find:

$$AK=A\left(\frac{K}{Y}\right)\gamma^{\omega/(\omega-1)}\left[1-\frac{(1-\gamma)^{\omega}}{\left(p_I/p\right)^{\omega-1}}\right]^{-1/(\omega-1)}\left(B_sL_s\right)\left\{\alpha\left(\frac{\alpha}{1-\alpha}\right)^{\sigma-1}\left(\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right)^{\sigma-1}+(1-\alpha)\right\}^{\sigma/(\sigma-1)}$$

Substituting AK according to the previous equation into the expression above for $\left(AK+B_uL_u\right)$, one may work out B_uL_u as a function of B_sL_s and the wage premium:

$$B_uL_u=B_sL_s\left(\frac{\alpha}{1-\alpha}\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right)^{\sigma}-A\left(\frac{K}{Y}\right)\left(B_sL_s\right)\left\{\alpha\left(\frac{\alpha}{1-\alpha}\right)^{\sigma-1}\left(\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right)^{\sigma-1}+(1-\alpha)\right\}^{\sigma/(\sigma-1)}$$

which one may substitute together with the expression above for $\left(AK+B_uL_u\right)$ into the labour share equation to obtain the labour share presented in equation (18):

$$LS^{IC}_{LCATP,LH,I}=\left(\frac{1}{1+\mu}\right)\left(1-A\left(\frac{K}{Y}\right)\gamma^{\omega/(\omega-1)}\left[1-\frac{(1-\gamma)^{\omega}}{\left(p_I/p\right)^{\omega-1}}\right]^{-1/(\omega-1)}\left[\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right]^{-1}(1-\alpha)^{\sigma/(\sigma-1)}\left\{1+\left(\frac{\alpha}{1-\alpha}\right)^{\sigma}\left[\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right]^{\sigma-1}\right\}^{1/(\sigma-1)}\right)$$

Expression (27)

Define the labour share in value added as:

$$LS^{IC,EB}_{LCATP,LH,I}=\frac{w_uL_u+w_sL_s}{Y}$$

Substituting in the above expression real wages for unskilled and skilled labour according to expression (25) and assuming that $\beta_u = \beta_s$ one gets:

$$LS_{LCATP,LH,I}^{IC,EB} = \frac{\left[\beta\left(\frac{Y}{L_u}\right) + (1-\beta)\left(\frac{MPL_u}{1+\mu}\right)\right]L_u + \left[\beta\left(\frac{Y}{L_s}\right) + (1-\beta)\left(\frac{MPL_s}{1+\mu}\right)\right]L_s}{Y} = 2\beta + \left(\frac{1-\beta}{1+\mu}\right)\left[\frac{MPL_u L_u + MPL_s L_s}{Y}\right]$$

Then substitute real wages for skilled and unskilled workers by the respective marginal productivity of skilled and unskilled labour (corrected by the markup) calculated on the basis of a production function like (13), thereby getting:

$$LS_{LCATP,LH,I}^{IC,EB} = 2\beta + \left(\frac{1-\beta}{1+\mu}\right)\left[\frac{\alpha B_u L_u (AK + B_u L_u)^{-1/\sigma} + (B_s L_s)^{1-(1/\sigma)}(1-\alpha)}{\alpha (AK + B_u L_u)^{(\sigma-1)/\sigma} + (1-\alpha)(B_s L_s)^{(\sigma-1)/\sigma}}\right]$$

Define $\frac{MPL_s}{MPL_u}$ as:

$$\frac{MPL_s}{MPL_u} = \frac{(1-\alpha) B_s (B_s L_s)^{-1/\sigma}}{\alpha B_u (AK + B_u L_u)^{-1/\sigma}}$$

This equation can be rearranged as:

$$(AK + B_u L_u) = \left(\frac{\alpha}{1-\alpha}\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right)^{\sigma} B_s L_s$$

Compute the capital-output ratio as:

$$A\frac{K}{Y} = \frac{AK}{\gamma^{\omega/(\omega-1)}\left[1 - \frac{(1-\gamma)^{\omega}}{(p_I/p)^{\omega-1}}\right]^{-1/(\omega-1)}\left[\alpha (AK + B_u L_u)^{(\sigma-1)/\sigma} + (1-\alpha)(B_s L_s)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}}$$

Substituting $(AK + B_u L_u)$ in the previous equation according to its value into the preceding one and working out AK we find:

$$AK = A\left(\frac{K}{Y}\right)\gamma^{\omega/(\omega-1)}\left[1 - \frac{(1-\gamma)^{\omega}}{(p_I/p)^{\omega-1}}\right]^{-1/(\omega-1)}(B_s L_s)\left\{\alpha\left(\frac{\alpha}{1-\alpha}\right)^{\sigma-1}\left(\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right)^{\sigma-1} + (1-\alpha)\right\}^{\sigma/(\sigma-1)}$$

Substituting AK according to the previous equation into the expression above for $(AK + B_u L_u)$, one may work out $B_u L_u$ as a function of $B_s L_s$ and $\frac{MPL_s}{MPL_u}$:

$$B_u L_u = B_s L_s \left(\frac{\alpha}{1-\alpha} \frac{B_u}{B_s} \frac{MPL_s}{MPL_u} \right)^{\sigma} - A\left(\frac{K}{Y}\right)(B_s L_s)\left\{ \alpha \left(\frac{\alpha}{1-\alpha} \right)^{\sigma-1} \left(\frac{B_u}{B_s} \frac{MPL_s}{MPL_u} \right)^{\sigma-1} + (1-\alpha) \right\}^{\sigma/(\sigma-1)}$$

which one may substitute together with the expression above for $\left(AK + B_u L_u\right)$ into the labour share equation to obtain the labour share presented in equation (23):

$$LS^{IC,EB}_{LCATP,LH,I} = \frac{w_u L_u + w_s L_s}{Y} = 2\beta + \frac{(1-\beta)}{(1+\mu)}\left(1 - A\left(\frac{K}{Y}\right)\gamma^{\omega/(\omega-1)}\left[1 - \frac{(1-\gamma)^{\omega}}{(p_I/p)^{\omega-1}}\right]^{-1/(\omega-1)}\left[\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right]^{-1}(1-\alpha)^{\sigma/(\sigma-1)}\left\{1 + \left(\frac{\alpha}{1-\alpha}\right)^{\sigma}\left[\frac{B_u}{B_s}\frac{MPL_s}{MPL_u}\right]^{\sigma-1}\right\}^{1/(\sigma-1)} \right)$$

Appendix 2: Comparative statics predictions of the labour share specification under different assumptions

Expression (4)

Let us consider the definition of the labour share in terms of the capital-output ratio given by expression (4) in the main text:

$$LS_{LATP} = 1 - \left[\alpha \left(\frac{K}{Y}\right)^{(\sigma-1)/\sigma}\right]$$

If we take the first derivative with respect to the capital-output ratio we get:

$$\frac{\partial LS_{LATP}}{\partial (K/Y)} = -\alpha \frac{\sigma - 1}{\sigma} \frac{1}{\left(\frac{K}{Y}\right)^{1/\sigma}} \text{ so if } \sigma > 1 \text{ then } \frac{\partial LS_{LATP}}{\partial (K/Y)} < 0 \text{; if } \sigma < 1 \text{ then } \frac{\partial LS_{LATP}}{\partial (K/Y)} > 0$$

If we take the second derivative with respect to the capital-output ratio we get:

$$\frac{\partial^2 LS_{LATP}}{\partial (K/Y)^2} = \alpha \frac{\sigma - 1}{\sigma^2} \frac{1}{\left(\frac{K}{Y}\right)^{(1+\sigma)/\sigma}} \text{ so if } \sigma > 1 \text{ then } \frac{\partial^2 LS_{LATP}}{\partial (K/Y)^2} > 0 \text{; if } \sigma < 1 \text{ then } \frac{\partial^2 LS_{LATP}}{\partial (K/Y)^2} < 0$$

This analysis tells us that, along the transitional dynamics, the labour share is decreasing and convex (increasing and concave) in the capital-output ratio if there is a high (low) degree of substitution between capital and labour, i.e., if *σ >1(σ<1)*.

Alternatively, one can link labour share dynamics off the steady state with capital deepening. Recall equation (5) in the main text:

$$LS_{LATP} = \frac{L * MP_L}{Y} = \frac{(1-\alpha)}{\alpha \left(\frac{K}{BL}\right)^{(\sigma-1)/\sigma} + (1-\alpha)}$$

If we take the first derivative with respect to the capital-labour ratio measured in efficiency units we get:

$$\frac{\partial LS_{LATP}}{\partial (K/BL)} = \frac{-\alpha(1-\alpha)\frac{\sigma-1}{\sigma}\left(\frac{K}{BL}\right)^{-1/\sigma}}{\left[\alpha\left(\frac{K}{BL}\right)^{(\sigma-1)/\sigma} + (1-\alpha)\right]^2} \text{ so if } \sigma > 1 \text{ then } \frac{\partial LS_{LATP}}{\partial (K/BL)} < 0 \text{; if } \sigma < 1 \text{ then } \frac{\partial LS_{LATP}}{\partial (K/BL)} > 0$$

If we take the second derivative with respect to the capital-labour ratio measured in efficiency units we get:

$$\frac{\partial^2 LS_{LATP}}{\partial (K/BL)^2} = \frac{\frac{1}{\sigma}\alpha(1-\alpha)\frac{\sigma-1}{\sigma}\left(\frac{K}{BL}\right)^{-(1/\sigma)-1}\left[\alpha\left(\frac{K}{BL}\right)^{(\sigma-1)/\sigma} + (1-\alpha)\right]^2 + 2\left[(1-\alpha)\alpha\frac{\sigma-1}{\sigma}\left(\frac{K}{BL}\right)^{-1/\sigma}\right]\left[\alpha\left(\frac{K}{BL}\right)^{(\sigma-1)/\sigma} + (1-\alpha)\right]\alpha\frac{\sigma-1}{\sigma}\left(\frac{K}{BL}\right)^{-1/\sigma}}{\left[\alpha\left(\frac{K}{BL}\right)^{(\sigma-1)/\sigma} + (1-\alpha)\right]^4}$$

Note that both the second term in the numerator and the denominator always have a positive sign, this implying that the sign of the second derivative above is given by the sign of the first term in the numerator. It is easy to see that if *σ >1,* then $\frac{1}{\sigma}\alpha(1-\alpha)\frac{\sigma-1}{\sigma}\left(\frac{K}{BL}\right)^{-(1/\sigma)-1}\left[\alpha\left(\frac{K}{BL}\right)^{(\sigma-1)/\sigma}+(1-\alpha)\right]^2>0$ and thus $\frac{\partial^2 LS_{LATP}}{\partial(K/BL)^2}>0$; conversely, if *σ<1* then, $\frac{1}{\sigma}\alpha(1-\alpha)\frac{\sigma-1}{\sigma}\left(\frac{K}{BL}\right)^{-(1/\sigma)-1}\left[\alpha\left(\frac{K}{BL}\right)^{(\sigma-1)/\sigma}+(1-\alpha)\right]^2<0$ and thus $\frac{\partial^2 LS_{LATP}}{\partial(K/BL)^2}<0$.

These calculations show that, along the transitional dynamics, the labour share is decreasing and convex (increasing and concave) in the capital-labour ratio -measured in efficiency units- if there is a high (low) degree of substitution between capital and labour, i.e., if *σ >1(σ<1).*

Expression (10)

Let us consider the definition of the labour share given by expression (10) in the main text:

$$LS_{LCATP}=\frac{(1-\alpha)}{\alpha\left(\frac{AK}{BL}\right)^{(\sigma-1)/\sigma}+(1-\alpha)}$$

If we take the first derivative with respect to capital-augmenting technical progress we get:

$$\frac{\partial LS_{LCATP}}{\partial A}=\frac{-\alpha(1-\alpha)\frac{\sigma-1}{\sigma}\left(\frac{AK}{BL}\right)^{-1/\sigma}\frac{K}{BL}}{\left[\alpha\left(\frac{AK}{BL}\right)^{(\sigma-1)/\sigma}+(1-\alpha)\right]^2}$$ so if *σ >1* then $\frac{\partial LS_{LCATP}}{\partial A}<0$; if *σ<1* then $\frac{\partial LS_{LCATP}}{\partial A}>0$

If we take the first derivative with respect to the capital-labour ratio measured in efficiency units we get:

$$\frac{\partial LS_{LCATP}}{\partial(K/BL)}=\frac{-\alpha(1-\alpha)\frac{\sigma-1}{\sigma}\left(\frac{AK}{BL}\right)^{-1/\sigma}A}{\left[\alpha\left(\frac{AK}{BL}\right)^{(\sigma-1)/\sigma}+(1-\alpha)\right]^2}$$ so if *σ >1* then $\frac{\partial LS_{LCATP}}{\partial(K/BL)}<0$; if *σ<1* then $\frac{\partial LS_{LCATP}}{\partial(K/BL)}>0$

This analysis tells us that the effects of capital-augmenting-technical progress and the capital-output ratio measured in efficiency units on the labour share have the same sign along the transitional dynamics, i.e., the labour share is decreasing (increasing) in capital-augmenting technical progress if there is a high (low) degree of substitution between capital and labour, i.e., if *σ >1(σ<1).* The same result applies to capital-augmenting technical progress.

Expression (12)

Consider the definition of the labour share given by expression (12) in the main text:

$$LS^{PC}_{LCATP,LH}=\frac{w_u^{PC}L_u+w_s^{PC}L_s}{Y}=1-(Ak)^{\rho}\left\{\alpha^{\varepsilon}+(1-\alpha)^{\varepsilon}l^{\frac{\varepsilon-\sigma}{\sigma}}\omega^{\varepsilon-1}\right\}^{\frac{\sigma\rho}{\varepsilon(\sigma-1)}}$$

Then, when the quantity of the two labour inputs and the wage premium remains unchanged,

$$\frac{\partial LS^{PC}_{LATP,LH}}{\partial k} = -\rho(Ak)^{\rho-1}\left\{\alpha^{\varepsilon} + (1-\alpha)^{\varepsilon} l^{\frac{\varepsilon-\sigma}{\sigma}} \omega^{\varepsilon-1}\right\}^{\frac{\sigma\rho}{\varepsilon(\sigma-1)}}$$

and $\frac{\partial LS^{PC}_{LATP,LH}}{\partial k} < (>) 0$ if $-\rho = -\frac{\eta - 1}{\eta} < (>) 0$ or if $\eta > (<) 1$.

Similarly, assuming that the labour supply is exogenously given, then first derivative of the wage share with respect to the wage premium is

$$\frac{\partial LS^{PC}_{LATP,LH}}{\partial(w_s/w_u)} = -(Ak)^{\rho}\frac{\sigma\rho(\varepsilon-1)}{\varepsilon(\sigma-1)}\left\{\alpha^{\varepsilon} + (1-\alpha)^{\varepsilon} l^{\frac{\varepsilon-\sigma}{\sigma}} \omega^{\varepsilon-1}\right\}^{\frac{\sigma\rho}{\varepsilon(\sigma-1)}-1}(1-\alpha)^{\varepsilon} l^{\frac{\varepsilon-\sigma}{\sigma}} \omega^{\varepsilon-2}.$$

Since $\frac{\sigma\rho}{(\sigma-1)}\frac{(\varepsilon-1)}{\varepsilon} = 1$, $\frac{\partial LS^{PC}_{LATP,LH}}{\partial(w_s/w_u)} < 0$.

Finally, for a given wage premium,

$\frac{\partial LS^{PC}_{LATP,LH}}{\partial l} = -(Ak)^{\rho}\frac{\sigma\rho(\varepsilon-\sigma)}{\sigma\varepsilon(\sigma-1)}\left\{\alpha^{\varepsilon} + (1-\alpha)^{\varepsilon} l^{\frac{\varepsilon-\sigma}{\sigma}} \omega^{\varepsilon-1}\right\}^{\frac{\sigma\rho}{\varepsilon(\sigma-1)}-1}(1-\alpha)^{\varepsilon} l^{\frac{\varepsilon-\sigma}{\sigma}-1} \omega^{\varepsilon-1}$ which is always negative since $-\frac{\rho(\varepsilon-\sigma)}{\varepsilon(\sigma-1)} = \rho - 1 = -\frac{1}{\eta} < 0$.

Expression (26)

Let us consider the definition of the labour share given by expression (26) in the main text:

$$LS^{IC,EB} = \frac{L * w^{IC,EB}}{Y} = \beta + (1-\beta)\frac{MPL}{(1+\mu)} * \frac{L}{Y} = \beta + (1-\beta)\frac{1}{(1+\mu)}\frac{L * MPL}{Y}$$

First, compute the partial derivative with respect to β:

$$\partial LS^{IC,EB} \big/ \partial\beta = 1 - \frac{1}{(1+\mu)}\frac{MPL}{Y/L} - \beta\frac{1}{(1+\mu)}\frac{\partial\left(MPL \big/ Y/L\right)}{\partial L^{*}}\frac{\partial L^{*}}{\partial\beta}$$

Note that, changes in β induce changes in the ratio of the marginal to the average product of labour insofar they affect the equilibrium level of employment. What do we know about the new labour market equilibrium? Coming back to graph 9, it is easy to see that an increase in β shifts the rent division curve rightwards towards the average product of labour and crosses the contract curve for higher levels of employment and the real wage. The new equilibrium now implies higher levels of both real wages and employment. Thus $\frac{\partial L^{*}}{\partial\beta} > 0$. Interestingly, unlike the right-to-manage framework, efficient bargaining can deliver higher real wages with no detriment for employment following an increase in the workers' bargaining power. Next, we ask ourselves about the sign of $\frac{\partial\left(\frac{MPL}{Y/L}\right)}{\partial L^*}$. For a CES specification like (7), $\frac{\partial\left(\frac{MPL}{Y/L}\right)}{\partial L^*} > 0$ if $\sigma > 1$ whereas $\frac{\partial\left(\frac{MPL}{Y/L}\right)}{\partial L^*} < 0$ if $\sigma < 1$. With this in mind, one can see that $\partial LS^{IC,EB} / \partial\beta > 0$ if $\sigma < 1$ whereas $\partial LS^{IC,EB} / \partial\beta < 0$ if $\sigma > 1$.

Second, compute now the partial derivative with respect to the reservation wage:

$$\partial LS^{IC,EB} / \partial RW = (1-\beta)\frac{1}{(1+\mu)}\frac{\partial\left(\frac{MPL}{Y/L}\right)}{\partial L^*}\frac{\partial L^*}{\partial RW}$$

The sign of this derivative therefore depends on the sign of the product $\frac{\partial\left(\frac{MPL}{Y/L}\right)}{\partial L^*}\frac{\partial L^*}{\partial RW}$. Again, changes in the reservation wage induce changes in the ratio of the marginal to the average product of labour insofar they affect the equilibrium level of employment. What do we know about the new labour market equilibrium? Coming back to graph 9, it is easy to see that an increase in the reservation wage leads to a leftwards shift of the contract curve. However, the rent division curve remains unaffected by changes in the reservation wage. As a result, the negotiated wages will increase, but by less than the full amount of the increase in the reservation wage. Thus the opportunity cost of not being employed has fallen, because the difference between the union wage and alternative remuneration has shrunk. The union can now offer its membership less assurance against the risk of not being employed, so negotiated employment falls in the new equilibrium. Thus $\frac{\partial L^*}{\partial RW} < 0$. As, for a CES specification like (7), $\frac{\partial\left(\frac{MPL}{Y/L}\right)}{\partial L^*} > 0$ if $\sigma > 1$ whereas $\frac{\partial\left(\frac{MPL}{Y/L}\right)}{\partial L^*} < 0$ if $\sigma < 1$, it is easy to see that $\partial LS^{IC,EB} / \partial RW > 0$ if $\sigma < 1$ whereas $\partial LS^{IC,EB} / \partial RW < 0$ if $\sigma > 1$.

Third, take the first derivative of (26) with respect to *μ:*

$$\partial LS^{IC,EB} / \partial\mu = -(1-\beta)\frac{1}{(1+\mu)^2}\frac{MPL}{Y/L} + (1-\beta)\frac{1}{(1+\mu)}\frac{\partial\left(\frac{MPL}{Y/L}\right)}{\partial L^*}\frac{\partial L^*}{\partial\mu}$$

The first term of the right-hand side is unambiguously negative. As regards the second term, a higher μ moves north-west up the contract curve while shifting the rent division curve downwards. This will have an ambiguous effect on wages, but will decrease employment unambiguously. Thus $\frac{\partial L^*}{\partial\mu} < 0$. As, for a CES specification like (7),

$\frac{\partial\left(MPL/_{Y/L}\right)}{\partial L^*} > 0$ if $\sigma > 1$ whereas $\frac{\partial\left(MPL/_{Y/L}\right)}{\partial L^*} < 0$ if $\sigma < 1$, it is easy to see that $\partial LS^{IC,EB}/_{\partial\mu} > 0$ if $\sigma < 1$ whereas $\partial LS^{IC,EB}/_{\partial\mu} < 0$ if $\sigma > 1$

Expression (30)

Let us consider the definition of the labour share given by expression (30) in the main text:

$$LS^{IC,EB} = \frac{L * w^{IC,EB}}{Y} = 1 - \frac{\kappa}{Y/L}$$

If we take the first derivative with respect to the workers' bargaining power we get:

$$\partial LS^{IC,EB}/_{\partial\beta} = \underbrace{\frac{\kappa}{(Y/L)^2}}_{(+)} \underbrace{\frac{\partial(Y/L)}{\partial L^*}}_{(-)} \underbrace{\frac{\partial L^*}{\partial\beta}}_{(+)} < 0$$

$$\partial LS^{IC,EB}/_{\partial RW} = \underbrace{\frac{\kappa}{(Y/L)^2}}_{(+)} \underbrace{\frac{\partial(Y/L)}{\partial L^*}}_{(-)} \underbrace{\frac{\partial L^*}{\partial RW}}_{(-)} > 0$$

$$\partial LS^{IC,EB}/_{\partial\mu} = \underbrace{\frac{\kappa}{(Y/L)^2}}_{(+)} \underbrace{\frac{\partial(Y/L)}{\partial L^*}}_{(-)} \underbrace{\frac{\partial L^*}{\partial\mu}}_{(-)} > 0$$

Note that the sign of $\frac{\kappa}{(Y/L)^2}$ is unambiguously positive, the signs of $\frac{\partial L^*}{\partial\beta}$, $\frac{\partial L^*}{\partial RW}$ and $\frac{\partial L^*}{\partial\mu}$ were discussed in the context of the algebra developed under Expression (26). As regards the sign of $\frac{\partial(Y/L)}{\partial L^*}$, note that, for a CES specification like expression (7) in the main text:

$$\frac{\partial(Y/L)}{\partial L^*} = \left[\alpha\left(\frac{AK}{BL^*}\right)^{(\sigma-1)/\sigma} + (1-\alpha)\right]^{[(\sigma-1)/\sigma]-1} \alpha\left(\frac{AK}{BL^*}\right)^{(\sigma-1)/\sigma} \frac{1}{L^*}(-1) < 0$$

Expression (34)

Let us consider the definition of the labour share given by expression (34) in the main text:

$$LS^{IC,EB,AC} = \frac{L * w^{IC,EB,AC}}{Y} = \beta + (1-\beta)\frac{[MPL - AC(\Delta L)]}{(1+\mu)} * \frac{L}{Y} = \beta + (1-\beta)\frac{1}{(1+\mu)}\frac{L * [MPL - AC(\Delta L)]}{Y}$$

Recall the definition of the labour share given by expression (26) in the main text:

$$LS^{IC,EB} = \frac{L * w^{IC,EB}}{Y} = \beta + (1-\beta)\frac{MPL}{(1+\mu)} * \frac{L}{Y} = \beta + (1-\beta)\frac{1}{(1+\mu)}\frac{L * MPL}{Y}$$

By comparing the two expressions above we see that:

$$LS^{IC,EB,AC} = LS^{IC,EB} - \frac{(1-\beta)}{(1+\mu)}\frac{L * AC(\Delta L)}{Y}$$

which one may rewrite in terms of lagged employment L_{-1} as:

$$LS^{IC,EB,AC} = LS^{IC,EB} - \frac{(1-\beta)}{(1+\mu)}\frac{(L_{-1} + \Delta L) * AC(\Delta L)}{Y}$$

We wish to show that the labour share is a decreasing function of the change in employment. The second term's derivative is given by the following expression:

$$-\frac{(1-\beta)}{(1+\mu)}\frac{[AC'(\Delta L)(L_{-1} + \Delta L) + AC(\Delta L)]Y - A[C(\Delta L)(L_{-1} + \Delta L)Y'(\Delta L)]}{Y^2}$$

Note that the term between brackets has the same sign as its numerator, which in turn can be written as:

$$AC''(\Delta L)(L_{-1} + \Delta L)Y + AC'(\Delta L)[Y - (L_{-1} + \Delta L)Y'(\Delta L) > 0]$$

which is unambiguously positive given the adjustment cost function specified under (31). We have therefore shown that, when adjustment costs are a convex function in employment, the labour share is a decreasing function of the change in employment.

Expression (35)

Let us consider the definition of the labour share given by expression (35) in the main text:

$$LS^{IC,EB,AC} = \frac{L * w^{IC,EB,AC}}{Y} = \beta + (1-\beta)\frac{[MPL - AC(\Delta L)]}{(1+\mu)} * \frac{L}{Y} + \frac{AC(\Delta L)}{Y}$$

Recall the definition of the labour share given by expression (26) in the main text:

$$LS^{IC,EB} = \frac{L * w^{IC,EB}}{Y} = \beta + (1-\beta)\frac{MPL}{(1+\mu)} * \frac{L}{Y} = \beta + (1-\beta)\frac{1}{(1+\mu)}\frac{L * MPL}{Y}$$

By comparing the two expressions above we see that:

$$LS^{IC,EB,AC} = LS^{IC,EB} - \frac{(1-\beta)}{(1+\mu)}\frac{L * AC(\Delta L)}{Y} + \frac{AC(\Delta L)}{Y}$$

which one may rewrite in terms of lagged employment L_{-1} as:

$$LS^{IC,EB,AC} = LS^{IC,EB} - \frac{(1-\beta)}{(1+\mu)} \frac{(L_{-1}+\Delta L) * AC(\Delta L)}{Y} + \frac{AC(\Delta L)}{Y}$$

We wish to show that the labour share is a decreasing function of the change in employment. We have already shown (see calculations under expression (34)) that the second term's derivative is unambiguously positive in the change in employment. We focus now on the third term, whose derivative with respect to a change in employment is given by the following expression:

$$\frac{AC'(\Delta L)Y - AC(\Delta L)Y'(\Delta L)}{Y^2}$$

Note that the term between brackets has the same sign as its numerator, which will be negative if and only if

$$AC'(\Delta L)Y < AC(\Delta L)Y'(\Delta L)$$

or

$$\frac{Y}{Y'(\Delta L)} < \frac{AC(\Delta L)}{AC'(\Delta L)}$$

We have therefore shown that, when adjustment costs are a convex function in employment, the labour share is a decreasing function of the change in employment.