The economic effects of unilateral reduction of emission permits in an integrated world economy: Theoretical and numerical analysis

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Abstract

This paper provides an international–economics rationale for the envisioned strengthening of the European emission trading system. Our hypothesis is that the foreign net asset position of an economic area influences its climate policy. We evaluate the hypothesis in a stylized two-country Overlapping Generations model with producer greenhouse gas emissions. We find that if one country unilaterally reduces her domestic emission permits level, this country faces an overall welfare loss independently of her foreign net asset position. However, the welfare loss is significantly smaller if the unilateral climate policy is performed by a net creditor like the EU–15.

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1 Introduction

With worldwide integrated (financial) capital markets and steadily integrating commodity markets the interdependency between national environmental policies is presumably increasing. As, however, the example of climate policy shows, growing policy interdependence is not met by forced international policy coordination but on the contrary unilateral policy approaches, like e.g. the envisioned strengthening of the European emissions trading system (ETS), are gaining prominence. The question arises whether there is any economic rationale for the unilateral approach to climate policy in an interdependent world economy.

To start with answering the main question, let us focus on the EU–US divide in climate policy. The surprising fact is that two equally developed economic areas perform such different climate policies. Disregarding politico-economic differences, it is thus natural to suggest that differences in the international position of each area might explain the divide in climate policy. As is well known, there are pronounced differences in the foreign net asset position between the US and other advanced countries: while the US (and the less developed countries) are the net debtors of the world economy, the other advanced countries (and oil-exporting countries) are net creditors (see IMF, 2006, 71). The question therefore arises whether the differences in national climate policies can be traced back to differences in the foreign net asset position of the involved countries.

Previously, the role of the foreign net asset position for the international effects of unilateral fiscal policy has been discussed by several authors as unprecedented levels of budget deficits in the USA coincided with a sharp appreciation of the dollar vis-à-vis other major currencies in the 1980ies (e.g., Feldstein, 1986; Frenkel and Razin, 1986; Zee, 1987). One major finding in this literature is that the effect of a unilateral government debt expansion in one of a two-country world economy on the real exchange rate (= external terms of trade in a two-country world) depends on her foreign net asset position (Zee, 1987): if Home is a net creditor, a government debt expansion by Home leads to an improvement in its terms of trade while its terms of trade deteriorate if Home is a net debtor.

Secondly, unilateral fiscal expansion affects also capital accumulation (Lipton and Sachs, 1983). Zee (1987) and Lin (1994) find that a higher level of government debt leads to lower steady state capital stocks in both countries while the steady state interest rates increase. Thirdly, as concerns the welfare effects of public debt expansion, Persson (1985) showed in a one-good, two-country OLG model that the impact of higher public debt on steady state welfare depends on the net foreign asset position of the more indebted country. In a two–good infinitely lived agent setting, Ono and Shibata (2005) decompose the overall welfare impacts caused by unilateral fiscal policy into three effects, a terms-of-trade effect, a capital accumulation effect (caused by changes in the interest rate), and a wealth effect (direct income loss due to a shift in the tax burden), leading to an ambiguous net welfare effect for Home and for Foreign.

The focus of this paper is thus to investigate the effects on terms of trade, capital accumulation and welfare caused by a unilateral approach to climate policy, namely a reduction in emission permits. In addition to these substantial questions, this paper focuses on methodological issues of applied dynamic policy modeling. In contrast to static CGE models, the existence of steady–state solutions and the dynamic stability of the intertemporal equilibrium dynamics must not simply be assumed in full–blown dynamic CGE model of the world economy. This is particularly true with respect to multi-country, overlapping generations (OLG) CGE models of the world economy which depart from the Small–Open–Economy and the Armington (1969) assumption. The reason is that even in small–scale two–country models of an interdependent world economy existence and stability issues are hitherto largely unexplored (see for some advances Brecher *et al.* (2005) for two–country ILA models and Farmer and Zotti (2007) for two–country OLG models).

In a closely related paper Farmer *et al.* (2008) provide sufficient conditions for the existence of multiple nontrivial steady states and the saddle–path of one nontrivial steady state in a two–country, two-good extension of Diamond's (1965) OLG economy with productive capital and internal government debt. To obtain an analytically tractable intertemporal general equilibrium, Farmer *et al.* (2008) assume identical preferences as well as identical production functions across countries. Only foreign net asset positions differ across countries as a consequence of the assumption of internationally varying per-capita public debt magnitudes. Farmer *et al.* (2008) find that the reduction of greenhouse gas emission permits in Home triggers off an upward jump of the terms of trade in the shock period followed by their continuous increase along the stable manifold towards the new steady state. Alongside with rising terms of trade of Home, capital accumulation in both countries decrease, significantly more in Home than in Foreign. Due to the log-linear specifications of the intertemporal utility function and the Cobb-Douglas specification of the production function, these comparative steady states results hold true independent of the foreign net asset position of Home. As concerns the effects on steady-state welfare of the Home household, the foreign net asset position does not impact on the sign of the welfare change but on its magnitude. If Home is a net creditor, the welfare loss is much smaller than in the case of Home being a net debtor.

This paper mainly departs from Farmer *et al.* (2008) by assuming that countries differ not only in the magnitudes of public debt per capita but also in production technology. In particular, we assume here that the production elasticity of capital in the Cobb-Douglas production function differs across countries (see Lin, 1994, for a similar specification). Although the assumption of different technologies does not lead to qualitatively different conclusions from those under identical technologies, the existence and dynamic stability of steady state solutions as well as the transitional dynamics from one to another steady state cannot be derived in an analytically explicit way. Hence, we have to resort to a theoretically inspired numerical analysis especially with respect to the transitional dynamics.

Utilizing the results of the numerical solution algorithm, we are able to show that for not too divergent production elasticities of capital the main results of the model with identical technologies can be qualitatively maintained. In particular, we find that the magnitude of Home's welfare loss of a unilateral reduction of emission permits in Home is significantly larger if Home is a net debtor than a net creditor. Applying this insight to the EU–US divide with respect to climate policy, helps to understand better economically why the EU–15 representatives press ahead with emission reduction objectives which the US is not ready to accept: the EU–15 as a foreign net creditor will not envisage large welfare losses while the US—should she reduce unilaterally emission permits—will.

This paper has seven sections. In the next section we provide a description of the two–country, two–good model with nationally tradable emission permits. This will be followed by the derivation of the intertemporal equilibrium dynamics and of the steady states in Section 3. Section 4 investigates the stability of the equilibria and analyzes the long–run effects, caused by the reduction in the permit volume in one country, on the terms of trade, and on domestic and foreign capital accumulation. The transitional effects of of such a unilateral permit reduction are analyzed in Section 5, while the welfare effects for both countries are investigated in Section 6. Section 7 summarizes our results and concludes.

2 The basic model

We consider a two–country two–good OLG model and a permit market for greenhouse gas emissions, whereby the two–country OLG model is an extension of Farmer and Zotti (2007). Each country's economy is composed of perfectly competitive firms and finitely lived agents. Each country (Home and Foreign) produces a specific good, which can be used for the purpose of consumption as well as for investment.[1]

The present paper extends the single country model of Ono (2002) towards two economically interdependent economies. In an closed economy OLG framework, Ono (2002) finds that a permit reduction policy can decrease both capital accumulation and environmental quality in the long run, even though pollution is reduced by levying permits. Jouvet *et al.* (2005*b*) investigate the effects of emission permits also in a closed economy and find that only auctioning permits is efficient, but not grandfathering. In a two–country OLG

[1]This assumption is a deviation of our model from the assumptions of the Heckscher–Ohlin model. Our model can be regarded as an OLG analogue to Ono and Shibatas' (2005) dynamic Heckscher–Ohlin ILA model.

model with a single commodity, Jouvet *et al.* (2005*a*) examine the equilibrium with an international market of tradable permits and show that the optimal allocation of emission permits across countries is proportional to efficiency labor. The present paper closes the research gap by assuming two nationally separated permit markets without emissions trading across countries.

2.1 Production and pollution

Production in each country is specified by a Cobb–Douglas production function with constant returns to scale. As in Ono (2002), total output in Home X_t is determined by three production factors, namely capital services K_t, labor services L_t, and pollution flow P_t.[2] Defining inputs and output in per capita terms $x_t \equiv X_t/L_t$, $k_t \equiv K_t/L_t$ and $p_t \equiv P_t/L_t$, yields per capita output as:

$$x_t = M\,(k_t)^{\alpha_K}\,(p_t)^{\alpha_P}\,, \tag{1}$$

where M denotes a productivity scalar. Total revenues of production (where the output is set as numeraire) are spent on factor costs of labor $w_t L_t$ and capital $q_t K_t$. Furthermore, following the specification of the permit market in Ono (2002), in each period emission quotas S are distributed free of charge to the firms. If the representative firm requires more allowances, it has to buy additional permits on the market for a price of e_t each. In case of excess permits, the firm gains revenues from selling them on the market. Assuming that labor supply L_t is normalized to one, the firms profit maximizing problem in per capita terms reads as follows:

$$\pi_t = M\,(k_t)^{\alpha_K}\,(p_t)^{\alpha_P} - q_t k_t - w_t + e_t\,(S - p_t.)\,. \tag{2}$$

$$\pi_t^* = M\,(k_t^*)^{\alpha_K}\,(p_t^*)^{\alpha_P} - q_t^* k_t^* - w_t^* + e_t^*\,(S^* - p_t^*.)\,.$$

[2]Ono (2002) shows that, by means of rescaling parameters, a production function which has constant returns to scale in using labor and capital as inputs and emission intensity as a scaling factor, can be transformed into a three–factor constant returns to scale production function with labor, capital and pollution as inputs.

The first order conditions of the firm in Home for an interior solution read as follows:

$$q_t = \alpha_K M \left(k_t\right)^{\alpha_K - 1} \left(p_t\right)^{\alpha_P} = \alpha_K \frac{x_t}{k_t}, \tag{3}$$

$$w_t = \left(1 - \alpha_K - \alpha_P\right) M \left(k_t\right)^{\alpha_K} \left(p_t\right)^{\alpha_P} = \left(1 - \alpha_K - \alpha_P\right) x_t, \tag{4}$$

$$e_t = \alpha_P M \left(k_t\right)^{\alpha_K} \left(p_t\right)^{\alpha_P - 1} = \alpha_P \frac{x_t}{p_t}. \tag{5}$$

Profit maximization implies that the firm's revenues net of the payments to production factors lead to a profit equal to the initial endowment of permits, $e_t S$ (Ono, 2002). This profit is collected by the government and reimbursed to the young households.[3]

Production in Foreign (denoted by an asterisk) is specified by a constant–returns–to–scale Cobb-Douglas production technology, too. Thus, total Foreign output in per capita terms is determined by $y_t^* = M(k_t^*)^{\alpha_K^*}(p_t^*)^{\alpha_P^*}$. The first order conditions in Foreign are:

$$q_t^* = \alpha_K^* M \left(k_t^*\right)^{\alpha_K^* - 1} \left(p_t^*\right)^{\alpha_P^*} = \alpha_K^* \frac{y_t^*}{k_t^*}, \tag{6}$$

$$w_t^* = \left(1 - \alpha_K^* - \alpha_P^*\right) M \left(k_t^*\right)^{\alpha_K^*} \left(p_t^*\right)^{\alpha_P^*} = \left(1 - \alpha_K^* - \alpha_P^*\right) y_t^*, \tag{7}$$

$$e_t^* = \alpha_P^* M \left(k_t^*\right)^{\alpha_K^*} \left(p_t^*\right)^{\alpha_P^* - 1} = \alpha_P^* \frac{y_t^*}{p_t^*}. \tag{8}$$

2.2 Intertemporal utility maximization and international asset allocation

Household preferences are identical across and within periods and countries. As is standard in Diamond (1965)–type OLG models, each generation lives for two periods, one working period and one retirement period. Furthermore, and in contrast to Ono (2002) and John and Pecchenino (1994), households consume in both periods. Lifetime utility depends on consumption during the working period, composed of the consumption goods of both countries, x_t^1 and y_t^1, which are weighted by expenditure shares ζ and $(1-\zeta)$, and consumption during the retirement period, x_{t+1}^2 and y_{t+1}^2. The time preference factor is

[3] In essence, this particular modeling of the permit system guarantees that the subsidy is non–distortionary.

denoted by β, $0 < \beta \leq 1$. For the sake of analytical tractability, household's preferences are represented by a log–linear utility function:

$$U_t = \zeta \ln x_t^1 + (1-\zeta) \ln y_t^1 + \beta \left[\zeta \ln x_{t+1}^2 + (1-\zeta) \ln y_{t+1}^2 \right]. \tag{9}$$

In maximizing intertemporal utility the young household is constrained by a budget constraint in each period of life. When young, wage income w_t, net of a lump–sum tax τ_t imposed by the government is spent by the household on consumption of the Home good x_t^1 and the Foreign good y_t^1, whereby the expenditures on the Foreign good in terms of the Home commodity are equal to y_t^1/h_t with h_t denoting the external terms of trade of Home (units of Foreign good per one unit of Home good). Furthermore, for transferring income to their retirement period, young households save in terms of capital k_{t+1} and in terms of bonds of Home b_{t+1}^H and of Foreign $b_{t+1}^{*,H}$ times $1/h_t$. From saving, the old household gains interest income, where i_{t+1} and i_{t+1}^* denote the interest rates in Home and Foreign. When old, the household spends interest income and capital on consumption, again for the Home and Foreign good (x_{t+1}^2 and y_{t+1}^2, respectively). Thus, the first period budget constraint is given by:

$$x_t^1 + \frac{1}{h_t} y_t^1 + s_t = w_t - \tau_t, \tag{10}$$

where savings are defined as

$$s_t \equiv k_{t+1} + b_{t+1}^H + (1/h_t)\, b_{t+1}^{*,H}. \tag{11}$$

After taking account of the no–arbitrage condition for the capital market in Home ($1+i_t = q_t, \forall t$), the second period budget constraint is given by:

$$x_{t+1}^2 + \frac{1}{h_{t+1}} y_{t+1}^2 = (1+i_{t+1}) \left[k_{t+1} + b_{t+1}^H \right] + \left(1 + i_{t+1}^* \right) \frac{1}{h_{t+1}} b_{t+1}^{*,H}. \tag{12}$$

Since government bonds are perfectly mobile across Home and Foreign, the real interest parity condition holds across both countries

$$\left(1 + i_{t+1}^* \right) \frac{h_t}{h_{t+1}} = (1 + i_{t+1}). \tag{13}$$

Taking account of (11) and (13), the first and second period budget constraint can be collapsed to the following intertemporal budget constraint for Home:

$$x_t^1 + \frac{1}{h_t} y_t^1 + \frac{x_{t+1}^2}{(1+i_{t+1})} + \frac{1}{h_{t+1}} \frac{y_{t+1}^2}{(1+i_{t+1})} = w_t - \tau_t. \tag{14}$$

Since Home and Foreign consumers are specified as having identical preferences, the representative household in Foreign maximizes the following intertemporal utility function

$$U_t^* = \zeta \ln x_t^{*,1} + (1-\zeta) \ln y_t^{*,1} + \beta \left[\zeta \ln x_{t+1}^{*,2} + (1-\zeta) \ln y_{t+1}^{*,2} \right] \tag{15}$$

subject to the two budget constraints

$$h_t x_t^{*,1} + y_t^{*,1} + s_t^* = w_t^* - \tau_t^*, \tag{16}$$

$$h_{t+1} x_{t+1}^{*,2} + y_{t+1}^{*,2} = \left(1 + i_{t+1}^*\right) \left(k_{t+1}^* + b_{t+1}^{*,F}\right) + h_{t+1} \left(1 + i_{t+1}\right) b_{t+1}^F, \tag{17}$$

where savings are defined by

$$s_t^* \equiv k_{t+1}^* + b_{t+1}^{*,F} + h_t b_{t+1}^F. \tag{18}$$

Maximizing (9) subject to (14), and (15) subject to (16)–(17) gives the optimal consumption quantities as follows:

$$x_t^1 = \frac{\zeta}{1+\beta} \left(w_t - \tau_t\right), \qquad x_t^{*,1} = \frac{\zeta}{1+\beta} \frac{\left(w_t^* - \tau_t^*\right)}{h_t}, \tag{19}$$

$$y_t^1 = \frac{1-\zeta}{1+\beta} \left(w_t - \tau_t\right) h_t, \qquad y_t^{*,1} = \frac{1-\zeta}{1+\beta} \left(w_t^* - \tau_t^*\right), \tag{20}$$

$$x_{t+1}^2 = \frac{\beta\zeta}{1+\beta} \left(1 + i_{t+1}\right) \left(w_t - \tau_t\right), \qquad x_{t+1}^{*,2} = \frac{\beta\zeta}{1+\beta} \left(1 + i_{t+1}^*\right) \frac{\left(w_t^* - \tau_t^*\right)}{h_{t+1}}, \tag{21}$$

$$y_{t+1}^2 = \frac{\beta(1-\zeta)}{1+\beta} \left(1 + i_{t+1}\right) \left(w_t - \tau_t\right) h_{t+1}, \qquad y_{t+1}^{*,2} = \frac{\beta(1-\zeta)}{1+\beta} \left(1 + i_{t+1}^*\right) \left(w_t^* - \tau_t^*\right). \tag{22}$$

Reformulating the first period budget constraint for s_t and substituting the optimal consumption quantities for x_t^1 and y_t^1 gives

$$s_t = \sigma \left(w_t - \tau_t\right), \quad \sigma \equiv \frac{\beta}{(1+\beta)}. \tag{23}$$

Denote total bonds issued in Home by b_t and in Foreign by b_t^*. Then market clearing for Home and Foreign bonds demands

$$b_t = b_t^H + b_t^F, \qquad b_t^* = b_t^{*,H} + b_t^{*,F}. \tag{24}$$

To eliminate w_t and τ_t in (23), we first write down the government budget constraints:

$$\tau_t + e_t S = i_t b_t, \qquad \tau_t^* + e_t^* S^* = i_t^* b_t^*. \tag{25}$$

The left hand side of (25) denotes the revenues from tax income and permit trading, while the right hand side gives the interest payments to the bond holders. Acknowledging the no–arbitrage condition for the capital market in Home ($1 + i_t = q_t, \forall t$), the market clearing for the permit market in Home ($p_t = S, \forall t$), and substituting for the firm's first order conditions (3)–(5) yields an expression for s_t which depends only on k_t and exogenously given parameters:

$$s_t = \sigma \left[(1 - \alpha_K) M (k_t)^{\alpha_K} (S)^{\alpha_P} - b_t \left(\alpha_K M (k_t)^{\alpha_K - 1} (S)^{\alpha_P} - 1 \right) \right], \tag{26}$$

and, similarly for Foreign, optimal savings s_t^* are a function of k_t^* only:

$$s_t^* = \sigma \left[(1 - \alpha_K^*) M (k_t^*)^{\alpha_K^*} (S^*)^{\alpha_P^*} - b_t^* \left(\alpha_K^* M (k_t^*)^{\alpha_K^* - 1} (S^*)^{\alpha_P^*} - 1 \right) \right]. \tag{27}$$

2.3 Market clearing and international trade

For the national markets, the following additional assumptions apply. The government runs a "constant–stock" fiscal policy and thus $b_{t+1} = b_t = b, \forall t$, and $b_{t+1}^* = b_t^* = b^*, \forall t$, respectively (as in Diamond, 1965). Without loss of generality, the rate of depreciation is set to be one, therefore the investment of the current period builds next period's capital stock. We further assume that there is no population growth.

To complete the model, further market clearing conditions have to be specified. First, the national product markets have to be cleared. The market clearing condition requires output per capita in Home to be equal to the sum of consumption per capita of working and retired households in Home and in Foreign, and capital per capita in period $t + 1$ which is built by investments of period t:

$$x_t = x_t^1 + x_t^2 + k_{t+1} + x_t^{*,1} + x_t^{*,2}, \quad \forall t, \tag{28}$$

while the product market clearing condition in Foreign reads as follows:

$$y_t^* = y_t^{*,1} + y_t^{*,2} + k_{t+1}^* + y_t^1 + y_t^2, \quad \forall t. \tag{29}$$

Clearing of the world "financial" capital market requires the supply of savings to be equal to the demand for savings (from (11), (18), and (24)):

$$s_t + \frac{1}{h_t} s_t^* = k_{t+1} + b + \frac{1}{h_t} \left[k_{t+1}^* + b^* \right], \quad \forall t. \tag{30}$$

Defining the net asset positions of Home and Foreign as

$$\phi_{t+1} \equiv k_{t+1} + b - s_t, \qquad \phi_{t+1}^* \equiv k_{t+1}^* + b^* - s_t^*, \tag{31}$$

gives the following relationship between Home's terms of trade and the net asset positions of Foreign and Home:

$$h_t = -\frac{k_{t+1}^* + b^* - s_t^*}{k_{t+1} + b - s_t} \equiv -\frac{\phi_{t+1}^*}{\phi_{t+1}}, \quad \forall t. \tag{32}$$

Since $h_t > 0$, either $\phi_{t+1} > 0$ and consequently $\phi_{t+1}^* < 0$, Home is a net debtor and Foreign a net creditor, or $\phi_{t+1} < 0$ and $\phi_{t+1}^* > 0$ which means that Home is a net creditor and Foreign a net debtor.

3 Intertemporal equilibrium dynamics and the steady state

In this section, the dynamic equations describing the intertemporal equilibrium of our two–country, two–good model are presented and the steady state is derived. Considering the two national no–arbitrage conditions of capital markets ($1+i_t = q_t, 1+i_t^* = q_t^*, \forall t$) and the firms' first order conditions (3) and (6) in the international interest parity condition (13), the equation of motion of the terms of trade follows

$$h_{t+1} = h_t \frac{\left(1 + i_{t+1}^*\right)}{\left(1 + i_{t+1}\right)} = h_t \frac{\alpha_K^* \left(k_{t+1}^*\right)^{\alpha_K^* - 1} \left(S^*\right)^{\alpha_P^*}}{\alpha_K \left(k_{t+1}\right)^{\alpha_K - 1} \left(S\right)^{\alpha_P}}. \tag{33}$$

By inserting the optimal savings functions (26) and (27) into the international capital market clearing condition (30), we obtain the second equation of motion:

$$h_t k_{t+1} + k_{t+1}^* = h_t \left[\sigma_0 \left(k_t\right)^{\alpha_K} - b \left(\sigma \ i_t + 1\right) \right] + \sigma_0^* \left(k_t^*\right)^{\alpha_K} - b^* \left(\sigma \ i_t^* + 1\right), \tag{34}$$

where $\sigma_0 \equiv (1-\alpha_K)\,\sigma M S^{\alpha_P}$ and $\sigma_0^* \equiv (1-\alpha_K^*)\,\sigma M\,(S^*)^{\alpha_P^*}$.

Multiplying the national product market clearing condition of Home (28) by h_t and the one of Foreign (29) by $\zeta/(1-\zeta)$, inserting optimal consumptions of households in Home and Foreign, and subtracting the second from the first gives the combined product market clearing condition:

$$h_t k_{t+1} - \frac{\zeta}{(1-\zeta)} k_{t+1}^* = h_t M\,(k_t)^{\alpha_K}\,(S)^{\alpha_P} - \frac{\zeta}{(1-\zeta)} M\,(k_t^*)^{\alpha_K^*}\,(S^*)^{\alpha_P^*}\,. \tag{35}$$

The dynamic system for the terms of trade, h_t, and for the capital stocks in Home and Foreign (k_{t+1} and k_{t+1}^* respectively) are thus described by Equations (33), (34), and (35).

A stationary state of the discrete dynamical system (33), (34) and (35) is defined by

$$(h,k,k^*) = (h_t,k_t,k_t^*) = \left(h_{t+1},k_{t+1},k_{t+1}^*\right).$$

Under the presumption of parameter sets which ensure the existence of at least one non–trivial steady state, these dynamic equations can be reduced to a system of three equations which determine the endogenous variables k, h and k^*:

$$KK: \quad h = -\frac{\phi^*}{\phi} \equiv -\frac{\left[k^* - M\,(k^*)^{\alpha_K^*-1}\,(S^*)^{\alpha_P^*}\,\sigma\left((1-\alpha_K^*)k^* - b^*\alpha_K^*\right) + (1-\sigma)b^*\right]}{\left[k - M\,(k)^{\alpha_K-1}\,(S)^{\alpha_P}\,\sigma\left((1-\alpha_K)k - b\alpha_K\right) + (1-\sigma)b\right]}, \tag{36}$$

and

$$GG: \quad h = \frac{\zeta}{(1-\zeta)}\frac{H^*}{H} \equiv \frac{\zeta}{(1-\zeta)}\frac{\left[M\,(k^*)^{\alpha_K^*}\,(S^*)^{\alpha_P^*} - k^*\right]}{\left[M\,(k)^{\alpha_K}\,(S)^{\alpha_P} - k\right]}, \tag{37}$$

where

$$k^* = \left(\frac{\alpha_K^*}{\alpha_K}\right)^{\frac{1}{1-\alpha_K^*}} \left(\frac{(S^*)^{\alpha_P^*}}{(S)^{\alpha_P}}\right)^{\frac{1}{1-\alpha_K^*}} (k)^{\epsilon}, \quad \epsilon \equiv \frac{1-\alpha_K}{1-\alpha_K^*}. \tag{38}$$

Equation (36) can be interpreted as the geometrical locus of all pairs (k,h) which assures international capital market clearing. The equilibrium locus of this market in $k-h$ space will be labeled, in accordance with Zee (1987, 613), as the KK-curve. Equation (37) represents the equilibrium condition for the combined product market and will be labeled GG-curve. In order to illustrate the KK– and GG–curves it is necessary to specify the model parameters. To remain within the theoretical focus of this paper, the parameters of the model are chosen to roughly replicate the stylized facts of the world economy (see Table 1).

Table 1: Parameter specifications

Param.	Value	Description
β	0.6	time preference factor
ζ	0.5	expenditure share of domestic consumption
M	5	total factor productivity
α_K	$0.325^{\dagger}(0.3; 0.35)$	production elasticity of capital in Home
$\alpha_K^{\star}$	$0.325^{\dagger}(0.35; 0.3)$	production elasticity of capital in Foreign
α_P	0.1	production elasticity of emissions in Home
$\alpha_P^{\star}$	0.1	production elasticity of emissions in Foreign
S	1	emission permits level issued in Home
$S^{\star}$	1	emission permits level issued in Foreign
b	$0.1^{\dagger}(0.4)$	per-capita bond stock in Home if $\phi < 0$ ($\phi > 0$)
$b^{\star}$	$0.4^{\dagger}(0.1)$	per-capita bond stock in Foreign if $\phi^{*} > 0$ ($\phi^{*} < 0$)

† Base case; alternative values are given in brackets.

Figure 1 depicts four typical configurations of KK- and GG-curves. Inspection of the slopes reveals that the KK-curve is U–shaped if $\phi < 0$ and inverted U–shaped if $\phi > 0$. The GG-curve is positively sloped if $\alpha_K < \alpha_K^{*}$ and negatively sloped if $\alpha_K > \alpha_K^{*}$.[4] The existence of two non–trivial steady states is proven in Appendix A.1. For all four types of parameter constellations there is a lower and a higher steady state capital stock, k^L and k^H, respectively. The lower steady states refers to the first intersection point of the KK– and GG–loci in Figure 1, while the higher steady state corresponds to the second intersection point. For the following analysis, we will primarily focus on the higher steady state because this will be the stable equilibrium.

[4]For $\alpha_K = \alpha_K^{*}$, the GG-curve is horizontal.

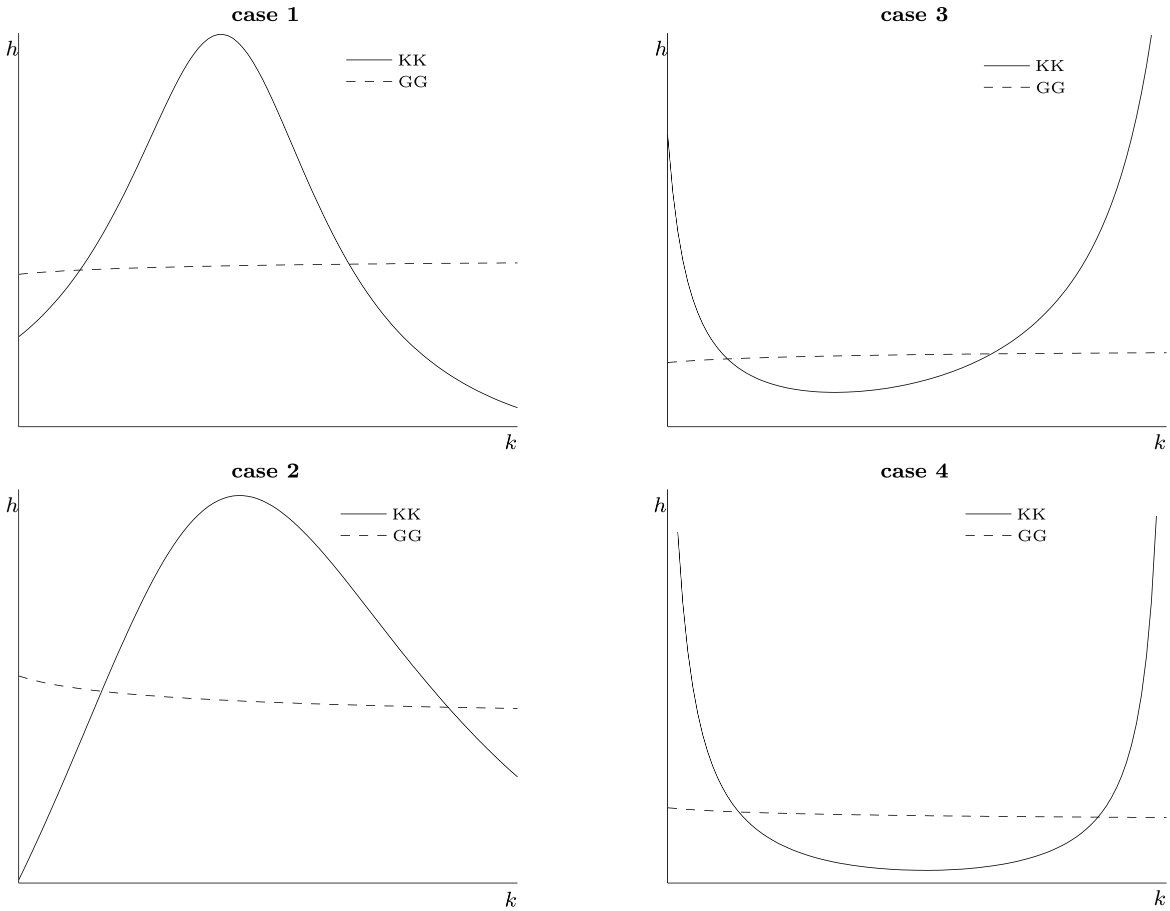

Figure 1: The KK- and GG-curves for case 1 ($\phi > 0, \alpha_K < \alpha_K^*$), case 2 ($\phi > 0, \alpha_K > \alpha_K^*$), case 3($\phi < 0, \alpha_K < \alpha_K^*$), and case 4 ($\phi < 0, \alpha_K > \alpha_K^*$).

4 Stability of steady states and the long–run effects of a unilateral permit reduction in Home

Being assured of the existence of two distinct steady–state solutions, the next step is to investigate the local stability of the two steady states. To this end, the equilibrium dynamics are linearly approximated in a small neighborhood of each of the two steady states. Due to the algebraic complexity of the Jacobian of the equilibrium dynamics around the steady states, the stability of the steady states can be proven only for small differences between α_K and α_K^* (see Appendix A.2).

For the parameter set as specified in Table 1, the calculation of the Jacobian matrix of the dynamic system in the two steady states indicates that in the steady state with the lower capital intensity (k^L) two eigenvalues are larger and one is less than unity, while in the steady state with the higher capital intensity (k^H) one eigenvalue is larger and two are less than unity. Thus, the former steady state is saddle path unstable while the latter is saddle path stable.[5] In Figure 1, this stable steady state associated with the higher capital intensity k^H can be found as the second point of intersection of the GG– and KK–curve (for all four cases).

Knowing that the steady state associated with k^H qualifies as being locally stable, we can now turn to the investigation of the long–run effects of a unilateral permit reduction in Home on the main variables of our model. To pursue this objective, we assume that Home implements a more stringent permit policy ($S \downarrow$) while the permit policy of Foreign remains unchanged at S^*. We further assume that the shock is unannounced and permanent such that the households and firms cannot act anticipatory prior to the shock (e.g., by adjusting their saving decision).

To determine the effects of a marginal reduction of S on the three dynamic variables, we totally differentiate (36), (36), and (38), with respect to S. This yields:

$$\begin{bmatrix} \phi & h\frac{\partial \phi}{\partial k} + \epsilon \frac{k^*}{k}\frac{\partial \phi^*}{\partial k^*} \\ (1-\zeta)H & (1-\zeta)h\frac{\partial H}{\partial k} - \zeta\epsilon\frac{k^*}{k}\frac{\partial H^*}{\partial k^*} \end{bmatrix} \begin{bmatrix} \mathrm{d}h \\ \mathrm{d}k \end{bmatrix} = \begin{bmatrix} \frac{\alpha_P}{1-\alpha^*}\frac{\partial \phi^*}{\partial k^*}\frac{k^*}{S} - h\frac{\partial \phi}{\partial S} \\ -[(1-\zeta)h + \frac{\alpha_P}{1-\alpha^*}\zeta\frac{k^*}{S}\frac{\partial H^*}{\partial k^*} \end{bmatrix} \mathrm{d}S \tag{39}$$

and

$$\mathrm{d}k^* = \epsilon \left(\frac{k^*}{k}\right)\mathrm{d}k - \alpha_P \frac{1}{1-\alpha_K^*}\left(\frac{k^*}{S}\right)\mathrm{d}S. \tag{40}$$

After defining the slopes of the KK- and GG-curve curve at the steady state by

$$\frac{\mathrm{d}h}{\mathrm{d}k}_{|KK} = -\frac{\left[h\frac{\partial \phi}{\partial k} + \epsilon\frac{k^*}{k}\frac{\partial \phi^*}{\partial k^*}\right]}{\phi}, \quad \frac{\mathrm{d}h}{\mathrm{d}k}_{|GG} = \frac{\zeta\epsilon\frac{k^*}{k}\frac{\partial H^*}{\partial k^*} - (1-\zeta)h\frac{\partial H}{\partial k}}{(1-\zeta)H} \tag{41}$$

and the shift of these curves caused by a change in S by

$$\frac{\partial h}{\partial S}_{|KK} = \frac{\frac{\alpha_P}{1-\alpha_K^*}\frac{\partial \phi^*}{\partial k^*}\frac{k^*}{S} - h\frac{\partial \phi}{\partial S}}{\phi}, \quad \frac{\partial h}{\partial S}_{|GG} = -\frac{\left[(1-\zeta)h\frac{\partial H}{\partial S} + \frac{\alpha_P}{1-\alpha_K^*}\zeta\frac{k^*}{S}\frac{\partial H^*}{\partial k^*}\right]}{(1-\zeta)H} \tag{42}$$

[5] For the case of equal production elasticities of capital in Home and Foreign, i.e. $\alpha_K = \alpha_K^*$, this result is analytically proven in Farmer *et al.* (2008, 32–33).

the solution of (39) by using Cramer's rule reads as follows:

$$\frac{\mathrm{d}h}{\mathrm{d}S} = \frac{\frac{\mathrm{d}h}{\mathrm{d}k}_{|KK}\frac{\partial h}{\partial S}_{|GG} - \frac{\mathrm{d}h}{\mathrm{d}k}_{|GG}\frac{\partial h}{\partial S}_{|KK}}{\frac{\mathrm{d}h}{\mathrm{d}k}_{|KK} - \frac{\mathrm{d}h}{\mathrm{d}k}_{|GG}}, \tag{43}$$

$$\frac{\mathrm{d}k}{\mathrm{d}S} = \frac{\frac{\partial h}{\partial S}_{|GG} - \frac{\partial h}{\partial S}_{|KK}}{\frac{\mathrm{d}h}{\mathrm{d}k}_{|KK} - \frac{\mathrm{d}h}{\mathrm{d}k}_{|GG}}. \tag{44}$$

Inspecting the signs of the slopes for the four different cases ($\phi \gtrless 0$, $\alpha_K \gtrless \alpha_K^*$) at the stable steady state associated with the higher capital intensity $k = k^H$, we observe the following, as summarized in Table 2.

Table 2: Slopes and shifts of KK- and GG-curves for the four cases

	$\frac{\mathrm{d}h}{\mathrm{d}k}_{\mid KK}$	$\frac{\mathrm{d}h}{\mathrm{d}k}_{\mid GG}$	$\frac{\partial h}{\partial S}_{\mid KK}$	$\frac{\partial h}{\partial S}_{\mid GG}$	$\frac{\mathrm{d}k}{\mathrm{d}S}$	$\frac{\mathrm{d}h}{\mathrm{d}S}$
case 1: $\phi > 0 \wedge (\alpha_K < \alpha_K^*)$	−	+	+	−	+	?
case 2: $\phi > 0 \wedge (\alpha_K > \alpha_K^*)$	−	−	+	−	+	−
case 3: $\phi < 0 \wedge (\alpha_K < \alpha_K^*)$	+	+	−	−	+	?
case 4: $\phi < 0 \wedge (\alpha_K > \alpha_K^*)$	+	−	−	−	+	−

Table 2 indicates that regardless in which of the four cases we are, the derivative of Home's capital intensity with respect to S, $\mathrm{d}k/\mathrm{d}S$, is always larger than zero, or in other words: Home's capital intensity decreases with a reduction of the permits volume in Home. However, this unambiguous result hinges on the presumption that in cases 2 and 3 for the denominator in (43) or (44) the following holds:

$$\left|\frac{\mathrm{d}h}{\mathrm{d}k}_{|KK}\right| > \left|\frac{\mathrm{d}h}{\mathrm{d}k}_{|GG}\right|.$$

But this condition is true in cases 2 and 3 in Figure 1, where the KK–curve is in both cases much steeper than the GG–curve.

On the other hand, the sign of $\mathrm{d}h/\mathrm{d}S$ which measures how strongly the terms of trade respond to a reduction of S is ambiguous in case 1 and 3, while negative in the remaining cases which means that Home's terms of trade improve with a reduction of the emission permits in Home. However, even the ambiguous cases 1 and 3 can be clarified since, first, we know that for $\alpha_K = \alpha_K^*$ the terms of trade of Home unambiguously rise with decreasing permits volume in Home (see Farmer *et al.*, 2008, 17) and second because the derivatives in (43) and (44) depend continuously on α_K^*, the sign of $\mathrm{d}k/\mathrm{d}S$ and $\mathrm{d}h/\mathrm{d}S$ does not change with a slight variation of α_K^* (see Appendix A.3).

These analytical results are also corroborated by the shifts of the GG– and KK–curve to their new intersection points in Figure 2: As a result of Home's permit reduction, the GG–curve is always shifted upwards while the KK–curve is shifted downwards (and to the left) if Home is a net debtor and upwards (and to the right) if Home is a net creditor. Combining the significantly larger leftwards shift of the KK–curve and the rather slight upward shift of the GG–curve, implies that in all four cases Home's new capital intensity is clearly lower than her pre–shock capital intensity (Figure 2), while the terms of trade increase too, but much less pronounced. As a consequence of the fall of Home's capital intensity, her real interest rate rises, and since real interest in Home and Foreign are equalized in steady state general equilibrium, Foreign's capital intensity declines, too.

To provide an economic rationale for the comparative steady state effects, let us focus on the case of Home as a net creditor to Foreign (right hand panels in Figure 2). First, in this case the slope of the KK–curve is positive since a rise in the terms of trade raises the value of net credit supplied by Home, measured in units of Foreign's output. To maintain equilibrium in the capital market, the demand for Foreign's and Home's capital must accordingly rise. Second, for given terms of trade, a reduction of Home's permit volume has a negative impact on Home's savings per capita and hence on world savings per capita. The capital market clearing condition (30) implies that k and/or k^* must be reduced. However, for $\alpha_K \neq \alpha_K^*$, the S shock has also an impact on the GG–curve which is positively sloped if $\alpha_K < \alpha_K^*$ and negatively sloped if the opposite is true. Again, for given terms of trade the reduction of Home's permit volume S has a negative impact on Home's net output available for domestic and foreign consumption. In order to restore the balance between the supply of Home's product to domestic and foreign consumers and their demand, Home's capital intensity has to increase, and thus the upward shift of the GG–curve.

As concerns the terms of trade effect of Home's permit reduction, it is worth noting that in contrast to the terms-of-trade consequences of higher government debt mentioned in the introduction, the terms-of-trade impacts of unilateral permits reduction do not depend on the pre–shock foreign net asset position of Home. The terms of trade increase both in the case of Home being a net creditor and a net debtor. The reason is that a reduction in S works predominantly as a supply shock for Home's economy (see equation (37)) while an increase in the government debt impacts primarily through international capital markets and hence on the demand side (see equation (36)). From equation (37), the GG–curve, the reason for the rise of h becomes apparent: the denominator declines more than the numerator or, in economic terms, the net supply of Home's product to Home and Foreign consumers H shrinks more than the net supply of Foreign's product H^* to Foreign's and Home's consumers. Home's product becomes more scarce and hence its relative price rises.

5 Transitional effects of a unilateral permit reduction

To investigate the transitional effects of a reduction in Home's permit volume S, recall that the shock is unannounced and permanent and that it occurs at the beginning of the transition period. The period in which the unilateral permit policy is introduced is denoted by $t = t_0$ and will be termed the shock period. To explain the rationale of the EU to follow a pro–active climate strategy, we furthermore assume that Home (the EU) is a net creditor ($\phi < 0$) while Foreign (the US) is a net debtor ($\phi^* > 0$). For investigating the transitional effects, we apply a numerical approximation algorithm to compute the stable manifold as briefly outlined in Appendix A.4.

While Home's and Foreign's capital stocks in t_0 remain at the initial steady state value, the terms of trade in the shock period change. In particular, the terms of trade jump upward in order to move onto the stable manifold (see Figure 3). After the initial jump, the terms of trade increase gradually towards the new, higher, steady state value. While the levels differ for different constellations of α_K and α_K^*, the qualitative pattern remains

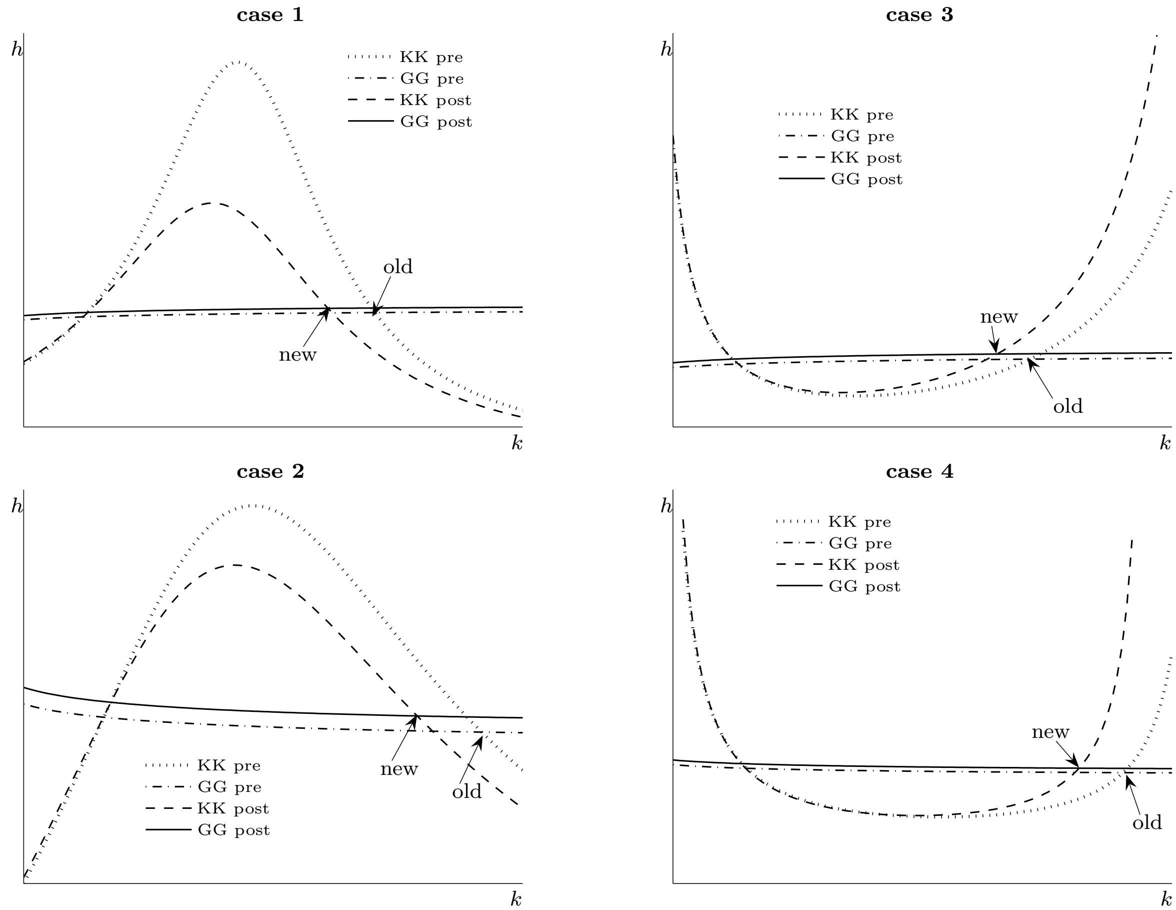

Figure 2: The shifts of the KK- and GG-curves for case 1 ($\phi > 0, \alpha_K < \alpha_K^*$), case 2 ($\phi > 0, \alpha_K > \alpha_K^*$), case 3($\phi < 0, \alpha_K < \alpha_K^*$), and case 4 ($\phi < 0, \alpha_K > \alpha_K^*$).

the same. Moreover, the structure of the transition path is not influenced by Home's net asset position.

To provide an economic rationale for the upward jump of the terms of trade in the shock period, assume on the contrary that h_{t0} does not respond to the shock. According to the firm's first order conditions, all factor prices and the production of Home's good decrease. As a consequence, young household's consumption of the domestic and the foreign good and old household's consumption fall. Since there is no change of S^*, in Foreign young household's consumption of the domestic and the foreign good are constant (Foreign's old household consumes slightly more of the Foreign and the Home good). Thus, Home's exports increase slightly while its imports sharply decline and thus Home's trade balance

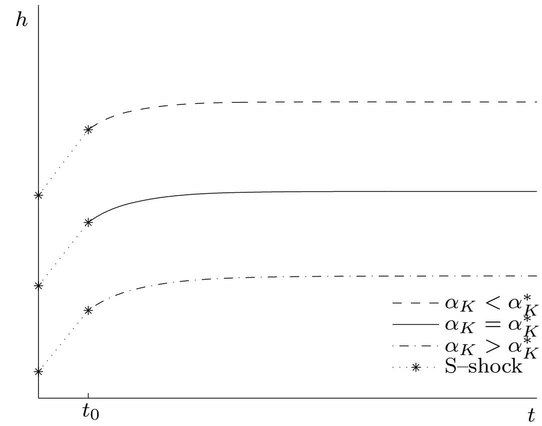

Figure 3: The transitional dynamics effects for the terms of trade of a permit reduction policy if Home is a net creditor ($\phi < 0$)

improves. Because Home is a net creditor ($\phi_{t0} < 0$), the current account in the shock period improves too, implying that the net asset position becomes more negative—Home becomes an even stronger net creditor. Since this trend cannot persist, Home's terms of trade have to improve and thus h_{t0} has to increase in the shock period.

Let us now turn to the transitional effects on capital accumulation. While the capital stocks are unaffected in the shock period, stocks in $t_0 + 1$ respond to the S shock, as depicted in Figure 4. We find two opposing effects: a negative direct effect of S decreasing the capital stock k_{t0+1} (holding h_t fixed), and a positive one induced by the improvement in the terms of trade. However, numerical investigation reveals that the first effect dominates the second one and thus Home's capital stock declines in the post–shock period as a consequence of a fall in S. On the other hand, in Foreign the direct effect on the capital stock is positive while the terms of trade effect is negative. Since the second effect dominates, a fall in S leads to a slight decline in Foreign's capital stock, too. But as Figure 4 reveals, the total effect on Home's capital stock is considerably stronger than on Foreign's one. Once the capital stocks k_{t0+1} and k^*_{t0+1} decreased, they monotonically fall towards their new, lower steady state values.

Regarding the influence of differences in the production elasticity of capital, Figure 4

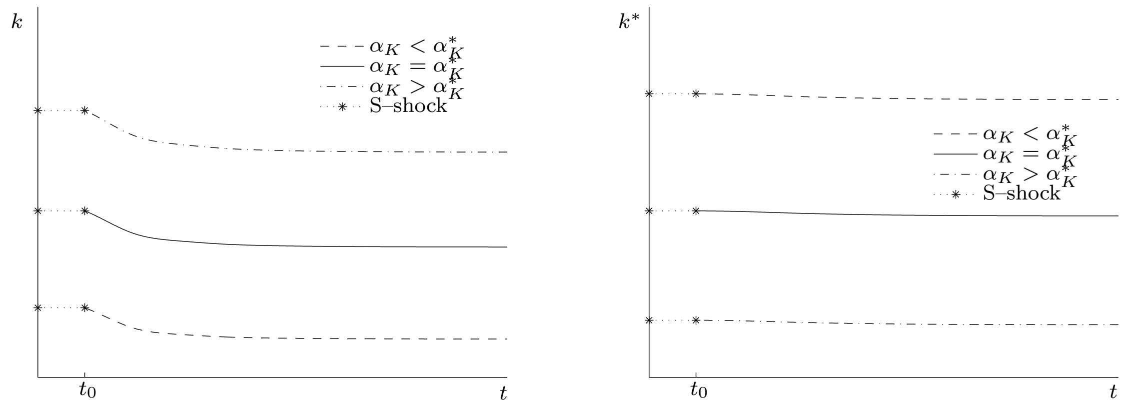

Figure 4: The transitional dynamics effects for capital stocks in Home (left panel) and Foreign (right panel) of a permit reduction policy if Home is a net creditor ($\phi < 0$)

reveals that their relative magnitude influences again the level of the stocks, but not the direction of change. The story is, however, a different one when comparing the effect of the country's net asset position: if Home is a net debtor, capital declines more sharply (to a lower new steady state value) than if she is a net creditor. Thus, if Home is a net creditor, a domestic unilateral policy is less detrimental for her capital accumulation than if she is a net debtor.

Finally, let us inspect the path of the permit price following a unilateral permit reduction. A reduction in S clearly reduces ceteris paribus the inputs to production, and hence Home's permit price increases sharply in the shock period, as illustrated by Figure 5. Thereafter, the permit price overshoots its new, higher, steady state value. Therefore, in the initial periods, the negative effects of a unilateral policy are most severe while later on the economy adjusts in terms of production. For Foreign, in the shock period the permit price remains unaffected, but thereafter there are spill–over effects through international trade. Thus, Foreign's permit price rises slightly and then declines towards its new steady state value which is below the pre–shock level.

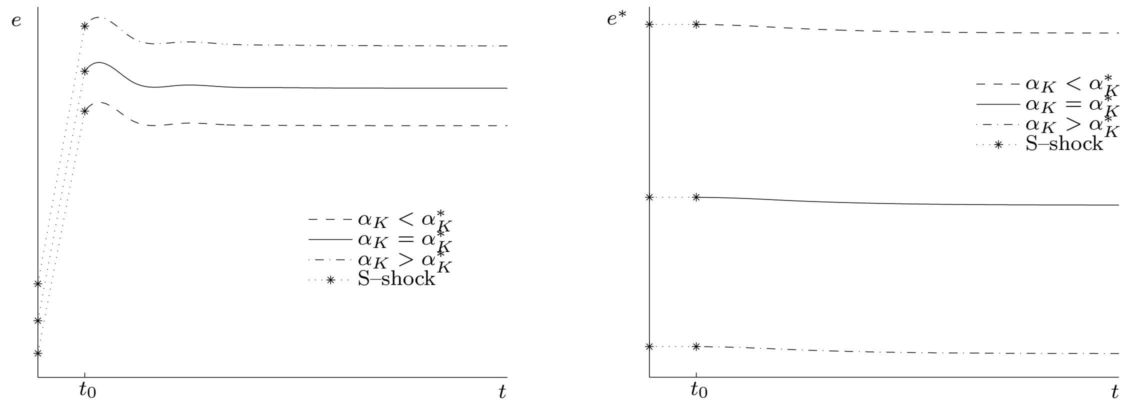

Figure 5: The transitional dynamics effects on Home's and Foreign's permit price of a unilateral permit reduction policy if Home is a net creditor ($\phi < 0$)

6 Welfare effects of a unilateral permit reduction

From the previous sections we know that a reduction in Home's permits volume has a positive effect on Home's terms of trade but negative consequences for capital accumulation. Hence, the question remains what the net effect of these two forces is—what is the impact on welfare? To derive the steady–state welfare effect of a shock in S, we define the indirect intertemporal utility function of Home as $U(x^1, y^1, x^2, y^2) \equiv V(w - \tau, 1 + i, h)$ and differentiate with respect to the dynamic variables. From the first order conditions of Home's utility maximization problem we know that $\partial U/\partial y^1 = \partial U/\partial x^1 (h)^{-1}$, $\partial U/\partial x^2 = \partial U/\partial x^1 (1 + i)^{-1}$, $\partial U/\partial y^2 = \partial U/\partial x^1 ((1 + i)h)^{-1}$. Using these FOCs and collecting similar terms, the welfare effect for Home is given by:

$$\frac{\mathrm{d}V}{\mathrm{d}S} = \frac{\partial U}{\partial x^1} \Bigg\{ \underbrace{\left[\frac{\partial (w-\tau)}{\partial k} \frac{\mathrm{d}k}{\mathrm{d}S} + \frac{\partial (w-\tau)}{\partial S} \right]}_{+} + \underbrace{\sigma \frac{w-\tau}{1+i} \left[\frac{\partial (1+i)}{\partial k} \frac{\mathrm{d}k}{\mathrm{d}S} + \frac{\partial (1+i)}{\partial S} \right]}_{-} + \underbrace{(1-\zeta) \frac{(w-\tau)}{h} \frac{\mathrm{d}h}{\mathrm{d}S}}_{-} \Bigg\}, \quad (45)$$

and, by a similar derivation, Foreign's welfare effect is given by:

$$\frac{\mathrm{d}V^\star}{\mathrm{d}S} = \frac{\partial U^\star}{\partial y^{\star,1}} \left\{ \underbrace{\frac{\partial(w^\star - \tau^\star)}{\partial k^\star} \frac{\mathrm{d}k^\star}{\mathrm{d}S}}_{+} + \underbrace{\sigma \frac{(w^\star - \tau^\star)}{(1+i^\star)} \frac{\partial(1+i^\star)}{\partial k^\star} \frac{\mathrm{d}k^\star}{\mathrm{d}S}}_{-} \underbrace{- \zeta \frac{(w^\star - \tau^\star)}{h} \frac{\mathrm{d}h}{\mathrm{d}S}}_{+} \right\}. \tag{46}$$

Thus, in line with the results for the case of unilateral fiscal expansion by Ono and Shibata (2005), a country's unilateral permit reduction affects its lifetime utility through three channels: the wealth effect, the interest effect, and the terms–of–trade effect. By investigating the signs of these factors in (45) and (46), we see that the derivative of Home's welfare with respect to S is positively influenced by wealth (net wages) and the terms of trade and negatively by the interest factor while Foreign's welfare is positively influenced by wealth and negatively influenced by both the interest factor and the terms of trade.

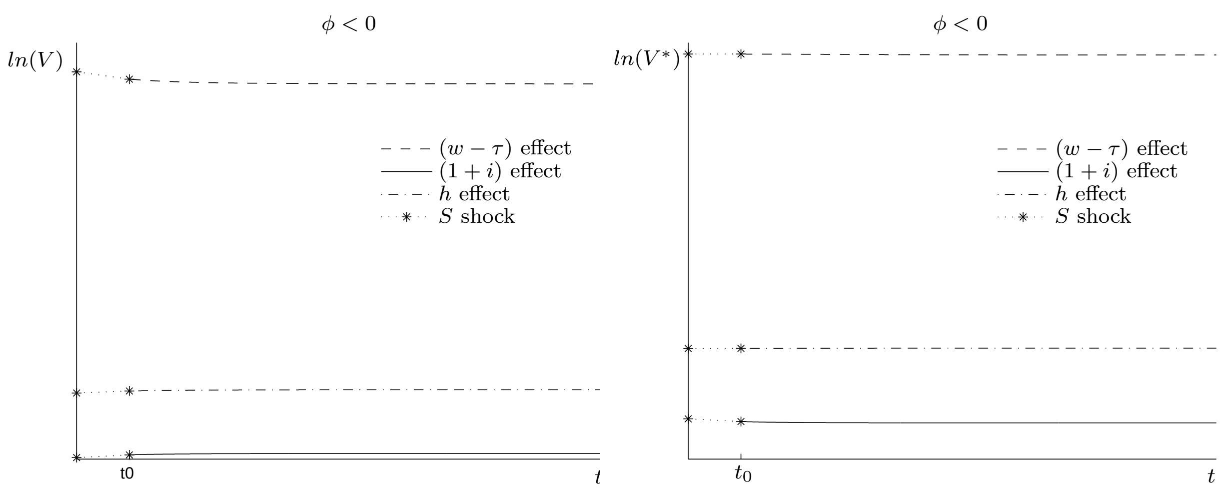

Figure 6: The transitional welfare effects for Home (left) and Foreign (right): the wealth, interest, and terms of trade effect

Figure 6 depicts these three effects for a decline in S along the transition path towards a new steady state.[6] The left graph, decomposing the net welfare for Home, shows that the wealth effect caused by an S–shock is clearly negative, while both the interest effect

[6]The relative size of these factors presupposes dynamic efficiency, i.e. $1 + i \geq 1$. A formal proof that under this condition Home's net welfare effect is negative is provided in Farmer *et al.* (2008) for the case of identical production elasticities of capital.

and the terms–of–trade effects are positive. The first effect, the wealth effect is caused by a decrease in Home's household lifetime net income (see the discussion in the previous section). The second effect is positive (the interest factor increases) and is caused by the reduced capital accumulation. Ono and Shibata (2005, 223) call this positive effect a foreign–asset or intertemporal macroeconomic effect, since this effect cannot appear in static trade models.[7] The third effect is the positive terms–of–trade effect which is familiar from the comparative steady state analysis above.

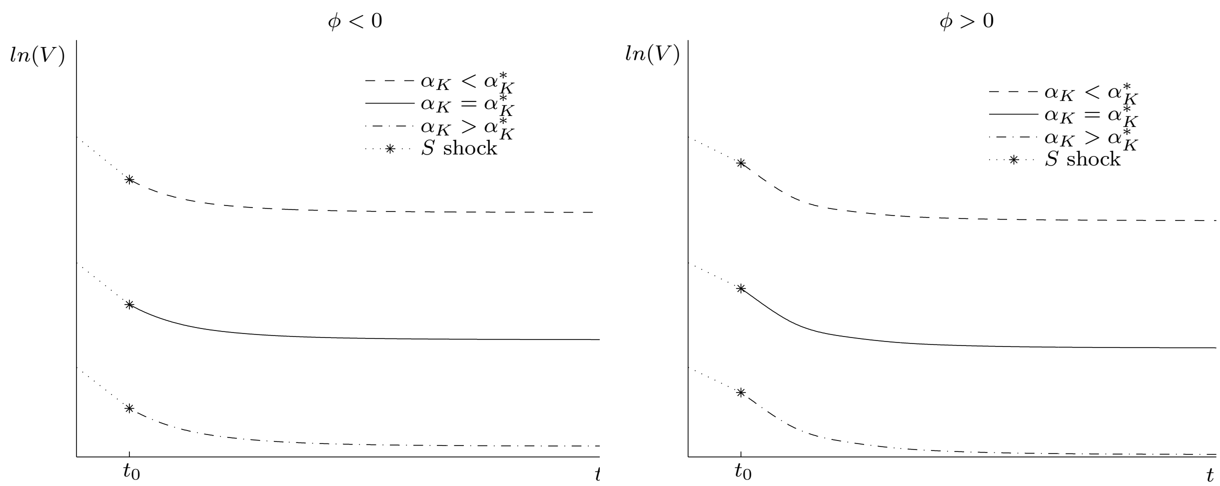

Figure 7: The transitional dynamics effects for Home's welfare of a permit reduction policy (left panel: $\phi < 0$, right panel: $\phi > 0$)

The consequences of Home's permit policy on Foreign's intertemporal welfare are illustrated in the right graph of Figure 6. For Foreign's welfare, we find similar, but less pronounced effects: Since there is no direct effect of the shock in S on $(w^\star - \tau^\star)$ and $(1 + i^\star)$, both the negative wealth effect and the positive interest effect caused by Home's unilateral permit policy are smaller (in absolute terms) for Foreign. Moreover, in contrast to Home, Foreign's terms of trade, $1/h$, deteriorate and thus Foreign's intertemporal welfare is reduced by this effect. Thus, while the wealth and the interest effects are considerably weaker for Foreign than for Home (hardly to be seen in Figure 6), the negative

[7]The foreign asset effect does not play any role in our model since in contrast to Ono and Shibata (2005) in our model the terms of trade in Home are independent of the foreign net asset position of Home (see (37)).

terms–of–trade effect strengthens the effect of Home's unilateral permit reduction on Foreign's welfare and hence also Foreign's welfare falls (see Figure 7). Consequently, even if the foreign country does not implement a more stringent permit policy it looses in terms of economic welfare—a result which contradicts the commonly used argument that a permit policy is detrimental only for domestic competitiveness. In an interdependent world economy, not acting and letting others act instead is thus not beneficial, but on the contrary, costly.

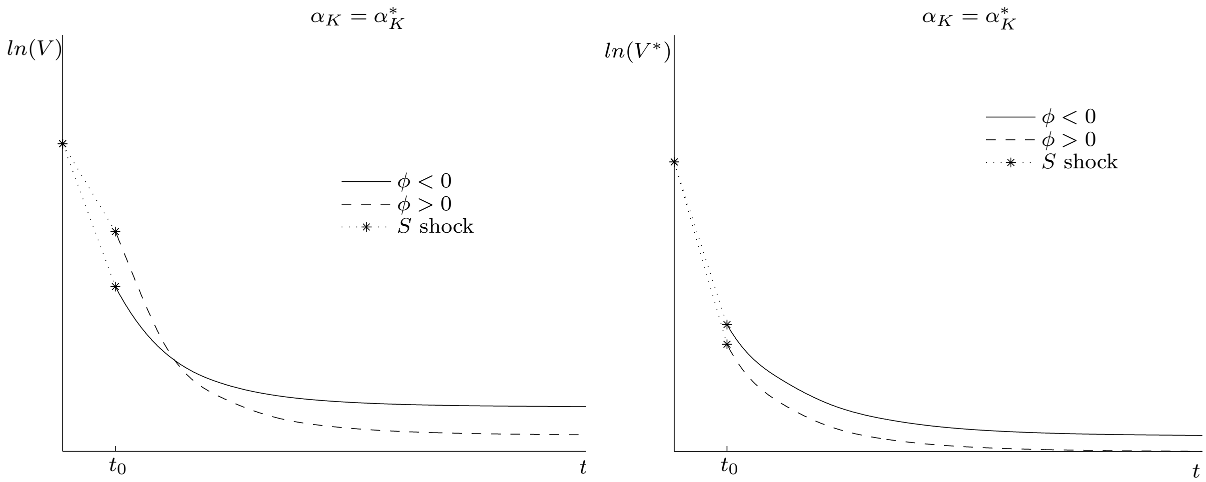

Figure 8: The transitional dynamics effects for Home's (left panel) and Foreign's (right panel) welfare of a permit reduction policy

Numerical investigation shows that the net welfare effect of a unilateral permit reduction by Home is negative (left graph of Figure 7). Moreover, comparison of the path reveals that if Home is a net creditor, then the net welfare effect is weaker than if she is a net debtor. By a similar argument, if Home is a net creditor, than the welfare effects for Foreign are smaller too than for the case if Home is a net debtor (right graph of Figure 7). Comparing the welfare effect for Home and Foreign, it is obvious that Home's welfare is reduced by more than that of Foreign regardless of Home's net asset position. However, a net creditor country, such as the EU, who is pursuing a unilateral permit reduction looses less in terms of economic welfare than if it were a net debtor country like the US. Finally, note that the magnitude of the production elasticity of capital ($\alpha_K \gtrless \alpha_K^*$) does not influence (qualitatively) this result, so in Figure 8 we illustrate only the case of identical elasticities across countries.

7 Conclusion

This paper investigates the effects of an unilateral reduction of emission permits in a two–country, two–good OLG model. After deriving the intertemporal equilibrium dynamics of the terms of trade, Home's and Foreign's capital intensities, we analyze numerically the impact of a unilateral permit reduction on the steady state and transitional paths of the key economic variables. We find that the terms of trade of the domestic economy immediately jump upwards in response to an unilateral permits reduction and continue to increase along the stable manifold towards the new steady state. Alongside with rising terms of trade, capital intensities in both countries go down, but by more in Home than in Foreign.

While these effects of unilateral emission permits reduction are independent of the net asset position of the countries, the effects on domestic household's welfare is more difficult to decide due to opposing effects. While the terms of trade improvement is welfare enhancing, a more stringent permit policy has welfare consequences via the factor prices, too: the wage rate declines, while the interest rate and the permit price increase. In total, for the dynamically efficient case, the welfare consequences caused by a permit reduction are negative for Home—and this gives an economic explanation why climate policy has been implemented with large hesitation. How strongly Home's welfare is reduced, depends on her net asset position, with the decrease of Home's welfare being largest when Home is a net debtor to Foreign.

For Foreign, both the wealth effect and the terms of trade effect caused by Home's permit reduction are negative (since Foreign's terms of trade deteriorate). On the other hand, Foreign's interest effect is positive. The strength of Foreign's welfare effect depends again on her net asset position, with the decrease of Foreign's welfare being largest when Foreign is a net debtor to Home.

Comparing the strength of the welfare decline (caused by a reduction of S in Home) between Home and Foreign, we find that the welfare decline in Foreign is always smaller than in Home. This result motivates two conclusions: one the one hand, unilateral environmental policy harms the domestic economy and thus explains why climate policy has been implemented cautiously in the past. On the other hand, the weaker negative welfare effect for Home being a net creditor helps to better understand why the EU has been more pro–active in pursuing climate policy than the US.

In terms of modeling approaches, we were able to show that different production elasticities of capital generalize the symmetrical model of identical elasticities across both countries without changing the dynamical properties of our model. While the form and the slopes of the equilibrium loci (GG–and KK-curve) depend on the relative size of the production elasticities (and, in addition, on Home's net asset position), the relative size of the elasticities change the pre– and post–shock steady states in a similar manner. Therefore, different constellations of α_K and α_K^* affect the levels of the three dynamic variables but not the qualitative results of a reduction in Home's permit level.

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A Appendix

A.1 Existence of steady states

This section is devoted to show how the existence of at least two non–trivial steady state solutions of the intertemporal equilibrium dynamics (33)–(35) can be proven. To this end, rewrite first equations (36)–(38) as follows:

$$k^* = \tilde{S}k^{\epsilon} \quad \text{where} \quad \tilde{S} \equiv \left[\frac{\alpha_K^*}{\alpha_K}\left(\frac{S^*}{S}\right)^{\alpha_P}\right]^{\frac{1}{1-\alpha_K^*}}, \tag{47}$$

$$h = \frac{\zeta}{1-\zeta}\frac{k^*}{k}\left[1+\left(1-\frac{\alpha_K}{\alpha_K^*}\right)\frac{MS^{\alpha_P}k^{\alpha_K-1}}{1-MS^{\alpha_P}k^{\alpha_K-1}}\right], \tag{48}$$

$$k = \bar{F}(k)+\Delta(k). \tag{49}$$

whereby

$$\bar{F}(k) = (1-\tilde{\alpha}_K)\sigma\frac{(1+i)}{\alpha_K} - \vartheta(k)(1+i) - \vartheta(k)(1-\sigma),$$

$$\Delta(k) = -\zeta\left(1-\frac{\alpha_K}{\alpha_K^*}\right)\frac{x}{1-\frac{(1+i)}{\alpha_K}}\phi,$$

with $(1-\tilde{\alpha}_K) \equiv \zeta(1-\alpha_K)+(1-\zeta)(1-\alpha_K^*)\frac{\alpha_K}{\alpha_K^*}$ and $\vartheta(k) \equiv \zeta b + (1-\zeta)b^*\frac{k}{k^*}$.

Next, consider the case of identical production technologies, i.e. $\alpha_K = \alpha_K^*$. Clearly, $\Delta(k) = 0$ and (49) reduces to

$$k = F(k) \equiv (1-\alpha_K)\sigma\frac{(1+i)}{\alpha_K} - \vartheta\sigma(1+i) - \vartheta(1-\sigma),$$

with $\vartheta \equiv (\zeta b + (1-\zeta)b^*\tilde{S}^{-1})$. Theorem 1 from Farmer *et al.* (2008, 13–14), which is reproduced as Lemma 1 below, provides sufficient conditions for exactly two strictly positive solutions of equation $k = F(k)$.

Lemma 1 *Let the parameter vector* $\omega = (\alpha_K, \alpha_P, \beta, \zeta, M, S, \vartheta)$ *be an element of the parameter space* $\Omega = [0,1]^4 \times \mathbb{R}^3_+$. *For any* $\omega \in \Omega$ *there exists* $\overline{\vartheta} \in \mathbb{R}_{++}$ *such that*

1. *for* $\vartheta < \overline{\vartheta}$ *there are one trivial* $(k = 0)$ *and two non–trivial steady states* k^L *and* k^H *with* $0 < k^L < k^H < \overline{k}$,

2. *for* $\vartheta = \overline{\vartheta}$ *there are one trivial and one non–trivial steady state, and*

3. *for* $\vartheta > \overline{\vartheta}$ *there is only the trivial steady state.*

Proof 1 *see Appendix A.1 in Farmer* et al. *(2008).*

The third step is to prove the existence of at least two strictly positive solutions of the equation $k = \bar{F}(k) + \Delta(k)$. The central insight here is that $\bar{F}(k) + \Delta(k)$ depends continuously on α^*_K.

Theorem 1 *For every parameter set* $\omega = (\alpha_K, \alpha^*_K, \alpha_P, \beta, \zeta, M, S, b, b^*) \in \Omega = [0,1]^5 \times \mathbb{R}^3_+$ *with* $|\alpha_K - \alpha^*_K|$ *sufficiently small some (non–unique)* $\bar{b}, \bar{b}^* > 0$ *exist such that for all* $b \in (0, \bar{b})$ *and* $b^* \in (0, \bar{b}^*)$ *there are at least two non–trivial steady state solutions* (h, k, k^*).

Proof 2 *For* $\alpha_K = \alpha^*_K$ *we know from Lemma 1 that for all* $\vartheta < \bar{\vartheta}$ *exactly two solutions* $0 < k^L < k^H$ *of* $k = \bar{F}(k) + \Delta(k)$ *occur. Since* $\bar{F}(k) + \Delta(k)$ *depends continuously on* α^*, *there is some interval* $\Lambda = (\alpha_-, \alpha_+)$ *such that for all* $\alpha^*_K \in \Lambda$ *at least two distinct solutions* $0 < \tilde{k}^L < \tilde{k}^H$ *exist*[8]. $\triangle$

A.2 Saddle–path stability of steady states

To prove the dynamic stability of a non–trivial steady state solution, we consider the Jacobian of the dynamic system (33)–(35) in a small neighborhood around both non–trivial steady state solutions. Again, we focus first on the case of identical production

[8] The analysis of $\bar{F}(k) + \Delta(k)$ shows, however, that for $\alpha_K < \alpha^*_K$ a third steady state $k^\infty > k^H$ exists.

elasticities of capital. Theorem 2 from (Farmer *et al.*, 2008, 15) claims that for $\vartheta < \bar{\vartheta}$, at the lower steady state solution, k^L, two eigenvalues of the Jacobian are larger than one and one eigenvalue equals $\alpha_K < 1$, while at the larger steady state solution of $k = F(k)$ two eigenvalues are less than one and one eigenvalue is larger than one. Hence, the lower steady state is saddle–path unstable while the larger steady state is saddle–path stable.

In considering the general case $\alpha_K \neq \alpha_K^*$, we focus again at a sufficiently small difference between α_K and α_K^*. Under this assumption, Theorem 2 of (Farmer *et al.*, 2008, 15) can be generalized as the following Theorem 2.

Theorem 2 *For every parameter set $\omega \in \Omega$ with $|\alpha_K - \alpha_K^*|$ sufficiently small some (non–unique) $\bar{b}, \bar{b}^* > 0$ exist such that for all $b \in (0, \bar{b})$ and $b^* \in (0, \bar{b}^*)$ the larger strictly positive solution of $k = \bar{F}(k) + \Delta(k)$ is saddle–path stable.*

Proof 3 *For $\alpha_K = \alpha_K^*$ see the proof to Theorem 2 of (Farmer* et al.*, 2008, 32–33) Again, since $k = \bar{F}(k) + \Delta(k)$ depends continuously on α^*, there is some interval $\Lambda_1 = (\alpha_-^1, \alpha_+^1)$ such that for all $\alpha_K^* \in \Lambda_1 \subset \Lambda$ the larger solution k^H of $k = \bar{F}(k) + \Delta(k)$ is saddle–path stable.*$\triangle$

A.3 Comparative Statics Effects

To derive the signs of the comparative statics effects of a shock in S on h and k, we generalize again the effects for the symmetrical case $\alpha_K = \alpha_K^*$.

Theorem 3 *For $|\alpha_K^* - \alpha_K|$ sufficiently small we know that*

$$\frac{\mathrm{d}h}{\mathrm{d}S} < 0 \text{ and } \frac{S}{k}\frac{\mathrm{d}k}{\mathrm{d}S} > \frac{S}{k^*}\frac{\mathrm{d}k^*}{\mathrm{d}S} > 0. \tag{50}$$

Proof 4 *Theorem 3 of Farmer* et al. *(2008, 17) shows that in a small neighborhood of k^H, $\mathrm{d}h/\mathrm{d}S < 0$ holds and since $\mathrm{d}h/\mathrm{d}S, [S/k][\mathrm{d}k/\mathrm{d}S]$ and $[S/k^*][\mathrm{d}k^*/\mathrm{d}S]$ are continuous functions of α_K^*, (50) holds for α_K^* not too distant from α_K.* $\triangle$

A.4 Computation of the (non-linear) saddle path in MATLAB

Noting that h_t is a jump variable and (k_t, k_t^*) are sluggish, a parameter shock in S at time t_0 leaves the initial capital intensities $k_{t_0}, k_{t_0}^*$ unaffected while the terms of trade in the shock period h_{t_0} are determined by condition $\lim_{t\to\infty} \|(h_t, k_t, k_t^*)\| < \infty$. To compute the dynamic variables $\left\{(h_t, k_{t+1}, k_{t+1}^*), t = t_0, t_{0+1}\right\}$ along the stable manifold towards the steady state, a two–step procedure is in order. First, linearize the dynamic system (33)–(35) at the larger steady state and write the linear approximation as $u_{t+1}^{(0)} = J_\psi(u)(u_t^{(0)})$ where $J_\psi(u) = \{j_{ij}\}_{i,j=1,2,3}$ denotes the Jacobian of the dynamic system (33)–(35) at a steady state. The elements of the Jacobian are:

$$
\begin{aligned}
j_{12} &= 1 + \tfrac{h}{k}(1-\alpha_K)j_{21} - \tfrac{h}{k^*}(1-\alpha_K^*)j_{31}, \\
j_{12} &= \tfrac{h}{k}(1-\alpha_K)j_{22} - \tfrac{h}{k^*}(1-\alpha_K^*)j_{32}, \\
j_{13} &= \tfrac{h}{k}(1-\alpha_K)j_{23} - \tfrac{h}{k^*}(1-\alpha_K^*)j_{33}, \\
hj_{21} + j_{31} &= (1-\alpha_K)\sigma MS^{\alpha_P}k_K^{\alpha} - b\sigma\alpha_K MS^{\alpha_P}k^{\alpha_K-1} - b(1-\sigma) - k, \\
hj_{22} + j_{32} &= h(1-\alpha_K)\sigma\alpha_K MS^{\alpha_P}k^{\alpha_K-1}\left(1+\tfrac{b}{k}\right), \\
hj_{23} + j_{33} &= (1-\alpha_K^*)\sigma\alpha_K MS^{\alpha_P}k^{\alpha_K-1}\left(1+\tfrac{b^*}{k^*}\right), \\
hj_{22} - \tfrac{\zeta}{1-\zeta}j_{32} &= h\alpha_K MS^{\alpha_P}k^{\alpha_K-1}, \\
hj_{32} - \tfrac{\zeta}{1-\zeta}j_{33} &= -\tfrac{\zeta}{1-\zeta}\alpha_K MS^{\alpha_P}k^{\alpha_K-1}, \\
hj_{21} - \tfrac{\zeta}{1-\zeta}j_{31} &= MS^{\alpha_P}k_K^{\alpha} - k.
\end{aligned}
$$

Second, calculate the solution $u_t^{(0)}$ of $u_{t+1}^{(0)} = J_\psi(u)(u_t^{(0)})$. Third, for some termination period $T > t_0$ solve the following optimization problem

$$
\begin{aligned}
&min_{u_T\in\mathbf{R}^3}\|u_T - u\| \\
&\text{subject to} \\
&u_{t+1} = \psi(u_t) \qquad t = t_0+1, \ldots, T-1 \\
&(k_{t_0}, k_{t_0}^*) >> 0 \quad \text{exogenously given.}
\end{aligned}
$$

where $u_t = (h_t, k_t, k_t^*)$ and $\psi : \mathbf{R}^3 \to \mathbf{R}^3$ is defined by the dynamic equations (33)–(35). As a guess for the initial value of u_t, $u_t^{(0)}$ can be used. Then, fmincon.m of the MATLAB Optimization–Toolbox does the job.