How does the minimum wage affect employment via on-the-job search intensity?

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Abstract

This paper studies the impact of the minimum wage in a segmented labor market describing career stages of workers. At the end of a learning-by-doing period, young workers paid the minimum wage quit unqualified jobs for better paid qualified jobs, according to an on-the-job search process with endogenous search intensity. A rise in the minimum wage reduces unqualified job creation and prompts workers to keep their unqualified job by reducing on-the-job search intensity. The balance between accession and separation flows, giving an ambiguous impact on youth employment, replicates and explains findings of several empirical studies. However, our macroeconomic result is consistent with the old view: a minimum wage rise reduces the overall employment and the output.

Keywords: Minimum wage; On-the-job search; Search intensity; Youth employment.

JEL Codes: J38, J42, J64.

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1 Introduction

The purpose of this paper is to highlight the impact of the minimum wage on sectorial and overall employment in a matching model with a segmented labor market. The segmentation allows us to distinguish two sectors: an unqualified sector, composed of unskilled newcomers becoming skilled on-the-job seekers at the end of a learning-by-doing process, all earning the minimum wage; a qualified sector exclusively composed of the former on-the-job seekers, remunerating (better paid) bargained wages. A rise in the minimum wage reduces unqualified job creation as well as search intensity of young on-the-job seekers. On the one hand, it becomes harder for unskilled workers to obtain an unqualified job, but on the other hand, skilled on-the-job seekers are less inclined to quit their unqualified job. The balance between the two effects is ambiguous, a higher stability of workers in the unqualified sector may therefore lead to an increase in unqualified employment. This result agrees with several empirical studies. However, this result is only sectorial. Our model shows that the macroeconomic effect of the minimum wage tallies with standard competitive models by revealing a lowering in overall employment as well as a cut in the output.

The literature dealing with the on-the-job search and job search intensity is not widespread. Holzer (1987) provides support to the theory of search unemployment by establishing that the young workers' on-the-job search is less efficient than searching unemployed. Unemployed search more intensively, receive and accept more offers. His results, which reinforce the idea that a higher search intensity leads to better job transitions, contrasts sharply with Blau and Robbins (1990) which show that employees are more inclined to generate offers and find new jobs than unemployed even if unemployed contact a larger number of firms. Although different, the two articles suggest that young employees are more concerned by job search than their older counterparts. In our paper, unemployed search full-time, their search intensity is thus maximal, which is not the case for on-the-job seekers. The unqualified sector can be seen as the young sector and the qualified sector as the senior sector. Therefore, only skilled workers of the unqualified sector search while on the job and are allowed to modulate their search intensity.

The impact of a mandatory minimum wage is a major issue of labor market public policy. One of the major work on the minimum wage is that of Card and Krueger (1995) who analyzed a set of 410 fast-food restaurants, whose workers are mainly young and unexperienced, in New Jersey and Pennsylvania following the 1992 increase in New Jersey's minimum wage. Their analysis showed that stores already paying more than the minimum wage were unaffected whereas employment at restaurants, which complied with the laws by increasing wages to the new minimum wage level, was increased. Instead of unemployment, the resultant higher costs were reflected through higher prices. Many empirical studies, such as Neumark and Wascher (2000), attempted to contradict this statement by reconsidering the data set. Card and Krueger's results, seemly neither consistent with standard competitive models nor entirely with monopsonistic models, are

still debated. Nevertheless, in their reply to Neumark and Wascher's challenging conclusion, Card and Krueger (2000) revised their study and restated that "The increase in New Jersey's minimum wage probably had no effect on total employment in New Jersey's fast-food industry, and possibly had a small positive effect." (p.1419). Such a paradox has been long discussed since then.

Recently, Cardoso and Portugal (2006) reconcile opposite results about the aggregate impact of the minimum wage on youth employment by analyzing young Portuguese worker flows, accessions and separations. Their aggregate analysis suggests that a sharp rise in the minimum wage does not seem to have a negative impact on employment for the specific young age group. However, a rising minimum wage has been changing job transitions. The major effect has been the reduction of separation rates, compensated for accession losses to new and continuing firms. Moreover, young workers are over-represented in firms closing down after the minimum wage rise.

In order to set up a theoretical framework being able to revisit empirical results, we use a matching model that segments labor market into two interdependent sectors. Sector 2 offers unqualified jobs to unskilled newcomers and skilled workers, they are paid the minimum wage. Sector-1 offers qualified jobs to skilled workers who bargain their wages. Firms are infinitely lived which is not the case for workers. The inflow of new workers entering the market exactly balances the outflow of departing workers. Obtaining first an unqualified job is the only gateway to better paid qualified jobs. Unskilled newcomers must thus obtain a sector-2 job, train themselves, become skilled and then apply for a sector-1 job while still on the job. Searching on the job is costly. Workers modulate their search intensity depending on their career expected profits. As soon as on-the-job seekers obtain a sector-1 job, they quit their previous sector-2 job.

To our point of view, the empirical puzzle about the impact of the minimum wage on employment can be disentangled through the issue of search intensity of young on-the-job seekers and their willingness to stay in their job when the minimum wage increases. The balance between accession and separation flows, operating in opposite direction, can generate an increase in youth employment, depending on workers' search intensity. An increase in the minimum wage reduces search intensity of sector-2 on-the-job seekers. The turnover rate of unqualified jobs falls as applicants to sector 1 are more inclined to keep their unqualified job. Our main result is the following: a minimum wage rise increases youth unemployment by reducing job creation in the unqualified sector, this rise also maintains skilled workers in their unqualified job by reducing their search intensity. The minimum wage rise penalizes unskilled employment but can also favor skilled employment in the unqualified sector. The balance between the two effects may therefore lead to an increase in youth employment, depending on the search intensity sensitiveness. However, an increase in sectorial employment does not necessary lead to an increase in the overall employment level or to an increase in the output. The sectorial analysis must be extended to a macroeconomic perspective in order to give a more complete answer than empirical studies did.

The paper is organized as follows. Section 2 outlines our framework, defines the search intensity equation and describes workers' career path. We define the labor force structure in stationary equilibrium in section 3. Section 4 solves the model equilibrium. Section 5 studies the impact of an increase in the minimum wage, first on youth employment, and second on the overall employment level. Finally, section 6 contains some concluding comments.

2 The Model

The labor market is segmented into a qualified sector (sector 1) and an unqualified sector (sector 2). Jobs are indestructible which means workers either voluntary quit for another job or definitively leave the labor market at rate m. Each worker who leaves the market is replaced with a newcomer. Firms are infinity-lived. All agents are risk-neutral and discount future payoffs at rate r ($r \geq 0$). The measure of the total labor force is constant and normalized to one. Time is continuous.

Sector-1 workers are skilled and bargain their wages, whereas sector-2 workers can be either unskilled (assimilated to newcomers) or skilled and earn the minimum wage. In a first step, (unskilled) newcomers must imperatively obtain a job in the unqualified sector (sector 2). After a learning-by-doing period, they become skilled enough to apply on qualified sector (sector 1) jobs. Sector-1 jobs being more attractive in terms of wage, sector-2 workers engage in an on-the-job search process, hoping they would obtain a sector-1 job. Searching while on the job is costly and this cost depends on workers' search intensity. Workers modulate their search intensity depending on their career expected profits. As soon as a sector-1 job is obtained, the sector-2 job is being instantaneously quitted. As no better situation can be expected, workers keep their qualified job until they permanently leave the labor market.

2.1 Accession to qualified jobs

The qualified sector can also be defined as the senior workers' sector as it is assumed that all labor market newcomers are young, unskilled and with the same potential/ability for work. Skilled workers are thus defined as workers who had trained themselves on their job. In the real world, workers have different skills correlated to their education level. This fact does not contradict our assumption regarding the existence of a mandatory unqualified stage required to access to the qualified sector. The length of stay in sector 2, required to apply for sector 1, depends on workers' education. The more the worker educated, the shorter her stay in sector 2.

As the searching behavior of a sector-2 worker applying for sector 1 determines the average time during which a sector-2 firm can keep its worker as well as the efficiency of the sector-1 matching process, it thus influences job creation in the two sectors.

2.1.1 Search intensity of sector-2 on-the-job seekers

At the end of the learning-by-doing process, sector-2 skilled workers become on-the-job seekers, applying for a better paid sector-1 job. The individual expected lifetime utility, denoted by $\widehat{W}_{2i}$ for a worker i, is a function of her own search effort, a_i. A higher search intensity makes the rate rate $\pi_1(a_i)$ of finding a sector-1 job higher. Searching on-the-job is costly and this higher intensity leads to an increase in the worker's search cost γ_i.

Only on-the-job seekers face search costs. Increasing the search intensity can, for example, lead to a reduction in the worker efficiency in her current job. The worker must thus makes an additional effort in order to maintain her productivity. The longer the devoted time to on-the-job search, the higher the search cost. The search cost is defined as:

$$\gamma_i = \gamma(a_i)$$

A search intensity of unemployed workers assumed to be equal to 1 guarantees the existence of an interior optimum solution. We use the following assumptions on the search cost function:

$$\gamma'(a_i) > 0 \quad , \quad \gamma''(a_i) > 0 \quad , \quad \gamma(0) = 0 \quad , \quad \gamma'(0) = 0 \quad \text{and} \quad \gamma(1) = +\infty$$

$\bar{w}_2$ is the exogenous minimum wage, earned by all sector-2 workers independently on their skill level. The net income of a searching worker is the difference between $\bar{w}_2$ and γ_i. Workers quit their sector-2 job as soon as they obtain a sector-1 job as well as when they permanently leave the market (at rate m). Let W_1 be the expected lifetime utility of a sector-1 worker, the individual utility of a skilled worker i, denoted by $\widehat{W}_{2i}$, satisfies:

$$r\widehat{W}_{2i} = \bar{w}_2 - \gamma(a_i) + \pi_1(a_i)\left[W_1 - \widehat{W}_{2i}\right] - m\widehat{W}_{2i} \tag{1}$$

Each worker i individually chooses the search intensity that maximizes her lifetime utility, therefore $\frac{\partial \widehat{W}_{2i}}{\partial a_i} = 0$. The optimal value of the search intensity of worker i thus satisfies:

$$-\gamma'(a_i) + \pi_1'(a_i)\left[W_1 - \widehat{W}_{2i}\right] = 0 \tag{2}$$

To make sense, we assume that the utility of a sector-1 worker is higher than the utility of a skilled sector-2 worker i, $W_1 > \widehat{W}_{2i}$.

The following section studies the matching process in sector 1 so that the transition rate can be defined.

2.1.2 Sector-1 matching process

In the qualified sector, the labor market tightness is defined in efficient search units. The average (endogenous) search effort of on-the-job seekers is $a \in [0, 1]$. Unemployed sector-1 workers search full-time for a sector-1 job, their search effort is thus equal to 1. Sector-1 tightness, defined as a function of workers' search intensity, is $\theta_1 = \frac{v_1}{1u_1 + a\hat{\ell}_2}$, with v_1 the number of vacancies in the qualified sector, u_1 unemployment in this sector and $\hat{\ell}_2$ sector-2 workers who search for a sector-1 job.

However, there is no possible separation between workers and firms before workers retire, which means sector-1 unemployment is a pure outside opportunity only taken into account in the wage bargaining process. Therefore, workers leave their sector-1 job exclusively when they permanently leave the labor market. Moreover, the only gateway to sector-1 job is sector-2 on-the-job search. The qualified sector tightness is thus defined as:

$$\theta_1 = \frac{v_1}{a\hat{\ell}_2} \tag{3}$$

where a is the average search intensity of sector-1 applicants.

The matching function is $h_1 = h_1(v_1, a\hat{\ell}_2)$. It is assumed to be increasing in both its arguments, concave and homogenous of degree 1. Therefore, the arrival rate of workers for sector-1 vacancies is:

$$q_1 = \frac{h_1}{v_1} = h_1\left(1; \frac{a\hat{\ell}_2}{v_1}\right) = h_1\left(1; \frac{1}{\theta_1}\right)$$

with $q'(\theta_1) < 0$.

Knowing that unemployed search full-time and that on-the-job seekers search at the average rate of a, the arrival rate p_1, of sector-1 job offers to unemployed sector-1 workers, is defined as:

$$p_1 = \frac{1}{a}\frac{h_1}{\hat{\ell}_2} = h_1(\theta_1; 1)$$

with $p'(\theta_1) > 0$.

The arrival rate, $\pi_1(a_i)$, of sector-1 job offers to a sector-2 worker i, depends on the ratio between her own search intensity and the average search intensity:

$$\pi_1(a_i) = \frac{a_i}{a}\frac{h_1}{\hat{\ell}_2} = a_i p_1$$

The individual search intensity does not affect the average search intensity and thus does not affect the sector-1 tightness θ_1. Following a rise in the worker i search intensity, a marginal increase of her transition rate is equal to:

$$\pi_1'(a_i) = p_1 \tag{4}$$

As they are identical, all workers have the same *ex post* behavior ($a_i = a$). Equation (2) can therefore be rewritten as:

$$\gamma'(a) = p_1\left[W_1 - \widehat{W_2}\right] \tag{5}$$

Equation (5) allows to determine the search intensity a of sector-2 skilled workers. Sector-2 skilled workers increase their search effort as long as the marginal search cost is below the marginal expected search gain. This search gain depends on the utility improvement associated with a sector-1 job and on the potential increase of transition rate in sector-1. Therefore, sector-1 tightness is a key variable for the optimal search intensity determination.

2.2 Asset values and private surplus in sector 1

We assume the sector-1 productivity to be sufficiently high to allow for a bargaining process leading to a negotiated wage above the minimum wage. Let J_1^F be the value of a filled sector-1 job. Because a match lasts until the worker leaves the labor market, the value of a filled job satisfies the following Bellman equation:

$$rJ_1^F = (y_1 - w_1) - m\left[J_1^F - J_1^V\right] \tag{6}$$

with J_1^V denoting the value of a sector-1 vacancy. We have:

$$rJ_1^V = -c_1 + q_1\left[J_1^F - J_1^V\right] \tag{7}$$

with c_1 being the cost to keep a vacancy open in sector 1.

Sector-1 vacancies are created as long as profit opportunities exists. This free-entry condition, that determines sector-1 job creation, implies:

$$J_1^V = 0 \tag{8}$$

When a worker gets a sector-1 job, she keeps it until she definitively leaves the labor market. Therefore, the lifetime utility of a sector-1 worker, denoted by W_1, satisfies:

$$rW_1 = w_1 - mW_1 \tag{9}$$

Sector-1 workers can never be unemployed. Nevertheless, during the wage bargaining process, the asset value of an unemployed sector-1 worker, denoted by U_1, is taken into account as a reservation utility. This utility which is a pure outside opportunity is thus defined as:

$$rU_1 = d + p_1\left[W_1 - U_1\right] - mU_1 \tag{10}$$

where d is the domestic output ($d < y_2 < y_1$) and p_1 would be the arrival rate of sector-1 job offers faced by unemployed sector-1 workers.

The (private) surplus of a match, denoted by S_1, is obtained by adding the individual surplus of a sector-1 firm (*i.e.* a firm that has chosen sector 1) and its worker. Under the free entry condition, this gives:

$$S_1 = \left[W_1 - U_1\right] + J_1^F \tag{11}$$

2.3 Sector-2 firms' profitability

In this unqualified sector, the matching function is $h_2 = h_2(v_2, u_2)$. It is assumed to be increasing in both its arguments, concave and homogenous of degree 1. The transition rate for a vacancy to be filled is determined in the usual way by $q_2 = q_2(\theta_2)$, where the sector tightness θ_2 is defined as:

$$\theta_2 = \frac{v_2}{u_2} \tag{12}$$

where v_2 is the number of vacancies and u_2 the number of unemployed. Properties of the matching function (Pissarides (2000)) allows us to know that $q_2'(\theta_2) < 0$ and to write the transition rate of sector-2 unemployed workers toward employment as $p_2(\theta_2) = \theta_2 q_2(\theta_2)$ with $p_2'(\theta_2) > 0$.

The asset value of a sector-2 firm with a vacant job, denoted by J_2^V, satisfies the following Bellman equation:

$$rJ_2^V = -c_2 + q_2\left[J_2^F - J_2^V\right] \tag{13}$$

where c_2 is the cost to keep a vacancy open in sector 2.

J_2^F is the asset value of a sector-2 job filled with an unskilled worker. Throughout the learning-by-doing process, unskilled workers cannot quit their sector-2 job for a sector-1 job. It is assumed that the learning-by-doing process comes to its end, at any time, for each worker, at rate λ. After a spell, whose average length is therefore $1/\lambda$, workers become skilled enough to apply for better paid sector-1 jobs, however sector-2 skilled workers (on-the-job seekers) still earn the minimum wage $\bar{w}_2$. We assume that sector-2 skilled workers' productivity remains y_2[1]. Therefore, sector-2 firms have nothing to earn from an improvement of workers' skills. When becoming skilled, workers have access to outside sector-1 opportunities. Therefore, the rate of separation between a sector-2 firm and its worker increases, and thus, the value of a filled sector-2 job depreciates.

Let $\widehat{J}_2^F$ be the value of a sector-2 job filled with a searching (hence skilled) worker, the asset value J_2^F of a sector-2 job filled with an unskilled worker is defined as:

$$rJ_2^F = (y_2 - \bar{w}_2) - m\left[J_2^F - J_2^V\right] - \lambda\left[J_2^F - \widehat{J}_2^F\right] \tag{14}$$

and the asset value of a sector-2 job filled with a skilled worker thus is:

$$r\widehat{J}_2^F = (y_2 - \bar{w}_2) - (m + ap_1)\left[\widehat{J}_2^F - J_2^V\right] \tag{15}$$

Sector-2 job creation is determined by the free entry condition. We have:

$$J_2^V = 0 \tag{16}$$

3 Labor force structure in stationary equilibrium

The total labor force is normalized to 1. Workers entering the labor market are identical. Then the coexistence of different career stages implies, to some extent, workers' heterogeneity.

In sector 1, employment is denoted by ℓ_1. In sector 2, u denotes unemployment, and total employment is divided into two subsets: unskilled workers' employment, denoted by ℓ_2, and (skilled) on-the-job seekers' employment, denoted by $\widehat{\ell}_2$.

[1]Results can be extended to the case where the productivity is higher for sector-2 skilled workers than for sector-2 unskilled workers as long as it remains below sector-1 skilled workers' productivity.

In steady state, entry flows balance exit flows at any stage of workers' careers. Flow equilibrium relationships that determine the labor force structure are, respectively for u, ℓ_2, $\widehat{\ell}_2$ and ℓ_1, the following:

$$m = u(m + p_2) \tag{17}$$
$$p_2 u = \ell_2(m + \lambda) \tag{18}$$
$$\lambda \ell_2 = \widehat{\ell}_2(m + ap_1) \tag{19}$$
$$ap_1 \widehat{\ell}_2 = m\ell_1 \tag{20}$$

This system exhibits the interactions between the two sectors in steady state. Whatever the stage of their career, workers definitively leaved the labor market at rate m. They are therefore replaced with newcomers (equation (17)). Unskilled (employed) workers become eligible applicants to sector-1 jobs at rate λ (*i.e.* when the learning-by-doing process comes to its end) (equation (18)). When leaving unskilled employment, workers either become (skilled) on-the-job seekers or retire from the labor market. On-the-job seekers either leave the market permanently or quit their current job for a job in sector 1 (equation (19)). Workers who obtain a sector-1 job keeps it until they definitely retire from the labor market (equation (20)).

Appropriate combination of the previous equations leads to the following labor force structure in steady state equilibrium:

$$u = \frac{m}{m + p_2} \tag{21}$$

$$\ell_2 = \frac{p_2 m}{(m + p_2)(m + \lambda)} \tag{22}$$

$$\widehat{\ell}_2 = \frac{\lambda p_2 m}{(m + ap_1)(m + p_2)(m + \lambda)} \tag{23}$$

$$\ell_1 = \frac{\lambda a p_1 p_2}{(m + ap_1)(m + p_2)(m + \lambda)} \tag{24}$$

The labor force structure depends on the main endogenous variables of the model (a, θ_1, θ_2).

Table 1: Direct effects of the main endogenous variables on the labor force structure

	u	ℓ_1	ℓ_2	$\widehat{\ell}_2$
a	0	+	0	-
θ_1	0	+	0	-
θ_2	-	+	+	+

Note that $\widehat{\ell}_2$ and ℓ_1 are the only variables which depend on the search intensity of on-the-job seekers. Obtaining a sector-2 job is the only way to obtain a sector-1 job later. Therefore, a fall in sector-2 job creation (decrease of θ_2) lowers employment in sector 2, as well as in sector 1 by decreasing the number of potential applicants (see comparative

static of Table 1). By opposition, a better transition rate for workers towards sector-1 jobs (ap_1) increases the number of the most productive jobs in the economy and reduces the on-the-job search duration, leading to a decrease in the number of skilled sector-2 workers.

4 Model equilibrium

4.1 Surplus sharing and job creation in sector 1

Job creation in sector-1 is determined by the free entry condition (8). Combining equations (6), (9) and (10), leads to the equation of the surplus S_1 generated by a filled sector-1 job:

$$S_1 = \frac{y_1 - d}{r + m + \beta p_1} \tag{25}$$

According to equation (25), the surplus of a match in sector 1 is a decreasing function of p_1, thus decreasing of tightness θ_1. In fact, an increase in the arrival rate of vacancies to searching workers raises the reservation utility (U_1) used in the bargaining process.

Surplus S_1 is then divided between both parties according to their bargaining power, β for the worker and $(1 - \beta)$ for the firm, with $\beta \in [0, 1]$. We thus have:

$$J_1 = (1 - \beta)S_1 \tag{26}$$

Using (7), (25) and (26), we obtain the sector-1 equilibrium equation:

$$0 = -c_1 + q_1(\theta_1)(1 - \beta)\frac{y_1 - d}{r + m + \beta p_1(\theta_1)} \tag{27}$$

This reduced form determines the θ_1 equilibrium value. Sector-1 tightness, defined by equation (3), is independent of sector 2 and is independent of the on-the-job search behavior. As in Pissarides (2000), θ_1 is an increasing function of all exogenous variables raising sector-1 firms profitability and conversely.

4.2 Bargained wage and optimal search intensity

As already noted, searching on the job is costly and the higher the search intensity, the higher the search cost. Therefore, their search intensity depends on their expected gain. This gain depends on the sector-1 bargained wage.

Wages are derived from static Nash bargaining (26) with (11). The wage equilibrium equation is given by:

$$w_1 = \frac{\beta y_1(r + m + p_1) + (1 - \beta)d(r + m)}{r + m + \beta p_1} \tag{28}$$

This wage, increasing with tightness θ_1, determines the expected gain of on-the-job seekers. Combining (28) with (5) yields the equation that determines the optimal on-the-job search intensity:

$$\gamma'(a) = \frac{p_1}{r + m + ap_1}\left[\frac{\beta y_1(r + m + p_1) + (1 - \beta)d(r + m)}{r + m + \beta p_1} - \bar{w}_2 + \gamma(a)\right] \tag{29}$$

Using (29), we show the following lemma.

Lemma 1. *A rise in sector-1 tightness increases the optimal search intensity (see appendix A for demonstration).*

The higher the sector-1 wage, the higher the search intensity of skilled sector-2 workers should be.

However, our model is consistent if and only if engaging in an on-the-job search process improves skilled workers' situation. The sector-1 bargained wage, depending on sector-1 productivity, must be sufficiently attractive compared with the minimum wage. For a skilled worker to engage in a searching process, her utility must necessary be above the utility of a sector-2 skilled worker who decides not to search. The expected lifetime utility of a sector-2 skilled worker who does not search while on the job is $\frac{\bar{w}_2}{r+m}$.

Equations (1) and (9) taken into account, a sector-2 skilled worker has better searching on the job for a sector-1 job, if and only if:

$$y_1 > \bar{w}_2 + (r+m)\left(\frac{\gamma(a)}{ap_1} + \frac{c_1}{q_1}\right) \quad \Leftrightarrow \quad w_1 > \bar{w}_2 + \frac{\gamma(a)}{ap_1}$$

Sector-2 skilled workers have better search a sector-1 job when w_1 is above the sum of the minimum wage and the mean search cost.

4.3 Job creation in sector 2

An appropriate substitution of (13), (14), (15) in addition with (16) gives rise to the sector-2 equilibrium equation:

$$0 = -c_2 + q_2 \frac{(y_2 - \bar{w}_2)(r+m+\lambda+ap_1)}{(r+m+\lambda)(r+m+ap_1)} \tag{30}$$

This equation defines θ_2 as a function of θ_1 and of a. Here, $\frac{\partial \theta_2}{\partial (ap_1)} < 0$. The sector-2 equilibrium equation shows θ_2 is a decreasing function of the transition rate ap_1 for workers to sector-1 job. In case that θ_1 and/or a increases, it would become easier for qualified sector-2 workers to find a sector-1 job. The value of sector-2 jobs would be reduced, less vacancies would be created and finally θ_2 would decrease.

4.4 Model recursive equilibrium

An equilibrium of the model can be defined as follow:

Definition. *An equilibrium of the labor market is a triplet $(\theta_1; a; \theta_2)$ determined by equations (27), (29) and (30).*

Notice that the model is partially recursive, (27) determines θ_1, (29) determines a as a function of θ_1 and (30) determines θ_2 as a function of a and θ_1. Other endogenous variables of the model such as the number of workers in each career stage, are deduced from variables θ_1, a and θ_2.

5 Minimum wage and employment

Since the last decade, the impact of the minimum wage on employment has strongly been debated in the empirical literature. Card and Krueger (1995)'s result, showing that an increase in the minimum wage may lead to an increase in unqualified employment, have opened the way for this literature. Among others, Neumark and Wascher (2000) have questioned Card and Krueger's result by reconsidering the data set. However, Card and Krueger (2000) have restated that a minimum wage rise may increase employment even if they have conceded that this rise most likely keeps employment unchanged.

Cardoso and Portugal (2006)'s study is particularly interesting. They obtain a similar result to Card and Krueger (1995). Their detailed analysis about young workers' flows exhibits two opposite effects of a minimum wage rise: one the one hand, young workers' accession to unqualified jobs is reduced, tending toward a lowering in youth employment; on the other hand, young workers keep there unqualified job longer, reducing job separations, tending toward an increase in youth employment.

Our model is an appropriate framework to account for the effect of the minimum wage on youth employment as described by Cardoso and Portugal. Our theoretical model explains the two opposite effect of the minimum wage through search intensity. Moreover, it allows us to extend the analysis to overall employment.

5.1 The minimum wage effect on unqualified employment: a sectorial analysis

Using (29) leads to the following result:

Result 1. *As a rise in the minimum wage does not change qualified firms' profitability, it has no impact on the qualified sector tightness. Conversely, a rise in the minimum wage does change unqualified firms' profitability, therefore, job creation in sector 2 is reduced.*

Proof. The minimum wage rise effect is deduced from equation (29), knowing that $\bar{w}_2$ has no impact on θ_1:

$$\frac{da}{d\bar{w}_2} = \frac{-p_1}{\gamma''(a)(r+m+ap_1)} < 0$$

Combining equations (13), (14), (15) and (30) lead to the following sector-2 equilibrium equation:

$$\frac{c_2}{q_2} = \frac{(y_2-\bar{w}_2)(r+m+\lambda+ap_1)}{(r+m+\lambda)(r+m+ap_1)} \Longleftrightarrow \frac{c_2}{q_2} = J_2^F$$

The impact of a minimum wage rise on job creation is defined by the following relationship:

$$\frac{d\theta_2}{d\bar{w}_2} = -\frac{dJ_2^F}{d\bar{w}_2}\frac{q_2^2}{cq_2'}$$

A necessary condition for the minimum wage to be binding (as we supposed it is in this model), is that firms have no interest to offer a higher wage[2]. It implies that $\frac{dJ_2^F}{d\bar{w}_2} < 0$.

[2]In spite of the reduction of profit per period $(y_2 - \bar{w}_2)$, firms could have better increase wages in order to prompt workers to keep their job.

So, we have:

$$\frac{d\theta_2}{d\bar{w}_2} < 0$$

□

As the minimum wage does not affect sector-1 bargained wage, sector-2 workers' gain, correlated to their probability of obtaining a sector-1 job, decreases with the minimum wage rise. This decrease in the potential gain reduces on-the-job search intensity. Transition of sector-2 workers toward sector-1 is reduced. In other words, it increases unqualified job turnover.

The previous comparative static allows us to pursue the study of the impact of the minimum wage on sector-2 employment. This sector tallies with jobs assigned to young workers studied by Cardoso and Portugal (2006).

The number of sector-2 unskilled workers decreases with a rise in the minimum wage:

$$d\ell_2 = \frac{m^2 p_2'}{(m+\lambda)(m+p_2)^2}\frac{d\theta_2}{d\bar{w}_2}d\bar{w}_2 < 0 \tag{31}$$

This result is consistent with the standard competitive theory. When the minimum wage rises, unqualified sector tightness decreases and job creation is reduced. It is harder for young unemployed to find a job. This observation complies with Cardoso et Portugal (2006). They indeed observe a reduction in job accession for young workers.

In our model, the reduction in sector-2 job accession, resulting from a cut in job creation, is neither discrepant with a rise in the number of skilled sector-2 workers nor with a rise in overall sector-2 employment. The reduction in sector-2 job accession may be more than balanced with a rise in the number of on-the-job seekers maintained in their sector-2 job (*i.e.* a reduction in separations). The two following derivatives, of the number of skilled sector-2 workers and of the overall sector-2 employment, clearly show this result:

$$d\widehat{\ell}_2 = \frac{\lambda m}{(m+\lambda)}\left[\frac{mp_2'}{(m+ap_1)(m+p_2)^2}\frac{d\theta_2}{d\bar{w}_2} - \frac{p_1 p_2}{(m+ap_1)^2(m+p_2)}\frac{da}{d\bar{w}_2}\right]d\bar{w}_2 \tag{32}$$

and

$$d(\ell_2 + \widehat{\ell}_2) = \frac{m}{(m+\lambda)}\left[\frac{mp_2'(m+\lambda+ap_1)}{(m+ap_1)(m+p_2)^2}\frac{d\theta_2}{d\bar{w}_2} - \frac{\lambda p_1 p_2}{(m+ap_1)^2(m+p_2)}\frac{da}{d\bar{w}_2}\right]d\bar{w}_2 \tag{33}$$

The impact of a rise in the minimum wage on sector-2 skilled workers' employment is uncertain. On the one hand, the reduction of the number of unskilled workers lowers the number of workers becoming skilled, and on the other hand, the minimum wage prompts skilled workers not to look efficiently for a qualified job. Skilled workers are less inclined to quit sector 2 for sector 1. They are more stable in their unqualified job. Again, our theoretical model coincides with Cardoso and Portugal's results. It offers a reasonable theoretical basis to Card and Krueger's results by suggesting that a rise in the minimum wage may increase youth employment.

When can such an increase happen? Looking at equations (32) and (33), the balance between two effects is clearly visible: the decrease of θ_2 tends to reduce unqualified employment whereas the decrease of a tends to raise unqualified employment. In the case search intensity is very sensitive to minimum wage variations, a reduction in on-the-job search intensity may thus balance or even exceed the decrease of unqualified job accession.

Proposition 1. *A rise in the minimum wage reduces unqualified job creation as well as turnover. This might lead to a rise in sector-2 employment.*

Our model has given an appropriate theoretical framework allowing us to formalize Cardoso and Portugal's interpretations and provide their empirical findings with a rational. Moreover, our model allows us to go beyond the sectorial analysis by extending our analysis to the impact of a minimum wage rise on employment at the macroeconomic level.

5.2 The minimum wage effect on global employment

We now focus on the effects of a rise in the minimum wage on unemployment and on sector-1 employment. Equations (21) and (24) lead to the following derivatives:

$$\frac{du}{d\bar{w}_2} = -\frac{mp_2'}{(m+p_2)^2}\frac{d\theta_2}{d\bar{w}_2} > 0 \tag{34}$$

and

$$\frac{d\ell_1}{d\bar{w}_2} = \frac{\lambda}{(m+\lambda)}\left[\frac{ap_1 mp_2'}{(m+ap_1)(m+p_2)^2}\frac{d\theta_2}{d\bar{w}_2} + \frac{amp_2}{(m+ap_1)^2(m+p_2)}\frac{da}{d\bar{w}_2}\right] < 0 \tag{35}$$

Equation (34) shows, without any ambiguity, a fall in global employment. This result is quite logical: sector 2 is the only gateway towards employment. A cut in sector-2 job creation thus narrows the gate and therefore reduces the number of upcoming on-the-job seekers. Sector 1 suffers twice from the minimum wage rise: the number of sector-1 applicants is reduced as well as on-the-job search intensity.

The more the minimum wage rises, the more the number of employees $(1-u)$ is reduced and the more unqualified (less productive) employment is favored. This phenomenon penalizes the global employment as well as the global output $((\ell_2 + \widehat{\ell_2})y_2 + \ell_1 y_1)$.

Proposition 2. *Despite a reduction in unqualified jobs turnover, a rise in the minimum wage necessarily raises overall unemployment.*

6 Conclusion

The theoretical model developed in our article allows to explain several recent empirical studies about the impact of a minimum wage rise on youth employment. Youth employment may remain unchanged but it may also increase. Thanks to an endogenous on-the-job search intensity, we formalize the mechanisms of accession and separation flows

described by Cardoso and Portugal in their 2006 empirical study. In spite of a reduction in unqualified job accession for unskilled young workers, the incentive of skilled workers to remain in their unqualified job may increase the number of minimum wage earners. However, this sectorial analysis does not explain the global impact of the minimum wage. Our macroeconomic analysis reveals that the reduction of unqualified job accession and the delay of skilled workers' career, affect the most productive sector in the economy by cutting qualified employment. As a result, unemployment increases and global output falls.

Our model framework has to be improved in order to better account for labor market mechanisms. Unqualified jobs, usually reserved to newcomers, are often precarious. Therefore, our assumption on the stability of unqualified sector's employment should be relaxed. We could thus show how job dismissals, even if optimal for firms, can introduce a negative externality in the overall economy.

A Appendix: tightness θ_2 impact on search intensity

The optimal search intensity increases with sector-1 tightness.

Proof. The optimal search intensity is such that

$$\gamma'(a) = \frac{p_1}{r+m+ap_1}\left[\frac{\beta y_1(r+m+p_1)+(1-\beta)d(r+m)}{r+m+\beta p_1} - \bar{w}_2 + \gamma(a)\right]$$

Around sector-1 stationary equilibrium, the optimal search intensity is:

$$\gamma'(a) = \frac{p_1}{r+m+ap_1}\left[y_1 - \frac{c_1}{q_1}(r+m) - \bar{w}_2 + \gamma(a)\right]$$

The partial derivative is:

$$\begin{aligned} &d\theta_1\left[\frac{r+m}{(r+m+ap_1)^2}\left(y_1 - \frac{c_1}{q_1}(r+m) - \bar{w}_2 + \gamma(a)\right) + \frac{\beta(1-\beta)(y_1-d)(r+m)}{(r+m+\beta p_1)^2}\right] \\ &= da\left[\frac{p_1^2}{(r+m+ap_1)^2}\left(y_1 - \frac{c_1}{q_1}(r+m) - \bar{w}_2 + \gamma(a)\right) - \frac{p_1}{r+m+ap_1}\gamma'(a) + \gamma''(a)\right] \end{aligned}$$

Replacing $\gamma'(a)$ by (29) and introducing w_1 lead to

$$\frac{da}{d\theta_1} = \frac{r+m}{(r+m+ap_1)^2\gamma''(a)}\left(w_1 - \bar{w}_2 + \gamma(a)\right) + \frac{\beta(1-\beta)(y_1-d)(r+m)}{(r+m+\beta p_1)^2\gamma''(a)} > 0$$

□

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