Career paths, unemployment and the efficiency of the labor market: should youth employment be subsidized?

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Abstract

This paper studies the implications of learning-by-doing on youth unemployment and market efficiency when workers benefiting from this kind of training experience search (while on the job) for a higher-skill job. Firms with low-skill jobs suffer from a hold up behavior by firms with high-skill jobs, causing a shortage of low-skill jobs and excessive youth unemployment. An optimal policy, consisting of taxing the output of high-skill jobs and subsidizing the output of low-skill jobs, restores market efficiency and reduces youth unemployment.

Keywords : Learning-by-doing, On-the-job search, Market efficiency, Youth unemployment, Optimal public policy.

JEL Codes : H21, J38, J64, J65.

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1 Introduction

The purpose of this paper is to study the implications of learning-by-doing on youth unemployment and market efficiency when workers benefiting from this kind of training experience search while on the job for a higher-skill job.

What should the Government do, when, as a consequence of learning-by-doing, workers' career paths take the form of a sequence of job-to-job moves from the lowest to the highest-skill jobs? We argue that firms with low-skill jobs suffer from a hold up behavior by firms with high-skill jobs, leading to insufficient low-skill job creation and an excessive level of youth unemployment. In order to restore market efficiency and reduce unemployment, the Government should tax the output of high-skill firms and subsidize the output of low-skill firms.

To assess the validity of this argument, we use a matching model that segments labor market into two interdependent sectors. Hierarchically, sector 2 offers low-skill jobs, and sector-1 high-skill jobs. Firms are infinitely lived and workers are eternally young as defined by Blanchard and Fisher (1989). The inflow of new workers entering the market exactly balances the outflow of departing workers.

When entering the labor market, new workers cannot obtain highly-paid jobs because they are not skilled enough. They can only expect a job in sector 2. While holding such a job, these inexperienced workers learn by doing. This learning process provides them with the skills needed to apply for a job in sector 1. Workers thus search (on the job) for a better-paid job, and once they find a sector-1 job, they quit their previous sector-2 job.

In our model, only low-skill workers (sector 2) search for another job while employed. This assumption is consistent with empirical studies that show that on-the-job search rates are higher for young unskilled workers and low-paid jobs. Skuterud (2005) reconstructs on-the-job search rates for Canada and the US according to age and sex. His results, which coincide with those of Pissarides and Wadsworth (1994) based on British data, suggest that on-the-job search rates actually decrease with age, and are visibly higher for young workers. Topel and Ward (1992) show that two thirds of young workers with more than one year's experience in the labor market leave their job within a year in most cases due to job-to-job transition rather than dismissal.

In both sectors, wage-setting proceeds from a Nash bargaining game and job creation results from the usual assumption of free entry. Our analysis focuses on the efficiency of the labor market. To that end, we adopt two assumptions. Firstly, for expositional simplicity, the interest rate is zero. Secondly, for methodological reasons, the Hosios condition is assumed to hold on both sub-markets. Under this condition, firms internalize what is known as the congestion effect. Our efficiency study thus focuses on the implications of the principal innovative features of our model, that is the learning-by-doing and on-the-job search processes.

We report three main results. First, in the *laissez-faire* equilibrium, job creation is too high in the high-skill sector. In other words, sector-2 firms suffer from a hold up behavior by sector-1 firms. In sector 1, firms do not internalize the true cost of recruiting a worker. Consequently, the implementation of a Taxes and Subsidies Policy (TSP), where taxes are levied on sector 1 and subsidies are allocated to sector 2, becomes a factor of motivation for job creation in sector 2. With this in mind, we develop a self-financed TSP, which allows the Government to achieve a social optimum. Finally, we show that this TSP also reduces unemployment. This means that, to achieve a social optimum, low-skill jobs are actually insufficient in number in a labor market *laissez-faire* equilibrium.

Related labor theory follows two lines of research. The first introduces on-the-job search into matching models with heterogeneous jobs and workers (see Gautier and Teulings 2006 for a brief review). One important issue is the extent to which on-the-job search reduces the mismatch between jobs and workers. The differentiation of agents may be either vertical, as in den Butter and Gorter (1999), or horizontal, as in Gautier et al. (2005) who use the circular model developed by Marimon and Zilibotti (1999). Whether vertical or horizontal, differentiation of agents is always exogenous. However, it seems that workers must first hold low-skill jobs in order to obtain high-skill jobs subsequently. Models with exogenous heterogeneity cannot account for this fact.

The second line of research examines the implications of the discontinuities in European employment protection laws when work involves a learning-by-doing process. Blanchard and Landier (2002) point out that fixed-term contracts (when combined with high employment protection as in France and Spain) incentivize firms to lay off experienced workers when the contract ends. This behavior ultimately leads to output loss and high youth unemployment, thus creating a motivation to relax labor contract regulation.

Contrary to our contribution, these papers do not allow for the on-the-job search factor and the labor market is not segmented. Firms therefore face no hold up threat, and the inefficiency of the labor market results exclusively from the institutions. From a theoretical point of view, our paper can be seen as a bridge between the two lines of research. However, we believe that it is of more than theoretical interest. It attempts to address an issue of very practical relevance: unskill job subsidies are often youth employment subsidies, and some countries (France being one) actually do subsidize youth employment. Our contribution provides this policy with a rationale.

The paper is organized as follows. Section 2 outlines our framework. We define a labor market decentralized (stationary) equilibrium in section 3. Section 4 studies market efficiency and states two main results: a decentralized equilibrium is efficient in terms of low-skill job creation but inefficient in terms of high-skill job creation; the *laissez-faire* situation is inefficient. In section 5, we exhibit a self-financed fiscal policy which leads to a social optimum and reduces unemployment. Finally, section 6 contains some concluding comments.

2 The Model

We study the implications of the learning-by-doing and on-the-job search processes in the following simplified environment. The economy consists of two types of agents, workers and firms. Firms are infinity-lived whereas workers have a finite life expectancy of $1/m$. Time is continuous and parameter m measures workers' labor market exit rate. Each worker who leaves the market is replaced with a newcomer. The measure of the total labor force is constant and normalized to one. All agents are risk-neutral and discount future payoffs at rate r ($r \geq 0$).

The labor market is segmented into two interacting sub-markets (sectors arranged into a hierarchy). Sector 2 offers low-skill jobs, while sector 1 offers high-skill jobs. When entering the labor market, new workers cannot find high-paid jobs, as they are not skilled enough. They therefore search for a job in sector 2. In a sector-2 job, workers earn wage w_2 and go through a learning-by-doing process. This learning-by-doing process enables them to develop the skills needed to work in sector 1. The expected duration of this process is denoted by $(1/\lambda)$. Beginner workers thus acquire the required skills at Poisson rate λ. Workers then search while on the job for a better paid job, and are thus called "on-the-job searchers". Finally, on finding a sector-1 job, they quit their previous sector-2 job and receive wage w_1 (with $w_1 > w_2$).

When entering the labor market, firms choose the sub-market i ($i = 1, 2$) in which they will operate. They then create a single job in their chosen sub-market. Frictions exist that prevent the instantaneous matching of jobs with workers. Firms thus have to pay a cost, c, in order to keep their vacancy open. When matched with a worker, jobs yield output y_i (with $y_1 > y_2$) and wages w_i are negotiated. Workers have a bargaining power of β and firms have a bargaining power of $(1 - \beta)$.

Concerning job creation, we adopt the usual assumption of free entry in both sectors. Firms freely enter the sub-market they choose, as long as profit opportunities exist. In equilibrium, all profit opportunities from new jobs are exploited, thus driving the value of vacant jobs, J_i^V, to zero. For $i = 1, 2$, we have:

$$J_i^V = 0 \tag{1}$$

2.1 Sector 1

Let us first describe the determination of an equilibrium in sub-market 1.

2.1.1 Asset values

A job can be either vacant or filled. Let J_1^F be the asset value of a filled sector-1 job. As a match lasts until the worker retires from the labor market, this value satisfies the following Bellman equation:

$$rJ_1^F = (y_1 - w_1) - m\left[J_1^F - J_1^V\right] \tag{2}$$

with J_1^V denoting the value of a sector-1 vacancy. We have:

$$rJ_1^V = -c + q_1\left[J_1^F - J_1^V\right] \tag{3}$$

where q_1 is the rate of arrival of on-the-job searchers (coming from sector 2) for sector-1 vacancies.

When a worker finds a job in sector 1, she keeps it until she leaves the labor market. Therefore, the expected lifetime utility of a worker in sector 1, denoted by W_1, satisfies:

$$rW_1 = w_1 - mW_1 \tag{4}$$

There is no unemployment in sector 1. However, in the bargaining process, the expected lifetime utility of an unemployed worker (in sector 1), denoted by U_1, is a reservation utility which must be defined. This asset value, which is a pure outside opportunity, satisfies:

$$rU_1 = d + p_1 [W_1 - U_1] - mU_1 \tag{5}$$

where d is the domestic output (with $d < y_2 < y_1$) and p_1 denotes the arrival rate of (sector-1) job offers, faced by on-the-job searchers.

2.1.2 Private surplus and wage setting

The (private) surplus of a match, denoted by S_1, is obtained by adding the individual surpluses of a sector-1 firm (i.e. a firm that has chosen sector 1) and its worker. Under the free-entry condition, this gives:

$$S_1 = [W_1 - U_1] + J_1^F \tag{6}$$

In this sector, wages are derived from static Nash bargaining. Surplus S_1 is then divided between both parties according to their bargaining power. We thus have:

$$W_1 - U_1 = \beta S_1 \tag{7}$$

$$J_1^F = (1-\beta) S_1 \tag{8}$$

2.2 Sector 2

Now, let us turn to sub-market 2.

2.2.1 Asset values

In sector 2, the value of a filled job changes when the learning period ends. Let $\widehat{J}_2^F$ be the value of a sector-2 job when matched with an on-the-job searcher. As the job becomes vacant when the worker enters sector 1, this asset value satisfies the following Bellman equation:

$$r\widehat{J}_2^F = (y_2 - w_2) - (m + p_1)\left[\widehat{J}_2^F - J_2^V\right] \tag{9}$$

Throughout the learning-by-doing process, beginner workers cannot quit sector 2. Therefore, the value of a sector-2 job when matched with a beginner, denoted by $\widetilde{J}_2^F$, satisfies:

$$r\widetilde{J}_2^F = (y_2 - w_2) - m\left[\widetilde{J}_2^F - J_2^V\right] - \lambda\left[\widetilde{J}_2^F - \widehat{J}_2^F\right] \tag{10}$$

In sub-market 2, the value of a vacancy is given by:

$$rJ_2^V = -c_2 + q_2 \left[\widetilde{J}_2^F - J_2^V \right] \tag{11}$$

where q_2 is the arrival rate of unemployed (new) workers for sector-2 vacancies.

In sector 2, the utility of workers also changes when the learning-by-doing process comes to an end. Workers then become on-the-job searchers, applying for a sector-1 job. Their expected lifetime utility, denoted by $\widehat{W}_2$, depends on the utility W_1 of a worker matched with a sector-1 job. We thus have:

$$r\widehat{W}_2 = w_2 + p_1 \left[W_1 - \widehat{W}_2 \right] - m\widehat{W}_2 \tag{12}$$

Beginner workers cannot enter sector 1 until the learning-by-doing process is completed. As they become on-the-job searchers at the rate λ, the expected lifetime utility of a beginner worker, denoted by $\widetilde{W}_2$, satisfies:

$$r\widetilde{W}_2 = w_2 + \lambda \left[\widehat{W}_2 - \widetilde{W}_2 \right] - m\widetilde{W}_2 \tag{13}$$

Finally, the lifetime utility of an unemployed (new) worker, U_2, satisfies:

$$rU_2 = d + p_2 \left[\widetilde{W}_2 - U_2 \right] - mU_2 \tag{14}$$

with p_2 denoting the arrival rate of (sector-2) job offers, faced by unemployed workers.

2.2.2 Private surplus and wage setting

The wage bargaining is based on the lifetime utility of a beginner worker, $\widetilde{W}_2$. The total private surplus S_2 of a match is then defined as follows:

$$S_2 = [\widetilde{W}_2 - U_2] + \widetilde{J}_2^F \tag{15}$$

According to Nash's rule, surplus S_2 is divided between both parties according to their bargaining strength (assumed to be the same as in sector 1)[1]. We obtain:

$$\widetilde{W}_2 - U_2 = \beta S_2 \tag{16}$$

$$\widetilde{J}_2^F = (1-\beta) S_2 \tag{17}$$

2.3 Flow equilibrium

In sector 1, employment is denoted by ℓ_1. In sector 2, u denotes unemployment, and ℓ_2 is total employment. The total employment in sector 2 is divided into two subsets: beginners' employment, denoted by $\widetilde{\ell}_2$, and on-the-job searchers' employment, denoted by $\widehat{\ell}_2$.

[1]Here, w_2 is not renegotiated when workers become on-the-job searchers. Results extend to the case in which wages are renegotiated at the end of the learning-by-doing process. Proofs are available from the authors upon request.

In steady state, entry flows balance exit flows at any stage of workers' careers. Flow equilibrium relationships that determine the labor force structure are the following:

$$m = u(m + p_2) \tag{18}$$

$$p_2 u = \widetilde{\ell}_2(m + \lambda) \tag{19}$$

$$\lambda \widetilde{\ell}_2 = \widehat{\ell}_2(m + p_1) \tag{20}$$

$$p_1 \widehat{\ell}_2 = m\ell_1 \tag{21}$$

This system exhibits the interactions between both sectors in steady state. Equation (19) shows that beginners exclusively comprise previously unemployed workers. Equation (20) proceeds from the learning-by-doing process. When leaving beginner employment, workers either become on-the-job searchers or retire from the labor market. On-the-job searchers either leave the market permanently or quit their current job for a job in sector 1 (equation (21)).

3 Decentralized Equilibrium

Let v_i denote vacant jobs in the labor sub-market i ($i = 1, 2$). The sub-market tightness of sector 1 is given by $\theta_1 = v_1/\widehat{\ell}_2$ and the sub-market tightness of sector 2 is given by $\theta_2 = v_2/u$.

Market frictions in sector-i are summarized in a constant-returns matching function that defines the arrival rate of workers for job vacancies $q_i(\theta_i)$ with $q_i'(\theta_i) < 0$ and the arrival rate of job offers for searching workers $p_i = \theta_i q_i$ with $p_i'(\theta_i) > 0$.

3.1 Equilibrium in Sector 1

For the determination of the sub-market tightness θ_1, the structure of the model is the same as the basic matching model (see Pissarides 2000). Therefore, the private surplus S_1 is determined as follows:

$$S_1 = \frac{y_1 - d}{r + m + \beta p_1} \tag{22}$$

According to equation (22), the surplus of a match in sector 1 is a decreasing function of rate p_1, and hence of tightness θ_1. An increase in the arrival rate of vacancies for searching workers raises the reservation utility (U_1) of workers in the bargaining process.

Substitution into equation (3) yields the sector-1 equilibrium equation. Using equation (7), we have:

$$0 = -c + q_1(\theta_1)(1 - \beta)\frac{y_1 - d}{r + m + \beta p_1(\theta_1)} \tag{23}$$

This reduced form determines the equilibrium value of θ_1. The tightness of sub-market 1 appears to be independent of sector 2. This is because firms freely enter the market, thus adjusting the number of vacancies to the number of on-the-job searchers. However, as we shall see below, employment on sub-market 1 does depend on the tightness of sub-market 2.

3.2 Equilibrium in Sector 2

Now let us turn to sector 2 (see appendixes for detailed calculus). A proper substitution of equations shows that, in steady state, private surplus S_2 is determined as follows:

$$\frac{(r+m+\lambda)(r+m+p_2\beta)}{r+m}S_2 = \frac{r+m+p_1+\lambda}{r+m+p_1}(y_2-d) + \frac{\lambda p_1 \beta}{r+m}S_1 \tag{24}$$

According to this equation, the surplus of a match in sector 2 (S_2) is an increasing function of the surplus of a match in sector 1 (S_1). This is because the expected lifetime utility of a beginner worker ($\widetilde{W}_2$) depends on all the future stages of her career path, hence on her surplus when finding a better-paid job. Taking into account the sector-1 equilibrium equation, surplus S_2 can be rewritten as (see appendix (A.1)):

$$\begin{aligned}\frac{(r+m+\lambda)(r+m+\beta p_2)}{r+m}S_2 &= \frac{r+m+p_1+\lambda}{r+m+p_1}(y_2-d) + \frac{\lambda p_1}{(r+m)(r+m+p_1)}(y_1-d) \\ &\quad -\frac{\lambda c}{r+m+p_1}\theta_1 \end{aligned}\tag{25}$$

This relationship plays a key role in the welfare study, as it shows that sector-2 firms perceive the effect of their entry decision on job creation in the other sub-market. Private surplus S_2 depends on tightness θ_1.

Finally, substitution of S_2 into the expression of the value of a vacancy yields the sector-2 equilibrium equation. This gives:

$$\begin{aligned}0 &= -c(r+m+\beta p_2(\theta_2)) \\ &\quad +q_2(\theta_2)\frac{(1-\beta)(r+m)}{(r+m+\lambda)(r+m+p_1(\theta_1))}\left[(r+m+p_1(\theta_1)+\lambda)y_2 + \frac{\lambda p_1(\theta_1)}{r+m}y_1 - \lambda c\theta_1\right] \\ &\quad -q_2(\theta_2)(1-\beta)d \end{aligned}\tag{26}$$

According to equation (26), the tightness of sub-market 2 is a function of the tightness of sub-market 1. Equilibrium in sector 2 depends on the equilibrium in sector 1.

3.3 Labor force structure

From the conditions for flow equilibrium (equations (18), (19), (20), (21)), we deduce the labor force structure as functions of sub-market tightness θ_1 and θ_2. We obtain:

$$u = \frac{m}{m+p_2} \tag{27}$$

$$\widetilde{\ell}_2 = \frac{p_2 m}{(m+p_2)(m+\lambda)} \tag{28}$$

$$\widehat{\ell}_2 = \frac{\lambda p_2 m}{(m+p_1)(m+p_2)(m+\lambda)} \tag{29}$$

$$\ell_1 = \frac{\lambda p_2 p_1}{(m+p_1)(m+p_2)(m+\lambda)} \tag{30}$$

θ_1 is independent of θ_2, but this does not mean high-skill employment is independent of job creation in the low-skill sub-market. Owing to the interactions between the two sub-markets, high-skill employment depends on the transition rates in sectors 1 and in sector 2.

3.4 General equilibrium

To sum up, a stationary decentralized equilibrium can be defined as follows:

Definition 1. *A decentralized equilibrium of the labor market is a pair* (θ_1, θ_2) *which satisfies equations (23) and (26).*

We deduce the equilibrium values for labor force structure from sub-market tightness θ_1 and θ_2.

4 Market efficiency

One important issue is the efficiency of such a decentralized equilibrium. Is *laissez-faire* an optimum? In particular, is job creation high enough in the low-skill labor sub-market? And, if not, what should be done? To answer this question, we first define a social optimum, then compare it with a labor market decentralized equilibrium.

4.1 Social optimum

Along the same lines as Hosios (1990) and Pissarides (2000), let us consider a social planner who is only subject to search frictions, and can redistribute income among agents at no cost. Restricting our study to the case where the interest rate reduces to zero[2], this planner should maximize the social surplus flow at steady state.

As $\theta_1 = v_1/\widehat{\ell}_2$ and $\theta_2 = v_2/u$, this social surplus, denoted by CS, can be written as follows.

$$CS = y_1\ell_1 + y_2(\widetilde{\ell}_2 + \widehat{\ell}_2) + du - c\theta_1\widehat{\ell}_2 - c\theta_2 u \tag{31}$$

As stated above (subsection 3.3), variables ℓ_1, $\widetilde{\ell}_2$, $\widehat{\ell}_2$ and u are functions of the pair (θ_1, θ_2) (see appendix A.2 for derivatives). The same therefore holds for CS. We thus have:

$$CS = CS(\theta_1, \theta_2)$$

A social optimum can thus be defined as follows:

Definition 2. *A social optimum is a pair* (θ_1, θ_2) *that maximizes the surplus* $CS(\theta_1, \theta_2)$.

This definition leads us to compute the derivative of $CS(.)$ with respect to both its arguments. The derivative of $CS(.)$ with respect to θ_1 can be written as follows (see appendix (A.3)):

$$\frac{\partial CS}{\partial \theta_1} = \frac{\lambda\, p_2 m}{(m+p_1)^2(m+p_2)(m+\lambda)}\Big[(1-\eta_1)q_1\,(y_1-y_2) - c(m+\eta_1 p_1)\Big] \tag{32}$$

[2]Efficiency results extend to a positive interest rate. Proofs are available from the authors upon request.

where $(1-\eta_1)$ is the elasticity of rate p_1. Equalizing this derivative to zero determines the θ_2 level that a social planner should set. Notice that the optimal value of θ_1 does not depend on θ_2.

The other derivative of $CS(.)$ can be written as follows (see appendix (A.3)):

$$\frac{\partial CS}{\partial \theta_2} = \frac{m}{(m+p_2)^2}\Bigg[\frac{q_2(1-\eta_2)m}{(m+p_1)(m+\lambda)}\left(\frac{\lambda\, p_1}{m}y_1 + (m+p_1+\lambda)y_2 - \lambda\, c\theta_1\right) \\ -q_2(1-\eta_2)d - c(m+\eta_2 p_2)\Bigg] \quad (33)$$

where $(1-\eta_2)$ is the elasticity of rate p_2. Equalizing this derivative to zero determines the θ_2 level that a social planner should set for a given level of θ_1. Contrarily to θ_1, the (partially) optimal value of θ_2 does depend on θ_1.

4.2 Efficiency of a decentralized equilibrium

We now study the efficiency of the *laissez-faire* situation by comparing the decentralized equilibrium with the social optimum. In this comparison, we will assume that the Hosios condition holds on both sub-markets. We then have:

$$\eta_1 = \eta_2 = \beta$$

The reason for adopting this assumption is mainly methodological. Here, what we want to emphasize are the implications for market efficiency of the innovative features of our model, that is, the learning-by-doing and on-the-job search processes. We already know that when the Hosios condition is not satisfied, firms do not internalize the so-called congestion effect of job creation.

Under the Hosios condition, sector-1 equilibrium equation (23) satisfies (for $r=0$):

$$c(m+\eta_1 p_1) = (1-\eta_1)q_1\,(y_1 - d)$$

Substitution into equation (32) shows that the derivative of the social surplus with respect to θ_1 has the same sign as:

$$-c(m+\eta_1 p_1) + (1-\eta_1)q_1(y_1-y_2) = -(1-\eta_1)q_1(y_2-d) < 0$$

This leads to the following proposition:

Proposition 1. *A decentralized equilibrium is* ***inefficient*** *in terms of job creation in sector 1, as the equilibrium value of sub-market tightness θ_1 is necessarily greater than its optimal value.*

The intuition behind this proposition is as follows. Sector-1 firms underestimate the hiring cost of a worker, as the true hiring cost is based on the productivity of a sector-2 firm, y_2, not the domestic output, d. Therefore, the creation of sector-1 vacancies is too high in a decentralized equilibrium. As expected, sector-2 firms suffer from a hold up behavior by sector-1 firms.

Let us now turn to sector 2. Under the Hosios condition, the equilibrium equation (26) can be rewritten as follows (for $r = 0$):

$$0 = -c(m + \eta_2 p_2) + q_2 \frac{(1-\eta_2)m}{(m+\lambda)(m+p_1)} \left[(m + p_1 + \lambda)y_2 + \frac{\lambda p_1}{m} y_1 - \lambda c \theta_1 \right] - q_2(1-\eta_2)d$$

Taking into account equation (33), this shows that:

Proposition 2. *A decentralized equilibrium is* ***partially efficient*** *in terms of job creation in sector 2, as the equilibrium value of sub-market tightness θ_2 maximizes the social surplus for the equilibrium value of sub-market tightness θ_1.*

The reason why we obtain this result is twofold. Firstly, sector-2 firms correctly evaluate the opportunity cost of hiring a worker who will leave for a sector-1 job. They know that they lose output y_2 when on-the-job searchers find a sector-1 job. Next, as private surplus S_2 depends on $\beta S_1 (= S_1 - \frac{c}{q_1})$, they internalize the costs of job creation in sector 1. Notice that this efficiency result does not mean that the equilibrium value of θ_2 is an optimum; θ_2 depends on θ_1 and we know that θ_1 is too high.

5 Optimal public policy

The *laissez-faire* situation is not an optimum. What, then, should the Government do? We now present a self-financed Taxes and Subsidies Policy (TSP) leading to a social optimum. In addition, this TSP appears to reduce unemployment. The same assumptions as above are adopted. The interest rate is equal to zero and the Hosios condition holds on both sub-markets.

5.1 Taxes, subsidies and efficiency

5.1.1 Taxing Sector 1

We now prove that the Government can decentralize the social optimum by implementing an appropriate fiscal policy.

We assume that sector-1 firms with a filled job pay a tax τ per period. Therefore, the Bellman equation becomes:

$$rJ_1^F = y_1 - w_1 - \tau - mJ_1^F$$

and private surplus S_1 is now given by:

$$S_1 = \frac{y_1 - d - \tau}{m + \beta p_1}$$

Therefore, the sector-1 equilibrium equation becomes:

$$0 = -c + q_1(1-\beta)\frac{y_1 - d - \tau}{m + \beta p_1} \tag{34}$$

If the Government sets:

$$\tau = y_2 - d \tag{35}$$

Equation (34) then coincides with the optimality condition in sector 1 derived from equation (32) (for $\beta = \eta_1$). In short, when paying tax $\tau = y_2 - d$, firms internalize the true hiring cost and thus job creation becomes efficient in sector 1. In addition, this result holds whatever the tax finances.

However, implementing this tax lowers (private) surplus S_1. Consequently, job creation in sector 2 is no longer efficient. This is a motivation for subsidizing the production of sector 2. But these subsidies must be financed. We thus have to show that the taxes which are levied on sector 1 exactly cover the subsidies needed to restore the efficiency of job creation in sector 2.

5.1.2 Subsidizing Sector 2

Let σ be the subsidy (per period) that a sector-2 firm receives from the Government as long as its job is filled. Subsidy σ must satisfy the budget constraint:

$$\tau \ell_1 = \sigma(\widetilde{\ell}_2 + \widehat{\ell}_2)$$

Combining equations (28), (29) and (30) gives subsidy σ as a function of rate p_1:

$$\sigma = \frac{\ell_1}{\widetilde{\ell}_2 + \widehat{\ell}_2}\tau = \frac{\lambda p_1}{m(m + p_1 + \lambda)}\tau \tag{36}$$

In the presence of this subsidy, we obtain the private surplus S_2 by replacing y_2 with $(y_2 + \sigma)$ in equation (24)(for $r = 0$):

$$\frac{(m+\lambda)(m+\beta p_2)}{m} S_2 = \frac{m + p_1 + \lambda}{m + p_1}(y_2 - d + \sigma) + \frac{\lambda p_1 \beta}{m} S_1 \tag{37}$$

Therefore, for a full TSP ($\tau = y_2 - d$), the sector-2 equilibrium equation becomes[3]:

$$0 = -c(m + \eta_2 p_2) + q_2 \frac{1-\eta_2}{m+\lambda}\left[(m+\lambda)y_2 + \frac{\lambda p_1 \beta}{m + \beta p_1}(y_1 - y_2)\right] - q_2(1-\eta_2)d \tag{38}$$

We can now state the following result:

Proposition 3. *With a full TSP, the decentralized equilibrium coincides with the social optimum.*

Proof. Let us consider the following expression, denoted by Φ:

$$\Phi = (m + p_1)[(m+\lambda)y_2 + \frac{\lambda p_1 \beta}{m + \beta p_1}(y_1 - y_2)]$$

This expression is equivalent to:

$$\Phi = m(m + \lambda + p_1)y_2 + \lambda p_1 y_1 - \lambda p_1 (y_1 - y_2) + \frac{\beta(m+p_1)}{m + \beta p_1}\lambda p_1 (y_1 - y_2)$$

[3] This equation is obtained by replacing y_2 with $(y_2 + \sigma)$ and (23) with (34) in Appendix (A.1.)

Using equation (34), we obtain (for $\tau = y_2 - d$):

$$\Phi = m(m + \lambda + p_1)y_2 + \lambda p_1 y_1 - m\lambda c\theta_1$$

Finally, substitution of Φ into equation (38) proves proposition 3 (see equation (33)). □

At first glance, this result looks surprising. It needs to be interpreted. We notice that as the tax makes tightness θ_1 become an optimum, sector-2 firms perfectly internalize the effects of their entry decision through the expression $(p_1\beta S_1)$ (see our comment on Proposition 2). The optimality of tightness θ_2 thus requires sector-2 firms to be compensated for what they lose when on-the-job searchers quit. This loss can be seen as a life annuity of an amount of $(y_2 - d)$ per period, ending at rate m. The lifetime expected loss is thus given by:

$$\Sigma = \frac{y_2 - d}{m}$$

It can thus be demonstrated that for job creation in sector 2, receiving the amount Σ when workers quit, is equivalent to receiving the subsidy flow $\sigma = y_2 - d$ for as long as jobs are filled. With the compensatory transfer Σ, the value of a sector-2 job when matched with an on-the-job searcher is given by:

$$r\widehat{J}_2^F = (y_2 - w_2) - (m + p_1)\left[\widehat{J}_2^F - J_2^V\right] + p_1\Sigma$$

Therefore, with transfer Σ, the private surplus $S_2(\Sigma)$ satisfies[4]:

$$\frac{(m+\lambda)(m+\beta p_2)}{m} S_2(\Sigma) = \frac{m + p_1 + \lambda}{m + p_1}(y_2 - d) + \frac{\lambda p_1 \beta}{m} S_1 + \frac{\lambda p_1 \Sigma}{m + p_1}$$

As (see equation (36))

$$\lambda p_1 \Sigma = (m + \lambda + p_1)\frac{\lambda p_1}{m(m + \lambda + p_1)}(y_2 - d) = (m + \lambda + p_1)\sigma,$$

private surplus $S_2(\Sigma)$ satisfies equation (37). This shows that transfer Σ is equivalent to subsidy flow σ.

5.2 Taxes, subsidies and unemployment

We have established that a full Taxes and Subsidies Policy can decentralize the social optimum. However, although the social surplus is the right theoretical criterion (insofar as the Government can redistribute the income at no cost), the advantages of such a policy may appear questionable if it leads to a rise in unemployment. To address this, we now prove that a TSP lowers unemployment as long as the private surplus per period of a match in sector 1 remains greater than $(y_1 - y_2)$. In other words, tax τ must be lower than $(y_2 - d)$ (see equation (34)).

In steady state, unemployment depends exclusively on the exit rate p_2. Therefore, all we need to know is the effect of a TSP on sub-market tightness θ_2. To that end, let us consider private surplus S_2.

[4] Detailed calculus follow the same line as Appendix (A.1).

Substitution of (36) into equation (37) shows that in the presence of a TSP, private surplus S_2 satisfies:

$$(m+\beta p_2)S_2 = y_2 - d - \frac{\lambda p_1}{(m+p_1)(m+\lambda)}(y_2 - d - \tau) + \frac{\lambda p_1 \beta}{m+\lambda} S_1$$

Or:

$$(m+\beta p_2)S_2 = y_2 - d + \frac{\lambda p_1}{(m+p_1)(m+\lambda)}(y_1 - y_2) - \frac{\lambda p_1}{(m+p_1)(m+\lambda)}(y_1 - d - \tau) + \frac{\lambda p_1 \beta}{m+\lambda} S_1$$

As:

$$S_1 = \frac{y_1 - d - \tau}{m + \beta p_1},$$

it results that:

$$(m+\beta p_2)S_2 = y_2 - d + \frac{\lambda p_1}{(m+p_1)(m+\lambda)}(y_1 - y_2) - \frac{(1-\beta)m\lambda p_1}{(m+p_1)(m+\lambda)} S_1$$

Using the equilibrium equation of sector 1 (equation (34)), we finally obtain:

$$(m+\beta p_2)S_2 = y_2 - d + \lambda \frac{p_1(y_1 - y_2) - mc\theta_1}{(m+p_1)(m+\lambda)} \tag{39}$$

According to equation (39), taxes and subsidies have no direct effect on the private surplus S_2. Their effect is conveyed through sub-market tightness θ_1. This property is used to formulate the following proposition.

Proposition 4. *Under the Hosios condition, a TSP lowers unemployment for all* $\tau < y_2 - d$.

Proof. From (34) and (39), we deduce that the sign of the derivative of S_2 with respect to θ_1 is given by the sign of (for $\eta_1 = \beta$):

$$\begin{aligned} q_1(1-\eta_1)(y_1 - y_2) - (m + \eta_1 p_1)c &= q_1(1-\eta_1)(y_1 - d - \tau) - (m + \eta_1 p_1)c \\ &\quad - q_1(1-\eta_1)(y_2 - d - \tau) \\ &= q_1(1-\eta_1)[\tau - (y_2 - d)] \end{aligned}$$

This shows that S_2 is a decreasing function of θ_1 for all $\tau < (y_2 - d)$. The same holds therefore for θ_2. Consequently, an increase in τ, that lowers θ_1 (see equation (34)), raises θ_2, leading to a fall in unemployment. □

This result is not very surprising. In line with our intuition, it means that, compared with the social optimum where market tightness is optimal in both sub-markets, there are too few low-skill jobs in the *laissez-faire* equilibrium because there are too many high-skill jobs. In other words, taxing high-skill jobs softens the threat of hold up faced by sector-2 firms, thus stimulating the creation of low-skill jobs. Consequently, youth unemployment falls.

6 Conclusion

In continental Europe, many countries face a high youth unemployment problem. In response to this situation, Governments often subsidize youth employment through different channels. Are these subsidies justified? And if so, how should they be financed? It could be argued that young workers entering the labor market naturally face higher unemployment rates (see Blanchard 2005) than their older counterparts. The youth unemployment problem thus arguably proceeds from bad overall market conditions and does not require any specific measure.

Assuming that beginners' work involves a learning-by-doing process, we have shown that such subsidies do improve market efficiency and reduce youth unemployment insofar as they are financed by taxes levied on the firms with high-skill jobs, i.e. the firms who take advantage of the learning-by-doing process.

Our analysis can be pursued along different lines. First, it could be useful to examine how the optimal public policy is affected when beginner workers receive a minimum wage, for with a binding minimum wage, job creation at the lower end of the market will no longer depend on the surplus of high-skill jobs. Another important issue is the need to protect the employment of beginner workers. Layoffs on the low-skill labor sub-market tend to delay the learning-by-doing process, leading to an exaggeratedly low employment level in the high-skill sub-market. Do firms internalize this effect when deciding on their layoff behavior?

A Appendix: Detailed calculus

A.1 Equilibrium in Sector 2

W_1 can be rewritten in terms of S_1 from (5) and (7):

$$W_1 = \beta S_1 + \frac{d + p_1 \beta S_1}{r+m} = \beta S_1 \frac{r+m+p_1}{r+m} + \frac{d}{r+m} \tag{40}$$

Equations (13) and (12) lead to:

$$(r+m+\lambda)\widetilde{W}_2 = w_2 + \frac{\lambda}{(r+m+p_1)}(w_2 + p_1 W_1) \tag{41}$$

$\widetilde{W}_2$ can be rewritten in terms of S_1 from (40) and (41):

$$(r+m+\lambda)\widetilde{W}_2 = w_2 + \frac{\lambda w_2}{(r+m+p_1)} + \frac{\lambda p_1 \beta S_1}{r+m} + \frac{\lambda p_1 d}{(r+m)(r+m+p_1)} \tag{42}$$

Equations (14) and (16) give:

$$U_2 = \frac{d + p_2 \beta S_2}{r+m} \tag{43}$$

The surplus of a beginner on a sector-2 job can be expressed as a function in S_1 and S_2 by combining equations (42) and (43):

$$\begin{aligned}(r+m+\lambda)\left[\widetilde{W}_2 - U_2\right] &= \frac{r+m+p_1+\lambda}{r+m+p_1} w_2 + \frac{\lambda p_1 \beta}{r+m} S_1 + \frac{\lambda p_1 d}{(r+m)(r+m+p_1)} \\ &\quad - \frac{r+m+\lambda}{r+m} d - \frac{p_2 \beta (r+m+\lambda)}{r+m} S_2 \end{aligned} \tag{44}$$

Equations (9) and (10) give:

$$(r+m+\lambda)\widetilde{J}_2^F = \frac{r+m+p_1+\lambda}{r+m+p_1}(y_2 - w_2) \tag{45}$$

The surplus of a sector-2 job (15) combined with (44) and (45) gives S_2 as a function of S_1:

$$\frac{(r+m+\lambda)(r+m+p_2\beta)}{r+m} S_2 = \frac{r+m+p_1+\lambda}{r+m+p_1}(y_2 - d) + \frac{\lambda p_1 \beta}{r+m} S_1 \tag{46}$$

The term $\frac{\lambda p_1 \beta S_1}{r+m}$ can be written as:

$$\begin{aligned}\frac{\lambda p_1 \beta S_1}{r+m} &= \frac{\lambda p_1 \beta}{(r+m)(r+m+\beta p_1)}(y_1 - d) \\ &= \frac{\lambda p_1 (y_1 - d)}{(r+m)} \frac{\beta(r+m+p_1) - (r+m) + (r+m)}{(r+m+p_1)(r+m+\beta p_1)} \\ &= \frac{\lambda p_1 (y_1 - d)}{(r+m)} \frac{(r+m+\beta p_1) - (1-\beta)(r+m)}{(r+m+p_1)(r+m+\beta p_1)} \\ &= \frac{\lambda p_1 (y_1 - d)}{(r+m)(r+m+p_1)} - \frac{\lambda p_1}{r+m+p_1} \frac{(1-\beta)(y_1 - d)}{r+m+\beta p_1}\end{aligned}$$

Using the equilibrium equation (23) in sector 1, it results that:

$$\frac{\lambda p_1 \beta S_1}{r+m} = \frac{\lambda p_1 (y_1 - d)}{(r+m)(r+m+p_1)} - \frac{\lambda c}{r+m+p_1} \theta_1$$

Finally, equation (46) can be rewritten as:

$$\frac{(r+m+\lambda)(r+m+\beta p_2)}{r+m}S_2 = \frac{r+m+p_1+\lambda}{r+m+p_1}(y_2-d) + \frac{\lambda p_1}{(r+m)(r+m+p_1)}(y_1-d) - \frac{\lambda c}{r+m+p_1}\theta_1$$

Substituting the equilibrium equation of sector 2,

$$\frac{c}{q_2} = (1-\beta)S_2,$$

in the previous equation, finally leads to the equilibrium equation:

$$0 = -c(r+m+\beta p_2) + q_2\frac{(1-\beta)(r+m)}{(r+m+\lambda)(r+m+p_1)}\left[(r+m+p_1+\lambda)y_2 + \frac{\lambda p_1}{r+m}y_1 - \lambda c\theta_1\right] - q_2(1-\beta)d$$

A.2 Derivatives of employment and unemployment levels

	ℓ_1	$\widetilde{\ell_2}$	$\widehat{\ell_2}$	u
θ_1	$p_1'\frac{\lambda p_2 m}{(m+p_1)^2(m+p_2)(m+\lambda)}$	0	$-p_1'\frac{\lambda p_2 m}{(m+p_1)^2(m+p_2)(m+\lambda)}$	0
θ_2	$p_2'\frac{\lambda p_1 m}{(m+p_1)(m+p_2)^2(m+\lambda)}$	$p_2'\frac{m^2}{(m+p_2)^2(m+\lambda)}$	$p_2'\frac{\lambda m^2}{(m+p_1)(m+p_2)^2(m+\lambda)}$	$-p_2'\frac{m}{(m+p_2)^2}$

A.3 Market efficiency

Impact of θ_1 on the social surplus:

$$\begin{aligned}
\frac{\partial CS}{\partial\theta_1} &= \frac{\partial\ell_1}{\partial\theta_1}y_1 + \frac{\partial\widehat{\ell_2}}{\partial\theta_1}y_2 - c\widehat{\ell_2} - c\theta_1\frac{\partial\widehat{\ell_2}}{\partial\theta_1} = 0\\
&= p_1'\frac{\lambda p_2 m}{(m+p_1)^2(m+p_2)(m+\lambda)}(y_1-y_2+c\theta_1) - c\frac{\lambda p_2 m}{(m+p_1)(m+p_2)(m+\lambda)} = 0\\
&= \frac{\lambda p_2 m}{(m+p_1)(m+p_2)(m+\lambda)}\left[\frac{p_1'}{m+p_1}(y_1-y_2+c\theta_1) - c\right] = 0\\
&= \frac{\lambda p_2 m}{(m+p_1)(m+p_2)(m+\lambda)}\left[\frac{p_1'}{m+p_1}(y_1-y_2) + \frac{p_1'}{m+p_1}c\theta_1 - c\right] = 0
\end{aligned}$$

The elasticity of p_1 with respect to θ_1 satisfies: $p_1' = (1-\eta_1)q_1$. The previous derivative can therefore be rewritten as:

$$\frac{\partial CS}{\partial\theta_1} = \frac{\lambda p_2 m}{(m+p_1)^2(m+p_2)(m+\lambda)}\left[(1-\eta_1)q_1(y_1-y_2) - c(m+\eta_1 p_1)\right] = 0$$

Impact of θ_2 on the social surplus:

$$\begin{aligned}
\frac{\partial CS}{\partial\theta_2} &= \frac{\partial\ell_1}{\partial\theta_2}y_1 + \left(\frac{\partial\widehat{\ell_2}}{\partial\theta_2} + \frac{\partial\ell_2}{\partial\theta_2}\right)y_2 + d\frac{\partial u_2}{\partial\theta_2} - c\theta_1\frac{\partial\widehat{\ell_2}}{\partial\theta_2} - u_2 c - c\theta_2\frac{\partial u_2}{\partial\theta_2} = 0\\
&= p_2'\frac{\lambda p_1 m}{(m+p_1)(m+p_2)^2(m+\lambda)}y_1 + y_2\left(p_2'\frac{m^2}{(m+p_2)^2(m+\lambda)} + p_2'\frac{\lambda m^2}{(m+p_1)(m+p_2)^2(m+\lambda)}\right)\\
&\quad -p_2'\frac{m}{(m+p_2)^2}d - p_2'\frac{\lambda m^2}{(m+p_1)(m+p_2)^2(m+\lambda)}c\theta_1 + p_2'\frac{m}{(m+p_2)^2}c\theta_2 - \frac{m}{m+p_2}c\\
&= \frac{m}{(m+p_2)^2}\left[\frac{\lambda p_1 p_2' y_1}{(m+p_1)(m+\lambda)} + \frac{p_2' m y_2}{m+\lambda} + \frac{p_2'\lambda m y_2}{(m+p_1)(m+\lambda)} - p_2' d - \frac{p_2'\lambda m c\theta_1}{(m+p_1)(m+\lambda)} + p_2' c\theta_2 - c(m+p_2)\right]
\end{aligned}$$

The elasticity of p_2 with respect to θ_2 satisfies: $p_2' = (1 - \eta_2) q_2$. The previous derivative can therefore be rewritten as:

$$\frac{\partial CS}{\partial \theta_2} = \frac{m}{(m+p_2)^2}\left[\frac{q_2(1-\eta_2)m}{(m+p_1)(m+\lambda)}\left(\frac{\lambda\, p_1}{m} y_1 + (m+p_1+\lambda) y_2 - \lambda\, c\theta_1\right) - q_2(1-\eta_2)d - c(m+\eta_2 p_2)\right] = 0$$

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