A GAME THEORY MODEL OF INVERSE PRICING STRATEGY

Ying Kong, Hsiao-Chan Wang
Department of Economics, York University
Toronto, Ontario, Canada

ABSTRACT

The inverse pricing strategy refers to the producer (or producers) "abnormally" increases its product price when its rivals decrease the price of theirs. This strategy has been observed at least in automobile and pharmaceutical industries. The objective of this paper is to use a two-stage game theory model to demonstrate why and when an inverse pricing strategy is sustainable in the market. The driving force in the model is that the market is segmented into two types of consumers, brand loyal consumers and price sensitive consumers. The brand loyal consumers will be only interested in one particular brand name product while the price sensitive consumers will adjust their consumptions to either brand name or non-brand name products based on market prices.The important finding of the research is that the strategy can be profitable as long as the cross-elasticity between two goods is within a certain range.

Key wards: Inverse Pricing, Two Stage Game, Auto Price Competition

1. INTRODUCTION

The market of automobile depends heavily on consumer's tastes and purchasing trends. Although the car companies do earn revenues from selling their vehicles to businesses and car rental companies through fleet sales, the direct consumer purchases are still the main source of revenue. For this reason, firms do thoughtfully taking into account the consumers' confidence in returning purchases.

The highly competitive industries generally earn low returns because the high competition drives down the market price. The automobile industry is considered as an oligopoly market and it minimizes the effect on price-based competition. The automakers understand and try to avoid the price-based competition because it does not necessarily lead to an increase in the market share. In the recent market structure, the competition between firms has seriously focused and based on introducing different types of promotion packages, such as preferred financing, intensifies rebates and long-term warranties, to make their products attractive. These promotion packages directly have influences on the price of the automobiles and critically play a role in the pricing-setting strategy in the market of automobile. A typical example is the "Employee Discount", which is a promotion to consumers practiced by three key American auto-producers starting in the summer of 2005. General Motors (GM) began its discount addicted ways in June of 2005 and extended to September with the announcement of its "Employee Discounts for Every One (EDEO) pricing scheme". This GM's employee discount program was a success because Ford and Chrysler were jumping on board. Ford joined GM by announcing a promotion so called the "Ford Family Plan" and Chrysler advertised its plan of "Employee Pricing Plus" immediately. The reason that both Ford and Chrysler exercised their promotion packages immediately after GM's success is understandable because GM had a 46.9 percent jump up in sales during the time of promotion, and which was the best selling period since 1986. GM sold more cars and light trucks in June than in any other months since September 1986.[1] This promotion package is attractive to some consumers who do not traditionally purchase automobiles from GM but are willing to try and also to many who do not necessarily need a new car but are buying due to the special offering. [2]

Nevertheless, Toyota Motor, one of the main competitors to the American automakers, reacted differently facing the fact that there was a deep price cut in the American car market. Toyota Motor surprisingly and unexpectedly announced a slightly price increase on many of its car models sold in United States at the time when Detroit's Big Three automakers were offering the 'Employee Discounts' to nearly everyone who walked in door. We call the strategy taken by Toyota Motor as the "Inverse Pricing Strategy". The inverse pricing strategy taken by Toyota Motor surprises most business analysts and economists, because the players' strategic reaction to a declined price set by their rivals should be lowering their price offering as well, according to the traditional *Bertrand* price competition game in an oligopoly market. The "Inverse Pricing Strategy" is also observed in the pharmaceutical market. Studies have shown that, when confronted by new generic entry into a market, the price of brand-name drugs increases, a finding contrary to what would normally occur when competition in a market increases (Frank and Salkever 1992, 1995; Caves, Whinston and Hurwitz, 1991, Kong 2004). This is phrased as "Generic Competition Paradox" in the literature (Scherer 1993).

Our study is using a two-stage game-theoretic model to demonstrate the conditions that an inverse pricing strategy is sustainable in the market. The driving force in this model is to segment the market into two types of consumers, the brand loyal consumers who only consume a particular brand and the price-sensitive consumers (or the non-loyal consumers) who consume different brands based on the price offering.

This study is related to two areas of research in the literature: the auto pricing and the brand loyalty. Verboven (1999) addresses a question whether and when the prices practiced on the basic products may differ from those on the premium products, such that should the products be sold with options or with add-ons. He concludes that only the brand rivalry model with limited consumer information predicts the result that the premium products have the larger percentage mark-up than the basic products; this provides that the brand rivalry is sufficiently intense. Leeflang and Wittink (2001) show the promotional expenditure decisions on a brand, as in any other marketing decisions, should be based on the expected impact on purchases and the consumption behaviours as well as on the likely reactions of the competitors. They also find that the elasticity of the competitive reaction is a positive function of the elasticity of cross-brand market share and is a negative function of the elasticity of own-brand market share. However, their research does not deeply cover the price strategy. Bloemer and Kasper (1995) identify two types of brand satisfaction: manifest satisfaction and latent satisfaction as well as two types of brand loyalty: true brand loyalty and spurious brand loyalty. They also investigate the relationship between the brand satisfaction and the brand loyalty, and show the relationship does not indeed depend on the type of satisfaction. The positive impact of manifest satisfaction on true brand loyalty is greater than the positive impact of latent satisfaction on true brand loyalty.

Another research done by Frank and Salkever (1992, 1995) perhaps is a little closer to our study. They develop a simple theoretic model to explain the possibility that the price of brand-name pharmaceutical increases with an increasing number of generic entrants, in which, a simple pharmaceutical market is segmented basing on the persistency of physicians' prescription patterns; some physicians are more likely to prescribe brand-name drugs while others would more often prescribe generics. The drug buyers are divided into two rigid groups: price-sensitive consumers as "disloyal" and price-insensitive consumers as "loyal". The demand for "loyal" consumers allows the brand-name drug producer to maintain its high price level, in the presence of lower-priced generic drugs. However, the difference between their research and ours is a fully game-theoretic model we have further developed. Besides, we clearly demonstrate the important role played by the factor of cross-elasticity to practice the inverse pricing strategy thought there are two types of consumers in the market. In other words, unlike the model developed by Frank and Salkever, we intensively prove the inverse pricing strategy is sustainable only if the cross elasticity of demand is relatively small.

The remaining of this paper is organized into three sections. Section 2 presents the general model of brand-name's price setting based on a two-stage game, section 3 demonstrates an example using the linear demand function and the constant marginal cost function, and concluding remarks are given in section 4.

2. THE GENERAL MODEL

2.1 Basic Assumptions

The first assumption of this model is to assume that there are two competing manufacturers producing differential but substitutable products, namely brand *A* and *B*. For example, the brand *A* product can be viewed as the American car and the brand *B* product as the Japanese car. In a car industry, it is quite obvious that two producers compete in prices in such market. The second assumption of this model is the market consists of two types of consumers, the brand loyal consumers who consume based on brand adoration and the brand non-loyal consumers who consume based on habitual pattern. A brand non-loyal consumer might switch to a different band very easily and can be attracted by another brand that offers a better deal. These consumers can be separated by the degree of true brand loyalty. The consumer surplus is supposed to be larger to the brand loyal consumers because they have a higher degree of the brand loyalty and hence the higher degree of satisfaction.

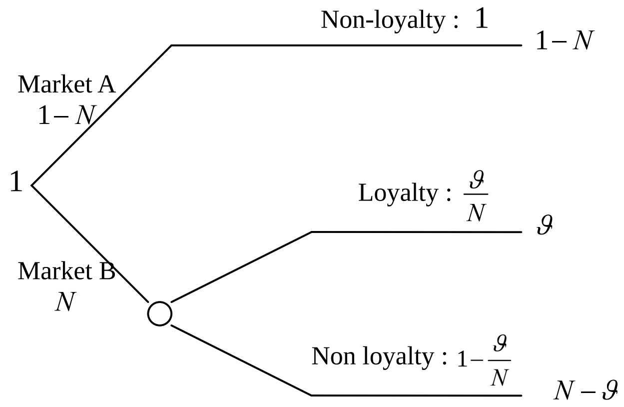
The game we construct in this study focuses on the reaction of one producer who observes the information such that the opponent changes its price and who then takes a rational strategy. Thus, two firms play a static *Stackelberg* price competitive game to share the market demand of this substitutable product.

2.2 The Two Stage Game

To specify the game we assume one of the two firms in the market acts as the first mover of the game who announces in the first stage and exercises a sale promotion with a low price offered in the second stage. This firm, namely firm *A*, can attract some consumers who are less loyal to a brand and can therefore increase its market share.

In the second stage, the strategies that firm B would take are to set a higher price or a lower price depending on the degree of brand loyalty and on the proportion of loyal and non-loyal consumers in the market to maximize its payoff after observing firm A's move. The following tree diagram, diagram I, explains the possible allocation of a representative consumer.

Diagram I: Game Tree



For simplicity, we normalize the total number of consumers to 1 without loosing generality, and (1–N) and N represent the proportion of consumers in market A and market B respectively. The parameter ϑ is the probability of consumers who are loyal to brand B while ($N-\vartheta$) is the joint probability of consumers who are non-loyal to brand B and are in market B. Hence, P(loyal | market B)= ϑ / N and P(non-loyal | market B)= $(1-\vartheta / N)$ represent the conditional probabilities of loyal and non-loyal consumers given the condition that they are in market B. These market features are summarized by table I.

Table I: Probability Distribution

	Loyal	Non-Loyal	
Market A	0	$1-N$	$1-N$
Market B	ϑ	$N-\vartheta$	N
	ϑ	$1-\vartheta$	1

In table I, the entries in the centre cells, $0, \vartheta$, $1-N$ and $N-\vartheta$ are all the joint probabilities. The first two are the joint probabilities of consumers who are loyal to brand B and are in market A and who are loyal to brand B and are in market B, respectively. The last two are the joint probabilities of consumers who are non-loyal to brand B and are in market A and who are non-loyal to brand B and are in market B, respectively. The demand functions for the loyal and the non-loyal consumers are denoted by D_L^B and D_{NL}^B , and the market demand functions for the two products can then be represented by:

$$
\begin{aligned}
Q_A &= D_{NL}^B\left(P_A, P_B\right) \\
Q_B &= \frac{\vartheta}{N} D_L^B\left(P_B\right) + \left(1 - \frac{\vartheta}{N}\right) D_{NL}^B\left(P_A, P_B\right)
\end{aligned}
\tag{1}
$$

The portion of two groups in market B is endogenized depending on the price level of firm A, such that if the price of brand A decreases, we expect the proportion of non-loyal consumers in the market of brand B decreases simultaneously due to switching. This implies the proportion of non-loyal consumers in market $B, (1 - \vartheta / N)$, decreases and the proportion of loyal consumers , ϑ / N, increases given a decline in price of brand A. To solve the *Nash* equilibrium of the game we start from the second stage. In stage two, firm B's best reaction function comes from:

$$
\begin{aligned}
P_B &= \arg\max\left\{\pi_B\right\} \\
&= \arg\max\left\{P_B Q_B - C(Q_B)\right\} \\
&= \arg\max\left\{P_B\left[\frac{\vartheta}{N} D_L^B\left(P_B\right) + \left(1 - \frac{\vartheta}{N}\right) D_{NL}^B\left(P_B, P_A\right)\right] - C\left[\frac{\vartheta}{N} D_L^B + \left(1 - \frac{\vartheta}{N}\right) D_{NL}^B\right]\right\}
\end{aligned}
\tag{2}
$$

The first order condition to maximize the profit function with respect to price choice, P_B, with $\theta = \vartheta / N$ can be expressed as

$$
\begin{aligned}
&\frac{\partial \pi_B}{P_B} = Q_B + \left(P_B - \frac{\partial C}{\partial Q_B}\right)\frac{\partial Q_B}{\partial P_B} = 0 \\
&\Rightarrow \theta D_L^B + (1 - \theta) D_{NL}^B + \left(P_B - \frac{\partial C}{\partial Q_B}\right)\left[\theta \frac{\partial D_L^B}{\partial P_B} + (1 - \theta)\frac{\partial D_{NL}^B}{\partial P_B}\right] = 0
\end{aligned}
\tag{3}
$$

We are interested in the price setting strategy by firm B given a decrease in the price of brand A. Viewing θ and D_{NL}^B as a function of P_A and total differentiating the first order condition (3) with respect to P_A yields[3]:

$$
\frac{dP_B}{dP_A} = \frac{\left(M \dfrac{\partial^2 C}{\partial Q_B^2} - 1\right) J - \left(P_B - \dfrac{\partial C}{\partial Q_B}\right) L}{M\left(2 - \dfrac{\partial^2 C}{\partial Q_B^2} M\right) + \left(P_B - \dfrac{\partial C}{\partial Q_B}\right) K}
\tag{4}
$$

where

$$
\begin{aligned}
M &= \theta \frac{\partial D_L^B}{\partial P_B} + (1 - \theta)\frac{\partial D_{NL}^B}{\partial P_B} \\
J &= \frac{\partial \theta}{\partial P_A} D_L^B + (1 - \theta)\frac{\partial D_{NL}^B}{\partial P_A} - \frac{\partial \theta}{\partial P_A} D_{NL}^B = \frac{\partial \theta}{\partial P_A}(D_L^B - D_{NL}^B) + (1 - \theta)\frac{\partial D_{NL}^B}{\partial P_A} \\
K &= \theta \frac{\partial^2 D_L^B}{\partial P_B^2} + (1 - \theta)\frac{\partial^2 D_{NL}^B}{\partial P_B^2}
\end{aligned}
$$

$$L=\frac{\partial \theta}{\partial P_A}\frac{\partial D_L^B}{\partial P_B}-\frac{\partial \theta}{\partial P_A}\frac{\partial D_{NL}^B}{\partial P_B}+(1-\theta)\frac{\partial^2 D_{NL}^B}{\partial P_A \partial P_B}=\left(\frac{\partial \theta}{\partial P_A}\right)\left(\frac{\partial D_L^B}{\partial P_B}-\frac{\partial D_{NL}^B}{\partial P_B}\right)+(1-\theta)\frac{\partial^2 D_{NL}^B}{\partial P_A \partial P_B}$$

Clearly, $\frac{\partial \theta}{\partial P_A}<0$, $\frac{\partial (1-\theta)}{\partial P_A}>0$, $\frac{\partial D_L^B}{\partial P_B}<0$, $\frac{\partial D_{NL}^B}{\partial P_B}<0$, $\frac{\partial D_{NL}^B}{\partial P_A}>0$ and $\frac{\partial^2 C}{\partial Q_B^2}>0$.

M is strictly negative owing to the negative impact of price on its own demand and *K* is strictly non-negative because of positive second order condion. The sign of *J* depends fully on i) the size of the cross elasticity of demand, $\partial D_{NL}^B / \partial P_A$, ii) the changes in proportion of brand loyal consumers to a change in price of product *A*, $\partial \theta / \partial P_A$, iii) and the sign of the demand difference between the loyal and the non-loyal consumers, $D_L^B - D_{NL}^B$. In addition, the sign of *L* depends on both the size of $\partial \theta / \partial P_A$, θ, and on the signs of $\left(\partial D_L^B / \partial P_B - \partial D_{NL}^B / \partial P_B\right)$, which is the difference of inversed slopes of two demand functions.

The sign of the change in brand-name price given a change in its differential product depends on the sign of both numerator and denominator of equation (4). First, the sign of demand difference $\left(D_L^B - D_{NL}^B\right)$ is expected to be positive because we expect a larger quantity demanded for the loyal consumers than for the non-loyal consumers in the market of brand *B*. The sufficient condition for *J* to be non-positive is:

$$\left|\frac{\partial D_{NL}^B}{\partial P_A}\right| < \left|\frac{\partial \theta}{\partial P_A}\frac{(D_L^B - D_{NL}^B)}{(1-\theta)}\right| \Rightarrow \gamma < \left|\frac{\partial \theta}{\partial P_A}\frac{\left(D_L^B - D_{NL}^B\right)}{\left(1-\theta\right)}\right| \qquad (5)$$

This inequality shows the cross elasticity of demand for the non-loyal consumers is relatively small. From another prospective, equation (5) also requires the term $\frac{\partial \theta}{\partial P_A}$ to be large and θ to be small. A large value $\frac{\partial \theta}{\partial P_A}$ implies most of the remaining consumers in market *B* are brand-loyalty such as a large value of θ_2. In other words, we expect an inverse pricing strategy to occur if most of the non-loyal consumers switch to market *A* facing a price promotion of a substitute product. The cross second order derivative is expected to be non-negative, i.e., $\partial^2 D_{NL}^B / \partial P_B \partial P_A \geq 0$, because the rate of non-loyal consumers switching from brand *B* to brand *A*, $\partial D_{NL}^B / \partial P_A$, would positively relate to the price of brand *B*. The term $\left(\partial D_L^B / \partial P_B - \partial D_{NL}^B / \partial P_B\right)$ is the difference between two inverse slopes and we expect the following relation for *L* to be non-positive is:

$$\left|\frac{\partial D_L^B}{\partial P_B}\right| < \left|\frac{\partial D_{NL}^B}{\partial P_B}\right| \qquad (6)$$

The explanation of inequality (6) is understandable and it satisfies the key assumption in this study, such that the non-loyal consumers are relatively more sensitive to a price change than the loyal consumers are. Both inequalities (5) and (6) ensure the numerator of equation (4) to be positive. Secondly, to discuss the sign of the denominator of equation (4), we expect the second order derivative of demand functions to be positive because the demand curves are assumed to be quasi-convex, i.e., $\partial^2 D_L^B / \partial P_B^2 \geq 0$ and $\partial^2 D_{NL}^B / \partial P_B^2 \geq 0$. Therefore, it is apparent that the sign of the first term in the denominator is negative and of the second term is positive in equation (4). Therefore, the sufficient condition for the denominator being negative is:

$$\left| M\left(2 - \frac{\partial^2 C}{\partial Q_B^2} M\right) \right| > \left| \left(P_B - \frac{\partial C}{\partial Q_B}\right) K \right| \tag{7}$$

Given the condition of (6), inequality (7) implies that the proportion of loyal consumers in brand *B* market before the decline in price *A* should not be too large. It is worthiness to mention here that inequality (7) will always be satisfied if the demand function is linear because all the second order derivatives are expected to be zero.
All inequalities (5), (6) and (7) bring to the following proposition.

Proposition 1
The price of brand B can be indeed rising while the price of brand A decreases if:

i) The factor of cross price-elasticity of demand for the non-loyal consumers is small.
ii) The rate of increase in the degree of brand-loyalty is sufficiently large.
iii) The proportion of loyal consumers initially in the market is sufficiently small.

The intuitive explanation of proposition 1 is precise and evident; it is no doubt that a parallel leftward shift in the demand curve of brand *B*, but also be true that a rotation in the curve because a price decrease in product *A* makes the product *B*'s demand function less price elastic. Consumers who continue buying brand *B* are those have a higher degree of brand loyalty and are unwilling to switch. There will be a relatively small amount of non-loyal consumers in market *B* who switch to brand *A* if the factor of cross price-elasticity of demand is relatively small. This shows the demand curve for the non-loyal consumers in brand *B* would shift in slightly. On the other hand, if the proportion of loyal consumers initially in the brand *B* market is relatively large, the producer of brand *B* may have no motivation to change its price given a price change in other brand because the effect on switching will then be relatively small. However, if the proportion of loyal consumers is initially relatively small such as the number of non-loyal consumers in the brand-name market before the decline in price of product *A* is much larger than the number of loyal consumers, the switching impact of the non-loyal consumers on the brand-name market would be large. If this is the case, firm *B* will be able to discriminate consumers into two types given the new price level of brand *A*, and will be beneficial to set the price at a higher level because firm B would have belief that most of the consumers remain in the market are brand-loyalty .

The above arguments are easily to be seen as we demonstrate in detail in the next section by assuming the linear demand function and the constant marginal cost function.

3. AN EXAMPLE

The general quadratic indirect utility functions for two types of consumers are expressed as:

$$V_{NL}^{i}(P_i,P_j)=v_{NL}^{i}\left(P_i,P_j\right)+Y=-\alpha_{NL}^{i}P_i-\alpha_{NL}^{j}+\frac{\beta_{NL}^{i}}{2}P_i^2+\frac{\beta_{NL}^{j}}{2}P_j^2-\gamma P_iP_j+Y \tag{8}$$

$$V_{L}^{i}(P_i)=v_{L}^{i}\left(P_i\right)+Y=-\alpha_{L}^{i}P_i+\frac{\beta_{L}^{i}}{2}P_i^2+Y \tag{9}$$

where i=A,B and v_{NL}^{i}, v_{L}^{i} are the indirect utility function for the non-loyal and the loyal consumers, Y is income and P_i is the price of brand i.

The brand A market consists of consumers who are non-loyal to brand B only, and the *Marshallian* demand function for brand A market is the result of first-order condition of equation (8):

$$Q_A=-\frac{\partial V_{NL}^{A}/\partial P_A}{\partial V_{NL}^{A}/\partial Y}=\alpha_{NL}^{A}-\beta_{NL}^{A}P_A+\gamma P_B \tag{10}$$

Brand B market consists of both types of consumers, loyalty and non-loyalty, with proportion ϑ/N and $(1-\vartheta/N)$. In the brand B market, the market demand function is the sum of two first-order conditions of equation (8) and (9) with their corresponding conditional proportions:

$$\begin{aligned}Q_B&=\frac{\vartheta}{N}\left(-\frac{\partial V_{L}^{B}/\partial P_B}{\partial V_{L}^{B}/\partial Y}\right)+\left(1-\frac{\vartheta}{N}\right)\left(-\frac{\partial V_{NL}^{B}/\partial P_B}{\partial V_{NL}^{B}/\partial Y}\right)\\&=\theta\left(\alpha_{L}^{B}-\beta_{L}^{B}P_B\right)+\left(1-\theta\right)\left(\alpha_{NL}^{B}-\beta_{NL}^{B}P_B+\gamma P_A\right)\end{aligned} \tag{11}$$

3.1 The Prior Price Discount Stage: *Bertrand* Game

Two manufacturers, firm A and firm B, produce differentiable products with some degree of substitution, and each firm sets its own price to maximize its profit by taking into account the other firm's price. The reaction function for firm B is derived from:

$$R_i(P_j)=\arg\max\left(P_iQ_i-C(Q_i)\right) \tag{12}$$

where $C(Q_i)$ is the cost function and is assumed to be represented by a constant marginal cost function. The total cost of production is:

$$C(Q_i)=cQ_i,\quad i=A,B$$

Then reaction functions of these two firms are:

$$R_A(P_B)=\arg\max\left(P_A-c\right)\left[\alpha_{NL}^{A}-\beta_{NL}^{A}P_A+\gamma P_B\right] \tag{13}$$

$$R_B(P_A)=\arg\max\left(P_B-c\right)\left[\theta\left(\alpha_{L}^{B}-\beta_{L}^{B}P_B\right)+\left(1-\theta\right)\left(\alpha_{NL}^{B}-\beta_{NL}^{B}P_B+\gamma P_A\right)\right] \tag{14}$$

Substitution for Q_A, Q_B and $C(Q_B)$ from (10), (11) and (12) into (13) and (14), the best reaction functions of the prices of two producers are

$$P_{Aj} = \frac{\alpha_{NL}^{A} + \gamma P_{Bj}}{2\beta_{NL}^{A}} + \frac{c}{2} \qquad (15)$$

$$P_{Bj} = \frac{\theta_j \alpha_L^B + (1-\theta_1)\left[\alpha_{NL}^B + \gamma P_{Aj}\right]}{2\left[\theta_j \beta_L^B + (1-\theta_j)\beta_{NL}^B\right]} + \frac{c}{2} \qquad (16)$$

where the subscript j=1 represents the stage prior to the price change of brand A. Equation (15) and (16) generate *Bertrand Nash* equilibrium prices for two firms

$$P_{A1} = \frac{\gamma\left[\theta_1\alpha_L^B + (1-\theta_1)\alpha_{NL}^B\right] + \left[2\alpha_{NL}^A + c\left(\gamma + 2\beta_{NL}^A\right)\right]\left[\theta_1\beta_L^B + (1-\theta_1)\beta_{NL}^B\right]}{4\beta_{NL}^A\left[\theta_1\beta_L^B + (1-\theta_1)\beta_{NL}^B\right] - (1-\theta_1)\gamma^2} \qquad (17)$$

$$P_{B1} = \frac{(1-\theta_1)\alpha_{NL}^A + 2\beta_{NL}^A\left\{\theta_1\alpha_L^B + (1-\theta_1)\alpha_{NL}^B + c\left[\theta_1\beta_L^B + (1-\theta_1)\beta_{NL}^B + \frac{1}{2}(1-\theta_1)\gamma\right]\right\}}{4\beta_{NL}^A\left[\theta_1\beta_L^B + (1-\theta_1)\beta_{NL}^B\right] - (1-\theta_1)\gamma^2} \qquad (18)$$

The parameter θ_1 represents the conditional probability of loyal consumers in market B before firm A decreases its price. The parameter of ϑ is a constant term before and after the price change because the total proportion of loyal consumers to non-loyal consumers is unchanged. However, the conditional probabilities $\theta = \vartheta / N$ and $(1-\theta) = (1-\vartheta/N)$ are affected because a change in price of brand A implies a change in the proportion of non-loyal consumers in market B. Accordingly, θ_1 depends on the total number of consumers in market B, N_1, before P_A changes, and $\theta_1 = \vartheta / N_1$.

3.2 The Post Price Discount Stage: Firm *B* as a Follower in a *Stackelberg* Game

The second stage is the stage where firm A exercises a sales promotion and firm B takes the price of product A as given and acts as a price follower to maximize its profit with respect to its own price. The reaction function of firm B is derived from:

$$R_B(P_A) = \arg\max\left(P_B Q_B - C(Q_B)\right) \qquad (19)$$

Maximization of the profit function for firm B in general with respect to P_B yields:

$$P_B = \frac{\theta\alpha_L^B + (1-\theta)\left[\alpha_{NL}^B + \gamma P_A\right]}{2\left[\theta\beta_L^B + (1-\theta)\beta_{NL}^B\right]} + \frac{c}{2} \qquad (20)$$

To discuss the effect of P_B in equilibrium when P_A decreases we differentiate equation (20) with respect to P_A. Since the parameter θ denotes the proportion of loyal consumers who are in the market of brand B and it is a function of P_A because when price of product A decreases, the proportion of consumers who are non-loyal to brand B, $(1-\theta)$, decreases at the same time, and therefore, θ increases. The total differentiation to equation (20) yields:

$$\frac{\partial P_B}{\partial P_A}=\frac{\frac{\partial\theta}{\partial P_A}\left[\alpha_L^B\beta_{NL}^B-\left(\alpha_{NL}^B+\gamma P_A\right)\beta_L^B\right]+(1-\theta)\gamma\left[\theta\beta_L^B+(1-\theta)\beta_{NL}^B\right]}{2\left[\theta\beta_L^B+(1-\theta)\beta_{NL}^B\right]^2} \tag{21}$$

We are interested in a negative relationship between two prices, such that firm *B* plays an inverse price strategy in a *Stackelberg* price setting game. To satisfy an inverse relationship, i.e., $\partial P_B/\partial P_A<0$, equation (21) implies the inequality condition as:

$$0<\gamma<\frac{-\frac{\partial\theta}{\partial P_A}\left(\alpha_L^B\beta_{NL}^B-\alpha_{NL}^B\beta_L^B\right)}{(1-\theta)\left[\theta\beta_L^B+(1-\theta)\beta_{NL}^B\right]-\frac{\partial\theta}{\partial P_A}\beta_L^B P_A} \tag{22}$$

Since we expect a steeper slope in demand function for the loyal consumers than for the non-loyal consumers, we have $\alpha_L^B>\alpha_{NL}^B$ and $\beta_L^B<\beta_{NL}^B$. Inequality (22) proves the proposition 1 once again using the linear demand function and constant marginal cost function.

To consider the sufficient condition for the second-stage price above the first-stage price, we first find the equilibrium price of product *B* in the stage 2, and then determine the price difference between the first and second stage. To attain the price difference of product *B*, we assume the price of product *A* after discount to be:

$$P_{A2}=(1-t)P_{A1}\text{, for } 0<t<1 \tag{23}$$

t is the discount rate in percentage, and P_{Aj} for *j*=1,2 denotes the equilibrium price of brand *A* pre and post a sales promotion. Substitution equation (23) and the conditional probability of loyal and non-loyal consumers post the sales promotion of $\theta_2=\vartheta/N_2$ into firm *B*'s reaction function (20), the equilibrium price of brand *B* in the second stage can be re-written as

$$P_{B2}=\frac{\theta_2\alpha_L^B+(1-\theta_2)\left[\alpha_{NL}^B+\gamma(1-t)P_{A1}\right]}{2\left[\theta_2\beta_L^B+(1-\theta_2)\beta_{NL}^B\right]}+\frac{c}{2} \tag{24}$$

When there is a sales promotion in the brand *A* market, consumers who are non-loyal to brand *B* switch their consumptions to brand *A*. The remaining ones in the brand *B* market are the loyal consumers and therefore the percentage of the loyal consumers in the brand *B* market after substituting becomes higher than before. As the total number of consumers in the market of brand *B* decreases, *N* becomes smaller, so that the conditional proportion of loyal consumers in market *B* increases, such that ϑ/N becomes larger; on the other hand, the conditional proportion of non-loyal consumers in market *B* declines, and $(1-\vartheta/N)$ becomes smaller. If we denote N_j for *j*=1,2 as the number of consumers in market *B* before and after a decline in the price of brand *A*, then $N_1>N_2$ and $\theta_1<\theta_2$. By substituting (17) into (24) and by comparing it with (18), we have:

$$\Delta P_B = P_{B2} - P_{B1}$$
$$= \frac{(\theta_2-\theta_1)\left[4\beta_{NL}^A\left(\alpha_L^B\beta_{NL}^B - \alpha_{NL}^B\beta_L^B\right) - \gamma\left(\gamma\alpha_L^B + \Phi\beta_L^B\right)\right] - t\gamma(1-\theta_2)\left\{\gamma\left[\theta_1\alpha_L^B + (1-\theta_1)\alpha_{NL}^B\right] + \Phi m_1\right\}}{2\left\{4\beta_{NL}^A m_1 - \gamma^2(1-\theta_1)\right\}m_2} \quad (25)$$

where $m_1 = \theta_1\beta_L^B + (1-\theta_1)\beta_{NL}^B$, $m_2 = \theta_2\beta_L^B + (1-\theta_2)\beta_{NL}^B$ and $\Phi = 2\alpha_{NL}^A + c\left(\gamma + 2\beta_{NL}^A\right)$.

To clearly identify the sign of ΔP_B is not an easy job; however, we can anticipate a relative direction of the change through the limit practice. We can examine two extreme cases in terms of γ. If γ is very small, such that $\gamma \to 0$, the second term in the numerator of equation (25) would approach to zero, so that the numerator will be always positive. In other words, the price of brand *B* definitely increases after the price discount of brand *A* if the degree of substitution is small. If γ is very large, the second term of the numerator as well as the second term in the first bracket will approach to an even larger number, so the value of the numerator will become negative. This implies that the pricing strategy for firm *B* is to decrease its price when observing a price discount in the brand *A* market if γ is very large. On the other hand, a small value θ_1 implies a large value of m_1 because $\beta_{NL}^B > \beta_L^B$ is expected; however, the effect of the magnitude of θ_2 on ΔP_B can not be seen very clearly based on equation (25). We will further demonstrate in detail both impacts of θ and γ on ΔP_B using numerical simulation in next subsection.

The intuitive explanation of inverse pricing strategy is now discussed and is illustrated in Figure 1. Prior to firm *A* practices a sales promotion, the producer of brand *B* faces a kinked demand curve of $D_B D_B^0 D_{B1}$.[4] We assume the equilibrium price set by firm *B*, before he observes a decrease in price of brand *A*, is relatively low in order to attract both types of consumers, and the optimal price is set at P_{B1} by equating the marginal cost to the marginal revenue. Some of the non-loyal consumers switch their demand from brand *B* to brand *A* when they notice a price discount or a sales promotion of product *A* and this switching behavior will lead to a downward shift and a leftward pivot in the lower part of the kinked demand curve to $D_B D_B^{0'} D_{B2}$ as shown in Figure 1. The marginal revenue curve, MR_{L+NL}, shifts and pivots to the left accordingly, hence, the new equilibrium occurs at a higher price level of brand *B*. The conditions for shifting and pivoting in the demand curve are explained next.

We demonstrate two critical factors, γ and θ, that determine the change in price of brand *B*. If the degree of substitution γ is relatively large, the demand curve will shift downward by a large amount, because the non-loyal consumers are very sensitive to the price change of other products, i.e., the cross price-elasticity of demand is large. The best strategy for firm *B* to take is to decrease his price immediately after observing firm *A*'s action in order to retain some of the non-loyal consumers from switching. On the other hand, the number of non-loyal consumers who switch to brand *A* is relatively small if γ is a small parameter, firm *B*'s reaction is then to maximize his profit by setting a slightly higher price as shown in Figure 2.

The other important factor that determines the price setting strategy taken by firm *B* is the size of the relative proportion of two types of consumers. To understand the impact of θ on the brand-name price, we highlight and demonstrate using two extreme cases. The fraction of the loyal consumers in the brand *B* market is denoted by θ, and these consumers have a high degree of brand loyalty and are unwilling to switch. The total number of loyal consumers in the brand *B* market is much larger than the total number of non-loyal consumers if θ is sufficient large, and the upper part of kinked demand curve will be much larger than the lower part of it as shown in Figure 2. If most of the consumers in the brand *B* market are loyal to its brand, the introduction of a sales promotion by firm *A* will have no impact on the selling decision taken by firm *B* because firm *B* mainly focuses his product selling on the loyal consumers, and the subsequent price will therefore remain unchanged at the original level. An extremely large θ implies a large fraction of loyal consumers in the brand *B* market and a small fraction of non-loyal consumers, and the optimal price set by firm *B* is beyond the non-loyal consumers' willingness to pay even if firm *B* wants to take into account the demand for both types of consumers. The equilibrium price and quantity, P_B and Q_B are then located on the upper part of the kinked demand curve as illustrated in Figure 2.

Nevertheless, firm *A* promotes a discount will have an effective impact on firm *B*'s pricing decision when θ_1 is sufficiently small. When θ_1 is small, a larger fraction of consumers in the brand *B* market are non-loyalty and a price decrease in brand *A* will result a large number of non-loyal consumers substituting from brand *B.* In other words, the total number of consumers in brand *B* market is shrinking much more under the case of small θ_1 than under the case of large θ_1, and the consumers retain in brand *B* market are those with higher degree of brand loyalty. Thus, the fraction of brand loyal consumers in brand *B* market becomes higher after the price of brand *A* decreases, i.e., $\partial\theta / \partial P_A < 0$, and the slope of the lower part of kinked demand curve becomes steeper, such that the price-elasticity of consumer demand becomes more inelastic. Firm *B* will therefore be beneficial to increase his price to P_{B2}, as illustrated in Figure 1, to extract the consumer surplus from the remaining consumers who have higher degree of brand loyalty and are less sensitive to a price change.

Figure 1: Small θ_1

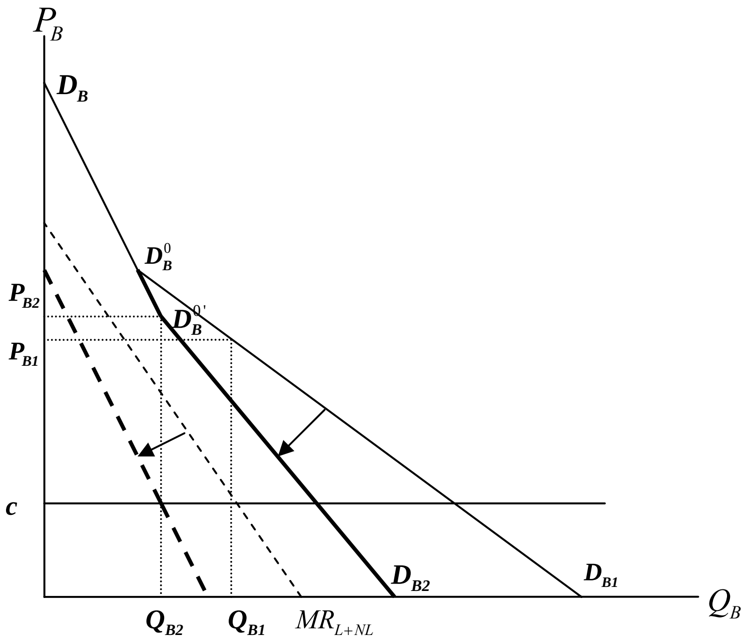
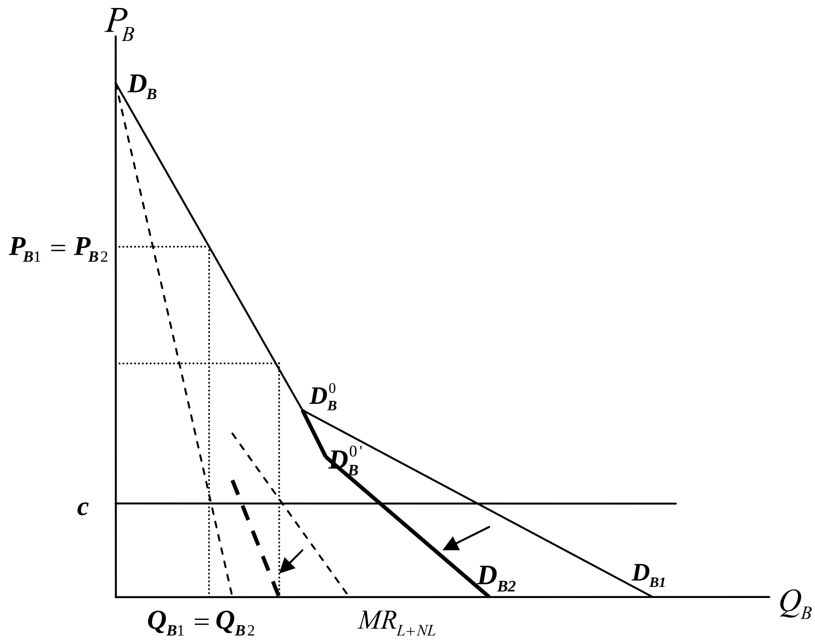


Figure 2: Extremely Large θ_1



3.3 Numerical Results

As mentioned earlier, the effect of the proportion of brand-loyalty consumers on the price change in market B can not be seen clearly in the general case. In this section, we provide numerical simulations to gain some insight into the conditional probability of brand-loyal consumers given who are in market B and the strategic behavior of firm B, and also the cross-elasticity and the strategic behavior of firm B.[5] To satisfy the conditions of quadratic indirect utility function and the expectation of a steeper slope in demand function for the loyal consumers, we assume $\alpha_L^B > \alpha_{NL}^B$, $\beta_L^B < \beta_{NL}^B$, and require $\beta_L^B \beta_{NL}^B - \gamma^2 > 0$ to ensure convexity. In addition, because of switching behavior that $N_2 < N_1$, we have $\theta_2 > \theta_1$.

Our model contains eleven parameters and we assign fixed numerical values for all α s, β s and other exogenous parameters with no particular important role in our context to demonstrate the underlying total and partial effects of the degree of substitution(γ) and the proportion of loyal consumers after the price of product decreases (θ_2) on the price change in market B (ΔP_B).

First we consider the joint effect by plotting ΔP_B on γ and θ_2 using equation (25) with $\alpha_{NL}^A = \alpha_{NL}^B = 80, \alpha_L^B = 90, \beta_L^B = 6, \beta_{NL}^A = \beta_{NL}^B = 10, c = 5$ for the baseline parameter for both demand functions and cost functions as shown in Figure 3. We further assign predetermined values for t and θ_1 which is assumed to be relative small, namely,

The rate of price discount incurred by firm A: $t = 0.15$

The proportion of loyal consumers in market B: $\theta_1 = 0.4$

To solve the nonlinear system (25), we substitute all of the values for exogenous parameters and generate one equation with three unknowns.

Figure 3: 3D Plot of inverse pricing

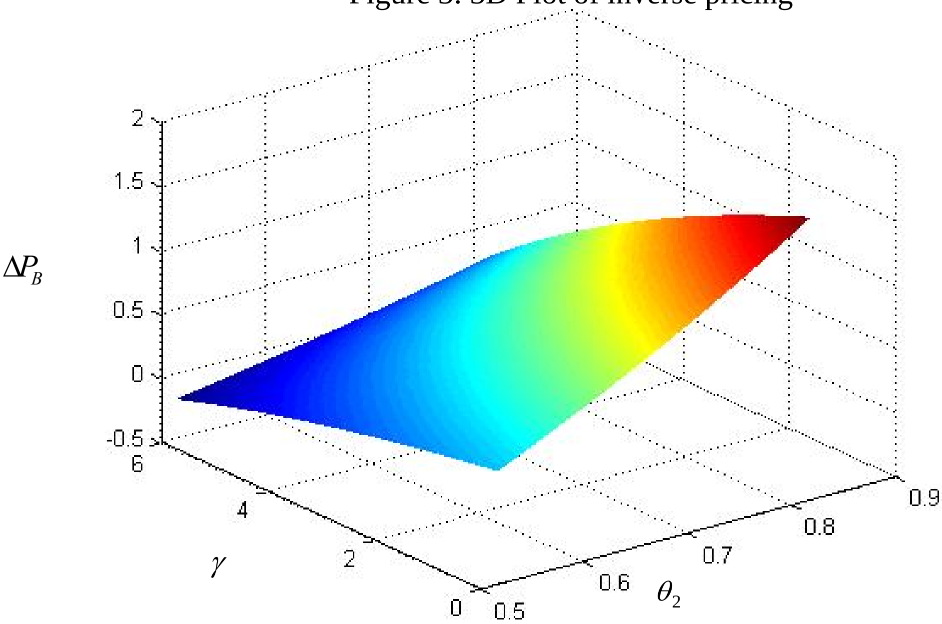


Figure 3 shows a three-dimension diagram of the effects of γ and θ_2 on ΔP_B. The resulting plot demonstrates a possible inverse pricing strategy such that Firm *B* would surprisingly raise its selling price knowing its opponent's move of promoting the sales, namely Firm *A*.

We then fix θ_2 at constant levels of 0.9, 0.8,.., 0.5 to compare the partial effect of the degree of substitution on the price change of Firm *B* as presented in Figure 4.

Figure 4: 2D Relationship Between ΔP_B and γ

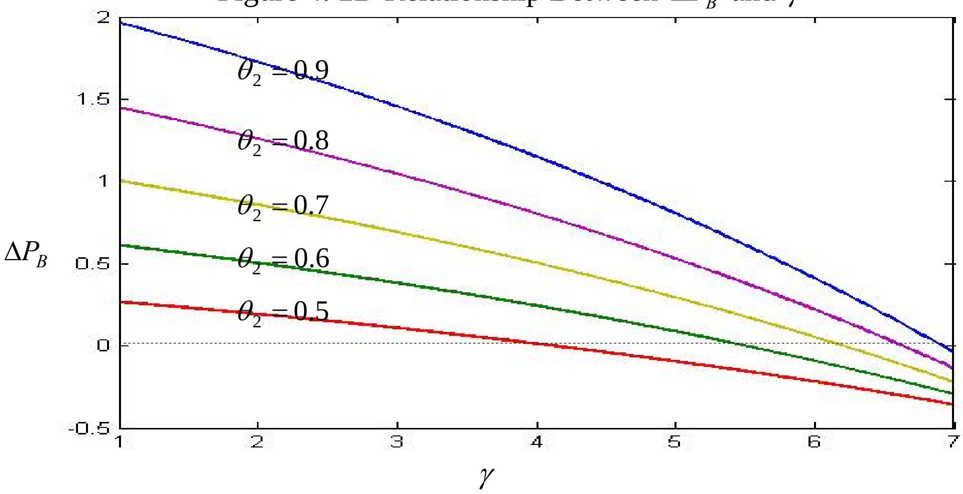


Figure 5: 2D Relationship Between ΔP_B and θ_2

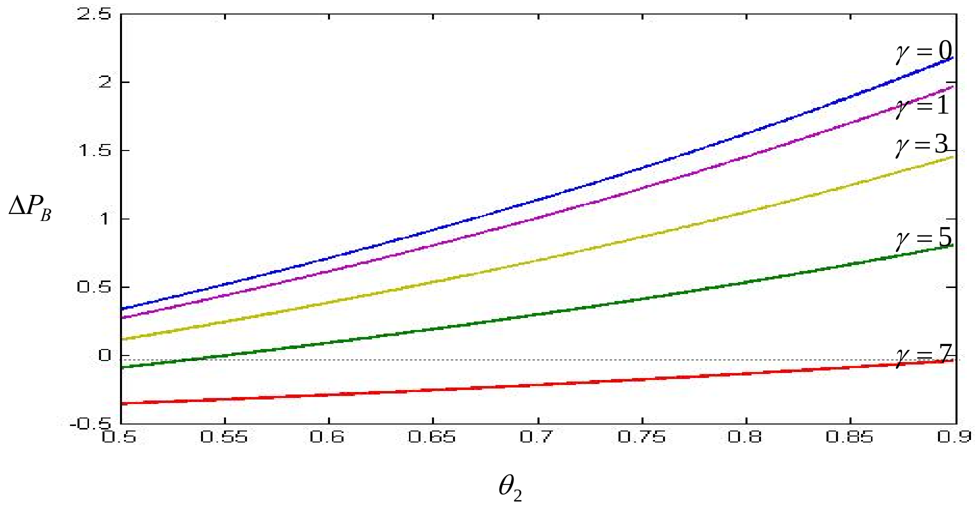


Figure 4 shows the degree of substitution at any given probability of loyal consumers varies inversely with the price change in brand *B* market. In other words, if the cross price-elasticity is small as two products are less substitute but the proportion of loyal consumers after price adjustment is large owing to the switching of non-loyal consumers, firm *B* may then not adjust its price the same way introduced in Bernard oligopoly competition as its opponent does. We are more likely to have a positive price change that firm *B* is more likely to increase its own price. When the consumers in market *B* are less sensitive to any price discounts announced by its rival, firm *B* has a higher chance of practicing an inverse price strategy. We also can easily understand that at any given degree of substitution γ, there is a positive relationship between the price change in brand *B* market and the probability of loyal consumers in market *B* as illustrated in Figure 5. The larger the value of θ_2 is, the larger the possibility is that firm *B* would raise its own product price while firm *A* exercises a price discount. This positive relationship between ΔP_B and θ_2 can be shown by a positive-sloping curve. *A* large value of θ_2 explains a large proportion of consumers in market *B* are loyal to the brand after the price decline of product *A*. When consumers are loyal to a certain brand, their willingness to pay for the brand they are loyal to is high, and also their willingness to switch is low, thus firm *B* can possible gain profit by charging a higher level of price. The price charged by firm *B* after facing a sales promotion of brand *A* is higher than the price charged prior to the promotion, namely, $\Delta P_B = P_{B2} - P_{B1} > 0$. For example, at $\gamma = 5$ in Figure 4, we have the curve $\theta_2 = 0.5$ below the zero line and all of other curves are above the zero line. This means when $\gamma = 5$, as long as the percentage of loyal consumers after the price decline of product *A* is larger than 55%, we would obtain a positive price difference, $\Delta P_B = P_{B2} - P_{B1} > 0$. When $\gamma = 5$ and $\theta_2 = 0.55$ in particular, we have $\Delta P_B = 0$ from Figure 4. This consists with the result in Figure 5 that the curve $\gamma = 5$ intersects the zero line at $\theta_2 = 0.55$ approximately. Both Figure 4 and Figure 5 obviously illustrate the results of partial relationships between θ_2 and ΔP_B and also γ and ΔP_B. These results explain and prove our theory of inverse pricing strategy holds under certain conditions. (1) When the degree of substitution is low that products in two markets are not close substitutes, (2) the initial probability of loyal consumers is small that firm has an incentive to retain the market of non-loyal consumers, and (3) when the probability of brand loyal consumers after the price adjusted by it opponent is large enough that remaining consumers are less likely to switch due to brand-loyalty, firms are possible to play an inverse pricing strategy, such that one firm would actually increase its price while its rival promotes a price discount.

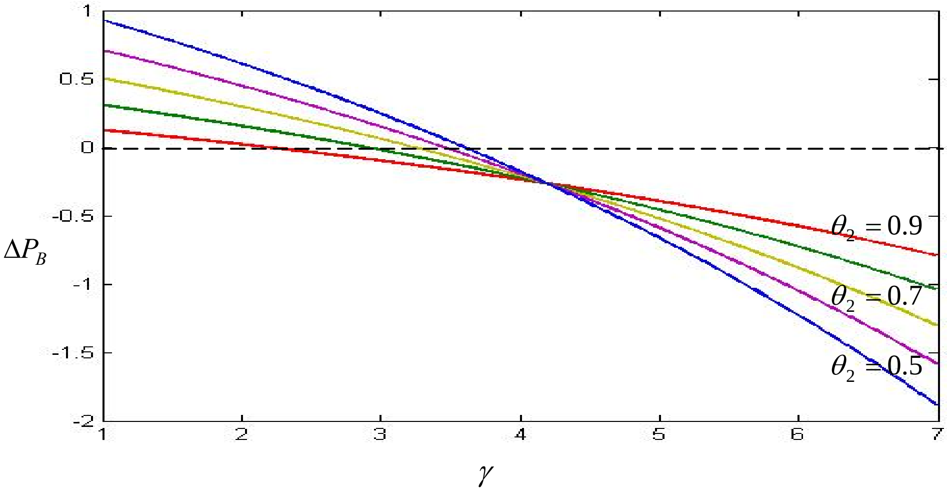
Lemma 1:

If all of the predetermined parameters are identified and are satisfied the convexity and non-negativity assumptions such that $\alpha_{NL}^{A} = \alpha_{NL}^{B} = 80, \alpha_{L}^{B} = 90$, $\beta_{NL}^{A} = \beta_{NL}^{B} = 10, \beta_{L}^{B} = 6$ *, and for the assumption of constant marginal cost,* $c = 5$ *and the rate of price discount,* $t = 0.15$ *. By assuming the probability prior to the price discount of loyal consumers in market B to be small,* $\theta_1 = 0.4$ *, then*

- *i)* *The price difference of brand B product prior and posterior to the price decline of product A varies inversely with the degree of substitution among two brands*
- *ii)* *The price difference of brand B product prior and posterior to the price decline of product A varies positively with the degree of brand-loyalty among two groups of consumers.*

It is worthwhile to mention that we could obtain another possible result for the relationship between ΔP_B and γ from simulation when the assigned values to the parameters are relaxed. However, the 3-dimentional figure is always consistent for any value of pre-determined parameters as long as the assumptions are made. In other words, by setting all parameters as random and by satisfying the convexity and positivity constraints, we could obtain a result as Figure 4 and also we could obtain one containing a crossing point within the possible range of γ as shown in Figure 6. The crossing point is always in the range of negative price change, $\Delta P_B = P_{B2} - P_{B1} < 0$, for any possible random integers. However, we are interested in the case of inverse pricing strategy that we only take into account the possible values of γ give the positive change in price of product *B*, and the crossing point is beyond this possible interval.

Figure 6: 2D Graph – Randomized Parameters



4. WELFARE ANALYSIS

For a discussion of total welfare changes, the sum of the change in consumer's surplus and the change in producer's surplus, we again assume the demand function being linear and the cost function is represented by a constant marginal cost function.

4.1 The Consumer Surplus

The change in aggregated consumer's surplus after a decrease in brand *A* price is represented by:

$$\Delta CS = \{(1-N_2)V_{NL2}^{A} - (1-N_1)V_{NL1}^{A}\} + \left\{N_2\left[\frac{\vartheta}{N_2}V_{L2}^{B} + \left(1-\frac{\vartheta}{N_2}\right)V_{NL2}^{B}\right] - N_1\left[\frac{\vartheta}{N_1}V_{L1}^{B} + \left(1-\frac{\vartheta}{N_1}\right)V_{NL1}^{B}\right]\right\} \tag{26}$$

Because brand *B* market consists of two types of consumers with the probability $\frac{\vartheta}{N}$ and $\left(1-\frac{\vartheta}{N}\right)$, the net consumer surplus in brand *B* market is the weighted sum of two indirect utilities corresponding to two types of consumers. If we re-arrange the terms in (26), we obtain the following:

$$\Delta CS = \vartheta[V_{L2} - V_{L1}] + (1-\vartheta)[V_{NL2} - V_{NL1}] \tag{27}$$

where the first bracket is the change in consumer's surplus of loyal consumers, and the second bracket represents the consumer surplus of non-loyal consumers. Substituting equation (1) and (2) into (27) to re-write (27) as:

$$\Delta CS = \vartheta\left\{(P_{B1} - P_{B2})\left[\alpha_L^B - \frac{1}{2}\beta_L^B(P_{B1} + P_{B2})\right]\right\} + (1-\vartheta)\left\{(P_{A1} - P_{A2})\left[\alpha_{NL}^A - \frac{1}{2}\beta_{NL}^A(P_{A1} + P_{A2}) + \gamma P_{B1}\right]\right\}$$
$$+ (1-\vartheta)\left\{(P_{B1} - P_{B2})\left[\alpha_{NL}^B - \frac{1}{2}\beta_{NL}^B(P_{B1} + P_{B2}) + \gamma P_{A2}\right]\right\}$$

(28)
The first bracket in equation (28) is the consumer's surplus change of loyal consumers in market *B*, and the second and the third brackets are the consumer's surplus change of non-loyal consumers in market *A* and *B*, respectively. Owing to our arguments that $P_{B2} > P_{B1}$ and $P_{A2} < P_{A1}$, the sign of the first and third brackets are negative, but the second term has a positive sign. Consumers in market *A* enjoy a lower price of brand *A* product and are better off, thus, the second term has a positive sign. In contrast, consumers who remain in market *B* when noticing a price discount of product *A* face a higher price of brand *B* and thus are worse off. Hence, the total change in consumer's surplus across two markets has an ambiguous sign and the sign depends again on the endogenous parameters θ_2 and γ. The simulation result for the relationship between two random parameters and the dependent variable (ΔCS) will be demonstrated in section 4.3.

4.2 The Producer Surplus

The total change in producer's surplus across two markets is the total changes in both firms' profits and can be represented as:

$$\Delta PS = \{(1-N_2)(P_{A2}-c)Q_{A2}-(1-N_1)(P_{A1}-c)Q_{A1}\}+\{N_2(P_{B2}-c)Q_{B2}-N_1(P_{B1}-c)Q_{B1}\} \quad (29)$$

By adding and subtracting $(1-N_1)P_{A2}Q_{A1}$ and $N_2P_{B1}Q_{B2}$ into Equation (29), total producer surplus across two markets can then be re-written as:

$$\begin{aligned}\Delta PS = &\{(1-N_1)(P_{A2}-P_{A1})Q_{A1}+(P_{A2}-c)[(1-N_2)Q_{A2}-(1-N_1)Q_{A1}]\}\\ &+\{N_2(P_{B2}-P_{B1})Q_{B2}+(P_{B1}-c)[N_2Q_{B2}-N_1Q_{B1}]\}\end{aligned} \quad (30)$$

where the first bracket in (29) or (30) represents the change in producer's surplus in market *A* and the second bracket is in market *B*. $(1-N_i)$ and N_i are the proportions of consumers in market *A* and *B*, respectively. We assume $N_2 < N_1$ because we expect a smaller number of consumers who remain in the brand-name market, namely market *B*, after a price discount of product *A*. The assumption of $P_{A2} < P_{A1}$ implies $Q_{A2} > Q_{A1}$ and the assumption of $P_{B2} > P_{B1}$ implies $Q_{B2} < Q_{B1}$, such as some of the non-loyal consumers originally in the brand-name market switch to market *A*. Equation (30) shows an ambiguous sign of a change in producer's surplus, where the first and the fourth terms possess a negative sign and the second and the third terms are positive. Once again, we will demonstrate the possible relationships among the two critical parameters and the total change in producer's surplus in next section.

4.3 Numerical Results: The Consumer Surplus and The Producer Surplus

As we mentioned in the previous two sections that the signs and the magnitudes of the consumer's surplus and the producer's surplus are complicated to be verified. In this section, we illustrate the possible relationships among the dependent variable and the two endogenous parameters and we also simulate the possible ranges of parameters for the change in consumer's and producer's surpluses. The following two figures, Figure 7 and

Figure 8, illustrate two three-dimensional plots for the consumer's surplus and the producer's surplus against the degree of substitution and the proportion of brand-loyalty in the post price change stage.

Figure 7: 3D Plot of Consumer Surplus

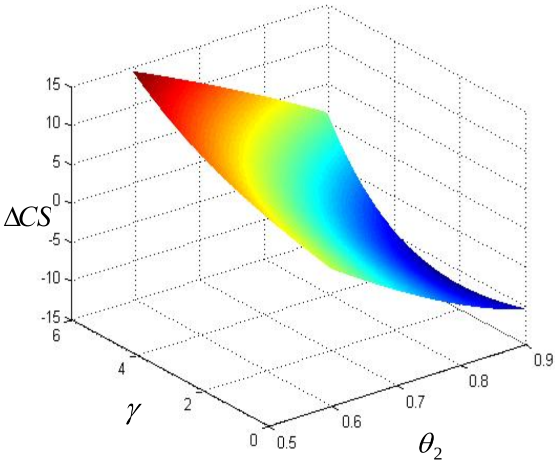
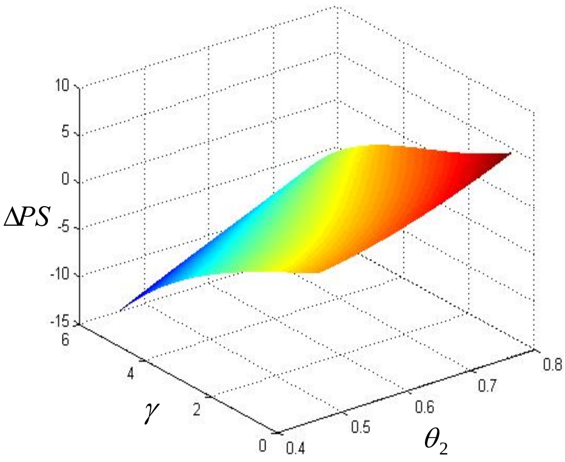


Figure 8: 3D Plot of Producer Surplus



We obtain a positive relationship between the degree of substitution (γ) and the total consumer's surplus change (ΔCS) because a large substitutability, such as two products are more likely to be identical, implies the switching behaviour to be active. A larger proportion of consumers who switch to brand *A* signifies more consumers benefit from a lower price charged, thus, it is more likely to have a positive change in aggregated consumer's surplus. In other words, the size of increasing in consumer's surplus in market *A* dominates the size of decreasing in consumer's surplus in market *B* when γ is sufficiently large. The above two figures, in contrast, show a negative impact of the posterior probability of loyal consumers in market *B* (θ_2) on the total change in consumer's surplus across two markets. When most of the consumers remain in market *B* are loyal to the brand because the non-loyal consumers have switched, it indicates a larger value of θ_2 and implies a larger portion of consumer's surplus being transferred to the producer's surplus. The magnitude of the negative change in consumer's surplus of market *B* rises with the value of θ_2, and hence the magnitude of the increasing in aggregated consumer's surplus shrinks with the value of θ_2. In addition, since the loyal consumer's willingness to pay is higher than the non-loyal consumer's, we expect a larger decline in consumer's surplus in market *B* than the increasing in consumer's surplus in market *A* when θ_2 is sufficiently large.

Figure 9: 2D Relationship Between ΔCS and γ

$\theta_2 = 0.5$

$\theta_2 = 0.6$

$\theta_2 = 0.7$

ΔCS

$\theta_2 = 0.8$

$\theta_2 = 0.9$

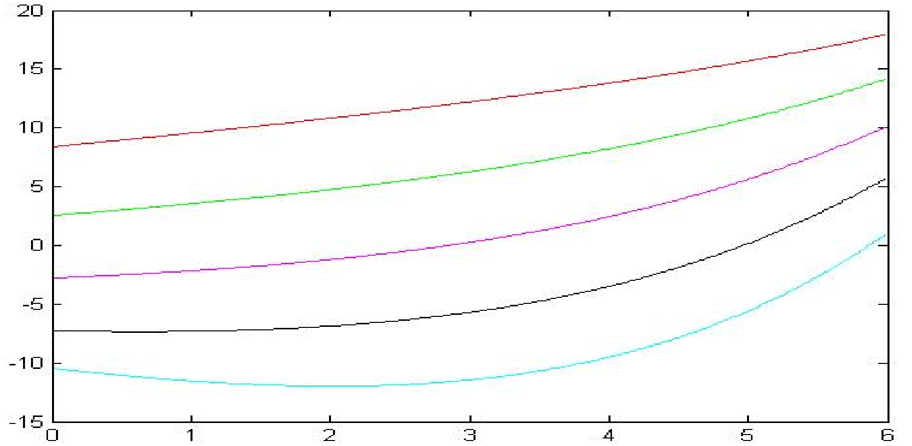
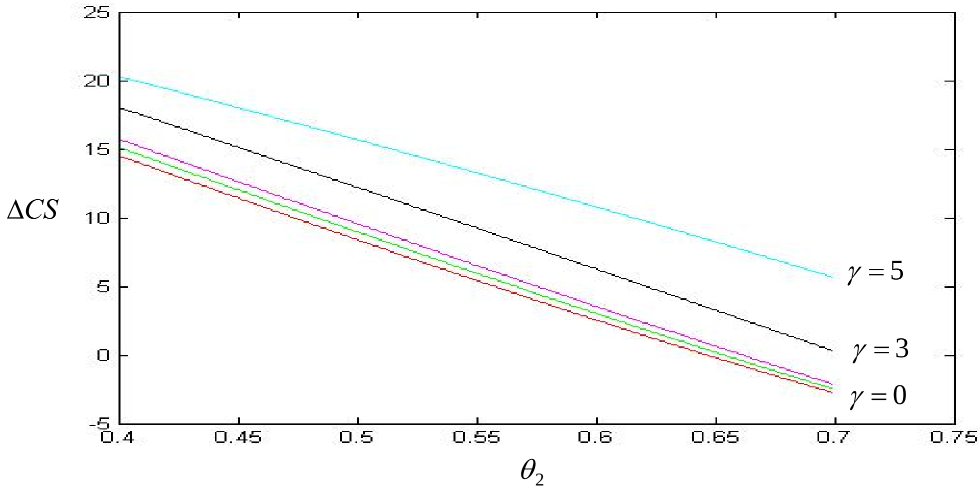


Figure 10: 2D Relationship Between ΔCS and θ_2



Lemma 2:

If the convexity and non-negativity assumptions are satisfied and the conditions of Proposition 1 are implied such as small value of γ and large value of θ_2, then the consumer's surplus is indeed decrease while the price of brand A decreases.

On the other hand, the impact of the cross-substitution on the change in producer's surplus is expected to be negative as illustrated in Figure 11 and Figure 12 because the larger the substitutability among two products is, the more is the switching behaviour of consumers among two markets fixing the percentage of brand loyalty. As more consumers switch to product *A*, there is a decline in aggregated producer's surplus across two markets owing to a price discount of product *A*. The last simulation result explains the relationship between the total change in producer's surplus and the remaining proportion of loyal consumers in the brand-name market. There shows a positive relation of the brand loyalty to the producer's surplus change at each given degree of substitution. This positive relation shows that when there is a large portion of loyal consumers remain in market *B*, the increase in producer's surplus associated with a change in price of good

A is more likely to be large. The larger the value of the brand-loyalty is (θ_2), the higher is the consumer's surplus in market *B* owing to the larger willingness to pay, thus the producer in market *B* can absorb more of the consumer's surplus by raising up its price. In other words, if the proportion of loyal consumers in market *B* is large posterior to the price decline of product *A*, i.e. $\theta_2 \to 1$, the producer's surplus associated with a price discount of product *A* is more likely to be positive and the consumer's surplus associated with a price discount of product *A* is more likely to be negative. There is a transfer of the consumer's surplus into the producer's surplus in the brand-name market.

Figure 11: 2D Relationship Between ΔPS and θ_2

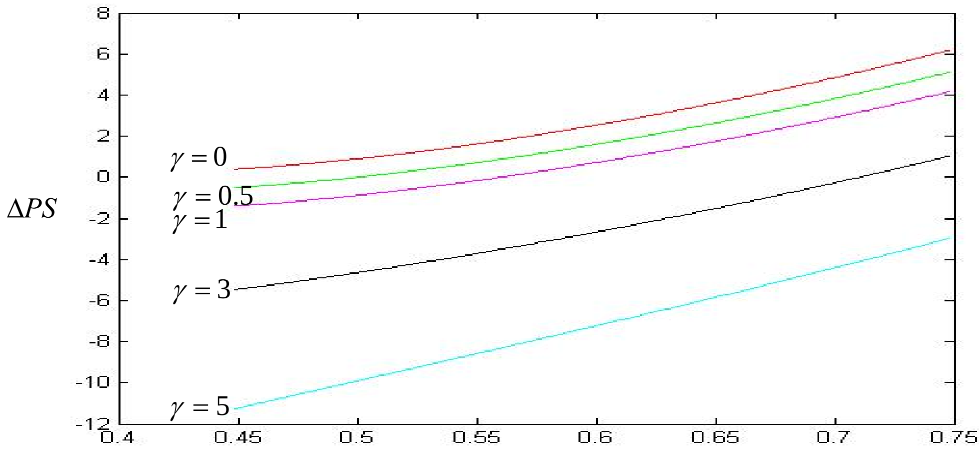
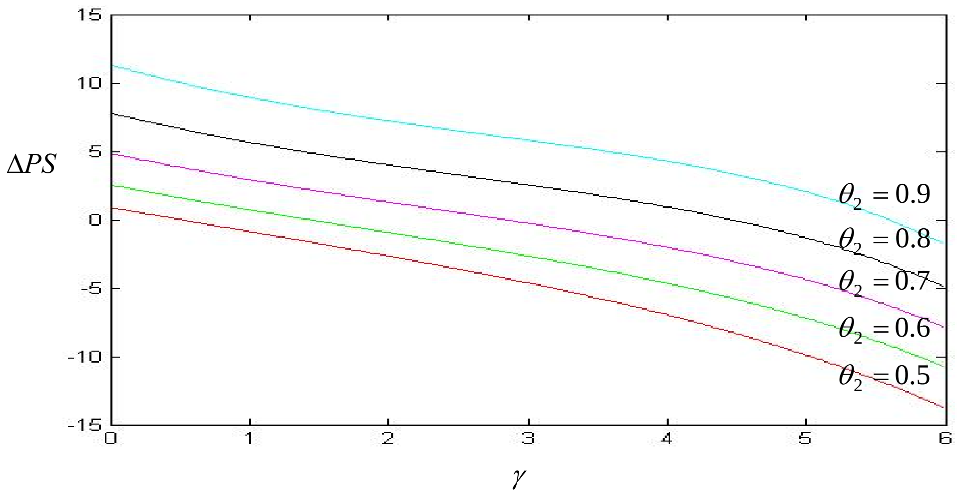


Figure 12: 2D Relationship Between ΔPS and γ



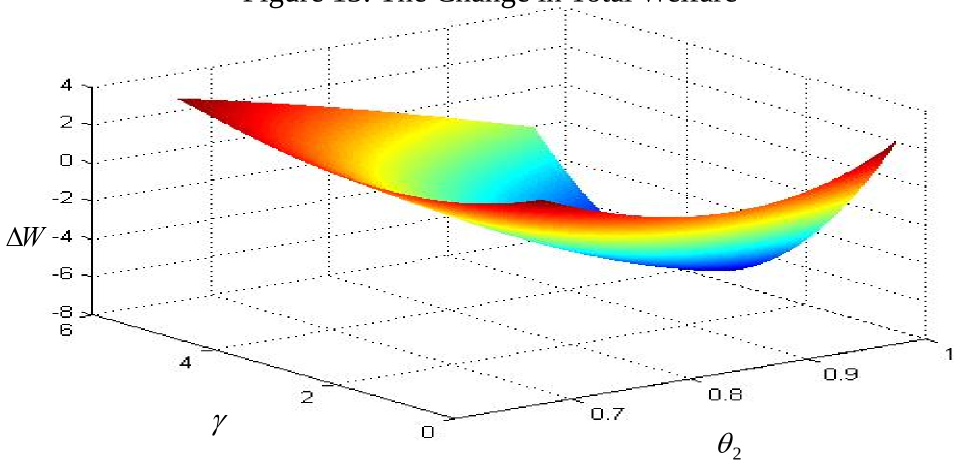
Lemma 3:

If the convexity and non-negativity assumptions are satisfied and the conditions of Proposition 1 are implied such as small value of γ and large value of θ_2, then the producer's surplus is indeed increase while the price of brand A decreases.

To discuss and demonstrate the total welfare change, we once more simulate a three-dimensional curve because the total change in social welfare may be difficult to calculate.

The total welfare change is the sum of the change in consumer's surplus and the change in producer's surplus, $\Delta W = \Delta CS + \Delta PS$. We have already calculated the change in consumer's surplus and the change in producer's surplus in equation (28) and (30). Thus, the total welfare change can be obtained by adding up Equation (28) and (30). We then simulate and plot the total welfare change against two random parameters, γ and θ_2, together to obtain the following figure, Figure 13. The result is the combination of Figure 9 and Figure 10, and this shows a parabola. This parabola shape illustrates the possible positive change in total welfare sorely depending on the value of these two random parameters.

Figure 13: The Change in Total Welfare



In an extreme case, there shows a positive social welfare change when the degree of substitution is extremely small and the proportion of loyal consumers after the price change is extremely large, i.e. $\gamma \to 0$ and $\theta_2 \to 1$. This is the case that two products are perfectly differentiated and consumers are perfectly identified, thus there is no occurrence of switching behaviour. Another example also illustrating a strictly increasing in total welfare associated with a price decrease of product A is when the degree of substitution is sufficiently large and the brand-loyalty of remaining consumers in market B is sufficiently small. This result is often seen in the market structure of perfect competition that two products are identical and consumers have no subjective preference of one over another.

Proposition 2:

For any value of pre-determined parameters satisfying convexity, non-negativity and constant marginal cost, and for any small value of θ_1 such that $0 < \theta_1 < \theta_2$, the change in total welfare can be positive if:

i) γ *is sufficiently small and* θ_2 *is sufficiently large.*

ii) γ *is sufficiently large and* θ_2 *is sufficiently small.*

By combining Proposition 1 and Proposition 2, we have the following lemma.

Lemma 4:

For any small value of θ_1 such that $0 < \theta_1 < \theta_2$, the inverse pricing strategy and the positive change in welfare may occur if and only if the cross price-elasticity is sufficiently small and the posterior degree of substitution is sufficiently large.

The main driving force in our paper is the abnormal strategic behaviour taken by the Japanese Automaker facing a price promotion of American Cars. In the automobile industry, products are differentiated and consumers tend to have their subjective preferences such as the brand-loyalty. Our finding shows the inverse pricing strategy may be applied and indeed this pricing strategy taken by an oligopolistic firm benefits the total welfare under certain conditions.

4. CONCLUSION REMARK

The strategy of inversing price to capture the loyal consumer's surplus by rising up its price is unusual but it has applied to the automobile and the pharmaceutical industries. This paper presents the first attempt to explain the "inverse pricing strategy" using a two-stage-game theoretic model. We examine the pricing strategy of an oligopolistic firm that believes there are at least some portions of consumers would always purchase its product. Under an assumption of segmenting two types of consumers with different degree of brand loyalty to a certain product, this paper shows the inverse pricing strategy would be sustainable when the certain market conditions are met. First we derive the conditions and restricts to the parameters for a possible inverse pricing behaviour, and we believe that when the cross-elasticity parameter is sufficiently small, the inverse pricing strategy is more likely to occur. Owing to an inconclusive result of the parameter for the degree of brand loyalty, we secondly simulate the correlated impacts of the degree of brand loyalty and the degree of cross-elasticity to the pricing reaction of this oligopolistic firm.

The simulation result shows that the higher probability of the inverse pricing strategy occurs when the proportion of loyal consumers after the price discount of its rival is larger in the market. The inverse pricing strategy could be applied to the automobile industry because firms in the automobile market produce differential products, such that the substitution between products is small and some of the consumers in this market have their subjective preferences on a specific brand, such that the degree of brand-loyalty is large.

In other words, this paper examines the fact that Toyota Motor adjusted its price up slightly on many of its car models sold in the United States while Detroit's Big Three automakers began their discount on their car models. Our studies have explained the abnormal price behaviour taken by a firm in an oligopoly market, namely the automobile and the pharmaceutical markets, and contradicted the fundamental pricing theory.

APPENDIX

The Proof of Equation (4)

We show in this appendix that equation (4) is derived by total differentiating the first order condition of firm *B*'s reaction function, equation (3), with respect to P_A:

$$\left\{\begin{array}{l}\left[(1-\theta)\frac{\partial D_{NL}^{B}}{\partial P_A}+\frac{\partial \theta}{\partial P_A}D_L^B+\frac{\partial(1-\theta)}{\partial P_A}D_{NL}^{B}\right]+\left(P_B-\frac{\partial C}{\partial Q_B}\right)\left[\frac{\partial \theta}{\partial P_A}\frac{\partial D_L^B}{\partial P_B}+\frac{\partial(1-\theta)}{\partial P_A}\frac{\partial D_{NL}^{B}}{\partial P_B}+(1-\theta)\frac{\partial^2 D_{NL}^{B}}{\partial P_A \partial P_B}\right]\\ -\left[\theta\frac{\partial D_L^B}{\partial P_B}+(1-\theta)\frac{\partial D_{NL}^{B}}{\partial P_B}\right]\left[\frac{\partial^2 C}{\partial Q_B^2}\left(\frac{\partial \theta}{\partial P_A}D_L+\frac{\partial(1-\theta)}{\partial P_A}D_{NL}^{B}+(1-\theta)\frac{\partial D_{NL}^{B}}{\partial P_A}\right)\right]\end{array}\right\}dP_A$$

$$+\left\{\begin{array}{l}\left[\theta\frac{\partial D_L^B}{\partial P_B}+(1-\theta)\frac{\partial D_{NL}^{B}}{\partial P_B}\right]+\left(P_B-\frac{\partial C}{\partial Q_B}\right)\left[\theta\frac{\partial^2 D_L^B}{\partial P_B^2}+(1-\theta)\frac{\partial^2 D_{NL}^{B}}{\partial P_B^2}\right]\\ +\left[\theta\frac{\partial D_L^B}{\partial P_B}+(1-\theta)\frac{\partial D_{NL}^{B}}{\partial P_B}\right]\left\{1-\frac{\partial^2 C}{\partial Q_B^2}\left[\theta\frac{\partial D_L^B}{\partial P_B}+(1-\theta)\frac{\partial D_{NL}^{B}}{\partial P_B}\right]\right\}\end{array}\right\}dP_B=0 \qquad (27)$$

By setting

$$M=\theta\frac{\partial D_L^B}{\partial P_B}+(1-\theta)\frac{\partial D_{NL}^{B}}{\partial P_B}$$

$$J=\frac{\partial \theta}{\partial P_A}D_L^B+(1-\theta)\frac{\partial D_{NL}^{B}}{\partial P_A}-\frac{\partial \theta}{\partial P_A}D_{NL}^{B}=\frac{\partial \theta}{\partial P_A}(D_L^B-D_{NL}^{B})+(1-\theta)\frac{\partial D_{NL}^{B}}{\partial P_A}$$

$$K=\theta\frac{\partial^2 D_L^B}{\partial P_B^2}+(1-\theta)\frac{\partial^2 D_{NL}^{B}}{\partial P_B^2}$$

$$L=\frac{\partial \theta}{\partial P_A}\frac{\partial D_L^B}{\partial P_B}-\frac{\partial \theta}{\partial P_A}\frac{\partial D_{NL}^{B}}{\partial P_B}+(1-\theta)\frac{\partial^2 D_{NL}^{B}}{\partial P_A \partial P_B}=\left(\frac{\partial \theta}{\partial P_A}\right)\left(\frac{\partial D_L^B}{\partial P_B}-\frac{\partial D_{NL}^{B}}{\partial P_B}\right)+(1-\theta)\frac{\partial^2 D_{NL}^{B}}{\partial P_A \partial P_B}$$

Equation (27) can then be simplified as follows:

$$MdP_B+JdP_A+M\left(1-\frac{\partial^2 C}{\partial Q_B^2}M\right)dP_B+\left(P_B-\frac{\partial C}{\partial Q_B}\right)KdP_B+\left(P_B-\frac{\partial C}{\partial Q_B}\right)LdP_A-M\frac{\partial^2 C}{\partial Q_B^2}JdP_A=0$$

$$\Rightarrow\left\{M\left(2-\frac{\partial^2 C}{\partial Q_B^2}M\right)+\left(P_B-\frac{\partial C}{\partial Q_B}\right)K\right\}dP_B=\left\{\left(M\frac{\partial^2 C}{\partial Q_B^2}-1\right)J-\left(P_B-\frac{\partial C}{\partial Q_B}\right)L\right\}dP_A$$

$$\Rightarrow\frac{dP_B}{dP_A}=\frac{\left(M\frac{\partial^2 C}{\partial Q_B^2}-1\right)J-\left(P_B-\frac{\partial C}{\partial Q_B}\right)L}{M\left(2-\frac{\partial^2 C}{\partial Q_B^2}M\right)+\left(P_B-\frac{\partial C}{\partial Q_B}\right)K}$$

Q.E.D

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NOTES

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[1] Ford, Chrysler Join GM in Offering Employee Discounts to Everyone , July 5, 2005, ConsumerAffairs.Com

[2] Central New York Business Journal July 8, 2005

[3] See Appendix for detailed proof

[4] This is a typical aggregated demand curve with two types of consumers indicating that any price above the kink will result in purchases by loyalty consumers only while prices below the kink will result in both types of consumers purchasing the good of brand *B*.

[5] The simulation result is done using Matlab program.