Framing Contingencies in Contracts*

Xiaojian Zhao†

September 2, 2008

Abstract

The paper develops a contracting model where the principal frames the contract when the agent is unaware of some contingencies, yet is aware that she may be unaware. We call the contract vague if the agent is still unaware of some contingencies after understanding the contract. We show that the optimal contract is vague if and only if the principal exploits the agent. Applying the model to an insurance problem, we show the insuree is free from exploitation if she slightly underestimates the unforeseen calamities. In a contracting problem, whenever the contractor is unaware of the force majeure event, she is always exploited by the employer. Then, we show that persuasive advertising of experience goods is exploitative. Lastly, a benevolent parent manipulates his kid's belief to make her more optimistic, and therefore overcomes the kid's self-control problem.

Keywords: Framing effects, contracts, unforeseen contingencies, unawareness, awareness of unawareness, insurances, force majeure clauses, persuasive advertising, self-control
JEL Classification: D86, D81, G22, K12, M37

*I am grateful to Syngjoo Choi, Jürgen Eichberger, Martin Peitz, Heiner Schumacher, Ernst-Ludwig von Thadden and Jidong Zhou for helpful discussions. I would also like to thank seminar participants at University of Mannheim, University College London, Spring Meeting of Young Economists 2008 at Lille, 5th Conference on Economic Design (SED 2008) at Ann Arbor in Michigan, International Conference on Policy Modeling (EcoMod 2008) at Berlin for their suggestions and comments. All remaining errors are mine. Financial support from the German Science Foundation (DFG) is gratefully acknowledged.

†CDSE, University of Mannheim, D-68131 Mannheim, Germany.
E-mail: xzhao@rumms.uni-mannheim.de.

"Probability judgments are attached *not* to events but to descriptions of events."

—— Amos Tversky and Derek J. Koehler (1994, p. 548)

1 Introduction

In many contracting environments, agents cannot be aware of all the contingencies in the future. In other words, there are some unforeseen contingencies to them. Consider an insuree buys some home insurance. If the insuree is only aware of one contingency of calamity, which is "fire", she is then unaware of many other possible calamities, say explosion, earthquake, lightning, storm, flood etc.

However, agents are aware that they *may* be unaware of something. Suppose the insurer has two types of contracts for the insuree: one compensates the insuree for her loss only at the contingency of fire, and the other compensates the insuree by the same amount of money in all calamity-contingencies. If the premiums of two contracts are identical, as one would expect, the insuree prefers the later contract.[1] Even though the insuree is only aware of "fire", she is aware that there may be potentially many unforeseen calamity-contingencies. The insuree can therefore classify the contingencies based on a general concept: "calamity". Although the insuree cannot spell out the residual unforeseen contingencies in the event "calamity", she is aware of the possibility that those contingencies exist.

Moreover, if the insurer describes the latter contract differently, say replacing the general term "calamity" by "fire, explosion, earthquake, lightning, storm, flood or other calamities", the insuree will be aware of those extra contingencies. If the insuree underestimates the existence of those contingencies when only "calamity" is mentioned, she is willing to pay more for the insurance after being more aware. Thus when the contingencies in the contract are framed differently, the insuree's preference over the same alternative (the contract or her outside option) is manipulated. This is a typical result of framing effect in contracting, which the standard contract theory abstracts from.

In the paper, we consider a bilateral contracting problem to explore how the principal (he) frames the contingencies in the optimal contract against the agent (she) who is unaware but aware of her unawareness. In contrast to the standard contract theory where the contract is reduced to a mapping from the realized contingency to the actions, the principal here additionally decides how to frame the contingencies in the contract, and contemplates whether or not to make the agent more aware. In the general setting, we use σ-algebra to model the richness of the agent's language. The agent can assign probability to an event only in her language. We call the contract vague if the agent is still unaware of some payoff-relevant contingencies after the principal proposes the contract. We show that the optimal contract is vague if and only if the principal exploits the agent.

The focus on the paper are the general framework for analyzing problems where the more aware principal contracts with the less aware agents, and the explanation

[1] Suppose the legal term "calamity" that covers "fire" is well-defined and verifiable. Furthermore, the insurance contract is perfectly enforced.

for vague terms in contracts. Moreover, we use the model to discuss several particular problems.

In an insurance problem, the insurer makes the unaware insuree fully aware if the insuree is aware that she is unaware of some unforeseen calamities and slightly underestimates their existence. Conversely, the insurer is silent on the insuree's unforeseen calamities. In one case, suppose the insuree underestimates the unforeseen calamities too much. The insurer then obtains a higher profit by providing low benefits in her unforeseen contingencies. In the other case, suppose the insuree overestimates the unforeseen calamities. The insurer benefits from raising both premium and benefit for the insuree in her unforeseen contingencies. However, the unforeseen contingencies are not so likely to occur. In both cases, the insurer exploits the insuree. Thus only a certain range of degrees of awareness of unawareness prevents the insuree from exploitation. Sometimes awareness of unawareness is valuable.

In a contracting problem with force majeure clauses, i.e., clauses which free some party from obligation when an unforeseen circumstance beyond the control of the parties occurs, such as war, strike, riot, crime, act of God (e.g., fire, flood etc.), we show that it is always optimal for the employer to propose a vague contract with a general term "force majeure" but not to describe the particular unforeseen contingencies, which promote the contractor's awareness, no matter how aware of her unawareness the contractor is. If the contractor underestimates the existence of the force majeure, the employer charges her for a higher transfer in the force majeure event. Conversely, the employer charges her for a higher transfer in the non force majeure event. In both cases, the extra transfer is more likely to occur than the contractor believes. Since the contractor is always exploited by the employer, the policy recommendation is promoting the contractor's awareness of the particular force majeure before contracting. The following suggestions are from Liblicense on the web:

> "To make sure that the parties know exactly what is and is not a legitimate excuse for failure to provide access to licensed materials, it would be better to specifically set forth the circumstances that excuse a failure of performance, rather than rely on a general force majeure clause."[2]

Then, we illustrate the persuasive advertising result of an experience good. We show that the firm has an incentive to make the consumer only aware of the good contingency of consumption if and only if the consumer underestimates both good and bad experiences. Because the advertisement raises the consumer's subjective valuation of the good, this is exactly the persuasive advertising result. However, the insuree's belief is wrong. Since the consumer puts too much weight on the good contingency, she is exploited in the objective world. In this sense, persuasive advertising of experience goods is exploitative.

Lastly, we show that a benevolent parent frames the contingencies in the future for his kid, and thus manipulates the kid's belief. This makes the kid more optimistic. The kid therefore overcomes her self-control problem.

[2]Liblicense: Licensing Digital Information (http://www.library.yale.edu~llicense/forcecls.shtml).

Related Literature:

Psychology:

Tversky and Kahneman (1974) originate the research on human judgment of probability for descriptive purpose in science. They argue that people use several heuristics to assess probability, and one of them is *availability*.[3] Tversky and Kahneman (1974, p.1127) argue:

> "There are situations in which people assess the frequency of a class or the probability of an event by the ease with which instances or occurrences can be brought to mind".

In peoples' minds, the probability judgment of an event depends on whether its instances can be retrieved. The availability of some instances is equivalent to awareness of some contingencies in our model. If a contingency is not available to the agent, we say the agent is unaware of the contingency, or the contingency is unforeseen by the agent.

A more relevant work is *support theory* by Tversky and Koehler (1994). They introduce an alternative theory of subjective probability that deviates from the Bayesian model by withdrawing the additivity of probability measure. The judged probability is modeled by the relative support values of the focal and alternative hypotheses. Empirical evidences suggest that the support function is *subadditive* for implicit disjunctions, that is, the probability judgment of an implicitly disjunctive event is smaller than the probability judgment of the same but explicitly unpacked event. One of the reasons for it is that unpacking an event enhances the availability of particular contingencies in the event. It shares a similar idea with the present model when the agent underestimates the existence of potential unforeseen contingencies, although there is some difference between these two approaches as we see below.[4]

Concerning the insurance-purchasing decision, Johnson et al. (1993) present some questionnaire evidences to show that illustration of vivid calamities increases the insuree's valuation of the insurance. In one application of the present paper, we explore systematically the insurance problem based on these psychological effects. However, we show that announcing vivid calamities is not always optimal for the insurer.

Modeling Unforeseen Contingencies:

Roughly speaking, the agent fails to foresee some event if she has not thought about it when she makes a decision. In economics, there are two main approaches to model unforeseen contingencies: decision-theoretic approach and epistemic approach. (See a survey by Dekel et al. (1998a))

Decision theoretic approach starts from the agent's preference or choice behavior without referring to the agent's true belief. The most relevant paper is Ahn and Ergin (2007). Unforeseen contingencies are modeled by generalizing standard subjective expected utility theory through partition-dependent framing effects. Ahn and

[3] See also the original paper on availability by Kahneman and Tversky (1973).

[4] Although, in a subsequent work, Rottenstreich and Tversky (1997) show that subadditivity is also valid for explicit disjunctions, the present paper abstracts from this effect and focuses only on implicit subadditivity. Our motivation is that unpacking of an implicitly described events can update peoples' awareness of some relevant contingencies, and is therefore more relevant to unforeseen contingencies in contracting problems.

Ergin (2007) also provide an axiomatic foundation of a generalized version of support theory introduced above. In spite of the relevance to our work, our paper cannot be captured by their model. For instance, in the insurance example in section 3, the agent's subjective probability for a vague contract and that for a non-vague contract are different, since there is a difference between announcement of the particular contingency "flood" and saying "other calamities". But, in Ahn and Ergin (2007), these two contracts have no difference for the agent, since the partitions of the set of contingencies in these two contracts are identical. Thus modeling unforeseen contingencies by partition-dependent framing effects loses some important considerations. Therefore, developing the decision theoretic foundation for the present model should be important for the future research.

In contrast, epistemic approach starts from the agent's belief. It directly models the knowledge of an event per se as a distinct event. If the agent fails to foresee an event, we say she is *unaware* of the event. Modica and Rustichini (1994) first study unawareness by epistemic approach. Later, Li (2006) and Heifetz et al. (2006) independently model unawareness that circumvent the impossibility result of non-trivial unawareness by Dekel et al. (1998b). Thus U (Unawareness) is possible to express. Considerable progress has been made such that Awareness of Unawareness (AU) is possible to be expressed. (See, e.g., Board and Chung, 2007) AU plays a role in our model. Agents are unaware of some future contingencies, while they are aware that they may be unaware of something. This changes the contracting result in many important aspects. For example, in our paper, an insuree is aware that there may be many potential unforeseen calamities. An appropriate degree of AU refrains the insuree from exploitation by the insurer.

Games with Unawareness:

Recently, many papers study games with unawareness. We only discuss those papers that are very relevant to our work. Ozbay (2008), and fundamentally Heifetz et al. (2008), studies strategic announcement of some contingencies. The difference from our work is on the agent's subjective probability of the newly announced contingencies. We assume that the agent can put correct weights on all contingencies she is aware of due to her ability to judge the frequencies of all vivid events. Furthermore, we do not require that the agent accepts only a justifiable contract that requires that agent's cognitive ability to reason the principal's profitability, as implicitly assumed in most bounded rationality literature. But our paper can be captured by Halpern and Rego (2006). Halpern and Rego (2006) provide a general setting for studying games with unawareness of actions (possibly the actions of the nature). AU of the agent is modeled by allowing some player to make a "virtual move". In our paper, although the agent cannot be aware of all particular contingencies in the general event, she believes that the nature can make some virtual move based on her subjective probability.

Unawareness and Contract Design:

Firstly, there are some papers on unawareness of endogenous variables, say actions of some contracting parties. Gabaix and Laibson (2006) study how the firm exploits the consumers who are unaware of later add-on prices. Zhao (2008) introduces unawareness into moral hazard problem, and analyses the value of awareness of additional actions. von Thadden and Zhao (2007) provide incentive design for an agent who is unaware of some choice possibilities.

Secondly, there are also some papers on unawareness of exogenous variables, say actions of nature (contingencies). Our paper belongs to this category. Besides it, Filiz-Ozbay (2008) incorporates unawareness into insurance contracts. Chung and Fortnow (2007) model a two-stage game of interaction between a contract (or law) writer and an interpreter. In Tirole (2008), a buyer is aware that the design sold by a seller may not be appropriate, and therefore invests some cognitive resources on thinking whether or not she is indeed unaware of something.

Other non-Bayesian Reasoning Models:

The paper belongs to the growing literature on interaction between a fully rational principal and a boundedly rational agent who uses a non-Bayesian learning rule. von Thadden (1992) studies a repeated contracting problem between a seller and a buyer who uses a non-strategic learning rule. Given the rule, in the long run, the buyer is free from exploitation. Piccione and Rubinstein (2003) model differences among consumers in their ability to perceive intertemporal patterns of prices. Spiegler (2006) shows that the patients using anecdotal reasoning suffer from the exploitation by quacks. Shapiro (2006) studies how a firm manipulates a consumer's memory of the consumption experience when consumers have imperfect recall. Mullainathan et al. (2007) discuss the principal's persuasion method by metaphor when the agent puts uncorrelated situations into one category.

The plan of the rest of the paper is as follows: In section 2, we provide the general model of framing contingencies in contracts in full details. Section 3 applies the model in an insurance problem. Section 4 presents a contracting problem with force majeure clauses. Section 5 discusses the persuasive advertising. Section 6 uses the model to view self-control problems. The last section concludes. For the ease of exposition, we put all the proofs in the appendix.

2 A Model

2.1 Language and Contracts

There are two parties involved in the contracting situation: a *principal* (he) P and an *agent* (she) A. The principal proposes a contract to the agent. The agent decides whether to accept it.

We assume that the principal is omniscient. He knows everything that the analyst knows. This assumption is strong but still plausible in situations where the principal is an experienced firm with many experts, whereas the agent is a naive individual (a consumer or an employee) who lacks sufficient contracting experience.

Let Ω denote a finite set of contingencies consisting of exclusive and exhaustive elements ω.

We assume that the agent is aware of only some contingencies in Ω. Let $K_0(\subset \Omega)$ denote the set of contingencies, which the agent is aware of. We call K_0 the agent's awareness. In terms of psychology, the elements in K_0 are the only available concrete scenarios in the agent's mind.

$X(\subset \Omega)$ represents a non-empty *general* event that is determined by a generic characteristic of contingencies. The characteristic leads to a dichotomic classification of payoff-relevant contingencies for the agent. Put it differently, X captures a general concept the agent understands, no matter whether or not the individual elements in X are available in the agent's mind. Similarly, its complement $X^C \neq \emptyset$ is also a general event. Both X and X^C are payoff-relevant to the agent. The economic meaning of this general event is captured by the agent's utility function, as we shall see later. Although it is more realistic to assume many general events based on other characteristics of contingencies, here we only focus on the most payoff-relevant one in the context under consideration, say "calamity" event for the insuree, or "good-experience" event for the traveler. Example 1 illustrates X and K_0 intuitively.

Example 1 *Let the set of contingencies be* $\Omega = \{no\ calamity,\ fire,\ flood,\ earthquake\}$. $X = \{fire,\ flood,\ earthquake\}$ *is a general event: "calamity". All calamity contingencies share the same characteristic that the insuree loses her assets, safety or health in these contingencies. However, the insuree is not necessarily able to list all the contingencies in* X. *Let* $K_0 = \{no\ calamity,\ fire\}$, *thus the agent is unaware of flood and earthquake. Figure 1 depicts* X *and* K_0 *graphically.*

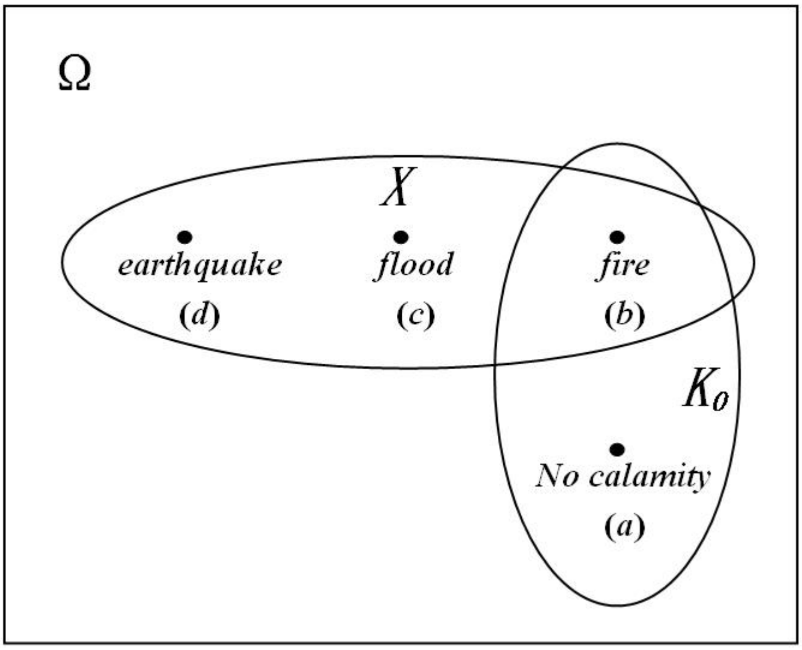

Figure 1: X and K_0 in Example 1

Given the set of contingencies Ω, a general event X and the agent's awareness K_0, we define the language for the agent to express events in a contract.

Definition 1 *The* ***language*** *of the agent with awareness* K_0 *is* $\mathcal{L}(K_0)$ *that is the smallest* ($\sigma-$) *algebra over* Ω *such that*[5]

1. $X \in \mathcal{L}(K_0)$ *and*

[5]There is no difference between $\sigma-$algebra and algebra here, since Ω is finite.

2. For all $\omega \in K_0$, we have $\{\omega\} \in \mathcal{L}(K_0)$.

If an event is in $\mathcal{L}(K_0)$, we say the event is *expressible* for an agent with awareness K_0. Property 1 reflects, although the agent may be unaware of some contingencies in X, she can express the general event X simply by an abstract term, say "calamity". Property 2 says, since the agent is aware of each contingency in K_0, she can express each singleton event $\{\omega\} \subseteq K_0$. Since Ω is finite, there is no difference between σ-algebra and algebra here. $\mathcal{L}(K_0)$ is closed under complements, intersections, and unions, which represent "not", "and", "or" in natural language. The set of expressible events is K_0-dependent. The larger the set K_0, the richer the σ-algebra. In words, the awareness of the agent determines the richness of her language. An example of $\mathcal{L}(K_0)$ is shown in Example 2.

Example 2 *Based on Example 1, for brevity, let $a \equiv$ no calamity, $b \equiv fire$, $c \equiv flood$, $d \equiv$ earthquake. We have $\Omega = \{a, b, c, d\}$, $X = \{b, c, d\}$, and $K_0 = \{a, b\}$.*

The agent's language is thus $\mathcal{L}(K_0) = \{\emptyset, \{a\}, \{b\}, \{c, d\}, \{a, b\}, \{a, c, d\}, \{b, c, d\}, \Omega\}$, that is the collection of all events the agent can express. For instance, the agent can express the event $\{a, c, d\}$ as "no calamities or calamities but not fire". However, say, the event $\{c\}$ is not expressible. To express it, the agent has to be aware of c or d.

In standard contract theory, a contract is reduced to a mapping $C : \Pi \mapsto S$ where Π is a partition of Ω and S is the choice set of the two parties.[6] The partitional domain of C is tantamount to the case where C is a complete contract in some incomplete contracts literature, although it is well-known that there is no agreed definition of incomplete contracts. Suppose that Π is not a partition. If $\cup_{E\in\Pi} E \neq \Omega$, there are gaps in C. (See Shavell, 2006) If $E \cap F \neq \emptyset$ for some E, $F \in \Pi$, there may be contradictions in C. (See Heller and Spiegler, 2008) In this paper, we focus only on complete contracts. Given the agent's language $\mathcal{L}(K_0)$, we can now model how the agent uses her language to form a contract.

Since, in general, not all events are expressible by the agent with awareness K_0, the partition Π is not arbitrary. Let $\Pi(K_0)$ denote the *finest* partition of Ω with respect to $\mathcal{L}(K_0)$. Formally, $\Pi(K_0)$ is a partition of Ω such that $E \in \mathcal{L}(K_0)$ for all $E \in \Pi(K_0)$ and there is no $E' \subset E \in \Pi(K_0)$ and $E' \neq \emptyset$ such that $E' \in \mathcal{L}(K_0)$. The following lemma explicitly describes the finest partition $\Pi(K_0)$.

Lemma 1 $\Pi(K_0) = \{\{\omega\} : \omega \in K_0\} \cup \{X \setminus K_0\} \cup \{X^C \setminus K_0\}$.

Proof. See Appendix A.1. ■

Lemma 1 shows the finest partition that the agent with awareness K_0 can express is the collection of all singleton events the agent is aware of and two residual unforeseen general events $X \setminus K_0$ and $X^C \setminus K_0$.

Using the finest partition $\Pi(K_0)$, we define the contract within the agent's awareness K_0 as follows:

Definition 2 *A **contract** with K_0 is a mapping $C^{K_0} : \Pi(K_0) \mapsto S$.*

The contract maps from the finest expressible event to their choice.

[6]Π is a partition of Ω if $\cup_{E\in\Pi} E = \Omega$ and $E \cap F = \emptyset$ for all E, $F \in \Pi$.

Example 3 *Based on Example 2, we have that* $\Pi(K_0) = \{\{a\}, \{b\}, \{c, d\}\}$ *is the finest partition. The contract with* K_0 *is*

$$C^{K_0} = \{(\{a\}, s_1), (\{b\}, s_2), (\{c, d\}, s_3)\} = \begin{pmatrix} \textit{no calamity} & \to s_1 \\ \textit{fire} & \to s_2 \\ \textit{other calamities} & \to s_3 \end{pmatrix},$$

where s_i *represents a particular choice.*

Of course, the principal can also announce some contingency out of K_0 to enlarge the agent's awareness. We will return to this point in section 2.3.

2.2 Probabilities

We define the *objective* probability space by $(\Omega, 2^\Omega, \mu)$ where μ is the objective probability measure on 2^Ω, which is the collection of all subsets of Ω. Since the principal is omniscient, he knows $(\Omega, 2^\Omega, \mu)$. Assume that $\mu(\{\omega\}) > 0$ for all $\omega \in \Omega$, so no contingency is trivially impossible.

However, the agent is unaware. Due to her limited language $\mathcal{L}(K_0) \subseteq 2^\Omega$, she is unable to judge the probability of all events in Ω. Moreover, her probability judgment of her expressible event may also be wrong, as a result of her unawareness. However, it is innocuous to assume that the agent has correct relative weights of the contingencies in K_0. Intuitively, the agent can objectively judge the frequency of each contingency within her awareness. Equivalently, by reducing one degree of freedom, we assume that the agent knows $\mu(\{\omega\})$ for all $\omega \in K_0$. In Example 1, the insuree is able to judge the frequency of "fire". Since the insuree is aware of fire, she can use some device, say Internet, to acquire the information.

However, the agent has her *subjective* weights on two residual unforeseen general events $X \setminus K_0$ and $X^C \setminus K_0$. For $Z \in \{X, X^C\}$, let $\alpha_Z(K_0)$, which is known by the principal, be a non-negative weight of the agent's residual unforeseen event in Z.[7] In other words, $\alpha_Z(K_0)$ represents the agent's degree of *awareness of unawareness* (AU) of unforeseen contingencies in Z. In Example 1, the insuree is aware that there may be some other calamities with her subjective frequency $\alpha_X(K_0)$, although the insuree cannot tell what they are exactly.

We make the following assumption on $\alpha_Z(\cdot)$.

Assumption 1 *If* $Z \subseteq K_0$*, then* $\alpha_Z(K_0) = 0$ *for* $Z \in \{X, X^C\}$*.*

Assumption 1 reflects that if the agent is aware of every contingency $\omega \in Z$, then she has a correct belief that there are no residual unforeseen contingencies in Z. It is natural that full awareness of contingencies in a general event implies no unforeseen contingencies in the event.

Thus from the agent's view, the probability space is $(\Omega, \mathcal{L}(K_0), \mu^{K_0})$. The agent's *subjective* probability measure is $\mu^{K_0} : \mathcal{L}(K_0) \mapsto \mathbb{R}_+$ such that

[7] The conjunction fallacy by Tversky and Kahneman (1983) shows that α_Z can be negative if the smaller event $Z \cap K_0$ is not available when the agent judges the probability of the larger event Z. However, since we have assumed that the agent is fully aware of contingencies in K_0, all contingencies in $Z \cap K_0$ are available to the agent. Thus $\alpha_Z(K_0) < 0$ is ruled out.

$$\mu^{K_0}(\{\omega\}) \equiv \frac{\mu(\{\omega\})}{\sum_{\omega \in K_0} \mu(\{\omega\}) + \sum_{Z \in \{X, X^C\}} \alpha_Z(K_0)} \text{ for all } \omega \in K_0 \text{ and}$$

$$\mu^{K_0}(Z \setminus K_0) \equiv \frac{\alpha_Z(K_0)}{\sum_{\omega \in K_0} \mu(\{\omega\}) + \sum_{Z \in \{X, X^C\}} \alpha_Z(K_0)} \text{ for all } Z \in \{X, X^C\}.$$

The agent can only assign probabilities to events within her language, and is able to assign probabilities to all events in her language. Furthermore, the agent's subjective probability measure depends on her awareness K_0. The agent uses a heuristic to judge the probability. She assigns a weight $\mu(\{\omega\})$ to each contingency in K_0. If the agent is aware of ω, that is, $\omega \in K_0$, then her probability judgment of $\{\omega\}$ is the ratio of the weight of ω to the sum of the weights of all foreseen contingencies and the residual unforeseen events. The agent's probability judgment of the residual unforeseen event is the ratio of the weight of the unforeseen event to the sum of all weights. The way how the agent forms probability is similar to support theory initiated by Tversky and Koehler (1994) and developed by Ahn and Ergin (2007). The difference has been discussed in the introduction.

Obviously, by Assumption 1, if $K_0 = \Omega$, then the agent's subjective probability measure is nothing but the objective one, that is, $\mu^{K_0} = \mu$.

If $\alpha_Z(K_0) = 0$, then $Z \setminus K_0$ is a completely unforeseen event. Since the agent gives zero weight to the residual event $Z \setminus K_0$, the agent believes that she is aware of every contingency in Z. Most applied unawareness papers are in this case where the agent is unaware and unaware of her unawareness. In Example 1, the insuree believes that fire is the mere calamity in this case.

If $\alpha_Z(K_0) = \sum_{\omega \in Z \setminus K_0} \mu(\{\omega\})$, then the agent has a correct belief of the weight to the residual unforeseen event $Z \setminus K_0$. The agent's degree of AU makes her behave *as if* she foresees $Z \setminus K_0$, although she cannot explicitly express the particular contingencies in $Z \setminus K_0$. This case is degenerate to the situation in which the agent only cannot describe the contingencies in $Z \setminus K_0$ as in Maskin and Tirole (1999) and Tirole (1999). In Example 1, the insuree is unaware of flood and earthquake, but she has a correct belief of the probability of the event that other calamities occur.

If $\alpha_Z(K_0) < \sum_{\omega \in Z \setminus K_0} \mu(\{\omega\})$, then the agent is aware that there may be potentially other contingencies in $Z \setminus K_0$, but *underestimates* their existence.[8]

Finally, if $\alpha_Z(K_0) > \sum_{\omega \in Z \setminus K_0} \mu(\{\omega\})$, the agent *overestimates* the existence of potential other contingencies in $Z \setminus K_0$.

The interpretation of different values of $\alpha_Z(K_0)$ is depicted in Figure 2.

2.3 Framing Contracts

In standard contract theory, we can reduce any contract to a mapping $C : \Omega \mapsto S$. However, it abstracts from the details of how the event in each clause is described.

[8] $\alpha_Z(K_0) \leq \sum_{\omega \in Z \setminus K_0} \mu(\{\omega\})$ with $Z \in \{X, X^C\}$ is nothing but the result of subadditivity of implicit disjunction by Tversky and Koehler (1994).

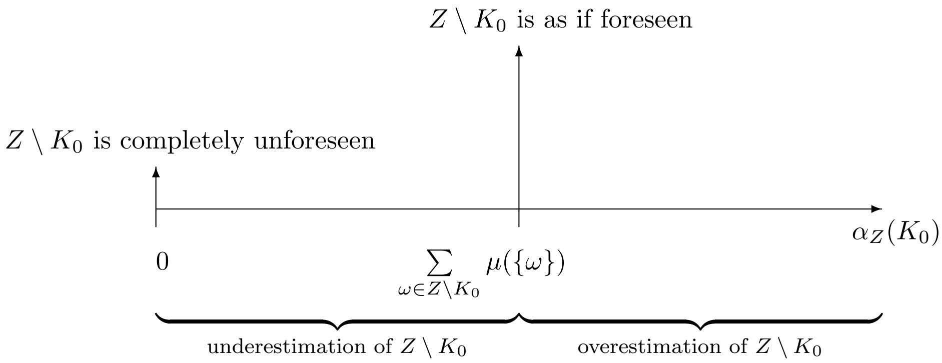

Figure 2: Interpretation of $\alpha_Z(K_0)$ with $Z \in \{X, X^C\}$

We consider the following two contracts following Example 1:

$$C_1 = \begin{pmatrix} \text{no calamity}, & s_1 \\ \text{fire}, & s_2 \\ \text{other calamities}, & s_3 \end{pmatrix}, C_2 = \begin{pmatrix} \text{no calamity}, & s_1 \\ \text{fire}, & s_2 \\ \text{flood}, & s_3 \\ \text{earthquake}, & s_3 \end{pmatrix}.$$

Although two contracts represent the same reduced mapping, they are framed[9] differently. C_2 makes the agent additionally aware of $flood$ and $earthquake$. Of course, in reality, besides $flood$ and $earthquake$ there are many other calamities. Then saying "other calamities" and "'flood or earthquake" are indeed different. In essence, we distinguish only two options here: expressing a general term of the event and listing all individual contingencies in this event.

In general, the principal can update the agent's awareness K_0 via the contract. The agent becomes aware of $\omega \notin K_0$ if ω is explicitly announced in the contract. Formally, we denote the awareness of the agent after reading the contract by K ($\supseteq K_0$). K consists of the contingencies which the agent is aware of after understanding the contract. Thus the principal chooses the *framing* K such that the agent's language becomes $\mathcal{L}(K)$, and the finest partition is refined to $\Pi(K)$. The principal can therefore enrich the language of the agent by framing contingencies.

Furthermore, the agent's subjective weights of the residual unforeseen events become $\alpha_Z(K)$. Roughly speaking, the new contingencies in the agent's mind change the agent's conjectural amount of the residual unforeseen contingencies. Consequentially, the agent's subjective probability measure becomes μ^K. This is not a standard Bayesian updating, since what is updated here is the probability space $(\Omega, \mathcal{L}(K), \mu^K)$ as a whole.

In contrast to the approach of biased belief about contingencies, the biased belief of the agent here is derived purely from the agent's awareness. More importantly, the principal can adjust the agent's biased belief only according to the way how the

[9]In general, framing effect says people's perception of an object will be different if the object is put into a different context, or described differently. Here, two insurance contracts have the same underlying mapping, but the insuree's preference is distorted by a different description of contingencies.

agent's awareness is updated.

Definition 3 *We call a contract C^K **vague** in Z if $Z \nsubseteq K$ where $Z \in \{X, X^C\}$.*

In other words, a contract C^K is vague in Z if the agent is still unaware of some contingencies in Z after C^K is proposed. We say a contract C^K is *vague* if C^K is either vague in X *or* in X^C. Moreover, we say a contract C^K is *less vague* than $C^{K'}$ if $K \supseteq K'$.[10]

Let the agent's von Neumann-Morgenstern (v.N.M) utility function be $u_A : \Omega \times S \mapsto \mathbb{R}$ such that

$$u_A(\omega, s) \equiv \begin{cases} u_A^X(s) & \text{for } \omega \in X \\ u_A^{X^C}(s) & \text{for } \omega \in X^C \end{cases}.$$

For $Z \in \{X, X^C\}$, $u_A^Z(s)$ represents the agent's utility level of choice s when a contingency $\omega \in Z$ occurs. The agent's v.N.M utility function is contingency-dependent. But it depends only on whether the contingency falls into X or not. Put differently, the utility function is "general-event-dependent". The difference between u_A^X and $u_A^{X^C}$ captures the economic meaning of the general event X. For example, let X denote the calamity event. $u_A^X(s) \neq u_A^{X^C}(s)$ reflects that the agent's utility of s in calamity is different from the utility of the same choice s when no calamity occurs. We also assume that the utilities of s are the same within X or X^C. In other words, if calamity happens the agent feels bad to the same extent, no matter it is a fire or a flood.

The agent's subjective expected utility of a contract C^K is therefore

$$\sum_{E \in \Pi(K)} \mu^K(E) \sum_{Z \in \{X, X^C\}} I_{E \subseteq Z} \cdot u_A^Z(C^K(E))$$

where $I_{E \subseteq Z}$ is an index function. If $E \subseteq Z$, we have $I_{E \subseteq Z} = 1$. Otherwise, $I_{E \subseteq Z} = 0$.

We denote the principal's v.N.M utility function by $u_P : S \mapsto \mathbb{R}$ that is contingency-independent. After all, whether or not the contingency s falls into the general event X is only payoff-relevant for the agent. Thus the principal's (objective) expected utility of C^K is

$$\sum_{E \in \Pi(K)} \mu(E) u_P(C^K(E)).$$

Since different problems have different restrictions on the choice of contracts, we denote the set of all *admissible* contracts with framing K by $\mathbb{C}^K$. The particular specification of $\mathbb{C}^K$ depends on the particular context under consideration.

The problem for the principal is therefore to design the optimal contract C^K, which includes the optimal framing K, subject to the agent's participation. It can be written formally as

[10] In contrast to contractual incompleteness that is a concept independent of the agents' awareness, the concept of vagueness depends on the agent's initial awareness K_0. For example, if $K_0 = \Omega$ (the agent is fully aware before contracting), then the contract can never be vague, because X, $X^C \subseteq K_0$.

$$\max_{K\supseteq K_0,\, C^K\in\mathbb{C}^K} \sum_{E\in\Pi(K)} \mu(E)u_P(C^K(E)) \tag{1}$$

$$\text{s.t.} \sum_{E\in\Pi(K)} \mu^K(E) \sum_{Z\in\{X,X^C\}} I_{E\subseteq Z}\cdot u_A^Z(C^K(E)) \geq \sum_{E\in\Pi(K)} \mu^K(E) \sum_{Z\in\{X,X^C\}} I_{E\subseteq Z}\cdot u_A^Z(\overline{s}).$$

On the right hand side of participation constraint of problem (1), $\overline{s}$ is the agent's outside option. Rejecting C^K means that the agent chooses $\overline{s}$ in each contingency. Since the expectation is determined by the agent's subjective probability $\mu^K(\cdot)$, the principal can also influence the agent's perception of her valuation of the outside option by choosing K.

We implicitly assume that the agent has no cognitive ability to infer the set of contingencies from the optimal contract. Her understanding of the set of contingencies is influenced only by the framing K. Thus we rule out the possibility that the agent can do the forward induction as in Heifetz et al. (2008).

The larger K, the richer the language the principal can use to express the events in the contract. However, the agent's subjective probability may be distorted in a direction that the principal dislikes. It brings us the general idea of a trade-off for choosing the optimal framing.

It is worth mentioning that it would be also interesting to study the problems richer than this two-stage game. However, different fields have different interesting considerations. At this given stage of the literature in the general contracting problem, we are restricted in this two-stage game benchmark. Richer models in the particular fields are worthy for the future research.

Slightly abusing notations, let $C^K(\omega)\equiv C^K(E)$ where $\omega\in E\in\Pi(K)$.

Definition 4 *We call a contract C^K **exploitative** if $\sum_{\omega\in\Omega}\mu(\{\omega\})u_A(\omega, C^K(\omega)) < \sum_{\omega\in\Omega}\mu(\{\omega\})u_A(\omega,\overline{s})$.*

In words, a contract C^K is exploitative if the agent's objective expected utility of C^K is lower than the objective expected utility of her outside option. Thus the judgment whether the agent is exploited or not is in terms of the objective probability, yet not the agent's subjective one.

To make things interesting, we make two additional assumptions:

Assumption 2 *If $C^K\in\mathbb{C}^K$, $K'\supseteq K$ and $C^K(\omega)=C^{K'}(\omega)$ for all ω, then $C^{K'}\in\mathbb{C}^{K'}$.*

Assumption 2 allows some natural flexibility on the set of admissible contracts. It says that if a contract is admissible, then any contract with a refined partition that has the same reduced mapping from the set of contingencies to the action space is also admissible. Put another way, for any vague contract that is admissible, the principal can also write a non-vague contract that shares the same reduced mapping.

Assumption 3 *The principal's tie-breaking rule is choosing one of the least vague C^K among the optimal contracts.*

In words, Assumption 3 says, whenever the principal is indifferent between making the agent more aware and being silent, the principal prefers the former. In most problems, since such tie-breaking situation is not generic, we can ignore it. Nevertheless, Assumption 3 is plausible in reality, because a less vague contract signals the principal's honest, specialty in his field. The principal has no incentive to shroud some contingencies unless he has a rent of doing so.

Given Assumption 1-3, we have the following proposition:

Proposition 1 *If C^K is an optimal contract, C^K is exploitative if and only if C^K is vague.*

Proof. See Appendix A.2. ■

Proposition 1 provides a necessary and sufficient condition that the principal exploits the agent. Hence, whenever there is a vague term in the contract, the agent must be exploited. Conversely, if the agent is exploited, the contract, which she accepted, must be vague. This proposition will be frequently used in the following sections.

The intuition of the "only if" part is straightforward. If principal exploits the agent, the participation constraint must be violated. This outcome cannot occur when the contract is non-vague (by Assumption 1). The intuition of the "if" part is as follows. Suppose an optimal contract is not exploitative. Then the agent will accept the contract when she is fully aware. By the tie-breaking rule in Assumption 3, the principal will choose a non-vague contract. Note that he principal is always able to do so due to Assumption 2.

3 Insurance Contracts

We consider a home insurance problem where an insurer as the principal proposes a contract to an insuree as the agent. Suppose the set of contingencies is $\Omega = \{a, b, c\}$. For simplicity, we assume there are only two contingencies of calamity: a and b. a is the contingency of "fire", and b is the contingency of "flood". The general event "calamity" is $X = \{a, b\}$, which is verifiable. If the event calamity occurs, it is either a fire or a flood. The residual contingency c is the contingency of "no calamity". Let the probability measure be $\mu(\{a\}) = p$, $\mu(\{b\}) = 1 - p - q$ and $\mu(\{c\}) = q$.

Before contracting, the insuree is fully aware of contingencies a and c while she is unaware of contingency b, that is, $K_0 = \{a, c\}$. But she is aware that there may be some other potential calamity contingencies of which she is unaware.

By Assumption 1, $\alpha_X(K) = 0$ for $b \in K$. In words, if b is announced in the contract C^K, the insuree will be fully aware of b and then correctly believes that there are no unforeseen calamities. In this case, the insuree understands the objective probability space $(\Omega, 2^\Omega, \mu)$ and assigns a correct probability to each event.

On the other hand, if b is not announced in the contract C^K, the insuree remains unaware of b. Let $\alpha_X(K) \equiv \alpha$ for $b \notin K$. α measures the insuree's degree of AU. She assigns a weight α to the unforeseen event $\{b\}$ while assigning weights p to a and q

to c, respectively. Thus her subjective probability of b is $\frac{\alpha}{\alpha+p+q}$ and her subjective probability of a and c are $\frac{p}{\alpha+p+q}$ and $\frac{q}{\alpha+p+q}$ respectively.[11]

By definition of subjective weights, we have $\alpha \geq 0$.

If $\alpha = 0$, then $\{b\}$ is completely unforeseen by the insuree. The insuree is extremely overconfident that she regards the set of contingencies as $\{a, c\}$ where a and c have probability $\frac{p}{p+q}$ and $\frac{q}{p+q}$ respectively. In contrast, $\alpha > 0$ captures the fact that the insuree is aware that she may be unaware of some other potential calamity contingencies.

If $\alpha = 1 - p - q$, then the insuree has a correct probability judgment. Everything is as if the insuree foresees $\{b\}$, although she cannot explicitly state "flood". If $0 < \alpha < 1 - p - q$, then the insuree is aware that she is unaware of something but underestimates their existence. On the other hand, if $\alpha > 1 - p - q$, the insuree overestimates the existence of potential other calamities.

The choice of the insurer in each contingency is $t \in \mathbb{R}$. t denotes a monetary transfer from the insuree to the insurer.[12] Let the monetary value of the house be $w_1 > 0$. If there is a calamity the value of the house reduces to $w_0 \in (0, w_1)$. Assume that the insurer is risk neutral and the insuree is risk averse. The v.N.M utility function of the insuree is $u(\cdot)$ over money where $u(\cdot)$ is a smooth, strictly increasing and strictly concave function, and satisfies Inada conditions ($u'(0) = \infty$ and $u'(\infty) = 0$).

Thus we have that the insuree's v.N.M utility function is

$$u_A(\omega, t) \equiv \begin{cases} u(w_1 - t) & \text{for } \omega = c \\ u(w_0 - t) & \text{otherwise} \end{cases}. \tag{2}$$

The insurer's v.N.M utility is $u_P(t) = t$.

There is no restriction on the insurer's choice t in each contingency. Thus the set of admissible contracts with K is the set of all contracts. Assumption 2 is therefore satisfied.

3.1 Case 1: Non-Vague Contracts

Firstly, we consider that the insurer proposes a contract where b is announced:

$$C^{\{a,b,c\}} = \begin{pmatrix} a, & t_a \\ b, & t_b \\ c, & t_c \end{pmatrix} = \begin{pmatrix} \text{fire}, & t_a \\ \text{flood}, & t_b \\ \text{no calamity}, & t_c \end{pmatrix}.$$

In this contract, the insuree gives the insurer the *net benefit* t_ω at contingency ω. Put it differently, the insurer charges the *premium* t_c to the insuree and transfers the *gross benefit* $t_c - t_a$ to the insuree when there is a fire and $t_c - t_b$ when there is a flood. The insurer's profit in expectation is therefore

$$pt_a + (1 - p - q)t_b + qt_c.$$

[11] More precisely, the insuree's subjective probability of a, b and c are $\frac{p}{\alpha+p+q+\alpha_{X^C}(\cdot)}$, $\frac{\alpha}{\alpha+p+q+\alpha_{X^C}(\cdot)}$ and $\frac{q}{\alpha+p+q+\alpha_{X^C}(\cdot)}$, respectively. But by Assumption 1, we have $\alpha_{X^C}(\cdot) = 0$.

[12] If $t < 0$, then it is equivalent to say $-t$ is the amount of transfer from the insurer to the insuree.

Since the flood contingency b is announced in $C^{\{a,b,c\}}$, the insuree becomes fully aware and insuree's probability judgment of the set of contingencies is objective. Her expected utility level of $C^{\{a,b,c\}}$ is

$$pu(w_0 - t_a) + (1-p-q)u(w_0 - t_b) + qu(w_1 - t_c).$$

The outside option of the insuree is not buying the insurance, that is, $\bar{t} = 0$. If she rejects the contract, she receives her objective utility level

$$pu(w_0 - \bar{t}) + (1-p-q)u(w_0 - \bar{t}) + qu(w_1 - \bar{t}) = (1-q)u(w_0) + qu(w_1).$$

The insurer maximizes his expected profit subject to the insuree's participation constraint, that is, he solves the following problem:

$$\max_{t_a,t_b,t_c} pt_a + (1-p-q)t_b + qt_c \tag{3}$$
$$\text{s.t. } pu(w_0 - t_a) + (1-p-q)u(w_0 - t_b) + qu(w_1 - t_c) \geq (1-q)u(w_0) + qu(w_1).$$

The problem degenerates to a standard insurance contract in which both insurer and insuree share the same probability judgment. The solution is characterized by $w_0 - t_a = w_0 - t_b = w_1 - t_c$ together with the binding participation constraint of problem (3). We therefore obtain the full insurance result. Since the insuree becomes fully aware, the solution is independent of α.

3.2 Case 2: Vague Contracts

Secondly, we consider that the insurer proposes a vague contract:

$$C^{\{a,c\}} = \begin{pmatrix} a, & t_a \\ b, & t_b \\ c, & t_c \end{pmatrix} = \begin{pmatrix} \text{fire,} & t_a \\ \text{calamity but not fire,} & t_b \\ \text{no calamity,} & t_c \end{pmatrix}.$$

Although the event lists in $C^{\{a,b,c\}}$ and $C^{\{a,c\}}$ have the same reduced mapping, they are framed differently. In $C^{\{a,c\}}$, flood is expressed as "calamity but not fire" while in $C^{\{a,b,c\}}$ flood is explicitly announced. After $C^{\{a,c\}}$ is proposed, the insuree remains unaware of b. The insuree believes that buying the insurance $C^{\{a,c\}}$ leads to a utility level

$$\frac{p}{p+\alpha+q}u(w_0 - t_a) + \frac{\alpha}{p+\alpha+q}u(w_0 - t_b) + \frac{q}{p+\alpha+q}u(w_1 - t_c).$$

If she rejects the contract, she believes that she receives her subjective utility level

$$\frac{p+\alpha}{p+\alpha+q}u(w_0) + \frac{q}{p+\alpha+q}u(w_1).$$

Thus the insurer solves the following problem:

$$\max_{t_a,t_b,t_c} pt_a + (1-p-q)t_b + qt_c \tag{4}$$
$$\text{s.t. } pu(w_0-t_a) + \alpha u(w_0-t_b) + qu(w_1-t_c) \geq (p+\alpha)\, u(w_0) + qu(w_1).$$

The solution is characterized by the following equation system:

$$pu(w_0-t_a) + \alpha u(w_0-t_b) + qu(w_1-t_c) - (p+\alpha)u(w_0) - qu(w_1) = 0, \tag{5}$$
$$(1-p-q)u'(w_0-t_a) - \alpha u'(w_0-t_b) = 0, \tag{6}$$
$$u'(w_0-t_a) - u'(w_1-t_c) = 0. \tag{7}$$

Equation (5) is nothing but the binding participation constraint of problem (4). Equation (7) implies that the insuree has the same final monetary value in contingency a and c, since she puts the correct relative weights on two contingencies. However, by (6), if $\alpha < 1-p-q$, then we have $u'(w_0-t_b) > u'(w_0-t_a)$ that implies $u(w_0-t_b) < u(w_0-t_a) = u(w_1-t_c)$. Therefore, if the insuree underestimates the existence of unforeseen calamities, she is under insured at b.

Let $\underline{\alpha}$ be a particular value of α such that equations (5)-(7) are satisfied and, additionally, $t_b = 0$. Formally, $\underline{\alpha}$ satisfies

$$pu(w_0-t_a) + \underline{\alpha} u(w_0) + qu(w_1-t_c) - (p+\underline{\alpha})u(w_0) - qu(w_1) = 0, \tag{8}$$
$$(1-p-q)u'(w_0-t_a) - \underline{\alpha} u'(w_0) = 0, \tag{9}$$
$$u'(w_0-t_a) - u'(w_1-t_c) = 0. \tag{10}$$

In other words, if $\alpha = \underline{\alpha}$, the insuree has zero *net* benefit at the unforeseen contingency b and thus is completely uninsured at the that contingency.[13] We now have the following lemmas.

Lemma 2 $\underline{\alpha} < 1-p-q$.

Proof. See Appendix A.3. ■

Lemma 2 says, in the situation where the insuree is completely uninsured at b, the insuree must underestimates the existence of unforeseen calamities.

Lemma 3 *$\alpha \geq \underline{\alpha}$ if and only if $t_b \leq 0$ in the solution of problem (4).*

Proof. See Appendix A.4. ■

Lemma 3 says under the condition that the insuree's degree of AU exceeds the level in which she is completely uninsured at b, the insuree always receives a positive net benefit at b in the solution of (4). Furthermore, this condition is also necessary for a positive net benefit at b.

Lemma 4 *The insurer's profit in the solution of problem (4) is increasing in α when $\alpha > \underline{\alpha}$ and decreasing in α when $\alpha < \underline{\alpha}$.*

[13] Put it differently, when $\alpha = \underline{\alpha}$, the insuree pays the premium t_c to the insurer. If b occurs, then the insurer returns the premium t_c back to the insuree.

Proof. See Appendix A.5. ■

Lemma 4 is surprising. It implies that the insurer gains his minimal profit when $\alpha = \underline{\alpha}$ if the contract is vague. In other words, the situation where the insuree is completely uninsured at b is the worst case for the insurer.

3.3 To Be or Not to Be Vague?

After obtaining the optimal contracts in two different framings, we now examine which framing is optimal. The following proposition provides the answer.

Proposition 2 *There exists $\alpha^* < \underline{\alpha}$ such that the insurer will announce b in the optimal contract if and only if $\alpha \in [\alpha^*, 1-p-q]$.*

Proof. See Appendix A.6. ■

Figure 3 depicts Proposition 2 graphically.

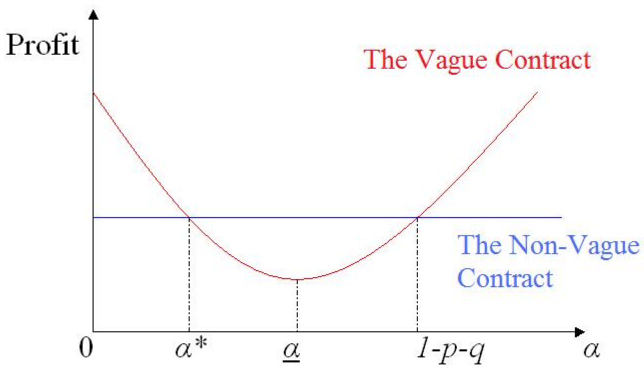

Figure 3: The profit curves in case 1 and 2 with different α in section 3

For example, let $p = q = \frac{1}{3}$, $u(\cdot) = \ln(\cdot)$, $w_0 = 1$ and $w_1 = 2$. The insurer will announce b in the optimal contract if and only if $0.1596 \leq \alpha \leq \frac{1}{3}$.[14]

The following corollary directly follows proposition 1 and proposition 2.

Corollary 1 *There exists $\alpha^* < \underline{\alpha}$ such that the insuree cannot be exploited by the insurer if and only if $\alpha \in [\alpha^*, 1-p-q]$.*

The interpretation of Proposition 2 and Corollary 1 is as follows.

If $\alpha > 1-p-q$, that is, the insuree overestimates the existence the other potential calamities, "flood" does not appear in the optimal contract. Since the insuree is over-worried about the potential unknown calamities, she puts a higher weight on the other calamity event. However, in the objective world, the calamity is not so likely to occur. Thus the insurer can charge a higher premium t_c to the insuree by raising $-t_b$. The insuree is therefore over-insured at contingency b. By corollary 1, the insurer exploits the insuree.

[14]Note that the result in case 1 is a special case of the result in case 2 when $\alpha = 1 - p - q = \frac{1}{3}$.

However, psychological evidences suggest that $\alpha \leq 1-p-q$. (See, e.g., Tversky and Koehler, 1994) It implies that the exploitative contract $C^{\{a,c\}}$ in case 2 when $\alpha > 1-p-q$ is not so likely to occur.

Johnson et al. (1993) provide some evidences to show that isolation of vivid causes of death increases the insuree's valuation of insurance. In this context, it means that, given the same gross benefits $t_c - t_a$ and $t_c - t_b$ in two contracts $C^{\{a,b,c\}}$ and $C^{\{a,c\}}$, the insurer can charge a higher premium t_c to the insuree in contract $C^{\{a,b,c\}}$ where the vivid flood contingency b is announced. It is indeed the case in our model when $\alpha < 1-p-q$.[15]

However, it does not imply that proposing $C^{\{a,b,c\}}$ is always optimal when $\alpha < 1-p-q$. It is because t_a and t_b are also endogenous variables for the insurer.

Particularly striking is that if $\alpha < \alpha^*$, that is, the insuree significantly underestimates the existence the other potential calamities, $C^{\{a,c\}}$ is better than $C^{\{a,b,c\}}$ for the insurer. The intuition is that the insuree believes that the event "calamity but not fire" is very rare. Then the insurer can provide a contract with a low gross benefit $t_c - t_b$ at flood. The insuree will accept the contract. However, her objective expected utility of the contract is lower than the objective utility level of the outside option. Thus the insurer earns a high profit by exploiting the insuree.

If $\alpha \in (\alpha^*, 1-p-q)$, that is, the insuree underestimates its existence but not too much, then "flood" appears in the optimal contract. The intuition is that α is not too low. There is no opportunity for the insurer to exploit the insuree by raising t_b. α is also not too high. There is no opportunity for the insurer to increase t_a and t_c while lowering t_b. By corollary 1, there is no exploitation in this contract. The insurer voluntarily does not exploit the insuree.

The main lesson is that if α is large enough (but still weakly less than the true probability $1-p-q$), the insurer will not propose a vague contract, and the insuree is not exploited. Although the insuree is unaware of the particular contingency b, because she is aware that she may be unaware of something, this makes her free from exploitation. Thus there is a value of certain degree of AU.

In Ozbay (2008) and Filiz-Ozbay (2008), the equilibrium concept requires the contract is *justifiable*, namely the contract is optimal for the insurer also from the insuree's view. Appendix A.7 shows that under the constraint of contractual justifiability the insurer will announce b in the optimal contract if and only if $\alpha \in (0, 1-p-q]$. Hence the role of AU is more significant: the insuree is free from exploitation whenever there is a positive degree of AU and weakly underestimates the unforeseen calamities.

4 Force majeure Clauses

We consider a situation where an employer as the principal proposes a contract to a contractor as the agent to fulfill a project. Let t, which is contractible, be the contractor's input to the project. The monetary cost of input t to the contractor is $c(t)$ where $c(\cdot)$ is a smooth, strictly increasing and strictly convex function with $c(0) = 0$, $c'(0) = 0$, $c'(\infty) = \infty$. Ex post, the monetary performance of the contractor

[15] The reason is that, by proposing $C^{\{a,b,c\}}$ in case 1, the insuree's subjective utility of the outside option becomes objective and is therefore lower than before. Fixing $t_c - t_a$ and $t_c - t_b$, the premium t_c can be larger by slightly lowering $-t_a$ and $-t_b$.

is nothing but t. The contractor has an initial wealth w. The contractor is risk averse and has a v.N.M. utility $u(\cdot)$ over money where $u(\cdot)$ is a smooth, strictly increasing and strictly concave function with $u'(0) = \infty$, $u'(\infty) = 0$. The employer who is risk neutral charges the contractor for p ex post. In the standard problem, the employer maximizes p subject to the contractor's participation, that is, he solves the following problem:

$$\max_{p,t} p$$
$$\text{s.t. } u(w + t - p - c(t)) \geq u(w).$$

Let the contractor's outside option be $\bar{p} \equiv 0$ and $\bar{t} \equiv 0$, thus her utility level of her outside option is $u(w)$ where $w > 0$ is the contractor's wealth. The solution to this problem is characterized by $c'(t^*) = 1$ and $p^* = t^* - c(t^*)$.

However, the simple situation above is in a world without *force majeure*, that is, there is no unexpected event beyond the control of contracting parties, such as war, strike, riot, crime, act of God (e.g., fire, flood, etc.). If a force majeure event occurs, the performance of the project will be jeopardized. Thus, in many contracting situations, the parties specify a force majeure clause in contract to release the contractor's obligations.

Suppose the set of contingencies is $\Omega = \{a, b\}$. The contingency a is the non force majeure contingency. For simplicity, we assume only one contingency of force majeure: b, say the contingency of "fire" in the workplace of the project. The general event "force majeure" is $X = \{b\}$. Let $\mu(\{a\}) = q$ and $\mu(\{b\}) = 1 - q$.

Before contracting, the contractor is fully aware of contingencies a while she is unaware of contingency b, that is, $K_0 = \{a\}$. But she is aware that there may be some unforeseen force majeure contingencies.

Again, we have $\alpha_X(K) = 0$ for $b \in K$, that is, if the fire contingency b is announced, the contractor will be fully aware of b, and then she comprehends the objective probability space (Ω, μ).

In contrast, $\alpha_X(K) \equiv \alpha$ for $b \notin K$. It captures the degree that the contractor is aware that she may be unaware of some particular force majeure contingencies if b is not announced. The contractor assigns a weight α to the unforeseen event force majeure $\{b\}$ while assigning weight q to contingency a. Thus her subjective probability of b is $\frac{\alpha}{\alpha+q}$, and her subjective probability of a is $\frac{q}{\alpha+q}$.

If a force majeure occurs, the contractor's performance in terms of money is zero, that is, the contractor's performance is totally destroyed in force majeure events. If there is no force majeure, the monetary performance equals to the input t.

The contractor's v.N.M utility function is therefore

$$u_A(\omega, t) \equiv \begin{cases} u(w - p - c(t)) & \text{for } \omega \in X \\ u(w + t - p - c(t)) & \text{for } \omega \in X^C \end{cases}.$$

The employer's v.N.M utility is $u_P(p, t) = p$.

The contractor implements the input t before the contingency is revealed. Thus t is forced to be identical in all contingencies. Formally, the set of admissible contracts with K is $\mathbb{C}^K = \{C^K : C^K(a) = (p_1, t) \text{ and } C^K(b) = (p_2, t)\}$. Note that Assumption 2 is satisfied here.

We now consider that the employer proposes a vague contract $C^{\{a\}}$ where b is not announced but a force majeure clause is specified:

$$C^{\{a\}} = \begin{pmatrix} a, & (p_1, t) \\ b, & (p_2, t) \end{pmatrix} = \begin{pmatrix} \text{not force majeure,} & (p_1, t) \\ \text{force majeure,} & (p_2, t) \end{pmatrix}.$$

Facing $C^{\{a\}}$, the contractor is still unaware of b. Since her outside option is $\bar{p} \equiv \bar{t} \equiv 0$, she believes that accepting $C^{\{a\}}$ leads to a utility level

$$\frac{q}{q+\alpha} u(w + t - p - c(t)) + \frac{\alpha}{q+\alpha} u(w - p - c(t))$$

and rejecting $C^{\{a\}}$ leads to a utility level

$$\frac{q}{q+\alpha} u(w + \bar{t} - \bar{p} - c(\bar{t})) + \frac{\alpha}{q+\alpha} u(w - \bar{p} - c(\bar{t})) = u(w).$$

Thus the employer solves the following problem:

$$\max_{p_1, p_2, t} qp_1 + (1-q)p_2 \tag{11}$$
$$\text{s.t. } \frac{q}{q+\alpha} u(w + t - p_1 - c(t)) + \frac{\alpha}{q+\alpha} u(w - p_2 - c(t)) \geq u(w).$$

It is straightforward to show that the solution of problem (11) is p_1^*, p_2^* and t^*, which are characterized by the following equation system:

$$c'(t^*) - q = 0,$$
$$(1-q)u'(w + t^* - p_1^* - c(t^*)) - \alpha u'(w - p_2^* - c(t^*)) = 0, \tag{12}$$
$$\frac{q}{q+\alpha} u(w + t^* - p_1^* - c(t^*)) + \frac{\alpha}{q+\alpha} u(w - p_2^* - c(t^*)) - u(w) = 0. \tag{13}$$

Observation 1 *If $\alpha < 1 - q$, then $u(w + t^* - p_1^* - c(t^*)) > u(w - p_2^* - c(t^*))$.*

We obtain observation 1 from equation (12). If $\alpha < 1 - q$, then $u'(w + t^* - p_1^* - c(t^*)) < u'(w - p_2^* - c(t^*))$. Thus we have $u(w + t^* - p_1^* - c(t^*)) > u(w - p_2^* - c(t^*))$. The condition $\alpha < 1 - q$ that is suggested by the psychology literature (See Tversky and Koehler, 1994) means that the contractor underestimates the existence of the force majeure. $u(w + t^* - p_1^* - c(t^*)) > u(w - p_2^* - c(t^*))$ is a result where the contractor is not fully "insured". The contractor is better off in the non force majeure event that is more likely in reality. Hence, besides hidden information and hidden action, underestimating unforeseen contingencies can be also a driving force for a non-full insurance outcome.

However, if b is announced in the contract, then the contractor shares the same probability measure as the employer. It is then equivalent for the employer to solve problem (11) when $\alpha = 1 - q$. The question is when the employer has an incentive to make the contractor have the correct belief. The following proposition provides a negative answer.

Proposition 3 *If $\alpha \neq 1 - q$, then proposing a vague contract $C^{\{a\}}$ is always better than a non-vague contract $C^{\{a,b\}}$ for the employer.*

Proof. See Appendix A.8. ■

Proposition 3 says that if the contractor is unaware of b, no matter how aware of her unawareness she is, the employer will be silent on the particular force majeure contingency fire and describes only a general force majeure event in the contract. Since the contract is always vague, by proposition 1, the contractor is always exploited. If $\alpha < 1 - q$, that is, the contractor underestimates the existence of force majeure, the employer exploits the contractor by charging a high p_2, which occurs more likely than the contractor believes. If $\alpha > 1 - q$, the employer exploits the contractor by charging a high p_1, which also occurs more likely than the contractor believes. Thus whenever the contractor is unaware of the particular force majeure contingencies, the employer can utilize the contractor's mis-perception.

For example, let $u(\cdot) = \ln(\cdot)$, $c(t) = \frac{1}{2}t^2$, $q = \frac{1}{2}$ and $w = 1$. Then the profit function of the employer which depends on α is depicted in Figure 4. Note that the result when b is announced is a special case of this result when $\alpha = 1 - q = \frac{1}{2}$. In Figure 4, we observe that two profit curves intersect at $\alpha = \frac{1}{2}$. Moreover, we find that the employer's profit when announcing b is always higher than not announcing it, and two curves are tangent at $\alpha = \frac{1}{2}$.

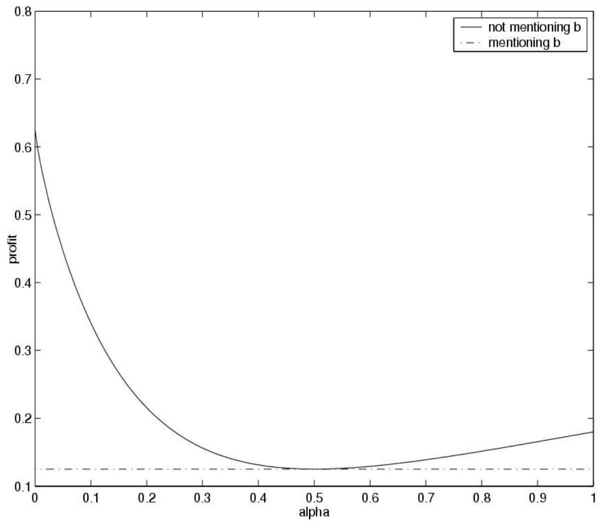

Figure 4: The profit curves when announcing b or not with different α in section 4

Proposition 3 presents a negative result. The only way to make the contractor free from exploitation is making the contractor aware in order to let her have a correct probability judgment. In contrast to the result in the insurance example, some certain degree of the contractor's AU cannot creates the incentive for the employer to make the contractor aware. Only ex ante full awareness of the contractor is valuable to her.

5 Persuasive Advertising

In this section, we consider a firm as the principal sells an experience good to a consumer as the agent.[16] Suppose now the firm provides a travel service. The set of contingencies is $\Omega = \{g, b\}$. The contingency g is the contingency of the consumer's good experience. b is the contingency of her bad experience. For simplicity, let announcing g in our language be a full description of the good contingency of travel, say an *advertisement* showing the most beautiful sites with sunshine. On the other hand, announcing b is the full description of the bad contingency, say expressing the possibility of a storm, theft and so on. The general event is a "good" experience $X = \{g\}$. Let the probability measure be $\mu(\{g\}) = q$ and $\mu(\{b\}) = 1 - q$.

The reason we focus on the experience good here is that, before contracting, the consumer knows nothing about the content of the travel. She is aware of no contingencies, that is, $K_0 = \emptyset$. In reality, travelers enjoy mainly the unknown experiences during the travel. A contingent plan under uncertainty would be uninteresting for the travellers. But the consumer has the general idea of events X and X^C, that is, a good experience and a bad experience.

By Assumption 1, we have that $\alpha_X(K) = 0$ for $g \in K$ and $\alpha_{X^C}(K) = 0$ for $b \in K$. If g (respectively b) is explicitly described in the contract, the consumer will be fully aware of g (respectively b), and assigns a correct weight $\mu(g) = q$ to g (respectively $\mu(b) = 1 - q$ to b) and zero weight to the non-existing residual event. We call the contract $C^{\{g\}}$ describing the contingency g a *positive* advertisement and $C^{\{b\}}$ a *negative* advertisement. Suppose for simplicity that the cost of advertisement is zero.

Let $\alpha_X(K) \equiv \alpha_g$ for $g \notin K$ ($\alpha_{X^C}(K) \equiv \alpha_b$ for $b \notin K$). Assume that α_g, $\alpha_b > 0$. It captures the fact that, if the contract is vague in the good experience X (respectively bad experience X^C), the consumer is aware that she may be unaware of some particular good contingencies (respectively bad contingencies).

The choice of the firm is a pair (p, t) where p is the monetary transfer from the consumer to the firm, or the price of travel and $t \in \{0, 1\}$ is the consumer's binary choice of accepting the travel or not. The cost of the travel is zero. The firm's utility is $u_P(p, t) = pt$. If $t = 1$, that is, the consumer accepts the contract, the firm receives the price p. Otherwise, he gets zero. Let $v > 0$ denote the consumer's valuation of the good experience. Assume the consumer's valuation of the bad experience is zero.

The consumer's v.N.M. utility function is therefore

$$u_A(\omega, p, t) \equiv \begin{cases} t(v - p) & \text{for } \omega = g \\ t(0 - p) & \text{otherwise} \end{cases}.$$

If the consumer accepts the contract ($t = 1$), the consumer's utility is her benefit from the travel v net of the price p when a good experience occurs. When a bad experience occurs, the consumer pays the price p but gains nothing.

The consumer has to decide whether or not to accept the contract before the contingency is revealed. Thus t and p must be identical in all contingencies. Formally,

[16] The experience good is a product or service whose payoff-relevant characteristics are difficult to know in advance. The typical examples are travel, movie, etc.

the set of admissible contracts with K is $\mathbb{C}^K = \{C^K : C^K(\omega_1) = C^K(\omega_2)$ for all $\omega_1 \neq \omega_2\}$. (Assumption 2 is satisfied.)

- Vagueness in both Good and Bad Experiences:

Firstly, we consider a case in which the firm does not advertise, that is, neither g nor b is announced. The contract is $C^{\emptyset}(\cdot) = (p, t)$. There is only one "catchall" clause in $C^{\emptyset}$ that is what to do no matter what happens.

The consumer believes that accepting the contract leads to a utility level

$$\frac{\alpha_g}{\alpha_g + \alpha_b} t(v - p) + \frac{\alpha_b}{\alpha_g + \alpha_b} t(-p).$$

On the other hand, let the consumer's outside option be $\bar{p} \equiv 0$ and $\bar{t} \equiv 0$. If she rejects the contract, she believes that she receives her utility level

$$\frac{\alpha_g}{\alpha_g + \alpha_b} \bar{t}(v - \bar{p}) + \frac{\alpha_b}{\alpha_g + \alpha_b} \bar{t}(-\bar{p}) = 0.$$

The firm therefore simply solves the following problem:

$$\max_{p,t} pt$$
$$\text{s.t. } \frac{\alpha_g}{\alpha_g + \alpha_b} t(v - p) + \frac{\alpha_b}{\alpha_g + \alpha_b} t(-p) \geq 0.$$

The solution to this problem is $p_1 = \frac{\alpha_g}{\alpha_g + \alpha_b} v$ and $t_1 = 1$. The firm charges the consumer the price $\frac{\alpha_g}{\alpha_g + \alpha_b} v$ and the consumer accepts it. The corresponding profit for the firm is $\pi_1 = p_1 = \frac{\alpha_g}{\alpha_g + \alpha_b} v$.

- Vagueness in Bad Experiences:

Secondly, we consider that the firm makes only the positive advertisement, that is, only g is announced. The contract is $C^{\{g\}}(\cdot) = (p, t)$, which frames the good experience differently. After understanding $C^{\{g\}}$, the consumer has a correct weight to contingency g. Then the firm solves the following problem:

$$\max_{p,t} pt$$
$$\text{s.t. } \frac{q}{q + \alpha_b} t(v - p) + \frac{\alpha_b}{q + \alpha_b} t(-p) \geq 0.$$

The solution to this problem is $p_2 = \frac{q}{q + \alpha_b} v$ and $t_2 = 1$. The corresponding profit of the firm is $\pi_2 = \frac{q}{q + \alpha_b} v$.

- Vagueness in Good Experiences:

Similarly, if the firm makes only the negative advertisement, the firm charges price $p_3 = \frac{\alpha_g}{\alpha_g + 1 - q} v$ and $t_3 = 1$. The corresponding profit of the firm is $\pi_3 = \frac{\alpha_g}{\alpha_g + 1 - q} v$.

- No Vagueness:

Lastly, if both g and b are announced, the contract $C^{\{g,b\}}(\cdot) = (p, t)$ is not vague. The consumer then understands the probability space. The solution for the firm is $p_4 = qv$ and $t_4 = 1$. The corresponding profit of the firm is $\pi_4 = qv$.

Proposition 4 *The contract $C^{\{g\}}$ is optimal for the firm if and only if $\alpha_g < q$ and $\alpha_b < 1 - q$.*

Proof. See Appendix A.9. ■

Proposition 4 says that if the consumer underestimates both positive and negative concrete scenarios of the good, then it is optimal for the firm to make only the positive advertisement. The intuition is straightforward. By making only the positive advertisement, the consumer puts a high weight on the good contingency g. The firm can therefore charge the highest price. Conversely, if it is optimal for the firm to make only the positive advertisement, the consumer necessarily underestimates both positive and negative contingencies of the good. Since we observe that in most advertisements only the good contingencies are announced in reality, we also confirm consumers' psychological characteristic that they underestimate both contingencies.

Consequently, the consumer's *subjective* valuation of the good is higher. This is a typical *persuasive advertising* result. However, the welfare implication is that the persuasive advertising on experience good is harmful to the consumers. Since $C^{\{g\}}$ is vague in $\{b\}$, by proposition 1, the consumer is exploited. In the result, $p = \frac{q}{q+\alpha_b}v$. The *objective* expected utility level of the consumer is $qv - p = \frac{qv}{q+\alpha_b}[\alpha_b - (1-q)] < 0$ since $\alpha_b < 1 - q$. The objective participation constraint of the consumer is violated. Thus such persuasive advertising hurts the consumers. In the standard persuasive advertising literature, it is difficult to judge the consumer's welfare change after persuasive advertising, since it is not clear we should use the utility before advertising or after it as the welfare criterion. This simple example suggests that neither of them should be the criterion since they are both subjective, but there exists an objective one known only by the firm and the fully aware consumers.

Hence, the policy recommendation is that, if competition among firms is not possible, the firm is required to report the bad contingencies compulsorily in the advertisement. For instance, it has been already mandatory to include a health warning in the Tobacco advertising in many countries.

However, if we extend the model by introducing Bertrand-competition among homogeneous firms, in equilibrium, all firms will choose $p = 0$. For every firm, each framing is possible to occur in equilibrium. The vagueness of the contract changes the consumer's ex ante subjective utility of contracts but plays no role in competition. Since $p = 0$, the consumer's objective expected utility is maximized irrespective of her ex ante subjective valuation of the good. Thus competition does not necessarily promote awareness of the consumer, but increases the consumer's welfare to the best extent.

6 Framing the Future and Self-Control Problems

In this section, we consider a benevolent principal encourages a present-biased agent to perform a long-run goal. For example, some parents want to stimulate their kid to study harder, and someone may want to encourage his friend to achieve an ambitious task. There is no conflict of interest between the principal and the agent. The principal's motivation of manipulating the agent's belief here is in order to help the agent overcome her self-control problem.

The set of contingencies is $\Omega = \{g, b\}$. The contingency g is the good contingency: the task is successful. b is the failure contingency. If the agent exerts efforts, the principal's utility (or the agent's objective utility) is $pv - 1$ where $v > 1$ is the benefit of the task, 1 is the cost of efforts and p is the objective probability of success. Let $K^0 = \emptyset$, that is, the agent initially knows nothing about the future. But she has a general event $X = \{g\}$ in mind. The agent knows that after making the effort something good or bad will occur in the future. Let $\alpha_X(K) \equiv \alpha_g$ for $g \notin K$ ($\alpha_{X^C}(K) \equiv \alpha_b$ for $b \notin K$). Assume that α_g, $\alpha_b > 0$ and $\alpha_g < p$, $\alpha_b < 1 - p$ as usual. Before contracting, the agent's subjective utility is $\beta \frac{\alpha_g v}{\alpha_g + \alpha_b} - 1$ where $\beta < 1$ represents the present bias of the agent.

Suppose $pv - 1 > 0$ and $\beta \frac{\alpha_g v}{\alpha_g + \alpha_b} - 1 < 0$. Thus the agent "should" exert efforts, but she is too lazy to do it. To make the thing interesting, we assume further that $\beta pv - 1 < 0$. That is, if the agent is fully educated about the future, she still prefers not performing the task because of her present bias. However, the following proposition provides a solution to the agent's self-control problem.

Proposition 5 *If $p\,(\beta v - 1) > \alpha_b$, only the contract $C^{\{g\}}$ overcomes the agent's self-control problem.*

Proof. Straightforward and omitted. ■

Proposition 5 says that if the self-control problem of the agent is not so severe ($\beta v - 1 > 0$ and it is large enough) and the both contingencies are substantially underestimated (p is actually large and α_b is small), then the principal can describe only the good scenario and shroud the bad scenario so as to motivate the agent. This manipulation of the agent's belief makes the agent more optimistic about the future, since the agent's subjective probability of success is $\frac{p}{p + \alpha_b} > p$. But this mis-perception can overcome the agent's self-control problem. A similar idea is also in Benabou and Tirole (2002) where overconfidence of one's ability is valuable.

7 Discussions and Conclusions

The paper provides a general model of framing contingencies in contracts against the agent's awareness of unawareness. We apply the model to four particular fields. The general policy recommendation is to promote the agent's awareness before contracting, since it makes the agent free from exploitation. Furthermore, this policy is robust to any subjective weights and thus does not require any knowledge of the weights of the policy maker. Besides it, if we include thinking cost as in Tirole (2008), the public announcement also saves the agent's cost of thinking about the unforeseen contingencies. Thus the deadweight cost of thinking is also avoided by promoting the awareness of the agent.[17]

[17] The conclusion depends on the assumption of the existence of an objective probability, and that the agent knows it if the agent is fully aware.

However, we abstract from hidden information and hidden action problems. For example, in the insurance example, there is no asymmetric information problem. If the insurees' degree of awareness of unawareness is heterogeneous, the insurer has to use some screening device to filter them out. In the contracting problem, the effort of the agent is observable. The moral hazard problem is ruled out. In short, each field is worthy to be extended in subsequent works. These further issues give us an outline for the future research in the particular fields.

A Appendix

A.1 Proof of Lemma 1

We define another $\sigma-$algebra $\mathcal{B}(K_0)$ that is the smallest $\sigma-$algebra over Ω such that $X \setminus K_0 \in \mathcal{B}(K_0)$, $X^C \setminus K_0 \in \mathcal{B}(K_0)$ and, for all $\omega \in K_0$, $\{\omega\} \in \mathcal{B}(K_0)$. Since the collection of $X \setminus K_0$, $X^C \setminus K_0$ and $\{\omega\}$ for all $\omega \in K_0$ is a partition of Ω, it is the finest partition of Ω with respect to $\mathcal{B}(K_0)$. It is left to show that $\mathcal{L}(K_0) = \mathcal{B}(K_0)$. It is equivalent to show that

1. $X \setminus K_0 \in \mathcal{L}(K_0)$,

2. $X^C \setminus K_0 \in \mathcal{L}(K_0)$ and

3. $X \in \mathcal{B}(K_0)$.

Firstly, since, for all $\omega \in K_0$, $\{\omega\} \in \mathcal{L}(K_0)$, we have $K_0 \in \mathcal{L}(K_0)$. Thus $K_0^C \in \mathcal{L}(K_0)$. Moreover, $X \in \mathcal{L}(K_0)$ implies $X \cap K_0^C \in \mathcal{L}(K_0)$. That is, $X \setminus K_0 \in \mathcal{L}(K_0)$.

Secondly, by the same argument above, we can show $X^C \setminus K_0 \in \mathcal{L}(K_0)$.

Finally, since $\{\omega\} \in \mathcal{B}(K_0)$ for all $\omega \in K_0$, we have $X \cap K_0 \in \mathcal{B}(K_0)$. Moreover, $X \setminus K_0 \in \mathcal{B}(K_0)$. Thus $X = (X \cap K_0) \cup (X \setminus K_0) \in \mathcal{B}(K_0)$.

□

A.2 Proof of Proposition 1

Firstly, we show the "only if" part. Suppose C^K is not vague but exploitative. Then $\mu^K(\{\omega\}) = \mu(\{\omega\})$ for all $\omega \in \Omega$ by Assumption 1. Since C^K is the optimal solution, the participation constraint of the problem (1) is satisfied. C^K is therefore not exploitative, a contradiction.

Secondly, we show the "if" part. Suppose C^K is vague but not exploitative. We have

$$\sum_{\omega\in\Omega} \mu(\{\omega\}) u_A(\omega, C^K(\omega)) \geq \sum_{\omega\in\Omega} \mu(\{\omega\}) u_A(\omega, \overline{s}). \tag{14}$$

We now define another contract $C^{K'}$ such that $C^{K'}$ is not vague ($K' = \Omega$) and $C^{K'}(\omega) = C^K(\omega)$ for all $\omega \in \Omega$. (By Assumption 2, such $C^{K'}$ exists.) Thus $C^{K'}$ gives the principal the same objective expected utility level as C^K does. Moreover, since $C^{K'}$ is not vague, we have $\mu = \mu^{K'}$ by Assumption 1. Thus the participation constraint of the problem (1) is satisfied because this constraint is nothing but (14). Hence $C^{K'}$ is also an optimal contract. However, $C^{K'}$ is less vague than C^K. It contradicts with the tie-breaking rule in Assumption 3.

□

A.3 Proof of Lemma 2

Let $\alpha = \underline{\alpha}$. By equation (10), we have $u(w_0 - t_a) = u(w_1 - t_c)$. Combining (8), we obtain $(p+q)u(w_0 - t_a) - pu(w_0) - qu(w_1) = 0$. $u(w_1) > u(w_0)$ yields $u(w_0 - t_a) > u(w_0)$. By (9), we therefore have $\underline{\alpha} < 1 - p - q$.

□

A.4 Proof of Lemma 3

We denote the solution of problem (4) as a function of α: $t_a(\alpha)$, $t_b(\alpha)$ and $t_c(\alpha)$. By equation (7), we have $u(w_0 - t_a(\alpha)) = u(w_1 - t_c(\alpha))$. Combining (5), we obtain

$$(p+q)u(w_0 - t_a(\alpha)) + \alpha[u(w_0 - t_b(\alpha)) - u(w_0)] = pu(w_0) + qu(w_1). \tag{15}$$

If $\alpha = \underline{\alpha}$, we then have

$$(p+q)u(w_0 - t_a(\underline{\alpha})) = pu(w_0) + qu(w_1). \tag{16}$$

Combining (15) and (16), we get

$$(p+q)[u(w_0 - t_a(\alpha)) - u(w_0 - t_a(\underline{\alpha}))] + \alpha[u(w_0 - t_b(\alpha)) - u(w_0)] = 0. \tag{17}$$

Firstly, let $\alpha > \underline{\alpha}$. Then, by equation (6) and (9), we get $\frac{u'(w_0 - t_a(\alpha))}{u'(w_0 - t_b(\alpha))} > \frac{u'(w_0 - t_a(\underline{\alpha}))}{u'(w_0)}$. It implies $\frac{u'(w_0 - t_a(\alpha))}{u'(w_0 - t_a(\underline{\alpha}))} > \frac{u'(w_0 - t_b(\alpha))}{u'(w_0)}$. Suppose $u(w_0 - t_b(\alpha)) \leq u(w_0)$. Then $u'(w_0 - t_b(\alpha)) \geq u'(w_0)$. Thus $\frac{u'(w_0 - t_a(\alpha))}{u'(w_0 - t_a(\underline{\alpha}))} > 1$. It implies $u(w_0 - t_a(\alpha)) < u(w_0 - t_a(\underline{\alpha}))$. But it makes the proved equation (17) impossible. Thus we must have $u(w_0 - t_b(\alpha)) > u(w_0)$.

Secondly, we can show $u(w_0 - t_b(\alpha)) < u(w_0)$ if $\alpha < \underline{\alpha}$ by the same argument. Thus $\alpha \geq \underline{\alpha}$ if and only if $t_b \leq 0$.

$\square$

A.5 Proof of Lemma 4

Let $\begin{pmatrix} f(t_a, t_b, t_c, \alpha) \\ g(t_a, t_b, t_c, \alpha) \\ h(t_a, t_b, t_c, \alpha) \end{pmatrix}$

$$\equiv \begin{pmatrix} pu(w_0 - t_a) + \alpha u(w_0 - t_b) + qu(w_1 - t_c) - (p+\alpha)u(w_0) - qu(w_1) \\ (1-p-q)u'(w_0 - t_a) - \alpha u'(w_0 - t_b) \\ u'(w_0 - t_a) - u'(w_1 - t_c) \end{pmatrix}.$$

Thus equation system (5)-(7) is equivalent to $\begin{pmatrix} f(t_a, t_b, t_c, \alpha) \\ g(t_a, t_b, t_c, \alpha) \\ h(t_a, t_b, t_c, \alpha) \end{pmatrix} = 0.$

Let $s(\alpha) \equiv \begin{pmatrix} t_a \\ t_b \\ t_c \end{pmatrix}$ be the solution of problem (4).

By implicit function theorem, we have

$$D_\alpha s(\alpha) = \begin{pmatrix} f_{t_a}(t_a, t_b, t_c, \alpha) & f_{t_b}(t_a, t_b, t_c, \alpha) & f_{t_c}(t_a, t_b, t_c, \alpha) \\ g_{t_a}(t_a, t_b, t_c, \alpha) & g_{t_b}(t_a, t_b, t_c, \alpha) & g_{t_c}(t_a, t_b, t_c, \alpha) \\ h_{t_a}(t_a, t_b, t_c, \alpha) & h_{t_b}(t_a, t_b, t_c, \alpha) & h_{t_c}(t_a, t_b, t_c, \alpha) \end{pmatrix}^{-1} \begin{pmatrix} f_\alpha(t_a, t_b, t_c, \alpha) \\ g_\alpha(t_a, t_b, t_c, \alpha) \\ h_\alpha(t_a, t_b, t_c, \alpha) \end{pmatrix}.$$

Now we use the following abbreviated notations. Let $a \equiv u'(w_0 - t_a)$, $b \equiv u'(w_0 - t_b)$, $c \equiv u'(w_1 - t_c)$, $x \equiv u''(w_0 - t_a)$, $y \equiv u''(w_0 - t_b)$, $z \equiv u''(w_1 - t_c)$, $u \equiv u(w_0 - t_b)$ and $v \equiv u(w_0)$. Hence, we get

$$D_\alpha s(\alpha) = \begin{pmatrix} -pa & -\alpha b & -qc \\ -(1-p-q)x & \alpha y & 0 \\ -x & 0 & z \end{pmatrix}^{-1} \begin{pmatrix} u-v \\ -b \\ 0 \end{pmatrix}$$
$$= \begin{pmatrix} \frac{-b^2z+yz(u-v)}{bxz+apyz-bpxz-bqxz+cqxy} \\ \frac{b(apz+cqx)+(u-v)xz(1-p-q)}{bxz\alpha+apyz\alpha-bpxz\alpha-bqxz\alpha+cqxy\alpha} \\ \frac{-b^2x+xy(u-v)}{bxz+apyz-bpxz-bqxz+cqxy} \end{pmatrix}.$$

Since the profit $\pi \equiv pt_a + (1-p-q)t_b + qt_c$, we have

$$\begin{aligned} D_\alpha \pi &= pD_\alpha t_a + (1-p-q)D_\alpha t_b + qD_\alpha t_c \\ &= -\frac{1}{(bxz+apyz-bpxz-bqxz+cqxy)\,\alpha}(vxz-uxz-abpz-bcqx+2puxz \\ &- 2pvxz+2quxz-2qvxz+abpqz+bcpqx-2pquxz+2pqvxz-quxy\alpha \\ &- puyz\alpha+qvxy\alpha+pvyz\alpha+abp^2z+bcq^2x-p^2uxz+p^2vxz-q^2uxz+q^2vxz \\ &+ b^2qx\alpha+b^2pz\alpha). \end{aligned}$$

By (7), we have $a=c$ and $x=z$. Replacing c and z by a and x respectively, we get

$$\begin{aligned} D_\alpha \pi = &-\frac{1}{(bx(1-p-q)+ay(p+q))\,\alpha}(vx-ux-abp-abq+2pux-2pvx \\ &+ 2qux-2qvx+2abpq-2pqux+2pqvx-puy\alpha+pvy\alpha-quy\alpha+qvy\alpha \\ &+ abp^2+abq^2-p^2ux+p^2vx-q^2ux+q^2vx+b^2p\alpha+b^2q\alpha). \end{aligned}$$

Since $x,y<0$ and $a,b>0$, we have $-\frac{1}{(bx(1-p-q)+ay(p+q))\alpha}>0$.
By (6), we have $b=\frac{1-p-q}{\alpha}a$. Substituting for b, we get

$$\begin{aligned} &(vx-ux-abp-abq+2pux-2pvx \\ &+ 2qux-2qvx+2abpq-2pqux+2pqvx-puy\alpha+pvy\alpha-quy\alpha+qvy\alpha \\ &+ abp^2+abq^2-p^2ux+p^2vx-q^2ux+q^2vx+b^2p\alpha+b^2q\alpha) \\ &= (v-u)(py\alpha+qy\alpha+x\,(p+q-1)^2). \end{aligned}$$

Since $x,\ y<0$, we have $py\alpha+qy\alpha+x\,(p+q-1)^2<0$. Thus we obtain that $D_\alpha\pi>0$ if and only if $u>v$.

By lemma 3, we have $D_\alpha\pi>0$ if and only if $\alpha>\underline{\alpha}$. Thus π is increasing in α when $\alpha>\underline{\alpha}$ and decreasing in α when $\alpha<\underline{\alpha}$.
□

A.6 Proof of Proposition 2

It is clear that the contract in case 1 is a special contract in case 2 when $\alpha=1-p-q$, thus the profit in case 1 is a constant which is independent of α. By lemma 4, the

insurer's profit in case 2 is increasing in α when $\alpha > \underline{\alpha}$ and decreasing in α when $\alpha < \underline{\alpha}$. Moreover, by lemma 2, we know $\underline{\alpha} < 1 - p - q$. Since α is non-negative, we have that when $\alpha \in [\alpha^*, 1 - p - q]$ for some $\alpha^* < \underline{\alpha}$, the contract in case 1 is more profitable for the insurer.
□

A.7 The Insurance Problem under Justifiability Constraint

Under the justifiability constraint, there must be a full insurance result ($w_0 - t_a = w_0 - t_b = w_1 - t_c$), and additionally (5) is satisfied. The insurer's profit is therefore

$$(1 - q)t_a + q(w_1 - w_0 + t_a)$$

where t_a is characterized by

$$u(w_0 - t_a) = \frac{p + \alpha}{p + \alpha + q} u(w_0) + \frac{q}{p + \alpha + q} u(w_1).$$

It is clear that the insurer's profit is increasing in α. In addition, if $\alpha = 1 - p - q$, the insurer's profit is the same as the profit for the optimal non-vague contract. Figure 5 depicts the insurer's profit as a function of α.

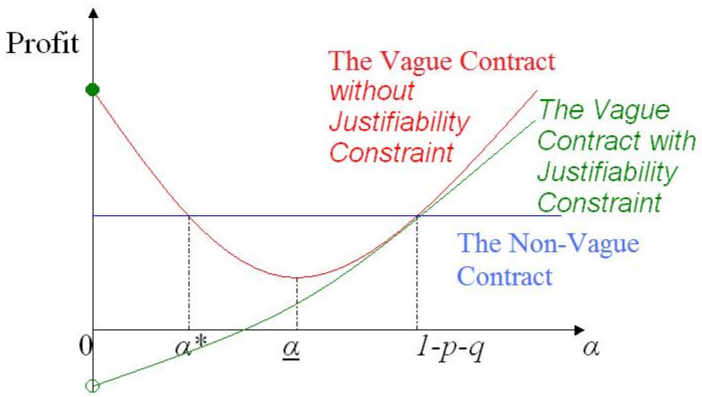

Figure 5: The profit curves for different α

Note that the profit curve with the justifiability constraint is weakly below the profit curve without it because of this additional constraint for the insurer.

However, when $\alpha = 0$, the insuree is completely unaware of the contingency b. Then every optimal contract without justifiability constraint is justifiable, because the insuree is equally insured at the contingencies, which she is aware of. Thus the profit for the vague contract with justifiability constraint is not continuous at $\alpha = 0$. We therefore conclude that under the constraint of contractual justifiability the insurer will announce b in the optimal contract if and only if $\alpha \in (0, 1 - p - q]$.
□

A.8 Proof of Proposition 3

Let $\begin{pmatrix} f(t,p_1,p_2,\alpha) \\ g(t,p_1,p_2,\alpha) \\ h(t,p_1,p_2,\alpha) \end{pmatrix} \equiv \begin{pmatrix} c'(t^*) - q \\ (1-q)u'(w+t^*-p_1^*-c(t^*)) - \alpha u'(w-p_2^*-c(t^*)) \\ qu(w+t^*-p_1^*-c(t^*)) + \alpha u(w-p_2^*-c(t^*)) - (q+\alpha)u(w) \end{pmatrix}$ $= 0$ and $s(\alpha) \equiv \begin{pmatrix} t^* \\ p_1^* \\ p_2^* \end{pmatrix}$ be the solution.

Now we abbreviate notations. Let $c \equiv c'(t^*)$, $k \equiv c''(t^*)$, $a \equiv u'(w+t^*-p_1^*-c(t^*))(1-c'(t^*))$, $b \equiv u''(w+t^*-p_1^*-c(t^*))$, $x \equiv u'(w-p_2^*-c(t^*))$, $y \equiv u''(w-p_2^*-c(t^*))$, $u \equiv u(w-p_2^*-c(t^*))$ and $v \equiv u(w)$.

By implicit function theorem, we have

$$\begin{aligned} D_\alpha s(\alpha) &= \begin{pmatrix} k & 0 & 0 \\ (1-q)b(1-c) - \alpha y(-c) & -(1-q)b & \alpha y \\ qa(1-c) + \alpha x(-c) & -qa & -\alpha x \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ -x \\ u-v \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ \frac{x^2 - y(u-v)}{-bx-aqy+bqx} \\ \frac{-aqx-(u-v)(b-bq)}{-bx\alpha-aqy\alpha+bqx\alpha} \end{pmatrix}. \end{aligned}$$

Since the profit is $\pi^*(\alpha) \equiv qp_1^*(\alpha) + (1-q)p_2^*(\alpha)$, we have

$$\begin{aligned} D_\alpha \pi^* =& qD_\alpha p_1^* + (1-q)D_\alpha p_2^* \\ =& q\left(\frac{x^2 - y(u-v)}{-bx-aqy+bqx}\right) + (1-q)\left(\frac{-aqx-(u-v)(b-bq)}{-bx\alpha-aqy\alpha+bqx\alpha}\right) \\ =& \frac{\alpha qvy - \alpha quy + \alpha qx^2 + bv - bu + 2bqu - 2bqv - aqx - bq^2u + bq^2v + aq^2x}{-bx\alpha - aqy\alpha + bqx\alpha} \end{aligned}$$

Since b, $y < 0$, all other variables are greater than 0, and $q < 1$, we obtain that the denominator $-bx\alpha - aqy\alpha + bqx\alpha = -bx(1-q) - aqy > 0$.

The numerator equals $(v-u)(\alpha qy + b(1-q)^2 + qx(\alpha x - (1-q)a))$. By equation (12), we have $\alpha x - (1-q)a = 0$.

If $\alpha > 1-q$, then, by equation (12), we have $a > x$. Then $u > u'(w+t^*-p_1^*-c(t^*))$. Thus we get $u > v$. We therefore have $(v-u)(\alpha qy + b(1-q)^2) > 0$. Therefore, $D_\alpha \pi^* > 0$. If $\alpha < 1-q$, the same, we obtain $D_\alpha \pi^* < 0$. Lastly, at $\alpha = 1-q$, we have $D_\alpha \pi^* = 0$.

Therefore, π gains its minimum at $\alpha = 1-q$.

□

A.9 Proof of Proposition 4

Firstly, we show the "if" part: Since $0 < \alpha_g < q$, $\alpha_b > 0$ and $v > 0$, we have $\pi_2 > \pi_1$. Because $0 < \alpha_b < 1-q$, $\alpha_g > 0$ and $v > 0$, we have $\pi_1 > \pi_3$. Thus $\pi_2 > \pi_3$. Lastly, since $0 < \alpha_b < 1-q$, $q > 0$ and $v > 0$, we have $\pi_2 > \pi_4$. The contract $C^{\{g\}}$ is therefore optimal for the firm.

Secondly, we show the "only if" part: Since π_2 is the highest profit of all, we have $\pi_2 > \pi_1$ and $\pi_2 > \pi_4$. In addition, $\pi_2 > \pi_1$ implies $\alpha_g < q$, and therefore $\pi_2 > \pi_4$ implies $\alpha_b < 1 - q$.
$\square$

References

[1] Ahn, D. and H. Ergin (2007). "Framing Contingencies," mimeo, University of California at Berkeley.

[2] Benabou, R. and J. Tirole (2002). "Self-Confidence and Personal Motivation," *Quarterly Journal of Economics*, 117(3), 871-915.

[3] Board, O. and K.S. Chung (2007). "Object-Based Unawareness," mimeo, University of Pittsburgh and University of Minnesota.

[4] Chung, K.S. and L. Fortnow (2007). "Loopholes,"mimeo, University of Minnesota and Northwestern University.

[5] Dekel, E., B. L. Lipman, and A. Rustichini (1998a). "Recent Development in Modeling Unforeseen Contingencies," *European Economic Review*, 42, 523-542.

[6] Dekel, E., B. L. Lipman, and A. Rustichini (1998b). "Standard State-Space Models Preclude Unawareness," *Econometrica*, 66(1), 159-173.

[7] Filiz-Ozbay, E. (2008). "Incorporating Unawareness into Contract Theory," mimeo, University of Maryland.

[8] Gabaix, X. and D. Laibson (2006). "Shrouded Attributes and Information Suppression in Competitive Markets,"*Quarterly Journal of Economics*, 121(2), 505-540.

[9] Halpern, J. and L.C. Rego (2006). "Extensive Games with Possibly Unaware Players,"in: *Proc. Fifth International Joint Conference on Autonomous Agents and Multiagent Systems*, 744-751.

[10] Heifetz, A., M. Meier and B.C. Schipper (2006). "Interactive Unawareness,"*Journal of Economic Theory*, 130, 78-94.

[11] Heifetz, A., M. Meier and B.C. Schipper (2008). "Dynamic Unawareness and Rationalizability," mimeo, University of California at Davis.

[12] Heller, D. and R. Spiegler (2008). "Contradiction as a Form of Contractual Incompleteness," *Economic Journal*, 118, 875-888.

[13] Johnson, E. J., J. Hershey, J. Meszaros, and H. Kunreuther (1993). "Framing, Probability Distortions, and Insurance Decisions," *Journal of Risk and Uncertainty*, 7, 35-51.

[14] Kahneman, D. and A. Tversky. (1973). "Availability: A Heuristic for Judging Frequency and Probability," *Cognitive Psychology*, 5, 207-232.

[15] Li, J. (2006). "Information Structures with Unawareness," mimeo, University of Pennsylvania.

[16] Maskin, E. and J. Tirole (1999). "Unforeseen Contingencies and Incomplete Contracts," *Review of Economic Studies*, 66, 83-114.

[17] Modica, S. and A. Rustichini (1994). "Awareness and Partitional Information Structure," *Theory and Decision*, 37, 107-124.

[18] Mullainathan, S., J. Schwartzstein and A. Shleifer (2007). "Coarse Thinking and Persuasion", *Quarterly Journal of Economics*, forthcoming.

[19] Ozbay, E. (2008). "Unawareness and Strategic Announcements in Games with Uncertainty," mimeo, University of Maryland.

[20] Piccione, M. and A. Rubinstein (2003). "Modeling the Economic Interaction of Agents with Diverse Abilities to Recognize Equilibrium Patterns," *Journal of European Economic Association*, 1, 212-223.

[21] Rottenstreich, Y. and A. Tversky (1997). "Unpacking, Repacking, and Anchoring: Advances in Support Theory," *Psychological Review*, 104, 406-415.

[22] Shapiro, J. (2006). "A "Memory-Jamming" Theory of Advertising," mimeo, University of Chicago.

[23] Shavell, S. (2006). "On the Writing and the Interpretation of Contracts," *Journal of Law, Economics and Organization*, 22, 289-314.

[24] Spiegler, R. (2006). "The Market for Quacks," *Review of Economic Studies*, 73, 1113-1131.

[25] Tirole, J. (1999). "Incomplete Contracts: Where Do We Stand?" *Econometrica*, 67, 741-781.

[26] Tirole, J. (2008). "Cognition and Incomplete Contracts," *American Economic Review*, forthcoming.

[27] Tversky, A. and D. Kahneman (1974). "Judgment under Uncertainty: Heuristics and Biases," *Science*, 185, 1124-1131.

[28] Tversky, A. and D. Kahneman (1983). "Extensional versus Intuitive Reasoning: The Conjunction Fallacy in Probability Judgment," *Psychological Review*, 90, 293-315.

[29] Tversky, A. and D. J. Koehler (1994). "Support Theory: A Nonextensional Representation of Subjective Probability," *Psychological Review*, 101, 547-567.

[30] von Thadden, E. (1992). "Optimal Pricing against a Simple Learning Rule," *Games and Economic Behavior*, 4, 627-649.

[31] von Thadden, E. and X. Zhao (2007). "Incentives for Unaware Agents," mimeo, University of Mannheim.

[32] Zhao, X. (2008). "Moral Hazard with Unawareness," *Rationality and Society*, forthcoming.