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THE ALCOHOL PRICE AND THE FLEX CARS

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Abstract

The "flex" car is an innovative automobile that is able to run with either gasoline or alcohol. For flex cars owners, there is perfect substitutability between the two kinds of goods. Differences regarding fuel prices will now depend on the proportions of alcohol, gasoline and flex cars in the total stock. Conversely, the demand for each type of car will also depend on the expected future prices of alcohol and gasoline (in addition to the car prices). We build a model that incorporates this feedback and shows the long term relationships between the two types of energies. It also explains the actual trend for the predominance of flex cars in the domestic market. The model reflects our findings that energy prices are tied in the long run and that causality runs stronger from gasoline to alcohol. The estimated error correction parameter is stable, implying that the speed of adjustment towards equilibrium remains unchanged. The latter result is probably due to a still small fraction of flex cars in the total stock (approx. 5%), despite the fact that its sales nearly reached 100% in 2006.

Keywords: Discrete choice, automobile, alcohol, gasoline, energy.

JEL Classification codes: Q41, Q42, Q43

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1 Introduction

As renewable energies are being developed as an alternative to oil, the understanding of their price behavior becomes increasingly interesting to producers, consumers, policy makers and environmentalists. This paper draws on the dynamics of hydrate alcohol prices (a type of ethanol used by automobiles) in Brazil relative to the price of its close substitute, gasoline. Alcohol and gasoline fueled cars have coexisted in Brazil since the 1980s. However, the Brazilian automobile industry launched an innovative technology in 2003: a car that is able to run on either gasoline or hydrate alcohol (a type of ethanol), but not a mix of both. Because of its flexibility in terms of the energy choice, some car producers have coined the term "flex" for this type of automobile and this has also become their popular name. Hence, we will refer to this type of car simply as flex.

Alcohol automobiles started to be sold in 1980. The degree of substitutability between hydrate alcohol (simply alcohol, hereafter) and gasoline during the 1980s and 1990s was limited: the owner of a gasoline automobile could only use this fuel and the same would apply to alcohol car owners. Price differences between the two types of fuel would induce car substitution. The decision to substitute automobiles would depend on the expected discounted present value of fuel cost savings and price differences between automobiles. Alternatively, a consumer could also adapt the engine of its alcohol car for gasoline and vice-versa in a garage. Nonetheless, this decision would involve not only a sunk cost but also higher costs of depreciation, because of technical problems resulting from the fact that the mechanics of an alcohol car were not designed to use gasoline and vice-versa. Put simply, changing the type of car or the engine in order to benefit from price differences per Km (between the two types of fuel) would involve a sunk cost and take time to happen.

On the other hand, the unique flex car allows for quick fuel substitution, which represents an advantage to the consumer. This seems to be reflected in the sales of new cars as the proportion of flex automobiles grew from 3.7% in 2003 to 21.6% in 2004 and reached 50.2% in 2005. Between 2006 and 2007 sales have increased substantially and are now very close to 100% (which corresponds to approximately two million cars). The share of flex cars in the total stock is also growing. Depending on the production decision of automobile industries, and in the absence of government regulations, this trend might be irreversible. As technology does spread across borders, the flex engine could also reach developed economies relatively soon, especially due to recent government talks (particularly between Brazil and USA) and prospects of a mounting demand for clean and renewable energy. Hence, the new technology could imply important changes concerning the prices of substitute energies to oil derivatives and the future of single fuel engines. For these reasons, our study might also be interesting and relevant for other countries besides Brazil.

Given the current state of technology, either the flex or the alcohol car fueled by alcohol have an inferior performance in terms of Km per liter than a car fueled by gasoline. The advantage for the owner of a flex car, as publicized by the car industry, is that a consumer can choose to buy one or the other type of energy in order to "profit" from differences in price per Km. Intuitively, and disregarding other non-economic motivations,[2] the consumer will decide which fuel to use depending on the price of alcohol relative to the gasoline's weighted by the car's loss of efficiency. A recommendation from the car industry, to ensure economic efficiency, is to buy alcohol only if its price is set below 70% of the gasoline price.[3] This proportion corresponds to the average loss of technical efficiency, as estimated by the car industry, when alcohol is used instead of gasoline.

In the following pages, we explain that this recommendation is rather a corollary of consumer's rational behavior, given that automobile owners are perfectly informed about their car's performance. We have also developed a partial equilibrium model representing the interaction between consumer's demand (for cars and fuel) and an oligopolistic supply side. This model helps to explain the price behavior of the two types of fuel and also the recent trend for an escalating proportion of flex cars in the total sales of new cars.

There are two main motivations for our work: one theoretical and the other empirical. The first relates to the modeling of a) the producer's price decision for a good that presents a discontinuous demand curve; b) the consumer's demand decision for alcohol, gasoline and flex automobiles and c) their interaction in order to determine prices in partial equilibrium. The second motivation refers to the examination of the data generating process of relative prices through the use of econometric procedures. Empirically, the substitutive character of alcohol and gasoline as alternative fuels may imply a cointegrating relationship between their price level and the stationarity of their relative price. If this relationship holds, variables will be tied in the long run and the understanding of the alcohol price cannot be disentangled from the behavior of gasoline. On the other hand, short-run dynamics can provide us with important information concerning the relationship between these prices. In fact, we suspect that the increase of flex cars in the total stock will result in faster adjustment towards an equilibrium relative price. This intuition will be further investigated for the pioneer case of Brazil.

Finally, the objective of this paper can be summarized in four main questions: How can we explain alcohol price dynamics? How can consumers' car buying decision be explained and how does it affect the market for fuel? Has the introduction of flex cars changed the nature of the dynamic process of relative prices? We believe that the answers provided in this paper will be relevant for academics, policy makers and environmentalists.

To the extent of our knowledge, there are no studies in the international literature that attempt to answer the questions posed above. It follows that the contribution of our paper is twofold: 1) the investigation of the short and long run behavior of the relative price and 2) the building of a model that bring interesting insights and explain relevant facts about the Brazilian automobile and fuel market.

The paper is divided as follows: in the next section we present the model emphasizing supply and demand conditions; in the subsequent section we discuss the long run properties of the model. In section 4 we show and discuss the results regarding the data

[2]Such as to avoid negative externalities imposed to others from the pollution of the smoke from gasoline.

[3]Another recommendation is to load the tank with one or the other type of fuel, but not a mix of both.

generating process of the relative price between alcohol an gasoline. Finally, we conclude.

2 The model

Let F_t, G_t and A_t be the fraction of flex, gasoline and alcohol cars in the total stock, respectively. The subscript t stands for time. In what follows, we will model the relationship between alcohol and gasoline prices at time t, based on the values of F_t, G_t and A_t. Instead of modeling this dependence in terms of energy prices, we do in terms of the Km (Kilometer) price of alcohol and gasoline. We define the Km price of alcohol as the monetary amount a consumer has to pay in order to run one Km exclusively with alcohol. An analogous definition follows for the Km price of gasoline. The advantage of proceeding in this manner will become clear in the next paragraph.

Let $p \mapsto D(p)$ be the direct aggregated demand for Km. In order to simplify the exposition, let us assume a monopolistic structure for the alcohol market.[4] Since there is a fraction of F_t flex cars and a fraction of A_t alcohol cars at time t, the alcohol monopolist (who sells Kms) faces the following demand function at time t, given that the Km price of gasoline at time t is g:

$$D_t(a) = \begin{cases} (A_t + F_t)D(a), & \text{if} \quad a \leq g \\ A_t D(a), & \text{if} \quad a > g \end{cases} \tag{1}$$

The demand function $a \mapsto D_t(a)$ in (1) reflects the rational behavior of flex car owners. In fact, as long as a (the Km price of alcohol) does not exceed g (the Km price of gasoline), both alcohol and flex car owners will demand exclusively alcohol (instead of gasoline)[5]. If a exceeds g, only alcohol car owners (i.e. the proportion A_t) will demand alcohol.

Representing $x \mapsto C(x)$ as the cost function of the alcohol monopolist in terms of Km production, the Km price of alcohol at time t will be given by a_* where

$$a_* = ArgMax_a \left\{ D_t(a)a - C(D_t(a)) \right\}$$

Due to (1), we see that a_* is a function of g, A_t and F_t. The function $(g, A_t, F_t) \mapsto a_*(g, A_t, F_t)$ depends on the functions $p \mapsto D(p)$ and $x \mapsto C(x)$. Several choices of $p \mapsto D(p)$ and $x \mapsto C(x)$ lead essentially to the same results that we are going to present. To come to the point, let us suppose that $p \mapsto D(p)$ and $x \mapsto C(x)$ are piecewise linear functions, that is, the former being linear and strictly decreasing for $0 \leq p \leq p_{max}$ and $D(p) \equiv 0$ for $p \geq p_{max}$ and the latter being strictly increasing, where

$$C(x) = cx \quad \text{and} \quad 0 < c < p_{max}$$

[4]This assumption is innocuous for our qualitative analysis. The results of our benchmark monopolistic model in comparison to one assuming localized oligopoly of the Hotelling type, are analogous [Hotelling, H. (1929)]. The assumption is used for simplification purposes as the oligopolistic model is cumbersome to present.

[5]We assume an asymmetric preference for alcohol when $a = g$. This is done just to facilitate the presentation of our results. The same qualitative results can be reached if we assume that the demand for energy of flex car owners is split into 50% alcohol and 50% gasoline when $a = g$.

The condition $c < p_{max}$ above just ensures that the alcohol supply might be positive. Otherwise, if the monopolist marginal costs already exceeds the maximum price of the Km, the model results would be trivial (there would be no supply of alcohol and 100% gasoline cars in the long run).

A straightforward computation leads to $a_*(g, F_t, A_t) = a_*\big(g, g_*(F_t/(F_t + A_t)\big)$ where

$$a_*\big(g, g_*(F_t/(F_t + A_t)\big) = \begin{cases} (c + p_{max})/2 & \text{if} \quad 0 \le g \le g_* \\ g & \text{if} \quad g_* < g < (c + p_{max})/2 \\ (c + p_{max})/2 & \text{if} \quad (c + p_{max})/2 \le g \end{cases} \tag{2}$$

where $g_* = g_*\big(F_t/(F_t + A_t)\big)$, the discontinuity point of $g \mapsto a_*\big(g, g_*(F_t/(F_t + A_t)\big)$, depends only on the proportion of flex cars in the stock of cars that are able to run with alcohol, i.e. $F_t/(F_t + A_t)$. We present g_* explicitly:

$$g_*\big(F_t/(F_t + A_t)\big) = (p_{max} + c)/2 - [(p_{max} - c)/2]\sqrt{F_t/(F_t + A_t)}$$

Due to $p_{max} > c$, the discontinuity point $g_*\big(F_t/(F_t + A_t)\big)$ is decreasing in $F_t/(F_t + A_t)$ as can be seen in Figure 1.

Note that for $0 < g < g_*$, the emerging Km price of alcohol is higher than the Km price of gasoline. For $g_* < g < (c+p_{max})/2$ both Km prices are equal and when $(c+p_{max})/2 < g$ it is more advantageous to use alcohol than gasoline. Thus, a consumer who is going to buy a new car and faces the choices F, G or A takes into account the expected variability of g, car prices and real interest rates.

The car market. One of our main goals is to model the functional dependence between alcohol and gasoline prices in the long term. In the short term (at time t), this functional dependence is already explained by the function $g \mapsto a_*(g, A_t, F_t)$, see (2). In the long run, however, the relationship between alcohol and gasoline prices should take into account the dynamics of (F_t, G_t, A_t). Hence, we need to show how the fractions of flex, alcohol and gasoline cars evolve in time. Suppose that the three types of cars are permanently available at prices Π_F, Π_G and Π_A which corresponds to the marginal cost of each type F, G and A, respectively (this assumption is reasonable if we assume that the supply of cars is structured as a price competition oligopoly of the Bertrand type, where firms have the same marginal costs and produce under constant returns to scale, see Mas-Colell et. al. (1995).

The dynamics of (F_t, G_t, A_t) depends on the consumers *discrete choices* [see Anderson et. al. (1992)] for either flex, alcohol, or gasoline cars. On the other hand, the consumer's discrete choice for a flex, gasoline or alcohol car depends on the expected future Km prices of each type of energy (gasoline and alcohol) during the whole car lifetime.[6]

[6] Even if the car is sold before it is fully depreciated, its value on the market will depend on the type of the car (flex, gasoline or alcohol) and the expected future prices of alcohol and gasoline during the car's remaining lifetime. By fully depreciated, we mean that the car cannot produce any Km out of energy.

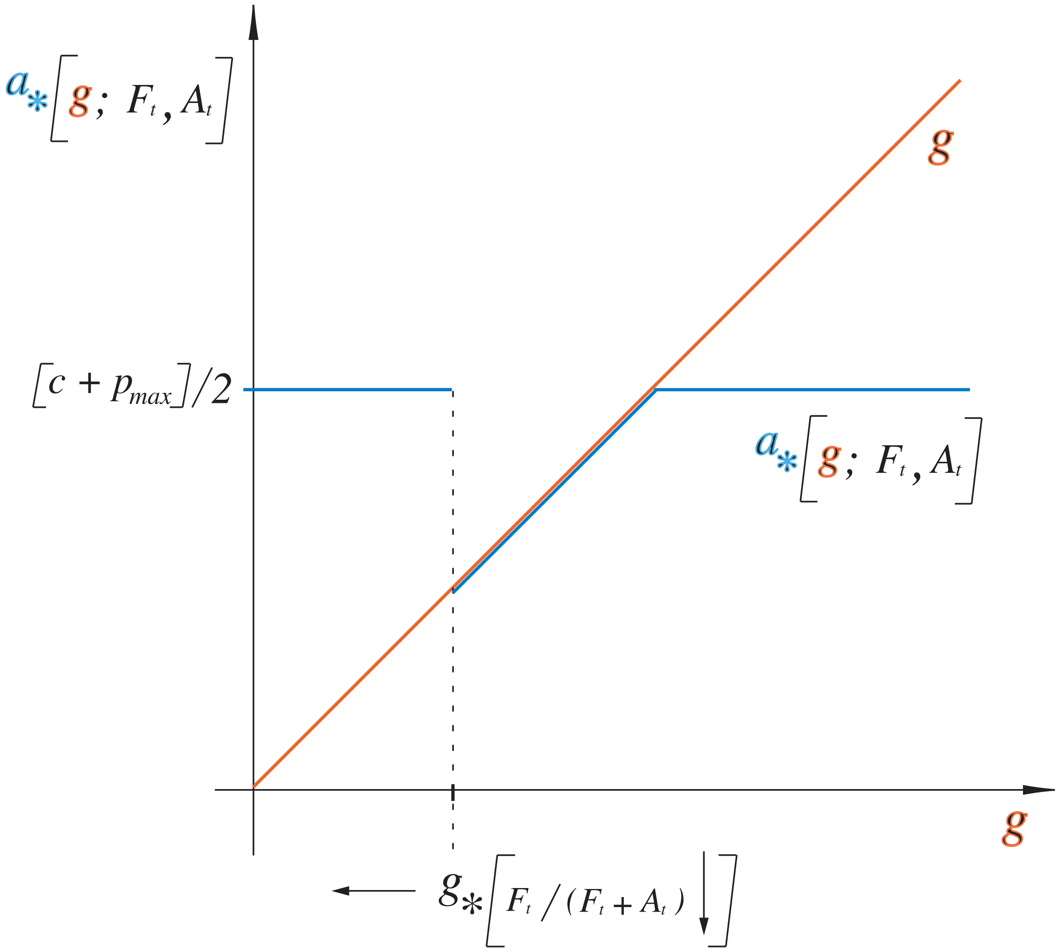


Figure 1: $g \mapsto a_*[\,g, F_t, A_t\,]\ \Big(\ = \ a_*[\,g, g_*(F_t/(F_t + A_t))\,]\Big)$ and $g \mapsto g$

Let us assume a finite population of consumers (car owners) labeled by $1, 2, \ldots n$ which does not vary over time. Suppose that each consumer keeps her car, and does not buy any other, during the whole cars's lifetime.[7] At the moment the car is fully depreciated it is subtracted from its respective stock: F, G or A (F = flex, G = gasoline, A = alcohol). Immediately after having discarded the old (depreciated) car, the consumer buys a new one that is added to its corresponding stock (F, G or A). The consumer uses this new car until it is fully depreciated, and so on.

We assume that all cars lifetimes are continuous, independent and identically distributed random variables, supposedly limited by a positive constant T_{max} (the maximum possible lifetime of a car). This assumption leads to two implications:

1. After the time T_{max}, all consumers have already faced the decision to buy a new car F, G or A (A if available).

2. At each real time $t \geq 0$, there is at most one consumer who changes the car at time t. The time interval between two subsequent car changes has positive range.

Recall that F_t, G_t and A_t are the corresponding fractions of flex, alcohol and gasoline cars at time $t \geq 0$. These fractions are well defined for each t at which no change occurs. For the model to be complete, we need to specify the values of F_t, G_t and A_t when a car change occurs at time t. Taking into account the second implication mentioned above, we define the values of F_t, G_t and A_t by the corresponding fractions immediately after the car change at time t.

Following, we explain which type of car (F, G or A) will be chosen at time t, given that a car change occurs at time t. For this purpose, let us assume that each consumer demands (in average) K kilometers per unit of time. The quantity K is a parameter that measures the long term expected consumption of Kms made by the consumers themselves. If a car is chosen at t, then its type is $x_t \in \{F, A, G\}$, where x_t minimizes the expected discounted costs $C_t(x)$, $x \in \{F, A, G\}$, where

$$C_t(x) = \mathbb{E}_t\Big(\Pi_x + \sum_{s=t+1}^{t+T} (1+r)^{t-s} K \, P_s^{(x)} \Big) \quad x \in \{F, A, G\} \tag{3}$$

$\mathbb{E}_t$ denotes the conditional expectation operator given all the information available at time t, Π_x is the price of the car type x ($x \in \{F, A, G\}$), T is the random life time of a car, $P_s^{(x)}$ the price for one kilometer at time s when the car type is x ($x \in \{F, A, G\}$) and r is the real interest rate.

Let $g_t = P_t^{(G)}$ be the Km price of gasoline at time t. Taking into account that the Km price of alcohol is a function of the Km price of gasoline, we have

$$P_t^{(G)} = g_t, \quad P_t^{(A)} = a_*(g_t, A_t, F_t), \quad P_F^{(t)} = Min\{g_t, a_*(g_t, A_t, F_t)\} \tag{4}$$

[7] This assumption does not impose any restriction in the modeling of the long term relationship between energy prices and the fractions of the cars. In fact, it does not matter whether the original owner sells or not a new car before the end of its lifetime. What is important here is that we consider only the acquisition of new cars and the discards of depreciated ones.

For simplicity, let g_t, $t = 0, 1, 2, \ldots$ be independent and identically distributed continuous random variables (whose variability depends on uncontrolled external shocks in the oil market - this approach can be extended to the case where $\{g_t : t = 0, 1, 2, \ldots\}$ is a stochastic process with memory). Let

$$f \quad \textit{be the density function of } g_0 \tag{5}$$

According to (3), (4), (5) and assuming that consumers do not internalize changes in (F_t, A_t, F_t) in their evaluation [this is consistent with a "myope" collective dynamics of agents, already suggested by Brock, W. and Durlauf, S. (2001) and Moshe, L. (2005)], it follows that

$$C_t(G) = \Pi_G + \lambda^{-1} \int_0^\infty g f(g) dg, \quad C_t(A) = \Pi_A + \lambda^{-1} \int_0^\infty a_*(g, F_t, A_t) f(g) dg$$

and

$$C_t(F) = \Pi_F + \lambda^{-1} \int_0^\infty Min\{\, a(g, F_t, A_t),\, g\} f(g) dg \tag{6}$$

where $\lambda^{-1} = K \sum_{t=1}^{\mathbb{E}(T)} (1+r)^{-t}$.

The computation of $C_t(G)$ stems from the fact that $g_0, g_1, g_2, \ldots$ are independent and identically distributed random variables having density function f and that the probability distribution of the cars' random lifetimes are homogenous in time t and independent of the random variables $g_0, g_1, g_2, \ldots$ Similarly, we deduced the computation of $C_t(A)$ and $C_t(F)$ as indicated in (6).

Let us assume the following preference order for the case when the expected costs coincides:

$$F \succ G \succ A \tag{7}$$

The above preference order means that: F is chosen when $C_t(F) \leq C_t(G)$ and $C_t(F) \leq C_t(A)$; G is chosen when $C_t(G) < C_t(F)$ and $C_t(G) \leq C_t(A)$ and A is chosen when $C_t(A) < C_t(F)$ and $C_t(A) < C_t(G)$.

Let us denote by D_t ($D_t \in \{F, G, A, \}$), the decision of a consumer who chooses a car of any type at time t. According to (2), (6) and (7), it follows that

$$D_t = \begin{cases} F & \text{if} \quad I_1(\gamma_t) \geq \lambda(\Pi_F - \Pi_A) \quad \text{and} \quad I_2 \geq \lambda(\Pi_F - \Pi_G) \\ G & \text{if} \quad I_1(\gamma_t) - I_2 \geq \lambda(\Pi_G - \Pi_A) \quad \text{and} \quad I_2 < \lambda(\Pi_F - \Pi_G) \\ A & \text{if} \quad I_1(\gamma_t) < \lambda(\Pi_F - \Pi_A) \quad \text{and} \quad I_1(\gamma_t) - I_2 < \lambda(\Pi_G - \Pi_A) \end{cases} \tag{8}$$

where $\gamma_t = g_*\big(F_t/(F_t + A_t)\big)$ and $I_1(\gamma_t)$, I_2 are the following integrals:

$$I_1(\gamma_t) = \int_0^{\gamma_t} ((c + p_{max})/2 - g) f(g) dg, \quad I_2 = \int_{(c+p_{max})/2}^\infty (g - (c + p_{max})/2) f(g) dg$$

In order to understand the relations implied by the decision rule (8), observe Figure 2. In this figure, we represent the case where the random variables $g_0, g_1, g_2, \dots$ are independent and identically distributed over the interval $[0, g_{max}]$. Let us measure the price of the Km in unities of g_{max}, that is $[0, g_{max}] = [0, 1]$. The value of the integrals I_1 $(= I_1(\gamma_t))$ and I_2 correspond to the indicated hachured areas in Figure 2. The average price of the Km of a flex car, denoted by P_F, corresponds to the area indicated below the hachured areas. If P_A and F_G are the average prices of the Km of the alcohol and gasoline cars, respectively, we have $P_A = P_F + I_1$ and $P_G = P_F + I_2$. It follows that $P_A - P_F = I_1$, $P_G - P_F = I_2$ and $P_A - P_G = I_1 - I_2$. The first line of decision rule (8) states the following: if the price difference between a flex and an alcohol car is compensated by the corresponding expected savings on Km prices, that is, by the difference $P_A - P_F$ $(= I_1)$, then the consumer will prefer a flex car instead of a alcohol one. If it also holds that the price difference between a flex and a gasoline car is compensated by the corresponding expected savings on Km prices $P_G - P_F$ $(= I_2)$, then the consumer will definitively decide for a flex car. The two subsequent lines of the decision rule (8) can be analogously explained.

Figure 3 illustrates the case when the next consumer decision is a flex car. In Figure 3, the two values of $\lambda(\Pi_F - \Pi_A)$ and $\lambda(\Pi_F - \Pi_G)$ correspond to two additional areas, that are subsets of the two hachured areas in Figure 2. Thus, $\lambda(\Pi_F - \Pi_A) < I_1$ and $\lambda(\Pi_F - \Pi_G) < I_2$, hence the next consumer decision will be a flex car.

Note that the advantage of a flex car over an alcohol one is given by the difference $I_1\big(g_*(\,F_t/(F_t + A_t))\,\big) - \lambda\big(\,\Pi_F - \Pi_A\,\big)$. The latter is represented by the difference of the areas seen on the left side of Figure 3. Since $x \mapsto I_1(g_*(x))$ is a decreasing function, the advantage of a flex over an alcohol car decreases when $F_t/(F_t + A_t)$ increases.

3 The long term behavior

In the previous section we defined a model revealing the links between alcohol and gasoline prices and the fractions of each type of car. In this section, we study the long term behavior of these variables in the model.

First of all note that $I_1(\gamma_t)$ are I_2 are non negative numbers. According to this, the following implication holds:

$$\Pi_F - \Pi_A \le 0, \quad \Pi_F - \Pi_G \le 0 \quad \Rightarrow \quad D_t = F, \ \forall \ t \ge 0$$

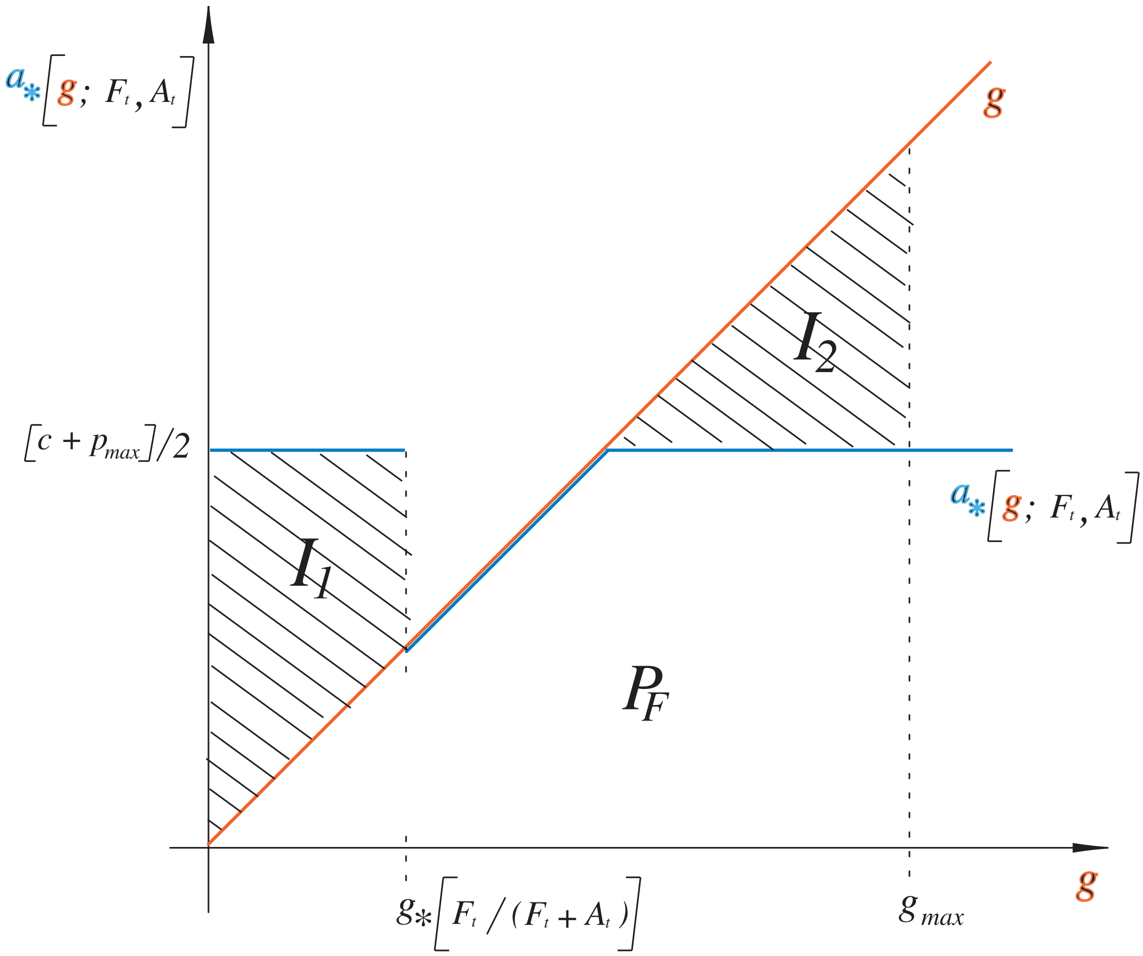
which necessarily implies

$$\Pi_F - \Pi_A \le 0, \quad \Pi_F - \Pi_G \le 0 \quad \Rightarrow \quad \lim_{t\to\infty} (F_t, G_t, A_t) = (1, 0, 0).$$ [8]

A more interesting case is

$$I_1(g_*(1)) < \lambda(\Pi_F - \Pi_A) < I_1(g_*(0)), \quad \Pi_F - \Pi_G \le I_2 \tag{9}$$

[8] Actually, we have: $\Pi_F - \Pi_A \le 0$, $\Pi_F - \Pi_G \le 0 \Rightarrow (F_t, G_t, A_t) = (1, 0, 0)$ for $t \ge T_{max}$.



$$P_A = P_F + I_1 \qquad P_G = P_F + I_2$$

Figure 2: P_F, P_G and P_A are the expected Km prices of flex, gasoline and alcohol cars, respectively.

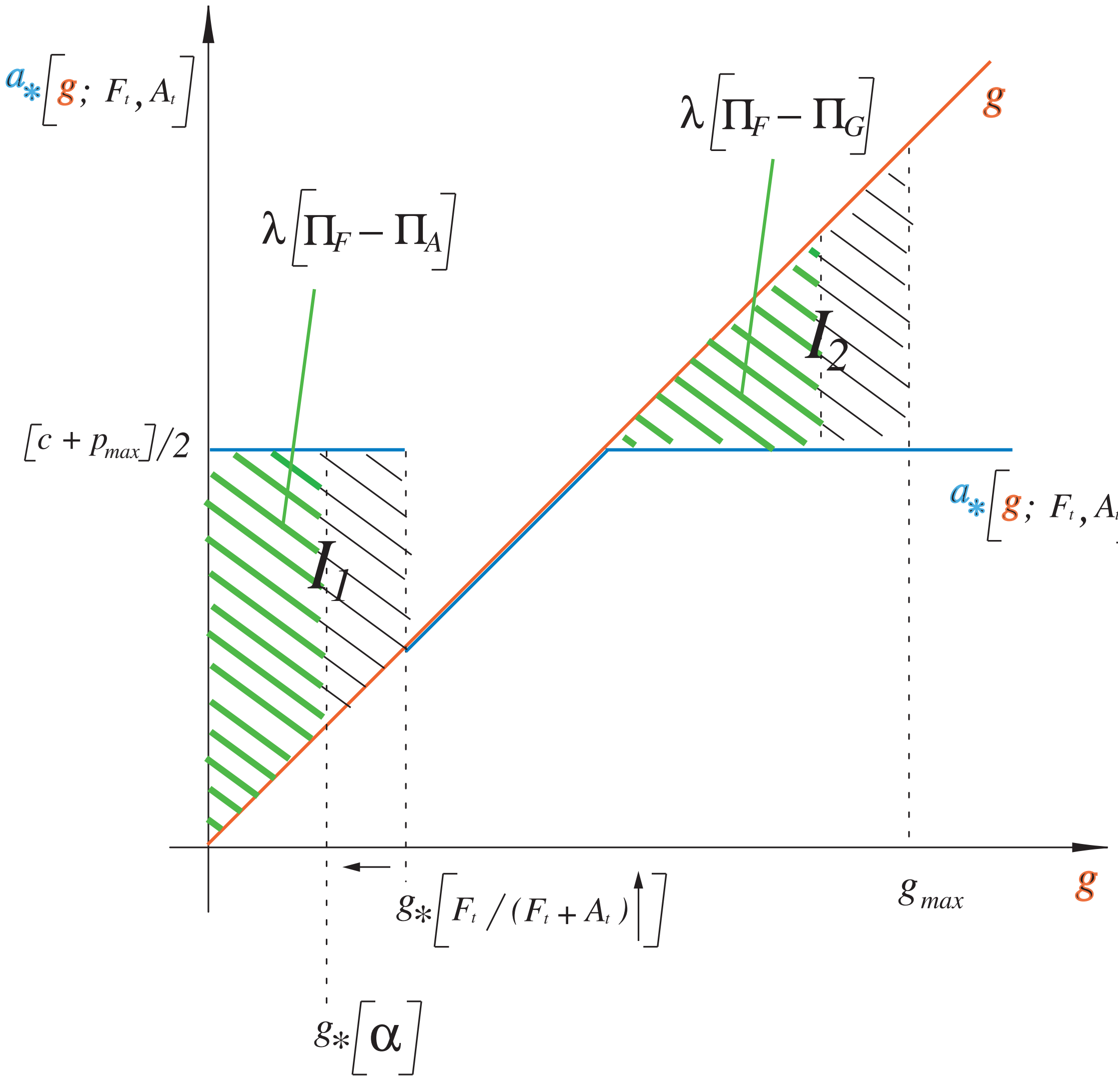


Figure 3: $\lambda(\Pi_F - \Pi_A)$ and $\lambda(\Pi_F - \Pi_G)$ correspond to the left and right small hachured areas respectively. I_1 and I_2 include the two small hachured areas and correspond to the total hachured areas left and right, respectively. A equilibrium is reached when $F_t/(F_t + A_t) = \alpha$.

For case (9), it holds that $G_t = 0$ and $F_t/(F_t + G_t) = F_t$ for $t > T_{max}$, and

$$\lim_{t\to n}\Big[\lim_{t\to\infty} I_1(g_*(F_t))\Big] = \lambda(\Pi_F - \Pi_A)$$

or more specifically: if condition (9) holds, then for $t \geq T_{max}$, we have

$$0 < g_*^{-1}\Big[I_1^{-1}\big(\lambda(\Pi_F - \Pi_A)\big)\Big] - 1/n \leq F_t \leq g_*^{-1}\Big[I_1^{-1}\big(\lambda(\Pi_F - \Pi_A)\big)\Big] + 1/n < 1$$

Let us summarize the asymptotical behavior of (F_t, G_t, A_t) under condition (9): $G_t \to 0$ and $F_t \to \alpha$, $A_t \to (1-\alpha)$. That is, gasoline cars disappears and the fraction of flex cars approaches a proportion α according to which there is no advantage of flex over alcohol cars and vice-versa. The fraction α is defined by $I_1(g_*(\alpha)) - \lambda(\Pi_F - \Pi_A) = 0$, as can be seen in Figure 3. From condition (9), it follows that $0 < \alpha < 1$.

Four possible cases summarize the asymptotical behavior for (F_t, G_t, A_t). Either $\lim_{n\to\infty}\big[\lim_{t\to\infty}(F_t, G_t, A_t)\big]$ equals:

$$(1,0,0),\ (0,1,0),\ (0,0,1) \text{ or } (\alpha,(1-\alpha),0), \quad \text{where } 0 < \alpha < 1$$

Let $(F_\infty, G_\infty, A_\infty) = \lim_{n\to\infty}\big[\lim_{t\to\infty}(F_t, G_t, A_t)\big]$. We present the table below, which shows the car price relations that lead to each value of $(F_\infty, G_\infty, A_\infty)$:

Relation	$(F_\infty, G_\infty, A_\infty)$
$\lambda(\Pi_F - \Pi_G) \leq I_2, \quad \lambda(\Pi_F - \Pi_A) \geq I_1(g_*(0))$	$(0,0,1)$
$\lambda(\Pi_F - \Pi_G) > I_2, \quad \lambda(\Pi_G - \Pi_A) > I_1(g_*(0)) - I_2$	$(0,0,1)$
$\lambda(\Pi_F - \Pi_G) \leq I_2, \quad I_1(g_*(1)) < \lambda(\Pi_F - \Pi_A) < I_1(g_*(0))$	$(\alpha, 0, 1-\alpha)$
$\lambda(\Pi_F - \Pi_G) \leq I_2, \quad \lambda(\Pi_F - \Pi_A) \leq I_1(g_*(1))$	$(1,0,0) \quad (*)$
$\lambda(\Pi_F - \Pi_G) > I_2, \quad \lambda(\Pi_G - \Pi_A) \leq I_1(g_*(0)) - I_2$	$(0,1,0)$

(10)

where $g_*(1) = c < (p_{max} + c)/2 = g_*(0)$ and $I_1(\gamma) = \int_0^\gamma((p_{max} + c)/2 - g)f(g)dg$ and $I_2 = \int_{(p_{max}+c)/2}^\infty (g - (p_{max} + c)/2)f(g)dg$ (c is the marginal cost of the Km for the alcohol monopolist).

3.1 The current state and long term prospects

The actual flow of flex cars in Brazil is approaching 100% of the total sales of new cars. Also, car prices do not vary significantly across types of fuel technology [the "standard" (alcohol and gasoline) and the innovative (flex)]. In terms of our model this corresponds to the case represented by line $(*)$ in the table above. If the relation expressed by line $(*)$ corresponds to reality, that is if $\lambda(\Pi_F - \Pi_G) \leq I_2$, $\lambda(\Pi_F - \Pi_A) \leq I_1(c)$ $(g_*(1) = c)$, then the model would, ceteris paribus, suggest 100% of flex cars in the long run. In this case, the dependence between the Km price of alcohol and gasoline approaches the function $g \mapsto a_*(g, c)$, where c (the marginal cost of the Km for the alcohol monopolist) is the limit of $g_*(F_t/(A_t + F_t))$ when $F_t/(A_t + F_t)$ approaches 1. We notice that $g_*(g, c) \equiv g$ holds for a large range of values of g.[9]

In the empirical part of the paper, we found results that lend support to the conclusion that prices are cointegrated. Hence, it seems natural to assume (in terms of our model) that $g_0, g_1, g_2, \dots$ oscillates over the solution set of $g = a_*(g, g_*(F_{t_0}/(A_{t_0} + F_{t_0})))$, where t_0 correspond to the moment where this investigation was made. Now, the solution set of $g = a_*(g, c)$ (given by the interval $[c, (c+P_*)/2]$) includes all the solutions $g = a_*(g, g_*(\gamma))$ for all fixed values of $\gamma \in [0, 1]$. Thus, if the probability distribution of g does not vary over time, then an even larger part of the support of the probability distribution of g will overlap the solution set of $g = a_*(g, c)$.

The above discussion suggests that the finding of cointegration is robust and will be corroborated as time passes. That is, if the probability distribution of the gasoline price does not change over time, the Km price of gasoline and alcohol tend to coincide in the long run. In other words,

$$P^a/e^a = \textit{Km price of alcohol} = \textit{Km price of gasoline} = P^g/e^g$$

where P_a is the price of alcohol, P_g is the price of gasoline and e^g, e^a stand for the efficiency of gasoline and alcohol, respectively. Efficiency is understood as the generation of Kms per liters of fuel. As indicated in the equation above, the Km price of alcohol and gasoline correspond to P^a/e^a and P^g/e^g, respectively. For example, if $P_g = 2US\$/liter$ and $e^g = 10Km/liter$, then the Km price of gasoline is $0.2US\$/Km$.

Note that $P^a/e^a = P^g/e^g$ is equivalent to

$$P^a = \beta P^g$$

where $\beta = e^a/e^g$.

In the empirical part of the paper we investigate the relation $P^a = \beta P^g$ both in the short and the long run. However, before showing the results of the tests, we will make some important remarks about the dynamics of our theoretical model.

[9] $g_*(g, c) \equiv g$ for $c \leq g \leq (c+P_*)/2$, where c is the marginal cost of Km and p_* is the reservation price of Km. If $0 \leq g < c$ or $g > (c + P_*)/2$, then $a_*(g, c) \equiv (c + P_*)/2$.

3.2 Dynamics and misleading policies

It is interesting to see that the actual trend, characterized by a permanent increase of F_t, is not necessarily irreversible. If the probability distribution of the random variable $g_0, g_1, g_2, \ldots$ changes, the actual trend may revert to a permanent decrease of F_t (and a permanent increase of G_t). In this context, some interventions which could seem favorable to ensure a market for alcohol, can be, in fact, innocuous.

In order to clarify this point, suppose, for example, that the car market is at the long term equilibrium where the flex cars dominate at time $t = 0$ ($F_0 = 1$). Suppose initially that $0 < \lambda(\Pi_F - \Pi_A) < I_1(g_*(1))$, $0 < \lambda(\Pi_F - \Pi_G) < I_2$ and $\Pi_G - \Pi_A = 0$. Let the random variable g_0 (the Km price of gasoline at time $t = 0$) be uniformly distributed over the interval $[0, g_{max}]$. As long as the preceding inequalities hold, we have $F_t = 1$, $t = 0, 1, 2, \ldots$ Now, suppose that at some time t_0 the gasoline price falls dramatically and remains permanently oscillating at a low level, such that g_t assumes another probability distribution for $t > t_*$. Assume a uniform distribution over the interval $[0, g'_{max}]$ with $0 < g'_{max} < g_{max}$, where according to it, the new values of I_1 and I_2 are such that $I_1(g_*(0)) > \lambda\Pi_G > I_1(g_*(1)) > I_2 = 0$.[10] Due to $I_2 = 0$ and $\lambda(\Pi_F - \Pi_G) > 0$, gasoline cars becomes more advantageous than flex cars, and due to $I_1(g_*(0)) - I_2 = I_1(g_*(0)) > 0 = \lambda(\Pi_G - \Pi_A)$ gasoline cars are also more advantageous than alcohol cars. Thus, from $t = t_0$ consumers start to demand only gasoline cars. In this case, gasoline cars would ultimately dominate the market, assuming that no intervention occurs.

In order to guarantee an alcohol market, suppose that the government sharply reduces the tax on alcohol cars (instead of reducing the tax on the flex car or on the consumption of alcohol) such that the alcohol car price Π_A now satisfies $\lambda\Pi_A < \lambda\Pi_G - I_1(g_*(1))$. The effect on the dynamics of (F_t, G_t, A_t) is interesting. The tax reduction (or subvention) on alcohol car price initially halts the demand for gasoline cars, and at the same time drives the demand for alcohol cars up. This might give the impression that this is the correct policy to guarantee the existence of the alcohol market. However, at a certain time t_1 ($t_1 > t_0$) this movement reverts definitively and gives rise to 100% gasoline cars.

The reason for this is the following. Since $\lambda\Pi_A < \lambda\Pi_G - I_1(g_*(1))$ and $I_2 = 0$, we have $I_1(g_*(1)) - I_2 < \lambda(\Pi_G - \Pi_A)$ and alcohol cars becomes more advantageous than gasoline ones. Since $I_2 = 0$, gasoline cars are more advantageous than flex cars. Thus, by transitivity, the tax reduction on alcohol cars will increase the demand for alcohol cars in the first moment. As long as $I_1(g_*(F_t/(F_t + A_t))) - I_2 < \lambda(\Pi_G - \Pi_A)$, flex and gasoline car owners change their cars for alcohol ones. This reduces $F_t/(F_t + A_t)$ even more.

Since $I_1(g_*(F_t/(F_t + A_t))) - I_2$ is decreasing in $F_t/(F_t + A_t)$, there will be a decision time t_1 where $I_1(g_*(F_{t_1}/(F_{t_1} + A_{t_1}))) - I_2 \geq \lambda(\Pi_G - \Pi_A)$.[11] Consequently, the consumer who decides at time t_1 chooses a gasoline car. Now, we do not know (deterministically) what happens with the fraction $F_t/(F_t + A_t)$ at time t_1. If the acquisition of a new gasoline car at time t_1 corresponds to the discard of an old flex car, then this fraction will

[10] In order to have $I_1(g_*(0)) > I_1(g_*(1)) > I_2 = 0$ it is sufficient that $g'_{max} < (c + p_{max})/2$

[11] That $I_1(g_*(F_t/(F_t + A_t))) - I_2$ reaches (or exceeds) $\lambda(\Pi_G - \Pi_A)$ follows from $I_1(g_*(0))) - I_2 > \lambda(\Pi_G - \Pi_A)$, where the latter inequality is guaranteed by $I_1(g_*(0)) > \lambda\Pi_G > I_1(g_*(1)) > I_2 = 0$.

decrease, but if it corresponds to the discard of an old alcohol car it will increase. Thus, in order to understand the dynamics of $(F_t, G_t A_t)$, we must know which types of cars will be discarded from t_1 onwards. Since we started the analysis from a long run equilibrium where the flex car dominates, and since alcohol cars were the ones most recently bought, their remaining lifetimes will be (on average) larger than that of the flex cars. Accordingly, with high probability,[12] the remaining flex cars will be the next ones to be discarded.

Following this argument, at the next car change after t_1, the fraction $F_t/(F_t + A_t)$ will (with high probability) satisfy $F_t/(F_t+A_t) = (F_{t_1}-1)/((F_{t_1}-1)+A_{t_1}) < F_{t_1}/(F_{t_1}+A_{t_1})$. Since $\gamma \mapsto I_1(g_*(\gamma))$ is decreasing, we have $I_1(g_*(F_t/(F_t + A_t))) - I_2 > I_1(g_*(F_{t_1}/(F_{t_1} + A_{t_1}))) - I_2 \geq \lambda(\Pi_G - \Pi_A)$. Thus, the following consumer will also decide for a gasoline car, and so on. After all flex cars have been changed to gasoline ones, say at t_2, the fraction $F_t/(F_t + A_t)$ $(= 0/(0 + A_t)$ remain at zero, whenever $A_t > 0$. Thus, after t_2, we have$I_1(g_*(F_t/(F_t + A_t))) - I_2 = I_1(g_*(0)) - I_2 > \lambda(\Pi_G - \Pi_A)$ leading to consecutive changes of the remaining alcohol cars to gasoline ones.

4 Empirical results

This part of the paper aims to shed some light on the actual behavior of prices. Alves and Bueno (2003) is the closest to our study in the international literature. Their work, which draws on Ramanathan (1999), focuses on the estimation of the demand function for gasoline. As explained by Alves and Bueno (2003), Brazil is the only major economy with a significant renewable substitute fuel to gasoline. Hence, one of the determinants of gasoline consumption is the alcohol price. Using annual data from 1984 until 1999 and the natural logarithm of gasoline per capita consumption as the dependent variable, they have estimated the cross price elasticities between gasoline and alcohol. Alves and Bueno (2003) found that the long run cross price elasticity was low (0.48) and interpreted these results as evidence that the degree of substitution between both goods was restricted. Their conclusion was "Although the cross-elasticity is positive, its absolute value is low. This is explained by the relatively high costs associated with changing from automobile engines from gasoline-fuel to alcohol-fuel" [Alves and Bueno (2003), p. 196]. As explained in the introduction, the flex technology has eliminated these costs.

Our empirical investigation consists in first applying unit root tests on the natural logarithm of the price levels and their first differences, in order to investigate the order of integration of these variables. Unit root tests on relative prices are performed, because if alcohol and gasoline are cointegrated, their relative price should be well approximated by a stationary process. Second, we tested for cointegration using the Engle and Granger (1987) "two step" procedure and the maximum likelihood tests proposed by Johansen (1991).

[12] We must make this statement in terms of probability, because car lifetimes are random variables. So, there is a small probability that the next discarded car is an alcohol one (instead of a gasoline). This causes a delay in the convergence of the proportions to (nearly) 100% of flex cars. An interesting question, regards the speed at which the process reaches the equilibrium (close to 100% of flex cars).

We have used two datasets in our tests. The first comprises higher frequency data[13] of **producer prices** compiled by two institutions. Alcohol data is provided by Cepea, the Centre for Advanced Studies on Applied Economics of the University of São Paulo. Gasoline was obtained with the Brazilian National Agency of Petrol (ANP). Cepea calculates the weekly average price of one liter of (hydrate) alcohol sold by the producer in the state of São Paulo while ANP compiles and provides average prices of one liter of gasoline sold by producers and importers in the southeast region of Brazil.[14] Data is for gasoline "A", which is not mixed with an additive, made out of ethanol. Hence, we avoided endogeneity by definition, since the gasoline sold in for the final consumer in Brazil (gasoline "C") is mixed with a proportion of 20% of ethanol. Both prices are without state taxes. Weekly data on prices from the first week of 2002 until the last week of 2006 were monthly averaged, hence, our period using high frequency data spans from 2002M1 until 2006M12. This is the largest sample period available for monthly data.

The second dataset was obtained from IPEA, the Institute of Applied Economic Research, which belongs to the Federal Government of Brazil. This is a low frequency, annual data of **consumer prices**, ranging from 1979 to 2004, which also corresponds to the longest period available. There are other related studies that also go along the lines of Ramanathan (1999) and Alves and Bueno (2003) in the domestic literature. They concentrate on the estimation of structural models for the supply and demand for gasoline and alcohol [for example, Marjotta-Maistro (2002)].

At this point, we need to make some important remarks about the database. Both the alcohol and gasoline markets were strongly controlled by the government until the end of the 1990s. In fact, cointegration before 1996 followed almost by definition as the market price of alcohol was set by the Federal government as a percentage of the gasoline price, which, in its turn, was also determined by the government. Because prices moved together for most part of the period spanning from 1979 until 1996, cointegration tests for that specific period would be meaningless. Gasoline and alcohol consumer prices were liberalized in 1996 and 1999, respectively. However, part of the market chain was still controlled by the government, especially gasoline producer prices, until 2002M1. The whole chain was liberalized from that month onwards, explaining the beginning of the higher frequency database.

Although we performed unit root tests using the annual dataset of relative prices, we must alert that results should be read with caution. On the other hand, we also understand that the sample period of the higher frequency data might be considered too small. However, we must point out that there is no objective criterion established for a

[13] The findings concerning the monthly series are based on data that is not seasonally adjusted. However when we use the seasonally adjusted one, using centered dummies, the results presented in the paper are qualitatively the same. They can be obtained with the corresponding author upon request.

[14] These prices are largely the same ones sold by producers (for distributers) located in the state of São Paulo. The southeast region of Brazil comprises four states, with São Paulo being the one with the most economic importance. Furthermore, Petrobras is the main producer and prices without state taxes vary by cents of Reais (the Brazilian currency), which corresponds to less than 1% of the total price. Furthermore, gasoline price increases across Brazilian states are analogous. Hence, we have annual prices of alcohol and gasoline producers for the state of São Paulo.

minimum period for cointegration tests to be meaningful, especially if we do not know for sure the length of the price cycles. We must stress that the choice of the databases, including sample period and frequencies, was exclusively determined by data availability. Although the database is not ideal, we understand that our tests can be very helpful to understand price dynamics during the liberalized period. The annual results serve to compare the controlled with the freed data generating process. If the reader is unconvinced by the empirical results of cointegration that are later presented and discussed, we redirect to the arguments put forward in the theoretical model, given its assumptions.

We first investigated the stationarity of the series. Table 2 presents the results of unit root tests. ADF statistics, estimated using monthly data, imply that the level of the natural logarithms of prices is $I(1)$ whereas its first difference is $I(0)$. Given the low power of the ADF tests, the result of a stationary alcohol and gasoline inflation is robust. In other words, if the unit root hypothesis is rejected using the ADF test, one can be confident that the series do not contain a trend. On the other hand, the $I(1)$ price level is in line with the findings of the related literature [see Alves and Bueno (2003) and, for oil prices, Cologni and Manera (2007)].

However, tests using annual data were not able to reject the null of a unit root either for the level or for the first differences. The latter result is not puzzling after a visual inspection of Graph 3 which plots the annual relative price. It is possible to note a break in the intercept during the end of the nineties coinciding with the liberalization of the alcohol prices to the consumer. As explained by Perron (1989), a date break could imply the non-rejection of the null of a unit root when the series in question is indeed stationary. The apparent break around 1997 contrasts with the no evident rupture in the graph using monthly data (Graph 1). Our suspicion of a break-level was confirmed using Perron (1997) tests, which have retrieved significant intercept dummies for the corresponding break dates. Perron (1997) test finds the date break endogenously (in our tests we used the method that maximizes the possibility of rejecting the null of a unit root) and is a more powerful complement for the ADF test when the series in question contain a break. Perron (1997) results, as shown in Table 3, lend support to the conclusion that first differences are stationary whereas the level is non-stationary.

Our first set of cointegration results are the stationarity tests performed on the relative price

$$\beta = P^a/P^g$$

where P^a and P^g represent the price level of alcohol and gasoline, respectively. If prices are cointegrated, then their relative price should be well approximated by a stationary process, i.e. the series would converge to an equilibrium. The advantage of this test is that it precludes any knowledge of the order of integration of the log-level variables. In Table 3, we present these results. Using monthly data, we found strong evidence of a stationary relative price and hence, cointegration between alcohol and gasoline. The equilibrium level estimated for this period [0.52] is below the [0.70] implied by the suggestion of the automobile industry. The explanation might be on the fact that alcohol cost structure is lower than gasoline's, and that demand has not yet pushed prices up. An investigation about the stability of the root can be found in Graph 4. The root seems to increase steadily after the end of 2003 and then becomes stable. Although this result apparently contradicts our intuition - that the flex will generate higher speed of convergence - it must be said that the small number of observations is probably driving these findings (since we are working with recursive estimates) and that the parameter is converging to its true conditional value. However, it must also be noticed that the share of flex cars in the market, although growing exponentially, is still small.

On the other hand, stationarity checks using annual data also suggest that there is cointegration between the two variables after the liberalized period. A plot of the relative price can be seen in Graph 3 and informal analysis would lead us to suspect that a break occurred at around the end of the 1990s. This contrasts with the inspection using monthly data in which no apparent break can be observed (see Graph 2). For these reasons, we performed Perron (1997) unit root tests on the relative price and found strong support for stationarity, which, as explained, can be interpreted as evidence of cointegration between alcohol and gasoline. The date break also coincides with the initial period of liberalized markets. For comparison, in Table 2 we also show the estimates of the root and equilibrium value for the whole period and also from 1999 onwards (when alcohol prices were liberalized).

We also examined the existence of cointegration by performing Engle and Granger (1987) tests on the log-level of prices. We first estimated an autoregressive distributed lag (ARDL) model. Lags were chosen to maximize the Schwarz Bayesian Criterion (SBC) criterion. Using the price of alcohol as the dependent variable, this criterion retrieved an ARDL (2,0). We also used the calibrated conservative strategy of the PcGets [see Hendry and Krolzig (2005) and Krolzig and Hendry (2001)] in order to check for the lag dynamics. It retrieved the same lag structure as the SBC, which tends to select a more parsimonious representation in comparison to the Akaike criterion, for instance.

After estimating the ARDL(2,0) model, we investigated causality between the two series based on Granger et al (2000)

$$\Delta p_t^a = \alpha_0 + \sigma_1(p_{t-1}^a - \hat{\beta}p_{t-1}^g) + \sum_{i=1}^{k} \alpha_{1i}\Delta p_{t-i}^a + \sum_{i=1}^{k} \alpha_{2i}\Delta p_{t-i}^g \tag{11}$$

$$\Delta p_t^g = \phi_0 + \sigma_2(p_{t-1}^a - \hat{\beta}p_{t-1}^g) + \sum_{i=1}^{k} \phi_{1i}\Delta p_{t-i}^a + \sum_{i=1}^{k} \phi_{2i}\Delta p_{t-i}^g \tag{12}$$

where p^a and p^g are the natural logarithms of P^a and P^g (the price levels of alcohol and gasoline, respectively). The subscript t stands for time and the Greek letter Δ represents first differences. The letters α_{1i}, α_{2i},ϕ_{1i}, ϕ_{2i} are coefficients, whereas α_0 and ϕ_0 are the intercepts. The term in parenthesis is the estimated (long run) cointegrating relationship from the ARDL model lagged in one period, in other words it is the lagged error correction mechanism; σ_1 and σ_2 are the speeds of adjustment and cointegration implies $|\sigma_1| + |\sigma_2| > 0$. Evidence of causality (in Granger sense) running from gasoline to alcohol prices implies rejecting $H_{01} : [\alpha_{21} = \alpha_{22} = \cdots = \alpha_{2k} = 0 \text{ and } \sigma_1 = 0]$, whereas if the hypothesis $H_{02} : [\phi_{11} = \phi_{12} = \cdots = \phi_{1k} = 0 \text{ and } \sigma_2 = 0]$ is rejected, then alcohol would

Granger-cause gasoline prices. If we reject both hypotheses, then there is bi-directional Granger-causality.

We do not show the results of the error correction representation of the ARDL (2,0) because in Table 4, we present estimates of the Granger causality tests, which provide information on both the short-run dynamics and the long-run ties between these two variables. Following the Engle and Granger (1987) two-step procedure, we performed unit root tests on the residuals of the estimated ARDL (2,0) and found that they were stationary. As seen in the bottom of the first part of Table 4, the ADF test statistic with one lag (which was chosen using a general to specific approach) is -5.81, a value that is highly significant (for instance, it is significant at the 1% critical level).

As can be seen in Table 4, there is support for cointegration since the parameter $\sigma_1$associated with the error correction variable (ECM_{t-1}) is also highly significant. For instance, about 30% of the deviations from the long run equilibrium relationship are eliminated every month by changes in the alcohol price. This parameter is relatively stable as can be seen in Graph 5, which allow us to conclude that the escalating flow of flex cars has not yet changed the price adjustment dynamics. One possible explanation is that the share of flex is still small (approximately, five per cent in 2006 , although this type of car dominates the automobile sales in Brazil).

Granger causality is found in one direction as implied by the Wald tests reported at the bottom of each part of Table 4. The value of the Wald test statistic is such that we were able to reject the null hypothesis of zero coefficients at the 1% significance level for the lags of gasoline in the alcohol equation. On the other hand, alcohol does not cause gasoline in the Granger-sense. This evidence lends support to the assumption that gasoline price is not determined by alcohol but is possibly exogenously decided in international commodity markets (as it depends on oil prices).

As there might be more than one cointegrating vector in the bivariate case, the standard theory suggests applying Johansen (1991) tests. These tests consist in estimating the matrix of coefficients from the structural model, which represent a feedback relationship between alcohol and gasoline prices. If the rank of this matrix is zero, i.e. there are no independent cointegrating vectors, the matrix is null and (11) and (12) will become a vector autoregression model in first differences. Our final set of results, based upon Johansen (1991), is shown in Table 5. The general conclusion corroborates the previous findings using Engle and Granger (1987) and the stationarity tests on relative prices. For instance, results using monthly data in Table 5 reveal that the values of both the maximum eigenvalue statistic and the trace statistic are able to reject the null hypothesis of no cointegrating relationship for the alternative of at least one. However, the hypothesis that there are two cointegrating vectors is rejected.

Overall, we found that prices are cointegrated and that causality (in the Granger sense) runs stronger from gasoline to alcohol. Hence, the conclusion is that relative prices are stationary which is also corroborated by our unit root tests. This means that there might be a higher degree of substitutability between both fuels than the one found by Alves and Bueno (2003). The speed of convergence to equilibrium remains stable as suggested by the recursive analysis. These stability checks allowed us to conclude flex cars have not yet changed price dynamics in a significant way.

5 Concluding Remarks

We performed a thorough analysis on the behavior of the relative price of alcohol and gasoline and the consumer's demand decision for automobiles when faced by two or more types of fuel technology. Theoretically, we filled a gap in the literature as we developed a model that took into account supply and demand conditions in both the fuel and car markets. We analyzed the interaction between the proportions of cars and demand for fuel, showing that alcohol and gasoline prices are tied in the long-run. Empirically, our work complemented the studies that have estimated demand elasticities for fuel and, especially in the case of Brazil, the cross price elasticity for alcohol. We shed light on the short and long run relationships between alcohol and gasoline prices.

A straightforward conclusion emerged from both the theoretical model and the empirical analysis regarding the dynamics of the alcohol price. The alcohol price cannot be understood if disentangled from the behavior of gasoline and also from the current state of the technologies that transform energy into Kms.

The model also captures the trend for the predominance of flex cars in the market. Interestingly, when all cars are flex, the model predicts that the relative price of alcohol will be a constant, equal to the technical marginal rate of substitution between the two goods (given that the gasoline price is neither too high nor too low).

According to our model a consumer buys a flex car in order to "benefit" from price differences. On the other hand, the increase on the proportion of flex cars will eliminate these differences. Then what will be the benefit of having a flex car in terms of economic efficiency in the long run? Our model solves this puzzle by showing that it is always going to be advantageous to have a flex car, assuming the price for all types of cars are equal and that there is a reasonable variation in gasoline prices. However, if the price of a flex car substantially exceeds the price of a standard automobile, then the model predicts a convergence to an equilibrium fraction for each type of car, implying that they will coexist.

Other interesting questions arise for future works. For example, what is the impact of the growing flex trend on society's welfare, if the car has a small technical disadvantage over standard ones when fueled with either gasoline or alcohol? On an empirical level our work suggests that new estimates of the cross-price elasticity will be interesting, especially when the proportion of flex cars increase considerably.

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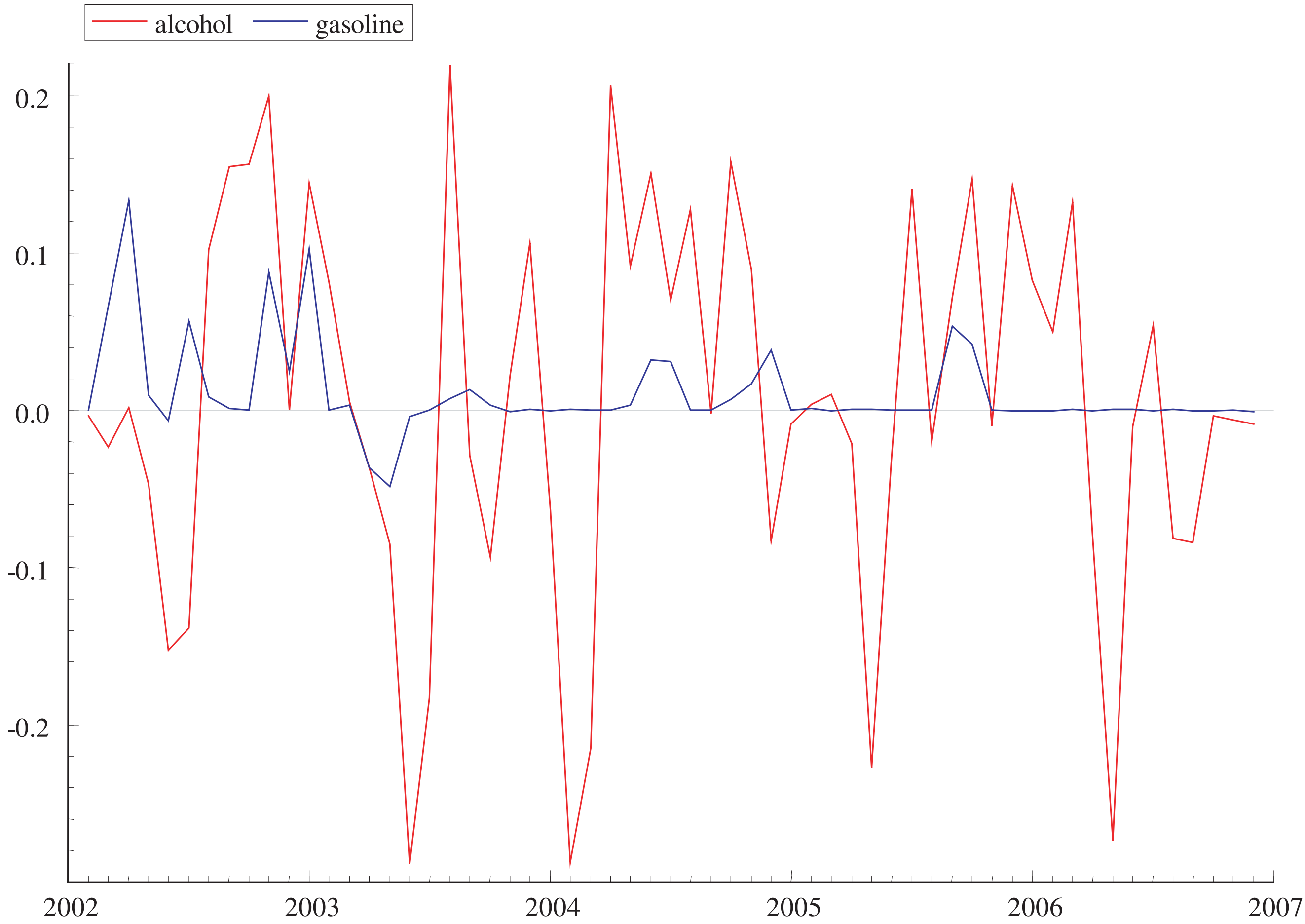
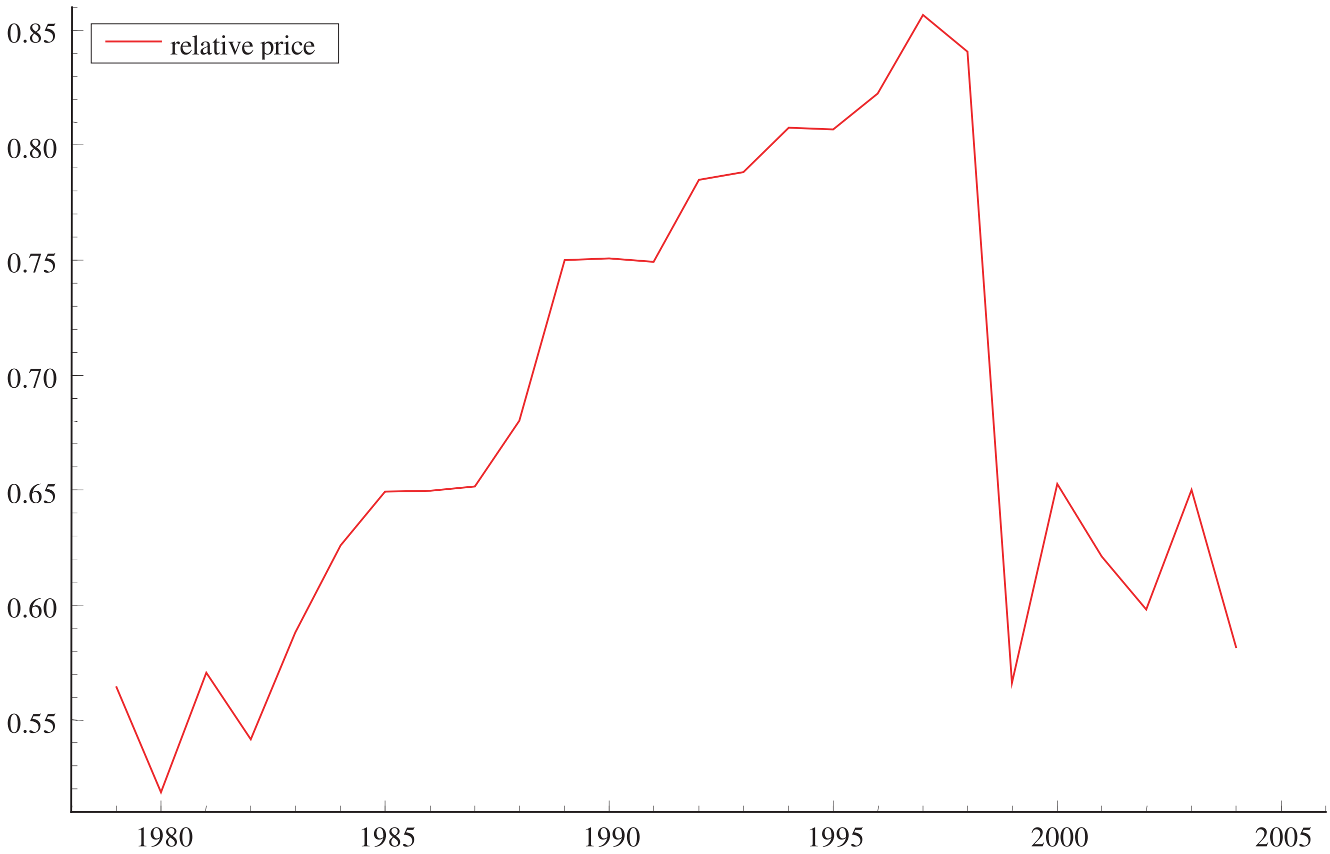
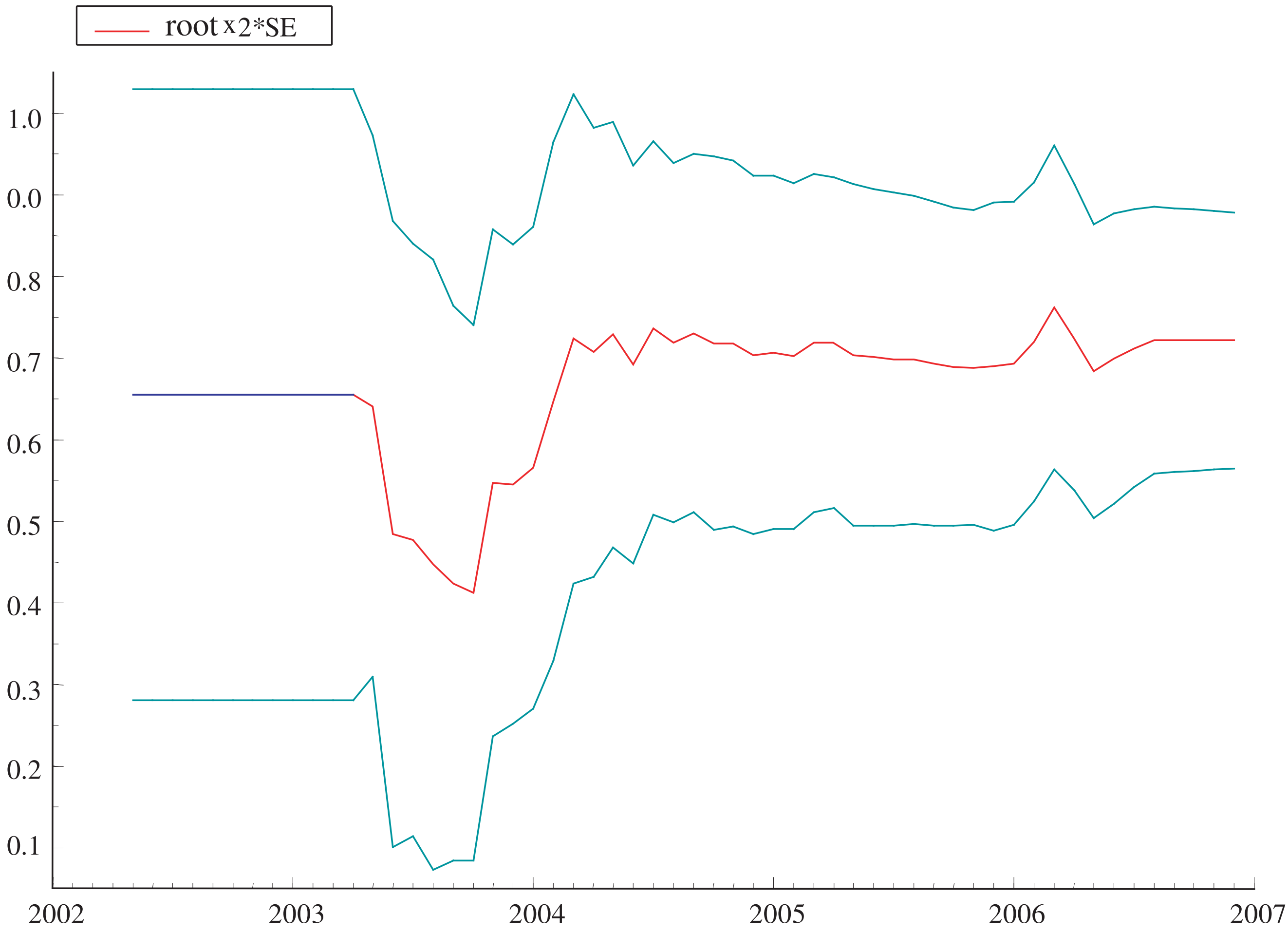


Graph 1: **Monthly Prices (first difference of the log-level).**

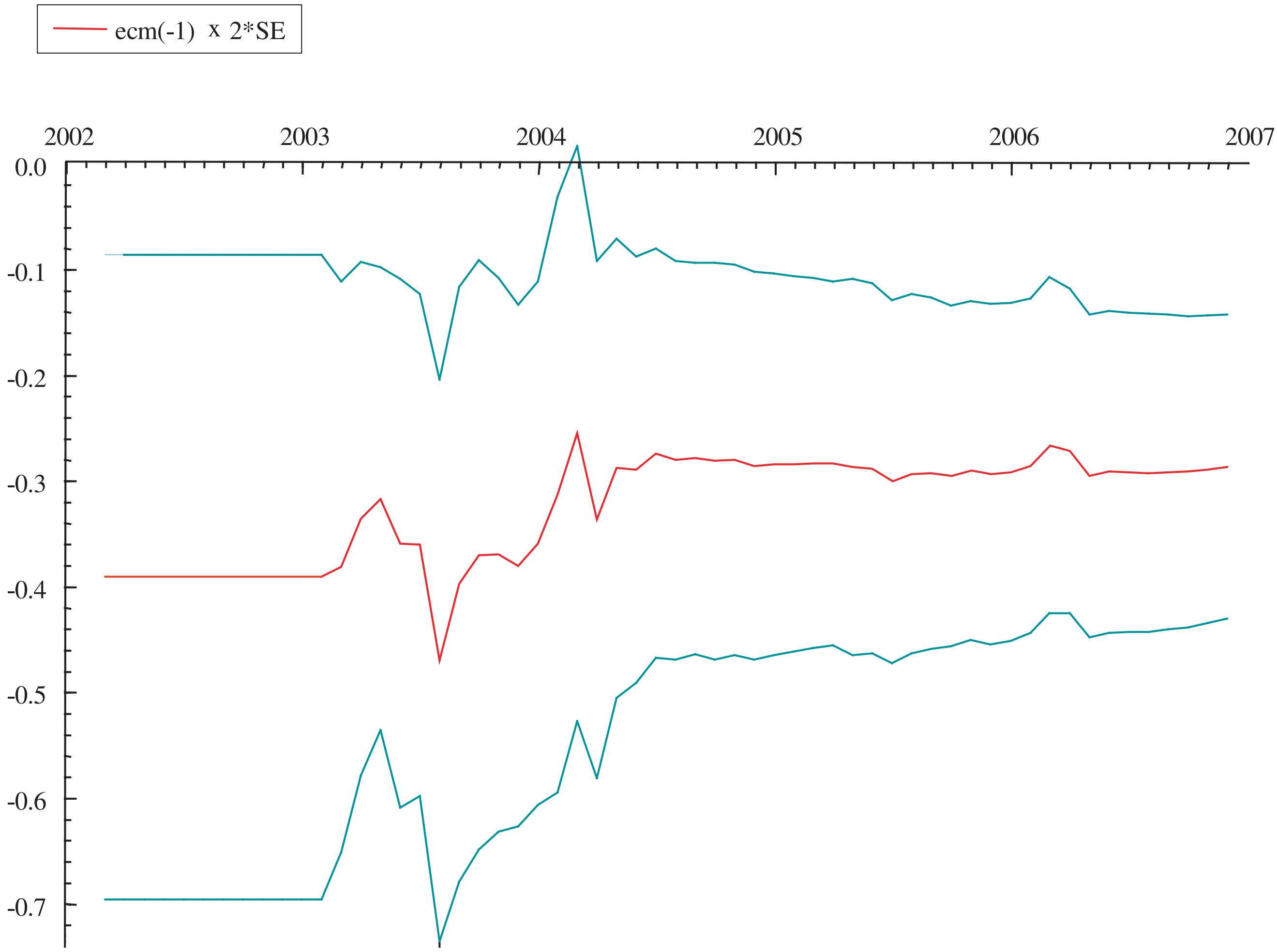
Graph 2: **Relative Prices - monthly data.**



Graph 3: **Relative Prices- annual data.**



Graph 4: **Stability of the Root - monthly data.** Forward recursive estimates, initial number of observations 12.



Graph 5: **Stability of the speed of adjustment σ_1.**

	Monthly data		Annual Data	
Variable	Nº of Lags	ADF statistic	Nº of Lags	ADF statistic
p^a	3	-2.49	1	-1.45
p^g	6	-1.95	1	-1.37
Δp^a	12	-3.15*	0	-1.93
Δp^g	12	-3.96*	0	-1.95

Table 1: **ADF Unit Root Tests.** Lags were selected through a general to specific approach. The variables p^a and p^g represent for the natural logarithm of the price level of alcohol and gasoline, respectively. The Greek letter Δ stands for the first difference. $*$ indicates rejection of the null of a unit root at the 5% confidence level.

	Nº of Lags	ADF statistic	Root	Half Life**	Intercept	Equilibrium
Monthly	3	-3.50*	0.72	≅ 2.1 months	0.15	0.52
Annual	0	-1.73	0.78	---	0.15	0.68
From 1999	0	-11.49*	-0.29	---	0.80	0.62

Table 2: **Relative Prices.** Lags were selected through a general to specific approach. The variables p^a and p^g represent for the natural logarithm of the price level of alcohol and gasoline, respectively. The Greek letter Δ stand for the first difference. $*$ indicates rejection of the null of a unit root at the 5% confidence level. $**$ half-life is calculated as $\ln(1/2)/\ln$ (estimated root).

Variable	Nº of Lags	t-statistic	Break Date
β	2	-9.12*	1997
p^a	1	-2.04	1988
p^g	1	-2.17	1988
Δp^a	0	-6.66*	1993
Δp^g	0	-6.74*	1993

Table 3: **Perron (1997) Unit Root Tests Using Annual Data.** Lags were selected through a general to specific approach. The variables p^a and p^g represent the natural logarithm of the price level of alcohol and gasoline, respectively. The Greek letter Δ stands for the first difference. Date breaks were chosen according to the method in which the t-statistic to test the null of a unit root is minimized. $*$ indicates rejection of the null of a unit root at the 1% confidence level.

Error Correction Model obtained from the ARDL(2,0), using Δp^a to estimate the cointegrating relationship (ECM_t)				
Dependent Variable	Regressor	Coefficient	Std. Error	p-value
Δp^a	Intercept	-0.23	0.07	[.003]
	Δp^a_{t-1}	0.41	0.13	[.003]
	Δp^a_{t-2}	-0.07	0.14	[.579]
	Δp^g_{t-1}	0.47	0.54	[.388]
	Δp^g_{t-2}	0.41	0.53	[.439]
	ECM_{t-1}	-0.29	0.07	[.003]
ADF tests on the residuals of the ARDL(2,0), ADF(1) = -5.81 , 95% critical value = -4.306				
Wald test of restrictions imposed on the coefficients Δp^g_{t-1}, Δp^g_{t-2} and ECM_{t-1}: Wald Statistic, χ^2 (3) = 11.465[.000]				

Dependent Variable	Regressor	Coefficient	Std. Error	p-value
Δp^g	Intercept	0.01	0.02	[.700]
	Δp^a_{t-1}	0.08	0.03	[.024]
	Δp^a_{t-2}	-0.03	0.03	[.354]
	Δp^g_{t-1}	0.28	0.13	[.038]
	Δp^g_{t-2}	-0.05	0.13	[.704]
	ECM_{t-1}	-0.00	0.02	[.991]
Wald test of restrictions Imposed on the coefficients Δp^a_{t-2}, Δp^a_{t-1} and ECM_{t-1}: Wald Statistic, χ^2 (3)= 2.563[.464]				

Table 4: **Granger et al (2000) Causality Tests using Monthly Data.**

Null Hypothesis	Alternative Hypothesis	Statistics	5% Critical Value	1% Critical Value
		λ_{max}		
$r = 0$	$r = 1$	**24.53**	15.67	20.20
$r \leq 1$	$r = 2$	8.77	9.24	12.97
		λ_{trace}		
$r = 0$	$r \geq 1$	**33.29**	19.96	24.60
$r \leq 1$	$r = 2$	8.77	9.24	12.97

Table 5: **Johansen Cointegration Tests using Monthly Data.** 58 observations were used to estimate a second order VAR. The assumption is that there is no deterministic trend in the data. Numbers in bold mean rejection of the null at the 1% critical level. The letter r is the number of hypothesized cointegrating equations. λ_{max} and λ_{trace} are the maximum eigenvalue statistic and the trace statistic, respectively.