Volatility forecasting and sign changes in currency returns

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This paper aims at finding the relation between the shifts in volatility and the changes in sign predictability for the daily changes in Euro/USD quotes. Previous research (Fatih Yilmaz, Sweet spots for directional trading, Bank of America) found certain relations between these two variables and identified optimal Sharpe ratio levels around 1.4 for a strong use in forecasting. The objective of this paper is to model the relation between these two variables in order to provide a dynamic process and then to test this process on real currency data.

The changes in currency rates are very important for a Forex trader. Besides forecasting the conditional distribution of the size of the future currency changes, a great deal of focus has been done in analyzing the changes in signs. Forecasting the sign probability of the future rate could provide us a powerful tool for taking currency positions.

The models used for the returns of the financial assets are constructed based on the stylized facts observed by many analysts, Cont (2001) among others. They are basically highlighting the facts that financial returns have fat tailed distributions and negative skewness generally, and also that the autocorrelations of returns are not statistically significantly different from 0, while the autocorrelations of volatilities are very significant up to a number of lags. These observations fed the need for models from the GARCH family and then the stochastic volatility. Hsieh (1991) looks at the power of these two models to filter the nonlinearities out of the data for a series of financial returns by using the BDS test to check for the randomness of the residuals. He found out that the stochastic volatility model performs better than the GARCH even if it is harder to compute. However the resulted benefit added by the stochastic volatility may not explain the need to use it against the GARCH due to the computation problems.

Lupu (2006) provides an analysis for the use of the jump-diffusion model as in Maheu and McCurdy (2003) for US dollar versus Euro rates by using the same BDS test.

In a recent paper Yilmaz (2007) explains that for an active manager, directional predictability necessitates ex-post returns (after adjusting for risk, skew and tails) that are better than a buy and hold strategy. This means that actively managed returns need to dominate underlying unconditional returns at the distributional level.

We will use the stochastic volatility model to filter the nonlinearity out of the data and then use the BDS test for the residuals in order to see if our model succeeded in extracting all the non-linear relations in the data. The resulting residuals will be used in a future paper to check if the stochastic volatility model can be used to forecast the sign of the returns.

1. The stochastic volatility model (SVOL)

The specification of the stochastic volatility model is the same as in Hsieh (1991). The log of the standard deviations of the returns follows an AR(p) process where p will be determined by an Akaike information criterium.

$$r_t = \sigma_t z_t$$
$$\log \sigma_t = \beta_0 + \sum_i \beta_i \log \sigma_{t-i} + v_t$$

where v_t follows a standard normal distribution. Hsieh (1991) evaluated this model as an alternative to the ARCH family models in the sense that it allows for the innovations v_t in the variance process. Hsieh analysed the existence of nonlinear deterministic processes in asset returns. Thus the residuals of the ARCH type models were tested for IID and Hsieh concluded that these models do not account for all the deterministic relations existing in the returns while the stochastic volatility model behaves better. As such the stochastic volatility model may be considered as the best fitted model. The objective of this paper provide a possible method for the implementation of the jump-diffusion model in options bound computation but the stochastic volatility model may provide an evidence for the performance of the jumps model by means of the BDS test.

2. Brock, Dechert, and Scheinkman (1987) – BDS test

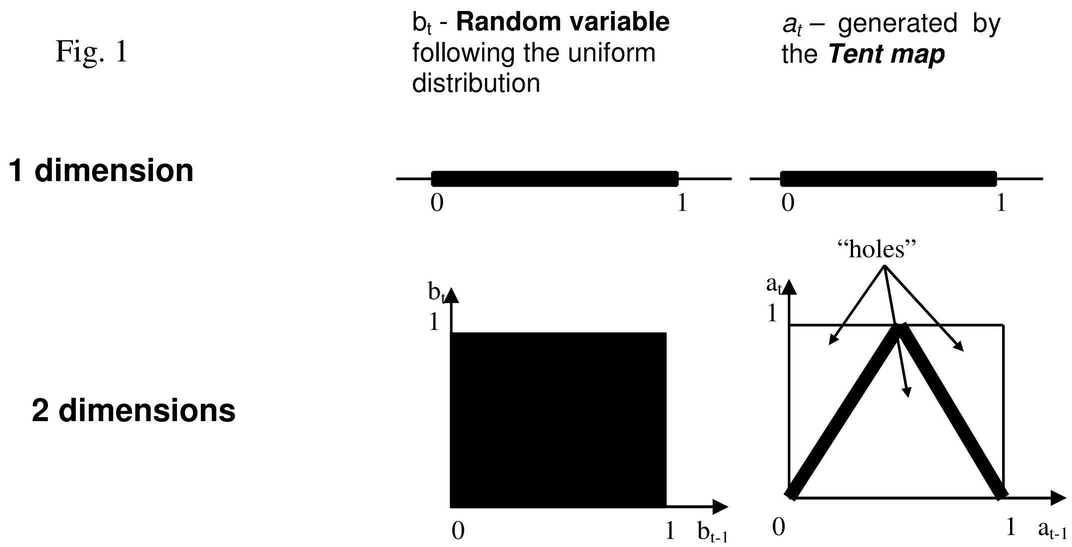
The BDS test is in close relation with the use of chaotic maps for time series inference. Chaos is a nonlinear deterministic process which “looks” random. According to Hsieh (1991), in general, chaotic maps are obtain by a deterministic rule: $x_t = f(x_{t-1}, x_{t-2}, \ldots)$ where f must be a nonlinear function and x_t is a scalar or a vector. The need to test for the existence of chaos in a certain time series revealed an interesting feature of a process following a chaotic map as opposed to a pure random variable – at a certain number of dimensions the random variable fills the space uniformly while the nonlinear function (the chaotic map) leaves certain areas uncovered. The tent map is the simplest form of chaotic map:

$$X_t = 2X_{t-1} \text{ if } X_{t-1} < 0.5$$

$$X_t = 2(1 - X_{t-1}) \text{ if } X_{t-1} > 0.5$$

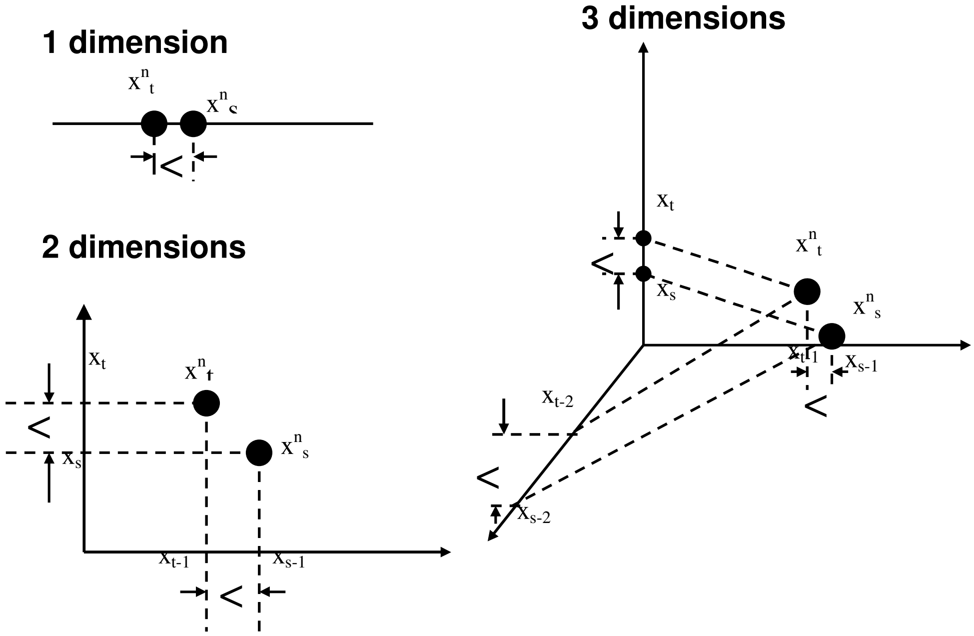
The tent map will fill all the space in the first dimension but leaves wide areas uncovered in the two-dimension space while the random variable following a uniform distribution in the (0,1) interval will fill all the space in both situations as in figure 1 below:

Fig. 1



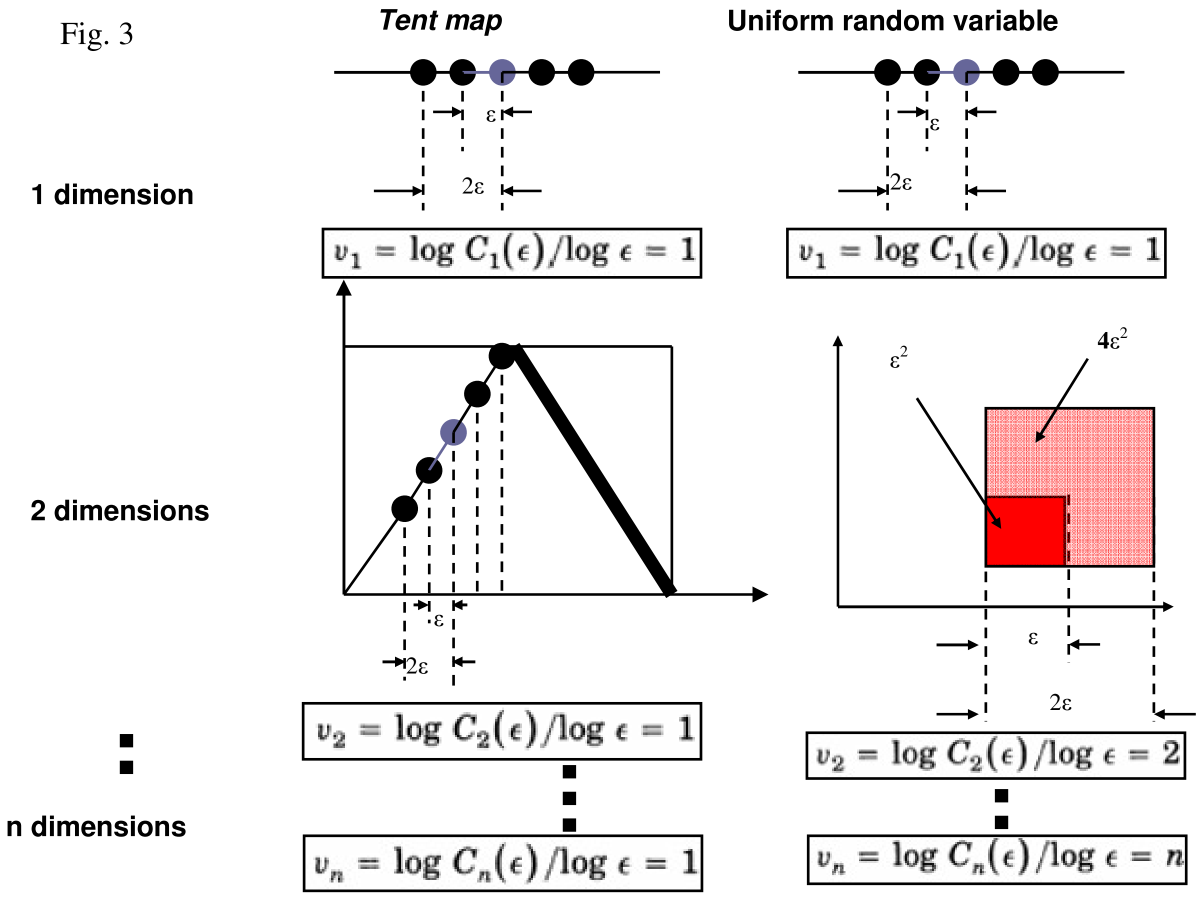
Thus, when the chaotic process becomes more complex we need to analyse the data in higher dimensions. A chaotic process can fill up the first n dimensions but leave large "holes" in the n+1 dimension. Grassberger and Procaccia (1983) derived the **dimension integral C(ε),** an instrument which counts the number of pairs of points close to each other (the distance between them is less than ε) in the analyzed dimension.

Fig. 2



The **correlation dimension** ($v_n = \log C_n(\varepsilon)/\log\varepsilon$) was developed by the same authors with the scope to measure how much space is "filled up" by a string of data. This instrument still does not provide a way to test for the existence of a nonlinear function in the process. Brock, Dechert, and Scheinkman (1987) developed the BDS test from the result that the correlation dimension should increase with the number of dimensions for a random variable and should remain constant or converge to a certain value for the nonlinear deterministic process. For the example with the tent map and the uniform random variable this result is provided in figure 3.

Fig. 3

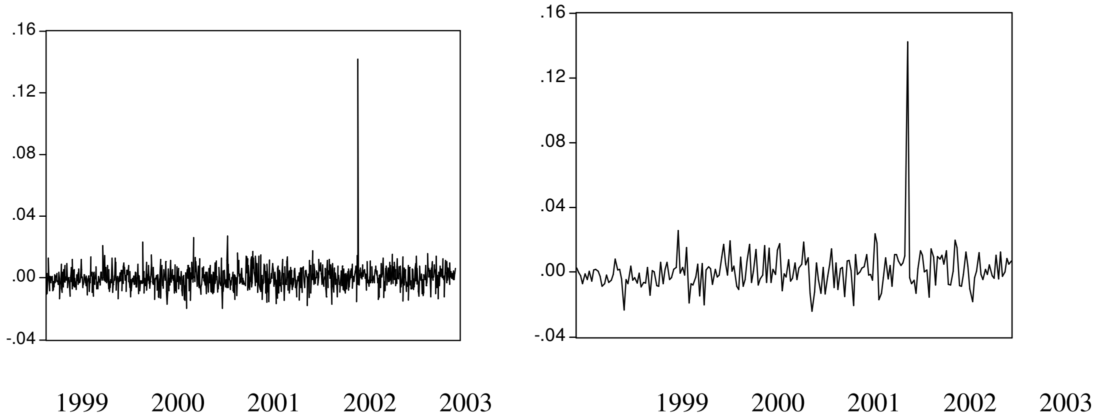


Thus the value of the correlation dimension for the case of the random variable should be $C_1(\varepsilon)^n$ (the value of the correlation dimension in the one-dimensional space at the power of n – the number of the dimensions in the analysed space). The BDS statistic analyses the difference between the value of the correlation dimension in the n^{th} -dimension space and the and $C_1(\varepsilon)^n$ and is asymptotically standard normal. Under the null hypothesis the process is a random variable, so the numbers analysed are IID, while under the alternative, the process is not IID – we have a certain deterministic function which generates the values of the process.

3. Application to exchange rates

The 1011 daily exchange Euro/USD rates were provided by quotes published by Saxo Bank, for the period the 4th of January 1999 until the 31st of December 2003. The parameters for the two models were computed from the log-returns. For the stochastic volatility model these daily quotes were used in blocks for the computation of 202 weekly returns volatilities. The jump-diffusion model used the weekly returns as the difference between the Friday log-return and the Monday log-return.

Fig. 3 **Daily log-returns** **Weekly log-returns**



The highest value (0.141713464118) occurred on the 3rd of January 2002 and it records the moment when the value of the Euro exceeded 1.

4. SVOL estimation

We will estimate the values of the parameters for the stochastic volatility process in order to use this process as a filter and apply the BDS test to the residuals. This is why we will first remove any linear deterministic dependence from the process. We compute an AR(p) process for the daily log-returns and use the residuals to compute the second moment dependence. The number of lags (p) was determined by the Akaike information criterion and an AR(3) was used. For the volatility we used an AR(10).

Table 1[1]

Returns

p	Akaike
1 lag	-6.875825
2 lags	-6.877685
3 lags	**-6.877944**
4 lags	-6.877383
5 lags	-6.875086
6 lags	-6.873464
7 lags	-6.873434
8 lags	-6.870474
9 lags	-6.869888
10 lags	-6.867183

Volatility

p	Akaike
1 lag	2.693677
2 lags	2.705715
3 lags	2.694237
4 lags	2.707740
5 lags	2.706011
6 lags	2.687764
7 lags	2.693174
8 lags	2.689740
9 lags	2.871425
10 lags	**2.683677**

Table 2 **AR(3) results for the log-returns**

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	7.24E-05	0.000244	0.296395	0.7670
C(2)	0.038072	0.031533	1.207382	0.2276
C(3)	0.057041	0.031477	1.812127	0.0703
C(4)	-0.054922	0.031504	-1.743337	0.0816

R-squared	0.007420	Mean dependent var	7.57E-05
Adjusted R-squared	0.004451	S.D. dependent var	0.007769
S.E. of regression	0.007751	Akaike info criterion	-6.877944
Sum squared resid	0.060263	Schwarz criterion	-6.858422

[1] Akaike info criterion was computed as *-2(l/T)+2(k/T)*, where *l* is the loglikelihood and *k* is the number of parameters. The best fit is provided by the minimum value of the Akaike.

Log likelihood	3467.045	Durbin-Watson stat	1.993582

AR(10) results for the log-returns

	Coefficient	Std. Error	t-Statistic	Prob.
K(1)	-6.798787	1.804949	-3.766747	0.0002
K(2)	0.050044	0.073062	0.684958	0.4942
K(3)	0.016673	0.073657	0.226365	0.8212
K(4)	-0.037252	0.073123	-0.509434	0.6111
K(5)	0.010065	0.072839	0.138179	0.8903
K(7)	0.116121	0.071223	1.630397	0.1047
K(8)	0.103323	0.071729	1.440476	0.1515
K(9)	-0.131853	0.071138	-1.853474	0.0654
K(10)	-0.065552	0.071689	-0.914394	0.3617
K(11)	0.182775	0.071899	2.542098	0.0119
R-squared	0.083090	Mean dependent var	-8.999275	
Adjusted R-squared	0.037748	S.D. dependent var	0.915709	
S.E. of regression	0.898259	Akaike info criterion	2.673962	
Sum squared resid	146.8503	Schwarz criterion	2.843623	
Log likelihood	-246.7003	Durbin-Watson stat	2.021411	

A bootstrap type test was used for the computation of the BDS on the residuals of the stochastic volatility model. We run 10 000 simulations for the stochastic volatility model and we compute the BDS on each of the residuals series from 2 up to 10 dimensions for an ε equal to the value of the standard deviation recorded for each series of residuals. We are interested in computing the number of rejections as a percentage of the total number of computations. We expect the BDS to be rejected 5% of the time if we consider a 95% confidence interval. This will provide evidence for IID residuals. The results presented in table 3 below show the percentage of the cases when the BDS was in the 95% confidence interval – when we do not reject the null.

Table 3

Number of dimensions	SVOL residuals	Random Number Generator sample
	The percentage of the situations when we accept the null for the BDS test	**The percentage of the situations when we accept the null for the BDS test**
2	**92.97**	**91**
3	**92.96**	**89.4**
4	**92.73**	**86.6**
5	**92.43**	**84.9**
6	**91.28**	**80.7**
7	**89.52**	**74**
8	**87.86**	**64.2**
9	**85.77**	**51.8**
10	**83.22**	**31.5**

We notice that the percentage is decreasing when we increase the number of dimensions which means that we have more rejections at higher dimensions. So we can conclude that the residuals behave more as high dimensional nonlinear deterministic processes. We should take into account though the pseudo-random number generator provided by the software package which is itself a chaotic map. For comparison we make 1000 draws from the normal distribution by using the pseudo random number generator in the software package. The results for a BDS test for 2 up to 20 dimensions are provided in table 4. We can see that they are behaving even worse than the results provided for the BDS test in our case.

The paper estimates the parameters of the stochastic volatility model according to Hsieh (1991) for the Euro/USD exchange rates from January 1999 until December 2003. The residuals form this model were tested with the BDS test for IID. We can say that the model is capable of capturing the most part of the deterministic component in the exchange rates leaving the residuals to account only for the random variability of the data. The next step in our research will be to adjust the stochastic volatility model in order to be able to come up with model that can forecast the median of the residuals obtained from the stochastic volatility model. This will provide a way to take into account the signs of the returns.

References:

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