Heterogeneous Income Distribution, Output Growth and Policy Transition under Non-Linear Dynamics and Multiple Equilibria: the Experience of Former Socialistic Countries

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A b s t r a c t: This paper studies policy transition under the condition of nonlinear dynamics and possibility of multiple equilibria in a system where output and income inequality endogenously depend on each other, while policy choice and political stability are assumed to be exogenous. The empirical model is estimated using dynamic panel of twenty transitional countries for the period of fifteen years at the first stage, while at the second one the estimated parameters are taken for the dynamical analyzes of the system in the space of output, income inequality and the change in income inequality. Two central questions are: *i*) what is the dynamics of such a system under the assumption that democracy fails to make redistributive policy endogenously dependant on the changes in income distribution; *ii*) whether there is a need for the government to control changing inequality in order to avoid both poverty traps and limit cycles. It is shown that for the three out of four regarded models the system has cyclical dynamics in two dimensions and non-cyclical in the third one for all the possible policy choices. All these models reveal the lower (in terms of output) of two possible equilibria as a local attractor, stable in three dimensions, whenever the fixed point is shifted by the policy change to a distance not large enough to put the system out of the basin of attraction. The higher of two equilibria, on the other hand, is saddle focus. As for the fourth model, it also displays two dimensional cyclical dynamics within the interval of steady state output which seems to be the most reasonable one in terms of pre-transitional output. However, in contrast to former models, here the system displays changing two-dimensional stability pattern as a result of the change in parameter of policy choice. This single model seems to reveal the possibility of Andronov-Hopf bifurcation and hence the limit cycles as a result of policy choice.

Keywords: income distribution, former socialistic countries, output growth, limit cycles, policy transition, poverty traps;

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[*] I would like to thank Thomas Aronsson, Kurt Bränäs, Jörgen Hellström, Levon Grigoryan, Tönu Puu, Tomas Sjögren and Magnus Wikström for valuable comments and suggestions. All the remaining mistakes are only mine.

Introduction

Initial conditions do not matter in a context of a neoclassical dynamic model, where there is only one balanced growth path toward unique stationary equilibrium. However, they do matter under the condition of nonlinear dynamics and possibility of multiple steady states. In such a system whenever external shocks occur, bringing the economy below the level of lower (stable or instable) equilibrium, it would become problematic for the system to recover without shocks (of opposite direction). In this case it is said that country is caught in a *poverty trap*, which is defined as a vicious cycle, where the economy suffers from persistent underdevelopment. For economic policy issues the escape of the Pareto-inferior equilibrium becomes essential.

Phenomenon of poverty trap arises in a system where two or more variables endogenously reinforce each other. The presence of such endogenously interdependent variables has an important implication to the stability of the system, since it makes the system very sensitive to external shocks, when even subtle changes in one of these factors can evoke a process of chain reaction and result in substantial effect. Other phenomena that are closely connected with this (non-linear) interaction of purely endogenous forces are limit cycles, which are defined as stable oscillations. What makes it important to recognize these oscillations is the fact that they appear as a result of variation in an exogenous parameter making the system to abandon its steady state and start to steadily oscillate. In the macroeconomic literature such an oscillation is related to Andronov-Hopf bifurcation.

This paper deals with policy transition under the conditions of non-linear dynamics and possibility of multiple equilibria in the economic system, where the change in output and income distribution endogenously depend on each other, while policy choice and political stability are assumed to be exogenous. The paper is motivated by the economic transition recently experienced by former socialistic countries. All the countries listed in *appendix i* have abandoned 'planned economy' and launched policy reforms which were to affect both output growth and income inequality. Although significantly differing in the historical and cultural context, these countries had pre-transitional output and inequality at least comparable to each other; another common feature was the strong believe that in the long-run all the reforms were to increase economic growth. Today, with a passage of less than twenty years, some of them are no longer comparable either in the income distribution pattern or in the output growth. This raises the problem of possible multiple equlibria and/or limit cycles.[1]

In current paper an empirical model for the economic transition is estimated using a dynamic panel of twenty countries within the period of fifteen years. Further the estimated coefficients are taken for the dynamical analysis of three-dimensional system in the space of *output, income distribution* and *the dynamic change in income inequality* to answer two central questions of this paper, which are:

-what is the dynamical pattern of income distribution and output growth in a systems, where the democracy is not strong enough to ensure that redistributive policy is endogenously determined by the dynamics of income distribution;

-whether policy choices in such models may lock the dynamical system under a poverty trap or within limit cycles; and whether there is a need for the government to control the change in income distribution when launching policy transitions in order to avoid both *to end up in the steady state inferior to the pre-transitional one* or *to be couth by the steady oscillation*.

There is a large body of scientific literature concerning to the policy choice and economic outcome for this group of countries. In general the list of variables considered as responsible for the outcome can be conditionally divided into three broad groups: initial conditions, stabilization

[1] this dangerous syndrome can be guessed indirectly by the number of political transitions that some of these countries has already undergone, and by the fact that in some of them such transitions still sound actual to the generation which otherwise should be 'sick and tiered ' of the' lifelong transition'.

and liberalization policy, international integration. Speed of liberalization was one of the central variables in mid 90's (Åslund et al. (1996) ;Denizer (1997)) and seems to be relevant even today (Merlevede (2003); Godoy and Stiglitz (2004)). Nonetheless, to my knowledge none of the papers concerned with speed of changes controls for the changing inequality as well, though countries were quite heterogeneous in the choice of both these variables. Moreover, Cornia and Popov (1997) included the 'changes in income inequality' as one of the four criteria's to analyze 'empiric archetypes of post-Soviet economies' and their 'long term growth potential'. However, to my knowledge the single paper that deals explicitly with the income inequality effect on output growth for the list of these countries is Sukiassyan (2004) reporting significant and negative effect of the inequality at the beginning of the period on the subsequent output growth.

Scientific literature on inequality determined by output growth for transitional countries also reveals contradictions. In a theoretical framework Ferreira (1999) predicted increasing inequality pattern as an outcome of transition. Aghion and Commander (1999) simulating inequality and growth on a general equilibrium model demonstrated that the time path of inequality may depend on the choice of policy parameters. Dahan and Tsiddon (1998) reveled Kuznets inverted U pattern. Hua Wan (2002) reported 'a (first) half U pattern'. Keane and Prasad(2002) and Kattuman and Redmond (1997) reported 'rollercoaster' pattern for Poland and Hungary respectively (the pattern was generalized to the experience of other transitional countries as well). Garner and Terrell (1998) revealed stable inequality in Czech and Slovak Republics.

There are also models introducing multiple equlibria concept into transitional countries literature. To list some of them Rosser et al (2003) looking into the 'unofficial' economy and income inequality effect; Kylimnyuk at al (2005) regarding dynamics of the share of agricultural production; and Aghion and Blanchard(1994) looking into the multiple equilibria in labor market can be mentioned.

In the empiric literature of inequality and output growth, on the other hand, although the idea of possible multiple equilibria seems first to be mentioned by Benabou(1996), the main concern of the previous and following papers were within initial distribution of income and subsequent economic growth. The findings are rather contradicting. Earlier papers (Alesina & Rodrik (1994); Perotti (1994,1996)) reported negative and significant correlation between these variables. Persson & Tabellini (1994) reported significant correlation for the developing countries and only 'presenting' negative effect in well established democracies; Barro (1999) reported negative correlation for the developing countries and no clear pattern for the developed ones. Forbes (2000), Li Hongyi & Zou(1998) concluded that inequality stimulates growth. Recently, Benerjee & Duflo(2003) demonstrated that this relation is not linear. Chambers (2003,2005) confirmed these findings. To my knowledge the single paper concerning with (unconditional) convergence in income inequality is Ravallion (2001) concluding that inequality converges both within and between countries.

In current paper to check the robustness of dynamic results analysis are curried out for four different estimated models (differing in the choice of data for inequality variable, list of the controlled variables and method of estimation). It is shown that for the three of regarded models system has cyclical dynamics in two dimensions and non-cyclical in the third one (which seems to be consistent with the dynamic pattern empiric data displaying more than one turning points in income inequality and/or output for at least half of the countries considered, see *appendix iii*) for all the chosen polices, whenever given choice has steady sate(s) and transition was started sufficiently close to it. Moreover, the fourth model reveals the same pattern whenever the policy choice is such that the ratio of the steady state output and observed sample minimum (which is for all the countries lower than the pre transitional level of 1989) is within the interval of [.024; 16.14]; for all the other cases this estimation displays non-cyclical dynamics in all the three dimensions. Nonetheless, for the three former models system is converging in two dimensions for all the possible steady states, which makes the lower (in the terms of output) of two possible steady states locally attractive in all the three dimensions, while the higher one is saddle focus.

Hence, when starting transition sufficiently close (to be kept within the basin of attraction) to this equilibria there is no need for the policy choice to pay additional attention to the changing inequality; while for the case of the higher equilibria the policy effect on the output is to be balanced by the policy effect on income inequality in order to be kept on the invariant plain: the single rout to the new steady state. In the case of the fourth model, it turns out that the cyclical convergence of the system may depend on the policy choice. In this single case it seems that the possibility of Andronov-Hopf bifurcation arises, i.e. the system can be locked within limit cycles as the result of policy choice.

For the larger perturbation and global dynamics more analysis are to be curried out, however within the scope of this paper two points can be already mentioned here. First of all, the estimated system is complicated enough to have periodic or even chaotic orbits whenever it leaves the basin of attraction. This makes gradual changes (or the shift of the fixed point) of lower risk. This may concern not only Cumulative Liberalization Index (one of the most contradicting variables in the literature of transitional economics) but also control in changing inequality (Poland for example was labeled as one of the 'advanced reformers', but at the same time has the better control over the changing inequality, in the contrast to FSU countries). Second, it seems that this is the change in income inequality rather than income distribution itself which determines the dynamic pattern of the system under the larger perturbation.

Finally, as long as two dynamic equations of output growth and change in income inequality were statistically estimated the effect that any of the regarded variables has on these factors can be of the particular interest itself.[2] Here, the two most surprising revealed facts are: i) the socio cultural factors such as the 'years under the planed economy' seem to have higher effect on the change of income inequality and, hence, indirect effect on the output growth; ii) from all the shock variables regarded the lowest level of governmental expenditures seems to be the single one that enters into the long run memory. Moreover, while governmental expenditures at the beginning of period have always negative (and almost always significant) effect on the output growth, the level of the lowest governmental expenditures has positive and statistically significant effect. Furthermore, the fourth model that displays a different dynamical pattern statistically differs from the one of other three models only by the inclusion of this variable. In addition, since the analysis seems to justify existence of at least two equilibria, the long-run outcome is to be non-linear in the policy choice. Moreover, because the lower of the two equalibria displays local stability in all the three dimensions it is important to have the policy choice such that the lower steady state is higher than the pre-transitional one, otherwise even though the attention is to converge to the better steady state the system in reality can be caught by the poverty trap.

There are two points making the topic of this research valid even within the light of the EU resent enlargement, which makes the term 'transitional countries' even more unclear. First of all, channels going between inequality and output growth are still vague, while policy reforms affecting both of these variables may become relevant in some other country; so that the experience of these countries is valuable enough to study and to take into account. Second, in the context of transitional countries it is called to ensure that none of them is locked under the poverty trap or within limit cycles as an outcome of a policy choice.

The paper is organized as follows. Section two reviews theoretical models of income inequality and output growth and briefly discusses the channels of possible nonlinearities. Section three presents the empirical model, econometric method and general results. Section four looks into the steady state and local dynamics as an outcome of policy choice. Finally, section five concludes.

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[2] Indeed, even when instrumenting time varying variables some unexpected results are noticed. For example the two variables that best of all predict foreign direct investments seems to be domestic rate of higher education and demography (in contrast to the governmental expenditures, inflation, wages and political stability)

2. Theoretical discussions. Possible channels of non-linearity

In general, literature on the economic growth and income distribution can be divided into two major groups.[3] The first one includes the so called socio-political models, regarding either distributional or social conflict as the third variable connected with both income inequality and output growth.[4] The argumentation is that increasing inequality either leads to higher redistribution (i.e. distortionary taxes) or political instability (i.e. higher investment risk). Rosser et al.(2003) can be thought of as a model with the so called 'social unrest' channel for transition countries.

The second group, labelled as 'wealth channel', mainly argues that the effect of the initial income distribution on the long run output growth crucially depends on the type of credit market. Under the assumption of incomplete credit market Aghion et al.(1999) demonstrates that long run output growth will be negatively correlated with initial inequality of income (wealth) distribution. Benerjee and Newman (1993) assumes occupational choice (hence income distribution) internally determined within the system. Galor and Moav (1993) considers the long run effect of income distribution on output growth in the system, where human and physical capital are regarded as complementarities. [5]

Nonetheless, even in an economy with no production externalities and under complete credit market income distribution will be endogenously determined in the system as a result of differences in elasticities of inter-temporal substitution for the agents with different incomes, when instantaneous utility is not of CRRA type. [6] On, the other hand, in such a system, the speed of convergence (divergence) will depend on the income distribution.[7] However, in an economy with complete credit markets the effect of the higher inequality on the output growth will crucially depend on the shape of instantaneous utility function. [8] This is the consequence of two facts. First, the channel form output growth to income inequality will go through individual risk tolerance which is responsible for the sensitivity of the individual consumption to a unit drop in the rate of return. The feedback will come from the aggregate risk tolerance, which is the average of individual ones. Second, since the aggregate risk tolerance is not weighted by individual incomes, and as long as the more unequal income distribution is, the further to the left will be shifted the income of 'median' individual, the aggregate risk tolerance will be more sensitive to

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[3]To point out papers on income distribution transitional dynamics two famous papers of Becker(1980) and Chatterjee(1992) can be mentioned. The former demonstrates that in the economy with heterogeneous time discount rate only most patient agents will end up with positive wealth. While, the latter shows that 'if the economy growing toward the steady state and preferences are such that marginal utility from consumption is infinite(finite) at some(all) positive(non-negative) consumption level(s), then the average propensity of agents is positively(negatively) related to their wealth'.

4 Alesina and Perotti (1995); Persson and Tabellini(1991); Benabou(1996); Benerjee and Duflo(2003)

[5] Aghion and Bolton(1997) and Matsuyama (2000) discuss conditions under which economic growth will be pro-poor

[6] As noticed in Sorger (2000), under the CRRA utility function $\left(\frac{\dot{c}}{c}\right)_t^i = \left(\frac{\dot{c}}{c}\right)_t^j$ as a consequence of Euler equation

[7] In the simplest Ramsay framework with heterogeneous wealth distribution the negative characteristic root is given by the equation $\lambda = \dfrac{\theta - \sqrt{\theta^2 + 4f''\left(\frac{1}{n}\sum_{i=1}^{n} k_i^*\right) \frac{1}{n}\sum_i \frac{u'(c^*)^i}{u''(c^*)^i}}}{2}$; where i=1,2..n refers to the i-th individual; θ is the time discount rate u(c) and f(k) are utility and production functions, respectively.

Hence, its eigenvector is not free of the income distribution. On the other hand, because $\theta > 0$, the positive root will always be of higher absolute value than the negative one. This will imply that the system asymptotically is to be caught with the eigenvector of diverging root, so that the only condition to converge is to start the transition on the eigenvector of negative root. So that any policy transition, which implies changes in steady state (per capita) consumption and capital, should chose initial conditions depending on the distribution of income.

[8] The economic literature of income inequality and output growth in such cases, referring to Kaldor (1957), assumes that higher inequality will foster economic growth. on the other hand, Sorger (2000) demonstrates for the one sector simple growth model with elastic labor supply that the steady state output will depend on the initial distribution of income, with higher (in)equality favoring steady state output when EIS is (large) low

the changes in the lower tail. This implies that when the utility function is such that savings (consumption) are more sensitive to the unit drop in return to aggregate accumulated capital for the agents with lower consumption, the higher inequality will lead to lower growth. It may be shown that the direction of such a relationship will depend on the relative magnitudes of first to forth derivatives of the instantaneous utility function. [9] Furthermore, there is nothing guaranteeing that these derivatives should be of the same relative magnitudes all over the utility function (i.e. for different levels of consumption). This, in turn, may cause nonlinearities in a relationship between income distribution and output.[10]

Moreover, in the case of saddle path transition, the distribution of income may also be responsible for the stability pattern.[8]Two papers Ghiglino and Sorger (2002) and Ghiglino and Olszak-Duquenne(2005) discuss the output dynamic and stability pattern under different wealth distributions (redistribution)[11]. The former demonstrates how the redistribution of wealth may lead the economy with positive output to a poverty trap with zero production. While the latter, concludes that more (un)equal wealth distribution may favour (in)stability when the aggregate risk tolerance is strictly (convex) concave. Ghiglino and Olszak-Duquenne (2001) demonstrates that heterogeneity of capital and labor distribution may cause fluctuations even in the system with homogenous preferences.

Since the shape of the individual utility function becomes of the crucial importance it may be worth to note that whether the assumption of non constant relative risk aversion is held in the real life, seems to be not clear yet. On the one hand, empiric research reveals that the poor agents seem to have lower intertemporal elasticity of substitution[12]; while on the other hand, theoretical concavity of consumption function requires the condition $\frac{u'''u'}{(u'')^2} = k > 0$ (which is the condition of HARA function family, including CRRA and CARA as well) to hold.[13,14]

In the context of transitional economies although the importance of political channel is of no doubt (and probably much more dramatic for transitional experience), this paper mainly concerns with a so called 'wealth effect'. The role of human capital accumulation is also disregarded.[15]

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[9]Since $\frac{\partial \dot{c}^i}{\partial k} = -f''(\sum_{i=1}^{n} k_t^i)\left(\frac{u'}{u''}\right)^i$;and $\frac{\partial^2 \dot{c}^i}{\partial k \partial c} = -f''(\sum_{i}^{n} k_t^t)\left(1 - \frac{u'_i u_i'''}{(u_i'')^2}\right)$ (*1).To have $\frac{\partial^2 \dot{c}^i}{\partial k \partial c} > 0$ (2.5) requires $\left(\frac{u'_i u_i'''}{(u_i'')^2}\right) < 1 \Leftrightarrow \frac{u_i'''}{u_i''} > \frac{u_i''}{u_i'}$ (*2); where i=1,2..n refers to the i-th individual

Actually, the left hand side of the expression (*2) is a measure of 'absolute prudence' as it is defined in Kimball (1990), while the right hand side is a measure of 'absolute risk aversion' as defined by Pratt (1964). Equation (*2) states that the positive aggregate capital accumulation will lead to higher increase in the consumption of poor agents (i.e. those with lower initial consumption) relative to the richer ones under the condition when the motivation 'to avoid a risk' dominates the motivation of 'precautionary savings'. Nonetheless, there is nothing guarantying that the inequality (*2) should hold with the same sign for different levels of consumption. . One possible mechanism of such a switching is 'buffer stock savings' theory stating that for different accumulated wealth (below or above targeted level) either 'prudence' or 'impatience' is to prevail. (see Carroll (1992)).

[10] Here nonlinear effect of the change in income inequality on the output growth may as well be the consequence of 'wealth distribution' channel, as 'political channel' as it is concluded in Benerjee and Duflo (2003)

[11] Sorger(2000),shows for the one sector simple growth model with elastic labor supply that the steady state output will depend on the initial distribution of income, with higher (in)equality favoring steady state output when EIS is (large)low

[12] see for example Atkinson and Ogaki (1996)

[13] see Carroll and Kimball (1996)

[14] Pratt and Zeckhauser (1987) showed that negative forth derivative forth derivative of the indirect utility function (along with $\upsilon'(x) \geq 0; \upsilon''(x) \leq 0 \, and \, \upsilon'''(x) \geq 0$) is a condition under which the risk premium for two independent risks combined will be more than the sum of the risk premium for each risk taken separately

[15] First of all, all the countries started transition with educational system (more or less the same) including free secondary and university education. Second, the institutional structure of the (pre)transitional process implies that

Besides, no explicit assumption is done either for the type of credit market[16], or for the shape of utility[17] and production functions.

3. Statistical estimation, Data and General Results

3.1 T he empirical model

The implemented empirical model imposes no special assumption about credit markets, utility or production functions. Indeed, it is a compromise between simplicity, on the one hand, and constraint imposed by both available data and minimum complication level required for the questions posed, on the other hand[18]. Thus, due to relatively short period of observation for long run trend analysis and because of missing observations (for some of the countries only two or three observations are available) the model is built as a dynamic panel of 20 countries rather than for any single country separately.

The estimated dynamical system may be described by following equations[19] :

$$\hat{Y}_{j,t} = \alpha_0 + \alpha_1 \times \ln(y_{j,t-1}) + \alpha_2 \times \ln^2(y_{j,t-1}) + \alpha_3 \times \hat{G}_{i,t-1} + \alpha_4 \times \ln(g_{j,t-1}) + \sum_{i=5}^{n} \alpha_i \times z_{i,j,t-1} + \varepsilon_{j,t} \quad (1)$$

$$\hat{G}_{j,t} = \beta_0 + \beta_1 \times \ln(g_{j,t-1}) + \beta_2 \times \hat{Y}_{j,t} + \beta_3 \times \ln(y_{j,t-1}) + \beta_4 \times \ln^2(y_{j,t-1}) + \sum_{i=5}^{n} \beta_i \times z_{i,j,t-1} + \xi_{j,t} \quad (2)$$

Where:

$$\hat{Y}_{j,t} = \frac{y_{j,t} - y_{j,t-1}}{y_{j,t-1}} \quad (3)$$

$$\hat{G}_{j,t-1} = \frac{g_{j,t-1} - g_{j,t-2}}{g_{j,t-2}} \quad (4)$$

and $\ln(y_{j,t}), \ln(g_{j,t}), z_{j,i,t-1}$ all are differenced with country average $(\ln(\bar{y}_j), \ln(\bar{g}_j), \bar{z}_{i,j}$ with j=1…20 denoting country). The only exception is $\ln^2(y_{j,t})$, which is differenced by $\ln(\underline{y}^j) = \ln(\min\{y_t^j\})$ where t=1…m; this is to avoid high correlation between $\ln(y_{j,t})$ and $\ln^2(y_{j,t})$. Here y and g denote inequality and output, respectively. For the growth rates, first $\hat{Y}_{j,t}, \hat{G}_{j,t-1}$ were calculated and thereafter differenced. Hence, these variables can be interpreted as deviation from country average growth rates. [20]

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human capital was less interesting driving force behind economic growth. For more details see Aghion and Commander (1999)

16 In general the transitional process in the sample countries may also be regarded as a transition form a system with no credit market to the one with complete(incomplete) credit marcet. Nonetheless, at least at the beginning of transition the incompleteness of Matsuyama (2000) type more probably can be ruled out by the nature of the system inherited. Thanks to the innate relative equality of wealth distribution, along with serious problems of public production sector, it seems reasonable to think that there was no initial capital threshold required for starting up a business. Hence, the appearance of these features with the course of time (i.e. existence of such constrain today) may have mainly policy choice rather than initial conditions implication.

[17] the single assumption is that it does not belong to the CRRA family

[18] for the credit markets the intention is to be as general as possible

[19] Alternatively equations (1) and (2) were estimated with term $\ln^2(g)$ instead of $\ln^2(y)$; this term did not gain statistically significant parameters for different estimated models.

[20] the main reason for calculating growth by formulas given in (3) and (4) instead of taking simple first differences is that in the latter case equations (1) and (2) can not be statistically estimated due to the scarcity of available data and the noise it contains. Initial output and inequality, on the other hand, are log-linearised in order to make it possible to approximate the left hand sides of (1) and (2) to continuous time derivatives of functions of output and inequality respectively.

Finally, z_i is a set of variables including both country specific dummies and indexes (like religion or political stability, etc.) as well as policy variables (inflation, government expenditures or wage, etc.). The full list of variables included in z_i may be found in appendix 1.ii.

3.2 Data

Inequality in equations (1) and (2) is measured by the Gini coefficient. This choice is due to the availability of data. One possible drawback of this index is that it usually gives more weight to the middle income class, while for the purpose of this paper indeces with higher weight to the lower income group may be more preferable. However, according to the fact that income was more or less normally distributed in all transitional countries prior to the transition reforms, and since most of reforms affected the middle class first of all, the Gini coefficient may serve as a good indicator of these changes.[21]

Two major sources of inequality data available for transitional countries are the WIIDER database on worldwide inequality and TransMONEE economic indicators. Since the latter contains longer uninterrupted time series for Gini coefficients for almost any single country, the preference is mainly given to this source of data. Actually, there are three different ways undertaken in an attempt to solve the trade off between consistency and number of observations. In the first case estimations are based on income Gini's solely. In this estimation Gini coefficients are mainly taken from TransMONEE 2005 report, while from WIIDER database (which itself contains different sources of information) data for missing observations is filled only in the case when it does not much differ from the observation available in TransMONEE database. Interpolations also are done only in the case when there is a single missing data and long time series are available from right and left sides, besides, for any single interpolation the loss of fit in model estimation is checked.

In the second case only earnings Gini from TransMONEE database is used. Although the model is to be built on the inequality of income rather than earnings, the former is in most cases significantly lower than the latter[22], this makes reasonable to believe that earnings inequality may serve as better approximation of income inequality than the data for income Gini available. This may imply that it is easier to follow inequality in earnings than in income. Actually, it turns out that this data series give the best fit for the model estimation.

In the last case the data for earnings inequality is used for most of the countries, while only for Poland, Hungary, Bulgaria and Romania incomes Gini are used, and Slovakia is added (all the data is taken only from TransMONEE database). The list of countries includes twenty former socialistic countries in the first and third estimation and only nineteen (with exclusion of Slovakia) in the second case. However, the data is still unbalanced in all the cases[23], it includes only two observations for Turkmenistan and maximum of fifteen observations for Czech Republic, Poland, and Slovenia.

The total number of observation is between 102 and 125 observations for different model specifications. The full list of the countries included and some descriptive statistics for endogenous variables of equations (1),(2) may be found in the Appendixes 1.i, 1.iii.

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[21]As is discussed in Hau Wan (2002) the Gini coefficient: satisfies the axioms of anonymity, income homogeneity, population homogeneity, and the transfer principle (according to Fields (2001)); and is the single ratio which is supposed by observed economic unit's behavior as it is based on non-individualistic or interpersonal utility functions, implying that Gini index allows for a much more realistic interpretation of both social welfare and social income inequality than the Teil, the generalized entropy and the Atkinson inequality measures (as is shown in Dagnum (1990)).

[22] Appendix 1.iii contains descriptive statistics for both of these coefficients for any given country and for all the included countries as well

[23] Data is unbalanced not only (although mainly) due to the missing observations in Gini coefficients, but in some cases there are missing observations for FDI, wages, employment or governmental expenditures

With exception of indeces such as Cumulative Liberalization Index (CLI, de Melo et al.; IBRD (1996)) or Political Stability (Kaufman et al. index) and foreign direct investments (UNCTAD data on net capital inflow as % of GDP) all the other macroeconomic indicators are also taken from the TransMONEE 2005 report. However, some of the data is filled from other sources (for example employment data is recalculated for countries with missing values using data available from UNISEF reports or other sources such as individual country reports on labor markets).Once again, since the indicator available in different sources for the same country and same year may differ significantly, only data close to the main source information is used.

The complete list of data and main sources of information can be found in Appendix 1.ii.

3.3 Econometric Method

The relative short period of data, its unbalanced nature and in some instance its quality imposes number of restrictions on the model and method of estimation. First of all, as it was already mentioned the dynamic panel of twenty countries rather than time series of any single country is estimated. The empirical literature on inequality and economic growth pays special attention to the choice of random or fixed effects models.[24] In this paper the fixed effect setting is preferred, due to its dynamical concern mainly and due to the belief that former-socialistic countries while having initial inequalities pretty close to each other, still differ significantly in historical and cultural background making differences in individual indicators systematic rather than random. Furthermore, since the style of the transitional reforms such as, for example, the pattern of privatization is to affect both equations of growth systematically, the procedure of the mean differencing for each country is also for the help in getting rid of these systematic differences. However, the main assumption here is that when adopting this or that style countries do not leave it later, which empirically seems reasonable.

Second, as one may notice from equations (1) and (2) the rate of change in inequality is taken by its first lag, this is due to the lack of good instruments available for the estimation of pure simultaneous system and high noise within inequality indeces. The choice of taking inequality change by its first lag in equation (1) rather than output growth lag in (2) lies on the ground of theoretical models with discrete time assuming that savings of this year become investment the next year, along with the assumption of this paper that the main channel between inequality and output goes through domestic investments. Empirically both of the equations were tested separately by simple OLS estimation with substituting current value of inequality (output) change by its first lag, it turns out that in explaining inequality growth rate by the first lag of output growth the gain in R^2 was 0,02 points, while in the case of substituting inequality growth by its first lag in output growth equation the gain was almost 0,10 points in R^2. While appearing due to technical constrains the first lag of inequality change creates, however, an opportunity to go deeper into the second order derivatives, which may be on the other hand justified by theoretical researches indicating that these are mainly second and third order derivatives which are responsible for determinacy of dynamics in models with heterogeneous distribution of wealth.[25]

Finally, while solving identification problem in estimation of simultaneous equations system, the first lag of endogenous variable of equation (2) when appearing in equation (1) may, alternatively, cause an autocorrelation problem entailing bias in estimated parameters. In an attempt to solve this problem following Arellano and Bover (1995) the matrix of instrumental variables was created as follows:

24 For discussions see for example Benerjee and Duflo (2003)

[25] see for example Ghiglino (2005); Ghiglino and Olszak-Duquenne(2001)

$$\mathbf{Vj}=\begin{vmatrix} v'_{j1} & 0' & \cdots & 0' \\ 0' & v'_{j2} & \cdots & 0' \\ \vdots & \cdots & \ddots & \vdots \\ 0' & 0' & 0' & a'_j \end{vmatrix}$$

where $v_{j,t} = z_{j,t-2}, \ln(y_{j,t-2}), \ln(g_{j,t-2})$ and $a'_j = \lfloor w'_j \quad \bar{v}_j \rfloor$ with w'_i including time invariant country specific variables, and $\bar{v}_i$ consisting of the mean values of time varying variables. Thereafter GMM estimation was used for the following equation system:

$$\hat{\bar{Y}}_{j,t} = \tilde{\alpha}_0 + \tilde{\alpha}_1 \times \ln(\bar{y}_{j,t-1}) + \tilde{\alpha}_2 \times \ln^2(\bar{y}_{j,t-1}) + \tilde{\alpha}_3 \times \hat{\bar{G}}_{i,t-1} + \tilde{\alpha}_4 \times \ln(\bar{g}_{j,t-1}) + \sum_{i=5}^{n} \tilde{\alpha}_i \times \bar{z}_{i,j,t-1} + \varepsilon_{j,t} \qquad (5)$$

$$\hat{\bar{G}}_{j,t} = \tilde{\beta}_0 + \tilde{\beta}_1 \times \ln(\bar{g}_{j,t-1}) + \tilde{\beta}_2 \times \hat{\bar{Y}}_{j,t} + \tilde{\beta}_3 \times \ln(\bar{y}_{j,t-1}) + \tilde{\beta}_4 \times \ln^2(\bar{y}_{j,t-1}) + \sum_{i=5}^{n} \tilde{\beta}_i \times \bar{z}_{i,j,t-1} + \xi_{j,t} \qquad (6)$$

where $\hat{\bar{Y}}_{j,t}, \hat{\bar{G}}_{j,t}, \bar{y}_{j,t}, \bar{g}_{j,t}, \bar{z}_{j,t}$ all denote the first differences.

While losing twenty two more observations, however, with this methodology it is still possible to identify all the parameters. Appendix S.vii reports $\tilde{\alpha}_i, \tilde{\beta}_i$ of estimated equations (5),(6) Non Linear Least Squares. As long as these parameters mainly are comparable with α_i, β_i parameters of equations (1),(2) [26] the further analysis is based on estimated α_i, β_i since mean differenced fixed effect seems more meaningful than the first differenced one.

Concerning variables included in $z_{i,t}$ since the main idea of equation system (1), (2) is to estimate structural parameters for the further dynamical analysis rather than to explain the change in output or inequality, $z_{j,i,t-1}$ includes variables which may have simultaneous effect on output and inequality. The attempt is done to incorporate variables usually controlled in inequality and economic growth empiric models (for both endogenous inequality as well as endogenous output growth) and transitional countries specific variables (such as speed of liberalization, initial macroeconomic distortions, etc.). For the time trend, since years of the *most intense changes*[27] differ among sample countries (Poland, Hungary and Romania had them in 1990, while Ukraine and Turkmenistan only in 1994) it seems reasonable to control any current year starting to account after the year of most intense changes for the given country individually rather than to control for the time trend in general (for 'pre intense change years' dummies are set to 0).

Here a general-to-simple method is adopted. Whenever variable z_i had both parameters α_i and β_i with P -value>.100, both of these parameters were jointly tested for the null hypothesis stating that they are jointly equal to zero. Whenever the hypothesis was not rejected under 90% probability the variable was taken away.

Nonetheless, there are still two major problems that may be caused by the exogenous vector $z_{j,i,t-1}$. First of all to avoid possible multicollinearity the raw correlation between variables is preliminary tested. Thus, due to the high correlation between government expenditures and budget deficit only the first one is kept. On the other hand, due to high correlation between inflation and (de Melo et al. (1997)) PRIN1 component describing initial macroeconomic distortions for transitional countries, the latter was taken by only three of its components, namely: initial dependence on trade, years under central planning, and dummy for the state.

A second potential problem with $z_{j,i,t-1}$ is the possible endogeneity of included policy or

[26] comparable here means in the dynamic contest of the paper, since for the dynamical analyzes the relative magnitude and signs of coefficients α_i, β_i mainly matters rather than the absolute value

[27] The 'definition' and schedule of the years of most intense changes is taken from Åslund at al (1996)

other time varying variables. The model assumes both policy choice and political stability are exogenous to output and/or inequality changes. This assumption, on the one hand, simplifies the empirical model (which is in some extent fairly complicated), while, on the other hand, sounds consistent to the main assumption that there is no enough democracy within countries, to ensure that redistribution policy is determined within the system. However, when distorted in the real life, this assumption may cause a bias. To discuss briefly, wages and employment during the transition depend not only on the marginal productivity of labor (hence on the level of accumulated domestic capital) as it is supposed by classical theory but also on the choice and speed of privatization.[28] Indeed, one of the noticeable differences between transition policies of former SU and Central and Eastern European countries was the choice of more flexible wages in the former case and more flexible employment in the latter one[29], even within FSU Baltic countries show significantly higher unemployment than the rest of the republics. On the other hand, the degree and efforts to control inflation, the speed and model of privatization, structure and size of governmental expenditures, in some extent depend either on initial distortions[30] or on the political bargaining power of certain groups of population.[31] While the former is obviously exogenous for this model, the latter is assumed to be exogenous and mainly determined by historical power gained by given groups of population and related to the cultural preferences rather than to output or inequality growth.

To get rid of this endogenaity all the time varying variables[32] are instrumented by their own and some other exogenous variables lagged values or dummies (here GMM 2SLS instrumentation is employed) first, then the predicted values are used in Non Linear Least Square to estimate equations (1), (2). However, since for some of the variables variation is predicted up to the 80%-85% for the others (such as governmental expenditures or wage growth) only 20%-25% is determined by the instruments, for the dynamical analyses both estimations (with instrumented and not-instrumented time varying variables) are used as alternatives.

Finally, due to the assumption of the main channel going from inequality to output through domestic savings and investments, domestic investments are not controlled and only foreign direct investments are included in $z_{j,i,t-1}$. Although it is natural to suspect that the inflow of FDI depends on output growth, the model assumes, that for the transition period it mainly depends on the exchange policy, economic openness of the country and political stability (all of them considerably differ among countries), so that the correlation between FDI and $\hat{Y}_{j,t}$ is not crucial.[33]

To check the robustness of results different models were estimated. This includes different data for inequality and different choice of exogenous variables. In the former case, income Gini and/or earnings Gini coefficients are regarded separately or jointly. In the latter case, general and simplified models are estimated, and the values of most intense shocks are added to time varying exogenous variables. In addition, parameters α_3 and β_2 were tested for the joint probability to be equal to zero and under the probability of 88% -95% (rejection probability differs for different models) null hypothesis was rejected. Furthermore, for the joint probability of five parameters: $\alpha_3, \alpha_4, \beta_2, \beta_3, \beta_4$ to be equal to zero, with F-statistics the hypothesis was rejected with 95%-99%

[28] for discussions see for example Ferreira (1999); Aghion and Commander(1999)

[29] see Tichit (2006)

30 as it is argued by De Melo et al. (1996)

[31] models of "rent seeking transition" Åslund et al. (1996); Milanovic (1995)

[32] this includes both policy variables and population variables such as population size or demography which may due to the high level of migration be influenced by both change in inequality and output growth

[33] Surprisingly, the raw correlation between FDI and other variables showed the highest inter-correlation between education (measured by enrollment in higher education) ' 0,58' and demography (measured by the share of population of workable age in the total population) ' 0,56'; while with GDPG the correlation is 0,27, inflation '-0,29' and Political Stability '0,10'.

certainty. Finally, α_2 and β_4, as the sources of non uniqueness of possible equilibria, were also jointly tested to the probability to be equal to zero, here again null hypothesis was rejected by 95%-99%. Alternatively Log Likelihood Ratio is checked to test these hypotheses, here H0 was rejected with 0,00 -0,070 degree of certainty in all the cases (Appendixes S.iii - S.vi report F statistics and Log Likelihood ratios for given models estimated).

3.4 General results

With respect of Gini coefficient there are three different models regarded. In the first case inequality is measured only by earnings Gini (EGINI), in the second case both earnings and income Ginis (IGINI) were considered, depending on which of these coefficients had higher mean value for a given country (the variable is labeled IEGINI), while in the third case only IGINI is used. It turns out that first two models perform better in terms of both R^2 and number of iterations required for the convergence. The model with IGINI, on the other hand displayed rather poor explanatory power (especially in the case of estimated equation (2)) and had significant problems with convergence. This is the main reason that the model with IGINI is not reported in general form. For the same reason shock variables are added to the model with EGINI.

Tables S.i-S.viii report the results on estimated models. In equation (1) both α_3, α_4 gained negative and significant, implying negative effect of both: inequality at the beginning of period as well as its change in previous year, on the current change in output. As for the output effect on the change in income inequality it has positive coefficient for the linear term and negative for the quadratic one. Output growth has positive and statistically significant effect on the increasing inequality.

Concerning variables included in vector $z_{j,i}$ probably the most noteworthy finding is that 'socio-cultural' variables such as 'years under the central planner' got higher statistical significance in equation(2) rather than (1), hence enter into economic growth mainly indirectly via their effect on changing inequality. The parameter of wage level is surprisingly in (1) always positive. Some of the initial distortions (such as pre-transitional trade dependence) seem to have long run effect. Time trend is not quite clear for the output growth, while for the income inequality is always positive.

Appendix S.ii reports estimated coefficients for the model with $z_{j,i,t}$ including values of policy variables at the year of most intense shocks (highest value of inflation that country j experienced etc.). It seems that the single shock variable having long-run implication was governmental expenditures. Moreover, though current level of the governmental expenditures is always negatively (and almost always significantly) correlated with subsequent growth, at the lowest level governmental expenditures have positive and statistically significant effect on output growth. This sounds in the some extent in well known spirit of Keynes. On the other hand, this is in the same line with Cornia and Popov (1998) concluding that private investments were not enough to substitute the governmental investments. Surprisingly, it seems to be not the highest level of inflation but rather the lowest level of governmental expenditures, that has long-ran implication on the change in income inequality as well. However, even when controlling for the maximal shock in variables responsible for income (or wealth) redistribution, such as intense inflation or lowest wages, employment and governmental expenditures, both of the coefficients α_2, β_3 are still statistically significant at 1% and have kept their signs.

4. Steady state, local dynamics and government policy

4.1 Steady states

Equation system (1)-(2) assumes the simplest form of nonlinearity when output enters into the growth equation by its quadratic form. Since in equation (1) and (2) coefficients of the quadratic term are statistically significant this form of nonlinearity is preserved in basic model for further analysis. For the 'representative' country with output and inequality changes described by deterministic parts of equations (1) and (2) it may be shown that the level of conditional convergence of output depends on predetermined level of policy variables included in **Z**. Indeed, the steady state values of y are to be defined by the following equation:

$\Lambda a^2 + \Psi a + \Omega = 0$ (7)

Where:

$$\Lambda = \alpha_2 - \frac{\alpha_2\beta_2 + \beta_4}{\beta_1 + \alpha_4\beta_2}(\alpha_4 + \alpha_3\beta_1)$$

$$\Psi = \alpha_1 + \alpha_3\beta_3 - \frac{(\beta_3 + \beta_2\alpha_1)(\alpha_4 + \alpha_3\beta_1)}{\beta_1 + \alpha_4\beta_2}$$

$$\Omega = \alpha_0 + \alpha_3\beta_0 - \frac{(\alpha_4 + \alpha_3\beta_1)(\beta_0 + \beta_2\alpha_0)}{\beta_1 + \alpha_4\beta_2} + \sum\left[(\alpha_i + \alpha_3\beta_i) - \frac{(\alpha_4 + \alpha_3\beta_i)}{\beta_1 + \alpha_4\beta_2}(\beta_i + \beta_2\alpha_i)\right] \times z_{j^.}$$

and $a = \ln(y)$

Hence whether there is one, two or no steady state at all depends on **Z**.

Moreover, because in the case of single steady state (when the discriminant of (7) is equal to '0') $\ln(y^*_j)>0$ ($= 0{,}5724\ ;0{,}694\ ;5{,}51$ *or* $\frac{y_j^*}{\underline{y}_j} \approx 1{,}77\ ;2{,}00\ ;248$ for three out of for tested models, namely: (appendixes) ***s.iii, s.iv*** and ***s.v*** respectively. And is negative =-0,667 for the model with GEXSH reported in appendix ***s.vi***)[34] it is possible to have both roots of (7) positive. The negative root, on the other hand will imply $\frac{y^*_j}{\underline{y}_j} < 1$; as long as $\underline{y}_j$ is lower than pre-transitional Y_{1989} for all the sample countries, the negative $y^*_{j,i}$ will imply convergence to the steady state of lower output level than pre-transitional Y_{1989}. Because of the parabolic shape of (7) lower $y^*_{j,1}$ will actually imply higher $y^*_{j,2}$, on the one hand, while on the other hand, the steady state level of output is to be nonlinear in policy variables and initial level of inequality. This makes dynamical analysis of higher interest.

As for steady state level of income inequality it is to be determined by the following equation:

$$\ln(g) = -\frac{\beta_0 + \beta_2\alpha_0}{\beta_1 + \alpha_4\beta_2} - \frac{\beta_3 + \beta_2\alpha_1}{\beta_1 + \alpha_4\beta_2} \times \ln(y) - \frac{\beta_2\alpha_2 + \beta_4}{\beta_1 + \alpha_4\beta_2} \times \ln^2(y) - \sum\left(\frac{\beta_i + \beta_2\alpha_i}{\beta_1 + \alpha_4\beta_2}\right) \times z_{j^*}.$$

[34] The log-liniarization of output and inequality index and further differencing by sample mean log value implies that $\ln(\tilde{y}_{j,t}) = \ln(y_{j,t}) - \ln(\underline{y}_j)$ or equally $\ln(\tilde{y}_{j,t}) = \ln\left(\frac{y_{j,t}}{\underline{y}_j}\right)$ and $\ln(\tilde{g}_{j,t}) = \ln\left(\frac{g_{j,t}}{\overline{g}_j}\right)$. Since in equations (1),(2) the squared term of the output is differenced by the observed minimum of country j, while the linear term by the sample mean $\ln(\tilde{y}_{j,t}) \equiv \ln(y_{j,t}) - \ln(\underline{y}_j) \Rightarrow \ln(y_{j,t}) - \ln(\overline{y}_j) = \ln(\tilde{y}_{j,t}) + \ln\left(\frac{\underline{y}_j}{\overline{y}_j}\right)$. So that the term $\psi \ln\left(\frac{\underline{y}_j}{\overline{y}_j}\right)$ can be added to the constant term.

4.2 Local dynamics

To check the robustness of dynamic results, analyses are conducted for four different estimations, namely: EGINI simplified with not instrumented policy variables (appendix S.2.iii); IEGINI simplified with policy variables instrumented (appendix S.2.iv); IGINI simplified with policy variables instrumented (appendix S.2.v)[35]; EGINI and GEX shock with policy variables not instrumented (appendix S.2.vi). It turns out that three former models reveal the same dynamic pattern and only model where governmental expenditures at the lowest level enter as an explanatory variable differs significantly by its dynamics.

As it is shown in appendix T.iii the system of equations (1), (2) can be approximated by the following system of differential equations:

$$\dot{f}(y_j(t)) = \alpha_0 + \alpha_1 \times f(y_j(t)) + \alpha_2 \times f^2(y_j(t)) + \alpha_3(e^{\varphi(t)} - 1) + \alpha_4 \times h(g_j(t)) + \sum_{i=5} \alpha_i z^*_{j,i} \qquad \text{(i.8)}$$

$$\begin{aligned}\dot{\varphi}(t) = {} & \beta_0 + \alpha_0\beta_2 + (\beta_1 + \beta_2\alpha_4)\times h(g_j(t)) + (\beta_3 + \alpha_1\beta_2)\times f(y_j(t)) + (\beta_4 + \beta_2\alpha_2)\times f^2(y_j(t)) + \\ & + (\beta_2\alpha_3 - 1)(e^{\varphi(t)} - 1) + \sum_{i=5}(\beta_i + \beta_2\alpha_i) z^*_{j,i}\end{aligned} \qquad \text{(i.7)}$$

$$\dot{h}(g_j(t)) = e^{\varphi(t)} - 1 \qquad \text{(i.6)}$$

Where equations (i.6)-(i.8) describe the laws of motion of income inequality, change in income inequality and output respectively.[36]

So that (from the same appendix) the Jakobian matrix of dynamical system (i.6)-(i.8) when linearized at steady states will look as following:

$$/J/= \begin{bmatrix} \beta_2\alpha_3 - 1 & \beta_1 + \beta_2\alpha_4 & \beta_3 + \alpha_1\beta_2 + 2(\beta_4 + \beta_2\alpha_2) \times f(y_{j,i}*) \\ 1 & 0 & 0 \\ \alpha_3 & \alpha_4 & \alpha_1 + 2\alpha_2 \times f(y_{j,i}*) \end{bmatrix} \qquad \text{(i.9.1)}$$

Where $y_{j,i}*$ is steady state value of output and i=1,2.

Two basic questions that are to be investigated in this part are: 1) what is the local dynamics[37] of the system (i.6)-(i.8); 2) whether this dynamics depends on: a) initial conditions; b) coordinates of the fixed point (which, in turn, is the function of policy variables included in Z).

There are two points that may be worth mentioning here. First of all, to make it easier to follow the analysis here and latter on in appendix T.ii all the simple and synthetic parameters of equations are calculated and reported for different estimations in a single table. Second, since all the different policy effects along with country specific variables enter into the same vector Z, when spiking about policy effect for the dynamical system (i.6)-(i.8), one should keep in mind that the different combinations of policy choice may end up with same z (as a scalar). Moreover, since Z also includes country specific variables, that gained statistically significant parameters for either of equations (1) or (2), the same combination of policy choice may actually have different final Z for

[35] Although this model has rather poor explanatory power, it is still interesting to see which kind of dynamical pattern it predicts, as long as it has income Ginis (not approximation by earnings Gini) as dependent variable

[36] since time dummy has not enter into any of analyzed models all the equations in the system (i-6)-(i.8) are autonomous

[37] 'local' means transition that a system starts sufficiently close to the fixed point in order to be kept within the basin of attraction, when the latter exists

the countries differing in these specific features. Hence, when speaking about Z the meaning is to be the general effect of all underlying policy and country descriptions included in Z.[38]

Matrix (i.9.1) has the following characteristic polynomial:

$$-\lambda^3 + (a_{11} + a_{33})\lambda^2 + (a_{13}a_{31} - a_{11}a_{33} + a_{12})\lambda + (a_{13}a_{32} - a_{12}a_{33}) = 0 \quad (8)$$

with:

$$\begin{aligned} a_{11} &\equiv \beta_2\alpha_3 - 1 \\ a_{12} &\equiv \beta_1 + \beta_2\alpha_4 \\ a_{13} &\equiv \beta_3 + \alpha_1\beta_2 - 2(\beta_4 + \beta_2\alpha_2) f(y^*_{j,i}) \\ a_{21} &\equiv 1 \\ a_{31} &\equiv \alpha_3 \\ a_{32} &\equiv \alpha_4 \\ a_{33} &\equiv \alpha_1 + 2\alpha_2 f(y^*_{j,i}) \end{aligned} \quad (8.1)$$

Or for the sake of simplification (8) can be rewritten as:

$$\lambda^3 + b\lambda^2 + c\lambda + d = 0 \quad (8.2)$$

Since equation (8) is of third degree in λ it is cubic equation that is to be investigated for clarification of the nature of characteristic roots of the system (i.6)-(i.8). Moreover, as it can be noticed from (8)- (8.2) all the parameters b, c and d are functions of $f(y^*_{j,i})$ (for details see appendix T.III; calculated components of parameters b, c and d can be also found in the appendix T.ii). Hence there are two steps that are taken in further analyses in the attempt to answer questions posed. At the first stage the core nature of characteristic roots (three real roots or one real root and two imaginary conjugates) is clarified (this will also imply (non)cyclical dynamic pattern of the system). At the second stage, the sign of the roots is analysed for the different $f(y^*_{j,i})$ implied by different policy choice.

For the cubic equation (8.2) to have one real and two imaginary roots the sufficient (but not necessary) condition is $\delta^2 = b^2 - 3c < 0$, in this case the left hand side of (8.2) will (geometrically) have no extreme points at all. More general condition for having two of roots in (8.2) as complex conjugates is a positive discriminant. As it is shown in appendix T.III equation (8.2) actually has positive discriminant for all the possible $f(y_{j,i}*)$'s in the three cases out of four investigated, while for the estimated model with GEXSH this is true only for fixed points that are located within interval: '-3.7538388 < f(y*) <2.77915528' (in terms of country minimum observed output this interval will require $.024 < \frac{y^*_j}{\underline{y}_j} < 16.14$). This implies that in the first three cases system (i.6)-(i.8) will have cyclical dynamic pattern in two dimensions and non-cyclical in the third one for all the policy choices whenever fixed point exists (equation (7) has real solutions) and economy starts its transition sufficiently close to it. In contrast, for the fourth case this dynamical pattern will be true only for the transition to fixed points situated within above mentioned interval, in all the other cases the system will have non-cyclical dynamics in all the three dimensions.

As for the sign of characteristic roots implying the (local) stability of fixed points, one can follow the sign (and its change) of the real root by the sign of determinant of (i.9.1). However, for the real part of imaginary roots the sign of determinant is of little help. Appendix T.IV analyses

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[38] Actually, vector Z is also to include $\alpha_1 \ln\left(\frac{\underline{y}_j}{\bar{y}_j}\right)$ and $(\beta_3 + \beta_2\alpha_1)\ln\left(\frac{\underline{y}_j}{\bar{y}_j}\right)$ where $\underline{y}_j$ and $\bar{y}_j$ for the country j are observed minimum and mean output, respectively.

the sign of real part for imaginary roots taking the mathematical formula of $re|\lambda_{2,3}|$ of cubic equation as a starting point. It is shown (in the same appendix), that for the first three estimations there are no intervals f(y*) implying positive real values of imaginary roots. For these estimations system (i.6)-(i.8) is spirally (locally) converging in two dimensions unconditional of initially conditions and/or policy choice. There is no possibility for the Andronov-Hopf style bifurcation for these estimations as well. Nonetheless, for the fourth estimated model the policy choice of z such as $f(y^*_{j,i}(\tilde{z}^*_j)) \cong -1.604$ seems to be (local) bifurcation point. Whenever parameter z^*_j passes through $\tilde{z}^*_j$ it may give a birth to the limit cycle dynamics for the system.

For the real root, on the other hand, all the three former models displayed negative (positive) sign (i.e. one dimensional (in)stability) for the lower(higher) of two $f(y^*_{j,i})$'s.[39]

Hence, to summarize, for the three dimensional dynamical system (i.6)-(i.8) in the space of *output, income inequality* and *the change in income inequality* , with estimated parameters as they are given in appendixes S.iii-S.v stable focus is the dynamic pattern for the lower $f(y^*_{j,i})$ and saddle focus for the higher one. In both of these cases the system is to display cyclical dynamics in two dimensions and non-cyclical in the third one when the transition is started sufficiently close to the new steady state. Nonetheless, for the lower steady state initial conditions will not matter as long as the system is locally stable in all three dimensions. While for the higher one, as long as the single stable transition path is situated on the invariant two-dimensional manifold of imaginary eigenvalues, whenever pre-transitionally the system was on this invariant plain to be kept there after reforms the following condition is to be hold:

$$\frac{\dot{h}(g_j(t))}{h(g_j(t))} = \frac{\dot{f}(y_j(t))}{f(y_j(t))} \qquad (9)$$

Substituting $\dot{h}(g_j(t))$ and $\dot{f}(y_j(t))$ by left had sides of (1.vi) and (1.v) respectively, expression (19) can be rewritten as:

$$\frac{h(g_j(t))}{f(y_j(t))} \times \left[\alpha_0 + \alpha_3\beta_0 + (\alpha_1 + \alpha_3\beta_3) \times f(y_j(t)) + \alpha_2 \times f^2(y_j(t)) + (\alpha_4 + \alpha_3\beta_1) \times h(g(t)) + \sum(\alpha_i + \alpha_3\beta_i) \times z_{j,i'}\right] =$$
$$= \beta_0 + \beta_2\alpha_0 + (\beta_3 + \beta_2\alpha_1) \times f(y_j(t)) + (\beta_4 + \beta_2\alpha_2) \times f^2(y_j(t)) + (\beta_1 + \beta_2\alpha_4) \times h(g_j(t)) + \sum(\beta_i + \beta_2\alpha_i) \times z_{j,i'}$$

(10)

This implies that for dynamical system which is initially on its saddle path toward the steady state, when new policy variable z' is introduced, to be kept on the converging rout it mast hold that:

$$\frac{h(g_j(t))}{f(y_j(t))} \times \left(\frac{\partial \dot{f}(y_j(t))}{\partial z'}\right) = \frac{\partial \dot{h}(g_j(t))}{\partial z'} \qquad (11)$$

Or equally:

$$\frac{h(g_j(t))}{f(y_j(t))}(\alpha' + \alpha_3\beta') = \beta' + \beta_2\alpha'$$

Or equally:

$$\left(\frac{h(g_j(t))}{f(y_j(t))} - \beta_2\right) \times \alpha' = \left(\frac{h(g_j(t))}{f(y_j(t))} - \alpha_3\right) \times \beta' \qquad (12)$$

Where α', β' are the effects of newly introduced policy on output and income inequality respectively, $\alpha_3 = \frac{\partial \dot{f}(y_j(t))}{\partial \dot{h}(g_j(t))}$ is an effect of inequality growth on output

[39] the 'brake' point for the model are f(y*)= 1,21;0,694;5,31 and '-12,29' respectively

growth, $\beta_2 = \frac{\partial \dot{h}(g_j(t))}{\partial \dot{f}(y_j(t))}$ is the marginal growth of inequality with respect to output growth; $f(y_j(t)) = \ln(y_j(t))$ and $h(g_j(t)) = \ln(g_j(t))$; finally $g_j(t)$ and $y_j(t)$ are current inequality and output respectively.

As for the model estimated and reported in appendix S.VI it is reasonable to believe that the dynamics here is also to be mainly of spiral in two dimensions and non-cyclical in the third one (as long as this dynamic pattern involves $.024 < \frac{y_j^*}{\underline{y}_j} < 16.14$). Nevertheless, dynamics (and control) is mach more complicated for this model as long as it involves possibility for bifurcation in the spiral dynamics, and hence the opportunities to end up within limit cycles.

Finally, to add couple of words about larger perturbation, it is obvious that more analyses are to be done in order to learn more about the global dynamics. However, there are at least two points that can be mentioned even within the scope of this paper. First of all, as long as system (i.6)-(i.8) is at least of the same degree of complexity as the famous system of Edward Lorenz, with larger perturbation it may have any possible trajectory including periodic or chaotic orbits. So that if dynamical system (i.6)-(i.8) is the proper approximation of the reality with respect of at least degree of complexity, then it is better to transform system 'gradually' in order to be kept within the basin of local attraction with higher degree of probability. This may in some sense add more light to the role of one of the most contradicting variables in transitional countries literature as CLI is. However, as long as the system (i.6)-(i.8) is to be regarded within the space of *output, income inequality* and *the change in income inequality*, it is reasonable to believe that the higher risk involved with higher speed of changes can be compensated by the proper control over the changing income inequality (once again one may compare experience of the Poland and countries emerged from FSU). Second, as long as the Jakobian matrix (i.9.1) for the larger perturbation is to look as (i.9) as it is described in appendix T.I, it seems that the dynamics of the system is to mainly be determined by the speed of change in income inequality, rather than inequality itself.

5. Conclusions

The current paper aimed to find out: i) *the dynamical pattern of inequality and output growth*; and ii) *the possibility for an economic system to be locked down under a poverty trap and/or wander in limit cycles as an outcome of a transition policy* under the assumption that there is no enough democracy to ensure that redistributive policy depends on the dynamics of income inequality.

It used the empirical experience of the countries which roughly some fifteen years ago had launched policy reforms in order to abandon planned economy for the market one (i.e. the former economic state for the better one). Starting from at least comparable conditions, today at the first glance they seem to have substantial gap in both output and inequality patterns. This raises a problem of possible existence of multiple equilibria and/or limit cycles.

To carry out the analysis at the first stage an empiric system of output and income inequality growths was econometrically estimated, while at the second stage the estimated coefficients were taken for the dynamical analysis. This analysis showed that there are at least two possible steady states with lower of them (in terms of output) to be locally stable in al the three dimensions (output, inequality and change in income inequality), while the higher one is saddle focus. In three cases (out of four regarded) the dynamics is cyclical in two dimensions and non-cyclical in the third one for all the possible police choices, when this choice has steady states and transition is started close enough to this steady state to be kept within the basin of attraction. In the forth case the same dynamical pattern is true when the policy choice is such that the steady state output, sample observed minimum output ratio for given country is within interval of

[.024;16.14]. For all the other steady states the dynamics is non-cyclical in all the three dimensions. Moreover, only for this model the possibility of Andronov-Hopf bifurcation rises.

For the larger perturbation and global dynamics more analysis are to be carried out, however within the scope of this paper two points can be already mentioned. First of all, the estimated system is complicated enough to have periodic or even chaotic orbits whenever it leaves the basin of attraction. This makes gradual changes of lower risk. Second, it seems that it is the change in income inequality rather than income distribution itself which determines the dynamic pattern of the system under the larger perturbation.

Although the analyzed results seem to be robust and close to the dynamical pattern of data available, there are still some significant limitations imposed by both strong assumptions and quality and availability of data. To discuss some of these limitations tree major ones probably should be mentioned as the most significant.

First of all, the period of fifteen years is rather short for the long-run analysis (although some of the sample countries managed to change not only several 'policy courses' but also several 'political systems' within this period). To solve this problem (and the problem of the shortage in data) the panel of the 20 countries was regarded. This raises another problem, which is the possible heterogeneity of the pattern. In the attempt to get rid of the country specific effects the sample was differenced by its mean, however there is nothing guarantying that countries differ only by the fixed effect but not by the slops as well. Moreover, due to the unbalanced nature of the data available the results can be rather sensitive to the number of observations available for different countries[40].

Second, it is often and quite fairly mentioned that these countries have experienced so many different shocks that it is rather difficult to find out which of them has which effect. These suspicions are in some extent confirmed by the rather low deterministic part especially in attempting to predict income inequality. The attempt was done so solve this problem by adding 'shock' levels of policy variables. This pointed out that the governmental expenditures at their lowest level seem to have long-ran effect. However, the problem unsolved is to a certain extent suspending in the air.

The third problem concerns to the assumption of the 'weak democracy' taking away the hypothetical role of the 'median voter' in a policy choice making process. This assumption being central for this paper may seem to be too strong for the real life. In an attempt to get rid of the possible feedback effect going from output and inequality growth to the policy choice, the last group of variables was instrumented and predicted values were used for the basic model estimation. This model displayed the same dynamics as an original one. Finally, this assumption in the real life can be much stronger than a 'simple exogenity' of a policy choice. Indeed, the political system itself may be endogenously determined within the system of output and inequality growth. Moreover, this group of endogenous variables may give a raise to the group of other variables also endogenous within the system such as corruption, army abuse etc. This undoubtedly important mechanism (which is not once mentioned in the relevant literature) can substantially alter the real dynamics of the system[41]. Along with the previously mentioned point this may raise the problem of the third and forth variables connected with both growth rates and omitted in the system.

[40] this problem is partially taken away by the fact that within different variables such as EGINI, INGINI and IEGINI, different countries had different weight (moreover, one may probably claim that different regions such as CEU or FSU got different weights), yet the analysis gain the same results

[41] this very important topic needs to have its own and deep investigation, while the purpose of this paper was to look into the long-run dynamical effect that 'uncontrolled' policy choice may have on an economic system , caused the solely by channels going through consumption and savings

1 .a. Statistical Appendix

i. List of the countries

1. Armenia
2. Azerbaijan
3. Belarus
4. Bulgaria
5. Czech Republic
6. Estonia
7. Georgia
8. Hungary
9. Kyrgyzstan
10. Latvia
11. Lithuania
12. Macedonia
13. Moldova
14. Poland
15. Romania
16. Russian Federation
17. Slovak Republic
18. Slovenia
19. Turkmenistan
20. Ukraine

1.*ii) List of all variables considered:*

Name	***description***	***Main source of data***
GDP	Ln(GDPt/GDP_{1989})	TransMONEE 2005 report
IGINI	Ln (Income Gini coeficient)	TransMONEE 2005 report, WIDER, in few cases interpolations are employed[*]
EGINI	Ln (Earnings Gini coeficient)	TransMONEE 2005 report
IEGINI	Ln (Earnings and Income Gini coeficient)[♣]	TransMONEE 2005 report
Country specific :		
Dummy variables and indaeces:		
SEA	0= landlocked; 1= having at least 1 sea	
RN (religion)	1=crestian; 0=muslim	
AR (area)	Ln (surface area [sk.km])	World Bank, *world development indicators*
NR (natural resorses)	0=poor;1=moderate; 2=rich	De Melo at al.(1997)
PS (political stability index)		Kaufman et al. index for 1996-2002
RZ	Dummy for the rubble zone	Rz=1 for all the FSU countries with exception of 3 Baltic Republics and Kyrgyzstan
CP (corruption index)		Corruption control index *World Bank, World Bank Policy Research Working Paper 3106*
YMIR(time dummy individual for any given country)	ij= 1 for the year of most intensive reforms for given country j ..2,3,4,..thereafter and 0 for all the years presiding the year of most intensive reforms	The year of most intensive reformes for individual country is taken from Åslund at al(1996)
WAR	1=was included into national conflict during transition[•]	
Planned economy effect , pre-transition distortions and transition specific variables:		
YEARL (years under the planned economy)	LN (years under planed economy)	De Melo at al.(1997)
STATE	0= no pre-transitional state institutions exist; 1=having some autonomy; 2=having pre-transitional state institutions	De Melo at al.(1997)
TD (trade dependence)	% of net export in pre-transitional GDP	De Melo at al.(1997)

[*] in all the cases logorimized Gini coefficient is taken as a number between 1and 100 (in contrast to 0and 1 interval widely used for Gini) regardless in which form the coefficient is reported in the data source

[♣] the data is taken only from TransMONEE report, however for the given year and given county the higher value of income or earnings Gini coefficient is used

[•] List of the countries involved in military conflicts includes Russia (for conflict in Chechnya) and Moldova (accounted for the unsolved conflict in Prednistrovia) in addition to the list of countries defined by Åslund at al. (1996) and widely used in transitional economies literature

PRIN2 (De Melo 1997 index of "initial level of development, resources and growth ")	Includes initial level of GDP; % of urbanized population; share of agriculture, industry and service; natural resources	De Melo at al.(1997)
CLI (cumulative liberalization index)		Worl Bank working papers De melo et al. (1996)
Population		
POPL (size of population)	Size of population with scale 1x1	TransMONEE
DEM (demography)	Share of population of 18-59 years (%)	TransMONEEE
EDL (education)	Higher education enrolment Unit Gross rates, per cent of population aged 19-24	TransMONEE
MIGR (migration rate)	Net migration per 100 000 of population	TransMONEE mainly[+]
Policy effect :		
AGR (share of agricultural product as% in GDP)	share of agricultural product as% in GDP	*De Melo et al.* (1996) for pre-transitional 1989 year and *World Bank development indicators report* for the year 2005, all the years in-between are linearly interpolated
Labor Market		
WG (wage growth rate)	(Wt-Wt-1)/Wt-1	TransMONEE mainly
W (real wage as proportion to pre transitional 1989 level)	Wt/ W1989	TransMONEE mainly
EMPL	Annual average number of employed as per cent of population aged 15-59	TransMONEE mainly, individual country reports
EMPLG (employment growth)	(EMPLt-EMPLt-1)/EMPLt-1	
Fiscal policy		
GEX (governmental expenditures)	% of GDP	TransMONEE mainly
Monetary policy		
INFLL (inflation)	Ln (Annual average per cent change in consumer prices)	TransMONEE mainly
Trade effect		
FDI	Annual FDI inward stock as % of GDP	UNCTAD data on net capital inflow as % of GDP
Policy Shock variables:		
INFLLSH	The highest level of INFLL for the given country	
GEXSH	The lowest level of GEX variable for given country	
WSH	The lowest level of W for the given country	
WGSH	The largest annual drop in wage level for given country	
EMPLSH	The lowest level of EMPL for given country	
EMPLGSH	The highest annual drop in the employment experienced by the individual country	

[+] variable is used only as instrument for POPL

iii. Descriptive statistics for endogenous variables

Country	income Gini		earnings Gini		GDP per capita (1,000$ at PPPs)		Dynamic* pattern	
	Level mean	annual Growth mean	Level mean	annual Growth mean	Level mean	annual Growth mean	Inequality	GDP
Within Countries								
Czech Republic	**22.1**	**0.589**	**25.177**	**2.540**	**12.4354**	**0.00517**	**U**	**U**
	[1.53]	[6.1913]	[2.5209]	[7.4578]	[0.0041]	[0.0818]		
Hungary	**24.5**	**1.639**	**32.086**	**5.177**	**10.2265**	**0.00517**	**R-C**	**R-C**
	[1.809]	[2.2541]	[3.8567]	[4.0751]	[1.7038]	[0.0948]		
Poland	**31.5**	**2.012**	**27.210**	**3.149**	**7.8489**	**0.01090**	**R-C**	**R-C**
	[3.0538]	[4.5920]	[3.4041]	[3.5312]	[1.7402]	[0.10233]		
Slovenia	**25.1**	**-0.338**	**28.800**	**3.164**	**13.6788**	**0.00855**	**/**	**U**
	[0.8957]	[3.9872]	[3.5476]	[11.0424]	[1.719]	[0.09419]		
Estonia	**36.6**	**-0.080**	**35.633**	**3.915**	**8.5639**	**0.00026**	**R-C**	**R-C**
	[3.5791]	[4.6172]	[5.52111]	[8.4310]	[2.554]	[0.13038]		
Latvia	**32.2**	**0.107**	**31.654**	**3.266**	**6.9204**	**-0.00842**	**R-C**	**R-C**
	[3.2584]	[1.0781]	[3.5302]	[12.7189]	[1.904]	[0.1419]		
Lithuania	**33.0**	**0.672**	**35.880**	**-0.517**	**8.2024**	**-0.01339**	**R-C**	**R-C**
	[3.4233]	[6.3257]	[3.8016]	[4.0773]	[1.685]	[0.1152]		
Bulgaria	**34.1**	**1.294**	**25.400**	**23.650**	**5.9136**	**-0.00743**	**U**	**R-C**
	[3.913]	[6.3681]	[3.2690]	-	[1.660]	[0.0848]		
Romania	**28.4**	**1.941**	**31.008**	**6.532**	**5.6094**	**-0.00514**	**U**	**Inc-U**
	[3.3516]	[8.8103]	[8.1878]	[8.1486]	[0.660]	[0.0859]		
FYR Macedonia	**31.1**	**2.594**	**26.323**	**2.348**	**6.1279**	**-0.01736**	**R-C**	****
	[2.2825]	[5.9965]	[1.8619]	[9.0824]	[0.593]	[0.065]		
Belarus	**24.5**	**-0.314**	**34.245**	**1.076**	**4.2872**	**-0.00611**	**R-C**	**R-C**
	[0.7939]	[4.0376]	[4.0428]	[6.7691]	[0.773]	[0.1024]		
Moldova	**40.5**	**-0.170**	**39.609**	**0.404**	**1.8377**	**-0.03550**	**R-C**	****
	[8.6719]	[0.3818]	[5.2665]	[8.2660]	[0.706]	[0.1469]		
Russia	**44.7**	**3.787**	**41.080**	**7.205**	**7.2412**	**-0.01814**	**R-C**	**Inv-U**
	[2.7809]	[8.9972]	[9.4246]	[11.2601]	[0.811]	[0.0819]		
Ukraine	**34.5**	**1.210**	**38.280**	**6.740**	**5.1055**	**-0.03039**	**U**	****
	[7.8673]	[11.7969]	[7.656]	[17.8981]	[1.291]	[0.1009]		
Armenia	**34.3**	**-0.097**	**35.186**	**7.305**	**2.2672**	**-0.01873**	**inv-U**	**inc-U**
	[8.5781]	[0.16965]	[7.2956]	[15.0394]	[1.654]	[0.1583]		
Azerbaijan	**33.7**	**12.026**	**44.200**	**1.380**	**2.7271**	**-0.02851**	**U**	****
	[3.0203]	[16.9340]	[7.2972]	[2.9881]	[0.673]	[0.14733]		
Georgia	**42.4**	**-0.017**	**39.200**	**8.440**	**2.1980**	**-0.03644**	**inv-U**	**/**
	[9.8374]	[0.0481]	[8.1873]	-	[1.003]	[0.1618]		
Kyrgyzstan	**37.6**	**-1.713**	**42.242**	**5.969**	**1.5746**	**-0.02224**	**R-C**	**R-C**
	[5.3794]	[5.4954]	[7.3827]	[16.1029]	[1.0320]	[0.10008]		
Turkmenistan	**24.3**	**0.054**	**24.450**	**5.290**	**3.7474**	**0.00017**	**U**	****
	[2.9866]	[0.3030]	[2.4569]	[30.3348]	[0.3757]	[0.2036]		
Slovakia	**25.6**	**2.072**	**-**	**-**	**9.8211**	**0.00109**	**U**	**R-C**
	[1.1021]	[4.2364]	-	-	[1.7019]	[0.1048]		
Between Countries								
Pre-transitional [1989]	**25.394**	-	**24.459**	-	**7.5687**	-	-	-
	[3.0181]	-	[3.3606]	-	[3.06025]	-	-	-
end of period observed [2004]	**33.913**	-	**38.717**	-	**8.0721**	-	-	-
	[6.4381]	-	[8.2296]	-	[4.9433]	-	-	-
For the period ove 1989-2004	**32.044**	**1.3633**	**33.561**	**5.1070**	**6.3167**	**-0.0108**	-	-
	[6.3974]	[2.8138]	[6.2188]	[5.1648]	[3.50232]	[0.01464]	-	-

*The description in last two columns is taken from Hua Wan (2002)

Appendix S.II

EGINI general with shock variables and policy variables instrumented

Parameter*	Estimate	Standard Error	t-statistic	P-value
A0	-40.5063	24.5404	-1.65060	[.099]
AGDP	-.774250	.287015	-2.69759	[.007]
ASEA	-5.20929	3.15212	-1.65263	[.098]
ATRD	-.047381	.100175	-.472985	[.636]
AFDI	1.93789	.739756	2.61963	[.009]
ACLI	.720287	.492981	1.46109	[.144]
AGEX	-.110548	.164403	-.672421	[.501]
AWG	-1.17763	1.06589	-1.10483	[.269]
AEMPLG	.154288	.069781	2.21104	[.027]
AEMPL	1.01453	1.35264	.750039	[.453]
ADEM	-54.0994	11.3942	-4.74798	[.000]
AGEXSH	-.041444	.045281	-.915246	[.360]
AWAR	.076931	.067873	1.13345	[.257]
AYMIR	-.149267	1.32892	-.112322	[.911]
APRIN2	. 173653	. 102531	1.69366	[.090]
AINFLLSH	1.27663	.781237	1.63412	[.102]
AINFLL	-.965945	.634488	-1.52240	[.128]
AWSH	7.84411	4.85110	1.61697	[.106]
AW	.144302	.087372	1.65158	[.099]
AEGINIGL	-.084732	.028752	-2.94701	[.003]
AEGINI	-.269398	.145500	-1.85152	[.064]
APOPL	. 218540	. 276600	.790093	[.429]
APS	. 140609	.0564678	2.49008	[.013]
ANR	. 379095	. 230751	1.64287	[.100]
ACP	-.708258	.651785	-1.08664	[.277]
AGDP2	. 211996	. 142000	1.49293	[.135]
AEMPLSH	-.707783	.441258	-1.60401	[.109]
AEDL	.167003	.046753	3.57202	[.000]
AAGR	.015235	.027338	.557275	[.577]
AYEARL	5.74857	3.39228	1.69461	[.090]
AAR	-.672594	.390938	-1.72046	[.085]
ARZ	-4.24438	2.29570	-1.84884	[.064]
B0	33.0322	33.7007	.980166	[.327]
BEGINI	-.713978	.092190	-7.74465	[.000]
BRN	-5.00446	5.82090	-.859741	[.390]
BWSH	-2.23082	1.47167	-1.51584	[.130]
BGEXSH	-.069707	.027941	-2.49483	[.013]
BPOPL	3.54944	5.08707	.697736	[.485]
BGDP	.626871	.132746	4.72232	[.000]
BEMPL	-6.64232	2.52471	-2.63092	[.009]
BWG	.313064	2.02707	.154442	[.877]
BPRIN2	-3.83412	2.49818	-1.53477	[.125]
BEDL	.054742	.843477	.064900	[.948]
BGEX	.259963	.611728	.424965	[.671]
BINFLLSH	.017542	.044793	.391621	[.695]
BINFLL	-.543901	1.21508	-.447624	[.654]
BDEM	35.5054	23.1034	1.53680	[.124]
BGDPG	.114588	.014436	7.93757	[.000]
BFDI	-.590809	1.38685	-.426008	[.670]
BEMPLGSH	1.27147	1.30338	.975513	[.329]
BEMPLG	-.621958	.130294	-4.77350	[.000]
BCLI	1.18794	.678158	1.75172	[.080]
BW	-.594392	.161709	-3.67569	[.000]
BPS	-3.70199	1.44859	-2.55558	[.011]
BWAR	.089532	.098109	.912586	[.361]
BTRD	.067977	.199804	.340219	[.734]
BNR	-2.46221	2.52791	-.974011	[.330]
BYEARL	4.79103	2.06794	2.31681	[.021]
BGDP2	-.306313	.359128	-.852934	[.394]
BEMPLSH	.150448	.601197	.250247	[.802]
BYMIR	.089780	.091619	.979927	[.327]
BAGR	.074282	.049957	1.48691	[.137]
BCP	-9.47923	1. 02478	-.924999	[.355]
BAR	1.13536	1.06894	1.06214	[.288]
BRZ	3.56107	3.56836	.997955	[.318]

*A--and B—parameters refer to GDP growth and IEGINI growth equations respectively

Appendix S.III

EGINI simplified with policy variables instrumented

Parameter*	Estimate	Standard Error	t-statistic	P-value
ASEA	.467672	.096879	4.82737	[.000]
AFDI	.121865	.044708	2.72578	[.006]
AEGINIGL	-.151055	.032987	-4.57920	[.000]
ATRD	-.025858	.052664	-.490992	[.623]
APRIN2	.023372	.082051	.284851	[.776]
AGEX	-062492	.255223	-.244851	[.807]
AINFLL	-.985296	.566814	-1.73830	[.082]
AW	1.28663	.454929	2.82819	[.005]
APS	.033925	.088776	.382138	[.702]
AEGINI	-.236885	.090661	-2.61285	[.009]
APOPL	.221657	.131750	1.68240	[.092]
AEDL	.042736	.139827	.305633	[.760]
AEMPLG	5.37819	1.10150	4.88263	[.000]
AGDP	-.360712	.073421	-4.91294	[.000]
AGDP2	.198009	.104204	1.90020	[.057]
ANR	.285085	.084559	3.37143	[.001]
AYEARL	-.016300	.026244	-.621110	[.535]
AWAR	-.481932	.153944	-3.13057	[.002]
AEMPL	-.555226	.133150	-4.16992	[.000]
AWG	.816901E-02	.896261	.911455E-02	[.993]
B0	14.6172	3.48722	4.19164	[.000]
BEGINI	-.828747	.072327	-11.4583	[.000]
BGDPG	.453926	.141685	3.20377	[.001]
BRN	-.057464	.035061	-1.63896	[.101]
BGEX	-1.59208	.603278	-2.63906	[.008]
BINFLL	.014973	.096896	.154522	[.877]
BW	-.481100	1.14172	-.421381	[.673]
BPRIN2	.075640	.161540	.468244	[.640]
BFDI	.078060	.101791	.766866	[.443]
BPS	-.126230	.152762	-.826322	[.409]
BTRD	-.408295	.104189	-3.91880	[.000]
BPOPL	.672423	.229069	2.93546	[.003]
BEDL	-.706448	.249980	-2.82602	[.005]
BEMPLG	2.09139	2.00515	1.04301	[.297]
BGDP	.859383	.191862	4.47918	[.000]
BGDP2	-.175961	.034055	-5.16702	[.000]
BNR	-.134611	.110788	-1.21504	[.224]
BYEARL	.053949	.044236	1.21957	[.223]
BWAR	-1.39695	.342104	-4.08341	[.000]
BEMPL	-.650065	.229860	-2.82810	[.005]
BWG	5.72065	2.05852	2.77901	[.005]

Number of observations = 102

Dependent variable: GDPG Dependent variable: EGINIG

R-squared = .822420 R-squared = .643692

*A--and B—parameters refer to GDP growth and IEGINI growth equations respectively

H_0	Log Likelihood Ratio	Prob.	F-statistics	Prob.
$\alpha_3 = \beta_2 = 0$	15.7508	[.000]	2.2705	[.110]
$\alpha_3 = \alpha_4 = \beta_2 = \beta_3 = \beta_4 = 0$	35.244	[.000]	2.7437	[.030]
$\alpha_2 = \beta_4 = 0$	26.3348	[.000]	7.4003	[.000]

Appendix S.IV

IEGINI simplified with policy variables not instrumented

Parameter*	Estimate	Standard Error	t-statistic	P-value
A0	-31.0	12.0	-2.58	[.010]
ASEA	1.42	.627	2.26	[.024]
ACLI	-.150	.057	-2.62	[.009]
AGEX	-.107	.053	-2.01	[.045]
AINFLL	-.518	.283	-1.83	[.067]
AW	8.39	2.79	3.01	[.003]
AEGINIGL	-.137	.039	-3.53	[.000]
AWG	6.38	1.64	3.90	[.000]
AEGINI	-.102	.034	-2.96	[.003]
APOPL	.634	.144	4.39	[.000]
AGDP	-.386	.045	-8.51	[.000]
AGDP2	.316	.146	2.16	[.031]
AEMPL	-1.70	.967	-1.76	[.079]
AWAR	-.011	.114	-.099	[.921]
AFDI	.082	.052	1.57	[.116]
AEMPLG	29.3	8.46	3.47	[.001]
AYEARL	-.711	2.31	-.308	[.758]
APS	2.63	.931	2.83	[.005]
AAGR	-.711	.205	-3.47	[.001]
B0	46.5	23.9	1.94	[.052]
BEGINI	-.385	.065	-5.91	[.000]
BRN	-.839	.201	-4.17	[.000]
BWG	-.124	.031	-3.95	[.000]
BGEX	-.140	.102	-1.37	[.169]
BINFLL	.186	.052	3.59	[.000]
BW	-.207	.074	-2.81	[.005]
BGDPG	.379	.131	2.88	[.004]
BFDI	.211	.104	2.02	[.043]
BCLI	.058	.120	.484	[.629]
BPOPL	-.011	.254	-.045	[.964]
BEMPLG	-.551	.215	-2.56	[.010]
BEMPL	-.293	.173	-1.69	[.090]
BGDP	.440	.098	4.49	[.000]
BGDP2	-.443	.289	-1.53	[.126]
BWAR	-.275	.228	-1.21	[.227]
BYEARL	8.93	4.88	1.83	[.067]
BPS	.170	1.87	.091	[.927]
BAGR	.158	.407	.389	[.697]

Number of observations = 125
Dependent variable: GDPG — Dependent variable: IEGINIG
R-squared = .788 — R-squared = .502
*A--and B—parameters refer to GDP growth and IEGINI growth equations respectively

H_0	Log Likelihood Ratio	Prob.	F-statistics	Prob.
$\alpha_3 = \beta_2 = 0$	5.37	[.070]	3.0211	[.050]
$\alpha_3 = \alpha_4 = \beta_2 = \beta_3 = \beta_4 = 0$	20.28	[.001]	2.9529	[.020]
$\alpha_2 = \beta_4 = 0$	5.80	[.050]	2.9299	[.060]

Appendix S.V

IGINI simplified with policy variables instrumented

Parameter*	Estimate	Standard Error	t-statistic	P-value
AEGINI	-.672	.367	-1.83	[.067]
AEGINIGL	-.683	.371	-1.84	[.065]
ASEA	-.164	.070	-2.35	[.019]
AFDI	3.59	1.02	3.53	[.000]
AGDP2	.230	.062	3.72	[.000]
AGEX	-1.18	.793	-1.49	[.136]
AINFLL	-.098	.036	-2.73	[.006]
APS	.096	.102	.947	[.344]
APRIN2	-.055	.092	-.595	[.552]
AEMPLG	2.98	1.18	2.53	[.012]
AGDP	-2.52	.430	-5.85	[.000]
AEMPL	.619	.976	.635	[.526]
ADEM	-.077	.069	-1.12	[.265]
ACLI	-.128	.050	-2.55	[.011]
AWAR	-.166	.159	-1.04	[.296]
AEDL	.014	.011	1.24	[.214]
ANR	.308	.090	3.43	[.001]
AAR	-.136	.038	-3.57	[.000]
ASTATE	-.011	.054	-.198	[.843]
B0	.283	.265	1.07	[.286]
BEGINI	-.427	.100	-4.25	[.000]
BFDI	-.329	.203	-1.62	[.106]
BGDP2	-.057	.031	-1.81	[.070]
BRN	-.130	.055	-2.36	[.018]
BGEX	.241	.244	.987	[.324]
BINFLL	.016	.740E-02	2.20	[.028]
BPS	-.012	.029	-.416	[.677]
BTRD	.099	.108	.923	[.356]
BPRIN2	.076	.034	2.25	[.024]
BEMPLG	-.306	.242	-1.27	[.206]
BGDP	.555	.134	4.13	[.000]
BEMPL	.096	.231	.416	[.677]
BGDPG	.023	.185E-02	12.3	[.000]
BDEM	-.425E-03	.017	-.025	[.980]
BCLI	.013	.010	1.28	[.199]
BWAR	.100	.045	2.22	[.026]
BEDL	-.220E-02	.309E-02	-.710	[.477]
BNR	.600E-02	.029	.209	[.834]
BAR	.011	.012	.929	[.353]
BSTATE	.048	.020	2.37	[.018]

Number of observations = 114

Dependent variable: GDPG   Dependent variable: IGINIG

R-squared = .747   R-squared = .225

*A--and B—parameters refer to GDP growth and IGINI growth equations respectively

H_0	Log Likelihood Ratio	Prob.	F-statistics	Prob.
$\alpha_3 = \beta_2 = 0$				
$\alpha_3 = \alpha_4 = \beta_2 = \beta_3 = \beta_4 = 0$				
$\alpha_2 = \beta_4 = 0$				

Appendix S.VI

EGINI simplified with policy variables not instrumented and GEXSH included

Parameter*	Estimate	Standard Error	t-statistic	P-value
A0	4.51	2.62	.72	[.085]
AFDI	.150	.041	3.68	[.000]
AEGINIGL	-.086	.032	-2.73	[.006]
ATRD	.023	.045	.520	[.603]
APRIN2	-.229	.074	-3.09	[.002]
AGEX	-.123	.058	-2.14	[.033]
AINFLL	-.769	.282	-2.73	[.006]
AW	1.18	.411	2.88	[.004]
APS	.021	.089	.235	[.814]
AEGINI	-.224	.077	-2.91	[.004]
APOPL	.384	.149	2.58	[.010]
AEDL	.167	.072	2.31	[.021]
AEMPLG	3.57	.969	3.68	[.000]
AGDP	-.289	.057	-5.07	[.000]
AGDP2	-.277	.220	-1.26	[.208]
AYEARL	-.061	.032	-1.92	[.055]
AWAR	-.654	.207	-3.16	[.002]
AEMPL	-.298	.078	-3.81	[.000]
AAR	-.018	.030	-.597	[.550]
AWG	.460	.153	3.01	[.003]
AGEXSH	.036	.845E-02	4.32	[.000]
B0	2.69	4.87	.553	[.581]
BEGINI	-.703	.087	-8.10	[.000]
BGDPG	.932	.149	6.24	[.000]
BRN	-.093	.029	-3.20	[.001]
BGEX	.016	.110	.150	[.881]
BINFLL	.106	.053	2.00	[.046]
BW	-2.98	.856	-3.48	[.000]
BPRIN2	.288	.150	1.93	[.054]
BFDI	-.060	.090	-.660	[.509]
BPS	-.268	.166	-1.62	[.106]
BTRD	-.218	.085	-2.57	[.010]
BPOPL	.289	.284	1.02	[.309]
BEDL	-.299	.137	-2.18	[.029]
BEMPLG	-2.75	1.88	-1.46	[.143]
BGDP	.532	.123	4.33	[.000]
BGDP2	-.081	.043	-1.87	[.061]
BYEARL	.112	.065	1.71	[.087]
BWAR	-.444	.393	-1.13	[.258]
BEMPL	.018	.147	.120	[.904]
B AR	.148	.068	2.19	[.028]
BWG	-1.29	.283	-4.58	[.000]
BGEXSH	-.038	.016	-2.32	[.020]

Number of observations = 103

Dependent variable: GDPG Dependent variable: EGINIG

R-squared = .816 R-squared = .563

*A--and B—parameters refer to GDP growth and EGINI growth equations respectively

H_0	Log Likelihood Ratio	Prob.	F-statistics	Prob.
$\alpha_3 = \beta_2 = 0$	5.8	[.050]	2.4916	[.090]
$\alpha_3 = \alpha_4 = \beta_2 = \beta_3 = \beta_4 = 0$	32.2	[.000]	2.7939	[.020]
$\alpha_2 = \beta_4 = 0$	8.2	[.020]	4.9631	[.010]

Appendix S.VII

Estimated parameters of equation system (5)-(6)

Parameter*	Estimate	Standard Error	t-statistic	P-value
A0	-57.7937	18.2867	-3.16042	[.002]
ASEA	.05337	.03011	1.77265	[.076]
AEGINIGL	-.11010	.03814	-2.88636	[.004]
AEGINI	-.23967	.06367	-3.76403	[.000]
AGDP	-.90268	.10360	-8.71285	[.000]
ANR	.01484	.01090	1.36089	[.174]
APS	.008457	.203552E-02	.415499	[.678]
ACLI	-.0148932	.0151263	-.984588	[.325]
AWAR	-.0273418	.01676	-1.63053	[.103]
ASTATE	.05277	.01414	3.73079	[.000]
APRIN2	.02159	.01108	1.94795	[.051]
AYEARL	-.18838	.05086	3.70371	[.000]
ATRD	.02906	.05908	.491939	[.623]
AAR	-.0180938	.574949E-02	-3.14704	[.002]
AYMIR	.122915E-02	.155340E-02	.791263	[.429]
AGDP2	.34446	.16140	2.13423	[.033]
AFDI	.092551E-02	.119332E-02	.775579	[.438]
AINFLL	-.097382	.145588	-.668887	[.504]
AW	.221265E-02	.065368E-02	3.38494	[.001]
APOPL	-.0962145	.25113	-.383122	[.702]
AEDL	.10936	.06367	1.71764	[.086]
AGEX	-.258594	.13270	-1.94871	[.051]
ACP	-.03190	.03645	-.875356	[.381]
ARZ	-.04695	.01965	-2.38962	[.017]
B0	1.08324	.543780	1.99206	[.046]
BEGINI	-1.41945	.068667	-20.6715	[.000]
BRN	-.063465	.068856	-.921714	[.357]
BGDPG	.095902	.015538	6.17193	[.000]
BGDP	.968804	.226944	4.26891	[.000]
BNR	-.031255	.030809	-1.01448	[.310]
BPS	-.099758	.055947	-1.78310	[.075]
BCLI	-.035900	.032766	-1.09564	[.273]
BWAR	-.061010	.032074	-1.90217	[.057]
BSTATE	-.017918	.023133	-.774596	[.439]
BPRIN2	-.686432E-02	.042549	-.161328	[.872]
BYEARL	.269080	.149216	1.80330	[.071]
BTRD	-.020220	.96357E-02	-2.09848	[.036]
BAR	.024660	.015881	1.55285	[.120]
BAGR	.110327E-02	.312493E-02	.353054	[.724]
BYMIR	-.646396E-02	.279883E-02	-2.30952	[.021]
BGDP2	-.138799	.294835	-.470768	[.638]
BFDI	-.536941E-03	.236304E-02	-.227224	[.820]
BINFLL	.138340E-02	.265272E-02	.521502	[.602]
BWPY	-.653881E-02	.119219E-02	-5.48471	[.000]
BPOPL	-.649889E-02	.464209	-.014000	[.989]
BEDL	-.191610	.120902	-1.58483	[.113]
BGEX	.336349E-02	.264883E-02	1.26980	[.204]
BCP	.124782	.074361	1.67807	[.093]
BRZ	.095484	.065992	1.44690	[.148]

Number of observations = 81

Dependent variable: DGDPG Dependent variable: DEGINIG

R-squared = .663544 R-squared = .790708

*A-and B—parameters refer to GDP growth and EGINI growth equations respectively

Appendix T.I

Derivation of Dynamic System (i.6)-(i.8)

In the equation system (1)-(2) if to assume the simplest linear dynamics in-between two (yearly) observed data available for income inequality and output, the left hand sides can be approximated to the $\frac{\partial f(y_j(t))}{\partial t} \cong \frac{y_{t+1} - y_t}{y_t}$ and $\frac{\partial h(g_j(t))}{\partial t} \cong \frac{g_{j,t+1} - g_{j,t}}{g_{j,t}}$

Where $f(y_j(t)) = \ln(y_j(t))$, $h(g_j(t)) = \ln(g_j(t))$.

For the dynamical system (1), (2) to be in steady state it must hold that:

$$\dot{h}(g_j(t+\Delta t)) - \dot{h}(g_j(t)) = 0$$

or from system (1)-(2) we get:

$$\dot{h}(g_j(t+\Delta t)) - \dot{h}(g_j(t)) = \beta_0 + \frac{\alpha_0}{\alpha_3} + (\beta_1 + \frac{\alpha_4}{\alpha_3}) \times h(g_j(t)) + (\beta_2 - \frac{1}{\alpha_3}) \times \dot{f}(y_j(t)) + (\beta_3 + \frac{\alpha_1}{\alpha_3}) \times f(y_j(t)) + (\beta_4 + \frac{\alpha_2}{\alpha_3}) \times f^2(y_j(t)) + \sum_{i=5} (\beta_i + \frac{\alpha_i}{\alpha_3}) z_j \quad \text{(i.1)}$$

Integrating the left hand side of (i.1) we will have:

$$\int \left[\dot{h}(g_j(t+\Delta t)) - \dot{h}(g_j(t))\right] dt = \ln(g_j(t+\Delta t)) - \ln(g_j(t)) = \ln\left(\frac{g_j(t+\Delta t)}{g_j(t)}\right) \quad \text{(i.2)}$$

Because we have $\frac{g_j(t+\Delta t) - g_j(t)}{g_j(t)} \cong \dot{h}(g_j(t))$; or equally $\frac{g_j(t+\Delta t)}{g_j(t)} - 1 \cong \dot{h}(g_j(t))$ (i.3)

Taking natural logarithm of both sides of (i.3) we will have:

$$\ln\left(\frac{g_j(t+\Delta t)}{g_j(t)}\right) \cong \ln(\dot{h}(g_j(t)) + 1) \quad \text{(i.4)}$$

Hence, from equations (i.2) or (i.3) if to define:

$$\dot{h}(g_j(t+\Delta t)) - \dot{h}(g_j(t)) \equiv \dot{\varphi}(t) \quad \text{(i.5)}$$

We will also have from (i.5) $\varphi(t) = \ln(\dot{h}(g_j(t)) + 1)$; or equally $\dot{h}(g_j(t)) = e^{\varphi(t)} - 1$ (i.6)

Finally, by substituting $\dot{f}(y_j(t))$ in equation (i.1) by the left hand side of equation (1) and rearranging we can rewrite (i.1) as:

$$\dot{\varphi}(t) = \beta_0 + \alpha_0\beta_2 + (\beta_1 + \beta_2\alpha_4) \times h(g_j(t)) + (\beta_3 + \alpha_1\beta_2) \times f(y_j(t)) + (\beta_4 + \beta_2\alpha_2) \times f^2(y_j(t)) + (\beta_2\alpha_3 - 1)(e^{\varphi(t)} - 1) + \sum_{i=5} (\beta_i + \beta_2\alpha_i) z_{j,i} \quad \text{(i.7)}$$

While from equations (1) and (i.6):

$$\dot{f}(y_j(t)) = \alpha_0 + \alpha_1 \times f(y_j(t)) + \alpha_2 \times f^2(y_j(t)) + \alpha_3(e^{\varphi(t)} - 1) + \alpha_4 \times h(g_j(t)) + \sum_{i=5} \alpha_i z_{j,i}$$

(i.8) Now for the dynamical system (i.6)-(i.8) when linearized at steady states, the Jakobian matrix will look as:

$$/j/= \begin{bmatrix} (\beta_2\alpha_3 - 1)e^{\varphi} & \beta_1 + \beta_2\alpha_4 & \beta_3 + \alpha_1\beta_2 + 2(\beta_2\alpha_2 + \beta_4) \times f(y^*_{j,i}) \\ e^{\varphi} & 0 & 0 \\ \alpha_3 e^{\varphi} & \alpha_4 & \alpha_1 + 2\alpha_2 \times f(y^*_{j,i}) \end{bmatrix} \quad \text{(i.9)}$$

Where $y^*_{j,i}$ is steady state value of output and i=1,2.

For steady state value of φ (t), when equation (i.6) holds, it implies that $\varphi(t) = 0$ when sufficiently close to the steady state as long as $\dot{h}(g_j(t)) = 0$. Hence the matrix (i.9) can be rewritten as:

$$/J/= \begin{bmatrix} \beta_2\alpha_3 - 1 & \beta_1 + \beta_2\alpha_4 & \beta_3 + \alpha_1\beta_2 + 2(\beta_4 + \beta_2\alpha_2) \times f(y_{j,i}*) \\ 1 & 0 & 0 \\ \alpha_3 & \alpha_4 & \alpha_1 + 2\alpha_2 \times f(y_{j,i}*) \end{bmatrix} \quad \text{(i.9.1)}$$

Appendix T.II

Cumulative table for all the simple and synthetic parameters in equations

Parameter	***Estimation***			
	S.2.iii EGINI Instrumented policy var	S.2.iv IEGINI not instrumented policy var	S.2.v IGINI instrumented policy var.	S.2.vi EGINI sock level for gex not instrumented policy var
Equation (i.6)				
α_2	-0.3607120	-0.3864460	-2.5163900	-0.2893990
α_1	0.1980090	0.3155660	0.2297610	-0.2769620
α_3	-0.1510550	-0.1367910	-0.6834720	-0.0862930
α_4	-0.2368850	-0.1015730	-0.6722320	-0.2240960
Equation (i.7)				
$\beta_3 + \beta_2\alpha_1$	-0.9362753	-0.4231526	-0.4427724	-0.9125917
$\beta_1 + \beta_2\alpha_4$	0.6956464	0.2940858	0.4976851	0.2623624
$\beta_4 + \beta_2\alpha_2$	-0.0901666	-0.0605839	-0.0514461	-1.0692767
$\beta_2\alpha_3 - 1$	-1.0685678	-1.0517851	-1.0155278	-1.0804152
Appendix T.iii				
Equation (iii.4.1)				
Θ	1.4292798	1.4382311	3.5319178	1.3698142
Λ	0.1980090	0.3155660	0.2297610	-0.2769620
Equation (iii.5.1)				
Ω	1.4268014	0.8698390	3.3383903	1.2479028
Φ	0.2252062	0.3401949	0.2684906	-0.2069628
Equation (iii.6.1)				
Ξ	0.5025139	0.1933968	1.4487480	0.3228975
Γ	-0.2067500	-0.1396863	-0.1363155	0.0131326
Diskriminant of charactiristic polinomial (equation (iii.9))				
χ	0.0006566	0.0008939	0.0004160	-0.0003214
ξ	0.0014468	0.0008921	-0.0066448	-0.0004603
ζ	0.0060806	0.0005685	0.0409195	0.0022774
ς	-0.0082307	-0.0001741	-0.1168592	0.0006250
ν	0.0157481	0.0002198	0.1324974	0.0097298
Appendix T.iv				
equation (iv.6)				
γ	-0.0195143	0.0016216	-0.3910247	0.0282539
ω	0.0951717	0.0301518	0.2014628	0.0810884
θ	0.0048225	0.0079152	-0.0417410	-0.0084864
ψ	0.0023003	0.0093110	0.0035938	-0.0062949
equation (iv.15)				
R	0.1153386	0.0872091	230.1461369	0.0913949
M	-0.1858015	-0.3245794	-262.8815045	0.2902501
K	0.1532254	0.5560148	131.8923653	0.4404728
N	-0.0858644	-0.5640723	-37.9392716	0.3951494
O	0.0374161	0.3692923	6.8393870	0.2220242
P	-0.0118359	-0.1592989	-0.7907702	0.0794063
S	0.0024732	0.0440192	0.0572369	0.0176513
T	-0.0002778	-0.0070449	-0.0023706	0.0022307
U	0.0000127	0.0005026	0.0000430	0.0001227
W	0.0000000	0.0000000	0.0000000	0.0000000

Appendix T.III

Discriminant of characteristic polynomial and characteristic roots of dynamical system (i.6)-(i.8)

Characteristic polynomial of the Jakobian matrix (i.9.1) will look as follows:

$$-\lambda^3 + (a_{11} + a_{33})\lambda^2 + (a_{13}a_{31} - a_{11}a_{33} + a_{12})\lambda + (a_{13}a_{32} - a_{12}a_{33}) = 0 \qquad \text{(iii.1)}$$

Where a_{ij} refers to the ij element of Jakobian matrix (i.9.1) with:

$$a_{11} \equiv \beta_2\alpha_3 - 1;\, a_{12} \equiv \beta_1 + \beta_2\alpha_4;\, a_{13} \equiv \beta_3 + \alpha_1\beta_2 + 2(\beta_4 + \beta_2\alpha_2)\, f(y^*_{j,i}) \qquad \text{(iii.2)}$$
$$a_{21} \equiv 1;\, a_{31} \equiv \alpha_3;\, a_{32} \equiv \alpha_4;\, a_{33} \equiv \alpha_1 + 2\alpha_2\, f(y^*_{j,i})$$

To simplify notation equation (iii.1) may be rewritten as general cubic equation:

$$\lambda^3 + b\lambda^2 + c\lambda + d = 0 \qquad \text{(iii.3)}$$

Where from (iii.1) and (iii.2) it holds that:

$$b \equiv 1 - \beta_2\alpha_3 - \alpha_1 - 2\alpha_2\, f(y^*_{j,i}) \qquad \text{(iii.4)}$$
$$c \equiv -\alpha_1 - \beta_1 - \alpha_3\beta_3 - \alpha_4\beta_2 - 2\, f(y^*_{j,i})(\alpha_3\beta_4 + \alpha_2) \qquad \text{(iii.5)}$$
$$d \equiv \beta_1\alpha_1 - \alpha_4\beta_3 + 2\, f(y^*_{j,i})(\beta_1\alpha_2 - \alpha_4\beta_4) \qquad \text{(iii.6)}$$

Further, to simplify the right hand sides of expressions (iii.4)-(iii.6) they can be rewritten as function of constant and steady state value of output as follows:

$$b \equiv \Theta - 2\Lambda f(y^*_{j,i}) \;\; \text{; with } \Theta \equiv 1 - \beta_2\alpha_3 - \alpha_1 \;\; \text{and } \Lambda \equiv \alpha_2 \qquad \text{(iii.4.1)}$$
$$c \equiv \Omega - 2\Phi f(y^*_{j,i}) \;\; \text{; with } \Omega \equiv -\alpha_1 - \beta_1 - \alpha_3\beta_3 - \alpha_4\beta_2 \;\; \text{and } \Phi \equiv \alpha_3\beta_4 + \alpha_2 \qquad \text{(iii.5.1)}$$
$$d \equiv \Xi + 2\Gamma f(y^*_{j,i}) \;\; \text{; with } \Xi \equiv \beta_1\alpha_1 - \alpha_4\beta_3 \;\; \text{and } \Gamma \equiv \beta_1\alpha_2 - \alpha_4\beta_4 \qquad \text{(iii.6.1)}$$

Since (iii.3) is cubic in λ the sufficient (but not necessary) condition for having one real and two imaginary characteristic roots is $\delta^2 = b^2 - 3c < 0$, in this case (iii.3) will (geometrically) have no extreme points. More general condition for having two of roots in (iii.3) as complex conjugates is positive discriminant. The latter will look as following:

$$\Delta = \frac{4c^3 - b^2c^2 - 18cb + 4db^3 + 27d^2}{108} \qquad \text{(iii.7)}$$

Using expressions (iii.4.1)-(iii.6.1), (iii.7) can be rewritten as:

$$\begin{aligned}\Delta = {} & f^4(y^*_{j,i})\left[-\frac{4}{27}\Lambda^2\Phi^2 - \frac{16}{27}\Gamma\Lambda^3\right] + f^3(y^*_{j,i})\left[\frac{4}{27}\Phi^2\Lambda + \frac{4}{27}\Lambda^2\Phi\Omega - \frac{8}{27}\Phi^3 + \frac{8}{9}\Theta\Lambda^2\Gamma - \frac{8}{27}\Xi\Lambda^3 - \frac{4}{3}\Lambda\Phi\Gamma\right] + \\ & + f^2(y^*_{j,i})\left[\frac{4}{9}\Omega\Phi^2 - \frac{1}{27}\Theta^2\Phi^2 - \frac{4}{27}\Theta\Phi\Omega - \frac{1}{27}\Omega^2\Lambda^2 - \frac{2}{3}\Xi\Phi + \frac{4}{9}\Xi\Theta\Lambda^2 - \frac{4}{9}\Theta^2\Lambda + \Gamma^2 + \frac{2}{3}\Theta\Phi\Gamma + \frac{2}{3}\Omega\Lambda\Gamma\right] + \\ & + f(y^*_{j,i})\left[\frac{1}{3}\Theta\Phi\Xi + \frac{1}{3}\Omega\Xi\Lambda + \frac{2}{27}\Theta^2\Gamma - \frac{2}{9}\Xi\Theta^2\Lambda + \Xi\Gamma - \frac{2}{9}\Omega^2\Phi + \frac{1}{27}\Theta^2\Omega\Phi + \frac{1}{27}\Omega^2\Lambda - \frac{1}{3}\Theta\Omega\Gamma\right] + \\ & + \left[\frac{1}{27}\Omega^3 - \Theta^2\Omega^2 - \frac{1}{6}\Theta\Omega\Xi + \frac{1}{27}\Xi\Theta^3 + \frac{1}{4}\Xi^2\right]\end{aligned} \qquad \text{(iii.8)}$$

Or for the sake of simplification expression (iii.8) can be rewritten as:

$$\Delta(f(y^*_{j,i})) = \chi f^4(y^*_{j,i}) + \xi f^3(y^*_{j,i}) + \zeta f^2(y^*_{j,i}) + \varsigma f(y^*_{j,i}) + \vartheta \qquad \text{(iii.9)}$$

The following table reports the roots of polynomial (iii.9) and summarizes intervals of positive (negative) discriminants for (iii.3) under the different estimations along with the nature of characteristic roots (implying (non)cyclical pattern in dynamics of the system (i.6)-(i.8) for the fixed points y* with different coordinates):

Estimation table	*Roots of polynomial (iii.9)*				*Sign of discriminant in-between roots*					*Characteristic Roots of (iii.3)*
	$f(\tilde{y}^*_{j,i,1})$	$f(\tilde{y}^*_{j,i,2})$	$f(\tilde{y}^*_{j,i,3})$	$f(\tilde{y}^*_{j,i,4})$	$-\infty$ $(f(\tilde{y}^*_{j,i,1}))$	$f(\tilde{y}^*_{j,i,1})$- $(f(\tilde{y}^*_{j,i,1}))$	$f(\tilde{y}^*$ $f(\tilde{y}^*$	$f(\tilde{y}^*$ $f(\tilde{y}^*$	$f(\tilde{y}^*$ $+\infty$	
S.2.iii *EGINI Simplified not instrumented policy variables*	Not real	Not real	Not real	Not real	+	+	+	+	+	*One real and 2 imaginary* roots for both fixed points independently of coordinates of y*
S.2.iv *IEGINI Simplified with instrumented policy variables*	Not real	Not real	Not real	Not real	+	+	+	+	+	*One real and 2 imaginary* roots for both fixed points independently of coordinates of y*

S.2.v *IGINI Simplified not instrumented policy variables*	Not real	Not real	Not real	Not real	+	+	+	+	+	*One real and 2 imaginary* roots for both fixed points independently of coordinates of y*
S.2.vi *EGINI and sock value for GEX Simplified with instrumented policy variables*	-3.7538388	2.77915528	Not real	Not real	-	+	-	-	-	1) *Three real roots* for the fixed points with $f(y^*)<-3.7538388$; and $f(y^*)>2.77915528$ 2) *one real and two imaginary roots* for the fixed point $-3.753838<f(y^*)<2.77915528$

Appendix T.IV

Real part of imaginary roots as a function of policy variables included in $z_{i,j}$ vector: two dimensional stability of system (i.6)-(i.8) and possibility of Hopf bifurcation

Andronov-Hopf bifurcation is defined as: 'a birth of a limit cycle from an equilibrium in dynamical system generated by ODEs, when the equilibrium changes stability via a pair of purely imaginary eugenvalues. The bifurcation can be *supercritical* or *subcritical*, resulting in stable or unstable (within an invariant two-dimensional manifold) limit cycle, respectively'

A standard formulation of Andronov-Hopf theorem states that a dynamical system: $\dot{x} = f(x, \beta),\ x \in \Re^n$ parametrized by a scalar parameter β, and havind isolated equilibrium $E_0 = x_0(\beta)$, undergoes Hopf bifurkation for $\beta = \beta_0$ (e.g. at: $x_0(\beta_0)$) if:

D.1 a simple pair of purely imaginary eigenvalues $\lambda(\beta_0),\ \overline{\lambda}(\beta_0)$ exist at $x_0(\beta_0)$ and no other eigenvalues real parts are zero;

D.2. the complex pair $\lambda(\beta),\ \overline{\lambda}(\beta)$, which becomes purely imaginary at β_0 satisfies the 'nonzero speed' condition: $\frac{d\ \mathrm{Re}(\lambda(\beta))}{d\beta}\Big|_{\beta=\beta_0} \neq 0$

Since for the dynamical system (i.6)-(i.8) $f(y^*_{j,i})$ is a function of scalar parameter z to ensure the first condition one needs to check whether there is such a $f(y^*_{j,i}(z^*))$ for which it holds that:

$\mathrm{Re}\lambda_{1,2}(f(y^*_{j,i}(z^*)))=0$; $im\lambda_{1,2}(f(y^*_{j,i}(z^*))) \neq 0$

and $\lambda_3(f(y^*_{j,i}(z^*))) \neq 0$

for the second requirement the following holds:

$$\frac{\partial\ \mathrm{Re}\ \lambda}{\partial z}\Big|_{z=z^*} = \left[\left(\frac{1}{12}\frac{\partial\Delta}{\partial f(y^*(z))}\Delta^{-1/2} - \frac{1}{6}\frac{\partial q}{\partial f(y^*(z))}\right)\left(q^2 - \Delta\right)^{-2/3} - \frac{1}{3}\frac{\partial b}{\partial f(y^*(z))}\right]\frac{\partial f(y^*(z))}{\partial z} \quad \text{(T.iv.d.2)}$$

Where Δ is as it is defined in (iii.9), q is as it is defined in (iv.5) below; and as long as the general formula for calculating $re\,|\lambda|$ looks as it is given in (iv.1)-(iv.3).

The general formula for calculating imaginary roots of cubic equation looks like:

$$\lambda_i = -\frac{u+v}{2} - \frac{b}{3} \pm i(u-v) \times \frac{\sqrt{3}}{2} \quad \text{(iv.1)}$$

Where
$$u = \sqrt[3]{-\frac{2b^3 - 9bc + 27d}{54} + \sqrt[2]{\Delta}} \quad \text{(iv.2)}$$

$$v = \sqrt[3]{-\frac{2b^3 - 9bc + 27d}{54} - \sqrt[2]{\Delta}} \quad \text{(iv.3)}$$

b, c, d are the coefficients defined in equation (iii.3) and Δ is the determinant of (iii.3) as it is defined in (iii.8).

Hence from (iv.1) it can be written that for the $re|\lambda_{2,3}|$=0 it must hold that:

$$(q - \Delta^{1/2})^{1/3} + (q + \Delta^{1/2})^{1/3} = -\frac{2}{3}b \quad \text{(iv.4)}$$

Where
$$q = -\frac{2b^3 - 9bc + 27d}{54} \quad \text{(iv.5)}$$

So that substituting (iii.4.1)-(iii.6.1) for b, c and d respectively (iv.5) can be rewritten as:

$$q = \gamma + \omega f(y^*_{j,i}) + \theta f^2(y^*_{j.i}) + \Psi f^3(y^*_{j,i}) \quad \text{(iv.6)}$$

Where:
$$\gamma = -\frac{1}{27}\Theta^3 + \frac{1}{6}\Theta\Omega - \frac{1}{2}\Xi \quad \text{(iv.7)}$$

$$\omega = -\frac{1}{3}\Theta\Phi - \frac{1}{3}\Lambda\Omega - \Gamma + \frac{2}{9}\Theta^2\Lambda \qquad \text{(iv.8)}$$

$$\theta = -\frac{4}{9}\Theta\Lambda^2 + \frac{2}{3}\Lambda\Phi \qquad \text{(iv.9)}$$

$$\psi = \frac{8}{27}\Lambda^3 \qquad \text{(iv.10)}$$

Raising both sides of equation (iv.4) into third degree the following can be written:

$$2q + 3(q-\Delta^{1/2})^{2/3}(q+\Delta^{1/2})^{1/3} + 3(q+\Delta^{1/2})^{2/3}(q-\Delta^{1/2})^{1/3} = -\frac{8}{27}b^3 \qquad \text{(iv.11)}$$

Further, equation (iv.11) can be reorganized as following:

$$2q + 3(q^2-\Delta)^{1/3}\left[(q-\Delta^{1/2})^{1/3} + (q+\Delta^{1/2})^{1/3}\right] = -\frac{8}{27}b^3 \qquad \text{(iv.12)}$$

Since from (iv.11) it follows that the expression in quadratic brackets in (iv.12) is equal to $\left(-\frac{2}{3}b\right)$ the latter equation can be reduced to $\frac{q}{b} + \frac{4}{27}b^2 = (q^2-\Delta)^{1/3}$ (iv.13)

Raising both sides of equation (iv.13) into third degree and rearranging components the following can be written:

$$q^3 - \frac{5}{9}b^3q^2 + \Delta b^3 + \frac{64}{27^3}b^9 + \frac{16}{243}qb^6 = 0 \qquad \text{(iv.14)}$$ [42]

Using the fact that all the components of (iv.14) are third to ninth degree polynomials in $f(y^*_{j,i})$ this expression can be rewritten as the ninth order polynomial in $f(y^*_{j,i})$. So that after taking care of all the calculations, one can right the following:

for $re|\lambda| = 0 \Rightarrow$

$$R + Mf(y^*) + Kf^2(y^*) + Nf^3(y^*) + Of^4(y^*) + Pf^5(y^*) + Sf^6(y^*) + Tf^7(y^*) + Uf^8(y^*) + Wf^9(y^*) = 0 \qquad \text{(iv.15)}$$

Where: $R = \frac{5}{9}\gamma^2\Theta^3 - \frac{64}{19683}\Theta^9 - \frac{16}{243}\Theta^6\gamma - \Theta^3\vartheta - \gamma^3$

$$M = \frac{10}{9}(\Theta^3\omega\gamma - 3\gamma^2\Lambda\Theta^2) + \frac{1152}{19683}\Theta^8\Lambda - \frac{16}{243}(\Theta^6\omega - 12\Theta^5\Lambda\gamma) - 3\omega\gamma^2 - \Theta^3\varsigma + 6\Theta^2\Lambda\vartheta$$

$$K = \frac{5}{9}(12\Lambda^2\gamma^2\Theta - 12\omega\gamma\Theta^2\Lambda + \omega^2\Theta^3 + 2\Theta^3\gamma\theta) - \frac{9472}{19683}\Theta^7\Lambda^2 -$$
$$- \frac{16}{243}(60\Lambda^2\Theta^4\gamma - 12\Theta^5\Lambda\omega + \Theta^6\theta) - 3\omega^2\gamma - 3\gamma^2\theta - \Theta^3\zeta + 6\Theta^2\Lambda\varsigma - 12\Theta\Lambda^2\vartheta$$

$$N = \frac{10}{9}\left(12\Theta\Lambda^2\gamma\omega - 4\gamma^2\Lambda^3 - 3\Lambda\omega^2\Theta^2 - 6\Lambda\Theta^2\theta\gamma + \Theta^3\psi\gamma + \Theta^3\theta\omega\right) + \frac{45056}{19683}\Theta^6\Lambda^3 -$$
$$-\frac{16}{243}\left(60\Lambda^2\Theta^4\omega - 160\Theta^3\Lambda^3\gamma - 12\Theta^5\Lambda\theta + \Theta^6\psi\right) - 6\omega\theta\gamma - \omega^3 - 3\psi\gamma^2 - \Theta^3\xi + 8\Lambda^3\vartheta - 12\Theta\Lambda^2\varsigma + 6\Theta^2\Lambda\zeta$$

$$O = \frac{5}{9}\left(12\Lambda^2\Theta\omega^2 + 24\Theta\Lambda^2\gamma\theta - 16\gamma\omega\Lambda^3 - 12\Lambda\Theta^2\gamma\psi - 12\Lambda\Theta^2\omega\theta + 2\Theta^3\omega\psi + \Theta^5\theta^2\right) - \frac{134144}{19683}\Theta^5\Lambda^4 +$$
$$+\frac{64}{243}\left(60\Theta^2\Lambda^4\gamma + 15\Lambda^2\Theta^4\theta - 40\Theta^3\Lambda^3\omega - 3\Theta^3\Lambda\psi\right) - 3\theta^2\gamma - 3\theta\omega^2 - 6\omega\psi\gamma - 12\Theta\Lambda^2\zeta + 8\Lambda^3\varsigma - \chi\Theta^3 + 6\Theta^2\Lambda\xi$$

$$P = \frac{10}{9}\left(12\Theta\psi\gamma\Lambda^2 - 4\Lambda^3\omega^2 - 8\gamma\Lambda^3\theta + 12\Lambda^2\omega\Theta\theta - 3\Lambda\Theta^2\theta^2 + \Theta^3\psi\theta - 6\Theta^2\Lambda\psi\omega\right) + \frac{448}{2187}\Theta^4\Lambda^5 +$$
$$+\frac{64}{243}\left(15\Theta^4\Lambda^2\psi - 40\Theta^3\Lambda^3\theta + 60\Theta^2\Lambda^4\omega - 48\Theta\Lambda^5\gamma\right) - 3\omega\theta^2 - 3\omega^2\psi - 6\gamma\psi\theta - 12\Theta\Lambda^2\xi + 8\Lambda^3\zeta + 6\Theta^2\Lambda\chi$$

$$S = \frac{5}{9}\left(24\Theta\Lambda^2\psi\omega + \psi^2\Theta^3 - 16\Lambda^3\psi\gamma - 16\Lambda^3\theta\omega + 12\Lambda^2\Theta\theta^2 - 12\Lambda\Theta^2\psi\theta\right) - \frac{323584}{19683}\Theta^2\Lambda^6 +$$
$$+\frac{64}{243}(60\Theta^2\Lambda^4\theta - 40\Theta^3\Lambda^3\psi + 16\Lambda^6\gamma - 48\Theta\Lambda^5\omega) - 6\omega\psi\theta - \theta^3 - 3\psi^2\gamma - 12\Theta\Lambda^2\chi + 8\Lambda^3\xi$$

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[42]Alternatively, for the real characteristic root of polynomial (iii.3) to be equal to zero the following must hold: $v + u = \frac{b}{3}$ (iv.4.1), so that equation (iv.14) in this case will look like:

$\frac{1}{27^3}b^9 - 8q^3 - \frac{2}{243}b^6q - \frac{5}{9}q^2b^3 + \Delta b^3 = 0$ (iv.14.1).One can check that polynomial (iv.14.1) has the single real root which coincides with the value of $f(y^*_{j,i})$ for which the determinant of Jakobian matrix (i.9.1) becomes zero.

$$T = \frac{10}{9}\left(12\ \Theta\Lambda^{2}\psi\theta - 8\Lambda^{3}\omega\psi - 4\Lambda^{3}\theta^{2} - 3\Theta^{2}\psi^{2}\Lambda\right) + \frac{262144}{19683}\Theta^{2}\Lambda^{7} + \frac{64}{243}\left(60\ \Theta^{2}\Lambda^{4}\psi - 48\ \Theta\Lambda^{5}\theta + 16\ \Lambda^{6}\omega\right) -$$
$$- 3\theta^{2}\psi - 3\omega\psi^{2} - 8\Lambda^{3}\chi$$
$$U = \frac{20}{9}\left(3\Theta\Lambda^{2}\psi^{2} - 4\Lambda^{3}\theta\psi\right) - \frac{131072}{19683}\Theta\Lambda^{8} + \frac{1024}{243}\left(\Lambda^{6}\theta - 3\Theta\Lambda^{5}\psi\right) - 3\psi^{2}\theta$$
$$W = -\frac{40}{9}\Lambda^{3}\psi^{2} + \frac{32768}{19683}\Lambda^{9} - \frac{1024}{243}\Lambda^{6}\psi - \psi^{3}$$

Nominally expression (iv.15) is ninth degree polynomial in $f(y^{*}{}_{j,i})$, however for some estimations the coefficients in front of higher degrees just cancel out so that it reduces to the seventh or eighth degree. Moreover, ninth degree polynomial implies just 'nominal' existence of nine roots; indeed, there may very well exist only one real root (for the even degree there can be no real root at all, geometrically implying that extreme points do not intercept x-axis). Abel-Ruffini's Impossibility theorem states that there is no general solution in radicals to polynomial equations of degree five or higher; nevertheless, this theorem does not assert that these polynomials are unsolvable, indeed they can be solved to any degree of accuracy by using numerical methods such as the Newton-Raphson method or Laguerre method[43]. The following table summarizes all the real and imaginary roots of polynomial (iv.6) for different estimations:

Estimation Table	*Real Roots of polynomial (iv.15)*		*Sign of $re\lvert\lambda\rvert$ inbetween real roots*			*Two dimensional stability pattern of fixed points*
	$f(\tilde{y}^{*}_{j,i,1})$	$f(\tilde{y}^{*}_{j,i,2})$	$-\infty$ $(f(\tilde{y}^{*}_{j,i,1}))$	$f(\tilde{y}^{*}_{j,i,3})$ $f(\tilde{y}^{*}_{j,i,4})$	$f(\tilde{y}^{*}_{j,i,4})$ $+\infty$	
S.2.iii *EGINI Simplified not instrumented policy variables*	No real root	No real root	-	-	-	*Stable in two dimensions* independently on coordinates of y^{*}
S.2.iv *IEGINI Simplified with instrumented policy variables*	No real root	No real root	-	-	-	*Stable in two dimensions* independently on coordinates of y^{*}
S.2.v *IGINI Simplified not instrumented policy variables*	No real root	No real root	-	-	-	*Stable in two dimensions* independently on coordinates of y^{*}
S.2.vi *EGINI and sock value for GEX Simplified with instrumented policy variables*	-3.782775640	-1.604617342	No imaginary root in this interval	+	-	1) *Unstable in two dimensions* for the fixed points with -3.7538388< f(y^{*})<-1.604617342 2) *Stable in two dimensions* for the fixed point with coordinates -1.604617342<f(y^{*})<2.77915528

Since for the fixed point at f(y*)=-3.782775640 system has tree real and no imaginary roots(e.g $im\ \lambda_{1,2}(f(y^{*}_{j,i}(z^{*}))) = 0$), the only candidate for the bifurcation point for all the estimated models is f(y*(z*))≈ -1.604617342(with $im\lambda_{1,2}(f(y^{*}_{j,i}(z^{*}))) = \pm 0{,}76401i$). What is now to be shown is that neither the single real root at z* nor $\frac{d\ \mathrm{Re}(\lambda(z))}{dz}\Big|_{z=z^{*}}$ are equal zero. For the former requirement the value of λ_3 can be followed either by the value of trace of (i.9.1) or calculated by (iv.4.1), in both case we will have $\lambda_3 \cong -0{,}48097$ hence differing from '0'. For the latter reqirement we will have $\frac{\partial\ \mathrm{Re}\ \lambda}{\partial z}\Big|_{z=z^{*}} \cong (-0{,}44349)\frac{\partial f(y^{*}(z))}{\partial z}$ hence as long as $\frac{\partial f(y^{*}(z))}{\partial z} \neq 0 \Rightarrow \frac{\partial\ \mathrm{Re}\ \lambda}{\partial z}\Big|_{z=z^{*}} \neq 0$.

[43] Because approximations are used to solve polynomial (iv.15) the resulted solutions will be also appreciations of zero point for real part of imaginary roots (this gives a point $re\lvert\lambda\rvert \approx 0$ with only ten zeros 'after the comma') .

References:

Åslund A., P. Boone and and S.Johnson (1996)'How to stabilize: lessons from post-communist countries', *Brooking Papers on Economic Activity* (1),217-313

Åslund, A. (2002)' The IMF and the ruble zone' *Comparative Economics*, XLIV, No 4,49-57

Aghion, Ph. And O.J. Blanchard (1994)' On the speed of transformation on Central Europe' *NBER Macroeconomics Annual*, 283-320

Aghion, P. and P. Bolton (1997),' A theory of trickle-down growth and development' *Review of Economic Studies*, 64 , 151-172

Aghion, Ph. And S. Commander (1999a) 'On the dynamics of inequality in the transition' *Economics of Transition*, 7(2), 275-298

Aghion, Ph., E. Caroli and C. Garcia-Penalosa (1999b) 'Inequality and economic growth: the perspective of new growth theory' Journal of Economic Literature, 37(4), 1615-1660

Alesina, A. and D.Rodrik (1994) 'Distributive Politics and Economic Growth' *Quarterly Journal of Economics*, 109(2), 465-90

Alesina, A. and R. Perotti (1995)' Income distribution, political instability, and ivestment' *Europian Economic Review* 40, 1203-1228

Arellano, M. (1990) ' Some tests of specification for Panel data: Monte Carlo evidence and an application to employment equation', *Review of Economic Studies* 58, 277-297

Arellano, M. and O. Bover (1995)' Another look at the instrumental variable estimation of error-components model', *Journal of Econometrics* 68,29-51

Atkinson, A. and M.Ogaki (1996) 'Wealth-varying intertemporal elasticities of substitution: evidence from panel and aggregate data' *Journal of Monetary Economics*, 38, 507-534

Azariadis, C. and J.Stachurski (2005) ' Poverty traps', chapter prepared for the *Handbook of Economic Growth*, by Aghion, P. and S.Durlauf, *Elsevier*

Barro, R. J. (2000)' Inequality and growth in a panel of countries' *Journal of Economic Growth*, 5: 5-32

Benabou, R. (1996) 'Inequality and growth' *NBER Working Paper* 5658

Benerjee, A.V and A. F. Newman (1993)' Occupational choice and process of development', *The Journal of Political Economy* 101(2), 274-298

Benerjee, A. and Duflo E. (2003)'Inequality and growth: what can the data say?', *Journal of Economic Growth* ,8, 267-299

Becker, A. (1980)' On the long-run steady state in a simple dynamic model of equilibrium with heterogeneous households', *The Quarterly Journal of Economics* 95 (2), 375-382

Carroll, Ch.(1992)' The buffer-stock theory of savings: some macroeconomic evidence' *Brooking Papers on Economic Activity*, 61-156

Carroll, Ch. And M. Kimball (1996) 'Notes and comments on the concavity of the consumption function' *Econometrica*, 64, No.4, 981-992

Chambers, D. (2003)'A semiparametric analysis of the role of inequality, investment and government expenditure in economic growth', Ph.D. dissertation

Chambers, D. (2005)'Inequality and growth: a semiparametric investigation', Ph. D. dissertation

Chatterjee, S. (1994)'Transtionalal dynamics and the distribution of wealth in a neoclassical growth model', *Journal of Public Economics* 54, 97-119

Commander, S., A. Tolstopiatenko and R.Yemtsov (1999) 'Cannels of redistribution. Inequality and poverty in the Russian transition' *Economics of Transition* 7(2), 411-447

Cornia, G. A. and V. Popov (1998) 'Transition and long-term growth: conventional versus non-conventional determinants', *Most* 8,7-32

Denizer, C., A. Gelb and M. de Melo (1996)'Patterns to transition from plan to market' *The World Bank Economic Review*, vol.10 No.3,397-424

_________..________(1997) 'Circumstance and choice. The role of initial conditions and policies in transition economies' World Bank Policy Research Working Paper,WPS 1866

Denizer, C. (1997)' Stabilization, adjustment and growth in transition economies', *World Bank Working Paper*, WPS1855

Fafchamps, M.(2003) 'Inequality and risk',*Oxford University working paper* No 2003/141,ISSN 1471-1498

Ferreira, F. (1999) 'Economic transition and the distribution of income and wealth', *Economics of Transition* 7(2), 377-410

Fischer, S., R. Sahay and C.A.Vegh (1996)' Stabilization and growth in transition economics: the early experience' *Journal of Economic Perspectives* 10(2), 45-66

Forbes, K.J.(2000)'A reestimation of the relationship between inequality and growth' *American Economic Review* 90(4),869-887

Förster, M. F.and I. G. Tóth (1998) 'The effects of changing labor markets and social policies on income inequality and poverty. Hungary and the other Visegrad countries compared', *LIS Working Paper 177,* p. 52

Förster, M. F., D.Jesuit and T. Smeeding (2002)'Rejional poverty and income inequality in Central and Eastern Europe: evidence from the Luxembourg Income Study', *LIS working paper* No.324

Galor,O. and J.Zeira (1993)' Income distribution and macroeconomics', *Review of Economic Studies*,60, 35-52
Galor,O. and O. Moav (2004)' From physical to capital accumulation: inequality and process of development '*Review of Economic Studies* 71(4), 1001-26
Garner, T.I. and K.Terrel (1998)'A Gini decomposition analysis of inequality in the Czech and Slovak Republics during the transition', *Economics of Transition* 6(1), 23-46
Ghiglino, Ch. And M. Olszak-Duquenne (2001) 'Inequalities and fluctuations in a dynamic general equilibrium model' *Economic Theory* 17,1-24
_______ and G. Sorger (2002) ' Poverty traps, inequality, and the wealth distribution' *Journal of Economic Theory*, 105, 120-139
_______and M. Olszak-Duquenne (2005a) ' On the impact of heterogeneity on indeterminacy' International Economic Review, 46 (1)
_______ (2005b)'Wealth inequality and dynamic stability' *Journal of economic theory* 124,106-115
Godoy, S. and J.E. Stiglitz (2004)' Growth, initial conditions, low and speed of privatization in transition countries: 11 years later' *NBER working paper*, w 11992
Guang Hau Wan (2002) 'Income distribution and growth in transition economies', *WIDER Discussion Paper* No.2002/104
Kaldor, N. (1956) ' Alternative theories of distribution' *The Review of Economic Studies* 23 (2), 83-100
______ (1957) ' A model of economic growth' *The Economic Journal* 67(268), 591-624
Kattuman, P. and G. Redmond (2001)' Income inequality in early transition: the case of Hungary 1987-1996', *Journal of Comparative Economics* 29(1), 40-65
Keane, M.P. and E. Prasad (2002)' Inequality, transfers, and growth: new evidence from the economic transition in Poland' *Review of Economics and Statistics* 84(2), 324-341
Kimball, M.S (1990a)' Precautionary saving in the small and in the large', *Econometrica* 58(1),53-73
_______(1990b) 'Precautionary savings and the marginal propensity to consume' NBER Working Paper No.3403
Kuznets, S. (1955) 'Economic growth and income inequality' *the American Economic Review*, 45(1), 1-28
Kylimnyuk D., L. Maliar and S.Maliar (2005)' A model of unbalanced sectoral growth with application to transition economies' *IVIE working paper*, WP-AD 2005-26
Li, Hongyi and Heng-fu Zou (1998)'Income inequality is not harmful for growth: theory and evidence' Review of Development Economics 2(3). 318-334
Lawson, C. and H.Wang (2006) 'Economic transition in Central and Eastern Europe and the former Soviet Union: which policy worked', *Bath University Working Paper*
Mardsen, J.E., Y. Kuznezov and L. Serovich (1998) "Elements of applied bifurcation theory" *Springer-Verlag NewYork, Incorporated*
Matsuyama, K. (2000)' Endogenous inequality' *Review of Economic Studies* 67, 743-759
Merlevede, B. (2003) 'Reform reversals and output growth in transition economies', *University of Gent, Working Paper* 2003/183
Micklewright, J. (1999) 'Education, inequality and transition' *Economics of transition* 7(2), 343-376
Milanovic, B. (1995) 'Poverty, inequality, and social policy in transition economies' *World Bank, Policy Research Working Paper* 1530
_________ (1999) 'Explaining the increase in inequality during transition' Economics of Transition 7 (2), 299-341
Perotti, R. (1993)' Political equilibrium, income distribution, and growth', *Review of Economic Studies* 60, 755-776
Persson, T. and G. Tabellini (1991)'Ia inequality harmful for growth? Theory and evidence', *American Economic Review* 48, 600-621
Pratt, J. W. (1964)' Risk aversion in the small and in the large', *Econometrica*, 32, 122-136
_________(1996)'Growth, income distribution and democracy', *Journal of Economic growth* 1,149-187
Polanec, S. (2004) 'Convergence at last? Evidence from transition Countries' *LICOS Center for Transition Economics*, Discussion Paper 144/2004
Pratt,J. and R.J. Zeckhauser (1987) ' Proper risk aversion' *Econometrica*,55, 143-154
Ravallion, M. (2000)' Inequality convergence' *Development Research group, World Bank*
_________ (2001)' Growth, inequality and poverty: looking beyond averages', *World Bank Policy Research* paper w/p 2558
Rosser,B.J, M. V. Rosser, and Ehsan Ahmed (2003)'Multiple unofficial economy equilibria and income distribution dynamics in systemic transition', *Journal of Post Keynesian Economics* 25(3), 425
Sorger,G. (2000) 'Income and wealth distribution in a simple model of growth' *Economic Theory* 16,23-42
Sukiassyan,G. (2004) 'Inequality and growth: what does the transition economy data say?' *Ph.D dissertation*
Tichit, A. (2006)' The optimal speed of transition', *European Journal of Political Economy* 22,349-369
Zeira, J. (1993)'Income distribution and macroeconomics', *Review of Economic Studies* 60,35-52
Zweimuller, J. (2000)' Inequality, redistribution, and economic growth' *Empirica* 27,1-20