Substitution between Young and Old Workers in an Ageing Context

Marcel Mérette
Department of Economics
University of Ottawa
mmerette@uottawa.ca

Prepared for the
International Conference on Policy Modeling
EcoMod 2007
University of Sao Paulo
Sao Paulo, Brazil, July 11-13

Abstract

Population ageing is expected to lead to a significant slowdown of the labour force growth rate. Among the factors that may offset the slowdown of the labour force is the anticipation of an increase in the wage rate with the shortage of labour that will accompany population ageing. We develop in this paper a computable overlapping generations model with technologies that assume imperfect substitution between young and old workers. As population ageing also involves a change in the age composition of workers with a greater representation of old workers, we show that if ageing increases the wage rate on average this may not be the case for old workers. In such a context, old workers may prefer to participate less in the job market and retire earlier.

Classification: F10, O50, C68

Key Words: Ageing, Retirement decision, Older Workers, Overlapping Generations

Introduction

Canada, like many developed countries, is facing a dramatic change in its population. One feature that may hurt standard of livings is the reduction in the growth of the labour force. The outcome of rising wage rate that may accompany the slowing growth of the labour force can encourage workers to postpone retirement and partly offset the expected slowdown of the labour force. This argument seems reasonable if the assumption of perfect substitution between the young and the old worker from a labour demand perspective is valid. However, the perfect substitution assumption may not correspond to the reality of the labour market. If the elasticity of substitution between the young and the old is less than infinite, then the impact of aging on the real wage rate may differ across the various age groups of workers. In fact, we may observe an increase in the average wage rate, but an absolute decline in the wage rate offered to older workers since they will be in abundance relative to other groups in the context of ageing. If so, the retirement age may decline further and the early retirement challenge is likely to be aggravated.

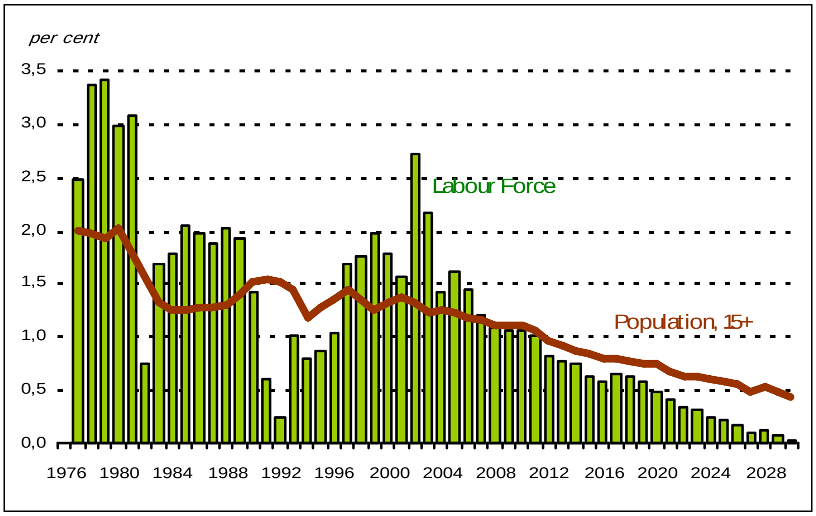
In this paper, we investigate the possible impacts of population ageing on real wage rates of workers of different age groups with technologies that assume imperfect substitution between young and old workers from the point of view of the firms. To this end, we use a computable overlapping generations general equilibrium model.

This paper is organized as follows. In Section 2, we review briefly the labour market challenges, especially in Canada, and some evidence on the imperfect substitutability assumption. Section 3 describes the computable general equilibrium model used for this study. Section 4 reports and discusses the simulation results of two simulation experiments. Section 5 concludes.

2. Older Workers and Labour Market Challenges in Canada

In Canada, the population 15+ has been declining since 2000 and will continue to decline over the next decades. Consequently, as illustrated in Figure 2.1 total labour force growth rate is expected to reach zero by 2030.

Figure 2.1. Growth in the Labour Force



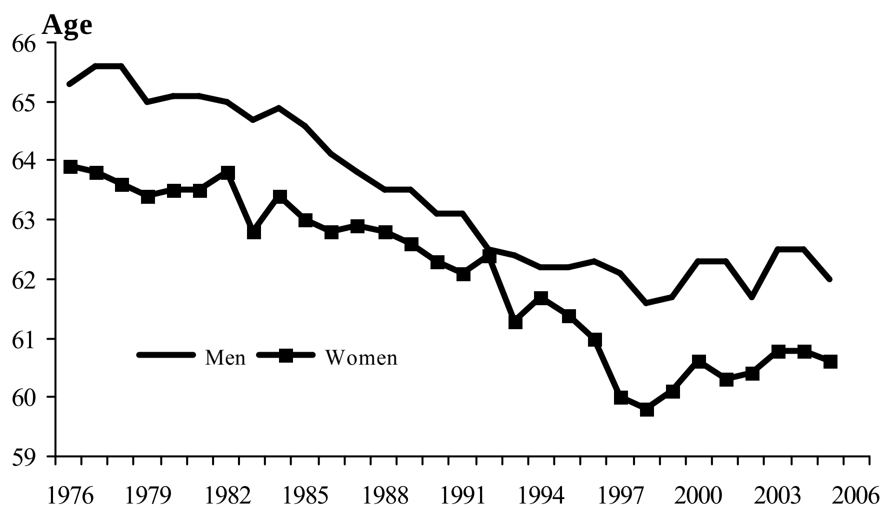
Source: Human Resources and Social Development Canada

The rising proportion of older workers due to population ageing can influence labour market pressures in two ways. First, assuming that retirement decisions do not change, retirement rates will rise as older cohorts of baby boomers reach retirement age. This in turn will reduce aggregate labour supply, exert rising pressures on the labour market and increase the cost of labour. In addition, if labour market adjustments are too slow in certain industries and occupations to accommodate rising replacement demand, labour and skill shortages may occur. Second, since older workers will represent a greater share of the workforce over the next decades, changes in retirement behaviour will have a larger impact on aggregate labour supply. For example, a trend towards early retirement would exacerbate the impact of population ageing on the labour market and the economy. If, on the other hand, older workers do decide to work longer, this may significantly mitigate the effects of ageing.

Over the past decades, despite a substantial increase in longevity, the trend in early retirement has become widespread in Canada. According to Statistics Canada, the average age of retirement has declined steadily in Canada over the period 1976 to 1998 for both men and women, from around 65 for men and 64 for women during the second half of the 1970s to 61.5 years for men and 60 for women in the late 1990s (see Figure 2.2). However, since 1998, the trend in retirement seems to have halted. In fact, from the bottom level observed in 1998, the average retirement age has tended to increase somewhat, ranging between 62 and 62.5 in 2003-2005 for men and around 61 for women.

Figure 2.2

Average Age of Retirement by Sex, 1976-2005



Source: Labour Force Survey, Statistics Canada

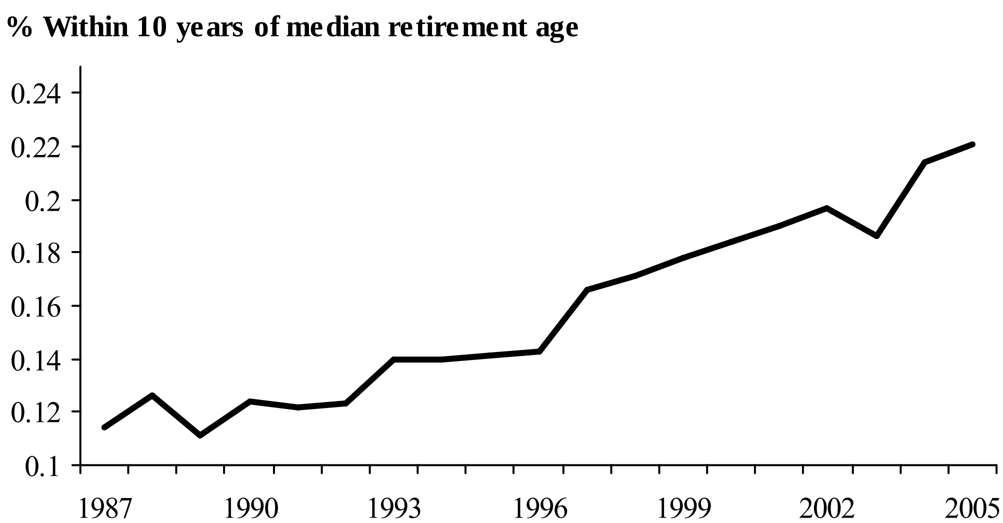
In order to estimate the size of the wave of retirement that is coming in Canada, we use of the near-retirement rate (NRR) that measures the percentage of workers who are within 10 years of the median retirement age. The NRR is influenced by two factors, the median age of retirement and the age distribution of the workforce. An increase in the median retirement age reduces the NRR, while a rise in the proportion of workers within 10 years of the median retirement age raises the NRR.

As can be shown in Figure 2.3, the NRR has increased continuously over the past 18 years. From 11.4% in 1987, the NRR has reached 22.1% in 2005. Between, 1987 and 1998, the rise in the NRR is due to both a reduction in the median retirement age and an increase in the proportion of older workers. After 1998, the median retirement age has stabilized or risen somewhat, thus contributing to a reduction in the NRR. However, despite this, workforce ageing has remained a dominating factor, leading to a continued increase in the NRR.

Over the next decades, it is evident that the rising proportion of older workers will continue to exert upward pressures on the NRR. What is less certain, however, is how retirement behaviours will evolve in the future. A gradual increase in the median retirement age, for example, might mitigate labour market pressures by smoothing the rise in the NRR.

Figure 2.3

The Near Retirement Rate, 1987-2005



Source: Labour Force Survey, Statistics Canada

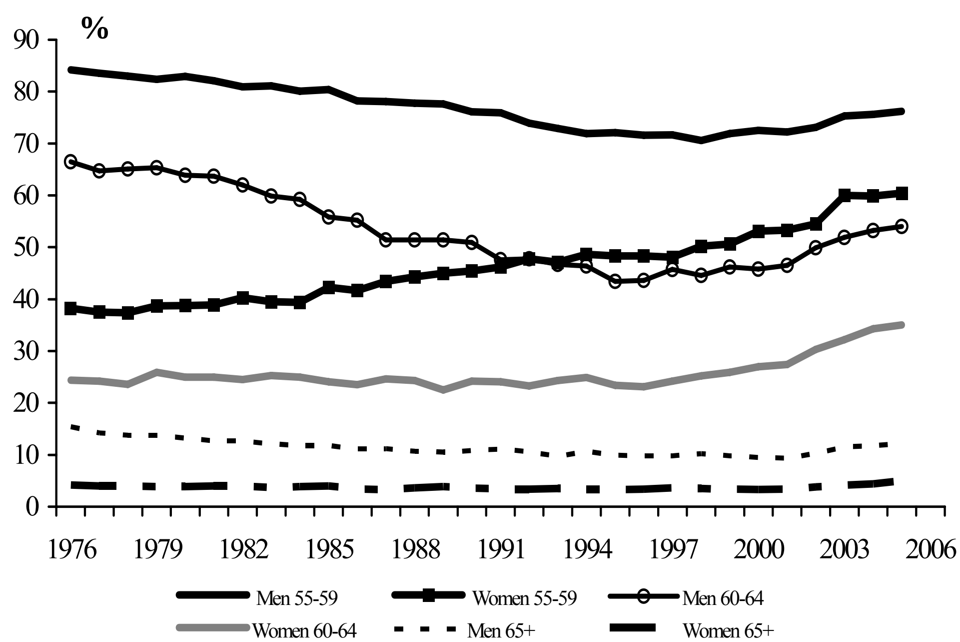
The median and average age of retirement is at best a proxy indicator of retirement behaviour. The measure reflects only those whose initial motivation for job separation was retirement. It does not account for workers who left the labour market because of an illness or a layoff and who never return. Moreover, it does not account for retirees who eventually return to the labour market. For these reasons, it is important to examine alternative measures, such as the labour force participation rate.

When we look at the participation rate of older workers (see Figure 2.4), we see that since the 1970s, the reduction in the participation rate is evident among older men, but not for women.

Indeed, between 1976 and 1998, the participation rate of men 55-59 has fallen from 84.2% in 1976 to 70.6% in 1998, while the participation rate of men 60-64 has declined even more, from 66.5% to 44.6%. Since 1998, the participation rate of men 55-59 and 60-64 has increased, reaching 76.2% and 54%, respectively, in 2005. By contrast, the participation rate of women aged 55-59 has increased continuously during the past 30 years, while the participation rate of women aged 60-64 has remained virtually flat between 1976 and 1998 and increased since. Therefore, the participation rate numbers not only suggest that early retirement trends have halted in 1998, but also that the participation rate of older workers has increased since.

Chart 2.4

Labour Force Participation Rate of Older Workers, by Gender, 1976-2005



Fougere *et al.* (2005) calculate that additional economic benefits of about 3.5% of real per-capita GDP would be generated with the marginal effect of workers working one more year. And economic gains would be achieved equivalent to 12% of real per- capita GDP if average retirement age could reach and maintain at 65. In sum, these results suggest that extending the effective retirement age can significantly offset the consequences of ageing, reduce pressures of labour market and increase the real GDP per capita.

However, according to Hamermesh (2001), the evidence indicates that there is substantial imperfect substitution between workers at different age. Card and Lemieux (2001) build a general framework in which different age and education groups are modeled upon imperfect substitutions for each other. They find that, in Canada, the U.S. and the U.K., the elasticity of substitution is about 4-5 range for different age groups. Wasmer (2001) finds an elasticity of substitution around 0.5 among age groups with different experience and knowledge. This 0.5 number means that substitution between young and old workers is far from perfect.

Under imperfect substitution between old and young workers, the labour demand becomes specific to the various age groups of workers. Hence the impact of population ageing on the wage rates may differ across workers of different age groups. It should be expect that with the upcoming demographic changes, the share of young (old) workers with respect to the total labour force will decline (increase). Hence the wage rate changes of a specific age group of workers will depend on the impact of population ageing on total labour force, but also on the relative share with respect to the total lalour force of this specific age group. Hamermesh (2001) mentions the possibility of a wage rise of prime-age workers accompanied by a decreasing wage for older workers. If so, the retirement age may decline further and the early retirement problem is likely to be exacerbated.

3. The Computable General Equilibrium Model

The computable overlapping generations model with endogenous labour supply was developed to analyze numerical property of the issue on population aging in the perspective of imperfect substitutability across workers of seven different age groups. Assuming an open economy background, the rest-of-the-world is specified by two reduced form equations, representing trade balance and importation demand. The small open economy produces a particular good which is an imperfectly substitute to the one manufactured by the rest-of-the-world. The model only considers the adult activity, so the adult life span of a typical age group is 70 years from the age 15 to 84 and divided by seven periods of 10years. In other words, the youngest individuals refer to the members of the 15-24 age group, while the oldest cohort are those people among the 75-84 age group. The way to divide those generations into seven groups (15-24, 25-34, 35-44, 45-54, 55-64, 65-74, 65-84) realistically reflects the potentially imperfect substitution across workers of different age groups.

a. Preliminaries on sets and indices

In each period (time is indexed by *t*), the last generation denoted by *gr* (*g* = *g*7584) fully retires and does not work, so the potential labour is available from the first six imperfectly substitute age groups of workers denoted by *gw* (*gw* = *g*1524, *g*2534, *g*3544, *g*4554, *g*5564, *g*6574). We first discuss the household problem, followed by the manufacturer problem, the rest-of-the-world, the government constraint, country's foreign trade and the equilibrium conditions.

b. Household g at time t

Households are specified as in an Allais-Samuelson overlapping generations framework, so the model is developed on the basis of the life-cycle theory of savings. Each adult individual joins the labour force at age 15 with perfect foresight and works for sequential six periods and fully retires at age 75, then dies at age 84. In each period, a new generation enters the workforce and the eldest generation *gr* dies. That is to say there are always seven generations alive at any point of time. Assumed to be fully dependent on their parents, younger children below 15 have no extra burden or benefits to their parents. A household's problem is maximizing an inter-temporal utility function of consumption and leisure subject to a lifetime income. The time-separable utility function takes the following constant-elasticity-of-substitution (CES) function form:

$$(1) \quad U_t = \frac{1}{1-\theta}\sum_{g-1}^{7}\left(\frac{1}{1+\rho}\right)^{g}\left(Cons_{t+g-1,g}^{1-\varepsilon} + \alpha_g Leis_{t+g-1,g}^{1-\varepsilon}\right)^{1-\theta/1-\varepsilon} \qquad \theta > 0, \alpha_g > 0,$$

where ρ symbolizes the pure rate of time preference, θ refers to the inverse of the inter-temporal elasticity of substitution σ (σ=1/θ), ε the inverse of the intra-temporal elasticity of substitution η (η =1/ε), and α_g is a parameter representing the intensity of household preferences for leisure relative to consumption.

The terms $Cons_{t,g}$ and $Leis_{t,g}$ respectively denote consumption and leisure of an member of age group *g* at time *t* and the value of leisure ranges from 0 to 1. Here it is easy to understand that (1-*Leis*) represents the participation rate in the workforce since *Leis* equals the portion of time distributed to leisure activity. Equation (1) indicates that the total welfare of one person is a weighted sum of seven periods of leisure and consumption from age group 15-24 at period *t* to age group 75-84 at *t+6*. With the addition of leisure into the utility function and the previous assumption of endogenous household's labour supply, we can investigate the reaction of labour supply, particularly the supply of older workers facing the upcoming demographic shift.

Assuming perfect capital markets and no borrowing constraints, the present value of the household wealth is the discounted sum of lifetime labour income $LifInc_{g,t}$ minus taxes but including public old-age pensions $Pens_{g.t}$:

$$(2)\,W_t = \sum_{g-1}^{7}\left[\frac{1}{1+R_{t+g-1}\left(1-\tau^K\right)}\right]^t \left[LifInc_{t+g-1,g}\left(1-\tau^w_{t+g-1}-contr_{t+g-1}\right)+\left(Pens_{t+g-1,g}\right)\right] \quad ,$$

where R_t represents the rate of interest, τ^K is effective tax rates on capital and τ^w is the tax rates on labour income, *contr* denotes the contribution rate to the pay-as-you-go pension system.

The defined labour income earned by working age group *gw* at period t and pension benefits are expressed in equation (3) and (4) respectively:

$$(3) \quad LifInc_{t,gw} = wage_{t,gw}\left(1-Leis_{t,gw}\right)$$

$$(4) \quad Pens_{t,gr} = PensR\sum_{gw} LifInc_{t-gr+gw,gw}$$

where $wage_{t,gw}$ is labour income, and retirees' pension benefits are the product of pension replacement rate *PensR* and labour income of the whole working life.

Substituting equation (3) and (4) into (2), we can see that the choices on leisure and consequently on labour market participation have significant effects on the amount of the future pensions and the household wealth. Through differentiating the household's utility function with respect to its lifetime budget constraint, we get a reservation wage that takes the future pension revenue into account and also considers the possibility that the proportion of time allocated to leisure may equal to upper bound 1. The reservation wage *ReWage* for each generation is written as:

(5) $$\mathrm{Re}Wage_{t,gw} = wage_{t,gr}\left(1-\tau_t^w - contr_t\right) + \prod_{s-t+gw}^{t+gr}\left(\frac{1}{1+R_s}\right)PensR \cdot wage_{t,gw} + \lambda_{t,gw}$$, where

$\lambda_{t,gw}$ is a kuhn-tucker multiplier and will not equal to zero if the individual chooses to retire in period *t*. This multiplier represents the extra money the individual would like to acquire in order not to retire from the workforce and supply a positive amount of labour. The contribution rate $contr_t$ presented in equation (5) implies the assumption that workers consider social security contributions as marginal taxes, which provide no additional benefits in return for additional taxes paid.

Before we state the household first order conditions, here we define a term as $Z_{t,gw}$ to avoid the messy look of the one of the following first order conditions:

(6) $$Z_{t,gw} = \left\{1+\alpha_{gw}^{\eta}\left[\frac{\mathrm{Re}Wage_{t,gw}}{\left(1+\tau_c\right)CP_t}\right]^{1-\eta}\right\}$$,

where CP_t refers to the composite price of output produced in Canada and in the rest-of-the-world. Now the intra-temporal and inter-temporal first order conditions of the household problem are generated as the following forms:

(7) $$Leis_{t,g} = \left[\frac{\alpha_g\left(1+\tau_c\right)CP_t}{\mathrm{Re}Wage_{t,g}}\right]^{\eta} Cons_{t,g}$$

(8) $$Con_{t+1,g+1} = \left[\frac{1+R_{t+1}\left(1-\tau_{t+1}^K\right)\left(1+\tau_t^{Cons}\right)CP_t}{\left(1+\rho\right)\left(1+\tau_{t+1}^{Cons}\right)CP_{t,g}}\right]^{\left(1/\theta\right)}\left(\frac{Z_{t+1,g+1}}{Z_{t,g}}\right)^{\frac{\eta-\sigma}{1-\eta}} Cons_{t,g}\,,$$

Equation (7) suggests that a decline in leisure relative to consumption can be caused by an increase in the reservation wage *ReWage*. Actually, with a change in the reservation wage, the percentage change in the ratio of leisure-consumption (*Leis/Cons)* equals to the intra-temporal elasticity of substitution η. For equation (8), we can see that the intra- and inter-temporal effects counteract each other. To be more precise, if the *ReWage* increases over time, the term Z will increase respectively and also the ratio (*Leis/Cons)* has a tendency to decline caused by the intra-temporal effect as mentioned in equation (7). On the other hand, from time to time, households begin to shift more weight to leisure by decreasing labour supply, which may be relatively high at households' earlier years since they are likely to supply more labour in their early years to take advantage of the higher accumulated wage. Thus leisure increases over time. Therefore, because of the offset factor, the net impact on the pattern of consumption over the lifecycle (*Cong+1 / Cong*) is unclear. There are three possibilities in this case:

(a) if the intra-temporal effect dominates (η>σ) when wage grows, equation (8) implies that consumption grows faster;

(b) in contrast, consumption grows more slowly when intra-temporal elasticity of substitution is less than the inter-temporal elasticity of substitution (η<σ);

(c) additionally, if the intra-temporal elasticity of substitution equals to the inter-temporal elasticity substitution (η=σ) or labour supply is exogenous ($\alpha_g = 0$), then equation (8) has another standard form:

$$(8') \qquad Con_{t+1,g+1} = \left[\frac{1 + R_{t+1}\left(1 - \tau_{t+1}^{K}\right)\left(1 + \tau_{t}^{Cons}\right)CP_{t}}{\left(1 + \rho\right)\left(1 + \tau_{t+1}^{Cons}\right)CP_{t+1}} \right]^{\sigma} Cons_{t,g} \quad ,$$

where the special case equation (8') states that in the condition that the price of consumption and the consumption tax rate stay constants, the consumption will increase over the life cycle if the interest rate net of capital taxes $R(1\text{-}\tau^{K})$ exceeds the pure rate of time preference ρ. And also the increase rate depends on the inter-temporal elasticity of substitution σ. Unlike the general case (8) which includes wage income on the pattern of consumption through inter- and intra-temporal substitution effects, the special case equation (8') does not involve the impact of wage income on the pattern of consumption over time since wage income does not influence the growth rate of consumption over the life cycle but does have impact on the consumption level through the budget constraint.

The pattern of consumption over the life cycle determines what amount of saving the households generate through the budget constraint. Households invest in physical capital, in bonds issued by firms and by the government, as discussed below.

c. Producers in Canada at time t

For producers' problem, inputs including capital services K_t^{dem} and labour services L_t^{dem} distinguished by the various age groups of workers $L_{t,g}$. Factors capital and labour are needed to produce the unique national good in quantity Q_t. Capital services and labour services are bought by paying the capital rental rate $\mathrm{Re}\,ntR_t$ and wage rate for the six age groups of workers $wage_{t,g}$ at the market prices. The aggregate price of labour is denoted by $WLdem_t$. Producers solve the following problem by minimizing the cost function:

(9) $$\underset{K_t^{inp}\ L_t^{inp}}{\textbf{Minimize}}\ \mathrm{Re}\,ntR_t \cdot K_t^{dem} + WLdem_t \cdot L_t^{dem},$$

subject to the following constraints (10) that represent the technology of the firm. Here we normalize the price of output produced in Canada P_t^{Can} to one for simplicity, so P_t^{Can} can be omitted in equation (10):

(10) $$Q_t = ScP \cdot K_t^{dem^{\alpha}} \cdot L_t^{dem^{1-\alpha}}$$

where equation (10) is the national production function taking a Cobb-Douglas form, using the capital input and labour input and specified with the scaling parameter *ScP* and expenditure shares α.

There is not much evidence that the age-composition of workers differs across sectors and across skills, so the assumption of a single production sector and a single skill for labour may be appropriate. Beckstead and Vinodrai (2003) point out that in the period of 1971 to 1996, the emergence and the growth of occupations linked to knowledge-based economy were widespread across regions and sectors of countries like Canada. Therefore, the well-believed saying that young workers are concentrated in the high-technology sectors may be not correct even though they are mostly hired by these occupations for their more connections to new technologies.

The first-order conditions from minimizing the producers' problem subject to the two constraints are as the forms below:

(11) $$\mathrm{Re}ntR_t \cdot K_t^{dem} = \alpha \cdot ScP \cdot Q_t \quad ,$$

(12) $$WLdem_t L_t^{dem} = (1-\alpha) \cdot ScP \cdot Q_t \quad .$$

Another producers' problem can be expressed as the following:

(13) **Minimize** $\sum_{gw} wage_{t,gw} \cdot L_{t,gw}$,

and the corresponding constraint is:

(14) $$L_t^{dem} = \left(\sum_{gw} \alpha p_{gw} \cdot L_{t,gw}^{\varphi} \right)^{1/\varphi} \quad ,$$

where the labour demand function based on the impact substitution among workers of different age group of workers $L_{t,g}$, is a CES function parameterized by share parameters α*p* and an elasticity of substitution $\sigma^{Ldem} = \frac{1}{1-\varphi}$

Through minimizing the labour costs subject to the labour constraint, we can derive the following relation as equation (14) and (15):

(14) $$L_{t,gw} = \alpha p_{gw} \left(\frac{WLdem_t}{wage_{g,t}} \right)^{\sigma^{Ldem}} \cdot L_t^{dem} \quad ,$$

(15) $$WLdem_t^{\left(1-\sigma_t^{Ldem}\right)} = \sum_{gw} \alpha p_{gw} \cdot wage_{t,gw}^{\left(1-\sigma^{Ldem}\right)}$$

where Equation (14) states that if its price ($wage_{t,gw}$) declines relative to the aggregate price of labour ($Wldem_t$), labour demand for age group *gw* increases with respect to total labour demand.

Also the percentage change in the ratio $\frac{L_{t,gw}}{L_t^{dem}}$ with respect to a change in $\frac{WLdem_t}{wage_{t,g}}$ depends on σ^{Ldem}. Finally, for the relation of the aggregate price of labour $Wldem_t$ and different age groups of workers $wage_{t,g}$, it is not difficult to understand that the aggregate price of labour $Wldem_t$ equals to a non-linear weighted sum of wages across age groups $wage_{t,gw}$.

d. The rest of the world at time t

The rest-of-the-world described in a reduced form is introduced here to close the model; its prices and income are treated as exogenous and hold constant. Its demand *D* for our small economy good is based on the region's sectoral competitiveness:

$$D_{can,row,t} = ScP_{can,row}^{D}\left(\frac{P_t^{row}}{P_t^{can}}\right)^{\eta_{row}}, \tag{16}$$

where P_t^{row} is the price of output produced in the rest-of-the-world. Considered the description of the reduced form and assumed the rest-of-the-world neither borrows nor lends internationally, we impose that its trade account is as always in balanced at all *t*:

$$P_t^{can} \cdot D_{can,row,t} = P_t^{row} \cdot D_{row,can,t} \tag{17}$$

e. The government at time t

The revenue of the central government comes from the taxes on labour and capital incomes, as well as on consumption expenditures. Government consumption Gov_t and interest payments on the debt constitute government spending. When central government runs a deficit, tax revenues come short of expenditures, the government issues new bonds, denoted as *Bond*, to satisfy its budget constraint. The following is the budget constraint of the government:

$$\begin{aligned} CP_t \cdot Bond_{t+1} + \sum_g Pop_{g,t}\Big[\tau_t^w\left(LifInc_{g,t} + Pens_{g,t}\right) + \tau_t^{Con} \cdot CP_t \cdot Cons_{t,g} + \tau_t^K \cdot R_{t-1} \cdot CP_{t-1} \\ \cdot Bij_{t,g} + \tau_t^K R_{t-1} CP_{t-1} Kij_{t,g}\Big] = CP_t \cdot Gov_t + \left(1 + R_t\right) CP_{t-1} Bond_t \end{aligned} \tag{18}$$

where $Pop_{g,t}$ represents the number of people living by age group g at period t. The population growth rate is assumed as exogenous. The terms $Kij_{t,g}$ and $Bij_{t,g}$ are the investment in physical capital and bonds respectively. Additionally, pay-as-you-go pension benefits are obtained by contribution rates on wage earnings:

$$\sum_{gr} Pop_{t,gr} \cdot Pens_{t,gr} = contr_t \sum_{gw} Pop_{t,gw} \cdot LifInc_{t,gw} \quad . \tag{19}$$

f. Country's foreign trade in goods at time t

Adding-up all individual demands, we get the aggregate demand for goods:

$$\sum_{g} Pop_{t,g} \cdot Cons_{t,g} + Inv_t + Gov_t \tag{20}$$

This demand can be allocated between domestic and the rest-of-the-world using the traditional Armington assumption. Goods of different regions are assumed differentiated in demand by their geographic origin although individual producers are treated as price takers. Therefore, a national importer using a CES ($D_{ii,j,t}; \alpha^{D}_{ii,j}, \sigma^{c}_{j}$) aggregator makes optimal choices on the basket of domestic and international goods in each sector. Then the composite price of output CP_t can be stated as a function of domestic and rest-of-the-world's producer price $P_{ii,t}$ (where *ii* refers here to a region of the world):

$$CP_t^{1-\sigma_j^c} = \alpha^{D}_{can,can} \cdot P_t^{can^{1-\sigma_j^c}} + \alpha^{E}_{row,can} \cdot P_t^{row^{1-\sigma_j^c}} \tag{21}$$

and the associated demand system can be expressed as:

$$D_{ii,j,t} = \alpha^{D}_{ii,j} \left(\frac{P^{c}_{j,t}}{P_{ii,t}} \right)^{\sigma_j^c} \left(\sum_{g} Pop_{j,t,g} \cdot Cons_{j,t,g} + Inv_{j,t} + Gov_{j,t} \right) \tag{22}$$

g. Equilibrium conditions

The equilibrium of the market clearing for goods is:

$$Q_{can,t} = D_{can,can,t} + D_{can,row,t} \quad . \tag{23}$$

And full employment of labour satisfies:

$$Pop_{t,gw} \cdot \left(1 - Leis_{t,gw}\right) = L_{t,gw} \quad . \tag{24}$$

The total capital demand is equal to the capital stock:

(25) $$StockK_{can,t} = K_{can,t}^{dem} \quad .$$

The physical capital stock is composed of investment Inv_t made by households in capital subtracts the depreciation at constant rate *DepR*:

(26) $$StockK_{t+1} = Inv_t + (1 - DepR)StockK_t \quad .$$

Returns to bonds equal to returns to capital since bonds and physical capital ownerships are perfectly substitutes. The interest rate R_t is thus equivalent to the one period expected rate of return on one unit of physical capital bought at time *t*-1:

(27) $$1 + R_t = [\mathrm{Re}\,ntR_t + (1 - DepR)]\left(\frac{CP_t}{CP_{t-1}}\right) \quad ,$$

At last, the equilibrium condition requires that total supply of assets in the financial market must be equal to the total demand:

(28) $$\sum_g Pop_{t,g} Lend_{t+1,g+1} = CP_t Bond_{t+1} + CP_t StockK_{t+1} \quad .$$

4. Simulation Results

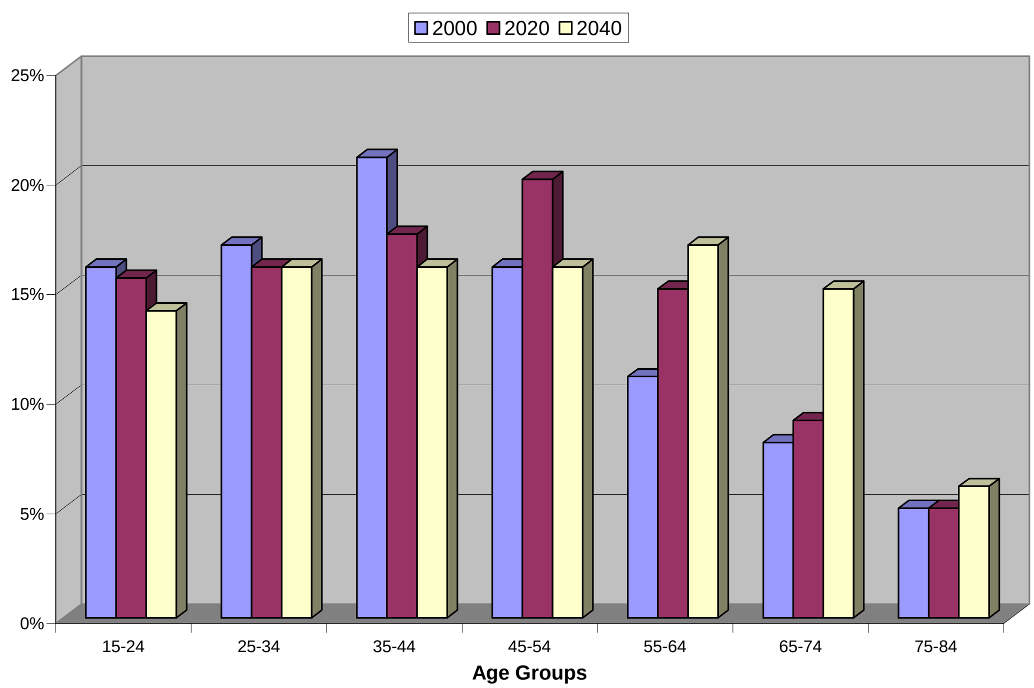
We conduct two experiments that differ only by the value of the elasticity of substitution for labour demand (σ^{Ldem}) used. The demographic shock on the two experiments is similar. The age structure for the whole period was determined by setting the birth rate for each year starting in 1930. To ensure a long run steady state, the model runs up to year 2200 and after 2070, we assume that the birth rate slowly stabilizes at a level that keeps total population constant. With regard to the demographic shift, Table 4.1 below reports the old-age dependency ratio imposed on Canada for the period of interest is in line with the one use in Fougère *et al.(*2005).

Table 4.1. Old-Age Dependency Ratio

Year	**2000**	**2010**	**2020**	**2030**	**2040**	**2050**	**2060**	**2070**
Canada	18.09	22.22	26.76	33.30	37.18	41.35	43.86	43.21

The progression of the old-age dependency ratio in the decades to come will also have important consequences on the evolution of the composition of the population and hence, on the composition of the labour force. As Figure 4.1 illustrates, the traditional demographic feature with a wide base of young people and a progressive narrowing through the oldest people at the peak, is already no longer applicable to Canada. The age profile of the Canadian population is now more concentrated among the 35-44 age group. In year 2020, it will be the 45-54 age group that will constitute the most important age group relative to total population. In fact, the relative size of this and the 55-64 age groups will be the only ones to increase significantly between year 2000 and year 2020. Finally it is worthy to note that the distribution of the population is evolving towards a quasi perfectly equal distribution across age groups of the labour force age (15-64) as projected for 2040. This suggests that the older workers will become relatively more abundant with ageing, whereas the younger workers will become relatively scarcer.

Figure 4.1. Size of the Age Groups Relative to Total Population



In Table 4.2 below, we state the values of several parameters, such as the inter-temporal elasticity of substitution, the intra-temporal elasticity of substitution, the elasticity of substitution for labour demand and the pension replacement rate ratio, which we used in the simulation experiments. Notice that the values of the elasticity of labour demand in the two scenarios range are 3.0 and 1.5.

Table 4.2: Parameter Values

<table>
<tr><td>Inter-temporal elasticity of substitution (σ)</td><td colspan="2">1.5</td></tr>
<tr><td>Intra-temporal elasticity of substitution (η)</td><td colspan="2">1.15</td></tr>
<tr><td rowspan="2">Elasticity of substitution for labour demand (σ^{Ldem})</td><td>First scenario</td><td>3.0</td></tr>
<tr><td>Second scenario</td><td>1.5</td></tr>
<tr><td>Replacement rate of wage income for pensions (*PensR*)</td><td colspan="2">0.25</td></tr>
</table>

The inter-temporal elasticity of substitution seems relatively high for an overlapping generations model. However, since every period of the model is 10 year long, the number implies a yearly elasticity of 0.15. This is more conservative than the number 0.25 used in Auerach and Kotlikoff (1987). The value of intra-temporal elasticity of substitution is smaller than the inter-temporal one, which implies a domination of the inter-temporal effect. Consequently leisure declines over time if wage income grows. Moreover, the replacement rate for pensions is a good approximation of the value used in the Canada Pension Plans and Quebec Pension Plans.

The elasticity of substitution for labour demand is usually assumed to be equal to infinite in the computable general equilibrium literature. However, Wasmer (2001) finds an elasticity of substitution around 0.5 among age groups with different experience and knowledge. Card and Lemieux (2001) find for Canada, the U.S. and the U.K., different age groups are close (elasticity of substitution in the 4-5 range), but not perfect substitutes for each other. Given the range of values found in the literature, we consider two scenarios and compare the results accordingly.

Population aging like assumed in this paper has two noteworthy features. First although the total number of workers increases over time, its relative size with respect to the total population increases slightly from year 2000 to 2010. After that the potential labour force becomes relatively scarcer up to 2060. Second, the size of older workers increases relative to the size of younger workers over time. With perfect substitution of labour demand across workers of different age groups, the unique wage rate in the economy will likely be determined by the relative size of the working age group (15-64) with respect to total population. In this case, for a given labour demand, we should expect a decline in the wage rate from 2000 to 2010 and thereafter an increase. If workers are imperfect substitutes from the labour demand perspective, each age group would have its own wage rate and the changes in the wage rate will differ across age groups. In particular, as the older workers are becoming relatively larger, we should expect their corresponding wage rate to decline relative to younger workers.

a. Baseline Scenario: $\sigma^{Ldem} = 3$

We first exam the simulation results of this general equilibrium model when the elasticity of substitution for labour demand equals to 3. We consider this scenario as our benchmark. We first report the evolution of the wage rate in percentage variations relative to year 2000 for different age groups of workers in Table 4.3.

Table 4.3: Real Wage Rates

(Changes in percentage terms relative to 2000)

Age Groups	2000	2010	2020	2030	2040	2050	2060	2070
15-24	0	0.12	0.26	0.52	0.73	0.85	0.85	0.83
25-34	0	-0.08	0.05	0.30	0.52	0.66	0.66	0.64
35-44	0	-0.70	-0.76	-0.53	-0.31	-0.16	-0.16	-0.18
45-54	0	-1.16	-1.84	-1.79	-1.59	-1.44	-1.42	-1.44
55-64	0	-1.34	-2.49	-3.05	-3.04	-2.89	-2.85	-2.85
65-74	0	-1.00	-2.37	-3.40	-3.95	-3.96	-3.92	-3.89

From Table 4.3, we can see that for 15-24 and 25-34 age groups, almost all the figures imply a positive percentage change of their real wage rate and that percentage with respect to year 2000 increases over time. Unlike the increasing trend of the real wage rates for young workers, the real wage growth rates for the relatively older age groups not only have the negative values, but also decline more and more significantly over the decades. These declines suggest a reduction in the opportunity cost of leisure for older workers.

To better estimate the opportunity cost of leisure, it is worthy to investigate the evolution of real wage net of taxes and the contribution rate on pensions. The following Table 4.4 gives the percentage changes of the real wage net of wage taxes and contributions relative to year 2000.

Table 4.4: Real Wage Rates Net of Taxes and Contributions

(Changes in percentage terms relative to 2000)

Age Groups	2000	2010	2020	2030	2040	2050	2060	2070
15-24	0	1.30	2.42	3.15	3.30	3.16	2.98	2.92
25-34	0	1.11	2.20	2.92	3.09	2.96	2.78	2.72
35-44	0	0.48	1.37	2.07	2.24	2.12	1.95	1.88
45-54	0	0.01	0.27	0.78	0.92	0.81	0.65	0.60
55-64	0	-0.18	-0.39	-0.52	-0.56	-0.67	-0.81	-0.84
65-74	0	0.19	-0.27	-0.88	-1.49	-1.77	-1.89	-1.90

Although Table 4.4 numbers exhibit similar trends to the numbers in Table 4.3, it is noticeable that the percentage changes in the net real wage rates are all larger. The possible reason may be due to the increasing working age-group population in period of 2000 to 2030 that permits new entry of taxation revenue for the government. Under the assumption of constant government expenditure over these decades, the payroll taxes imposed on each worker decline. This pushes up real wage rates net of taxes and contributions and makes the net real wage rates for younger workers growing faster and that for older workers declining at a slower pace.

Net real wage rates reflect the real opportunity cost of labour. In Table 4.5 we report labour participation rate percentage changes. These changes are the result of inter-temporal and intra-temporal substitution plus income effects in the optimization problems of the various cohorts.

Table 4.5: Labour Participation Rates

(Changes in percentage terms relative to 2000)

Age Groups	2000	2010	2020	2030	2040	2050	2060	2070
15-24	0	0.07	0.21	0.29	0.27	0.18	0.12	0.11
25-34	0	0.15	0.36	0.45	0.38	0.26	0.20	0.20
35-44	0	0.05	0.33	0.47	0.40	0.24	0.17	0.17
45-54	0	0.01	0.18	0.38	0.35	0.19	0.07	0.07
55-64	0	0.07	0.25	0.37	0.40	0.23	0.05	-0.01
64-75	0	0.90	1.33	1.45	1.26	1.09	0.89	0.75

From Table 4.5, first we can see a common trend of increasing labour participation rates from year 2000 to year 2030. Then in most cases, the participation rates decline. Notice it declines more rapidly for older workers. Second, the increase in participation rates is larger for the 25-34 and 35-44 age groups at each period. The exception is for the 65-74 age group, but in this age category, the initial equilibrium participation rate is so small, any increase would represent a more significant one in percentage terms.

b. Second Scenario: $\sigma^{Ldem} = 1.5$

With an elasticity of substitution for labour demand equals to 1.5, workers of different age groups are more heterogeneous than the two previous scenarios. Hence, not surprisingly, when we look at the percentage changes in the real wage rates and in the net of taxes and contribution real wage rates (see Table 4.6 and Table 4.7), the differences across age groups are much more important. In particular, notice that the changes of the net of taxes and contributions real wage rate are negative only for the older workers.

Table 4.6: Real Wage Rates

(Changes in percentage terms)

Age Groups	2000	2010	2020	2030	2040	2050	2060	2070
15-24	0	0.71	1.37	1.97	2.36	2.54	2.55	2.54
25-34	0	0.38	0.98	1.59	1.99	2.18	2.18	2.16
35-44	0	-0.70	-0.41	0.17	0.58	0.78	0.78	0.75
45-54	0	-1.51	-2.27	-2.00	-1.62	-1.40	-1.38	-1.41
55-64	0	-1.77	-3.29	-4.02	-3.92	-3.70	-3.65	-3.66
64-75	0	-1.05	-2.81	-4.15	-4.88	-4.88	-4.82	-4.80

Table 4.7: Real Wage Rates Net of Taxes and Contributions

(Changes in percentage terms relative to 2000)

Age Groups	2000	2010	2020	2030	2040	2050	2060	2070
15-24	0	1.90	3.55	4.64	5.03	5.02	4.93	4.92
25-34	0	1.56	3.16	4.25	4.65	4.65	4.56	4.54
35-44	0	0.47	1.74	2.79	3.20	3.22	3.12	3.09
45-54	0	-0.35	-0.17	0.56	0.95	0.98	0.91	0.88
55-64	0	-0.62	-1.21	-1.51	-1.41	-1.37	-1.41	-1.42
64-75	0	0.12	-0.711	-1.64	-2.39	-2.58	-2.61	-2.59

The numbers in Table 4.6 and Table 4.7 reflects a higher demand for younger than for older workers. Hence, wage rates are up for younger and down for older workers. Moreover, the wage rates increase in a fast speed for younger workers and decline dramatically for older workers over the decades. In this sense, although there is an increase in the wage rate on average because of population aging, this may not be the case for old workers. Consequently labour market reaction to population aging may not generate economic incentives for old workers to return to or stick to the workforce and postpone retirement. Notice as well that in contrast to the second scenario, the differences of percentage wage rate changes between the young and the old in a certain decade are much greater than the differences in the baseline scenario.

Table 4.8: Labour Participation Rates

(Changes in percentage terms relative to 2000)

Age Groups	2000	2010	2020	2030	2040	2050	2060	2070
15-24	0	0.38	0.76	0.95	0.95	0.86	0.83	0.83
25-34	0	0.39	0.83	1.02	1.00	0.89	0.87	0.88
35-44	0	0.04	0.44	0.66	0.62	0.49	0.46	0.49
45-54	0	-0.20	-0.15	0.02	0.01	-0.14	-0.20	-0.17
55-64	0	-0.27	-0.51	-0.76	-0.84	-1.01	-1.11	-1.11
64-75	0	0.56	0.19	-0.56	-1.31	-1.62	-1.73	-1.78

Not surprisingly, over the periods, the participation rates are all positively growing for workers of age 15 to 44. For older workers, however, almost the figures are negative. Additionally, the growth rates of participation rates for the younger workers rise over time, but decline further for older workers. Under this scenario, the decreasing cost of leisure provides incentives for the earlier retirement and drives old workers out of the workforce. In this case, the participation rates of older workers would not increase but decline, in contrast to what the literature suggests.

5. Conclusion

In this paper, we develop a computable overlapping generations model under the assumption of imperfect substitution between young and old workers. This assumption determines that from the labour demand perspective, each age group would have its own wage rate and the changes in the wage rate will differ across age groups. We impose the same demographic shock and conduct two experiments differing by the value of the elasticity of substitution for the labour demand σ^{Ldem} used.

Through comparing the two scenarios, first of all, we show that the higher the value of the elasticity of substitution of the labour demand, the more important the size of the total labour force in determining the wage rates. In other words, the increase in the homogeneity across workers of different age groups implies smaller differences of percentage wage rate changes between the young and the old workers at different points in time. Secondly, we show that although population aging increases the wage rate on average because of an eventual slowdown in the labour force, this may no be the case for old workers. Consequently, labour market reaction to population aging may not generate economic incentives for old workers to return to or stick to the workforce and postpone retirement. In such a context, there may imply a new trend of early retirement.

References

Auerbach, A.J. and L.J. Kotlikoff (1987), “Dynamic Fiscal Policy”, Cambridge UK: Cambridge University Press.

Card, David and Lemieux, Thomas (2001), "Can Falling Supply Explain the Rising Return to College for Younger Men? A Cohort-Based Analysis”, *Quarterly Journal of Economics 116*, May 2001a, pp. 705-46.

Fougère, Maxime, Harvey Simon, Jean Mercenier et Marcel Mérette, (2005) “Population Ageing and the Effective Rate of Retirement in Canada”, Working Paper 2005 A-03, HRSDC-IC-SSHRC Skills Research Initiative.

Hamermesh, Daniel (2001), “Older workers in the coming labor “shortage”: implications of labor demand”, prepared for the Roundtable on the Demand for Older Workers, Brookings Institution, March 2001, unpublished.

OECD (1998), “Workforce ageing in OECD countries”, Chapter 4 in *Employment Outlook: June 1998*, OECD: Paris, pp. 123-152.

Statistics Canada (2005), “Population Projections for Canada, Provinces and Territories, 2005 to 2031”, Catalogue No. 91-520-XIE.

Wasmer, Étienne (2001), “Between-Group Competition in the Labour Market and the Rising Returns to Skill: US and France 1964-2000”, Working Paper, CEPR, No. 2798.