# Modeling a two pillar Phillips Curve for the Euro area using state space and non-linear models[1]

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## Abstract

Among the tools that economists use to analyze inflation developments in the euro area, the money-money augmented Phillips curve or the "two pillar" Phillips curve has reached a prominent place. The reason for this is that this type of model formalizes the two-pillar strategy followed by the ECB and at the same time it takes into account the fact that inflation expectations are important in modeling inflation. With this type of model it is possible to decompose inflation into contributions stemming from the output gap and core money growth, which proxies for inflation expectations. It has, however, often been argued that reduced form models, such as the Phillips curve, tend to be unstable. Thus, I estimate a "two-pillar" Phillips curve for the euro area using methods that allow for time-varying parameters.

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## 1. Introduction

There is a long tradition of using the Phillips curve to model the relationship between inflation and some measure of excess demand, either as a tool for forecasting, analyzing business cycles or analyzing monetary policy. Empirically, however, the validity of the traditional Phillips curve model has been questioned due to the disappointing performance of this reduced-form model to forecast inflation out- of-sample. In particular, it has been found that in the last decade or so the output gap has lost its power to explain inflation.

This parameter instability in the reduced form version of the Phillips Curve has been explained by a number of different non-competing theories. Some of the most appealing explanations for parameters instability besides the most basic Lucas Critique (see Turner 1997) are: 1) non-linearities (Schaling 1998), stemming from the fact the effect of the output gap on inflation is larger in the up turn or when the economy is at close to full employment than in the downturn or during recessions. 2) Changes in the structure of the economy or in the monetary policy regime and in the way inflation expectations are formed 3) More recently, some authors have explained the declining sensitivity of inflation to measures of domestic economic conditions with the increasing importance of global factors (Borio and Filardo (2006) IMF (2006)).[3] 4) Model instability could also be due to a misspecified model. Indeed, recent developments in the literature have yield the so called New Keynesian Phillips Curve which starts from the premise that inflation is mostly driven by inflations expectations. (Galí and Gertler (1989)).

Another more recent extension of the traditional Phillips Curve has focused on the usefulness of the Phillips curve to forecast inflation by including the low frequency elements of monetary development as a way of joining inflation expectations and the belief that inflation is a monetary phenomenon. This type of model introduced by Neumann (2003) and Gerlach (2004) attempts to formalize the monetary policy strategy followed in the euro area. The idea behind the money-money augmented Phillips curve or the "two pillar" Phillips curve is that money measured at low frequencies may contain information to explain inflation beyond that of the output gap. In particular, core money

[3] This last argument has been refuted on both theoretical (Ball 2006) and empirical grounds (Gnan and Valderrama 2006).

growth enters the equation as a proxy for inflation expectations (see Neumann (2003), consistent with the ECB paradigm that in the long run inflation is ultimately a monetary phenomenon.

Several authors have found that indeed including money in the Phillips curve improves the performance of the model. Gerlach (2004) and Neumann (2003) estimate "money-money augmented" or "two-pillar" Phillips curve and find that filtered money growth contains information useful for forecasting future prices that is not already embedded in a similarly filtered measure of inflation. Assenmacher-Wesche and Gerlach (2006d) have shown that this model fits also US, Japan and UK data. See Assenmacher-Wesche and Gerlach (2006b) to a detailed recount of this type of models.

Since the two-pillar Phillips Curve is a reduced form model as the traditional Phillips Curve, which has been often found to show parameter instability, there are reasons to expect that the two-pillar Phillips Curve may also have time varying parameters. Thus, the goal of this paper is to investigate whether the two-pillar Philips curve also suffers from parameter instability. Moreover, an attempt is made to determine whether parameter instability stems from the changing relationship between the output gap and inflation alone, or whether the relationship between monetary growth and inflation has also changed over time, as it is postulated by critics of the ECB.

The next section describes the reduced form Phillips curve that is used to test the hypothesis that inflation in the euro area is best modeled with a two-pillar Phillips curve using methods that allow for regime switches and time-varying parameters. The third section shows the results of estimating the two-pillar Phillips curve with three different econometric methods that account for time varying parameters: rolling regressions, Kalman filter estimation and Markov switching regressions. The fourth part presents conclusions and policy implications.

## 2. Model Setup and Data

The starting point of our analysis is a traditional output gap Phillips curve, where inflation $\pi_t^T$ is measured as the year on year growth rate of the HICP and the output gap $\tilde{y}$ is estimated by a Hodrick- Prescott filter of real GDP[4]:

$$\pi_t = \beta_1 \pi_{t-1} + \beta_2 \pi_{t-2} + \beta_3 \tilde{y}_t + \varepsilon_t \qquad (1)$$

As mentioned before, Neumann (2003) and Gerlach (2004) have proposed a money-money augmented or two-pillar Phillips curve which basically tries to model the two-pillar strategy followed by the Eurosystem within the Phillips curve framework. With this type of model it is possible to decompose inflation into contributions stemming from money, the output gap and other variables such as import prices or oil prices. The most important feature of this model is that only low frequency movements of money growth are relevant to explain inflation. Thus, the proposed model reflects the paradigm followed by the Eurosystem in which money explains medium to long term developments in inflation while the output gap and some other supply shock (changes in commodity prices in our case) explain short run developments in inflation. The theoretical justification behind the two-pillar Phillips curve is that inflation expectations are driven by the long term trend of money growth or core money growth.

In the meantime, there have been a few papers that have exploited the idea of the two-pillar Phillips curve on one hand, and the combination of low frequency and high frequency data on the other hand. Although, there are a number of different specifications of the two-pillar Phillips curve, none has proved to be superior to the other one. For the sake of simplicity we use one which has been used for forecasting purposes, and take the one proposed in Hoffman (2006)

$$\pi_t = \beta_1 \pi_{t-1} + \beta_2 \pi_{t-2} + \beta_3 \tilde{y}_t + \beta_4 \Delta m_t^T + \varepsilon_t \qquad (2)$$

Where core money growth $\Delta m_t^T$ is measured as in Hofmann (2006) by filtered money growth rates.[5]

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[4] All data is taken from the AWM Database over the EABCN Network.

[5] Hofmann (2006) uses trend money growth as the growth rate of a one-sided Hodrick-Prescott filter of M3. See page 20.

We additionally estimate a specification of the two-pillar Phillips curve taking into account a supply shock $comm$, which captures changes in world commodity prices

$$\pi_t = \beta_1 \pi_{t-1} + \beta_2 \pi_{t-2} + \beta_3 \tilde{y}_t + \beta_4 \Delta m_t^T + \beta_5 comm_t \tag{3}$$

For the sake of completeness an money augmented Phillips curve (without core money growth) but including commodity prices is also estimated:

$$\pi_t = \beta_1 \pi_{t-1} + \beta_2 \pi_{t-2} + \beta_3 \tilde{y}_t + \beta_5 comm_t + \varepsilon_t \tag{4}$$

From chart 1 we see clearly that since the mid 1980's the long run trend of inflation has fallen compared to the first two decades of our example. The charts show also, that while the output gap does not follow this change in inflation, core money growth tends also to show a lower long run trend. This observation, which is not particular only to the euro area, has also brought up the idea of the "flattening" of the Phillips curve. There are various arguments for this behavior, on one hand some authors argue that globalization has contributed to this flattening but here the verdict is still out.[6] Another, explanation is that monetary policy has been successful in anchoring inflations expectations. In general, it may be easier to think that in the almost four decades of data in the sample there have been different monetary policy regimes, which give rise to changes in the parameters of the equation.

## 3. Results

In the following we explore the issue of parameter instability or time-varying parameters by three different methods, rolling regression with a fixed sample size, a Kalman filter specification and a Markov-switching specification as in Hamilton (1989). The reason we use three methods is to test the robustness of the results. The first two methods are standard in the literature to model time varying parameters. Markov switching models are, on the other hand, well suited to capture possible non-linearities in the data. For each of the three methods proposed we show results for the four specifications described above.

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[6] See Ball (2006), Gnan and Valderrama (2006) and Iakova (2007)

### 3.1. Rolling Regressions

We start exploring the issue of parameter instability by estimating the equation above with rolling regressions with a fix window size of 80 observations. In order to compare the estimated parameters we show the estimated parameters when the whole sample is estimated (148 observations) and when the equation is estimated only for the period 1990 to 2006.

Based on these estimations, we see that the parameter of the output gap is consistently higher at the beginning of the sample, when it reaches values between 0.06 and 0.1 depending on the specification used, and falls also consistently close to 0.02 towards the end of the sample. Moreover, looking at the estimates at different points in time we see also that the significance of the coefficient of the output gap changes strongly depending on the time period under estimation.

Taking a look at the parameter of core money growth we see that there is also some variation, but the trend is not declining at the end of the sample, it rather falls temporarily around the years 1996 to 2001, and increases again to the levels obtained at the beginning of the sample. Thus, the parameter of money growth varies within a very narrow range, and that it is almost always significant.

### 3.2. State-space model: Kalman filter

A more formal way of estimating time varying parameters is using state space models such as the Kalman Filter. In this case our model above will be estimated using a signal and a state equation (see ZEW 2001, for an intuitive explanation of the Kalman Filter). Thus, to illustrate we rewrite equation (3) in which we let both the output gap and core money growth to vary over time while setting lagged inflation and commodity prices fix This makes sense, given the results from the rolling window regressions.

Thus our estimated model consists of a signal equation and two state equations, in which the parameters of both the output gap and core money growth are assumed to follow random walks:

Signal equation: $\pi_t = \beta_1 \pi_{t-1} + \beta_2 \pi_{t-2} + sv_1 \tilde{y}_t + sv_2 \Delta m_t^T + \beta_3 comm_t + (\text{var} = \exp(\beta_4))$

State equations:
$$sv_1 = sv_{1_{t-1}} + (\text{var} = \exp(\beta_5))$$
$$sv_2 = sv_{2_{t-1}} + (\text{var} = \exp(\beta_5))$$

The results of the four different specifications confirm that output gap is not significant in this model, taking even negative values towards the end of the sample, while the parameter of core money is positive and significant. The signal equations show that when core money is not included in the specification then output gap has an even lower degree of significance.

In chart 2 we show the smooth estimate of the coefficient for the output gap for the four specifications proposed and in chart 3 the smooth estimate of the coefficient for core money growth for the two specifications in which money is included. These results confirm what was observed in the preceding section, that the sensitivity of inflation to output gap has declined significantly, while the estimates of core money growth have varied also over time, but there is no recognizable declining (or increasing trend). In particular, these results suggest that the sensitivity of core money growth increased up to 1975, then again between 1980 and 1985 and has increased since 2000. This result is interesting for monetary policy given that the first two periods correspond to periods of increasing inflation rates.

### 3.3. Markov switching regressions

Another way of modeling time varying parameters is to apply Markov switching regime models developed originally by Hamilton (1989). In this type of models, parameters switch according to an unobservable state variable, which is estimated along with the model parameters, and which is assumed to follow a Markov switching process.

Thus, our equation (3) from above would be written as:

$$\pi_t = \upsilon(s_t) + \beta_1(s_t)\tilde{y}_t + \beta_2(s_t)\Delta m_t^T + \beta_3(s_t)comm_t + \varepsilon_t$$
$$\varepsilon_t \sim i.i.d.N(0, \Sigma(s_t)),$$

where $s_t$ represents the unobservable state variable which takes one out of 2 values, $s_t = k, k = 1,2$, in each period and is assumed to follow a first-order Markov process. The probability that regime *j* prevails in *t* depends on the regime prevailing in *t-1*, $\Pr(s_t = j | s_{t-1} = i) = \eta_{ij}$. In the following, the $2 \times 2$ conditional transition probabilities are gathered in the transition matrix $\eta$:

$$\eta = \begin{bmatrix} \eta_{11} & \eta_{12} \\ \eta_{21} & \eta_{22} \end{bmatrix}.$$

with the obvious restriction $\eta_{iK} = 1 - \sum_{j=1}^{K-1} \eta_{ij}$ .

The results show two interesting patterns. One hand, using only the output gap (with or without commodity prices) can be characterized by two regimes which resemble a structural break dated at around 1984, which is consistent with the results obtained from the Kalman filter. Second, the coefficient in regime 1 for these specifications is significant in regime 1 at stands at 0.0024, while in regime 2, it is significant only at the 10% level of significance and its value stands at 0.0009. Thus, these results confirm, that the sensitivity of inflation to the output gap has declined significantly over time.

Results for the two versions of the two-pillar Phillips curve show, that there are also two regimes. These are more difficult to characterize given that the differences in the size of the parameters for both the output gap and core money growth, do not differ significantly between regimes. (See Table 1).

## 4. Conclusions

The idea of using a two pillar Phillips curve or money augmented Phillips curve to describe or quantify the monetary policy strategy followed by the ECB is implemented in this paper by taking into account that for a relative extensive period, the parameters of the Phillips curve may have changed over time. Based on different methods, we find that indeed, the parameter of the output gap has significantly diminished, and in particular has lost its explanatory power for inflation.

We also confirm results by other authors that core money growth or in other words movements of money measured at low frequencies contribute to explain inflation, and in some periods has even more explanatory power then the output gap. An interesting finding is that while the parameter of the output gap has tended to become insignificant, the parameter of core money growth has varied during the period under study in a narrow range and there is no recognizable declining trend in the explanatory power. The estimates suggest that the parameter of core money growth was higher during the high inflation periods at the end of the 1980s. According to estimates from a Markov switching regressions suggest, on the other hand, that we are currently in the same regime as in the period starting in the mid 1980s.

Our results shows that these facts are robust to the model specification (traditional Phillips curve, money augmented Phillips curve or two-pillar Phillips curve) as well as to the method used to estimate the time-varying parameters.

The policy implication of these results is that while one has to recognize that there has been changing relationship between inflation and the output gap, the relationship between inflation and core money growth seems to have remained relatively stable over the last 4 decades.

The most obvious next step is to extend the model to forecast inflation. Further research, also would include some sensitivity and further robustness tests.

## References


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# APPENDIX

## Data

HICP: yoy growth rates

Output gap: Hodrick Prescott filter of real GDP

Core money growth: Hodrick Prescott filter of yoy growth rate of M3

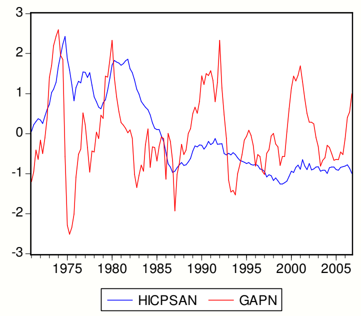
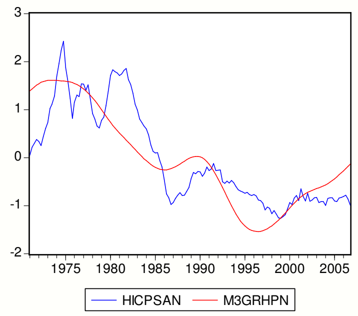
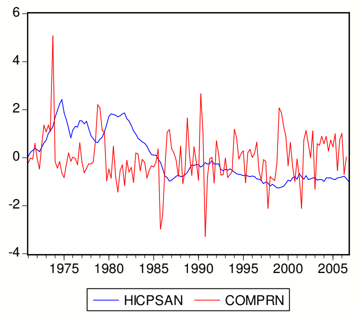
Commodity prices: yoy growth rates

Sample: 1970:1 to 2006:4

Quarterly data.

Source: AWM Database

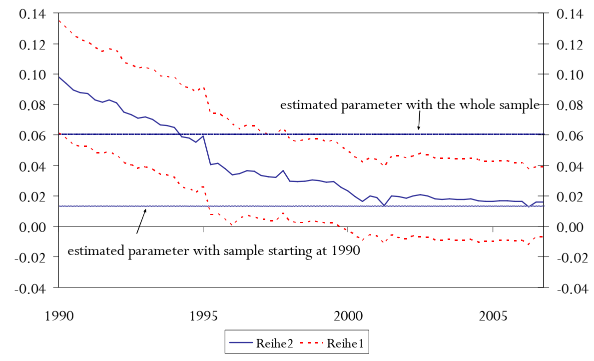
**Chart 1**

| **Inflation and output gap** | **Inflation and core money growth** | **Inflation and commodity prices** |
|---|---|---|
|  |  |  |

## Rolling Regressions

### Chart 2

#### Smooth estimates of $\beta_3$ for Equation (1)

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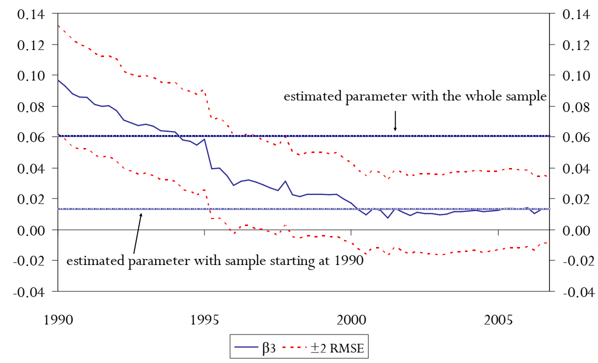
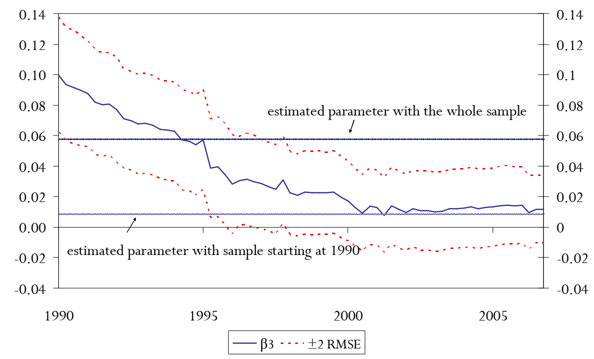
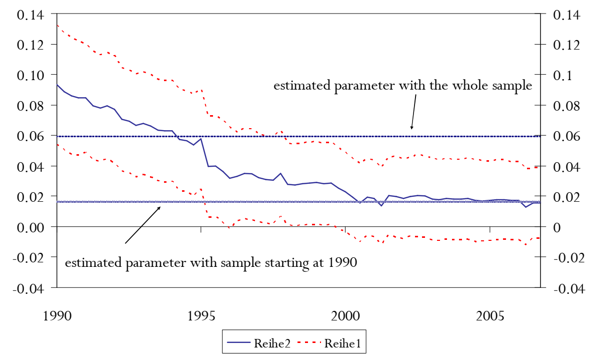



#### Smooth estimates of $\beta_3$ for Equation (3)

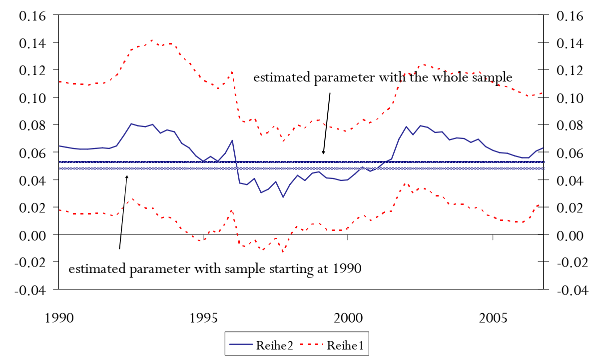



#### Smooth estimates of $\beta_3$ for Equation (4)

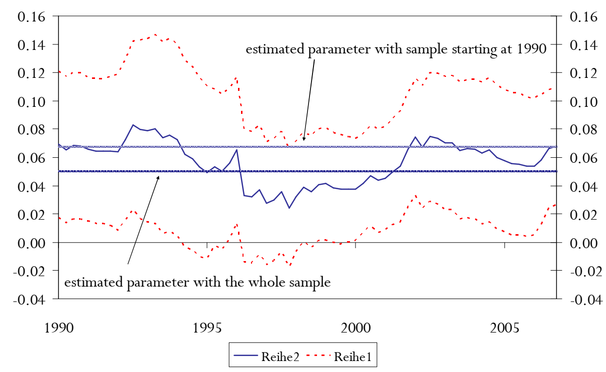



### Chart 3

#### Smooth estimates of $\beta_4$ for Equation (2)



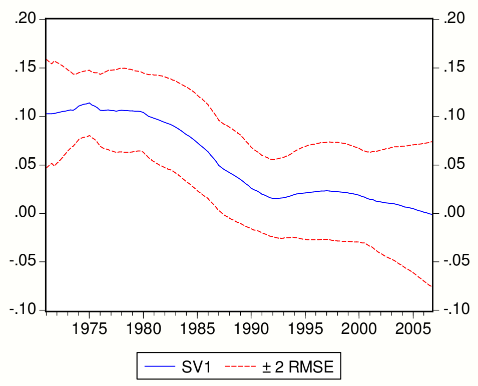

#### Smooth estimates of $\beta_4$ for Equation (3)



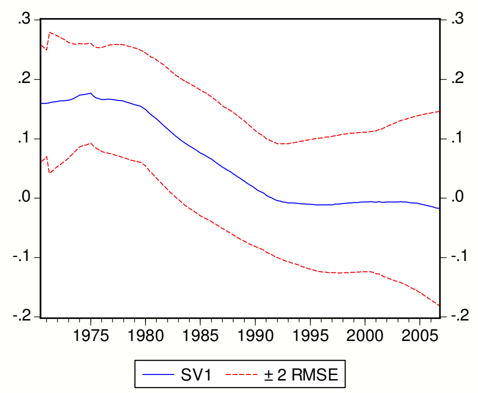
## Kalman Filter

### Chart 4

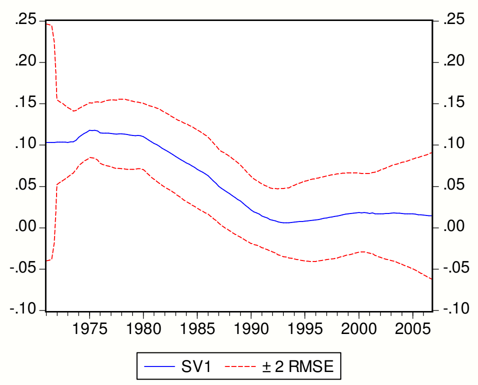
**Smooth estimates of $sv_1$ for Equation (1)**



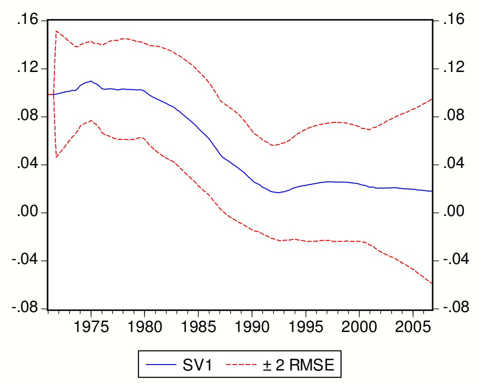

**Smooth estimates of $sv_1$ for Equation (2)**




**Smooth estimates of $sv_1$ for Equation (3)**

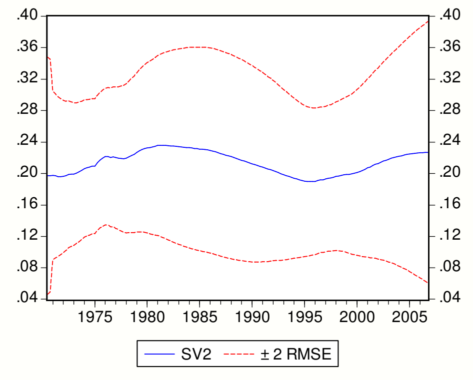



**Smooth estimates of $sv_1$ for Equation (4)**

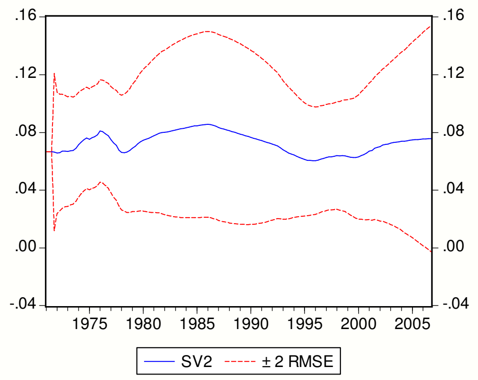



### Chart 5

**Smooth estimates of $sv_2$ for Equation (2)**




**Smooth estimates of $sv_2$ for Equation (3)**



## Markov switching regressions

**Chart 6**

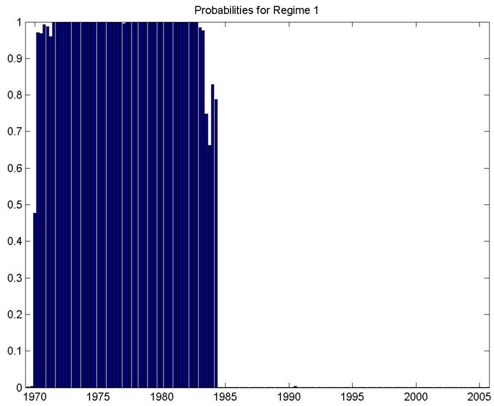
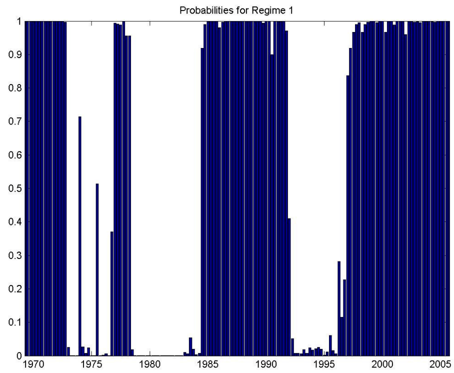
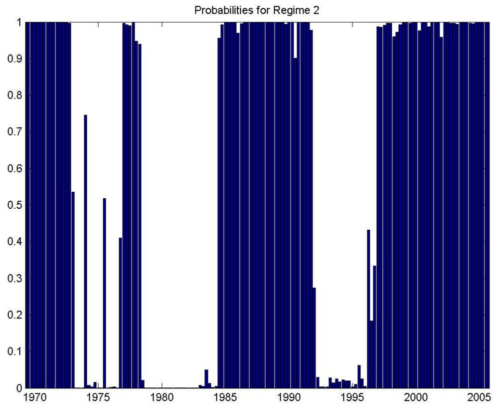
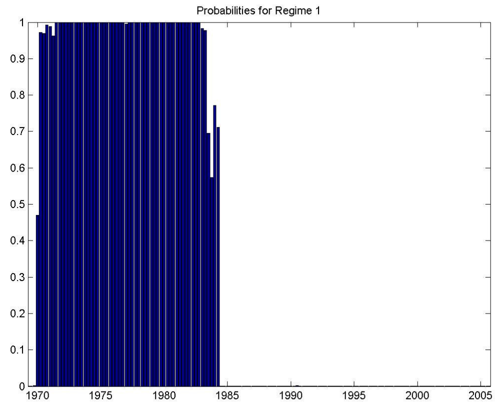
| **Probabilities of regime 1 for Equation (1)** | **Probabilities of regime 1 for Equation (2)** |
| --- | --- |
|  |  |
| **Probabilities of regime 1 for Equation (3)** | **Probabilities of regime 1 for Equation (4)** |
|  |  |









**Table 1**

| | Output Gap | | | | Core money growth | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | regime 1 | | regime 2 | | regime 1 | | regime 2 | |
| | | coeff. | coeff. | t-stat | coeff. | t-stat | coeff. | t-stat |
| Equation 1 | 0.0024 | 5.65 | 0.0009 | 1.79 | | | | |
| Equation 2 | 0.0016 | 4.47 | 0.0026 | 6.98 | 0.0056 | 15.24 | 0.0066 | 17.43 |
| Equation 3 | 0.0015 | 4.43 | 0.0029 | 7.76 | 0.0054 | 15.36 | 0.0065 | 17.62 |
| Equation 4 | 0.0027 | 5.65 | 0.0009 | 1.78 | | | | |