# Debt Maturity in a Small Open Economy under Inflation Target 

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#### Abstract

This work analyzes several issues about public debt maturity in a general equilibrium model of a small open economy under inflation target. Following a risk premium shock, the households of this economy suffer higher wealth losses if they finance the government with longer maturity nominal bond. This happens due to surprise inflation and because of the foregone returns not earned in a longer position. This result may explain the difficulty faced by several Treasuries of emerging economies to extend the debt maturity in moments of confidence crisis. Our simulations also indicate that stronger commitment to stable inflation helps a Treasury willing to extend debt maturity since it reduces wealth losses. The simulations are all carried in Dynare for Matlab.


Key words: Debt Maturity, Risk Premium, Monetary Policy, Inflation Target, New Open Economy Macroeconomics, Emerging Economy.

## JEL classification:E60, F41, H63

## 1 Introduction

This paper is motivated by the short term lending bias observed in many emerging market countries, especially in Latin America. We show that when an economy is prone to confidence
crises, households rationally demand a premium to compensate for this risk. This raises the cost to a Treasury of borrowing long, and leads to the observed short term debt maturity structure. In order to understand this problem, we study the impact of alternative government debt maturity strategies in a small, open dynamic general equilibrium model. We subject the economy to an exogenous risk premium shock. The exchange rate depreciates immediately and the Central Bank must increase the interest rate to control inflation. Surprise inflation reduces consumer wealth because it depresses the real return on nominal bonds. This wealth effect is worse when the household holds longer maturity bonds because the foregone return is greater. We show that when an economy is prone to confidence crises, investors will hold longer bonds only if they are compensated for this possible wealth loss. This extra compensation increases the fiscal authority's borrowing cost, and it in turn reacts by shortening the maturity structure.

We construct an environment a with nominal friction, but note recent work on debt maturity by Angeletos (2002), Buera and Nicolini (2004) and Shin (2006) that consider a classical scenario with fully flexible prices. The first two showed that the optimal allocation can be implemented by carefully choosing the debt maturity, implying that there is no need to rely on state contingent bonds to replicate the optimal allocation of Lucas and Stokey (1983). The optimal maturity is obtained by appropriately issuing bonds of at least as many maturities as the number of states. The result is an optimal maturity that is state invariant, implying that a country's Treasury would not need to actively manage the maturity structure.

Shin (2006) observed that despite the insight provided by Angeletos and Buera and Nicolini, it is well known that active management is conducted by every Treasury. Shin derived an active debt management strategy that could also implement the optimal allocation, by using a Markov chain with an asymmetric transition matrix to capture how fast government expenditure rises and falls. This matrix is then used to obtain the active maturity strategy with a short term bond and a consol. Calibration for England, during the $18^{\text {th }}$ century, reveals that Shin's strategy matches British Treasury policy. He verifies that, during peace times when government consumption was lower, more consols were issued while short period bonds were accumulated. A reverse operation was conducted as war broke out and government spending was "medium". As war reached its climax and expenditure was higher, the initial situation was again observed, but not necessarily
with the same intensity.
Even though we also study debt maturity, the economic environment considered here is different from those of previous authors. In particular, our world is one in which nominal frictions and monetary policy matter. The reason for this is that we wish to consider situations of recent interest, where interest rate movements to control inflation affect the bond returns.

In a set up closer to ours, Benigno and Woodford (2006) analyze optimal inflation targeting under alternative fiscal regimes. They argue that if the maturity structure proposed by Angeletos and Buera and Nicolini could be effectively implemented, the central bank should target a zero inflation rate in every period. This would happen because such a maturity structure is designed to compensate for all sources of shocks in the economy while leaving the consumer's consumption constant across states. Although Benigno and Woodford do not extend their analysis to analyze the impact of maturity structure in an economy with nominal frictions, they are skeptical regarding the practical use of the solution proposed by Angeletos and Buera and Nicolini due to its complexity. Another reason is high sensitivity to parameterization.

Our work constitutes, in this sense, an effort to analyze debt maturity in a more realistic scenario where monetary policy matters. We are able to address the impact of debt maturity in a dynamic stochastic general equilibrium model, which is the framework used by Benigno and Woodford and most of the recent work on monetary policy. Despite the distance from the setup used by Shin, our conclusions also provide insight into his results.

The classical reference on the effects of debt maturity in a monetary economy is the work of Tobin (1963). He considers monetary policy in a closed economy in which the Treasury and the Central Bank face the task of determining the supply and demand of short and long term nominal debt. A higher amount of short term debt outstanding creates an incentive for banks to lend more resources, since this asset is converted into money. This is the main channel through which maturity affects the economy, justifying debt management. The other reason for concern about maturity is to minimize the cost of public sector liabilities. Our analysis suggests an important cost minimization role for maturity management, but we identify the wealth effect as the main channel through which maturity affects household decisions. ${ }^{1}$

[^0]In our model, an exogenous increase in a country's risk premium causes the exchange rate to depreciate through the risk adjusted uncovered interest rate parity condition. This raises present and future inflation through its impact on costs. The Central Bank, committed to inflation targeting, increases the nominal interest rate. Household wealth drops as higher inflation depresses the real value of bond returns. An intertemporal negative wealth effect is added if the government finances the public sector mainly through two period bonds. This extra impact is due to the opportunity cost of being locked into two period bonds and hence unable to benefit from higher interest rates. Lower wealth reduces consumption and increases labor supply, which results in higher output.

This negative wealth effect can explain the tendency of emerging economies Treasuries to shorten debt maturity as a confidence crisis is more likely to happen. We argue that the prospect of higher losses makes households ask for a higher term premium to hold long bonds, which ultimately increases the fiscal sector's financial expenditure. Although it is recognized that longer debt maturity reduces the likelihood of a self-fulfilling debt crisis (Cole and Kehoe (1996) and depresses the sovereign's risk premium (Gapen, Gray, Lim, and Yingbin (2005)), the reduction in the maturity due to an increase in the term premium seems plausible according to our simulations.

The same argument may be used to explain the results obtained by Shin (2006) in an environment of fully flexible prices. When a government starts to elevate expenditure due to the announcement of a war, households expect the interest rate to increase. But they must sell their consol to obtain a higher return right from the beginning of the elevation in the interest rate. This implies that households would only hold a consol if they were compensated for this risk, which would also imply higher financial costs for the British Treasury. The Treasury's reaction is to shorten the maturity structure of outstanding debt by optimally replacing expensive long term debt with relatively cheaper short term debt.

The rest of the paper proceeds as follows:
We present the baseline model in the next section. It features similar structure presented by Benes and Vavra (2003) for the Czech Republic and by Hamann, Julio, Restrepo, and Riascos (2004) (HJRR hereafter) for Colombia.
rate has the potential to produce inflationary pressure through its effect on public debt. If this possibility is present, the cost minimization role of debt management also has a direct effect on inflation by not allowing the debt to increase so much.

In section 3 we obtain the steady state and calibrate the model for the Brazilian economy. We allow for two alternative maturity structure: in case A the government sells only short maturity bonds and in case B it sells one and two period bonds. In section 4 we expose the economy to an exogenous risk premium shock and verify a larger negative wealth effect when the household finances the government using longer maturity bonds. This implies that if a crisis is more likely to happen, the household would want to obtain long run financing from the government (borrow), while she would finance the government (save) by buying short bonds. This is exactly the situation Shin (2006) describes for England; when government expenditures increased due to war, the interest rate rose and the British sold mainly short bonds while holding consols.

We also contrast the implications of alternatives central bank reactions to inflation. In the presence of a more conservative central bank, the difference between short and long maturity debt composition is reduced. This happens because inflation becomes smaller when the interest rate reacts more intensively to inflationary oscillations, which minimizes the losses with the nominal bonds. This indicates that a more credible and rigid central bank may also favor the extension of the public debt maturity profile.

The paper has three appendices. In the first we derive an analytical linearized equation for the household budget constraint in order to show the impact of maturity on wealth. In the second we assume zero outstanding debt and two other maturity possibilities. In case C the Treasury is a creditor in short maturity bonds and a debtor in long bonds. In case D the opposite situation is studied. Our conclusions remain unchanged under this alternative scenario.

In the third appendix we present a simpler version of the model without capital. Although the dynamics for this alternative environment perform very poorly when confronted with the actual paths for output following a risk premium shock, our conclusions about the impact of debt maturity remain the same, which shows the robustness of our analysis.

We left aside important issues about debt maturity (like its direct relation to the sovereign risk premium and default probability) to facilitate our understanding of how it affects the dynamics of an economy that is exposed to a risk premium shock. Incorporating these features would certainly provide more insight on the relevance of debt maturity.

## 2 The Economic Environment

The economy has a Central Bank, a Fiscal Sector, and a representative household who owns a wholesale firm and a measure one of retail firms.

The wholesaler acts in a perfectly competitive international market by producing a homogeneous tradable good that can be exported, consumed internally by retailers, or saved as capital. Purchasing power parity (PPP) always holds for this good.

The retailers compete among themselves in a monopolistically competitive market. Each of them is specialized in the production of a unique good that is demanded by the domestic government and by the household. An individual retailer buys the homogeneous good from the wholesaler and differentiates it using a costless technology. A nominal friction is introduced by allowing for Calvo pricing in the inflation rate: in each period, a fraction of retailers choose prices optimally, while the remaining fraction update prices according to past inflation. The inflation rate rule, rather than in price levels, guarantees a more realistic path for inflation as it yields a hump shaped pattern over time.

The household derives utility from leisure and consumption of a Dixit-Stiglitz composite good. She finances the government by buying one and two period's nominal bonds, and is allowed to borrow in the international financial market. She also makes a capital (investment) decision.

The fiscal sector consumes differentiated goods supplied by the retailers and collects revenue by taxing consumption and labor income. Debt is financed through domestic sales of one and two period nominal bonds. We impose a fiscal and a maturity rule. The fiscal rule determines the public debt and expenditure. The maturity rule specifies the debt management strategy that remains constant throughout our experiments. The impact of maturity is studied by allowing for different initial (steady state) maturity composition. Finally, the Central Bank uses a forward looking interest rule to target inflation.

We "close" the small open economy by allowing for an external debt elastic risk premium function, as proposed by Schmitt-Grohe and Uribe (2003).

### 2.1 The Wholesale Firm

This firm produces a tradeable good $Y$ that can be sold domestically to retailers or exported. Its price is subject to purchasing power parity (PPP), i.e., $P_{t}^{Y}=S_{t} P_{t}^{*}$, where $P_{t}^{Y}$ and $P_{t}^{*}$ are the prices of homogeneous good $Y$ in the domestic and foreign markets, respectively. $S_{t}$ is the nominal exchange rate (the domestic price of one unit of the foreign currency). For simplicity, we assume that $P_{t}^{*}=1$ for every period $t$, implying zero inflation in the foreign country. This assumption allows us to write the PPP relation as

$$
\begin{equation*}
P_{t}^{Y}=S_{t} \tag{1}
\end{equation*}
$$

The firm's objective is to maximize current profit $\Pi_{t}^{Y}$ after paying nominal wages $W_{t}$ and real rental of capital $r_{t}$ for the use of labor $\left(L_{t}\right)$ and capital $\left(K_{t}\right)$, respectively. The following CobbDouglas production function is used

$$
\begin{equation*}
Y_{t}=K_{t}^{\alpha} L_{t}^{1-\alpha} \tag{2}
\end{equation*}
$$

where $0 \leq \alpha \leq 1$. The maximization problem is

$$
\begin{equation*}
\underset{K_{t}, L_{t}}{M a x}\left\{P_{t}^{Y} K_{t}^{\alpha} L_{t}^{1-\alpha}-W_{t} L_{t}-P_{t}^{Y} r_{t} K_{t}\right\}, \tag{3}
\end{equation*}
$$

implying the following first order optimality conditions for capital and labor, respectively,

$$
\begin{gather*}
\alpha \frac{Y_{t}}{K_{t}}=r_{t}  \tag{4}\\
(1-\alpha) \frac{Y_{t}}{L_{t}}=\frac{W_{t}}{P_{t}^{Y}} .
\end{gather*}
$$

Letting $P_{t}$ be the domestic CPI, as defined by ??, the last relation may be rewritten as

$$
\begin{equation*}
(1-\alpha) \frac{Y_{t}}{L_{t}}=\frac{w_{t}}{q_{t}} \tag{5}
\end{equation*}
$$

where $w_{t}=\frac{W_{t}}{P_{t}}$ is the real wage and $q_{t}=\frac{P_{t}^{Y}}{P}=\frac{S_{t}}{P}$ is the real exchange rate (the relative price of the tradable against the non-tradable good).

### 2.2 Household

In each period the representative household chooses $c_{i t}$, which is the consumption of good $i$ in period $t$. Aggregation of all $i \in[0,1]$, using a standard Dixit-Stiglitz aggregator, yields

$$
\begin{equation*}
C_{t}=\left(\int_{0}^{1} c_{i}^{\frac{\theta-1}{\theta}} d i\right)^{\frac{\theta}{\theta-1}}, \tag{6}
\end{equation*}
$$

which implies the following consumer price index (CPI),

$$
\begin{equation*}
P_{t}=\left(\int_{0}^{1} p_{i}^{1-\theta} d i\right)^{\frac{1}{1-\theta}} . \tag{7}
\end{equation*}
$$

$-\theta$ represents the price elasticity of demand for each good $c_{i}$. The household chooses her amount to borrow abroad by supplying international bonds $F_{t}^{S}$. She also trades nominal government bonds expiring at time $t+1$ and $t+2\left(b 1_{t, t+1}^{D}\right.$ and $b 2_{t, t+2}^{D}$, respectively). The return on one period domestic bond expiring at $t+1$ is $i_{t}$, while the two period government bond pays $i 2_{t}$ at $t+2$. The international asset pays a constant risk free return equal to $i_{t}^{*}=i^{*}$ for all $t$.

The household also chooses investment $J_{t}$, which determines the capital stock $k_{t+1}$ by the following law of motion

$$
\begin{equation*}
k_{t+1}=(1-\delta) k_{t}+J_{t}, \tag{8}
\end{equation*}
$$

where $\delta$ is the depreciation rate of capital. Finally, she chooses the amount of composite good $C_{t}$ to consume and the amount of labor to supply, $l_{t}$.

In any period $t$, her budget constraint is given by

$$
\begin{aligned}
& \left(1+\tau^{c}\right) P_{t} C_{t}+P_{t}^{Y} J_{t}+b 1_{t, t+1}^{D}+b 2_{t, t+2}^{D}+\left(1+i^{*}\right)\left(1+r p_{t-1}\right) S_{t} F_{t-1}^{S} \leq \\
& \leq\left(1-\tau^{l}\right) W_{t} l_{t}+P_{t}^{Y} r_{t} k_{t}+\left(1+i_{t-1}\right) b 1_{t-1, t}^{D}+\left(1+i 2_{t-2}\right) b 2_{t-2, t}^{D}+ \\
& \\
& \quad+S_{t} F_{t}^{S}+\Pi_{t}^{Y}+\int_{0}^{1} \pi_{i t} d i
\end{aligned}
$$

where $\tau^{c}$ and $\tau^{l}$ are tax rates on consumption and on labor revenue, $r p_{t}$ is the country's risk premium, and $\pi_{i}$ indicates the profit of retailer $i$. The term $\left(1+i^{*}\right)\left(1+r p_{t-1}\right)$ makes clear that
the return on foreign bonds is adjusted for domestic risk. Another important aspect of this budget constraint is that capital expenses are stated in terms of the price of the homogeneous good $\left(P_{t}^{Y}\right)$, implying that capital is a tradeable good subject to purchasing power parity. The nominal exchange rate $S_{t}$ multiplies the foreign bond $F_{t}^{S}$ (on the left hand side of the budget constraint) because the selling (or buying) of the international asset happens at time $t$. Similarly, $\left(1+i^{*}\right)\left(1+r p_{t-1}\right) S_{t} F_{t-1}^{S}$ is the amount received (paid) at time $t$ for the international transaction occurred at $t-1$.

The consumer's problem is to choose sequences $c_{i t}, C_{t}, l_{t}, J_{t}, k_{t+1}, b 1_{t, t+1}^{D}, b 2_{t, t+2}^{D}, F_{t}^{S}$ to maximize the expected present value of her lifetime utility, subject to capital accumulation (??), budget constraint, consumption aggregator (??), and given initial values for the capital stock $\left(k_{t-1}\right)$, government bonds ( $b 1_{t-1, t}^{D}$ and $b 2_{t-2, t}^{D}$ ), and foreign bonds $\left(F_{t-1}^{S}\right)$. The problem can be simplified after substituting (??) into the budget constraint, implying the following problem:

$$
\begin{equation*}
\left.\underset{\left\{c_{i t}, C_{t}, l_{t}, k_{t+1}, b 1\right.}{\operatorname{Max}} \underset{t, t+1}{D}, b 2_{t, t+2}^{D} F_{t+1}^{S}\right\}_{t=0}^{\infty}, ~ E 0 ~ \sum_{t=0}^{\infty} \beta^{t}\left[\frac{C_{t}^{1-\sigma}}{1-\sigma}-\chi l_{t}\right] \tag{9}
\end{equation*}
$$

subject to $i$ ) (??);
ii) $\left(1+\tau^{c}\right) P_{t} C_{t}+P_{t}^{Y} k_{t+1}+b 1_{t, t+1}^{D}+b 2_{t, t+2}^{D}+\left(1+i_{t-1}^{*}\right)\left(1+r p_{t-1}\right) S_{t} F_{t-1}^{S} \leq$

$$
\begin{align*}
& \leq\left(1-\tau^{l}\right) W_{t} l_{t}+P_{t}^{Y} r_{t} k_{t}+(1-\delta) P_{t}^{Y} k_{t}+ \\
& +\left(1+i_{t-1}\right) b 1_{t-1, t}^{D}+\left(1+i 2_{t-2}\right) b 2_{t-2, t}^{D}+S_{t} F_{t}^{S}+\Pi_{t}^{Y}+\int_{0}^{1} \pi_{i t} d i \tag{10}
\end{align*}
$$

iii) and given $k_{t-1}, b 1_{t-1, t}^{D}, b 2_{t-2, t}^{D}$ and $F_{t-1}^{S}$.

The following restrictions are also necessary: $0<\sigma \leqslant 1$ and $\chi>0$.
The first order optimality conditions imply the following relations:

$$
\begin{gather*}
c_{i t}=C_{t}\left(\frac{p_{i t}}{P_{t}}\right)^{-\theta}  \tag{11}\\
C_{t}^{-\sigma}=E_{t} C_{t+1}^{-\sigma}\left[\beta \frac{\left(1+i_{t}\right)}{\frac{P_{t+1}}{P_{t}}}\right] \tag{12}
\end{gather*}
$$

$$
\begin{gather*}
E_{t} r_{t+1}=E_{t}\left[\frac{1+i_{t}}{\frac{P_{t+1}^{Y}}{P_{t}^{Y}}}-(1-\delta)\right]  \tag{13}\\
C_{t}^{\sigma}=\frac{\left(1-\tau^{l}\right)}{\chi\left(1+\tau^{C}\right)} w_{t}  \tag{14}\\
\left(1+i_{t}\right)=\left(1+i^{*}\right)\left(1+r p_{t}\right) E_{t} \frac{S_{t+1}}{S_{t}}  \tag{15}\\
\left(1+i 2_{t}\right)=E_{t}\left(1+i_{t}\right)\left(1+i_{t+1}\right) \tag{16}
\end{gather*}
$$

Equation (??) shows how the household chooses each good $c_{i}$. Relation (??) is the Euler equation for the composite good $C_{t}$, with $-\frac{1}{\sigma}$ being the intertemporal elasticity of substitution for consumption. Equation (??) shows that the expected return on investment made at time $t$ must be enough to cover capital depreciation and the expected real interest rate $\frac{1+i_{t}}{P_{t+1}^{Y} / P_{t}^{Y}}$. The labor-leisure trade off is represented by equation (??). Notice that labor is not present in this equation because we follow the RBC literature and assume indivisible labor, implying that production will increase due to new hiring instead of extending the working hours of employed workers. The last two equations reflect no arbitrage conditions on the international and domestic bond markets. Equation (??) is the risk adjusted uncovered interest parity which is a no arbitrage condition between the foreign and domestic bond markets. According to this relation, the domestic nominal interest rate must equal the return on foreign bonds, adjusted for the domestic risk of default and for the expected change in the nominal exchange rate. Finally, equation (??) shows what the two periods nominal bond should pay to make households indifferent between holding it or holding a one period bond.

### 2.3 Fiscal Sector

The fiscal sector is divided into two branches. The first is the fiscal authority, responsible for making standard government decisions about expenditure and public debt. The second consists of a public debt manager, responsible for implementing rules regarding the composition of the debt maturity structure.

The fiscal authority only buys non-tradable goods, implying an expenditure of $p_{i t} g_{i t}$ for each good $i$ demanded. The Dixit-Stiglitz aggregator $G_{t}$ for every good $g_{i}, i \in[0,1]$, has the same
properties as the one used for the household, i.e.,

$$
\begin{equation*}
G_{t}=\left(\int_{0}^{1} g_{i}^{\frac{\theta-1}{\theta}} d i\right)^{\frac{\theta}{\theta-1}} . \tag{17}
\end{equation*}
$$

The demand for each of these goods is obtained by minimizing the expenditure on every good $i$ subject to the previous aggregator, leading to

$$
\begin{equation*}
g_{i t}=G_{t}\left(\frac{p_{i t}}{P_{t}}\right)^{-\theta} . \tag{18}
\end{equation*}
$$

By taxing household consumption and labor income, the government obtains the following nominal tax revenue: $\tau_{t}=\tau^{c} P_{t} C_{t}+\tau^{l} W_{t} l_{t}$. A possible public sector deficit is financed through the sale of nominal bonds to domestic households. These bonds expire either in 1 or 2 periods. The short period nominal bond $\left(b 1_{t, t+1}^{S}\right)$ pays the current nominal interest rate $i_{t}$ in period $t+1$, while the two period bond ( $b 2_{t, t+2}^{S}$ ) pays the nominal interest rate $i 2_{t}$ at time $t+2$.

Combine revenue and expenditure to write the government's nominal budget constraint as

$$
\tau_{t}+b 1_{t, t+1}^{S}+b 2_{t, t+2}^{S}=P_{t} G_{t}+\left(1+i_{t-1}\right) b 1_{t-1, t}^{S}+\left(1+i 2_{t-2}\right) b 2_{t-2, t}^{S} .
$$

Defining $B 1_{t, t+1}^{S} \equiv \frac{b 1_{t, t+1}^{S}}{P_{t}}$ and $B 2_{t, t+2}^{S} \equiv \frac{b 2_{t, t+2}^{S}}{P_{t}}$ as the real values of the respective bonds issued at time $t$, the real budget constraint becomes

$$
\begin{equation*}
T_{t}+B 1_{t, t+1}^{S}+B 2_{t, t+2}^{S}=G_{t}+\frac{1+i_{t-1}}{1+\pi_{t}} B 1_{t-1, t}^{S}+\frac{1+i 2_{t-2}}{\left(1+\pi_{t-1}\right)\left(1+\pi_{t}\right)} B 2_{t-2, t}^{S} \tag{19}
\end{equation*}
$$

where

$$
\begin{equation*}
T_{t}=\tau^{c} C_{t}+\tau^{l} w_{t} l_{t} \tag{20}
\end{equation*}
$$

is the real tax revenue.
The end of period real outstanding debt is $D_{t} \equiv B 1_{t, t+1}^{S}+B 2_{t, t+2}^{S}+\frac{\left(1+i 2_{t-1}\right)}{\left(1+\pi_{t}\right)\left(1+i_{t}\right)} B 2_{t-1, t+1}^{S}$, where the last term is due to the two period bond sold at $t-1$ with interest accrued up to time $t$. $D_{t}$ is the amount the government would have to pay, at the end of period $t$, if it wanted to cancel the total debt.

At this point there are two fiscal sector equations (?? and ??), but must determine 3 variables: $G_{t}, B 1_{t, t+1}^{S}$, and $B 2_{t, t+2}^{S}$. We do this by imposing a fiscal rule that determines $G_{t}$ and a debt management strategy that specifies the amount supplied of each bond.

We assume that government consumption is adjusted to keep real outstanding debt constant at its steady state level,

$$
\begin{equation*}
D_{t}=D . \tag{21}
\end{equation*}
$$

This simple fiscal rule is used to facilitate future analysis, but our conclusions are not affected by alternative rules.

Debt management deals with the composition and strategies regarding the maturity structure of the public debt. We only consider one rule for dealing with maturity. Under this rule, the debt manager follows a passive strategy as the real amount of two period bonds remains constant:

$$
\begin{equation*}
B 2_{t, t+2}=B 2 \tag{22}
\end{equation*}
$$

Since this rule does not determine the initial steady state $B 1$ and $B 2$, we also need to specify these values. This will be done later as we analyze the dynamics of the model and the impact of different maturity composition.

### 2.4 Retailers

The economy has a unit measure of $i$ retailers who act as intermediaries. Each of them buys, at price $P^{Y}$, a quantity $y_{i}^{R D}$ (with $R D$ standing for retailer $i$ 's demand) of the homogeneous good $Y$, which is later transformed into the differentiated good $y_{i}$ by means of a costless technology. The good is then sold to the government and household at price $p_{i}$ in a monopolistically competitive market. The demand $y_{i}^{D}$ facing each monopolist at time $t$ equals $y_{i t}^{D}=c_{i t}+g_{i t}$, which can also be expressed as $y_{i t}^{D}=\left(C_{t}+G_{t}\right)\left(\frac{p_{i t}}{P_{t}}\right)^{-\theta}=y_{i t}^{R D}$. The last equality arises because the retailers only buy the homogeneous good in enough quantity to meet the demand for his differentiated product.

These retailers are subject to Calvo pricing, so they cannot always charge the optimal price. In particular, in every period there is a random draw that defines the fraction $\varepsilon$ of firms that will
follow a pricing rule, and the fraction $1-\varepsilon$ that will be able to charge optimally. The firms following the rule update price at time $t$ according to the previous period CPI inflation, i.e.,

$$
p_{i t}^{r u l e}=p_{i t-1} \Delta P_{t-1}
$$

where $\Delta P_{t}=\frac{P_{t}}{P_{t-1}}$. It follows that the intertemporal problem for each retailer is

$$
\max _{\left\{y_{i \tau}, p_{i t}\right\}_{t=\tau}^{\infty}} E_{t} \sum_{\tau=t}^{\infty} \zeta_{t, \tau} \varepsilon^{\tau-t} y_{i \tau}^{d}\left(p_{i t} Z_{t, \tau}-P_{\tau}^{Y}\right)
$$

subject to sequences of demand functions

$$
y_{i \tau}^{D}=\left(C_{\tau}+G_{\tau}\right)\left(\frac{p_{i \tau}}{P_{\tau}}\right)^{-\theta}
$$

where $\varsigma_{t, \tau}$ is the rate at which the retailer discounts values from period $\tau$ to $t$, and $Z_{t, \tau} \equiv \frac{P_{\tau-1}}{P_{t-1}}$.
Let $m_{t}$ denote the optimal price of good $i$ relative to the CPI, i.e. $m_{t} \equiv \frac{p_{i t}}{P_{t}}$. The problem's solution can be expressed as

$$
\begin{equation*}
m_{t}=\frac{\theta}{\theta-1} \frac{\Gamma_{t}}{\Omega_{t}} \tag{23}
\end{equation*}
$$

where

$$
\begin{equation*}
\Gamma_{t}=\left(C_{t}+G_{t}\right) q_{t}+E_{t}\left[\frac{\varepsilon}{\left(1+i_{t}\right) / \Delta P_{t+1}}\left(\frac{\Delta P_{t+1}}{\Delta P_{t}}\right)^{\theta} \Gamma_{t+1}\right] \tag{24}
\end{equation*}
$$

and

$$
\begin{equation*}
\Omega_{t}=\left(C_{t}+G_{t}\right)+E_{t}\left[\frac{\varepsilon}{\left(1+i_{t}\right) / \Delta P_{t+1}}\left(\frac{\Delta P_{t+1}}{\Delta P_{t}}\right)^{\theta-1} \Omega_{t+1}\right] . \tag{25}
\end{equation*}
$$

The presence of the real exchange rate $q_{t}$ in (??) captures the pass-through from the exchange rate to the optimal price charged by firms.

### 2.5 Central Bank and Inflation

CPI aggregator (??) can be rewritten as $P_{t}=\left[\varepsilon\left(p_{t}^{r u l e}\right)^{1-\theta}+(1-\varepsilon)\left(m_{t} P_{t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$. If we divide both sides by $P_{t}$ and define the CPI inflation at period $t$ to be $\pi_{t}=\Delta P_{t}-1$, we obtain an
expression showing how CPI inflation is determined in each period,

$$
\begin{equation*}
1+\pi_{t}=\left[\varepsilon\left(\Delta P_{t-1}\right)^{1-\theta}+(1-\varepsilon)\left(m_{t} \Delta P_{t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{26}
\end{equation*}
$$

We assume that the only role of the independent central bank is to manipulate the nominal interest rate to keep CPI inflation as close as possible to its target level $\bar{\pi}$. The central bank is assumed to choose the interest rate according to the following forward looking rule:

$$
\begin{equation*}
\left(1+i_{t}\right)=(1+\bar{i}) E_{t}\left[\frac{\left(1+\pi_{t+1}\right)}{(1+\bar{\pi})}\right]^{\rho} \tag{27}
\end{equation*}
$$

where $\bar{i}$ is the nominal interest rate that makes $\pi=\bar{\pi}$. Parameter $\rho$ reflects the strength of the central bank reaction to expected deviations of future inflation from its target $\bar{\pi}$.

### 2.6 Risk Premium

One of the difficulties for constructing small open economy models is that the foreign country (or the rest of the world) is not modeled, which brings a serious problem for determining the supply and demand of the international asset. In our model, the household borrows on the international financial market by supplying bond $F^{S}$, but the model does not specify the demand for these assets. This is a common feature in most small open economy models. As a consequence, following a perturbation in the economy, the interest rate accumulates over higher (or smaller) levels of debt and there is no other equation that prevents it from exploding (or imploding). It follows that $F_{t}$ would feature a unit root following any shock. This problem is aggravated by the fact that the model is solved for approximations around the steady state of each variable. Clearly, the presence of an explosive variable does not allow the system to be solved.

This problem has been recognized by several authors and different approaches to "close small open economy models" have been suggested. ${ }^{2}$ Schmitt-Grohe and Uribe (2003), and Nason and

[^1]Roger (2006) use the strategy of adding a debt elastic risk premium to the model, which is basically the same as recognizing a demand for domestic assets in foreign markets since the risk premium affects the bond's price. We follow their approach and allow for the presence of a commonly used risk premium function:

$$
\begin{equation*}
r p_{t}=r p \exp \left(\iota \frac{F_{t}^{D}}{F}+\sigma_{r p} \nu_{t}\right) \tag{28}
\end{equation*}
$$

where $\nu_{t}$ is an iid normally distributed exogenous shock with mean 0 and variance 1 ; rp and $F$ are, respectively, the steady state values of the risk premium and external debt. The parameter $\iota$ reflects how the risk premium reacts to deviations of $F$ from its long run equilibrium, while $\sigma_{r p}$ is the standard deviation of the exogenous shock.

### 2.7 Market Clearing and Absorption

Equilibrium requires that demand equals supply in every market. In the case of each differentiated good and in the factor markets we need the following conditions to be satisfied:

$$
\begin{gather*}
y_{i t}^{R D}=y_{i t}^{D}=c_{i t}+g_{i t} \text { for every } i \in[0,1]  \tag{29}\\
L_{t}=l_{t}, \text { and }  \tag{30}\\
K_{t}=k_{t} \tag{31}
\end{gather*}
$$

The following is also required to hold in the bond markets:

$$
\begin{gather*}
F_{t}^{D}=F_{t}^{S} \equiv F_{t}  \tag{32}\\
B 1_{t, t+1}^{D}=B 1_{t, t+1}^{S} \equiv B 1_{t, t+1}, \text { and }  \tag{33}\\
B 2_{t, t+2}^{D}=B 2_{t, t+2}^{S} \equiv B 2_{t, t+2} \tag{34}
\end{gather*}
$$

All the previous conditions need to be satisfied at any period $t \geqslant 0$. Since the homogeneous good is traded with the rest of the world, its equilibrium condition can only be obtained after
aggregating the entire economy, which results in the following equation for domestic absorption:

$$
\begin{equation*}
Y_{t}=C_{t}+J_{t}+G_{t}+\left(1+i f_{t-1}\right) F_{t-1}-F_{t} . \tag{35}
\end{equation*}
$$

The last relation can also be written as $Y_{t}=C_{t}+J_{t}+G_{t}+T B_{t}$, where $T B_{t}$ is the trade balance.

## 3 Steady State and Calibration

A more realistic baseline calibration, where public debt is different from zero, is presented next. The case of zero debt is also considered in order to facilitate our analytical conclusions and to check for robustness. For similar reasons we present, in the appendix, a simpler version of the model without capital and investment decisions.

### 3.1 Baseline Calibration

Although each period corresponds to a quarter of a year, in the next two tables we present parameters and economic relations on a yearly basis. Table ?? reports the exogenous parameters values and their sources. Table ?? reports the implied parameters and allocations based on the steady state of our model with the values reported in table ?? . The model is calibrated to match the Brazilian economy.

The capital/GDP ratio of 2.825 is an average of 2.98 and 2.67 , the values used by Araujo and Ferreira (1999) and Bugarin and Paes (2004), respectively. The investment/GDP of 0.2068 is the average from the first quarter of 1999 to the first quarter 2005 computed by the Instituto Brasileiro de Geografia e Estatistica ${ }^{3}$ (IBGE). Steady state capital accumulation equation (??) implies $J=\delta K$, resulting in a quarterly capital depreciation of capital of $\delta=\frac{J / Y}{K / Y}=0.0183$ (or $7.52 \%$ in one year).

The annual CPI inflation target was set at $5 \%^{4}$, the foreign risk free interest rate is $4 \%$ a year ${ }^{5}$, and the quarterly domestic nominal interest rate is $17.73 \%$. This last value corresponds to the

[^2]Table 1: Exogenous Parameter and Economic Relations

| Parameter | Description | Value | Source |
| :---: | :---: | :---: | :---: |
| K/Y | Capital/GDP | 2.825 | Araujo-Ferreira (1999) |
|  |  |  | and Bugarin-Paes (2004) |
| D/Y | Pub. Debt/GDP | 0.53 | IBGE |
| J/Y | Inv/GDP | 0.2068 | IBGE |
| G/Y | Gov't cons./GDP | 0.20 | IBGE |
| $i^{*}$ | Int'l. risk-free rate | $4 \%$ | Gali and Monacelli (2005) |
| $i$ | Home nominal rate | $17.73 \%$ | Central Bank Brazil |
| $\pi$ | CPI target | $5 \%$ | Central Bank Brazil |
| $\theta$ | Markup | 5 | HJRR (2004) |
| $\epsilon$ | Retailers under rule | 0.65 | HJRR (2004) |
| $\rho$ | C. Bank reaction | 1.5 |  |
| $\delta$ | Depreciation | $7.52 \%$ | $\frac{K / Y}{J / Y}$ |
| $L$ | Labor supply | $1 / 3$ | Bugarin et al. (2005) |

average from the third quarter of 1999 to the first quarter of $2005 .{ }^{6}$ The nominal exchange rate variation equals the inflation rate in the steady state. The uncovered interest parity condition (??) can then be used to obtain the risk premium, which is $r p=0.078$ (or 780 basis point spread over the international risk free rate $)^{7}$. The risk adjusted foreign interest rate becomes if $=0.121$.

In a symmetric equilibrium steady state, each retailer charges the same price $p=p_{i}$, meaning that the price aggregator $P$ will also equal $p$, implying that $m=1$. As a result, $q=\frac{P^{Y}}{P}=\frac{\theta-1}{\theta}=0.8$, after setting $\theta=5$. This last parameter corresponds to a mark up of $25 \%$ over the marginal cost, and it is the value used by $\operatorname{HJRR}$ (2004) for the Colombian economy. ${ }^{8}$

The consumer's Euler equation implies an intertemporal discount rate of $\beta=\frac{(1+i)}{(1+\pi)}=0.8919$ (or 0.9718 in a quarter). The capital Euler equation, which in steady state implies $r=\frac{1}{\beta}-1+\delta$,

[^3]Table 2: Implied Parameters and Economic Relations

| Parameter | Description | Value | Source |
| :---: | :---: | :---: | :---: |
| $\beta$ | Discount factor | 0.8919 | $\frac{(1+i)}{(1+\pi)}$ |
| $\Delta s$ | Exch. rate change | $5 \%$ | $\Delta s=\pi$ |
| $r p$ | risk premium | 0.078 | $\frac{(1+i)}{(1+\Delta S)\left(1+i^{*}\right)}-1$ |
| $i f$ | risk adj. int'l return | 0.121 | $(1+r p)\left(1+i^{*}\right)-1$ |
| $r$ | real return on K | 0.1964 | $\frac{1}{\beta}+\delta-1$ |
| $\alpha$ | fraction of capital | 0.54 | $r \frac{K}{Y}$ |
| $q$ | $\frac{P^{Y}}{P}$ | 0.8 | $\frac{\theta-1}{\theta}$ |
| $w$ | real wage | 6.04 | $(1-\alpha) q \frac{Y}{L}$ |
| $Y$ | GDP | 5.41 | from the model |
| $C$ | Consumption | 3.21 | from the model |
| $J$ | Investment | 1.12 | from the model |
| $K$ | Capital | 61.14 | from the model |
| $F$ | External debt | 0.0 | assumption |
| $D$ | Government Debt | 2.87 | from the model |
| $\Gamma$ | from opt. pricing | 9.32 | from the model |
| $\Omega$ | from opt. pricing | 11.65 | from the model |
| $T$ | Tax Collection | 1.17 | from the model |
| $G$ | Gov't spending | 1.08 | from the model |
| $\tau^{l}$ | Labor inc. tax rate | 0.2211 | Coherent to |
| $\tau^{c}$ | Cons. tax rate | 0.2243 | Bugarin-Paes $(2004)$ |
| $\chi$ |  | 1.1974 | to keep $L=1 / 3$ |

together with the wholesaler's first order condition with respect to capital, implies $r=0.0473$ and $\alpha=0.5347{ }^{9}$, respectively. The representative household is assumed to spend one third of her time in productive activities $l=L=\frac{1}{3}$, which is the value used by Bugarin et al (2005) based on research conducted by the Brazilian Institute of Geography and Statistics. From the production function we have that $\frac{L}{Y}=\left(\frac{Y}{K}\right)^{\frac{\alpha}{\alpha-1}}=0.06161$, which can be used with the firm's labor decision, $w=(1-\alpha) q^{Y} \frac{Y}{L}$, to obtain wage $w=6.04142$ and GDP $Y=5.41047$.

Public expenditure and debt, both as fractions of GDP, were set at 0.2 and 0.53 , respectively, implying a tax revenue/GDP ratio of $0.21538^{10}$. Table 9 in Bugarin and Paes (2004) shows that

[^4]consumption tax revenue is approximately $61 \%$ larger than the income tax revenue. Keeping this same relation we find that consumption and labor tax revenues (as fractions of GDP) are 0.13306 and 0.08232 , respectively. These values imply a tax rate on labor revenue of $22.118 \%\left(\tau^{l}=0.22118\right)$ and on consumption of $22.992 \%\left(\tau^{C}=0.2243\right)^{11}$. We use these results to find that consumption as fraction of the GDP is 0.5932 .

After assuming a logarithmic specification for consumption ( $\sigma=1$ ), one can use the labor leisure trade off, equation ??, to obtain the value for $\chi: \chi=\frac{\left(1-\tau^{L}\right)}{\left(1+\tau^{C}\right)} \frac{w}{C}=1.1974$.

## Maturity

It remains to determine the initial debt composition. We consider two exogenous debt compositions, chosen as benchmarks:

Case $A$ : There is only short run debt, implying $B 1=D$ and $B 2=0$. In each period the government has the following real financial expenditure: $B_{t}=B 1_{t-1, t} \frac{\left(1+i_{t-1}\right)}{\left(1+\pi_{t}\right)}+B 2_{t-2, t} \frac{\left(1+i 2_{t-2}\right)}{\left(1+\pi_{t-1}\right)\left(1+\pi_{t}\right)}$. According to case A , this value is equal to $B=\frac{1}{\beta} B 1$.

Case B: The amount expiring due to long term debt is 4 times larger than that of short term debt: $B 1_{t-1, t} \frac{\left(1+i_{t-1}\right)}{\left(1+\pi_{t}\right)} 4=B 2_{t-2, t} \frac{\left(1+i 2_{t-2}\right)}{\left(1+\pi_{t-1}\right)\left(1+\pi_{t}\right)}$. This implies a steady state in which $4 \beta B 1=B 2$, which results in $B 1=0.3227, B 2=1.2545$, and $B=1.6601$.

Notice that except for values of $B, B 1$, and $B 2$, the remaining steady state values of the economy do not change with the debt composition since the outstanding debt $D$ is not affected by the maturity. This implies that steady state government spending, which is equal to $G=T+D\left(1-\frac{1}{\beta}\right)$, does not vary with maturity. Similarly, the reaction functions do not depend on the composition implemented by the debt manager. These two features guarantee we are dealing with different but comparable situations.

Maturity compositions A and B were chosen solely for expositional purposes. Our conclusions are not affected by using other values, which is verified in the simulations conducted in appendix 3.

[^5]
## 4 Impulse Response Analysis

In this section we analyze the dynamic properties of our model as the economy is exposed to an exogenous increase in the risk premium. We then compare the paths implied by alternative maturity compositions. In the impulse response analysis, $t=0$ is the moment the shock hits the economy.

### 4.1 Risk Premium Shock

Figure ?? shows the impact of a one standard deviation positive risk premium shock in case A: there is only a one period nominal bond and $D / Y=0.53$.

The uncovered interest parity condition implies that the expected return on domestic bonds must increase to compensate for the higher risk. As it is standard in international finance models, this happens through an overshooting of the nominal exchange rate at $t=0$, which guarantees higher expected return at $t=1$. The currency devaluation raises retailers' cost, since the homogeneous good $Y$, used in the production of each good $y_{i}$, is subject to purchasing power parity. This is the channel by which currency depreciation elevates domestic inflation. The central bank reacts to higher inflation by elevating the nominal interest rate. The movements just described are shown in the first row of figure ??.

A higher expected real interest rate reduces present consumption (according to Euler equation (??)) as households save more. A higher interest rate also reduces investment at $t=0$ and GDP at $t=1$. At $t=0$, however, production slightly increases because 1 ) there is no change in the amount of capital used, and 2) the wage/exchange rate relation falls due to currency depreciation and smaller real wages.

On impact the fiscal sector is affected by smaller tax collection following the decline in the economic activity. However, financial expenditure also falls due to higher inflation that reduces the real return of the bonds expiring at $t=0$. The impact of surprise inflation is captured by the government's real financial expenditure (defined by $B_{t}=B 1_{t-1, t} \frac{\left(1+i_{t-1}\right)}{\left(1+\pi_{t}\right)}$ ) which behavior is plotted in the third graph/third row of figure ??. According to the previous equation, at $t=0$ inflation $\left(\pi_{t=0}\right)$ is the only variable affecting the government's financial expenditure. A higher real interest


Figure 1: Impulse response functions following a risk premium shock: baseline calibration, $B 1=D$, and $B 2=0$.
rate is responsible for the elevation of $B_{t}$ from $t=1$ onwards.
Since we assumed that the fiscal authority follows a fiscal rule that keeps outstanding debt constant, government spending depends on the magnitude of tax collection and financial expenses. This is easily verified after substituting the debt rule into the government's budget constraint, which leads to $G_{t}=T_{t}+\left(1-\frac{1+i_{t-1}}{1+\pi_{t}}\right) D$. If the increase in $\pi$ is not large enough to compensate for smaller tax collection, the fiscal authority has to reduce $G$. This is exactly the case observed in our economy: at $t=0, T$ is smaller ( -0.0006 ) and financial spending does not fall as much ( -0.0002 ), forcing a reduction in $G_{t}(-0.0004)$.

At $t=1$, tax revenue falls and government financial spending increases, forcing a huge drop in G. $T$ falls because of smaller consumption and bad economic performance. Government financial
spending increases due to the absence of more surprise inflation and mainly due to a higher real interest rate. This combination of factors is what determines the drop in $G$ to keep debt unchanged. In the last two plots/third row of figure ?? we can see that the financial spending dictates the primary surplus needed to keep debt constant.

On the external front, trade balance increases (and external debt, $F$, falls) on impact due to smaller domestic absorption $(C, G$, and $J$ ) and higher GDP. This situation is reversed at $t=1$, since lower output and higher investment impact the trade balance negatively, elevating external debt. From $t=2$ onwards, the country needs to run a trade surplus to service the external debt, which is also affected by the elevation in the interest rate. Notice that higher external debt is responsible for keeping the risk premium above its steady state level. ${ }^{12}$

### 4.2 Sensitivity Analysis: $\rho=1.5$ and $\rho=3$

Increasing the central bank reaction from $\rho=1.5$ to $\rho=3$ does not produce any qualitative difference to our economy, but it certainly improves our understanding of the model. In figure ?? we contrast the dynamics of some selected variables when $\rho=1.5$ (dotted lines) and $\rho=3$ (solid lines). In figure ?? we plotted the difference of prices and allocations of an economy with $\rho=1.5$ and the case where $\rho=3$ (i.e. $X(\rho=1.5)-X(\rho=3)$, where $X(\rho)$ refers to either a price or an allocation under the central bank reaction $\rho$ ). It follows that positive values are associated with larger values when $\rho=1.5$.

The first thing to observe in figure ?? is that a larger central bank reaction produces smaller inflation. This happens despite the lower increase in the nominal interest rate and is based on expectation of smaller inflation in the future. In this sense a tougher (and credible) central bank can achieve lower inflation with smaller increments of the nominal interest rate. The expected real interest rate $\frac{1+i_{t}}{E\left(1+\pi_{t+1}\right)}$, however, remains higher for $\rho=3$, as we can observe in the first column/second row of figure ??

Given the uncovered interest parity condition, it may seem strange that a smaller increase in the nominal exchange rate (which happens for $\rho=3$ ) produces smaller devaluation of the domestic

[^6]

Figure 2: Comparing impulse responses under different central bank reactions: $\rho=1.5$ and $\rho=3$
currency (third column/first row of figure ?? ). But we also need to take into account that in the presence of a more conservative central bank the risk premium does not increase as much for reasons we will soon explain.

The impact of $\rho=3$ on the rest of the economy varies over time. At $t=0$, when the shock hits, higher real interest rate ( $\rho=3$ ) causes larger drop in consumption, in investment and in capital $(C(\rho=1.5)-C(\rho=3)>0, J(\rho=1.5)-J(\rho=3)>0$, and $K(\rho=1.5)-K(\rho=3)>0)$ as we can verify in figure ??. Also, because of higher real interest rate, the fiscal authority pays a larger amount to honor its debt (last plot in figure ?? ), forcing a larger drop in government spending $(G)$, which is shown by the positive initial values in the last graph of figure ?? ; $G(\rho=1.5)-G(\rho=3)>$ 0 . It is then obvious that the domestic absorption falls more intensively in the presence of a


Figure 3: Comparing impulse responses under alternative central bank reaction.
more conservative central bank, which contributes to a higher trade balance surplus at $t=0$; $T B(\rho=1.5)-T B(\rho=3)<0$.

At $t=0$ we also observe that GDP is affected only by the adjustment in the labor market (third and fourth graphs/first row of figure ??, respectively), since capital used at $t=0$ had been decided at $t=-1$. Furthermore, the employment of extra hours in the production process is impacted by the behavior of the wage/exchange rate relation (second column/first row of figure ??) according to the wholesaler optimality condition equation ?? ; $(1-\alpha) \frac{Y_{t}}{L_{t}}=\frac{w_{t}}{q_{t}}$. It follows that the smaller currency devaluation observed in the presence of a more conservative central bank favors a more moderate increase of the GDP at $t=0$.

In the external front, a lower trade balance under $\rho=1.5$ is responsible for a smaller reduction in the external debt; $F(\rho=1.5)-F(\rho=3)>0$. For $t \geq 1$, higher external debt (under $\rho=1.5$ )
requires a higher trade surplus to keep the balance of payment constant. As a consequence, we observe a more elevated risk premium under a more moderate central bank reaction.

Going back to the production side of the economy, we notice that the GDP gap between both policies reduces together with the capital gap. The capital difference, on the other hand, reduces because investment under $\rho=3$ has to be higher for $t \geq 1$ given the larger drop in capital at $t=0$ under this tougher monetary reaction. As a result, employment will also fall over time, as we can verify in the third column/first row of figure ?? .

### 4.3 The Impact of Debt Maturity

Debt maturity compositions $A$ and $B$ produce very similar paths. In order to distinguish both cases, we construct series of the difference between their adjustment processes for $\rho=1.5$ and $\rho=3$. These series, plotted in figure ??, were constructed by subtracting equilibrium prices and allocations of maturity $A$ (with only short bonds) from those of maturity $B$ (with short and long bonds). For instance, in the first row/first column of figure ?? we have the difference between the government financial spending of maturity A subtracted from the same variable under maturity B.

The difference in adjustments under alternative portfolios is caused by the wealth effect. At $t=0$, surprise inflation reduces the real value of the entire stock of public debt when only the short bond is sold. The same reduction happens over the fraction of bonds expiring at $t=0$ when long bonds are also in the portfolio. It follows that, at $t=0$, inflation reduces more intensively the public sector financial burden when only short maturity bonds are sold, which is why we observe a positive value (at $t=0$ ) in the first plot/first row of figure ??. Such a positive value shows a larger reduction in $B_{t=0}$ in the presence of short maturity bond (case A).

To understand what happens to family wealth, remember that households finance government debt. The reduction in fiscal sector real financial spending caused by inflation corresponds to financial losses for households. It follows that at $t=0$, households suffer higher losses if they finance the government by relying exclusively on short bonds.

At $t=1$ onwards, however, this situation is reversed: households are better off (government worse off) if they hold more short term bonds (case $A$ ). We see this in figure ?? by noticing that the difference in government financial spending becomes negative, reflecting higher revenue for the


Figure 4: Differences in the impulse responses (baseline scenario): $\rho=1.5$ in solid line and $\rho=3$ in dotted line.
families under case $A$.
We understand better what is happening after verifying that government debt service is higher (at $t=1$ ) when only a one period bond is present, since a higher interest rate applies to all debt. When a long term bond is also traded, the two period bonds sold at $t=-1$, and expiring at $t=1$, are still not affected by the elevation of the interest rate at $t=0$. The real return on these bonds is actually reduced due to inflation in $t=0$ and $t=1$, which makes their presence more attractive for the Treasury, but not for families. It is due to these reasons that we observe negative values for the difference in the government financial spending from $t=1$ onwards, which corresponds to a better situation for households if they held a shorter maturity portfolio.

It is now important to link wealth effect to the remaining variables in the economy. We observe
higher consumption (second column/first row in figure ?? ) when the family's financial revenue is higher, which is the situation associated with a shorter maturity portfolio: $C^{\text {long }}-C^{\text {short }}$ is always negative. Notice that this happens despite the better financial result, at $t=0$, in the presence of the longer portfolio. This reflects the forward looking behavior of households that anticipate higher returns when the portfolio is shorter.

The impact of longer debt maturity on consumption can be further assessed after linearizing some equations of our system around the economy's steady state (see appendix 1), which leads to the following equation for the household's budget constraint:

$$
\begin{equation*}
\widehat{C}_{t}=\lambda_{1} \widehat{B}_{t}+\lambda_{2}\left(\widehat{B 2}_{t-1, t+1}+\widehat{i 2}_{t-1}-\widehat{\pi}_{t}-\widehat{i}_{t}\right)+\Sigma_{t} \tag{36}
\end{equation*}
$$

where

$$
\Sigma_{t}=\lambda_{3}\left(\widehat{q}_{t}+\widehat{Y}_{t}\right)+\lambda_{4}\left(\widehat{\pi}_{t}-\widehat{\pi}_{t-1}\right)-\lambda_{5} \widehat{q}_{t}+\lambda_{6}\left[\frac{1}{\beta}\left(\widehat{q}_{t}+\widehat{K}_{t}+\beta r \widehat{r}_{t}\right)-\left(\widehat{K}_{t+1}+\widehat{q}_{t}\right)\right]
$$

and

$$
\widehat{B}_{t}=\frac{\beta}{\beta+\frac{B 1}{B 2}}\left(\widehat{B 1}_{t-1, t}+\widehat{i}_{t-1}-\widehat{\pi}_{t}\right)+\frac{\frac{B 1}{B 2}}{\beta+\frac{B 1}{B 2}}\left(\widehat{B 2}_{t-2, t}+\widehat{i 2}_{t-2}-\widehat{\pi}_{t-1}-\widehat{\pi}_{t}\right) .
$$

$\Sigma_{t}$ is a new created variable only used to simplify the linearized equation for $\widehat{C}_{t}$, as it can be checked in the appendix 3 . All the variables belonging to $\Sigma_{t}$ are multiplied by parameters that do not depend on the debt composition.

Notice that the term in parentheses multiplying $\lambda_{2}$ (in equation ??) captures the impact of the two periods bond, expiring at $t+1$, on present consumption. Since $\lambda_{2} \equiv \frac{B 2}{C\left(1-\tau^{c}\right)}$ is positive, surprise inflation ( $\widehat{\pi}_{0}$ ) and the consequent elevation of the interest rate ( $\widehat{i}_{0}$ ) reduce consumption at $t=0 .{ }^{13}$ Furthermore, because $\lambda_{2} \equiv \frac{B 2}{C\left(1-\tau^{c}\right)}$, the higher presence of two period bonds reduces present consumption by a larger amount.

This can be easily seen if we rewrite $\widehat{C}_{t}$ only considering the variables that change at $t=0$, which results in $\widehat{C}_{0}=-\frac{\lambda_{0}}{\beta} D \widehat{\pi}_{0}-\frac{\lambda_{0}}{\beta} B 2 \widehat{i}_{0}+\Sigma_{0}$, where $\lambda_{0} \equiv \frac{1}{C\left(1-\tau^{c}\right)}$ is positive. Since $\widehat{\pi}_{0}$ and $\widehat{i}_{0}$ are

[^7]positive, a larger amount of two periods bond produces higher drop in consumption at $t=0$. We also notice that surprise inflation $\widehat{\pi}_{0}$ does not affect current consumption in the special case where $D=0$.

Although debt maturity affects $\Sigma_{t}$, the impact is not as direct as in the previous analysis. For instance, $\lambda_{3}, \lambda_{4}, \lambda_{5}$, and $\lambda_{6}$ are all independent of the maturity structure (see appendix 1 ), suggesting that the differences in the dynamics due to alternative maturities should not be greatly affected by the behavior of $\Sigma_{t}$. We observe this in appendix 3, where we construct a simpler version of the model without capital and investment decisions (which corresponds to a case in which $\lambda_{6}=0$ ) and conclude that our previous findings are remain valid.

Going back to the impact of maturity on other variables of the economy, we observe a lower wage when consumption is also smaller according to the labor-leisure trade-off (equation ??). In this case a shorter maturity portfolio would imply in smaller reduction of real wages (third column/first row of figure ??), influencing firms to hire more labor and produce more in the presence of longer maturity bonds (first and second columns/second row of figure ??).

The impact of maturity on government consumption is similar to that on households, but with the opposite sign. If the Treasury relies more extensively on long bonds, fiscal sector real financial spending is smaller, which allows government consumption $G$ to be higher, as observed in the second row/third column of figure ??.

We are cautious in interpreting the impact of maturity on inflation, the interest rate and exchange rate devaluation, as they are determined by the ad-hoc relation between external debt and the risk premium. As far as the model is concerned, higher government spending under longer debt maturity contributes to a smaller trade balance, or more elevated external debt, making the risk premium higher, which affects inflation, the exchange rate and the interest rate in the same direction.

At this point it should be clear that households would achieve higher welfare, following a risk premium shock, if they held shorter maturity bonds. This is observed in figure ?? by the larger consumption and lower hours worked when only short bonds are present in the portfolio. We can quantify the welfare differences between the alternative portfolio compositions if we use the simulated consumption and hours of work to compute household utility. In order to compute
this utility, we first recover both series by adding the deviations of $C_{t}$ and $L_{t}$ to their respective steady state. We can then plug these values into the utility function and compare the results. After adding the utilities from $t=0$ to $t=50$ we obtained $U($ short $)=1.024660$ and $U($ long $)=$ 1.024648. Despite the tiny difference, we observe higher utility following a risk premium shock, when households have shorter maturity portfolios. This should not be a surprising result, given the short maturity bias observed in emerging economies during confidence crises.

## The British Treasury strategy in the $18^{\text {th }}$ century

Although we have considered the effect of maturity following a risk premium shock, the central argument about debt maturity, which is the wealth effect, can guide us in other analysis. For instance, Shin (2006) was able to match the British Treasury maturity strategy, in the $18^{\text {th }}$ century, after calibrating the transition probability matrix of a Markov chain to reproduce the change in government spending. He observed that during peace time, the British treasury would extend the debt maturity, which would eventually be shortened as a war was announced. According to our analysis, if households anticipate a higher interest rate during war time, they would not hold long maturity (perpetuities) in their portfolio, ceteris paribus, as they would be locked in a position that would not allow them to earn higher returns. This induces an increase in the term premium to hold long bonds, which elevates the financial costs for the British Treasury if it insists on selling consols.

## 5 Conclusions

We analyze the effects of debt maturity in a dynamic general equilibrium model that is calibrated to match the Brazilian economy. Following an exogenous risk premium shock, the exchange rate depreciates and obligates the central bank to increase the interest rate to control inflation. Surprise inflation reduces consumer wealth as it depresses the real return on nominal bonds. An even worse wealth effect is observed when the household holds longer maturity bonds. This is a consequence of the opportunity cost of not being able to obtain higher interest returns, since she is locked with bonds that expire next period. The optimal labor-leisure trade off induces the household to accept
a lower wage and work more intensively following a large drop in wealth. In this sense the GDP would not fall as much as if the Treasury had longer maturity liabilities.

For a similar, but opposite reason, government consumption would be higher if a larger proportion of its debt was financed through longer maturity bonds. This implies that, following a confidence crisis, the fiscal sector of an emerging economies would face easier adjustments if the debt maturity was longer. But since maturity shortening actually happens as crises become more likely, our exercise suggests that the premium placed by households, in order to hold longer bonds, is sufficiently high to induce a short run maturity bias.

Our exercises also suggested that the Treasury would more easily extend the debt maturity if a more conservative central banker is in office. This happens because potential losses produced by surprise inflation would be more moderate according to our simulations. In this sense households would require a smaller premium to hold long bonds under a more rigid inflationary control.

Finally, we observe that our analysis can be used to explain historical debt maturity strategies in very different situations to the one described here. For instance, the finding of Shin (2006) that the British Treasury reduced bond maturity during war times suggests that households were aware that the extra government spending would raise interest rates, and cause a negative wealth effect, if they held longer maturity bonds.

For future research it would be interesting to analyze the impact of other debt instruments on the economy. This is an important issue given the widespread use of indexed bonds. In emerging market economies, it would also be useful to understand how the economy is affected by inflation indexed and exchange rate indexed bonds. Blanchard (2004) and Favero and Giavazzi (2004) have recently considered these issues in other contexts.

## 6 Appendix

This appendix, which is divided in three subsections, complements information presented in the chapter 3 of this dissertation. In the first subsection we show the derivation of equation ?? . In the second subsection we present impulse response analysis and compare the presence of short and long maturity debt assuming a steady state with zero public debt. Finally, in the last subsection, we construct an alternative model to that presented in chapter 3. This new model does not incorporate physical investment. The conclusions obtained in chapter 3 do not change even after modifying the economic environment, which shows the robustness of our analysis.

## Linearization

First order linearization of a function $f\left(x_{t}\right)$ around its steady state level $f(x)$ is conducted according to the following Taylor approximation: $f\left(x_{t}\right)=f(x)+\frac{\partial f\left(x_{t}\right)}{\partial x_{t}} d x_{t}$, where the derivative is evaluated at steady state levels. We define the percentage deviation of a variable $x$ from its steady state as $\widehat{x}_{t}=\frac{d x_{t}}{x}=\frac{x_{t}-x}{x}$.

Our main goal here is to simplify the system so we are able to see the impact of debt maturity on household's decisions. In particular, we want to show the impact of maturity on household's wealth, which naturally leads us to work with her budget constraint.

Let $Z_{t}$ be the household's wealth, at time $t$, which we define as

$$
Z_{t}=B_{t}+\frac{\left(1+i 2_{t-1}\right)}{\left(1+\pi_{t}\right)\left(1+i_{t}\right)} B 2_{t-1, t+1}+\left(1-\delta+r_{t}\right) q_{t} K_{t}-\left(1+i^{*}\right)\left(1+r p_{t-1}\right) q_{t} F_{t-1}
$$

Adding $\frac{\left(1+i 2_{t-1}\right)}{(1+\pi t)\left(1+i_{t}\right)} B 2_{t-1, t+1}$ to the left hand side of the budget constraint allows us to write it as

$$
\begin{equation*}
C_{t}+q_{t} K_{t+1}+D_{t}-q_{t} F_{t}=W_{t} L_{t}+T_{t}+Z_{t}+\Pi_{t}^{Y}+\Pi_{t}^{R} \tag{37}
\end{equation*}
$$

where $\Pi_{t}^{R}=\frac{1}{P_{t}} \int_{0}^{1} \pi_{i t} d i$ corresponds to aggregate profit of all retailers. We reach the following
equation after solving (??) for $C_{t}$ and linearizing it:

$$
C \widehat{C}_{t}=L d w_{t}+w d L_{t}-d T_{t}+d Z_{t}+d \Pi_{t}^{R}-q d K_{t+1}-K d q_{t}+F d q_{t}+q d F_{t}
$$

where we used the fact that $d D_{t}=0$ (due to the fiscal rule) and $d \Pi_{t}^{Y}=0$, since in a perfect competitive market profit is always zero. We proceed to find the linearized expressions for the components of $d C_{t}$. Starting with $d Z_{t}$ we have

$$
\begin{aligned}
d Z_{t}=B \widehat{B}_{t}+\frac{B_{2}}{\beta}\left(\widehat{B 2}_{t-1}+\widehat{i 2}_{t-1}-\widehat{\pi}_{t}\right. & \left.-\widehat{i}_{t}\right)+ \\
& +\frac{q K}{\beta}\left(\widehat{q}_{t}+\widehat{K}_{t-1}+\beta r \widehat{r}_{t}\right)-\frac{q F}{\beta}\left(\widehat{r p}_{t-1}+\widehat{q}_{t}+\widehat{F}_{t}\right) .
\end{aligned}
$$

The aggregate profit of retailers is the sum of profits from those who are able to choose price optimally and those who follow the pricing rule: $\Pi_{t}^{R}=\left(C_{t}+G_{t}\right)\left\{\varepsilon \frac{\left(1+\pi_{t-1}\right)}{\left(1+\pi_{t}\right)}+(1-\varepsilon) m_{t}-q_{t}\right\}$. In steady state the aggregate profit becomes $\Pi^{R}=(C+G) \frac{1}{\theta}$, after using $q=\frac{\theta-1}{\theta}$. Linearization of $\Pi_{t}^{R}$ results in

$$
d \Pi_{t}^{R}=\frac{1}{\theta} C \widehat{C}_{t}+\frac{1}{\theta} G \widehat{G}_{t}+(C+G)\left[\varepsilon\left(\widehat{\pi}_{t-1}-\widehat{\pi}_{t}\right)+(1-\varepsilon) \widehat{m}_{t}-\frac{\theta-1}{\theta} \widehat{q}_{t}\right] .
$$

After adding and subtracting $\frac{\left(1+i 2_{t-1}\right)}{\left(1+\pi_{t}\right)\left(1+i_{t}\right)} B 2_{t-1, t+1}$ from the left hand of the government budget constraint (??) and applying the definition of $D_{t}$ and $B_{t}$ we reach $G_{t}=T_{t}+D_{t}-B_{t}-$ $\frac{\left(1+i 2_{t-1}\right)}{\left(1+\pi_{t}\right)\left(1+i_{t}\right)} B 2_{t-1, t+1}$. Its linearization delivers $\widehat{G}_{t}=\frac{1}{G} d T_{t}-\frac{B}{G} \widehat{B}_{t}-\frac{B 2}{\beta G}\left(\widehat{B 2}_{t-1}+\widehat{i 2}_{t-1}-\widehat{\pi}_{t}-\widehat{i}_{t}\right)$, since $d D_{t}=0$. Linearization of tax collection results in $d T_{t}=\tau^{c} C \widehat{C}_{t}+\tau^{l} w L\left(\widehat{w}_{t}+\widehat{L}_{t}\right)$.

After using $\widehat{w}_{t}+\widehat{L}_{t}=\widehat{q}_{t}+\widehat{Y}_{t}$ and substituting all the previous linearized equations into (??) we obtain ?? :

$$
\widehat{C}_{t}=\lambda_{1} \widehat{B}_{t}+\lambda_{2}\left(\widehat{B 2}_{t-1, t+1}+\widehat{i 2}_{t-1}-\widehat{\pi}_{t}-\widehat{i}_{t}\right)+\Sigma_{t}
$$

where

$$
\Sigma_{t}=\lambda_{3}\left(\widehat{q}_{t}+\widehat{Y}_{t}\right)+\lambda_{4}\left(\widehat{\pi}_{t}-\widehat{\pi}_{t-1}\right)-\lambda_{5} \widehat{q}_{t}+\lambda_{6}\left[\frac{1}{\beta}\left(\widehat{q}_{t}+\widehat{K}_{t}+\beta r \widehat{r}_{t}\right)-\left(\widehat{K}_{t+1}+\widehat{q}_{t}\right)\right]
$$

and

$$
\widehat{B}_{t}=\frac{\beta}{\beta+\frac{B 1}{B 2}}\left(\widehat{B 1}_{t-1, t}+\widehat{i}_{t-1}-\widehat{\pi}_{t}\right)+\frac{\frac{B 1}{B 2}}{\beta+\frac{B 1}{B 2}}\left(\widehat{B 2}_{t-2, t}+\widehat{i 2}_{t-2}-\widehat{\pi}_{t-1}-\widehat{\pi}_{t}\right) .
$$

We also have that $\lambda_{0}=\frac{1}{C\left(1+\tau^{c}\right)}, \lambda_{1}=\lambda_{0} B, \lambda_{2}=\lambda_{0} \frac{B 2}{\beta}, \lambda_{3}=\frac{\chi}{\theta-1} L\left[\theta+\frac{\tau^{l}}{\left(1-\tau^{l}\right)(\theta-1)}\right], \lambda_{4}=$ $\epsilon \frac{\theta}{\theta-1} \lambda_{0}(C+G), \lambda_{5}=\lambda_{0}(C+G)$, and $\lambda_{6}=\lambda_{0} K$.

## Alternative Scenario ( $D=0$ )

Here we consider simulations assuming zero government debt in steady state, $D=0$. This implies that the steady state value for tax revenue is equal to government expenditure; $G=T=1.08$. Under this scenario, tax rates and $\chi$ are, respectively, $\tau^{l}=0.2053, \tau^{C}=$ 0.2082 , and $\chi=1.2379$.

Zero debt does not necessarily mean absence of bond trading. Since steady state public debt is equal to $D=B 1+B 2\left(1+\frac{1}{\beta}\right)$, the government may take opposite positions in the bonds market such that $D=0$.

We again analyze the impact of debt maturity assuming two alternative initial compositions:

Case $C: \frac{B 2}{Y}=-0.2$, implying $B=1.1135, B 1=2.1956$, and $B 2=-1.0821$; the government is a debtor in short bonds, but a creditor in long bonds.

Case $D: \frac{B 2}{Y}=0.2$, implying $B=-1.1135, B 1=-2.1956$, and $B 2=1.0821$. This is the opposite case.

According to the previous discussion on the impact of the debt maturity structure, one should expect that consumption would be higher if the household was a debtor in long bonds $(B 2<0)$ and a creditor in short bonds $(B 1>0)$, which is the situation we find under Case


Figure 5: Difference in Impulse Responses (Alternative Scenario): maturity D - maturityC C.

After simulating the response of the economy under both cases, we observe in figure ?? the same pattern observed in figure ??, when public debt was positive and the shorter maturity portfolio guaranteed higher financial revenue and higher consumption for the household. Figure ?? shows that the consumer is better off when she is a short creditor and a long debtor. In this sense the zero outstanding debt does not change the nature of our conclusion, since different wealth effects is driven by the debt composition.

## Model with no Capital

Except for the absence of capital and investment decisions, the next model is the same as our baseline environment. We modified, however, the nature of the labor leisure trade-off. Instead of having indivisible labor, the household's utility function is $U_{t}=\log C_{t}-\frac{l_{t}^{1+\chi}}{1+\chi}$, with $\chi>0$. The intertemporal maximization problem then becomes:

$$
\begin{equation*}
\underset{\left\{C_{t}, l_{t}, b 1_{t, t+1}^{D}, b 2_{t, t+2}^{D} F_{t+1}^{S}\right\}_{t=0}^{\infty}}{\operatorname{Max}} E_{0} \sum_{t=0}^{\infty} \beta^{t}\left[\log C_{t}-\frac{l_{t}^{1+\chi}}{1+\chi}\right] \tag{38}
\end{equation*}
$$

subject to
i) $\left(1+\tau^{c}\right) P_{t} C_{t}+b 1_{t, t+1}^{D}+b 2_{t, t+2}^{D}+\left(1+i_{t-1}^{*}\right)\left(1+r p_{t-1}\right) S_{t} F_{t-1}^{S} \leq\left(1-\tau^{l}\right) W_{t} l_{t}+$

$$
\begin{equation*}
\left(1+i_{t-1}\right) b 1_{t-1, t}^{D}+\left(1+i 2_{t-2}\right) b 2_{t-2, t}^{D}+S_{t} F_{t}^{S}+\Pi_{t}^{Y}+\int_{0}^{1} \pi_{i t} d i \tag{39}
\end{equation*}
$$

ii) given $b 1_{t-1, t}^{D}, b 2_{t-2, t}^{D}$ and $F_{t-1}^{S}$.

The difference in the first order conditions is that the euler equation for capital (equation ??) disappears, while the labor leisure trade-off becomes

$$
\begin{equation*}
C_{t} l_{t}^{\chi}=\frac{\left(1-\tau^{l}\right)}{\left(1+\tau^{C}\right)} w_{t} \tag{40}
\end{equation*}
$$

We set $\chi$ to be 3, as in Gali and Monacelli (2005), so that labor supply elasticity is $\frac{1}{3}$.
The wholesale firm now produces according to

$$
\begin{equation*}
Y_{t}=L_{t} \tag{41}
\end{equation*}
$$

Profit maximization implies

$$
\begin{equation*}
w_{t}=q_{t} \frac{Y_{t}}{L_{t}} \tag{42}
\end{equation*}
$$

Market clearing conditions and aggregation imply

$$
\begin{equation*}
Y_{t}=C_{t}+G_{t}+T B_{t} \tag{43}
\end{equation*}
$$

Table 3: Implied Parameters and Economic Relations - Model without Capital

| Parameter | Description | Value | Source |
| :---: | :---: | :---: | :---: |
| $\mathrm{D} / \mathrm{Y}$ | Pub. Debt/GDP | 0.53 | IPEA |
| $\mathrm{G} / \mathrm{Y}$ | Gov't cons./GDP | 0.25 | IBGE |
| $\chi$ | inverse labor supply elast | 3 | Gali, Monacelli (2005) |
| $w$ | real wage | 0.8 | $q \frac{Y}{L}$ |
| $Y$ | GDP | 0.94 | from the model |
| $L$ | hours worked | 0.94 | from the model |
| $C$ | Consumption | 0.70 | from the model |
| $D$ | Government Debt | 0.50 | from the model |
| $\Gamma$ | from opt. pricing | 2.03 | from the model |
| $\Omega$ | from opt. pricing | 2.54 | from the model |
| $T$ | Tax Collection | 0.25 | from the model |
| $G$ | Gov't spending | 0.23 | from the model |
| $\tau^{l}$ | Labor inc. tax rate | 0.1268 | Coherent to |
| $\tau^{c}$ | Cons. tax rate | 0.2186 | Bugarin-Paes (2004) |

In the calibration of the model we continued to assume zero external debt in steady state, which also implies in zero trade balance; $F=T B=0$. Since total output is only divided between private and government consumption, we decided to increase the government participation to $25 \%$ of the GDP so that private consumption would not be too large when compared to actual data. Tax rates were computed as before. In table ?? we present parameters and economic relations that differ from those used in the baseline model that were reported in tables ?? and ??.

Under maturity $\mathrm{A}, B=0.509, B 1=0.495$, and $B 2=0$. Under maturity $B, B=0.287$, $B 1=0.06$, and $B 2=0.217$. Figure ?? shows the impulse responses following a one standard deviation increase in the risk premium under maturity case $A$. Notice that the new system behaves similar to the baseline scenario in which investment and capital decisions were


Figure 6: Impulse Responses Following a Risk Premium Shock: no investment, no capital, $B 1=D$, and $B 2=0$.
present. However, based on empirical analysis provided by Uribe and Yue (2006) and by the second chapter of this dissertation, we should expect a deep decline in the product following a risk premium shock, which is not observed. In this sense excluding capital and investment decisions seems to be a great departure from reality.

Despite the different responses in GDP, our conclusions about the impact of debt maturity does not change regardless of having capital in the model. This is verified in figure ?? where we show the dynamics of maturity B minus those of maturity A. Observe that the graphs plotted in figure ?? are very similar to those of figure ?? and ??, when capital and investment were present.


Figure 7: Difference in Impulse Responses (no capital and investment decision): maturity $B$ maturity $A$

## References

[1] Alesina, A., Prati, A., Tabellini, G., 1990. "Public Confidence and Debt Management: A Model and a Case Study of Italy", in Dornbusch, R., and Draghi, M., Public management: theory and history. Cambridge University Press, Cambridge, USA.
[2] Amaral, A., 2002. "Uma avaliacao do risco politico", O Estado de Sao Paulo, July 1, 2002.
[3] Angeletos, G.M., 2002. "Fiscal Policy with Non-Contingent Debt and the Optimal Maturity Structure", Quarterly Journal of Economics, 117, pp. 1105-1131.
[4] Araujo, C. H., Ferreira, P. C., 1999. "Reforma Tributaria, Efeitos Alocativos e Impactos de Bem-Estar", Revista Brasiliera de Economia, 53, pp. 133-166.
[5] Benes, J., Vavra, D., 2003. "Developing a new generation core model for the FPAS of the CNB", Draft, Czech National Bank, Prague, Czech Replublic.
[6] Benigno, P., Woodford, M., 2006. "Optimal Inflation Targeting Under Alternative Fiscal Regimes", NBER Working Paper, No. 12158 (April).
[7] Blanchard, O., 2004. "Fiscal Dominance and Inflation Targeting: Lessons from Brazil", NBER Working Paper, No. 10389 (March).
[8] Buera, F., Nicolini, J. P., 2004. "Optimal Maturity of Government Debt without State Contingent Bonds", Journal of Monetary Economics 51, pp. 531-554.
[9] Bugarin, M., Paes, N., 2004. "Impactos de Longo Prazo de Reformas Tributárias Alternativas à Luz de um Modelo de Equilíbrio Geral Dinâmico com Agentes Heterogêneos", Anais do Encontro Brasileiro de Econometria XXVI, Joao Pessoa, Brasil (December).
[10] Chang, R., Velasco, A., 2001. "A Model of Financial Crises in Emerging Economies", The Quarterly Journal of Economics, 116, pp. 489-517.
[11] Cole, H.,Kehoe, T., 1996. "A Self-Fulfilling Model of Mexico's 1994-95", Journal of International Economics, 41, pp. 309-330.
[12] Cole, H., Kehoe, T., 2000. "Self-Fulfilling Debt Crisis", Review of Economic Studies, 67, pp. 91-116.
[13] Favero, C., Giavazzi, F., 2004. "Inflation Targeting and Debt: Lessons from Brazil", CEPR Working Paper No 4376.
[14] Froot, K., Rogoff, K., 1995. "In: Grossman, G., Rogoff, K. (Eds.). Perspectives on PPP and long-run real exchange rates", in Handbook of International Economics, Vol. III. North-Holland, Amsterdam, pp. 1647-1688.
[15] Gali, J., Monacceli, T, 2005. "Monetary Policy and Exchange Rate Volatility in a Small Open Economy", Review of Economic Studies, Vol 72, No. 3, pp. 707-734.
[16] Gapen, M., Gray, D., Lim, C., Xiao, Y., 2005. "Measuring and Analyzing Sovereign Risk with Contingent Claims", IMF Working Paper, No. 155 (August).
[17] Garcia, M., Rigobon, R., 2004. "A Risk Management Approach to Emerging Market's Sovereign Debt Sustainability with an Application to Brazilian Data", NBER Working Paper, No 10336 (March).
[18] Hamann, F., Julio, J., Restrepo, P., Riascos, A., 2004. "Inflation targeting in a small open economy: the Colombian case", Draft, Conference on Inflation Target held at the Atlanta FED (October).
[19] Laxton, D., Pesenti, P., 2003. "Monetary Rules for Small, Open, Emerging Economies," Journal of Monetary Economics, 50, pp. 1109-1146.
[20] Loyo, E.H.M., 1999. "Tight Money Paradox on the Loose: A Fiscalist Hyperinflation.", Draft, Kennedy School of Government (June).
[21] Lucas, R., Stokey, N., 1983 "Optimal fiscal and monetary policy in an economy without capital ", Journal of Monetary Economics, Vol. 12, No. 1, pp. 55-93.
[22] Nason, J., Rogers, J.H., 2006. "The Present Value Model of the Current Account Has Been Rejected: Round Up the Usual Suspects", Journal of International Economics, Vol. 68 (January), pp. 159-187.
[23] Reinhart, C., Rogoff, K., Savastano, M., 2003. "Debt Intolerance ", NBER Working Paper 9908, August 2003.
[24] Schmitt-Grohe, S., Uribe, M., 2003. "Closing Small Open Economies Model", Journal of International Economics 61, pp. 163-185.
[25] Shin, Y., 2006. "Managing the Maturity Structure of Government Debt", Draft, University of Wisconsin (February)
[26] Tobin, J., 1963. "An Essay on Principles of Debt Management: Fiscal and Debt Management Policies", Draft, presented at the Commission on Money and Credit, 1963.
[27] Uribe, M., Yue, V., 2006. "Country spreads and emerging economies: Who drives whom?", Journal of International Economics 69, pp. 6-36.


[^0]:    ${ }^{1}$ We do not consider the situation described by Loyo (1999) and Blanchard (2004) where a higher interest

[^1]:    ${ }^{2}$ For instance Schmitt-Grohe and Uribe (2003), Nason and Roger (2006), among others. Schmitt-Grohe and Uribe summarize five different procedures to "close small open economy models" that have been considered in the literature: (1) an endogenous intertemporal discount factor; (2) a debt-elastic interest rate premium (which is used in the current paper); (3) a convex portfolio adjustment cost; (4) complete asset markets; and (5) lack of stationary inducing features.

[^2]:    ${ }^{3}$ IBGE is responsible for computing, among other things, the Brazilian National Income Accounts.
    ${ }^{4}$ The average target annual inflation rate, from the first quarter of 1999 to the first quarter of 2005 is $4.97 \%$. (Source: Banco Central do Brasil)
    ${ }^{5}$ Gali and Monacelli (2005).

[^3]:    ${ }^{6}$ Although the inflation target started in January 1999, we exclude the interest rates in the first two quarters of this year because they largely reflect the reaction of the central bank to a currency attack in early January, which implied average rates of $31.5 \%$ and $28.6 \%$ for the first and second quarters, respectively. Similarly, the interest rates from the $4^{\text {th }}$ quarter of 2002 to the $3^{\text {rd }}$ quarter of 2003 were not used when computing the average, since during this period the central bank was far from the announced inflation target as a result of a confidence crisis.
    ${ }^{7}$ The average from the the third quarter of 1999 to the first of 2005 , excluding the crisis period previously mentioned, was 745 . This spread is our proxy for risk premium.
    ${ }^{8} \mathrm{I}$ am not aware of studies that try to estimate mark up in Brazil, but $25 \%$ seems to be in a safe range. For instance, Laxton and Pesenti (2003) find that the mark up ranges from $10 \%$ to $70 \%$ for small open economies, depending on the country and on the study. They use $\theta=6$ in their paper, which corresponds to a $20 \%$ markup.

[^4]:    ${ }^{9}$ This value is not far from other studies for Brazil. In particular, Bugarin and Paes(2004) find 0.4378 , while Araujo and Ferreira (1999) find 0.49.
    ${ }^{10}$ Based on IBGE data, the average government expenditure as a fraction of GDP, from the first quarter of 1999 to a similar period in 2005 , was 0.192 . For the same period, the average public sector outstanding debt as a fraction of GDP was 0.533.

[^5]:    ${ }^{11}$ The highest income tax on households in Brazil is $27.5 \%$, which is charged on incomes exceeding 10 minimum wage. An intermediate income group pays $15 \%$ in taxes. The consumption tax rate is harder to compare with our results given the large variety in the rates charged across products and states.

[^6]:    ${ }^{12}$ An important issue we do not explore is the direct impact of maturity on the risk premium, which could be done by making the risk premium a function of $B 1$. We avoid this extension to focus exclusively on channels of debt maturity that do not depend on the addition of $a d$-hoc features to the model.

[^7]:    ${ }^{13}$ At $t=0, \widehat{B 2}_{t-1}$ and $\widehat{i 2}_{t-1}$ are equal to zero. From period $t=1$ onwards, $\widehat{i 2}_{0}-\widehat{i}_{1}=\widehat{i}_{1}+\frac{1}{2} \widehat{i}_{0} \widehat{i}_{1}$, where the last multiplication is the consequence of a second order Taylor approximation of the expectation operator. This is in accordance with Uribe and Schmitt-Grohe (2004) solution method that is used by Dynare.

