Dynamic General Equilibrium Model with Uncertainty: Uncertainty regarding the future path of the economy

Abstract

This paper outlines the conceptualisation and preliminary results of a simple Dynamic General Equilibrium model with uncertainty. In forward-looking dynamic Computable General Equilibrium (CGE) models economic agents are endowed with perfect foresight, so both consumers and firms anticipate any exogenous shocks and adjust their maximising behaviour from the first time period onwards. Perfect foresight then would appear to negate any uncertain response to a shock. Taking a simple model with Ramsey economic growth dynamics, this paper illustrates a frame work that incorporates uncertainty by allowing alternative future time paths resulting in uncertainty in the model. The base line growth path will be contrasted with simulations where the probability of the shock varies both in magnitude and over time.

1. Introduction

The concept of risk has been examined from many different disciplines: from an economics perspective, from a sociological perspective, from a financial perspective. Risk is a complex construct. Risk has been defined in many different ways. One frequently cited definition of risk is Knight's definition (1921). He defines risk as "measurable uncertainty". Risk is a pervasive part of all actions. As MacCrimmon and Wehrung point out (1986), "along with death and taxes, risk is one of the certainties of life". Some risks are dramatic such as natural disasters or crossing a busy street, while others are much more insidious such as borrowing money or working in a polluted environment. Some risk might affect many people, while other risk may be isolated to one person. Within the realm of economics, risk has played a role in various theories from consumer theory to the producer theory and investment theory. In expected utility theory, individuals make choices among risks, each represented by a probability distribution. Individuals differ in their willingness to bear risk. Some are risk averse, some are risk loving, and some are risk neutral. An individual who prefers a certain given outcome to a risk scenario with the same expected outcome is described as being risk averse. The individual is risk neutral if indifference always holds and risk loving if the preference is reversed. The curvature of the utility function reflects their risk attitude. The next section (Section 2) outlines the nature of risk and uncertainty contained in CGE models and draws out the implicit assumptions regarding risk in CGE models, which are traditionally deterministic in nature. Section 3 assesses previous research that has attempted to incorporate risk and uncertainty into CGE models. Section 4 in this paper describes the explicit treatment of risk in a CGE model involving the creation of multiple future paths for the model, where agents are able to predict each path and make decisions, given an element of risk aversion, in the presence of this uncertainty. Section 5 concludes.

2. Computable General Equilibrium Models and Implicit Risk

2.1 CGE Modelling

The section outlines the basic characteristics of the different types of CGE models: the static model, the dynamic recursive model, the single sector dynamic forward-looking model and the multi-sector dynamic forward-looking model. For each type of model, the implicit assumptions of risk are outlined.

2.1.1. The Within Period CGE Model (Static / Intra-temporal Model)

The static (within period) CGE model follows the interactions and relationships of a market economy and solves for a set of prices including production prices, factor prices and exchange rate and levels of production that clear all markets. The static model recreates an Arrow-Debreu (1954) general economic equilibrium model. The model contains a representative consumer. Each consumer has an initial endowment of the N commodities and a set of preferences resulting in demand functions for each commodity. Market demands are the sum of all consumers' demands. Commodity market demands depend on all prices and satisfy Walras' law. The total value of consumer expenditures equals consumer incomes, at any set of prices. Technology is described by constant returns to scale production functions. Producers maximize profits. The zero homogeneity of demand functions and the linear homogeneity of profits in prices (i.e. doubling all prices double money profits) imply that only relative prices are of any significance in such a model. The absolute price level has no impact on the equilibrium outcome (Rutherford and Paltsev, 1999).

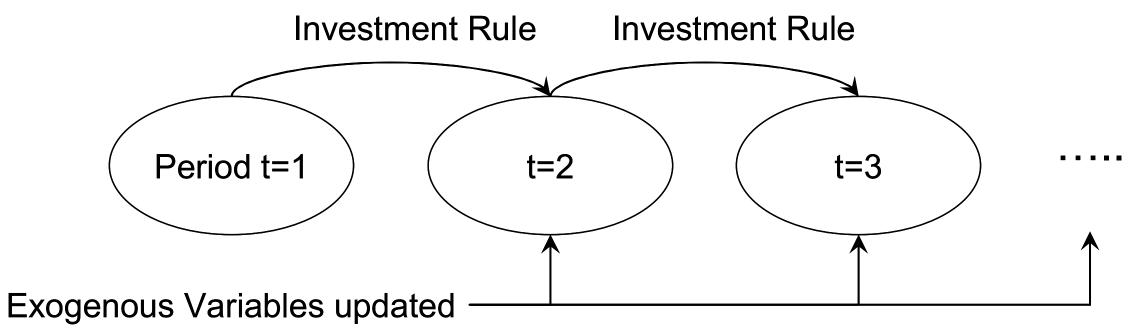
Equilibrium in this model is characterized by a set of prices and levels of production in each industry such that the market demand equals supply for all commodities. Since producers are assumed to maximize profits, and production exhibits constant returns to scale, this implies that no activity (or cost-minimizing technique for production functions) does any better than break even at the equilibrium prices. Mathiesen (1985) has shown that an Arrow-Debreu model can be formulated and solved as a complimentarity problem. Demand for and supply of goods and services readjust until all excess demands and excess supplies are eliminated through changes in prices. The production function is specified into terms of labour and capital and the amount of each type of these inputs employed by a producer in a particular sector is based on the sector specific production technology and input prices. The most common functional form tends to be a Constant Elasticity of Substitution (CES) function to express this relationship. Perfectly competitive markets operate to determine these equilibrium prices. Additionally, in equilibrium, no sector earns above-normal profits, markets clear for all factors and products, and, in an open economy, the value of imports for intermediate use and final demand equals the value of export earnings.

In terms of implicit risk, the return on capital captures all the inherent risk associated with the investment and owners of capital are paid an appropriate return, given the level of risk. Elasticities capture the trade-off between the choice of various products and of the inherent risk associated with the curvature of the utility functions. In such models, there is no obvious place for risk or uncertainty to be factored into the model.

2.1.2 Dynamic Recursive Model

A dynamic recursive model involves solving a model for period t (the intra-temporal model) and then solves the model for t+1 and so on. What links the models together is the investment rule. However, the investment rule then determines to a large extent, the results of the model. The inter-temporal (between-period) model links the static (intra-temporal or within-period) models by updating the variables that are exogenous in the base year from one period to the next. This process can be seen in the figure below.

Figure 1 Dynamic Recursive Model



The "problem" with dynamic recursive models is that expectations are not taken into account. For the most part, economic agents behave as in a one-shot example many times. In recursive models, agents lack a coherent representation of expectations. In reality, although economic agents do not have full information regarding the future, they behave as if the future matters. Recursive models tend to be characterised by "overshoot and collapse" behaviour. Given the fact that these models are essentially "one-shot" model advanced one time period and updated with the values of the exogenous variables for the next time period, the assumptions regarding risk are similar to the intra-temporal model, that is, the interest rate (return to capital) reflects all the inherent risk associated with saving and investing.

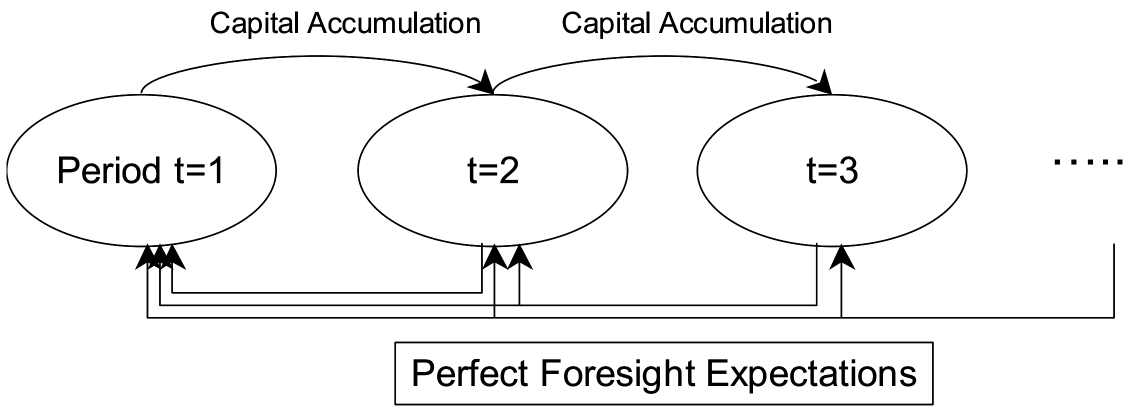
2.1.3 Single-Sector Dynamic Forward-Looking Model

The dynamic forward-looking computable general equilibrium model depicts the circular flow of output, income and expenditure in the goods and factor markets accounting for price-based backward and forward linkages across various production sectors for the economy over the entire model horizon. In each time period, a representative household, who is endowed with labour and capital, supply these factors of production to firms. In turn, the firms use these inputs to produce goods and services. As owners of the factors of production, households are compensated according to their marginal contribution for each factor they provide to production. Income earned from work and/or from supplying capital is then either spent on current consumption of domestic or foreign products, or saved for future consumption. Firms, then, use those savings to purchase a composite investment good, which replace depleted capital and add to their capital stock. Ex post total investment equals the ex-ante amount of savings in the economy.

Economic agents follow maximising behaviour, that is, households and firms make optimal choices given their inter-temporal budget constraints. Households maximises the present value of their lifetime utility, firms maximise the value of their profits. In every period, prices adjust to guarantee equilibrium in the model so that demand equals supply. Labour is perfectly mobile across sectors in the model. Labour will flow to an industry with a higher marginal revenue product from one with a lower marginal revenue product until the demand and supply adjust from price changes, in this case, the price being the prevailing wage rate.

Demand for and supply of goods and services re-adjust until all excess demands and excess supplies are eliminated through changes in prices. Perfectly competitive markets operate to determine these equilibrium prices. Additionally, in equilibrium, no sector earns above-normal profits, markets clear for all factors and products, and the value of imports for intermediate use and final demand equals the value of export earnings. Including a public sector, any changes in taxes will change economic behaviour and eventually market prices via the model's equilibrium conditions.

Figure 2 Dynamic Forward Looking Model: Infinite Horizon



In contrast to the dynamic recursive model, the dynamic forward looking model does not just have a rule that links one time period to the next but capital is accumulated in each future time period. Further, firms maximise the net present value of their profits and consumers maximise their net present value of their utility. They have rational expectations about future time periods and can "see" the future. Decisions made in period t=1 (and subsequent time periods) take into consideration events that occur in future time periods. Economic agents can adjust to shocks before they occur.

A representative consumer maximises the present value of their lifetime utility:

$$\max \sum_{t=1}^{\infty} \left(\frac{1}{1+\rho} \right)^{t} U(c_t) \qquad \textbf{(1)}$$

Where t denotes the time period

ρ denotes the discount factor or individual time-preference parameter

U denotes the utility function

c_t denotes consumption in period t

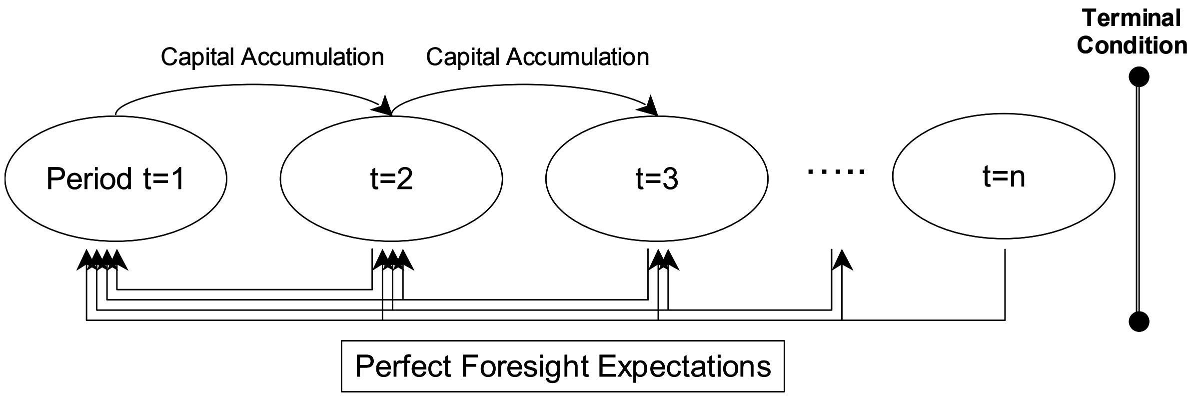
2.1.4 Terminal Condition

The implementation of a dynamic general equilibrium involves three steps. These steps are calibration of model parameters, replication of the benchmark economy, and computation of transitional dynamics in relation to external shocks or policy changes in the model economy (the counter-factual). Numerical models can only be solved for a finite number of periods. An adjustment needs to be made to produce a model which approximates over the infinite horizon. If there was no adjustment process then all capital would be consumed in the last period and nothing would be invested. Lau *et al.* (2002) propose a method for approximating the infinite horizon equilibria with endogenous capital accumulation. This method has become the standard way of approximating infinite horizon general equilibrium models. Their central idea is to split the intertemporal utility function into two parts. This can be written as:

$$U = \sum_{t=1}^{T} \beta^t U(C_t) + \sum_{t=T+1}^{\infty} \beta^t (C_t) \qquad \textbf{(2)}$$

The second term in this utility function collapses into a constant term. The dynamic forward looking model can be seen graphically in the following way:

Figure 3 Dynamic Forward Looking Model: Finite Horizon



Differing from the dynamic forward looking infinite horizon model, the finite horizon model is solved for a certain number of time periods after which the terminal condition then is operationalised. This method gives the model tractability.

In a single sector model, the two sub-problems are linked through the capital stock in period T+1. Having decomposed the model, a good terminal approximation occurs where the capital stock in period T+1 is close to the optimal value in the infinite-horizon part of the model. If the "true" value of the capital stock in the post-terminal period is known then the true consumption and saving paths can be calculated in the interim transitional period. After an external shock, however, the "true" value of the capital stock in the post-terminal period may not be known. In this case, what Lau *et al.* (2002) recommend is, rather than to impose the long-run steady-state value of capital stock (where the model horizon may be extraordinarily long for it to converge to the steady state), the state variable, K_{T+1}, can be determined as part of the equilibrium calculation by targeting the associated control variable, I_T. This is done by imposing a constraint that defines how terminal investment grows. Gross investment in the terminal period is therefore determined by the size of the capital stock in the terminal period, the steady state growth rate, and the rate of capital depreciation. Rutherford (2004) has suggested the following:

Terminal investment growth set equal to the long-run steady-state growth rate:

$$\frac{I_T}{I_{T-1}} = 1 + \gamma \qquad \textbf{(3)}$$

Terminal investment growth rate set equal to the growth rate of consumption:

$$\frac{I_T}{I_{T-1}} = \frac{C_T}{C_{T-1}} \qquad \textbf{(4)}$$

Terminal investment growth rate set equal to the growth rate of aggregate output:

$$\frac{I_T}{I_{T-1}} = \frac{Y_T}{Y_{T-1}} \qquad \textbf{(5)}$$

Lau *et al.* (2002) argue that this state-variable targeting method is a superior method to the one outlined by Barr and Manne (1967). The latter method involves an increased weight on utility of consumption in the terminal period, and a constraint on investment in the terminal period. However the state variable targeting method has two advantages over the techniques based on optimisation methods. State variable targeting provides a more precise approximation of the infinite horizon equilibria. In other words, the model is more efficient (takes fewer periods to approximate the infinite horizon saddle path) when the state variable targeting method is used) Further, state variable targeting does not require ex ante specification of the growth rate of the terminal period or impose a specific capital stock in the post-terminal period. This will be the method this research follows because it is therefore suitable for models with endogenous growth where the terminal stock is not determined ex ante.

For both the single-sector dynamic forward-looking model with the infinite and finite horizon versions, risk is implicit as it is with the static and dynamic recursive models.

2.1.5 Multi-Sector Equivalent of Dynamic Inter-temporal Model

This section explains the difference between the single-sector and multiple-sector dynamic intertemporal model. The change from homogenous capital to heterogeneous capital has more implications than just adding a sector-specific subscript. In this model, heterogeneous capital can introduce exogenous risk premia.

Firms invest, using savings. The market rental rate of capital is determined by market forces, the supply of and demand for capital. Total investment demand equals the use of investment goods from domestic and imported sources. Economy-wide, a composite investment good is derived from the final investment demand column from the IO table. The composite investment good is allocated to sector-specific investment so that the marginal productivity of capital is equal across sectors. Investment opportunities are arbitraged when the net rate of return from each sectorally differentiated investment does not exceed the rate of interest. When investment is undertaken in that sector the net rate of return in that sector will equal the rate of interest. These relationships can be expressed in the following equations:

$$R_{i,t} - \delta_i \leq r_t \quad \textbf{(6)}$$

$$I_{i,t} \geq 0 \quad \textbf{(7)}$$

$$I_{i,t}(R_{i,t} - \delta_i - r_t) = 0 \quad \textbf{(8)}$$

Where $R_{i,t}$ = Gross of depreciation rate of return in sector *i* at time *t*

δ_i = Sector-specific depreciation rate

r_t = rate of interest at time *t*

This arbitraging condition means that sectors with high gross return and lower depreciation rate generate more gross investment demand. In the steady-state, investment will grow at the same rate in all sectors, and the return to capital will be equalised across all sectors. However, during the transitional phase, it is possible for the net return in one sector to fall below that of another. As a result, investment can be shut down in the low return sector. Similar to the single-sector model, assets depreciate. Gross sectoral investment increases the capital stock as well as replaced depreciated capital.

$$K_{i,t+1} = K_{i,t}(1 - \delta_i) + I_{i,t} \quad \textbf{(9)}$$

Different form the other models; the multi-sector dynamic forward looking model can introduce a risk premia between the different rates of return on sector-specific investment. Examples of applied models where this has been introduced can bee found in Section 3 of this paper. Nevertheless, this neoclassical risk is exogenous. Hence CGE models are deterministic in nature. Applied general equilibrium models ignore the stochastic elements that may affect both the production and consumption sides of the economy. With forward-looking models, perfect foresight negates the possibility of endogenous risk as producers and consumers would foresee the risk in the future and change their behaviour accordingly.

Based on these limitations it might be argued that there is no room for risk to be incorporated into a CGE model. The CGE methodology does not allow it. The next section reviews a selection of instances where elements of risk have been incorporated into applied models.

3. Addressing the Issue of Risk and Computable General Equilibrium Models: A Review of the Literature

3.1 Modeller Uncertainty

There have been several ways to the issue of risk and uncertainty have been addressed concerning CGE modelling. Uncertainty can originate within the economic system or uncertainty can occur as part of the modelling process, that is, modeller uncertainty. Modeller uncertainty relates to the risk of reporting incorrect results or the uncertainty relating to exogenous parameters which have been inputted into the model. The modelling process may involve uncertainty regarding the true value of the model parameters, such as elasticities. Elasticities are sometimes applied to CGE models from one region or points in time that have been estimated econometrically from datasets from different regions or different time periods, that is, there is a mismatch between the data sample and the source of variation in the econometrics and the policy experiment explored in the CGE model (Hertel *et al.*, 2004). Systematic sensitivity analysis, via Monte Carlo analysis or Guassian Quadrature procedure (DeVuyst and Preckel, 1997) is a way to account for this type of uncertainty in CGE models. This technique has been used by Blake (2005), for example.

Modellers may also been uncertain about the results obtained in their CGE model simulations. There is a risk of reporting inaccurate results. Because the source data for CGE models are usually Input-Output tables and Social Accounting Matrices (SAMs) from a particular benchmark year, the assumed production functions and consumer preference functions are calculated deterministically by a process of calibration rather than being estimated econometrically. As such, t-ratios and confidence intervals that can be used for statistical testing do not exist hence there is uncertainty over the accuracy of results. Research exists that attempts to validate results usually through econometric techniques. Valenzuela *et al.* (2005), in an attempt to validate results from the global CGE model, GTAP, employ the method of stochastic simulation to reflect random production variability for the commodity, wheat. They model uncertainty in wheat output using shocks derived from a times-series (ARMA) model of wheat production to measure the randomness inherent in annual output. Repeatedly solving a CGE model, while sampling from the residuals of the times-series model, creates a distribution that imitates the corresponding market price changes for wheat, by region. The standard deviations based on these model results are compared to the observed outcomes for annual wheat price changes in order to validate the model. They find that the simulated outcomes for some regions are remarkably close to the observed outcomes but for other regions the model does not perform as well. Two other pieces of research to employ similar methods: Gehlhar (1997) and Liu *et al.* (2004), where the former uses a backcasting simulation to evaluate the validity of GTAP model results versus observed outcomes concerning East Asian economic growth in the 1980s. Gehlhar finds that the CGE model performs adequately at the qualitative level (direction of change in trade share), but is weak in its predictive power. Liu *et al.* build on Gehlar's approach and develop and approximate likelihood function to assess the quality of model performance over a 6 year period, 1986 to 1992.

3.2 Economic Risk

In economic theory, there has been an attempt to incorporate risk into the economic system in a CGE model in two ways: through the introduction of risk premia and through assumption regarding economic agents' expectations.

3.2.1 Risk Premia

McKibbin and Wilcoxon (1999), in their G-Cubed model, incorporate exogenous risk premiums in their inter-temporal dynamic multi-sector multi-region CGE model. They do this through the full integration of real and financial markets. With the assumption of perfectly integrated asset markets across regions, the expected returns on loans (interest rates plus risk premiums) denominated in one region (currency) is equal to the expected returns in another region (currency) adjusted by the exchange rate so there is no arbitrage. Within each economy, the expected returns to each type of asset are equalized by arbitrage, adjusting for adjustment costs of physical capital stock and exogenous risk premiums. In long run equilibrium the return of capital across sectors is the same, yet in the short run, simulations can allow for arbitrage and hence risk premiums across different capital assets. Country risk has also been modelled through exogenous risk premiums using a dynamic CGE model (Malcolm, 1998). The standard GTAP model assumes that the global bank equalizes risk-adjusted rates of return so that the risk-adjusted rates for all regions are equal to a weighted average of returns around the world. Malcolm (1998) explicitly defines this risk premia and hence examines the effects of changes in these risk premia. In these multi-sector models, the risk premiums are exogenous.

One method of endogenising risk into a CGE model has been developed by Arndt and Tarp (2000). They employ a CGE model to analyse the interactions between agricultural technology improvement, risk, and gender roles in agricultural production in Mozambique. They introduce a particular type of "technology" risk into the model, assuming that a safety first strategy is pursued, that is, they assume that households aim to produce a certain exogenous amount of cassava (the crop of interest in the study) for risk reduction purposes only. Once resources have been allocated to produce a minimum amount of cassava, resources are then distributed to other agricultural and non-agricultural activities according to the market. The safety first risk-aversion strategy is applied by adding an endogenous variable which serves as a risk premium. This risk variable enters in two functions in the model: the factor demand function and the factor income equation with the risk premium in the numerator of the factor demand function and the risk premium is in the denominator of the factor income equation. This means that if a risk premium exists (>1), factor demand for the commodity will be higher than in the risk-less pure profit maximizing position and factor income will be lower than in the risk-less scenario. Arndt and Tarp conclude there are considerable differences in production and price movements for cassava between the risk and no risk scenarios.

3.2.2 Incomplete Information

Uncertainty can be viewed in the context of incomplete information – a market inefficiency. The lack of information regarding the future may give producers will an incentive to supply too much of some products and too little of others. Alternatively, consumers may not purchase a product even though they would benefit. One method of simulating incomplete information has been to contrast static expectation (incomplete information), where consumer and producers have full information in each current time period but know nothing of the future, to rational expectations (perfect information) where consumers and producers have perfect knowledge of both current and future market conditions.

The issue of uncertainty with regards to future information is explored by Arndt and Bacou (2000). Taking a relatively standard CGE model of Mozambique, these authors explore the value of climate forecast information that operates and interacts at a farm level, at a marketing system level and at a full economy level. Under the premise that predictable droughts are less damaging than randomly occurring droughts, three simulations are modelled. This first simulation involves an unanticipated drought (droughts being simulated by a Hicks-neutral technology decline), where agricultural activity is fixed and can not be adjusted based on realized climate outcomes. The second and third simulation allow farmers and both farmers and marketing agents, respectively, to react to a received perfect climate forecast. Results show that reduction in risk associated with a perfect forecast results in lower decreases in real GDP and welfare as resources are reallocated from drought intolerant to more drought tolerant activities.

Following along the same lines but in a different context Adams *et al.* (2001) explore the issue of timing and announcement of policy changes within a dynamic CGE model. The two scenarios modelled involve, first, the introduction of a once-off quota without any previous announcement on the production of pigs in the Danish economy and the second scenario involves an announced gradual phased in production quota. Not surprisingly, the adjustment path of the economy is smoother when the policy is announced compared to the surprise policy implementation. The key factor is the relationship between investment and expectations. In the 'announced' scenario investors correctly anticipate future adjustments in prices and rental rates on capital when making investment decisions and the capital stock starts to adjust from the start of the simulation. In the 'surprise' scenario, expectations are static and investors adjust fully only when the quota is implemented (rather than announced). With the announced policy, scenarios that contract the economy occur earlier and more gradually so whether the announcement or surprise implementation is preferable depends on agent's attitude to risk and their implicit discount rate. The more risk averse they are or the lower they discount future consumption, the more likely it is they prefer an announced policy.

Focusing on international capital mobility, Ianchovichina *et al.* (1999) develop a disequilibrium approach for a dynamic multi-sector multi-region general equilibrium model. As well as modelling exogenous risk premia as McKibbin and Wilcoxon do, the key feature of this model is that there are errors in investors' assessment of potential returns to capital. They argue that investors' expectations are "sticky" and that when the observed rates of return change, investors are uncertain whether this change is temporary or permanent. It is only with a lag that they adjust their expectations. Initially, investors make small adjustment, and if the change in the rate of return persists, they make further changes in their expectations until the expected rate equals the observed rate of return. This was the explanation for the Asian financial crisis of 1997. The authors argue that the developments in East Asia reflect the fact that investors have not foreseen correctly returns to capital. They argue that this can be represented through a simple recursive solution procedure to mimic investment theory of adaptive expectations. They argue that the limitation of forward-looking inter-temporal models is the assumption of perfect foresight of returns to capital. In this case, the financial crisis would imply investors did not have perfect foresight.

Boussard *et al.* (2002), as well as allowing for imperfect expectations, examine the issue of agricultural trade liberalization, adding instability in the model by endogenising risk through lags in delivery, and risk aversion. Uncertainty is introduced into the model through a production lag in the agricultural sector. Picking up the work done by Ezekiel (1938), who developed cobweb theorem, the researchers specify a lag between the production and consumption decision for the agricultural sector. The market equilibrium is between the previous year's production and current consumption. Production decisions are taken on the basis of expected prices, rather than equilibrium prices. Equilibrium prices are used as inputs, expectations are important, in this model, only for next year's production. In turn, income in the current year depends heavily on expectations for the future year; implying firms can gain or lose. As such, firms bear risks. In sum, Boussard *et al.* introduce risk, imperfect information and production lag in the agricultural sector in a standard CGE model to model uncertainty. They find, in contrast to the classical perfect foresight model where global gains are associated with trade liberalization, the model with risk aversion, imperfect information and a production lag in the agricultural sector shows negative changes in real income. Imperfect information constrains the economy from reaching its optimal. In a later piece of work, Boussard *et al.* (2004) explain how the endogenous risk differs from exogenous risk. Neoclassical risk is exogenous, it is delivered form above, outside the model. The behaviour of agents has no relationship o the level of risk involved and cannot influence the degree of risk. Endogenous risk is a consequence of expectation errors. These errors are inconsistent with the rational expectations hypothesis.

In the CGE risk literature a dynamic recursive model is implemented to replicate incomplete information about the future. However, one weakness with this research is that the dynamic recursive model has some inherent problems. While the researchers cited above argue that a dynamic forward-looking model does not allow the existence of imperfect information or errors in expectations, to argue economic agents do not make any decisions based on what they know about the future and / or attempt to, say smooth consumption or production is to err in the other direction. In the next section, an alternative method is outlined that treats risk explicitly through creating multiple future paths for the model, with agents able to predict each path and make decisions in the presence of this uncertainty.

As CGE models have become more sophisticated, there have been efforts to model elements of risk and uncertainty. There have been two main categories: modeller uncertainty, where there is uncertainty regarding exogenous parameters and results in general and attempts to allow for uncertainty in these cases; and uncertainty regarding future expectations where agents' behaviour under static expectations is contrasted with expectation regarding perfect foresight expectations. The next section describes a different way to incorporate uncertainty into a CGE model.

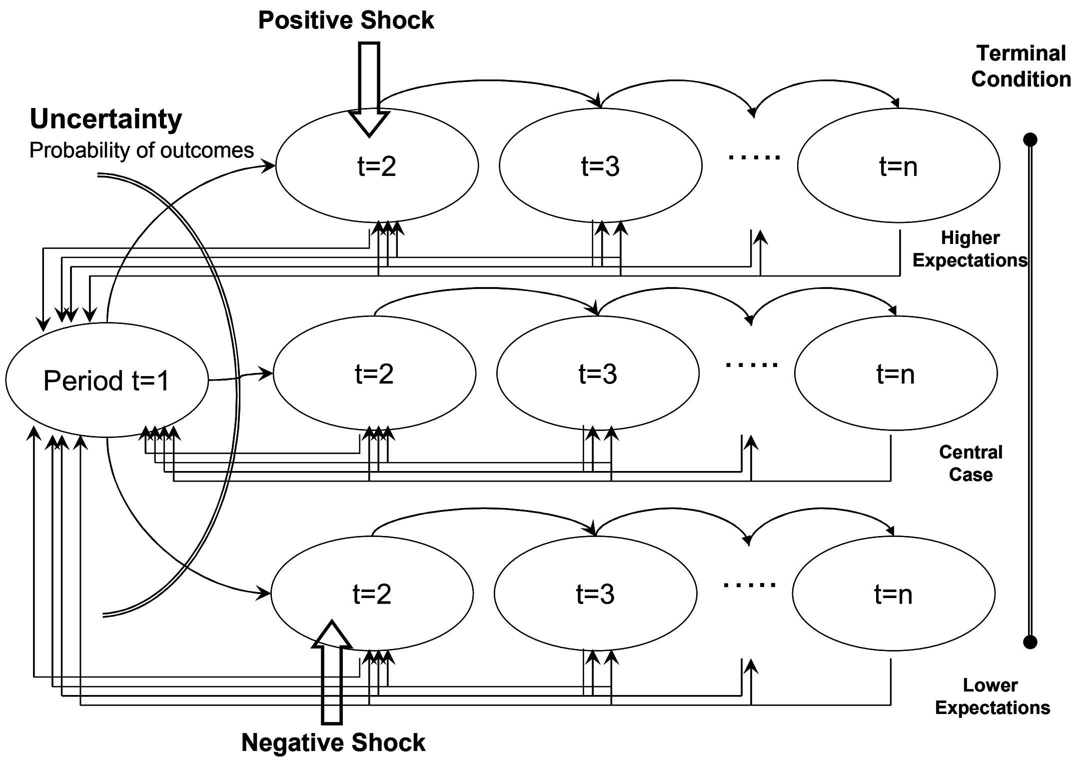
4. Uncertainty Regarding Future Paths

4.1 Conceptualisation

In this dynamic forward-looking model, uncertainty will enter the model through the uncertainty in an exogenous shock. An exogenous variable, for example an exports, can be simulated to vary stochastically (with a designated probability) about the base year mean. The standard assumption of rational expectations will hold so that the representative consumer, firms, and government are endowed with perfect foresight, and so anticipate any exogenous shocks and adjust their maximising behaviour from the second time period (the period in which the uncertainty, but not necessarily the shock will occur). The uncertainty in the model involves the choice of future paths by consumers and firms. Given this uncertainty, how do consumers and firms change their behaviour? For example, in the partial equilibrium literature, firms will under-invest in sectors where there is uncertainty. Do these results hold in the general equilibrium context? It may even be that the uncertainty may be so high as to stifle investment such that, it may be unprofitable to invest in the current time period. The economic impacts of the timing (when the shock occurs and how long the uncertainty remains) as well as the magnitude of the exogenous shocks will also be analysed.

A graphically representation of the conceptual model is shown in Figure 4. Several dynamic forward looking models are possible. One scenario may be that baseline growth path is contrasted with a positive shock and a negative slump. These impacts are then followed through to the *n*th time period. Expectations are consistent throughout so that in economic agents have higher expectations under the positive shock scenario from the first time period, even though the positive shock does not take place until period *t=2*. Uncertainty is represented by the likelihood of a positive shock or a negative shock to occur at any given time period.

Figure 4 Dynamic Forward Looking Model: With Uncertainty



4.2 Application

This section of the paper outlines how the dynamic forward-looking model with uncertainty is operationalised. A simple model featuring three sectors (Agriculture, Manufacturing and Service), 2 factors of production (Capital and Labour) in an open economy is used to illustrate how uncertainty will work. The SAM of this fictitious economy is displayed in Table 1 below.

Table 1. SAM

	INTERMEDIATE DEMAND			FINAL DEMAND		
	Agriculture	Manfacturing	Service	Consumption	Investment	Exports
Agriculture	100	100	50	250	50	200
Manfacturing	50	150	100	350	300	100
Service	50	100	200	800	50	100
Imports	200	100	100			
Capital	150	400	250			
Labour	200	200	600			

The model can be thought of as consisting of two parts. The first time period can be viewed akin to a static model where there is only one possible state of the economy. The second part of the model is the dynamic part where there are different paths that the economy can take. The model is intended to be as standard as possible so as to highlight the uncertainty feature. The representative consumer is endowed with primary factors (labour, capital). The value of exports equal the value of imports so there is a trade balance. After the representative consumer sells primary factors, and purchases back capital in the terminal period, the remaining income is devoted to private consumption.

4.2.1 Production

Firms are assumed to be price takers who choose variable inputs and its level of investment in order to maximise profits. Each industry is modelled using the constant elasticity of substitution (CES) family of functions, which includes Leontief, Cobb-Douglas and Constant Elasticity of Transformation (CET) functions. Each production sector $Q_{i,t}$ produces two types of commodities: domestic goods $D_{i,t}$ and goods for export $X_{i,t}$. These goods are assumed to be imperfect substitutes, and they have a constant elasticity of transformation. For production, each sector uses capital, labour, and intermediate goods. As such, the sector's *i* production function is

$$Q_{i,t} = \mathrm{g}(\mathrm{D}_{\mathrm{i,t}}, \mathrm{X}_{\mathrm{i,t}}) = \mathrm{f}(\mathrm{K}_{\mathrm{i,t}}, \mathrm{L}_{\mathrm{i,t}}, \mathrm{A}_{\mathrm{i,j,t}})$$

where g is output transformation function, and f is input transformation function. Output transformation is assumed to be the constant elasticity of transformation (CET):

$$Q_{i,t} = \Theta\left(\delta_i^e D_{i,t}^{\frac{\eta-1}{\eta}} + (1-\delta_i^e) X_{i,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta-1}{\eta}}$$

where

$Q_{i,t}$ = output

$X_{i,t}$	=	Exports
$D_{i,t}$	=	Domestic production
η	=	the elasticity of transformation in total supply
δ_i^e	=	the calibrated share of exports
Θ	=	the calibrated shift parameter in the transformation function

Overall market clearing in the product market means the Armington aggregate is used for private consumption, investment, and as an intermediate input for production.

$$\mathrm{A}_{\mathrm{i,j,t}} = CC_{i,t} + I_{i,t} + \sum_{j}^{n} IO_{i,j,t}$$

where:

$\mathrm{A}_{\mathrm{i,j,t}}$ = the Armington CES aggregate CES aggregate of domestic production and imports

$CC_{i,t}$	=	composite consumption
$I_{i,t}$	=	composite investment
$\sum_{j}^{n} IO_{i,j,t}$	=	Demand for intermediate inputs

For input combination, we have a Leontief aggregation of factors of production. Capital and labour enters as a Cobb-Douglas value-added aggregate. Intermediate inputs from different sectors enter as a Leontief aggregate into a sector's i production function:

$$\mathrm{f}(\mathrm{K}_{\mathrm{i,t}}, \mathrm{L}_{\mathrm{i,t}}, \mathrm{A}_{\mathrm{i,j,t}}) = \min\left\{ B_i L_i^{\alpha_i} K_i^{(1-\alpha_i)}, \min\left\{ \frac{A_{i,1}}{a_{i,1}}, \frac{A_{i,2}}{a_{i,2}}, \ldots, \frac{A_{i,j}}{a_{i,j}} \right\} \right\}$$

An intermediate input to a sector *i* from a sector *j* is an Armington aggregate of domestic output and import. Users regard these goods as imperfect substitutes, and these goods are assumed to have a constant elasticity of substitution (CES).

$$Q_{i,t} = \Omega \left(\delta_i^m D_{i,t}^{\frac{\gamma-1}{\gamma}} + (1-\delta_i^m) M_{i,t}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma-1}{\gamma}}$$

where

$Q_{i,t}$	=	output
$M_{i,t}$	=	Imports
$D_{i,t}$	=	Domestic production

γ = the elasticity of substitution between domestic goods and services and imported goods and services

δ_i^m = the calibrated share of imports

Ω = the calibrated shift parameter in the substitution function

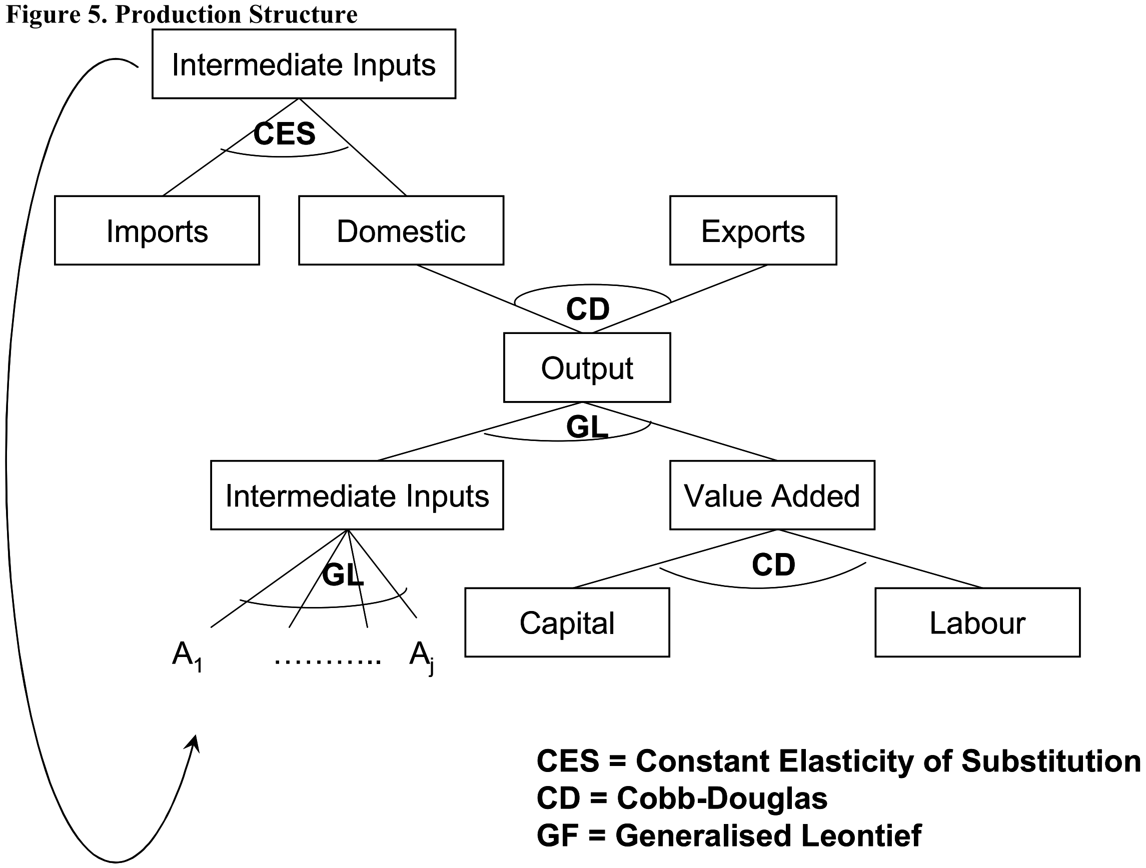
Each firm's production technology is represented by a constant elasticity of substitution (CES) function.

$$VA_{i,t} = A_i^o \left((1-\delta_i^o)(K_{i,t})^{\frac{1}{\sigma_i^o}} + \delta_i^o (L_{i,t})^{\frac{\sigma_i^o - 1}{\sigma_i^o}} \right)^{\frac{\sigma_i^o}{\sigma_i^o - 1}}$$

Where $VA_{i,t}$ is the gross value added of sector i, A_i^o reflects the level of technology in the production function, $K_{i,t}$ and $L_{i,t}$ are the amounts of capital and labour used in sector i, δ_i^o is the share parameter of labour in production, and σ_i^o is the elasticity of substitution and the subscript indicates that the parameters apply to 'output'. This is a constant returns to scale production function.

The intra-temporal (static) production can be depicted as in Figure 5.

Figure 5. Production Structure



4.2.2 Consumption

The representative consumer has an endowment of primary factors of production: capital and labour. They demands investment and consumption goods. The investment sector output are exogenous while consumption demand is determined by utility maximizing behaviour. Consumer utility consists of a Cobb-Douglas utility index defined over Armington aggregation of domestic and imported commodities

$$C = \prod_{i=1}^{n} c_i^{\alpha_i}$$

where

$$c_{i,t} = \mathrm{X}\left(\delta_i^m CD_{i,t}^{\frac{\gamma-1}{\gamma}} + (1-\delta_i^m) CM_{i,t}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma-1}{\gamma}}$$

where

$c_{i,t}$	=	output
$CM_{i,t}$	=	Imported production of consumption good
$CD_{i,t}$	=	Domestic production of consumption good
γ	=	the elasticity of substitution between domestic goods and services and imported goods and services
δ_i^m	=	the calibrated share of imports
X	=	the calibrated shift parameter in the substitution function

4.2.3 Initial Values for Capital and Investment

To calibrate the model the benchmark level of capital (K_0) and the real rate of return (R_0) need to be calculated, given the benchmark capital stock (KS_0) and benchmark level of investment (I_0) such that:

$$K_0 = I_0 * \frac{(1 - SAME * DEP)}{(GROW + DEP)}$$

where DEP is the depreciation rate, GROW is the constant growth rate of the economy in the steady state and *SAME* the same-period proportion of new capital that comes on line in the same period.

$$R_0 = \frac{KS_0}{(K_0 + SAME * I_0)}$$

4.2.4 Growth Rates for Quantities and Prices

Assuming the base period of an economy is on a steady-state growth path, then all quantities including capital, labour, consumption and output grow at the same constant rate, *GROW*. For example, for labour:

$$L_t = L_0(1+GROW)^t$$

Likewise, given a constant interest rate and constant depreciation rate, all future prices are designed in terms of present value: The present value of the wage W_t is:

$W_t = \left(\frac{(1-R_0)}{(1-DEP)} \right)^t$ where DEP is the depreciation rate and R_o is the real rate of return.

4.2.5 Capital and Investment

The consumer faces two constraints. The first constraint is that total output produced in the economy can either be consumed or invested, I (saved). The second constraint is that capital depreciates at the rate, δ. Hence,

$$c_t = F(,L_t,K_t) - I_t \qquad \textbf{(10)}$$

and

$$K_{t+1} = K_t(1-\delta) + I_t \qquad \textbf{(11)}$$

where K denotes capital and $F(\bullet)$ is the production function.

Capital can either be bought or rented. This is reflected in the production of capital so the current period capital earns the return R_t and next periods capital earns PK_{t+1}. Likewise for investment, current period investment earns PK_t and investment in the next time period earns PK_{t+1} which then discounted and depreciates.

4.2.6 Adding Uncertainty

Taking this dynamic forward-looking CGE model, in period t = 2, there is uncertainty on which path the economy might follow, given an exogenous shock. The next step is to introduce a number of different paths that the economy might take. Clearly, it is not possible to model all possible future time paths. The number of possible paths chosen might that span a feasible solution. Let p be the number of possible path, then $\Phi(p)$ is the probability that a specific path is taken. It is a necessary condition that $\sum_{1}^{P} \Phi(p)$, that is, that the probability of the sum of the paths sum to 1. This occurs after period t = 1, that is, when the exogenous shock occurs. From the second time period onwards, all the sectors and quantities need to be multiplied by the probability that this particular path is chosen. The relationships described above (Section 4.1) exist for the first time period and in each path $\Phi(p)$, where K^* indicates the first time period and $K_{\Phi,t}$ indicates capital in period t for path Φ. For example the capital accumulation equation now becomes:

$$K_{\Phi,t=2} = K^*(1-\delta) + I^* \quad \textbf{(12)}$$

$$K_{\Phi,t+1} = K_{\Phi,t}(1-\delta) + I_{\Phi,t} \quad \textbf{(13)}$$

where * in Equation 12 denotes first-year values and Equation 13 represents the capital accumulation equation from the second period onwards.

On the consumer side, utility given by Equation 1 is now dependent on the time path chosen is represented by:

$$U_{\Phi} = \sum_{t=1}^{\infty}\left(\frac{1}{1+\rho}\right)^t C_{\Phi,t} \quad \textbf{(14)}$$

$$U_{\Phi} = C^* + U^* \quad \textbf{(15)}$$

$$U^* = \left[\sum_{\Phi}^{P}\Phi(p)U_{\Phi}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \quad \textbf{(16)}$$

Where U^* is first period utility
C^* is first period consumption
ρ is the discount factor or individual time-preference parameter
U_{Φ} is discounted utility, and
σ is the risk aversion parameter

5. Conclusion

CGE models have traditionally tended to be deterministic in nature. Recently, there have been attempts to add stochasticity into the model in various ways. Understandably, there are limitations to modelling uncertainty in this way. It is not possible to model all future possible future time paths. One area to develop the research further is to determine which future time paths are more important to model; nevertheless, the model explicitly includes risk and agents' reaction to risk. This paper outlines one way to add uncertainty into a dynamic forward-looking model. More forward, the model needs further testing on the dataset of a 'real' economy.

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