China's Agricultural Production Insurance

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Abstract: China's agricultural plays an important role in its whole economy. China's agricultural production, however, suffers the natural disasters almost every year, such as flood, drought, storm, hail, plant diseases and insect pests, which incur big loss in China's agricultural production. A sound agriculture insurance system is, therefore, needed to protect farmers from the natural disasters loss. Unfortunately, China's agriculture insurance is weak. This paper aims to design optimal insurance contracts for agricultural production under symmetric information and asymmetric information (adverse selection and moral hazard) by solving optimal programming models.
Key words: Agricultural production, Insurance, Optimal programming model

I. Background

Agricultural production insurance, as a stabilizer of risk, plays an important role in government policies of supporting agricultural production in most countries. Government subsidy on agricultural production insurance is adopted commonly and is becoming one of the focuses of agriculture policies in most countries the world. China is an agricultural country and its agriculture sector still takes big share in the whole economy. China's agricultural production, however, suffers the natural disasters almost every year, such as flood, drought, storm, hail, plant diseases and insect pests, which incur big loss in China's agricultural production. A sound agriculture insurance system is, therefore, needed to protect farmers from the natural disasters loss. Unfortunately, China's agriculture insurance is still weak.

Table 1.1. Economic and Technical Indicators of Insurance Companies Funded with Chinese and Foreign Capital (100 million Yuan)

Year	Total		Property Insurance Companies		Agriculture Insurance		Life Insurance Companies	
	Premium	Claim & payment	Premium	Claim & payment	Premium	Claim & payment	Premium	Claim & payment
1985	25.73	12.54	21.61	12.33	0.43	0.53	12.33	0.21
1986	42.35	18.88	30.77	17.02	0.78	1.06	11.58	1.87
1987	67.14	27.66	43.73	22.12	1.00	1.26	23.41	5.83
1988	94.76	36.99	60.51	28.02	1.16	0.92	34.25	8.97

1989	122.91	48.51	82.92	39.43	1.30	1.07	39.99	9.08
1990	155.76	68.30	106.68	45.65	1.92	1.67	49.08	22.65
1991	209.71	114.29	146.54	83.82	4.55	5.42	63.17	30.46
1992	335.15	159.13	241.29	114.75	8.17	8.15	93.86	44.38
1993	456.87	254.76	371.02	217.47	5.61	6.47	85.95	37.39
1994	376.42	230.36	233.28	174.09	5.04	5.39	143.13	56.27
1995	453.32	236.39	292.41	181.75	4.96	3.65	160.90	54.64
1996	538.33	305.42	323.52	203.75	5.74	3.94	214.81	101.67
1997	772.71	247.15	382.23	214.69	5.76	4.19	390.48	32.46
1998	1255.97	531.68	505.74	289.51	7.15	5.63	750.23	242.17
1999	1406.17	508.02	527.22	279.70	6.32	4.86	878.95	228.32
2000	1598	526	608	308	4	3	990	218
2001	2109	597	685	333	3	3	1424	264
2002	3054	707	780	403	5	4	2274	304
2003	3880	841	869	476	5	3	3011	365
2004	4318	1004	1125	579	4	3	3194	426
2005	4932	1137	1283	691	7	6	3649	446

Source: China Statistical Yearbook

As shown in Table 1.1, China's agriculture insurance developed fast in the period of 1982-1992 and reached its peak in 1992 with 817 million Yuan. After that, it went down until 2004 and then recovered in 2005. Nevertheless, the scale of agriculture insurance is too small taking account of its role in whole economy. This is mainly caused by the following reasons.

1) High frequency of natural disasters and then high loss ratio in agriculture insurance. Table 1.3 shows that average area shares of covered and affected out of sown are 31.95% and 17.03% respectively, areas ratio of affected to covered is 53.02%. High frequency of natural disaster and high area ratio of affected to covered bring high risk for agricultural production and high loss ratio in agriculture insurance. As shown in Table 1.2, Loss ratio in agriculture insurance is much higher than those in other insurances, with average loss ration 91.49% in 1985-2005, and even higher than 100% in seven years. Such high loss ration plus 20% of management cost make china's agriculture insurance loss for a long time, which force insurance companies shrink agriculture insurance business and reduce agriculture insurance supply.
2) Low farming income and comparative cost disadvantage in agricultural production and then weak demand for agriculture insurance. Typically, farmers are risk-averse. However, insurance premiums are still high compared with their income level, which greatly frustrate their enthusiasm to buy agriculture insurance. Table 1.2 shows that premium share of agriculture insurance out of property insurance is merely 1.58% in 1985-2005 averagely, with 3.39% of highest share in 1992.
3) Insufficient support from government and inappropriate role played by the government. In China, agriculture insurances are mainly run by commercial insurance companies and lack in government support It is well known that agriculture insurance, to some extent, is public goods, which means that government can and should play some roles in order to regulate agriculture insurance market to reach optimal equilibrium. Usually, government functions in agriculture insurance are defined by law. But in China, this kind of definition is not clearly in

related laws. That what can be done or should be done by government in agriculture insurance is not clearly defined can make government act at discretion, which may have some negative impact on the development of agriculture insurance market.

Table 1.2. Loss ratio of insurance in China (%)

Year	Total loss ratio	Loss ratio in Property Insurance	Premium share of agriculture Insurance	Loss ratio in agriculture Insurance	Loss ratio in life Insurance
1985	48.74	57.06	1.99	123.26	1.70
1986	44.58	55.31	2.53	135.90	16.15
1987	41.20	50.58	2.29	126.00	24.90
1988	39.04	46.31	1.92	79.31	26.19
1989	39.47	47.55	1.57	82.31	22.71
1990	43.85	42.79	1.80	86.98	46.15
1991	54.50	57.20	3.10	119.12	48.22
1992	47.48	47.56	3.39	99.76	47.28
1993	55.76	58.61	1.51	115.33	43.50
1994	61.20	74.63	2.16	106.94	39.31
1995	52.15	62.16	1.70	73.59	33.96
1996	56.73	62.98	1.77	68.64	47.33
1997	31.98	56.17	1.51	72.74	8.31
1998	42.33	57.24	1.41	78.74	32.28
1999	36.13	53.05	1.20	76.90	25.98
2000	32.92	50.66	0.66	75.00	22.02
2001	28.31	48.61	0.44	100.00	18.54
2002	23.15	51.67	0.64	80.00	13.37
2003	21.68	54.78	0.58	60.00	12.12
2004	23.25	51.47	0.36	75.00	13.34
2005	23.05	53.86	0.55	85.71	12.22
Average	40.36	54.30	1.58	91.49	26.46

Note: the numbers in this table are calculated from Table 1.

Table 1.3. Areas Covered and Affected by Natural Disaster (1000 ha)

	Total areas sown	Areas covered	Areas affected	Area share of covered out of sown (%)	Area share of affected out of sown (%)	Area ratio of affected to covered (%)
1978	150104	50790	24457	33.84	16.29	48.15
1980	146380	44526	29777	30.42	20.34	66.88
1985	143626	44365	22705	30.89	15.81	51.18
1989	146554	46991	24449	32.06	16.68	52.03
1990	148363	38474	17819	25.93	12.01	46.31

1991	149586	55472	27814	37.08	18.59	50.14
1992	149007	51333	25859	34.45	17.35	50.38
1993	147741	48829	23133	33.05	15.66	47.38
1994	148241	55043	31383	37.13	21.17	57.02
1995	149879	45821	22267	30.57	14.86	48.60
1996	152381	46989	21233	30.84	13.93	45.19
1997	153969	53429	30309	34.70	19.69	56.73
1998	155706	50145	25181	32.20	16.17	50.22
1999	156373	49981	26731	31.96	17.09	53.48
2000	156300	54688	34374	34.99	21.99	62.85
2001	155708	52215	31793	33.53	20.42	60.89
2002	154636	47119	27319	30.47	17.67	57.98
2003	152415	54506	32516	35.76	21.33	59.66
2004	153553	37106	16297	24.17	10.61	43.92
2005	155488	38818	19966	24.97	12.84	51.43
Average				31.95	17.03	53.02

Source: China Statistical Yearbook

II. Optimal contract of agricultural production insurance under symmetric information

We design mathematical programming models to analyze the effects of agricultural production insurance when natural disaster occurs. Firstly, we design a mathematical programming model under symmetric information as a baseline model. Then, we extend the baseline model under asymmetric information.

We assume that

1) Farmers and insurances face commercial insurance market.
2) Insurances are risk-neutral and farmers are risk-averse.
3) The probability that a natural disaster occurs is π, area share of affected out of sown is α, yield loss is β, the growth rate of cost in the prevention of natural disaster is γ, the growth rate of agricultural products composite price is ρ.
4) Insurance contract between farmers and insurance companies are signed under symmetric information.

Suppose that a farmer’s areas sown are A hectares, yield in normal year (state θ_0) is Y mt/ha, production cost is Z yuan/ha, agricultural products composite price is P yuan/kg. Then in normal year, the farmer’s profit from agricultural production is

$$W_0 = A\,(YP - Z). \qquad (2.1)$$

In abnormal year (suffering some natural disaster) (state θ_1), the farmer’s profit from agricultural production is

$$W_1 = A\,(1-\alpha)(YP(1+\rho) - Z) + A\alpha\,(Y(1-\beta)P(1+\rho) - Z(1+\gamma)). \qquad (2.2)$$

Therefore, the risk in agricultural production faced by the farmer is

$$[W_1, \pi\,;\, W_0, (1-\pi)],$$

The farmer’s expected profit from agricultural production is

$$W = \pi\, W_1 + (1-\pi)\, W_0.$$

Suppose that the farmer's utility from his profit of agricultural production is

$u(\cdot)$, $u(0\cdot) = 0$, $u'(\cdot)>0$, $u''(\cdot)<0$;

Then, the farmers' expected utility is

$E(u(W)) = \pi u(W_1) + (1-\pi) u(W_0)$.

Figure 2.1 shows us the farmer's indifferent curve and iso-profit line of expected utility on plane (W_0, W_1). On the indifferent curve of expected utility, we have

$D(E(u(W))) = \pi u'(W_1)dW_1 + (1-\pi) u'(W_0)dW_0 = 0$,

Then

$$\frac{dW_1}{dW_0} = -\frac{1-\pi}{\pi}\frac{u'(W_0)}{u'(W_1)} \text{。} \qquad (2.3)$$

This is the marginal rate of substitution between two states (θ_0 and θ_1).

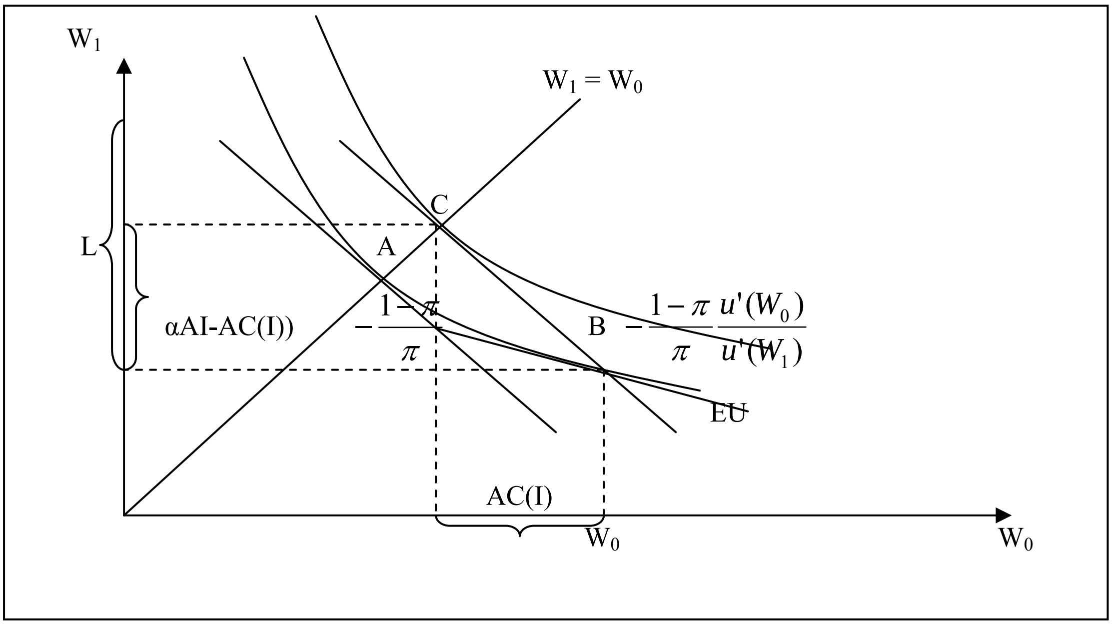

Figure 2.1 Insurance contract under symmetric information

The indifferent curve of expected utility is convex towards the origin since we assume that farmers are risk–averse. The slope at a point on the indifferent curve that is upper right to point A is steeper than $-(1-\pi)/\pi$. Therefore, $u'(W_1) < u'(W_0)$. On the opposite side, the slope at a point on the indifferent curve that is lower left to point A is flatter than $-(1-\pi)/\pi$. Therefore, $u'(W_1) > u'(W_0)$.

On iso-profit line, $W_1 = W_0$. This line is also called safety line because the farmer's profit is unchanged no matter which state occurs. At the point that is upper left to iso-profit line, $W_1 > W_0$, the farmer is over-insured. At the point that is lower right to iso-profit line, $W_1 < W_0$, the farmer is under-insured.

It is obvious that $u'(W_1) = u'(W_0)$ on iso-profit curve. Therefore

$$\left.\frac{dW_1}{dW_0}\right|_{W_1=W_0} = -\frac{1-\pi}{\pi}, \qquad (2.4)$$

That is, at the intersectant point, A, between the indifferent curve and the iso-profit line, the marginal rate of substitution between two states is equal to the ratio of probabilities between two states. At the same time, when $W_1 = W_0$ we obtain from (2.1) and (2.2) that

$$\frac{\rho - \alpha\beta(1+\rho)}{\alpha\gamma} = \frac{Z}{YP} = S \qquad (2.5)$$

Where S = Z / YP is the ratio of input to output. To ensure that the ratio is positive, we must have

$$\frac{\rho}{1+\rho} > \alpha\beta \qquad (2.6)$$

Therefore, to have the profits from agricultural production under two states be unchanged, this ratio must satisfies (2.5) and (2.6). Usually, these constraints are difficult to be satisfied. The farmer's expected profit will be unchanged when the farmer increases his output value under state θ_0 (or under state θ_1) while his loss in output value under state θ_1 (or under state θ_0) satisfy (2.3). In Figure 2.1, a farmer has the same utility expectation at point B and A. However, his output value is higher at point B than at point A under state θ_0, his output value is lower at point B than at point A under state θ_1. If the changes in output value under two states do not satisfy (2.3), the farmer's expected utility curve will shift. Utility expectation will be lower when the curve is closer to the origin point O, and vice versa

A farmer can diversify his risk by buying insurance. He can increase his expected utility in this way, though at some cost.

Let B be a farmer's initial profit. The farmer's loss in profit from agricultural production when suffering some natural disaster is (by (2.2))

$L = -AYP\rho + \alpha A(YP\beta + YP\beta\rho + Z\gamma)$.

If the farmer pays C(I) Yuan premium per hectare to an insurance company for A hectares, and he can claim I Yuan per hectare when suffering some natural disaster with affected area share α, i.e., total claim αAI Yuan, then his production profits under two states are

$W_{I0} = W_0 - AC(I) = A(YP - Z) - AC(I)$,

$W_{I1} = W_1 + \alpha AI - AC(I) = A(1-\alpha)(YP(1+\rho) - Z) + A\alpha(Y(1-\beta)P(1+\rho) - Z(1+\gamma)) + \alpha AI - AC(I)$.

Suppose that an insurance company is risk neutral with zero expected profit under competitive insurance market. Then the insurance company's iso-profit line on plane (W_0, W_1) is

$\pi(AC(I) - \alpha AI) + (1-\pi)AC(I) = 0$.

Hence, $\frac{AC(I) - \alpha AI}{AC(I)} = -\frac{1-\pi}{\pi}$.

That is, the income ratio under two states, or the marginal rate of substitution under two states, is equal to the negative ratio of probabilities of the two states. The farmer's optimum is attained at point C because he maximizes his expected utility at the point in the insurance company's iso-profit line. This can also be proven by solving the following optimal problem

$\max_I E(u(W, C(I), I) = \pi u(W_{I1}) + (1-\pi) u(W_{I0})$,

s.t $\pi(AC(I) - \alpha AI) + (1-\pi)AC(I) = 0$.

Let λ be the multiplier, then FOC is

$\pi u'(W_{I1})(\alpha A - AC'(I)) + (1-\pi)u'(W_{I0})(-AC'(I)) + \lambda(\pi(AC'(I) - \alpha A) + (1-\pi)AC'(I) = 0$,

i.e.,

$\pi(\alpha - C'(I))(u'(W_{I1}) - \lambda) = (1-\pi)C'(I)(u'(W_{I0}) - \lambda)$.

From the constraint condition, we have

$$C(I) = \pi\alpha I.$$

Therefore

$$C'(I) = \pi\alpha.$$

Then

$$u'(W_{I1}) = u'(W_{I0}),$$

by the property of utility function, we have

$$W_{I1} = W_{I0}.$$

Then

$$W_0 = W_1 + \alpha AI, \text{ or } \alpha AI = L.$$

Hence

$$AC(I) = \pi\alpha AI = \pi L.$$

The farmer's expected utility is

$$\begin{aligned} & E(u(W, C(I), I) \\ & = \pi u(W_1 + \alpha AI - AC(I)) + (1-\pi) u(W_0 - AC(I)) \\ & = \pi u(W_0 - \pi\alpha AI) + (1-\pi) u(W_0 - \pi\alpha AI) \\ & = u(W_0 - \pi\alpha AI) \\ & = u(\pi(W_0 - \alpha AI) + (1-\pi) W_0) \\ & = u(\pi W_1 + (1-\pi)W_0) \\ & > \pi u(W_1) + (1-\pi) u(W_0) = E(u(W)). \end{aligned}$$

That is, the farmer's expected utility increases after he buy insurance because his risk in agricultural production is diversified by insurance company. When he suffers from natural disaster, his insurance claim is

$$I = L/\alpha A = (-\rho/\alpha + \beta + \beta\rho)YP + Z\gamma, \qquad (2.7)$$

and he pay insurance premium per hectare.

$$C(I) = \pi\alpha I = \pi\alpha ((-\rho/\alpha + \beta + \beta\rho)YP + Z\gamma)。 \qquad (2.8)$$

III. Optimal contract of agricultural production insurance under asymmetric information

Equation (2.7) and (2.8) shows that an insurance company can offer an insurance contract for each farmer under symmetric information since he can distinguish farmer's risk state. Of course, that is just the solution under ideal situation, we use it as baseline. In reality, the insurance company can estimates the probability that a natural disaster occurs, π, from time series data. However, information on affected area share, α, on yield loss, β, on the growth rate of cost in the prevention of natural disaster, γ, or on the growth rate of agricultural products composite price, ρ, may be asymmetric to the insurance company. In this case, the insurance company cannot design an insurance contract for each farmer because he doesn't know the farmer's values of those parameters used in (2.7) and (2.8).

Usually, there are two types of asymmetric information problems. One is so-called adverse selection problem which appears when a farmer holds his private information on the parameters used in (2.7) and (2.8) before an insurance contract is signed. Another is so-called moral hazard which exits when a farmer's action is not verifiable, or when the farmer receives private information after the insurance contract has been signed.

3.1 Adverse Selection

With the same natural disaster, farmers may have different probabilities to incur loss. Suppose there are two types of farmers: ones who have high probability to incur loss, and ones who have low probability to incur loss. Let π^H and π^L be the probabilities for these two types of farmers respectively, $1 > \pi^H > \pi^L > 0$. An insurance company offers types of insurance contracts to the farmers: insurance premiums are $C^H(I^H)$ and $C^L(I^L)$ Yuan per hectare, insurance claims are I^H and I^L Yuan per hectare. Other parameters are assumed to be the same as those in baseline. The insurance company is still assumed to be risk neutral, and has zero expected profit in competitive insurance market. Then, the profits for the two types of farmers are

$$W^H_{I0} = W_0 - AC^H(I^H) = A\,(YP - Z) - AC^H(I^H),$$

$$W^H_{I1} = W_1 + \alpha AI^H - AC^H(I^H)$$

$$= A\,(1-\alpha)(YP(1+\rho) - Z) + A\alpha\,(Y(1-\beta)P(1+\rho) - Z(1+\gamma)) + \alpha AI^H - AC^H(I^H),$$

and

$$W^L_{I0} = W_0 - AC^L(I^L) = A\,(YP - Z) - AC^L(I^L),$$

$$W^L_{I1} = W_1 + \alpha AI^L - AC^L(I^L)$$

$$= A\,(1-\alpha)(YP(1+\rho) - Z) + A\alpha\,(Y(1-\beta)P(1+\rho) - Z(1+\gamma)) + \alpha AI^L - AC^L(I^L).$$

Their expected utilities after buying insurance are

$$E(u(W^H, C^H(I^H), I^H) = \pi^H\, u(W^H_{I1}) + (1-\pi^H)\, u(W^H_{I0}),$$

$$E(u(W^L, C^L(I^L), I^L) = \pi^L\, u(W^L_{I1}) + (1-\pi^L)\, u(W^L_{I0}).$$

The slopes in indifferent curve of the expected utilities are

$$\frac{dW^H_{I1}}{dW^H_{I0}} = -\frac{1-\pi^H}{\pi^H}\frac{u'(W^H_{I0})}{u'(W^H_{I1})},$$

$$\frac{dW^L_{I1}}{dW^L_{I0}} = -\frac{1-\pi^L}{\pi^L}\frac{u'(W^L_{I0})}{u'(W^L_{I1})}.$$

In iso-profit lines of the two types of farmers, we have

$$\frac{dW^H_{I1}}{dW^H_{I0}} = -\frac{1-\pi^H}{\pi^H},$$

$$\frac{dW^L_{I1}}{dW^L_{I0}} = -\frac{1-\pi^L}{\pi^L}.$$

Since $1 > \pi^H > \pi^L > 0$, we have

$$\frac{dW^L_{I1}}{dW^L_{I0}} < \frac{dW^H_{I1}}{dW^H_{I0}} < 0.$$

That is, the indifferent curve of high risk farmer’s expected utility is flatter than that of low risk farmer’s expected utility. By the property of indifferent curve, there will be only one point of intersection, if there is.

If π^H and π^L are known by farmers and the insurance company, the two types of farmers will be offered the optimal insurance contracts at point C^H and point C^L (see Figure 3.1), respectively.

These contracts attain pareto optimum because both insurance company and farmers attain optimum.

This will not be true when π^H and π^L are farmers' private information. The insurance cannot offer insurance contracts at point C^H and point C^L for type H farmers and type L farmers respectively because the company doesn't know π^H and π^L.

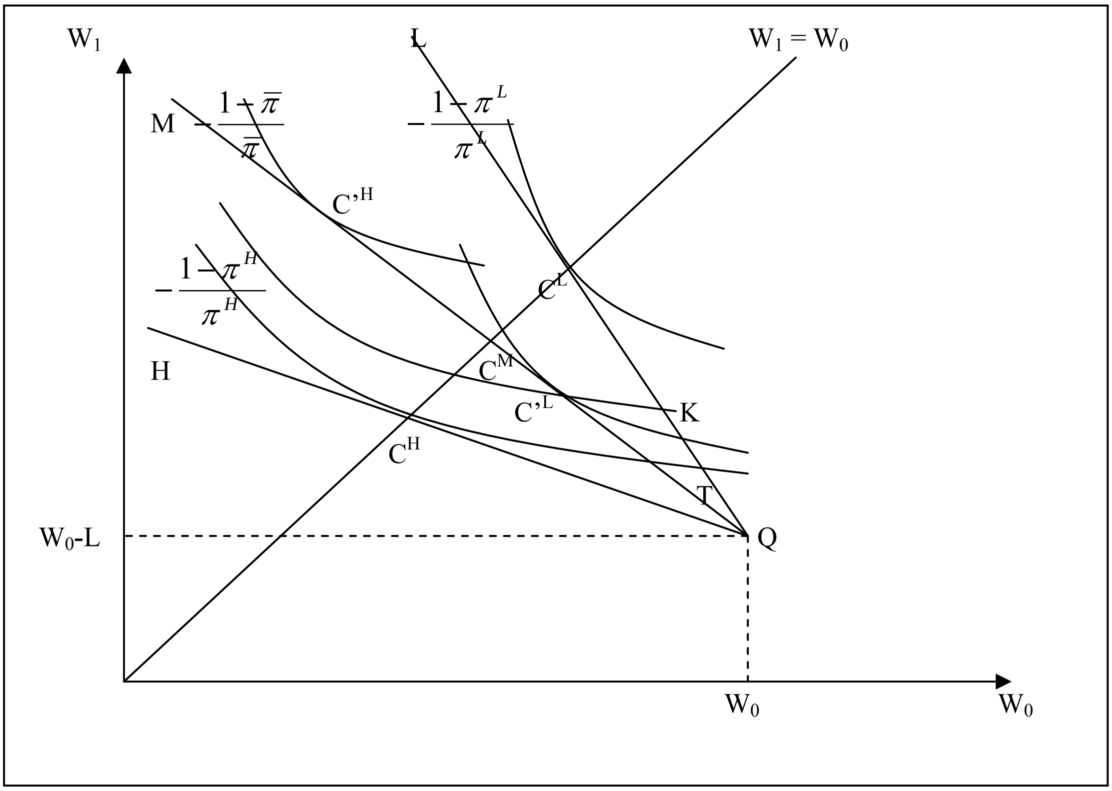

Figure 3.1 Insurance contract under asymmetric information

If the insurance company knows the ratio of high risk farmers to low risk farmers R:(1-R), the company can calculate an average probability $\bar{\pi}$:

$$\bar{\pi} = R\pi^H + (1-R)\pi^L.$$

Since $\pi^H > \bar{\pi} > \pi^L$, the insurance company's iso-profit line is in-between of the iso-profit line QH for high risk farmers and the iso-profit line QL for low risk farmers, i.e., the line QM shown in Figure 3.1. In this case, the insurance company offers insurance contract at point C^M (in farmer's iso-profit line, i.e., full insurance). This contract is welcomed by high risk farmers because they can obtain higher expected utility at point C^M than at point C^H, but will be frustrated by low risk farmers because they will obtain lower expected utility at point C^M than at point C^H. Therefore, high risk farmers will surge into the insurance market but some low risk farmers will quit, which will bring insurance company loss. This situation will be finally known by the insurance company. The insurance company has to increase insurance premium or decrease insurance claim. This change will force lower risk farmers quit. The final result of this procedure will be that only highest risk farmers buy insurance.

If the insurance company design different insurance contracts along his iso-profit line QM and let farmers to choose, high risk farmers will choose the contract at point C'^H while low risk farmers will choose the contract at point C'^L. In this case, high risk farmers will be over-insured and low risk farmers will be under-insured. If a farmer buys insurance at point C'^H, he informs the insurance company that he is a high risk farmer. However, high risk farmers prefer C'^L to C'^H because they can get more expected utility at C'^L than at C'^H. Hence, high risk farmers will pretend to be low risk farmers and buy insurance at point C'^L.

Therefore, the insurance company can offer only same contract to all farmers. This kind of contract is so-called pooling contract. However, point C'^L is not competitive equilibrium point. If another company offers insurance contract at point K, low risk farmers will prefer K to C'^L, high risk farmers prefer C'^L to K because they are farther from fully-insured at K than at C'^L though they have the same expected utility. As a result, this company attracts low risk farmers and leaves high risk farmers to other companies. At last, the contract for low risk farmers will located at point in the line QL, and the contract for high risk farmers will be located at point at which line QH is tangent with high risk farmers' indifferent curve of expected utility.

Therefore, to attain a competitive equilibrium point, an insurance company has to design two types of insurance contracts, and farmers will choose their type contract in their own interests. The problem that the insurance company has to solve is

$$\max_{I^H, I^L} \ \pi^L u(W^L_{I1}) + (1-\pi^L) u(W^L_{I0}) \quad (3.1.1)$$

$$\text{s.t.} \quad \pi^H u(W^H_{I1}) + (1-\pi^H) u(W^H_{I0}) \geq \pi^H u(W^L_{I1}) + (1-\pi^H) u(W^L_{I0}), \quad (3.1.2)$$

$$\pi^H u(W^H_{I1}) + (1-\pi^H) u(W^H_{I0}) \geq u(W_1), \quad (3.1.3)$$

$$\pi^H (AC^H(I^H) - \alpha A I^H) + (1-\pi^H) AC^H(I^H) = 0, \quad (3.1.4)$$

$$\pi^L (AC^L(I^L) - \alpha A I^L) + (1-\pi^L) AC^L(I^L) = 0. \quad (3.1.5)$$

(3.1.1) aims to maximize low risk farmer's expected utility. (3.1.2) aims not to induce high risk farer to pretend to be low risk farmer when the insurance company offers two types of contracts, ($C^H(I^H)$, I^H) and ($C^L(I^L)$, I^L), i.e., incentive compatibility constraint. (3.1.3) is participation constraint. (3.1.4) and (3.1.5) are the insurance company's zero profit line.

From (3.1.4) and (3.1.5), we have

$$C^H(I^H) = \pi^H \alpha I^H,$$

$$C^L(I^L) = \pi^L \alpha I^L.$$

Then the optimal problem becomes

$$\max_{I^H, I^L} \ \pi^L u(A(1-\alpha)(YP(1+\rho) - Z) + A\alpha(Y(1-\beta)P(1+\rho) - Z(1+\gamma)) + (1-\pi^L)\alpha A I^L) + (1-\pi^L) u(A(YP - Z) - \pi^L \alpha A I^L) \quad (3.1.1)'$$

$$\text{s.t.} \quad \pi^H u(A(1-\alpha)(YP(1+\rho) - Z) + A\alpha(Y(1-\beta)P(1+\rho) - Z(1+\gamma)) + (1-\pi^H)\alpha A I^H) + (1-\pi^H) u(A(YP - Z) - \pi^H \alpha A I^H)$$
$$\geq \pi^H u(A(1-\alpha)(YP(1+\rho) - Z) + A\alpha(Y(1-\beta)P(1+\rho) - Z(1+\gamma)) + (1-\pi^L)\alpha A I^L) + (1-\pi^H) u(A(YP - Z) - \pi^L \alpha A I^L), \quad (3.1.2)'$$

$$\pi^H u(A(1-\alpha)(YP(1+\rho) - Z) + A\alpha(Y(1-\beta)P(1+\rho) - Z(1+\gamma)) + (1-\pi^H)\alpha A I^H) + (1-\pi^H) u(A(YP - Z) - \pi^H \alpha A I^H)$$
$$\geq u(A(1-\alpha)(YP(1+\rho) - Z) + A\alpha(Y(1-\beta)P(1+\rho) - Z(1+\gamma))). \quad (3.1.3)'$$

Let λ, μ be the Lagrange multipliers of (3.1.2)', (3.1.3)', then FOCs are (assuming that $I^H > 0$, $I^L > 0$)

$$((\lambda + \mu)\pi^H(1-\pi^H)\, \alpha A(u'(W^H_{I1}) - u'(W^H_{I0})) = 0, \quad (3.1.6)$$

$$(\pi^L(1-\pi^L) - \lambda\pi^H(1-\pi^H))\alpha A(u'(W^L_{I1}) - u'(W^L_{I0})) = 0. \quad (3.1.7)$$

It is obvious that the equality of (3.1.2)' must be hold at a competitive insurance market. Hence

$\lambda > 0$, then $\lambda + \mu > 0$, and then by (3.1.6)

$$u'(W^H_{I1}) = u'(W^H_{I0}).$$

By the property of utility function, we have

$W^H_{I1} = W^H_{I0}$, and writing it as W^H.

Then in the insurance contract for high risk farmers, the insurance claim per hectare is

$$I^H = L/\alpha A = (-\rho/\alpha + \beta + \beta\rho)YP + Z\gamma. \quad (3.1.8)$$

The insurance premium per hectare is

$$C^H(I^H) = \pi^H \alpha I^H = \pi^H \alpha\, ((-\rho/\alpha + \beta + \beta\rho)YP + Z\gamma). \quad (3.1.9)$$

In (3.1.7), if $u'(W^L_{I1}) = u'(W^L_{I0})$, then by the property of utility function and (3.1.2), we must have $W^H_{I1} = W^H_{I0} = W^L_{I1} = W^L_{I0}$, then $C^H(I^H) = C^L(I^L)$, $I^H = I^L$, and then $\pi^H = \pi^L$, it is contradictive with initial assumption. That is,

$$u'(W^L_{I1}) \neq u'(W^L_{I0}).$$

We can solve I^L from $u(W^H) = \pi^H u(W^L_{I1}) + (1-\pi^H) u(W^L_{I0})$, and then to get $C^L(I^L)$.

Thus, we get a separating contract. In this contract, a high risk farmer ise offered the same contract as that in baseline, i.e., the contract at point C^H in Figure 3.1. A low risk farmer is offered a contract at point T in Figure 3.1, at which he get lower expected utility than at point C^L. This loss is caused by asymmetric information. This is actually the cost that the low risk farmer wants to distinguish himself from high risk farmers, and prevent high risk farmers from pretending to be low risk farmers.

3.2 Moral Hazard

In section 3.1, we assumed that farmers will have different probabilities to incur loss when facing the same natural disaster. Here, we assume that the probability to incur loss and the growth rate of agricultural products composite price, i.e., π and ρ, are the same for all farmers and public information. By the analysis in baseline given above, area share of affected out of sown, yield loss, the growth rate of cost in the prevention of natural disaster, i.e., α, β and γ, which are depended on farmer's endeavor to prevent natural disasters, will determine the values of insurance premium and claim. By (2.7) and (2.8), both C(I) and I are increasing functions of α, β and γ. From $C(I) = \pi\alpha I$ we have $I > C(I)$. Therefore, farmers will not do as less as possible to prevent natural disasters in his own interest.

Suppose that α, β, γ are α^G, β^G, γ^G in the case that a farmer take active activity to prevent a natural disaster, and α, β, γ are α^B, β^B, γ^B in the case that a farmer does not take active activity to prevent a natural disaster, insurance premium and claim per hectare are I^B and $C^B(I^B)$. $\alpha^G < \alpha^B$, $\beta^G < \beta^B$, $\gamma^G > \gamma^B$, $C^G(I^G) < C^B(I^B)$, $I^G > I^B$.

A farmer's profits from agricultural production under two states are

$$W^G_{I0} = W_0 - AC^G(I^G) = A\,(YP - Z) - AC^G(I^G),$$

$$W^G_{I1} = W^G_1 + \alpha^G A I^G - AC^G(I^G)$$
$$= A\,(1-\alpha^G)(YP(1+\rho) - Z) + A\alpha^G\,(Y(1-\beta^G)P(1+\rho) - Z(1+\gamma^G)) + \alpha^G A I^G - AC^G(I^G),$$

and

$$W^B_{I0} = W_0 - AC^B(I^B) = A\,(YP - Z) - AC^B(I^B),$$

$$W^B_{I1} = W^B_1 + \alpha^B A I^B - AC^B(I^B)$$
$$= A\,(1-\alpha^B)(YP(1+\rho) - Z) + A\alpha^B(Y(1-\beta^B)P(1+\rho) - Z(1+\gamma^B)) + \alpha^B A I^B - AC^B(I^B).$$

If an inactive farmer pretends an activity farmer, then his profit is

$W^B_{I0}' = W_0 - AC^G(I^G) = A(YP - Z) - AC^G(I^G)$,

$W^B_{I1}' = W^B_1 + \alpha^B AI^G - AC^G(I^G)$

$= A(1-\alpha^B)(YP(1+\rho) - Z) + A\alpha^B(Y(1-\beta^B)P(1+\rho) - Z(1+\gamma^B)) + \alpha^B AI^G - AC^G(I^G)$.

From an insurance company's iso-profit line, we have

$C^G(I^G) = \pi\alpha^G I^G$,

$C^B(I^B) = \pi\alpha^B I^B$.

The insurance company's problem is to solve

$$\max_{I^G, I^B} \ \pi\, u(W^G_{I1}) + (1-\pi)\, u(W^G_{I0}) \qquad (3.2.1)$$

$$\text{s.t.} \ \pi\, u(W^B_{I1}) + (1-\pi)\, u(W^B_{I0}) \geq \pi\, u(W^B_{I1}') + (1-\pi)\, u(W^B_{I0}'), \qquad (3.2.2)$$

$$\pi\, u(W^B_{I1}) + (1-\pi)\, u(W^B_{I0}) \geq u(W^B_1). \qquad (3.2.3)$$

Let λ, μ be the Lagrange multipliers of (3.2.2), (3.2.3), then FOCs are (assuming that $I^G > 0$, $I^B > 0$)

$$\pi(1-\pi)\alpha^G (u'(W^G_{I1}) - u'(W^G_{I0})) = \lambda\,(\pi(\alpha^B - \pi\alpha^G)\, u'(W^B_{I1}') - \pi(1-\pi)\alpha^G\, u'(W^B_{I0}')) \qquad (3.2.4)$$

$$(\lambda+\mu)\pi(1-\pi)\alpha^B (u'(W^B_{I1}) - u'(W^B_{I0})) = 0. \qquad (3.2.5)$$

With same reason as in section 3.1, $\lambda+\mu > 0$, hence

$u'(W^B_{I1}) - u'(W^B_{I0}) = 0$.

Then

$W^B_{I1} = W^B_{I0}$.

That is, the insurance contract for an inactive farmer is located at the point in his iso-profit line. When a natural disaster occurs, the inactive farmer's insurance claim per hectare is

$$I^B = L/\alpha^B A = (-\rho/\alpha^B + \beta^B + \beta^B\rho)YP + Z\gamma^B, \qquad (3.2.6)$$

The insurance premium is

$$C^B(I^B) = \pi\alpha I^B = \pi\alpha^B((-\rho/\alpha^B + \beta^B + \beta^B\rho)YP + Z\gamma^B). \qquad (3.2.7)$$

Since $\alpha^G < \alpha^B$,

$\pi(1-\pi)\alpha^G (u'(W^G_{I1}) - u'(W^G_{I0})) < \lambda\,\pi(1-\pi)\alpha^G (u'(W^B_{I1}') - u'(W^B_{I0}'))$,

i.e.,

$u'(W^G_{I1}) - u'(W^G_{I0}) < u'(W^B_{I1}') - u'(W^B_{I0}')$.

Since $C^G(I^G) < C^B(I^B)$, $I^G > I^B$,

$W^B_{I0}' = W^B_{I0} + A(C^B(I^B) - C^G(I^G))$,

$W^B_{I1}' = W^B_{I1} + \alpha^B A(I^G - I^B) + A(C^B(I^B) - C^G(I^G))$

$> W^B_{I0} + \alpha^B A(I^G - I^B) + A(C^B(I^B) - C^G(I^G))$

$> W^B_{I0}'$,

then

$u'(W^B_{I1}') - u'(W^B_{I0}') < 0$,

$u'(W^G_{I1}) - u'(W^G_{I0}) < 0$.

Hence

$W^G_{I1} > W^G_{I0}$.

That is, active farmers will be over-insured in order to encourage their active activities to prevent natural disasters.

From (3.2.4) we know that $\lambda \neq 0$, then (3.2.2) is binding. From

$\pi\, u(W^B_{I1}) + (1-\pi)\, u(W^B_{I0}) = \pi\, u(W^B_{I1}') + (1-\pi)\, u(W^B_{I0}')$

we can solve I^G, then to get $C^G(I^G)$.

Therefore, when there are moral hazard problems, the optimal insurance contracts should be that:

the contracts for inactive farmers are the same as those in baseline, while the contracts for active farmers are over-insured. Two types of farmers will choose the contracts designed for them in their own interests. The over-insured contracts for active farmers are the cost of the insurance company to distinguish farmers and to encourage farmers active activities to prevent natural disasters.