MONTE CARLO SIMULATIONS APPLIED TO AN ENDOGENOUS GROWTH MODEL WITH EQUALITY

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Abstract: Recent empirical papers (Barro, 2000; Banerjee and Dufflo, 2003; Forbes, 2000; Panizza, 2002 and Voitchovsky, 2005) have highlighted the negative relationship between the growth rate of per capita Gdp and the level of equality in developed economies. In this paper we design an endogenous model to explain how an increase in the level of income inequality may have a positive effect on economic growth. We include equality as an argument that increases the utility of the agents: a larger degree of equality in society may be optimal from the point of view of the utility of consumers, but it entails lower growth. Hence, in order to evaluate more in deph the behaviour of the model we conduct an extensive Monte Carlo experiment. This procedure simulates different settings to obtain the distribution of the growth rate and the parameter values. The growth rate predicted lies within plausible values and come up with results that are also stochastic in nature. Numerical implementation of the proposed approach is illustrated on a sample test system. Finally, we speculate on the economic implications of these results and discuss the misleading interpretation arising from the adoption of some political economy considerations against inequality.

Key words: equality, economic growth, Monte Carlo simulations.

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1. Introduction.

The presence of large degrees of inequality within a country or across countries has always been a cause of concern for researchers, policymakers, and ample sectors of the society. In addition, according to the U.N in its Human Development Report, the last decades have been affected by divergences in poverty and inequality across countries[1]. Dollar and Kraay (2002) examine a sample of developing countries that have grown rapidly in the last decades and conclude that growth has reduced poverty but has not altered the relative distribution of income –because growth has improved the standard of living of all the population-. The changes in poverty do not necessarily alter the degree of inequality in a particular country. Following these findings, it is not strange that an abundant literature on the issue of inequality has flourished in recent years. Part of these studies have been elaborated within the framework of the New Growth Theory, since it seems plausible to believe in a link between inequality and growth. In effect, investigators have been analyzing whether a faster growth rate entails a more even distribution of income within a country or across countries or, instead, it increases inequality because the gains from growth are received only by a small fraction of the population, either pertaining to a particular nation or to a group of countries of the world. The reverse link has also been explored, by means of formulating a basic question: is inequality harmful for economic growth or, instead, more unequal societies tend to grow faster?

Regarding the first issue, Kuznets (1955), in a classical paper, asserted that inequality first increases and later decreases during the process of economic development, that is, the explanation of the Kuznets curve. This hypothesis has been tested extensively[2]. Recent papers that find support for the Kuznets hypothesis are Eusufzai (1997) and Savvides and Stengos (2000). Barro (2000) also reports some findings that lead him to conclude that the Kuznets curve is a well-established empirical regularity.

[1] Inequality and poverty are not the same. This statement may seen trivial, but these terms are often confused especially by the non-specialists. For a discussion of differences between concepts see Sala-I-Martín (2002).

[2] For a revision see Bruno et al. (1998).

Nonetheless, a new paper by Sala-i-Martín (2005) posits that global income inequality has been reduced between 1980-1998. In the paper there is a careful distinction and observation between within country inequality and across country inequality. The author confirms that within country inequality in several countries (USA, UK, Australia and China) has increased. However, across countries differences in per capita income have narrowed remarkably, especially due to the dramatic improvement in living conditions in China and India, offsetting the within countries effect. Dollar and Kraay (2002) claim that growth has not clearly increased inequality for a sample of 80 countries over four decades, since the income of the poor rises one-for one with the overall growth.

As we can tentatively conclude examining the literature (further revision in section 2) the link from growth to inequality is, at least, controversial. Nevertheless, this brief comment on the link from growth to inequality seemed adequate in order to center the issue of this paper; the paper will focus in the second aspect, i.e. the impact that inequality may exert on economic growth.

This topic has been covered in a number of recent studies, but mostly in the context of political economic models. Some of them will be commented below. Here, we want to assess the issue form the point of view of standard growth models. Hence, this paper designs an endogenous growth model, which accounts for the relationship between economic growth and equality. The model extends Barro (1990) in one main feature: the inclusion of a variable of welfare, which is used as a proxy for equality. Government resources are allocated to infrastructure and to social services (welfare). As the share of Government spending that is allocated to welfare increases, a crowding out effect occurs due to fewer resources allocated to infrastructure, which decrease production. Welfare spending is needed in society because it is included in the utility function. This paper provides a possible explanation to a recently discovered empirical fact: in rich countries, there is a negative correlation between equality and economic growth (Barro, 2000; Forbes , 2000 and Bengoa and Sánchez-Robles 2005). Additionally, once the model has been developed we carry out a calibration exercise to know which parameter values better fit the evidence. We use Monte Carlo techniques to

predict the distribution of the variables of the model. Our model forecasts a growth rate that lies within the quantitative features founded in the empirical studies.

The paper is organized as follows: section 2 motivates the paper by presenting the links from inequality to grow. Section 3 designs an endogenous growth model that accounts for the negative impact that equality may exert on growth. In section 4 the model is tested to obtain its behaviour under different assumptions and the results are discussed. Section 5 offers some conclusions and suggests further research.

2. The links from inequality to growth.

As it was stated briefly in the introduction, the links between inequality and growth are rather complex. There is no consensus in the theoretical literature or in the empirical studies about how a country´s level of income inequality predicts its subsequent rate of economic growth. The sign of the correlation between both variables is also unclear. We can find analyses, at the theoretical and the empirical level, posing either a positive or a negative link from inequality to growth. The theories examining the relationship between income distribution and growth may be classified in two large groups[3]. The first category comprises the set of contributions that predict a positive link from inequality to growth; this hypothesis may be traced back to Keynes (1920) and was later on pursued by Kaldor (1956), Bourguignon (1981), Benabou (1996b) and Galor and Tsiddon (1997). On the other side, in turn, we may classify the theories that sustain a negative impact of inequality on growth following the market imperfection argument (Banerjee and Newman, 1993; Perotti, 1993; Aghion and Bolton, 1997) or the political instability and socio-political unrest (Alesina and Perotti, 1996; Benhabib and Rustichini, 1996; Aghion, Caroli and García-Peñalosa, 1999; Aghion, Banerjee and Piketty 1999).

In the context of *political economy models*, there are a large number of papers that explore the relationship between the variables analyzing the behavior of voters. More unequal societies[4], in which the mean income is higher than the median, will favor more redistribution and this process will entail slower growth. (Dolmas and

[3] For thorough surveys see Benabou 1996b and Aghion *et al*, 1999.

[4] There is an interesting exception to this claim: the so-called POUM hypothesis. If poor people anticipate large future rents, they may oppose redistribution. For a discussion and presentation of some evidence, see Benabou and Ok (1998).

Huffman, 1997 and Milanovic, 2000). In other words, pre-tax inequality will increase the demand for redistribution, and these pressures may lead the policymakers to design a complex interplay of taxes and transfers that, ultimately, could distort the economy and reduce growth (Barro, 2000)[5].

As it was said above, the empirical evidence on whether more equal countries tend to grow faster or slower is somehow controversial. The widespread belief in the 90s – backed up by some of the papers mentioned above, among others - was that inequality was harmful for growth. This idea, however, was challenged recently by some influential papers, Forbes (2000), Barro (2000) and Voitchovsky (2005).

Forbes (2000) featured the current belief that income inequality has a negative relationship with economic growth. She found, using panel estimation with an improved data set on income inequality, that an increase in the level of income inequality have positive effects on subsequent economic growth in the short and medium term. This relationship is robust to the use of alternative model specifications, across samples and variable definitions.

Barro (2000) analysed the different behavior between equality and growth depending on the stage of country development. To contrast that, he divided the group of countries in two sub-samples. For the low-income group the connection between inequality and growth is negative. However, in the high-income sub-sample the link turns out to be positive -for developed countries an increase in the level of inequality has a positive correlation with growth-.

A new empirical study developed by Voitchovsky (2005) investigates the importance of the shape of the income distribution as a determinant of economic growth in a panel of 25 countries. The study suggests that inequality at the top end of the distribution is positively associated with growth. In other words, at the top of distribution rich individuals represent the main source of savings in the economy and large investors might also be able to spread the risk of their investment and could

[5] There are also contributions that pose that the impact of inequality on growth can change over time as a country develops. Galor and Moav (2002) claim that, at early stages of development, inequality may be positively correlated with the rate of growth, since the drive engine of growth at that point is physical capital accumulation. In turn, this kind of investment will typically be accomplished by those fractions of the population that enjoy a higher propensity to save. Later on, though, growth may rest on human capital accumulation, which is instead favored by a larger degree of equality.

receive a higher rate of return. According with the author, these factors imply that higher inequality at the top end of the distribution may promote economic growth, as it boosts funds available and investment.

Finally, Bengoa and Sanchez-Robles (2005) contribute to the existing empirical evidence by presenting additional results. Two different samples were explored over the last three decades using Arellano and Bond technique applied to panel data. For the medium income nations, the relationship between equality and growth seems to be hump shaped. In the second sample, made up by high income nations the connection is unambiguously negative, more equality is detrimental for growth. Our empirical results suggest that the impact of equality on growth may be different at various stages of development.

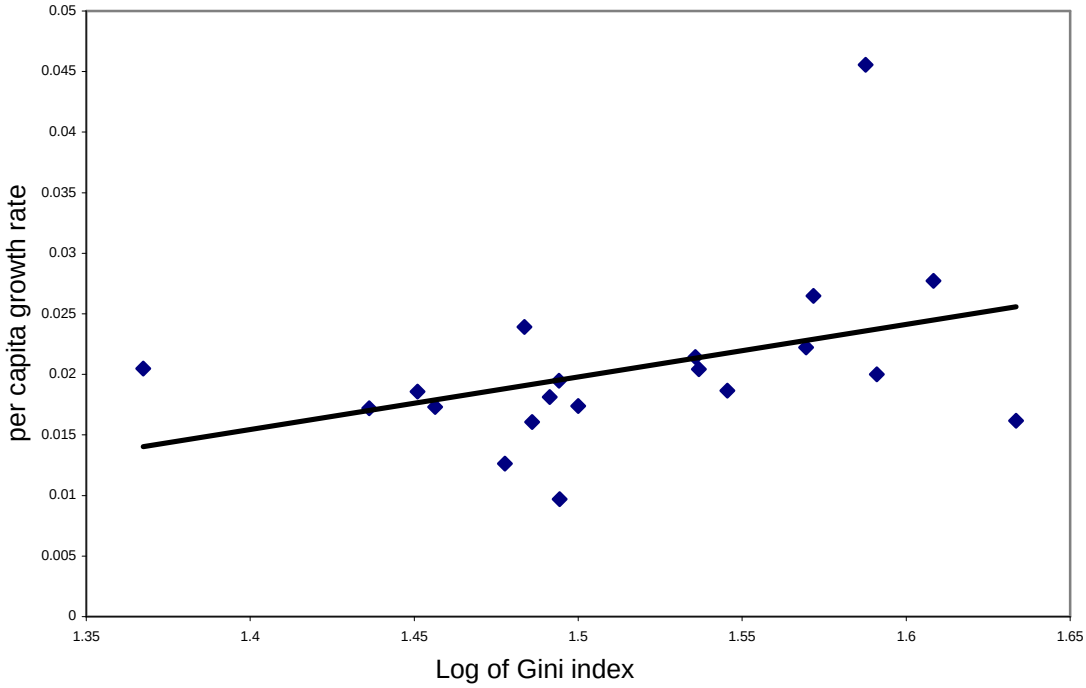
One possible explanation, for these results, is that for rich economies, more equality may damage growth since it desincentivates the undertaking of risky projects by individuals. In addition, the social payments provided by the State to reduce inequality decrease the amount of resources that are available for investment in productive capital, thus affecting growth negatively. Another possible implication is that political instability is more common in LDCs than in advanced countries, and it is also generally true that financial and insurance markets are fairly developed in rich countries, so the credit constrains problems will be more severe in the first group than in the second. In most LDC countries, therefore, it may be true that equality enhances growth.

However, the argument is not that clear for developed countries, since their inhabitants already enjoy reasonable levels of welfare and social stability. In these last instances, the government intervention intended to increase equality in the population may have perverse effects, because the distortion introduced by the taxes that finance social programs could damage efficiency. In other words, the crowding out effect exerted by the need to finance large expenses in social activities may jeopardize growth.

Following these arguments, this study provides a theoretical explanation to this last empirical finding (that could seem, at first sight, counterintuitive): i.e. more equality entails a lower rate of growth in countries that have achieved a certain level of development.

In order to offer a quantitative documentation that address for this fact, Figure 1 provide some information obtained when looking at a sample of 20 developed countries[6]. Figure 1 shows the connection between the (log) Gini index in the middle of seventies, and the growth rate of per capita GDP over the years 1978-2002. The connection between the Gini index at the beginning of the period and the growth rate is clearly positive. The graphic suggests a positive connection between higher levels of inequality (as captured by the Gini indexes) and the rate of growth of per capita GDP.

Figure 2.Inequality and growth,developed countries, 1978-2002



Source: Deininger and Squire and World Bank.

Table 1 reports the value of the index for 20 developed countries in the sample, together with some summary statistics. The mean value of the Gini index for the 20 countries considered is 33.16. As it was to be expected, this average is below the one reported for other samples made up of both developed and developing countries (Barro, 2000; Forbes, 2000). The country exhibiting the largest level of inequality is France, while the lowest value is attained in UK. USA is slightly above the average. Nordic countries (with the exception of Norway) have lower than average levels of inequality,

[6]Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, UK and USA.

according to Table 1. The reverse phenomenon is observed in the Mediterranean countries and Ireland.

Table 1. Gini indexes in the mid seventies, developed countries

Australia	34.33	Italy	39.00
Austria	31.20	Netherlands	28.60
Belgium	28.25	New Zealand	30.04
Canada	31.62	Norway	37.30
Denmark	31.00	Portugal	40.58
Finland	30.45	Spain	37.11
France	43.00	Sweden	27.31
Germany	30.62	Switzerland	31.22
Greece	35.11	UK	23.30
Ireland	38.69	USA	34.42

Mean	33.16
Standard deviation	4.98
Maximum	43.00
Minimum	23.3

Source: DS data basis and own elaboration

The empirical results provided by recent papers confirm the intuition obtained from the Figure. The Gini index displays a positive correlation with the rate of growth in these studies (Forbes, 2000; Barro 2000; and Bengoa and Sanchez-Robles, 2005) employing different econometric techniques. The magnitude of the coefficient for the sub-sample of high-income countries (around 0.04-0.05) is rather similar in all the econometric estimations. Of course, it is plausible that a new econometric analysis would be necessary in order to derive more conclusions. The debate continues in the empirical literature because it seems that the effect of income inequality on growth depend on the econometric method and the data considered. Although, according to some empirical evidence and, mostly for developed countries, inequality may be positively correlated with growth.

In this paper, we intend to provide some theoretical background to previous econometric results. This kind of link was detected in samples made up of developed countries; therefore, it would not be perhaps a sound strategy to base the model on the assumption that different levels of income within a country diverge in their saving rates. As Galor and Moav (2002) point out, these differences tend to disappear over time inasmuch as countries develop. The other alternative –to base the study on the crowding out effects of social expenditure– looks like a more promising avenue for constructing this type of models[7].

In this regard, one line that can be pursued is to consider equality as an argument in the utility function of consumers. This procedure has not been used much, to our knowledge[8], yet is appealing for several reasons. First, it provides a rational explanation for the existence of mechanisms that reduce inequality but also diminish the rate of growth. One could ask how it is possible that rational agents would choose this sort of policies. By entering equality in the utility function the answer is straightforward: for some nations it can be indeed optimal to grow less but have smaller income disparities because in this way they attain a larger level of utility and of total welfare. Thus, for example, the large Welfare States (in comparison to US) that many European countries keep become justified even from the point of view of economic rationality.

Second, this procedure has not been widely explored by the literature on fiscal policy and growth. This literature has dealt extensively with the issue of public expenditure, starting with the seminal contribution of Barro (1990). The modeling strategy adopted by most papers is to consider public expenditure (for example, infrastructure) as a productive input that enters the production function of the economy together with private capital and (possibly) labor. However, not all categories of public expenditure are productive *stricto sensu*. In other words, public expenditure does not affect the economy exclusively via the production function. Sometimes it increases the utility of consumers without having a clear effect on productivity. Examples are national parks, the maintenance and free access to beaches or the subsidized entrance to

[7] The lack of incentives to work brought about by too generous systems of Social Security – for instance, large unemployment benefits - is also an important point, which should not be dismissed as a potential explanation of the negative connection between equality and growth. We neglect this effect here, however, and leave it for future research, because the model presented in this paper does not include labor-leisure decisions for simplicity.

[8] One exception is Olszewski and Rosenthal (2004).

museums. And the fact that a particular type of public expenditure is not productive does not mean necessarily that it should not be provided.

These reasons seem compelling enough to try to address this issue with a model that captures equality differently from other models: i.e. entering the preference for equality into the utility function. This exercise will be described next.

3. The Endogenous Growth Model with Equality.

Next we shall present an endogenous model that intends to capture the kind of crowding out considerations alluded to before and posit ultimately, in a very simple way, a negative relationship between equality and growth. The basic intuition is that social security payments intended to provide equality -or other categories of non-productive public expenditure- have to be financed, and this diverts resources from other activities. The main conclusion of the model could match rather well the current *statu quo* in most western countries as far as this problem is concerned. They could as well provide theoretical support for the recent empirical results mentioned above that document a negative impact of equality on growth.

The model is based upon the seminal contribution of Barro (1990). The basic assumptions of the model will be described first, and next we shall proceed to obtain an expression for the rate of growth of the economy.

3.1. Assumptions.

3.1.1. Preferences.

Households maximize the present discounted value of future utility from now to infinity. The utility function is basically of the standard CRRA type, common in models of economic growth, but we introduce a new term, S. S may be considered as a proxy of equality, as brought about by public programs of welfare. Therefore it may include pensions and other kind of social benefits. The main rationale for taking the latter as a proxy for the former is as follows: if the resources devoted to these kinds of programs are large, then the degree of equality will increase in the society. Alternatively, S may also reflect other categories of public expending that do not have impact on production but increase the utility of agents (For example, the public maintenance of museums and its direct consequence, the low price charged to visitors).

The utility function is Cobb Douglas in the two arguments, consumption, C and a publicly provided public good, S, and concave in both. An analogous utility function has been used by Olszewski and Rosenthal (2004). Notice that people in this country have a clear preference for equality. In turn, this preference for equality can be attributed to the fact that more egalitarian societies enjoy lower sociopolitical unrest, higher levels of personal safety or just to political and ideological reasons. In this economy the agents do care about issues that are related to the provisions of public services and the amount of transfers they receive from the State, and ultimately to the level of equality in society.

Notice that State payments increase utility not because they allow the agent to consume more (in this case the model would imply that he is getting back what he is paying out of taxes to increase consumption, and S would be tantamount to C) but rather because they are a proxy of equality: the agent represented in the model is happier if a larger degree of equality (as captured by a higher value of S) is present in the society.

Another important caveat is in order here. It could be argued that a model that refers to equality should allow for heterogeneous agents, and make explicit the income distribution prevailing in the economy. Fortunately, there is an interesting result by Caselli and Ventura (2000) that is relevant for the analysis carried out here. Caselli and Ventura (2000) show that the methodological expression of the representative agent provides valid results if the utility function is homothetic, as it is the case in this model. This assumption does not rule the possibility of agents' heterogeneity; it means, rather, that the aggregation of all consumers *behaves as* the average consumer. Therefore the modeling device of a representative agent can be used in this framework, simplifying greatly the subsequent theoretical apparatus without affecting the main conclusions. On the other hand, the fact that preferences are homothetic, there are not market imperfections and taxes are proportional (as we will see below) ensures that the representative agent assumption can be used in this case[9] without altering the basic features of the model.

[9] We are indebted to Ronald Benabou for pointing it out.

The relative weight of both arguments in the utility function, as captured by parameter β need not be the same.

The rest of the parameters are the standard in these types of settings: ρ is the rate of time preference and σ represents the intertemporal elasticity of substitution among periods. There is no population growth in this economy for simplicity and we normalize initial population to 1. Hence the analysis in aggregate and in per capita terms is the same. We have already suppressed the argument t in order to alleviate notation.

$$U(0) = \int_0^{\infty} e^{-\rho\, t} u(C,S)\, dt \quad (1)$$

$$u(C,S) = \frac{\left(C^{\beta} S^{1-\beta}\right)^{1-\sigma} - 1}{1-\sigma} \quad (2)$$

3.1.2. Technology.

Output Y is composed of one sort of final good, which is sold in competitive markets. Two inputs enter in the production function of the economy: private capital K and public expenditure G. Some caveats are in order here. First, (and as in Barro, 1990) there is no labor in the production function for simplicity. This assumption is harmless, though, as long as we think of private capital as an aggregate of physical and human capital. G encompasses infrastructure, understood as all kinds of public expenditure that have a positive impact in the productivity of private capital. Second, G is considered here a flow rather than a stock. This point is also innocuous: if we assume that public investment is proportional to the public stock of capital, then the analysis will be similar in both cases. Infrastructure in this model is a publicly provided rival good, not subject to congestion for simplicity[10].

Therefore the production function can be written as equation (3).

[10] For models that deal explicitly with congestion, see Barro and Sala-i-Martin, 1992b.

$$Y = AK^{\alpha}G^{1-\alpha} \quad (3)$$

$$0<\alpha<1$$

Notice that the production function is also concave in both inputs. Nonetheless, since it exhibits constant returns to scale in K and G together, it will be able to display endogenous growth. In other words, it is an AK function, in which capital is understood as a composite of private and publicly provided goods. A is an index of efficiency in a broad sense: i.e. it can include not only technological progress *stricto sensu* but also the quality of institutions, the lack of distortions, the degree of financial efficiency, and other variables that are not captured already by the provision of infrastructure.

3.1.3. Dynamics of Private Capital.

Output net of taxes (to which we shall refer below) is devoted to saving and consumption. As it is usual in this type of setting, the law of motion of private capital K represents net investment as the difference between gross investment and break-even investment.

$$\dot{K} = (1-\tau)Y - C - \delta K \quad (4)$$

In which τ is the constant tax rate, included in the equation for reasons that will be apparent below, and δ is the rate of depreciation of private capital.

3.1.4 Public Sector Behavior.

The government finances public expenditure out of taxes. We are assuming a balanced budget, which is a reasonable assumption in the long run, following Barro (1990), and no capital inflows from abroad. Public expenditure is devoted to infrastructure and social security payments, in a proportion given by the parameter μ. More in particular, the government behavior can be described by the following set of equations:

$$G+S=\tau Y \quad (5)$$

$$G=\mu\tau Y$$

$$S=(1-\mu)\tau Y$$

It is reasonable to assume that the parameter μ will be linked to the preferences of the individuals, at least in democracies[11]. In particular, higher preferences for equality in the individuals (higher 1-β) will induce the policymakers to devote more resources to social expenditure, resulting in a higher (1-μ). In the more general case, $\mu = f(\beta)$ with $f' > 0$.

Now, perhaps, it is easy to grasp the intuition behind equations 4 and 5. Total output is devoted to consumption and saving. However, the agents are only able to channel to private investment the quantity of production that is left over after taxes are paid. This explains the term (1-τ) in equation 4. In turn, the revenues collected by the government are allocated to public expenditure in both categories, G and S.

3.2. Discussion of The Model.

3.2.1. The Competitive Solution.

As it shall be detailed below, there is an externality in this model, and therefore the market planner solution is not Pareto optimal, a social planner's outcome being superior. We shall compute the market solution. Since individuals own the firms in this economy and there is an only asset (capital), solving the model in a simplified way, (the so-called producing-families approach) is identical to build the general equilibrium setting. For simplicity we shall follow the first procedure, applying optimal control theory in the usual fashion in order to obtain the rate of growth of the relevant variables.

Individuals maximize (1) subject to the budget constrain given by (4). C and K

[11] It is beyond the scope of this paper to fully endogenize the parameter μ, and, although no doubt interesting, it is not crucial for the basic results of this model. For a contribution that treats this issue in detail see Olszewski and Rosenthal (2004).

are the control and state variables, respectively, for this problem. We can set up the present value Hamiltonian and compute the first order conditions. If we suppose, as usual, that the economy has at the starting point some amount of both sorts of capital, and if we add the usual transversality condition (TVC), the dynamics of the economy over time will be described by the following system of non linear differential equations with a boundary condition (eq. 6 below).

$$H = e^{-\rho t} \frac{\left(C^{\beta} S^{1-\beta} \right)^{1-\sigma} - 1}{1-\sigma} + \lambda \left[(1-\tau) Y - C - \delta K \right]$$

$$H_C = 0 \Rightarrow e^{-\rho t} \beta C^{\beta(1-\sigma)-1} S^{(1-\beta)(1-\sigma)} = \lambda \quad (6)$$

$$H_K = -\dot{\lambda} \Rightarrow \lambda \left[(1-\tau) MPK - \delta \right] = -\dot{\lambda}$$

$$TVC \quad lim_{t \to \infty} \lambda K = 0$$

Where MPK is the marginal productivity of private capital.

Standard procedures in the analysis of this kind of models – i.e. taking logs and derivatives and substituting the ratio G/K by its value as described by equation 5 – allow to come up with a closed form solution for the rate of growth of the economy.

$$\frac{\dot{C}}{C}^{market} = \frac{1}{1-\beta(1-\sigma)} \left[(1-\tau) A^{\frac{1}{\alpha}} \alpha (\mu\tau)^{\frac{1-\alpha}{\alpha}} - \delta - \rho \right] \quad (7)$$

This model is of the AK type, as it was said above, and hence all relevant variables in this economy grow at the same rate (for a proof see Barro and Sala-i-Martín, 1998), given by eq. 7. The interpretation is the usual in this kind of models: i.e. the economy will grow overtime whenever the (after tax) marginal productivity of

private capital, net of depreciation, exceeds the rate of time preference. The larger the willingness to smooth consumption (as captured by a larger σ), the smaller the rate of growth. Since the production function is homogeneous of degree one in K, G, the driving force of this economy is the interplay between K and G. Intuitively, investment in K entails higher G via the balanced budget assumption, (and also higher S) and therefore larger levels of Y. The lack of diminishing returns in K, G, considered together prevents growth from stopping, and delivers endogenous positive growth in the steady state (in fact, as it is well known, these models lack transitional dynamics and describe an economy that is always at the steady state). However, if the society requests a large amount of public revenues to be devoted to social security (in other words, if μ is smaller), holding τ constant, the rate of growth will be smaller. Therefore equation 7 posits a negative connection between the degree of equality in this economy (as captured by a large S due to a little μ) and the rate of growth. Notice that despite this negative connection between S and growth, agents will not choose a corner solution with no S and maximum growth because this solution would not be optimal from the point of view of utility. In other words, utility is maximized in the model (provided that the optimal control problem is solved properly) even though growth is not maximized.

The relationship between the size of the government, as measured by τ, and the rate of growth, is still quadratic, as in Barro (1990) and in Sala-i-Martin (1997).

4. The Monte Carlo Simulations.

We have pursued a calibration exercise in order to explore more in depth the behavior of the model. Traditional calibration procedures use only a limited – and typically discrete - number of parameter values. To test the model within a wider range of parameter values and check its robustness we have opted for a different strategy: we have assumed that the parameters follow (normal) probability functions rather than discrete values. Next, we have applied Monte Carlo techniques to simulate the model. This procedure, as far as we know, is widespread in other areas of economics, such as finance, but has not been commonly used to test these kinds of models.

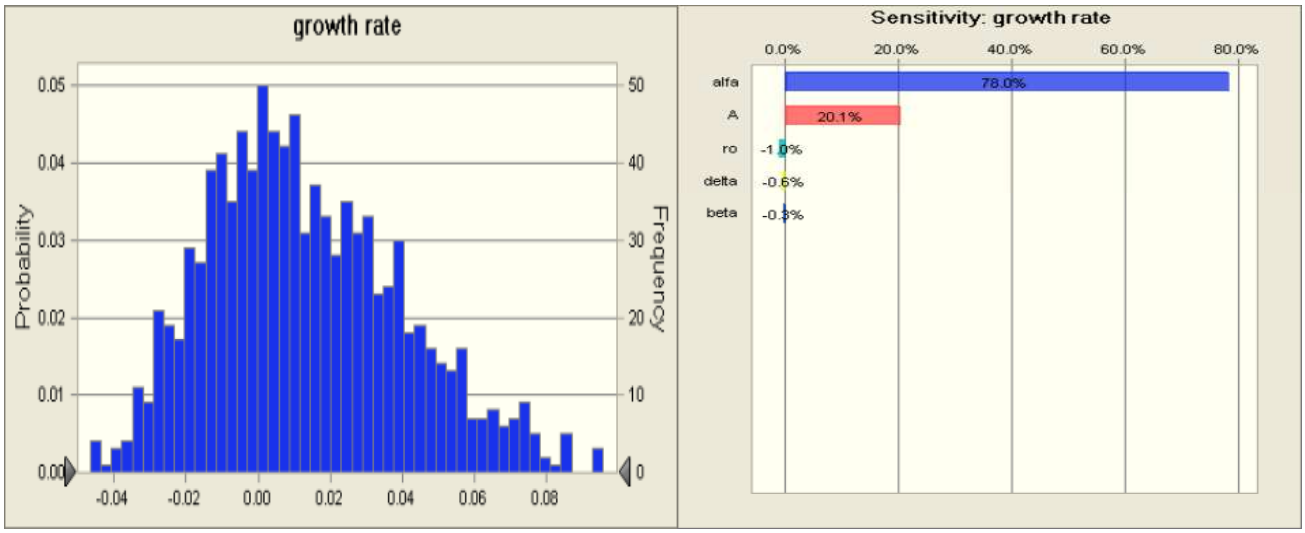
Table 2 summarizes the baseline case. For the different parameters, we have chosen values of the mean that are in line with those commonly used in the literature (A 0.7, α 0.7, β 0.7, δ 0.05, ρ 0.08) Figure 2 shows the distribution of the growth rate of the economy. Basic statistics are reported also in Table 2. The growth rate may vary between -0.05 and 0.15 the mean value is 0.01. The growth rate is negative for this scenario in the 20% of the cases. Figure 2, also displays a sensitivity chart. It shows the contribution of the various parameters to the variance of the growth rate. We can see that α is the parameter that impacts most in the variability of the growth rate, since it is responsible of 78% of the variance of growth. The impact of A is 20.1%, whereas the effect of ρ, δ and β is negative and of 1%, 0.6% and 0.3% of magnitude.

Table 2. Forecast and Assumptions. Sensitivity analysis.

VARIABLES	Growth Rate	A	ρ	δ	α	β
Mean	0.01	0.70	0.08	0.05	0.7	0.7
Standard Dev.	0.03	0.10	0.01	0.01	0.07	0.07
Selected range	-0.05, 0.15	0.4-1	0.06-0.1	0.03-0.07	0.4-0.9	0.4-0.9

Note: The parameters follow a normal distribution

Figure 2. Simulation Result 1 and Sensitivity Chart: Growth Rate.



Next, we have allowed for a larger value of A. When we assume a N(1,0.1), the growth rate varies between –0.04 and 0.23. Its average value is now 0.06, growth being positive from the 10th percentile. Sensitivity values change slightly, but orders of magnitude are basically the same as in the first case (see figure 3).

Figure 3. Simulation Result 2 and Sensitivity Chart: Growth Rate.

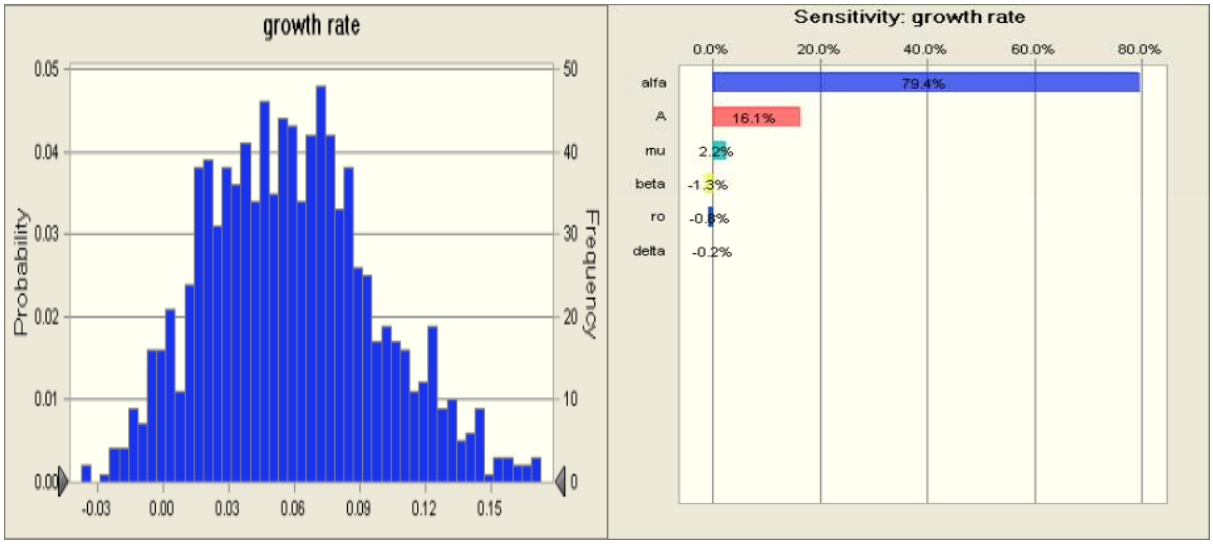
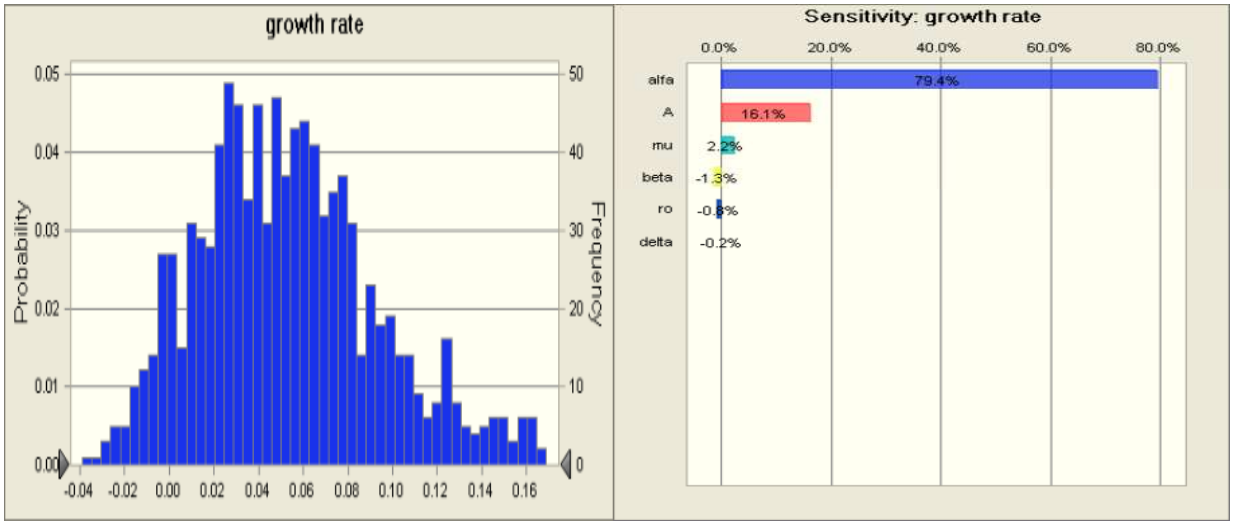


Figure 4 shows that if we allow μ to vary as well following a normal distribution with mean 0.7 and standard deviation 0.08, the average growth rate becomes 0.05. The impact of μ in its variance is 2.2%. The following step has been to compute the optimal tax rate that maximizes growth in this last scenario and the decision variable value is 0.32. This result fix the average tax rate applied in many developed countries.

Figure 4. Simulation Result 3 and Sensitivity Chart: Growth Rate.



Once we have the optimal value of the decision variable using the standard parameters, what it is interesting to know is how a change in the parameter β (that

accounts for the individual's preference of equality) affects the tax rate and the growth rate of the economy? If we apply an increase in the preference for equality (higher 1-β) reducing β from 0.7 to 0.5 -distributed as it is showed in Table 3- the optimal decision variable (τ) is 0.28. The simulation procedure predicts an average growth rate of 0.01. This result makes sense: if the preference for social goods increases in the economy (as suggested by a lower β) then the tax rate necessary to maximize growth experiments a slightly reduction. The consequences are a decreasing effect over productive public expenditure (G) in favor of equality (S) and a strong reduction of the per capita growth rate (see Table 3).

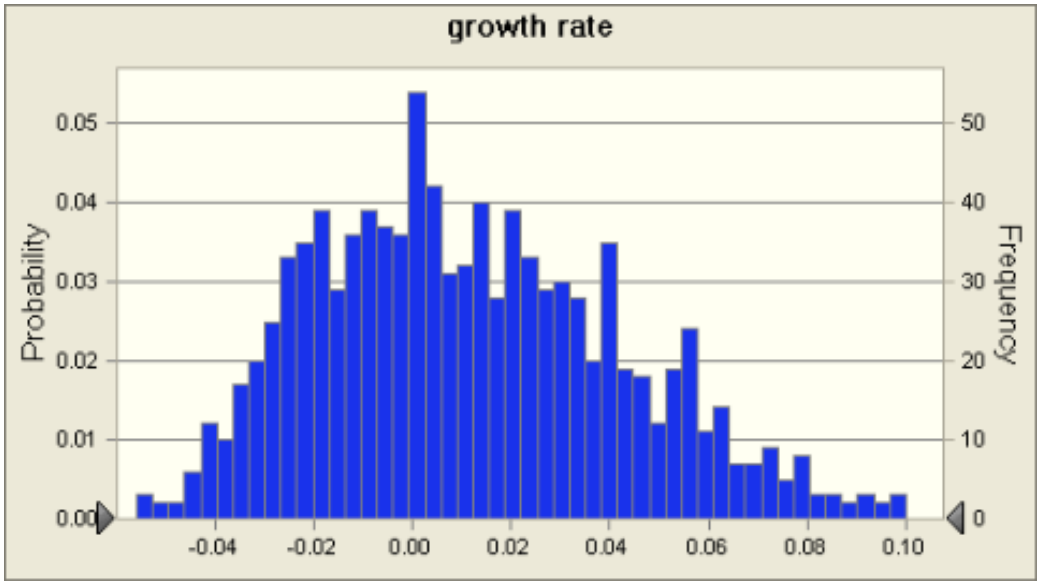
Table 3. Forecast, Decision Variable and Assumptions. Sensitivity analysis.

VARIABLES	Growth Rate	Decision Variable ι	A	ρ	δ	α	β
Mean	0.01	0.28	0.7	0.08	0.05	0.7	0.5
Standard Dev.	0.03	-	0.10	0.01	0.01	0.07	0.07
Selected range/variable bounds	-0.06, 0.15	0.2-0.40	0.6-1	0.06-0.1	0.03-0.07	0.4-0.9	0.2-0.7

Note: The parameters follow a normal distribution

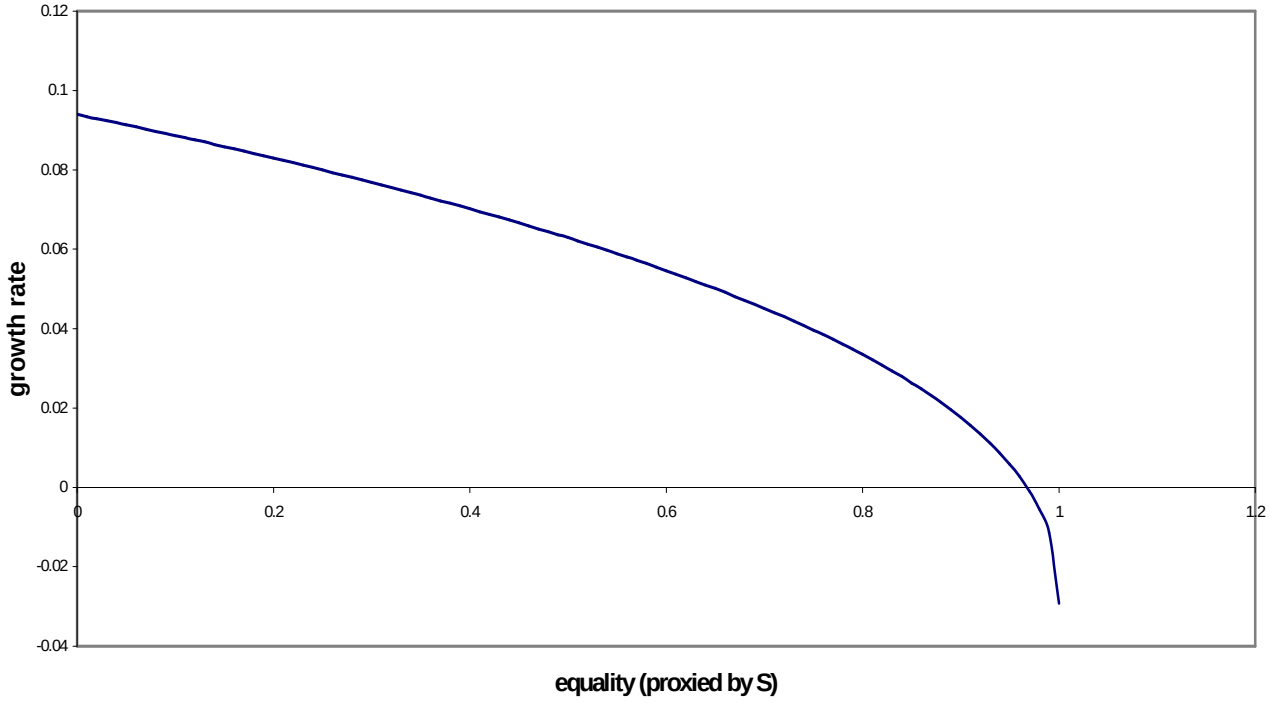
We can observe these results, clearly, in the Figure 5. When the individual's preference for social goods increases, the growth rate distribution moves to the left.

Figure 5. Simulation Result 5: Growth Rate.



Finally, Figure 6 shows the link between the degree of equality in the economy (proxied by S) and the rate of growth, for a constant τ of 0.3 (common in many European countries). Notice that the connection is monotonically decreasing. More resources devoted to promote equality exert a crowding out effect in all the domain of S, because this variable does not enter in the production function (but only in the utility function).

Figure 6. Equality and growth



5. Policy Implications And Concluding Remarks.

This paper has designed a simple model of public activity and growth that intends to provide a theoretical background for a puzzling empirical regularity observed in the data: i.e. the negative connection between equality and growth which is found for developed countries. The basic idea underlying the model is that social payments, intended to reduce inequality, reduce the amount of resources that the government is able to devote to investment in productive capital. In particular, the rate of growth depends positively on the fraction of public expenditure devoted to infrastructure (up to a certain point) and negatively on the amount of resources absorbed by social programs. The model can be understood in a broad sense, hence providing a simple explanation to for an empirical regularity documented in the literature: the negative link between

public consumption and growth. It also helps explain why it can be optimal for some country to choose a larger level of equality at the cost of lower growth.

The results of the simulations reported in this paper, with different parameters values, fix well with the reality observed in many developed countries. The model fitness establishes that, with an optimal tax rate around 0.3, a preference for equality with a higher share of government expenditure devotes to satisfy individual's equality preference, could exert a crowding-out effect. The result predicts a decreasing effect in the growth rate of GDP per capita.

Of course, this is a positive implication of the model. We are not posing the question of whether societies should devote more or less resources to social programs. However, governments and voters – especially in European countries – should be aware of this intertemporal trade-off between high levels of growth, high levels of per capita income and a large degree of equality in the population. This is particularly interesting concerning the recent discussion about heavy welfare systems in Europe, with particular relevance in a period of quite slow growth in these countries.

Finally, this study suggests the need for a further study of the differences between poor and rich nations that lead to inverse relationships between equality and economic growth in these two groups. Additionally, it is necessary to carry out more empirical work that investigates how these two variables and their determinants are interconnected.

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APPENDIX. The social planner's solution

The social planner maximizes the same utility functions that the individuals do. He can choose G and S optimally, and therefore he is subject to the resources constraint given by (4'). In addition, he explicitly takes into account the balanced budget assumption. In terms of the analysis of the model, this implies that he plugs the balanced budget condition into the law of motion of private capital. The control variables for his problem are C, G and S. The stock variable is again K. Taking the first order conditions for this new problem (we shall not rewrite the first order consumption for C again, since is the same as in the markets' solution) and proceeding along the same lines as before, we get expression 10, which entails a larger rate of growth since $\alpha<1$ by assumption.

$$\dot{K} = Y - C - G - S - \delta K \qquad (4')$$

$$H_G = 0 \Rightarrow AK^{\alpha}(1-\alpha)G^{-\alpha} = 1 \qquad (8)$$

$$H_S = 0 \Rightarrow e^{-\rho t} C^{\beta(1-\sigma)}(1-\beta)S^{(1-\beta)(1-\sigma)-1} = \lambda$$

$$\Rightarrow \frac{C}{S} = \frac{\beta}{1-\beta} \qquad (9)$$

$$H_K = -\lambda \Rightarrow \lambda\left[(1-\tau)A(\mu\tau A)^{\frac{1-\alpha}{\alpha}} - \delta\right] = -\lambda$$

$$\frac{\dot{C}}{C}^{planner} = \frac{1}{1-\beta(1-\sigma)}\left[(1-\tau)A^{\frac{1}{\alpha}}(\mu\tau)^{\frac{1-\alpha}{\alpha}} - \delta - \rho\right] \qquad (10)$$