The Optimal Monetary Policy Rule for the European Central Bank

Paolo Gelain[†]
Department of Economics – University of Pisa

January 2007

Abstract. In this paper we derive the optimal monetary policy rule for the European Central Bank (ECB), solving a minimization problem in which the Central Bank minimizes a (quadratic) loss function (whose arguments are inflation, output gap and the interest rate lag), subject to the constraint given by a model representing the economy which the Bank refers to. We used the algorithm to solve stochastic discounted optimal linear regulator problems. The interest rate is the policy instrument and the policy rule we derive shows the following main features: 1) the coefficient suggested as response of the interest rate to the current inflation is less than one, hence smaller then what indicated by the well known Taylor rule; 2) it is optimal for the ECB to allow for a high degree of policy gradualism or interest rate smoothing. Moreover, the use of our derived optimal rule guarantees much less variability of inflation and output gap responses to an interest rate shock.

† e-mail: paologelain@yahoo.it p.gelain@ec.unipi.it

CONTENTS

APPENDICES

INTRODUCTION

In this paper we want to derive the optimal monetary policy rule for the European Central Bank (ECB). Loosely speaking we want to derive a rule which can be used by the ECB to optimally set the interest rate, which we are going to assume the instrument used to implement monetary policy, in response to the variations of the main variables we consider relevant for the ECB, i.e. inflation and output gap (as a measure of the economic activity), in terms of minimization of a loss function which characterizes the preferences of the central bank about the variability of those variables. The economic environment will be the constraint of the ECB.

The main reason why we think that it is worth developing this paper is that we found just one paper which derives an optimal rule for the ECB (Peersman and Smets (1998)) like us. However, the sample period that has been used goes from 1975 to 1997, like in all the others works regarding the monetary policy rules for the euro area, in which however rules are estimated rather that optimally derived. This approach may suffer of the drawback that the answer to the question "does it make sense to do monetary policy analysis with respect to a Central Bank taking into account a period in which that Central Bank does not exist?" is "probably it does not". In fact, we will show that our results imply quite different policy suggestions for the ECB with respect to those of Peersman and Smets (1998). In particular, those differences crucially depends on the changed features of the euro area in the recenr years, especially in terms of persistence of inflation. Actually our sample is from 1995 to 2003, and apparently it is interested by the same kind of critique, but in what follows we will extensively explain why we think that it is not true.

Moreover, there is another good reason to develop our work. There are some papers (see for instance Breuss (2002), Galí et al. (2004), and Ullrich (2003)) which estimate rules trying to overcome the problem of the sample period, focusing on years when ECB "is at work", i.e. after 1999. However, they may suffer of the fact that data are not enough to estimate properly the model of the economy. Besides, we did not find anyone deriving the optimal rule.

Hence, taking into account these two first motivations, we see our sample as a good compromise between the need to have enough observations and a reasonable historical period considered.

Further, my work is different from the others in two respects. In fact, since before 1998 euro area did not exist, there are no proper data for it. The most part of datasets are constructed aggregating the variables of interest of the single European countries, often considering 3 or 5 representative countries. These are forms of approximation which may lead to lack of reliability. Hence we have decided to adopt the dataset created by Fagan et al. (2001) who propose the Area Wide Model in which 11 European countries are taken into account and aggregated following the “Index method” (see the paper for further details). What it is important to note is that our geographic euro aggregate better reflects the euro area composition.

In the end, since data before 1995 are not reliable, whatever aggregation one uses, my data set has the advantage of being more reliable than the ones used in other works.

The approach we are going to use to derive the optimal rule is not new in the literature. It is a simple application of the algorithm to solve stochastic discounted optimal linear regulator problems (see Ljungqvist and Sargent (2004)). Basically the problem consists in a minimization of a (quadratic) loss function, which is the objective function of the monetary authority (ECB in our case) subject to the constraint represented by the economy which the authority refers to.

Solving that problem, it is possible to derive the optimal policy rule, i.e. a rule which can be used by the ECB to optimally set the interest rate given the state of the economy and the relationships between the economic variables representing it.

We develop five sections. In the first one we briefly describe the role and the objectives of the ECB. The second section is devoted to present and estimate the model of the European economy, which will be the constraint of the maximization problem. In the third section, after giving the motivation to consider the loss function we considered, we set it and the state space representation in order to derive the optimal rule; then we derive it. The fourth section concerns the analysis of the optimal rule, its dynamic properties, its macroeconomic performance and its comparison with the Taylor rule. In the fifth, we briefly compare the optimal behaviour of ECB with what it has really done. Some concluding remarks close this work.

1. THE ECB: OBJECTIVES, RULES AND MONETARY POLICY

Article 105 of the Maastricht Treaty of 1992 clearly establishes which is the aim of the European Central Bank. It states that "*the primary objective of the ESCB*[1] *is to maintain price stability. Without prejudice to this objective, it shall support the general economic policies in the Community with a view to contributing to the achievement of the objectives of the Community* [which include a high level of employment and sustainable and non-inflationary growth]. *Furthermore, the ESCB shall act in line with the principle of an open market economy with free competition*".

Although clear enough, the article is quite vague for what concerns the definition of price stability. In fact, in October 1998 the Governing Council of the ECB defined price stability as "*a year-on-year increase in the Harmonised Index of Consumer Prices (HICP) for the euro area of below 2%*" and added that price stability "*was to be maintained over the medium term*". The Governing Council confirmed this definition in May 2003 following a thorough evaluation of the ECB's monetary policy strategy. On that occasion, the Governing Council clarified that "*in the pursuit of price stability, it aims to maintain inflation rates below but close to 2% over the medium term*".

What it is important to underline at this point for the purposes of our analysis is that the ECB has been charged with the responsibility of the maintenance of price stability, but at the same time it has to care about other objectives, like economic growth.

Having defined a target for inflation, may imply that ECB works in a regime of inflation targeting. As a matter of fact definition of inflation targeting is not unique since many give different and sometimes conflicting definitions, but there are some criteria which one has to look for to define inflation targeting. In particular, quoting Mishkin (2001) "*[i]nflation targeting is a recent monetary policy strategy that encompasses five main elements: 1) the public announcement of medium-term numerical targets for inflation; 2) an institutional commitment to price stability as the primary goal of monetary policy, to which other goals are subordinated; 3) an information inclusive strategy in which many variables [among which inflation forecast may have an important role] 4) increased transparency of the monetary policy strategy through communication with the public and the markets about the plans, objectives, and decisions of the monetary authorities; and 5) increased accountability of the central bank for attaining its inflation objectives*".

[1] European System of Central Banks.

In this work we don't want to investigate whether or not the ECB is an inflation targeter. In particular because "*while there are many similarities between the ECB's strategy and strategies of other central banks using inflation targeting, the ECB decided not to pursue a direct inflation targeting strategy in the sense discussed above for a number of reasons*"[2], basically related to the problems associated with inflation forecast.

For what concerns the policy rules, we want to highlight, although not properly correct, that to an inflation target regime is associated a policy rule which the central bank follows to achieve its targets. As it is known, the policy process is too complex to be represented by a simple monetary rule, but apart the general consensus of central bankers on the fact that policy rules can be a good guidance for monetary policy, also the EBC does not seem to totally reject the usefulness of monetary policy rules. In fact, in its Monthly Bulletin of October 2001 (ECB 2001, p. 38), the ECB states: "*The emphasis … on rule-guided monetary policy … is generally welcome [because] it provides a salutary antidote to the perennial risks of a discretionary, ad hoc approach to monetary policy*". This statement by itself provides enough motivation for the search of the monetary policy rule of the ECB.

[2] European Central Bank (2004).

2. THE MODEL OF THE EUROPEAN ECONOMY

2.1. Motivations

There is a number of motivations to estimate a simple model like the one we propose later on. Most of them are underlined in Rudebucsh and Svensson (1998), hence in reporting them we will draw heavily from that paper. Moreover, we will add a couple of reasons to strengthen our argument.

First of all, our model is composed of just two equations, an IS curve and a Phillips curve which represent the demand and the supply side of the economy respectively. This allows for more transparent results, but it may create problems for what concerns the richness of the dynamic incorporated in it. However we will see that from a comparison with richer models (i.e. VAR) our model performs in a satisfactory way. This kind of comparison may also be good in terms of fit to data, since VAR is a tool that allows data to speak as much as possible.

Second, simple models capture the spirit of many policy oriented macroeconometric models. If this is true for many central bankers, as it can be seen in the 11 models described in the central bank comparison project for the Bank for International Settlements[3], it is also quite true for the ECB which has not a formal model[4] used for the euro area analysis, although some economists of the Institute provided some of them, and as we will see later on half of them are based on simple structural equations.

In the end we can add the following reasons. First we argue that simple models are in line with the recent macroeconomic theory regarding the New Keynesian general equilibrium models[5]. In fact, despite their derivation, which is something mathematically very articulated and refined, they end up with two equations, i.e. the forward looking version of both the IS and Phillips Curve.

Second, a very simple argument may be that the main aim of this work is not to estimate a model of the euro area, but rather to derive the optimal policy rule for the ECB.

[3] See Bank for International Settlements (1995).
[4] This means that it has not a model that everyone knows it is the ECB model, like for instance the FRB/US model (substituted by the MPS model in 1996) which is the Federal Reserve Board's main quarterly macroeconometric model.
[5] The two masterpieces on the subject are Clarida, Gali and Gerlter (1999) and Goodfriend and King (1997).

2.2. Theoretical issues

The model I want to estimate is a small macroeconometric model. The following theoretical formulation reflects the next empirical results, and this is why some lags are missing:

$$\pi_{t+1} = \alpha_1 \pi_t + \alpha_3 \pi_{t-2} + \alpha_4 \pi_{t-3} + \gamma_1 y_t + \varepsilon_{t+1} \qquad (1)$$

$$y_{t+1} = \beta_1 y_t + \beta_3 y_{t-2} + \delta_1 (i_t - \pi_t) + \delta_3 (i_{t-2} - \pi_{t-2}) + \delta_4 (i_{t-3} - \pi_{t-3}) + \eta_{t+1} \qquad (2)$$

where π_t is the annual rate of growth of the consumption expenditure deflator[6] (*infpcd* in figure 2) expressed in percentage points. The choice of this measure of inflation rate derives from the fact that as we told before, the measure to which the ECB refers to maintain price stability is the Harmonised Index of Consumer Prices (HICP), hence the rate of growth of that index (*infhicp* in figure 2) should be the variable to consider in the estimation. However, we were not able to find a seasonally adjusted index, and then we decided to use the other measure for the following reasons: first of all, as it is possible to see from figure 2, the pattern of *infpcd* is very similar to that one of *infhicp*, but with less variability. The confirmation that the former is a good approximation of the latter is given by the high positive correlation between the two (about 0.81, see table 1). Moreover, that correlation is also higher than the correlation of *infyed* (the rate of growth of GDP deflator) with *infhicp* (about 0.62), suggesting that *infpcd* is better, even though *infyed* is usually used by others authors. Then, y_t is the deviation of the quarterly real GDP from the potential output[7] in percentage points, i.e. $100[(y_t - y^*)/y^*]$. For what concerns i_t, it is the quarterly short term (three months) nominal interest rate at annual rate expressed in percentage points. Hence, $(i_t - \pi_t)$ is a sort of ex-post real interest rate[8], which we will indicate with r_t. In the end, ε_t and η_t are random disturbances which are supposed to be white noise processes, i.e. i.i.d. with zero mean and constant variance σ^2_ε and σ^2_η.

---

[6] In Rudebusch and Svensson 1998, like in others, the annual rate of growth is obtained, using quarterly data, by $400(\ln p_t - \ln p_{t-1})$. Our inflation is simply obtained by $100[(p_t - p_{t-4})/p_{t-4}]$.

[7] In Fagan et al. (2001) the output gap is defined as the ratio of actual output to potential output, which is based on an aggregate Cobb-Douglas production function with constant returns to scale and Hicks-neutral technical progress. For this, trend total factor productivity has been estimated within-sample by applying the Hodrick-Prescott filter to the Solow residual derived from the production function.

[8] In some previous articles, the real interest rate is approximated using the difference between the four-quarter average nominal rate (i.e. $\Sigma^3_{j=0} i_{t-j}$) and the four-quarter inflation computed in the same way. We tried the estimation with this kind of approximation and the results were pretty much the same.

All variables are demeaned before estimation, hence no constants appear in the equations.

The first equation is a Phillips Curve. It can be viewed as a supply curve since it relates the inflation rate with its own lags and the lags of the output gap. What it is important to note is that the expectations are assumed to be adaptive. This formulation is compatible with the idea of a constant equilibrium value of the real output, which we assume here on computing the potential output, if the Phillips Curve is vertical in the long run. This is assured if the sum of the parameters of the lagged values of the inflation in equation 1 is equal to one. Later on we will see that this hypothesis is not rejected by our data.

Moreover, there are some arguments that it is useful to report to justify the use of adaptive expectations, rather than other forms (e.g. rational ones). In the first instance, some central bankers (like for instance Alan Blinder, Federal Reserve Governor) appear more comfortable with backward looking version, as demonstrated by many models used by them. Second, Taylor (1993) and Bofim and Rudebusch (1997) sustain that in a period of transition to a new monetary policy, the adaptive expectations may be more realistic than the rational ones, since people are try to learn the new central bank behaviour. The same kind of argument can be extended to the ECB, since it is a relative young institution and the agents may be still trying to learn about its policy. As a matter of fact, the objective of the ECB is clear enough to allow agents to form expectations about its policy. However, sometimes it is not clear if the ECB monetary policy is either discretionary or linked to some form of commitment[9]. In the end, for the US economy it has been found that backward looking models require relatively more aggressive policies with at most moderate inertia; rules that are optimised for such models tend to perform reasonably well in forward looking models, while the reverse is not necessary true[10]; Adalid et al. (2005) found same results for the euro area.

What we also report is the fact that although a large literature is in favour of the New Keynesian Phillips curve[11], which is the version which explicitly takes into account for rational expectations, there are many papers in which autoregressive Phillips curves are tested against forward looking versions and they cannot reject the hypothesis that the former holds[12].

Equation (2) is a IS Curve, which represents the demand side of the economy. It relates the output gap with its own lags and with the lags of the real interest rate. The latter synthesizes the transmission mechanism of the monetary policy to the real economic activity

[9] See for instance Monacelli (2005).

[10] See Bryant et al. (1993), Taylor (1999), Levin et al. (1999).

[11] See for instance Gali and Gerltler (1999) and Gali (2001) for what concerns the estimation of the New Keynesian Phillips Curve.

[12] Among others, Fanelli (2005) and the references therein. They are similar but the latter is an extension of the former since it uses the same database but with more years, part of which is exactly the years of our sample.

and to the price level. As a matter of fact the transmission mechanism is more complicated in the euro area, as can bee seen in figure 3, involving also possible effects of changes in money markets rates on financial asset prices, such as share prices or exchange rate which in turn act on the saving and investment decisions of households and firms. Moreover wealth and income effects can arise from changes in asset prices. Expectations on future inflation rate are also an important channel. However the same figure shows that there is a clear and direct link between the official interest rate and the markets rates. This allows us to consider the interest rate as the instrument used by the ECB to conduct monetary policy. To further confirm that short term interest rate is a good proxy for the ECB policy instruments, we can look at figure 4 to verify that the ECB can really control the market interest rate. As a matter of fact, the evolution of the latter is closely related to the main refinancing operation rate, which is the official rate used for the implementation of the monetary policy, i.e. it is the rate which the ECB control directly.

Our estimation refers to the period 1995Q1 to 2003Q4. The choice of the period is motivated by different reasons. On the first place, data from Fagan et al. were available only up to 2003. Moreover, the reliability of the data is assured just from 1995.

On the other hand, all the works done on the derivation of an optimal rule for the ECB up to now are based on a sample period starting in the 1970s and ending in the 1990s. Basing on such a long period, estimates can be more accurate, but one may wonder whether it makes sense to do monetary policy analysis for a period during which the Central Bank did not exist. My sample period is interested by the same kind of critique since ECB was born in 1999. However, some characteristics of the economic variables during the period 1995 1998 suggest that European countries were already following a sort of unique monetary policy which features were very close to the current ones, at least as long as the objectives are concerned. The Maastricht Treaty of 1992 accelerated the process of monetary convergence imposing criteria on the level of interest an inflation rate. This is confirmed by figure 5 where are depicted the monetary market interest rates of Germany, France and Italy. The high positive correlation (see table 2) between those rates can also suggest that monetary policies were similar in those countries. In the end, the high variances of the output gaps and the low variances of the inflation rates (see table 3) may imply equal monetary policy objectives, i.e. that policies were more inflation stabilization oriented.

In the end, no important structural changes has occurred in the euro area during our sample period, except for the creation of the euro area, but this doesn’t seem to have created change in the European economy[13], since the process of monetary integration had starting to taking place several years before.

2.3. Empirical results

To obtain our results we mainly use a general to simple approach based on the information criteria comparison. Moreover, we put attention on the individual statistical and economical significance of the coefficients, on the goodness of fit (R^2 and adjusted R^2), on the correlation between actual and fitted values and on the appropriateness of the tests statistics regarding autocorrelation (LM test) and normality of OLS residuals and presence of heteroskedsticty. A Reset test has also been implemented to test eventual misspecifications (even though it can have low power).

Our estimation results are (standard errors in parenthesis):

$$\pi_{t+1} = \underset{(0.1462)}{1.034}\pi_t - \underset{(0.2239)}{0.595}\pi_{t-2} + \underset{(0.1969)}{0.391}\pi_{t-3} + \underset{(0.05868)}{0.091}y_t + \varepsilon_{t+1} \tag{3}$$

obs.=36, R^2=0.791288, Adjusted R^2=0.768926, σ_ε=0.185, AR (1-3) test: F(3,25)=0.44800 [0.7209], Normality test: $\chi^2(2)$=1.0371 [0.5954], heteroskedasticity test: F(8,19)=1.1938 [0.3533], RESET test: F(1,27)=0.79620 [0.3801], correlation between actual and fitted values: 0.88957.

$$y_{t+1} = \underset{(0.1109)}{1.234}y_t - \underset{(0.1085)}{0.38}y_{t-2} - \underset{(0.1129)}{0.274}(i_t - \pi_t) + \underset{(0.2120)}{0.503}(i_{t-2} - \pi_{t-2}) - \underset{(0.1596)}{0.315}(i_{t-3} - \pi_{t-3}) + \eta_{t+1} \tag{4}$$

obs.=36, R^2=0.909528, Adjusted R^2=0.896125, σ_η=0.2674, AR (1-3) test: F(3,24)=1.0820 [0.3755], Normality test: $\chi^2(2)$=1.9964 [0.3685], heteroskedasticity test: F(10,16)=0.48865 [0.8737], RESET test: F(1,26)=0.81920 [0.3737], correlation between actual and fitted values: 0.95358.

Rejection of null hypothesis is marked with * and ** for 5 and 1 per cent significance level respectively.

Estimation has been done separately using OLS, but similar results could be obtained estimating a system of equations, since the cross-correlations of the residuals are essentially zero.

Both the estimations display a good fit to data since R^2s are very high and close to 1. Also the correlation with fitted data is good. All tests suggest absence of autocorrelation, of non

[13] See for instance Clausen and Hayo (2002).

normality in the residuals and of heteroskedasticity. This allows us to rely upon inference drawn from the standard errors. In particular, all coefficients are highly statistically significant, except for the first lag of output gap in equation (3) which however is significant at 10 per cent significance level. For a comparison of our results with those of others authors in terms of significance of the coefficients see table 4.

As for the restriction on the lagged values of inflation in equation (3), the χ^2 statistics is $\chi^2(1) = 3.17189$ [0.0749], hence we cannot reject the null hypothesis of a vertical Phillips Curve.

Two main features emerge from these results. First, inflation displays a quite high persistence. In fact, the sum of all the coefficients of the lags is equal to 0.83. This is in line with the recent findings of the Inflation Persistence Network, whose analysis focus on measuring and comparing patterns of price setting and inflation persistence in the euro area[14], and in this respect inflation is more persistent that what found by Peersman and Smets (1998). In fact, they obtain a sum of the autoregressive coefficients equal to 0.74 (see table 4). Nevertheless, we have to underline at this stage that here the meaning of persistence has to be interpreted as the time needed to a variable to come back to its equilibrium value after a shock, and in this other respect, inflation is less persistent, because in my case it comes back much earlier than in Peersamn and Smets (see fig.6b and the next paragraph for further details).

On the other hand, there is a low role of the output gap in explaining inflation. In fact the coefficient is low and not highly significant.

These two characteristics will be fundamental in order to obtain my results in terms of optimal rule.

2.4. Comparison with other models

The estimation of simple models characterized by the presence of only few variables may have some drawbacks. In particular it may be subject of the following critiques:

a) it can be a poor representation of the economic system which it refers to, failing to capture its salient characteristics;

b) it may lack to account for the right dynamic relationship between variables.

[14] See Angeloni et al. (2005).

In order to avoid the first drawback it may be useful to compare our model with other estimated structural models regarding the same economic environment, while the second objection may be faced by considering an unrestricted VAR.

As already said, the ECB has not a formal model used for economic analysis. As summarized in Adalid et al. (2005), structural models for the euro area are essentially four:

1) the Coenen-Wieland model (see Coenen and Wieland, 2000) is a small-scale model of aggregate supply and aggregate demand which is designed to capture the broad characteristics of inflation and output dynamics in the euro area.

2) The Smets-Wouters model (see Smets andWouters, 2003) is an extended version of the standard New-Keynesian DSGE closed-economy model with sticky prices and wages. The model is estimated by Bayesian techniques using seven euro area macroeconomic time series: real GDP, consumption, investment, employment, real wages, inflation and the nominal short-term interest rate.

3) The Area-Wide Model (see Fagan et al., 2001) is a medium-size structural macroeconomic model that treats the euro area as a single economy. It has a long-run neo-classical equilibrium with a vertical Phillips curve but with some short-run frictions in price and wage setting and factor demands.

4) The Dis-aggregate Model of the euro area used in Angelini et al. (2002) and Monteforte and Siviero (2002) is a multi-country version of the simple backward-looking two equations model in Rudebusch and Svensson (1999). It consists of an aggregate supply equation and an aggregate demand equation for each of the three largest economies in the euro area; i.e., Germany, France and Italy.

There are two types of comparisons we can implement. With backward-looking models (i.e. the first one and the fourth one) we can compare both the coefficients and the impulse response functions (especially for what concerns the response to monetary policy shocks), while with forward-looking model just in terms of impulse response functions.

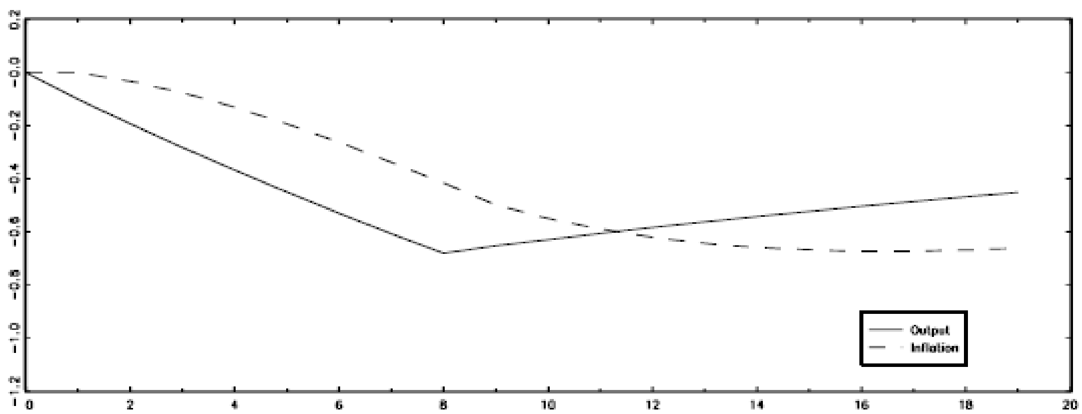
The coefficients of the comparable model are reported in table 4. Other model estimates of the euro area are reported together with one of the United States. The sample period of those estimations is larger and it precedes ours. This last fact may justify the differences in the IS curve estimation, even though Coenen and Weiland (2000) obtain similar results and the model for United States as well. The dynamics of the interest rate is also quite different (richer) from all others.

As for the Phillips Curve, differences are clear. Our equation shows that inflation displays much less inertia then the previous ones, i.e. shocks that hit inflation are less persistent.

Again, this is in line with the recent findings of the Inflation Persistence Network. They found that "…an unexpected monetary policy change has a short-lived effect on euro area real output, which peaks around 4-6 quarters after the shock and then dissipates relatively quickly. By contrast, the aggregate price level in the economy is affected more gradually but permanently… An interpretation of this evidence is that the inflation process in the euro area is subject to higher degree of rigidity, possibly due to a less competitive environment, more extensive price regulation or other formal or informal constraints on price setters". In fact looking at the impulse response functions[15] in figure 6b, we can easily see that all the responses have the expected sign (in particular those of output and inflation to interest rate). The response of output gap reaches a pick after three quarters and it comes back to the equilibrium after eleven quarters; whereas inflation reverts its negative path after 6-7 quarters, to reach the equilibrium after only about 20 quarters. Similar responses are obtained by richer structural models, e.g. either Smets and Wouters (2003) or Fagan et al. (2001), at least from the point of view of the signs of the responses.

A comparison with Peersman and Smets (1998) is due. It is clear from the following graphic drawn from that paper that in terms of time needed to inflation (and to output) to come back to the equilibrium after an interest rate shock inflation is less persistent in my case:

Figure 7



Responses of output and inflation to an interest rate shock in the EU5 as reported in Peersman and Smets 1998.

[15] Since our model has not an interest rate equation, to compute the impulse responses function we have estimated a constrained trivariate VAR(y, π, r) allowing for no restrictions on the interest rate equations (see afterward in the text). Moreover, test on over-identified restrictions doesn't reject them.

Impulse response functions in figure 6b can be compared with the impulses from a VAR[16] with output gap, inflation and real interest rate[17] as endogenous variables with four lags[18]. In fact our structural model can be viewed as a two restricted equations from the trivariate VAR. Figures 6a and 6b shows that the estimation of a parsimonious model doesn't lead to loss in the dynamic features of the system, given that most of the responses are pretty much the same. Moreover, the response of the inflation to interest rate is even more satisfactory, since it has the right sign, contrary to the one of the VAR.

As a final comparison between our structural model and the VAR, table 5 below reports the information criteria of our equations and those of the single equations of the VAR. The main criteria on which we base the comparison are the Schawrz (SC) and Akaike Information Criteria (AIC). As a matter of completeness we report also the Hannan-Quinn (HQ) and Final Prediction Error (FPE) criteria. The formers are functions of the residual sum of squares and they are differentiated by their degrees of freedom penalty for the number of parameters estimated. The SC more heavily penalizes extra parameters. As shown in table 5, the structural model's inflation equation is favoured over the VAR's inflation equation by both the SC and AIC (even by the other criteria). The SC favours the structural output equation, while the AIC favours the VAR. Overall, the information criteria do not appear to view our structural model restrictions unfavourably.

Table 5. INFORMATION CRITERIA

	AIC	SC	HQ	FPE
Structural inflation equation	-12.3805	-12.1973	-12.3198	4.20508e-006
VAR's inflation equation	-12.0170	-11.3757	-11.8044	6.43514e-006
Structural output equation	-11.6954	-11.4663	-11.6195	8.35389e-006
VAR's output equation	-11.8709	-11.2296	-11.6583	7.44756e-006

Both the highest absolute value for negative numbers and the smallest value for positive numbers suggest that the associated equation is preferable.

[16] The impulse response functions are obtained considering a Cholesky decomposition, i.e. according with the order of the variables chosen in the estimation this implies that shocks to interest rate have not a contemporaneous effect on output and inflation, but just lagged. The same kind of decomposition is used in Peersman and Smets (2001).

[17] Usually the impulse response function to analyse an interest rate shock when this is used as a monetary policy instrument is computed from a VAR in which the nominal interest rate appears as endogenous variable. Therefore, using here the real one doesn't give us different results.

[18] The number of lags has been chosen in accordance with the maximum lag of the structural equations, which is exactly four.

2.5. Stability tests

The analysis of the stability of the parameter is essential to avoid being "victims" of the so called Lucas Critique (1976). In fact in his article, Lucas stated that "*given that the structure of an econometric model consist of optimal decision rules of economic agents, and that optimal decision rule vary systematically with change in the structure of series relevant of the decision maker, it follows that any change in policy will systematically alter the structure of econometric models*". In other words, quantitative evaluations of alternative economic policies based on reduced form (like the one we are proposing) do not provide any useful piece of information and may even be misleading.

In particular, the Lucas Critique is particularly strong when considering backward looking models. In fact, one of the remedy is to build models with rational expectations (or forward looking models), since those models seem safe from the critique, although Estrella and Fuhrer (1999) provide empirical evidence for greater stability of estimations based on backward-looking rather than on forward looking models.

Some test the Lucas critique using the *exogeneity* and the *super-exogeneity* tests proposed by Engle, Hendy and Richard (1983), but recent works (see e.g. Lindé (1999) among others) underline the lack of power of super-exogeneity tests.

In the end, to justify the tests we are going to propose, we quote Lubik and Surico (2006) who argue that "*tests for parameter stability in backward-looking specifications or reduced forms of macroeconomic relationships typically fail to reject the null of structural stability in the presence of well-documented policy shifts. This evidence would support the conclusion that policy changes are … 'modest' enough not to alter the behaviour of private agents in a manner that is detectable by the econometrician. A further implication is that backward looking monetary models of the type advocated by Rudebusch and Svensson (1998), which perform well empirically, are safe to use in policy experiments*".

Since it seems that there is no consensus on whether Lucas critique is important or not and on which tests to use to test it, we proceed with some common tests to check the stability of our model. Tests we will implement are particularly useful because they are based on the idea that a regime change might take place slowly, and at un unknown point in time, or that the regime underlying the observed data might simply not be stable at all, contrary to the more common tests on structural breaks such as the Chow test.

The main test which we rely upon is the test proposed by Hansen (1992). This test is based on the algebraic properties of the cumulative sum of the least square estimation. For particularly high[19] values of the test the null hypothesis of model stability is rejected. Table 6 below reports the Hansen statistics and they indicate stability for both the equations.

Table 6. HANSEN TESTS

	Variance	joint	Individual parameters tests
Inflation's equation	0.21700	0.54632	
α_1			0.056182
α_3			0.039964
α_4			0.081917
γ_1			0.061552
Output gap's equation	0.025497	0.58377	
β_1			0.048403
β_3			0.043172
δ_1			0.18656
δ_3			0.10825
δ_4			0.13735

To further confirm this first results, we proceed in computing some recursive tests. In particular, figures 7a and 7b show the 1-step residual bordered by $0\pm2\hat{s}_t$ over M,...,T, where $\hat{s}_t$ is the standard error and M and T are two points inside the sample. In our case they are 1997Q3 and 2003Q4 respectively. Points outside the 2 standard-error region are either outliers or are associated with coefficient changes, and as it is possible to see there are not outside points for both equations. Then, we propose three variants of the Chow test: 1-Step Chow tests, Break-point Chow tests and Forecast Chow tests. All of them are computed for all t = M,…,T. The last two are distinguished from the fact that they are computed from the end to 1 and from the beginning to the end. They are distributed as F(1,t-k-1), F(T-t+1,t-k-1), F(t-M+1,M-k-1), where k is the number of regressors. As it is in general for the recursive analysis, results are better reported using graphics; in fact figures 7a and 7b report also those associated with the tests just described. For what concerns the output gap equation, all tests indicate that the null hypothesis of stability during the period cannot be rejected[20] at 1 per cent significance level. Test at 5 per cent show similar results, except for just a rejection regarding the 1-step Chow tests at t = 2000Q1, but this cannot compromise our conclusion of substantial stability of the parameters. Turning on the inflation equation, all the recursive tests assure

[19] Hansen provides asymptotic critical values for the test of model consistency. In PcGive the rejection of the null hypothesis is marked with an asterisk and here as well.

[20] Tests are scaled by 1-off critical values from the F-distribution at any selected probability level as an adjustment for changing degrees of freedom, so that the significant critical values become a straight line at unity. In other words, if the test line crosses that strait line it means rejection of the null at the significance level indicated in the up-left corner label.

great parameters' stability over the sample period, failing to reject all the null hypothesis at 5 per cent.

We can conclude that stability is assured and monetary policy analysis can be done.

Fig. 7a. OUTPUT GAP'S EQUATION STABILITY TESTS

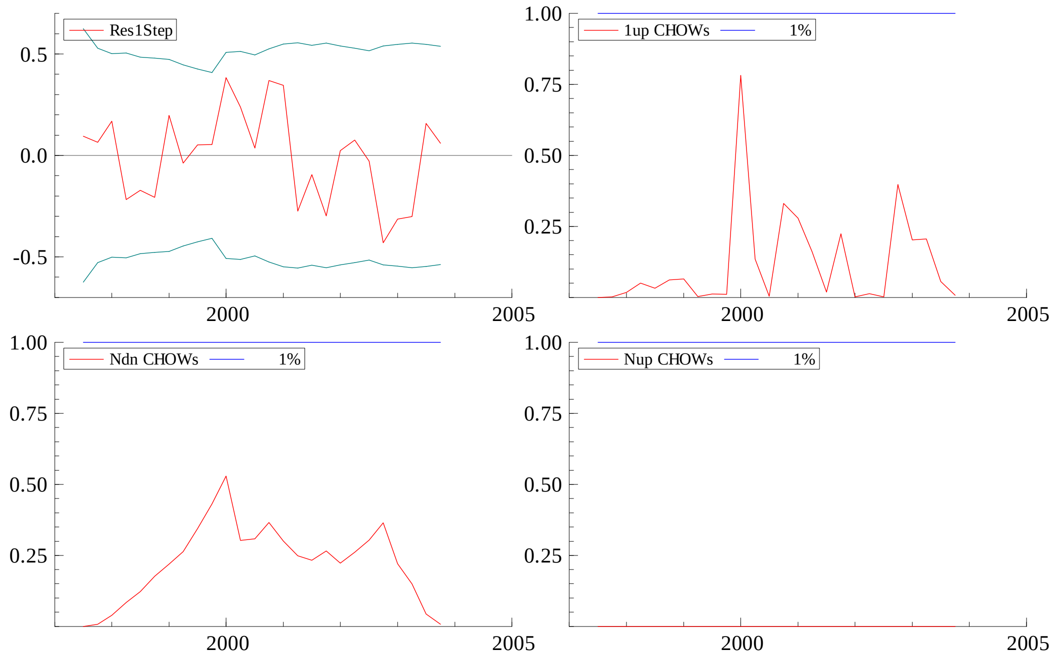
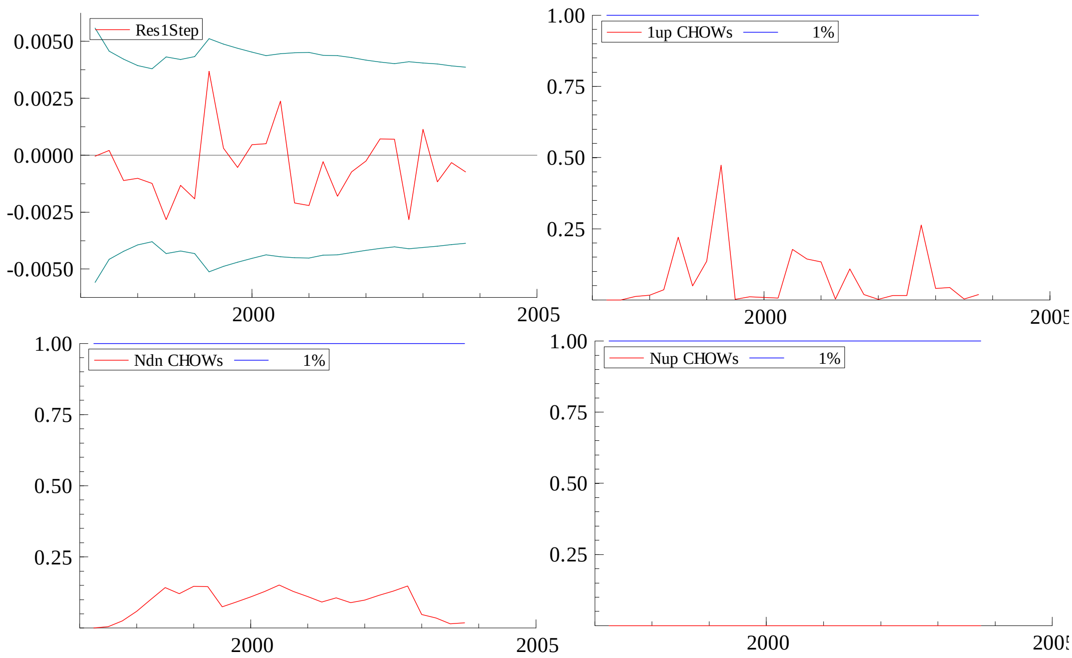


Fig. 7b. INFLATION'S EQUATION STABILITY TESTS



3. COMPUTATION OF THE OPTIMAL RULE

3.1. The model

The computation of an optimal policy rule passes through the maximization (or minimization) of an objective (loss) function by the Central Bank in which some variables are taken as objectives subject to a constraint. The choice of the objectives which a Central Bank has to care about in its loss function is not straightforward. The most common variables considered in the literature are inflation rate, some measures of the economic activity (e.g. output gap), and first differences of interest rate to include the interest rate smoothing[21] preferences. In fact this is what we propose below.

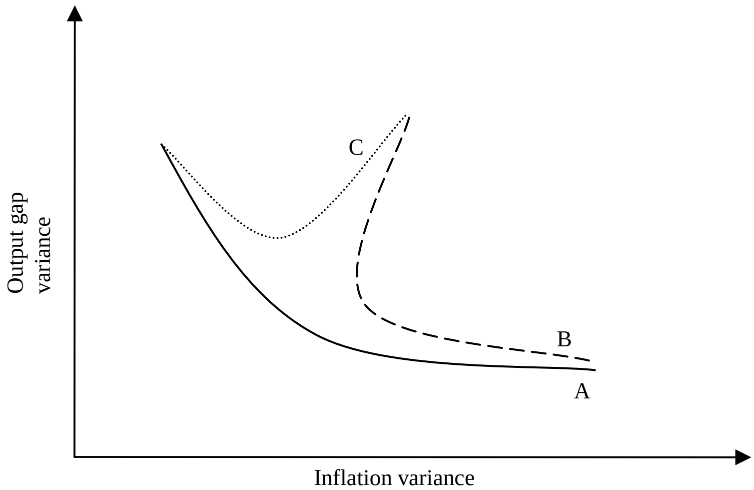
However, some other variables may be justified case by case. In particular, if the central bank has different targets beyond those of inflation and output stabilization, the deviations from those other targets may enter in the loss function. For instance, exchange rate is usually a good candidate for relatively open economies. Nevertheless, as suggested by Taylor (?), for the United States including the exchange rate in the loss function (or in the policy rule, which is the same) worse the macroeconomic performances of the policy rule, measured in terms of fluctuations of inflation and output gap around their targets. Moreover Ball (1999), Svensson (2000) and Taylor (1999) find that including the exchange rate was quite useless in terms of improvement of economic performances. This is because assuming rational expectations the effects of a variation of the exchange rate today is reflected in a change in the expectations on future output and inflation, which in turn leads to a change in the expectation of the future interest rate, which lastly implies a change in the actual interest rate. Our model has not rational expectations, but there is another explanation which can be used: the short run fluctuations of exchange rate cannot have a large effect the inflation so that no Central Bank reaction is necessary.

Another important variable which may be targeted is some monetary aggregate. This is particularly important for the ECB since, as already, said its policy has a target on the rate of growth of M3. However, as showed in Rudebusch and Svensson (2000), when there is some positive weight on money-growth stabilization, the resulting combination of inflation and output-gap variability will be inefficient. As a matter of fact, in deriving the optimal rule by taking explicitly into account the monetary aggregate target and by evaluating the efficiency frontier, they obtain the results showed in figure 8 below. “A” is the efficiency frontier in the

[21] Synonyms: policy gradualism, policy inertia.

case of mix inflation-output gap target, “B” and “C” are the efficiency frontiers in the case of mix inflation-money growth target (no weight on output gap) and of mix output gap-money growth target (no weight on inflation) respectively. It follows that for intermediate weights on inflation stabilization, output-gap stabilization, and money-growth stabilization, the corresponding combination of inflation and output gap variance will be in the interior of the area enclosed by the three curves in figure 8, implying the inefficiency above described.

Fig. 8. EFFICIENCY FRONTIERS



Moreover, looking at the behaviour of the M3 annual growth rate (figure 9) we can see that most of the time it does not meet the target level of 4.5%. This can be interpreted as a complete failure of the money growth objective. Since this would be a too strong statement, it is most likely that ECB doesn’t care too much about that objective. As a matter of fact, the ECB lowered interest rates several times during this period and it explained the increase in M3 was due to several special circumstances, that is, portfolio shifts and an increase in precautionary money demand. However, it is difficult to implement a strategy based on monetary aggregates if these special effects occur over longer periods.

In the end, Gerlach (2004) states that *“…[he is] not aware of any studies that find that money growth impacts on the ECB’s interest rate setting. Indeed, it is commonly argued that the ECB does not react to money growth*”.

Now we can proceed with the presentation of the objective function. This part is largely taken from Rudebusch and Svensson (1998). In our analysis, we will interpret “inflation targeting” as having a loss function for monetary policy where deviations of inflation from an explicit inflation target are always given some weight, but some weight will bi given to other targets too. In particular, for a discount factor δ (which has not to be confused with the parameters of the output gap equation), 0 < δ <1, we consider the intertemporal loss function in quarter t,

$$E_t \sum_{r=0}^{\infty} \delta^r \mathrm{L}_{\mathrm{t+r}} \, , \tag{5}$$

where the period loss function is

$$L_t = \pi_t^2 + \lambda y_t^2 + \nu (i_t - i_{t-1})^2 \tag{6}$$

Here variables have the same previous meaning. In fact, variables in the loss function are generally interpreted as deviations from a given constant target, except for the interest rate which is explicitly considered a deviation of its own lagged value. In the estimation of our model above, we used demeaned variables in the regression to avoid including constants in the equations. The average inflation rate in our sample period is about 1.9 per cent. Given the target level for inflation of the ECB of 2 per cent, we can conclude that the π_t has the same meaning[22]. This explanation will be clearer when we will derive the optimal policy rule. Moreover, λ, $\nu \geq 0$ are the weights on output stabilization and interest rate smoothing, respectively. We will refer to all variables as the goals variables. As defined in Svensson (1997), "strict" inflation targeting refers to the situation where only inflation enters the loss function ($\lambda = \nu = 0$), while "flexible" inflation targeting allows other goal variables (non zero λ or ν). Conversely we can define a strict output targeting the situation in which only output enters the loss function, i.e. zero weight on inflation and $\nu = 0$, although this kind of regime is not of a particular interest, since a strict output targeting Central Bank should not be sustainable from an inflation objective point of view, objective that every central bank usually has[23].

In order to set and solve the minimization problem we have to re-write the model of the economy and the objective function in a tractable form, i.e. the state space form.

[22] In Rudebusch and Svensson (1998) they were obliged to make distinction between the meaning of π_t in the two equations.

[23] South American countries (e.g. Argentina) can be good examples of that kind of situation.

3.2. State-space representation

The model formed by the equations (1) and (2) may be represented in the following matrix form (state space representation):

$$X_{t+1} = AX_t + Bi_t + \omega_{t+1} \tag{7}$$

The 10x1 vector X_t of state variables, the 10x10 matrix A, the 10x1 column vector B, and the 10x1 column disturbances vector ω_t are given by:

$$A = \begin{bmatrix} 1.034 & 0 & -0.595 & 0.391 & 0.091 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.274 & 0 & -0.503 & 0.315 & 1.234 & 0 & -0.38 & 0 & 0.503 & -0.315 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$X_t = \begin{bmatrix} \pi_t \\ \pi_{t-1} \\ \pi_{t-2} \\ \pi_{t-3} \\ y_t \\ y_{t-1} \\ y_{t-2} \\ i_{t-1} \\ i_{t-2} \\ i_{t-3} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -0.274 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad \omega_t = \begin{bmatrix} \varepsilon_t \\ 0 \\ 0 \\ 0 \\ \eta_t \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Furthermore, it is convenient to define the 3x1 vectors of goal variables. It fulfils

$$Y_t = C_x X_t + C_i i_t \tag{8}$$

where the vector Y_t, the 3x10 matrix C_x and the 3x1 column vector C_i are given by:

$$Y_t = \begin{bmatrix} \pi_t \\ y_t \\ i_t - i_{t-1} \end{bmatrix} \qquad C_i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C_x = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

This latter specification allows us to do the cross product between i_t and the vector X_t (as we will see later on in equation (10)) and to write the loss function as:

$$L_t = Y_t^{'} K Y_t \tag{9}$$

where the 3x3 K matrix is given by:

$$K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \nu \end{bmatrix}$$

and all these state space representations lead us to write the problem of the central bank like a stochastic discounted optimal linear regulator problem in which the loss function (9) has to be minimize subject to the constraint of the economy (7), i.e.:

$$\max_{\{i_t\}} - E_0 \sum_{t=0}^{\infty} \delta^t \left\{ Y_t^{'} K Y_t \right\} = \max_{\{i_t\}} - E_0 \sum_{i=0}^{\infty} \delta^t \left\{ X_t^{'} R X_t + 2 i_t W X_t + i_t^{'} Q i_t \right\} \tag{10}$$

$$\text{s.t.} \qquad X_{t+1} = A X_t + B i_t + \omega_{t+1}$$

where

$$R = C'_x K C_x \qquad W = C'_i K C_x \qquad Q = C'_i K C_i$$

3.3. Solving the model

This problem can be solved using the Dynamic control theory. In appendix A we report a brief description of the dynamic programming method which relies upon an equation called Bellman equation, which usually has an unknown functional form. Here, we consider the class of dynamic programming problems in which the return function (equation (10)) is quadratic and the transition function (equation (7)) is linear. This specification leads to the optimal linear regulator problem, for which the Bellman equation can be solved quickly using linear algebra. Furthermore, in the derivation of the optimal rule we consider the special case in which the return function and the transition function are both time invariant and the problem is not stochastic, because this facilitates the calculus and the results are the same, thanks to the *certainty equivalence principle*[24]. The only difference is in the expression of the value function which has not the *d* term in the nonstochastic case (see next first line).

The starting point is making an initial guess on the functional form of the value function V(X)[25]. This guess is that it has a quadratic form of the type V(X) = - X'PX - d, where P is a semidefinite symmetric matrix, *d* is $[\delta(1 - \delta)^{-1} \operatorname{tr}(P\Sigma_{\omega\omega})]$[26] and *tr* is the trace of the matrix P times the covariance matrix of disturbances vector ω.

Using transition law to eliminate next period's state, the Bellman Equation becomes:

$$-X'PX = \max_i \left\{ -X'RX - 2iWX - i'Qi - (AX + Bi)'P(AX + Bi) \right\} \tag{11}$$

The first-order necessary condition of the maximum problem on the right hand side of equation (11) is[27]:

$$(Q+\delta B'PB)i = -(W+\delta B'PA)X \tag{12}$$

---

[24] It states: the decision rule that solves the stochastic optimal linear regulator problem is identical with the decision rule for the corresponding nonstochastic linear optima regulator problem.

[25] It expresses the optimal value of the original problem, starting from an arbitrary initial condition of states variables. See appendix A for further mathematical details.

[26] It can be shown that for δ = 1, V(X)= $\operatorname{tr}P\Sigma_{\omega\omega}$.

[27] To derive that first order condition the following matrices properties have been used: $\frac{\partial x' Ax}{\partial x} = (A + A')x$; $\frac{\partial y' Bz}{\partial y} = Bz$; $\frac{\partial y' Bz}{\partial z} = B' y$.

which implies a rule of the type:

$$i = FX \qquad (13)$$

where F = -inv(Q+δB'PB)(W+δB'PA) and it is a 1x10 vector which contains the optimal response coefficient of the interest rate to each element of the vector X.

Substituting the optimizer (13) into the right hand side of (11) and rearranging gives:

$$P=R+\delta A'PA-(W'+\delta A'PB)(inv(Q+\delta B'PB))(W+\delta B'PA) \qquad (14)$$

This equation is called the algebraic *matrix Riccati equation*. Under particular condition, it has a unique positive semidefinite solution[28], which is approached in the limit as $j \rightarrow \infty$ by iteration on the following matrix Riccati difference equation:

$$P_{t+j}=R+\delta A'P_jA-(W'+\delta A'P_jB)(inv(Q+\delta B'P_jB))(W+\delta B'P_jA) \qquad (15)$$

starting from $P_0 = 0$.

The results of optimal rule under different weights (K matrices) are reported in the following table:

Table 7. DIFFERENT VECTORS F

Λ	0.1	0.1	0.5	0.5	1	1	5	5	100	100
N	0.01	0.1	0.01	0.1	0.01	0.1	0.01	0.1	0.01	0.1
π_t	2.3377	1.0332	1.5148	0.9875	1.2925	0.9786	1.0659	0.9826	1.0034	0.9984
π_{t-1}	-1.1794	-0.5542	-0.7893	-0.5774	-0.6938	-0.5750	-0.6015	-0.5673	-0.5770	-0.5746
π_{t-2}	-1.0859	-0.5210	-0.9932	-0.6970	-0.9619	-0.7734	-0.9266	-0.8791	-0.9162	-0.9138
π_{t-3}	1.6757	0.8511	1.0917	0.8207	0.9377	0.7995	0.7811	0.7577	0.7380	0.7370
y_t	2.6713	1.4622	2.9124	2.0163	2.9814	2.2971	3.0550	2.8027	3.0760	3.0595
y_{t-1}	-0.3496	-0.3312	-0.3296	-0.3568	-0.3228	-0.3556	-0.3150	-0.3338	-0.3126	-0.3140
y_{t-2}	-0.7434	-0.4703	-0.8290	-0.6200	-0.8534	-0.6918	-0.8795	-0.8183	-0.8870	-0.8829
i_{t-1}	0.6802	0.8026	0.6209	0.7510	0.6024	0.7143	0.5820	0.6308	0.5760	0.5794
i_{t-2}	0.6942	0.3480	0.8240	0.5249	0.8620	0.6210	0.9031	0.8065	0.9150	0.9084
i_{t-2}	-0.6162	-0.3899	-0.6872	-0.5139	-0.7074	-0.5735	-0.7291	-0.6783	-0.7353	-0.7319

[28] Having eigenvalues in A of modulus less than unity is a sufficient condition.

The numerical values associated with the weights have not a particular meaning by their own. What it is important is the relative weight between objectives. For instance, since the weight on inflation is always 1, a weight of 0.1 on the output gap objective means that the Central Bank puts one tenth of weight on it with respect to inflation stabilization. A weight of 5 implies that output stabilization is five times more important that inflation stabilization, and so on. Moreover, a weight of 0.01 is like to say zero importance of that target, and 0.1 for output can be considered a quasi-strict inflation targeting regime, which we here consider as strict.

What it is important to note about those results, is that the coefficient suggested as response of the interest rate to the current inflation (first row) approximately one, except for insignificant weights of interest rate smoothing and low weights on output gap. This is in contrast with the results of Peersman and Smets (1998) who found a coefficient of 0.34 in what they considered the benchmark optimal policy rule, i.e. equal weights on inflation and output stabilization and half a weight on interest rate smoothing, which is:

$$i_t = 0.34\pi_t + 0.17\pi_{t-1} + 0.09\pi_{t-2} + 0.05\pi_{t-3} + 1.17y_t + 0.12y_{t-1} + 0.56i_{t-1} \qquad (16)$$

This seems to contrast also with the Taylor (1993)[29] principle, i.e. with the Taylor principle; this states that the response of interest rate to inflation has to be more that one to one[30]. This is a principle which is valid for forward looking expectations models. Hence our optimal rule seems to capture the backward looking nature of our problem.

For what concerns the response to current value of output gap, we found an optimal response much higher than suggested by Taylor, even 4 – 5 times more accentuated. In any case, Peersman and Smets (1998) found similar results, although with a coefficient of 1.17, for the benchmark case.

Now the problem consists in how to choose a rule among all the optimal ones reported in the table for the different weights. A possible choice may be clear in answering the following question: which rule should the ECB follow if it wants to principally pursuit the objective of price stability, and without prejudice to this objective also care about the output stability (as actually it emerges from statutes and treaties), and allow for a certain degree of gradualism in

[29] He suggested the following rule: r = p + 0.5y + 0.5(p - 2) + 2, where *r* is the federal funds rate, *p* is the rate of inflation over the previous four quarters *y* is the percent deviation of real GDP from a target.
[30] For forward looking models the principle of more that one-to-one is also necessary to guarantee the uniqueness of the general equilibrium.

the monetary policy (as it merges from its experience[31])? In order to reach all those objective the weights that have to be chose are λ = 1 and ν = 0.1, or in other words the ECB has to set the interest rate according with the following rule (forth column of table 7):

$$i_t = 0.98\pi_t - 0.58\pi_{t-1} - 0.7\pi_{t-2} + 0.82\pi_{t-3} + 2.02y_t - 0.36y_{t-1} - 0.62y_{t-2} + 0.75i_{t-1} + 0.52i_{t-2} - 0.51i_{t-3} \quad (17)$$

A first observation is worth making. As we already said, our optimal rule implies a response of approximately one-to-one current inflation. This is the main result of my analysis. Since the economic features of the euro area have changed a lot in the last years, also the policy suggestions have changed accordingly. In fact, using the same argument of Peersman and Smets (1998), if the inflation is less persistent as in our case, this may be interpreted as higher credibility of the inflation target and this implies in turn that the central bank will need to lean relatively less against inflation.

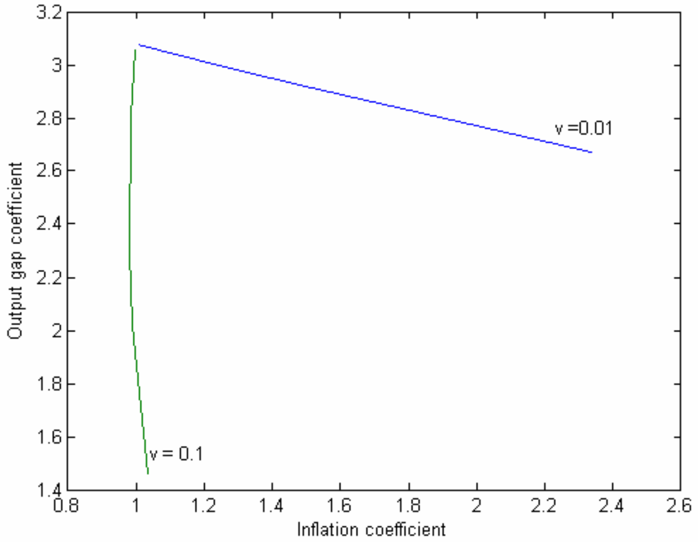
Secondly, we found that the coefficient on current value of output gap is higher than both the coefficient on inflation and of the Taylor rule suggestion. From the literature it is not clear as for the inflation which has to be the right magnitude of the coefficient on output gap, but our result is for instance in line Peersman and Smets (1998). This may be explained by the relative low importance of it in the role to stabilize inflation implied by our model.

Moreover, we can note that the coefficient on the first lag of interest rate in equation (17) is equal to 0.75, suggesting a high persistence, i.e. a high propensity for ECB to smooth the interest rate. In some cases (e.g. in Peersman and Smeets (1998)), this may reflect the fact that the weight put in the loss function on interest rate smoothing is high, but in our case we obtain similar results in terms of persistence even if we put very low weight on it, as it is possible to note looking at the eighth row of table 7, in which responses of i_t to i_{t-1} are in the range of 0.5760 – 0.6802 when ν = 0.01.

Finally, another important analysis regarding the short run concerns the relationship between the coefficients on inflation and output gap when one of them changes, given a certain weight on interest rate smoothing. This relation is easily viewable in figure 10; for the short run rule, it shows the relationship between the change in the optimal response coefficient of interest rate to the current value of inflation and output gap, given a certain weight on interest rate smoothing, i.e. the first and fifth row of table 7 above.

[31] For the interest rate smoothing preferences of ECB see Miguel C. (2006).

Fig. 10. EFFICIENT OPTIMAL FEEDBACK RULE COEFFICIENTS RELATIONSHIP



For ν = 0.01, an increase in the output gap weight even very small requires a very big variation in the coefficient of inflation to allow for the policy rule to be still optimal. Contrary, with a bigger weight on interest rate smoothing (ν = 0.1), which is the case for the ECB, the relation reverts, i.e. whatever the weight on the output gap, it is always optimal to respond one-to-one to current inflation.

4. ANALISYS OF THE OPTIMAL RULE

4.1 Efficiency Frontier

In the previous paragraph we have derived the formula to compute the vector F of the optimal responses of interest rate to the state and goals variables and their lags. Now, for any given optimal vector we can compute the numerical value of the loss experimented by the Central bank associated with that particular rule.

The following computation is necessary for the derivation of the loss. Hence, for any vector F, the dynamic of the model follows[32]:

$$X_{t+1} = MX_t + \omega_{t+1} \tag{18}$$

$$Y_t = CX_t \tag{19}$$

where the matrices M and C are given by

$$M = A + BF \tag{20}$$

$$C = C_x + C_iF \tag{21}$$

Moreover, looking at the equation (5), we can see that when $\delta \rightarrow 1$ the sum in that equation becomes unbounded. It consists in two components, however, one corresponding to the deterministic optimization problem when all shocks are zero, and one proportional to the variance of the shocks[33]. The former component converges for $\delta = 1$ (because the terms approach zero quickly enough), and the decision problem is actually well defined also for that case. For $\delta \rightarrow 1$, the value of the intertemporal loss function approaches the infinite sum of unconditional means of the period loss function, $E[L_t]$. Then, the scaled loss function $(1-\delta)E_t\sum_{r=0}^{\infty}\delta^r L_{t+r}$ approaches the unconditional mean $E[L_t]$. It follows that we can also define the optimization problem for $\delta = 1$ (which we assume in the following analysis, on the basis of the fact that in the literature a value of δ equal to 0.99 is usually assumed) and then interpret the intertemporal loss function as the unconditional mean of the period loss function, which equals the weighted sum of the unconditional variances of the goal variables:

[32] It is obtained with simple substitution of equation 14 into equations 8 and 9 respectively.

[33] It is possible to demonstrate that $E[L_t] = \text{trace}(K\Sigma_{\omega\omega})$.

$$E[L_t] = Var[\pi_t] + \lambda Var[y_t] + \nu Var[i_t - i_{t-1}] \quad (22)$$

In the end, for any given rule F that results in finite unconditional variances of the goal variables, the unconditional loss fulfils:

$$E[L_t] = E[Y_t'KY_t] = trace(K\Sigma_{YY}) \quad (23)$$

where Σ_{YY} is the unconditional covariance matrix of the goal variables (see appendix B).

The following table presents the values of the loss function and the standard deviation of inflation and output stabilization computed as explained above:

Table 8. VARIANCES AND LOSS VALUES

	ν=0.01				ν=0.1			
λ	σ_π	σ_y	σ_r	L	σ_π	σ_y	σ_r	L
0.1	0.12033	0.13313	0.87346	0.14238	0.13523	0.17921	0.32302	0.18645
0.3	0.12598	0.10306	0.83486	0.16525	0.13537	0.12865	0.41809	0.21577
0.5	0.12869	0.09616	0.83461	0.18512	0.13545	0.11314	0.47591	0.23962
0.7	0.13027	0.09350	0.83821	0.20411	0.13558	0.10589	0.51666	0.26137
1	0.13169	0.09177	0.84353	0.23190	0.13573	0.10033	0.56078	0.29214
2	0.13368	0.09027	0.85418	0.32277	0.13598	0.09400	0.64466	0.38846
3	0.13445	0.08995	0.85924	0.41289	0.13607	0.09209	0.68982	0.48134
5	0.13510	0.08977	0.86400	0.59261	0.13613	0.09076	0.73988	0.66396
10	0.13562	0.08969	0.86806	1.04126	0.13616	0.09000	0.79261	1.11545
100	0.13612	0.08967	0.87211	9.11197	0.13617	0.08967	0.86255	9.18952
Taylor•					0.18197	0.53847	0.15122	0.46633
P&S1••					0.15227	0.26270	0.36890	0.32051
P&S2•••					0.15020	0.22089	0.25097	0.28574

The loss values of all the special cases (except the pure inflation target case) reported in the table are computed imposing λ=0.5 and ν=0.1. In this way, I can measure the loss experimented by the ECB in following a rule different from what its preferences are (or should be).

P&S = Peesrman and Smets (1998).

The column with the loss value function (L) is obtained by $\sigma_\pi+\lambda\sigma_y+\nu\sigma_r$.

• Taylor values are obtained by imposing restrictions on the elements of the vector F, i.e. imposing that all the elements are zero except the first (f_1) equal to 1.5 and the fifth (f_5) equal to 0.5.

•• Obtained imposing f_1=1.53 and f_5=1.58.

••• Obtained imposing equation (16).

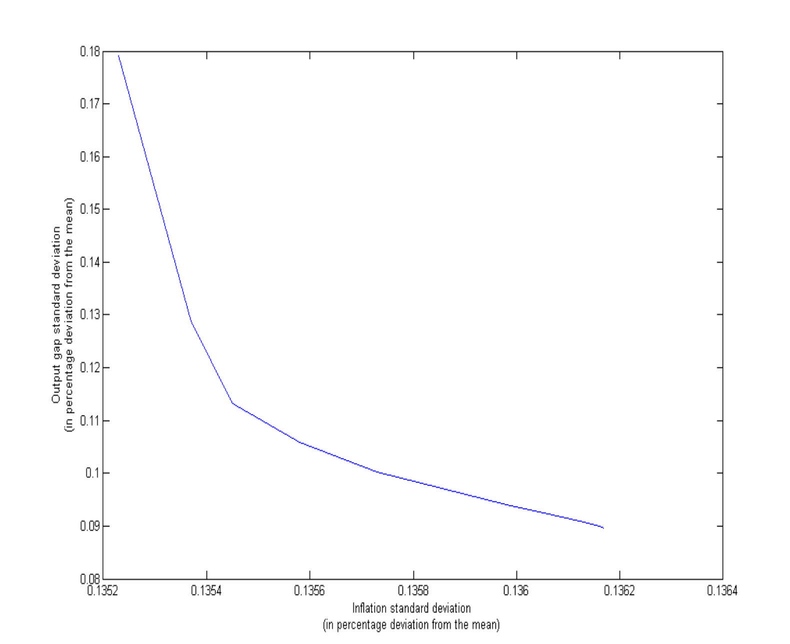
In the first place, we can note that the total loss corresponding to the case ν = 0.01 is always less than the case of ν = 0.1. However, this does not seem a good reason to choose the first alternative. In fact, bearing in mind the sense of my exercise, I am searching what it is optimal for the ECB if it wants to principally care about inflation, but also about output stability and interest rate smoothing.

However, what we suggested as the optimal rule for ECB (equation (17)) shows a total loss equal to 0.23962, which is better of the most part of the remaining cases. In particular, it is even better than the Taylor rule which implies an increase in the loss of about 95 per cent with respect to the optimal rule.

In order to compare my rule with the rules derived by Peersman and Smets I should compute as they do the restricted rules. Nevertheless, it is more convenient to calculate the loss implied by their rules (last two rows of table 8) and compare it with the loss implied by my rule. As it is clear, both of them are worst than mine.

In the end, we want to highlight that there is a trade off between the two objectives of output stabilization and inflation stabilization. This is clearer from figure 11 of the efficient frontiers.

Fig. 11. EFFICIENCY FRONTIER



It is possible to note that for values of inflation variability lower than 0.13, the curve becomes very steep, i.e. the output variability increases a lot. In other words, only extremely uneven consideration for inflation would lead the ECB to choose a monetary policy rule which generates output gap variability outside the range 0.09 – 0.11.

4.2. Dynamic response

As in the previous paragraphs, to include an interest rate equation in our model we have to estimate a VAR(y, π, r) model[34] and then imposing restrictions on the equations for y and π established by the estimated model (1) – (2), and the equation for r as established by my optimal rule (equation(17)), the Taylor rule and the P&S rule. The comparison is in terms of response functions.

The three following figures report the responses of output gap and of inflation to a shock on interest rate for these three cases.

Fig. 12a. IMULSE RESPONSE FUNCTION UNDER OPTIMAL RULE

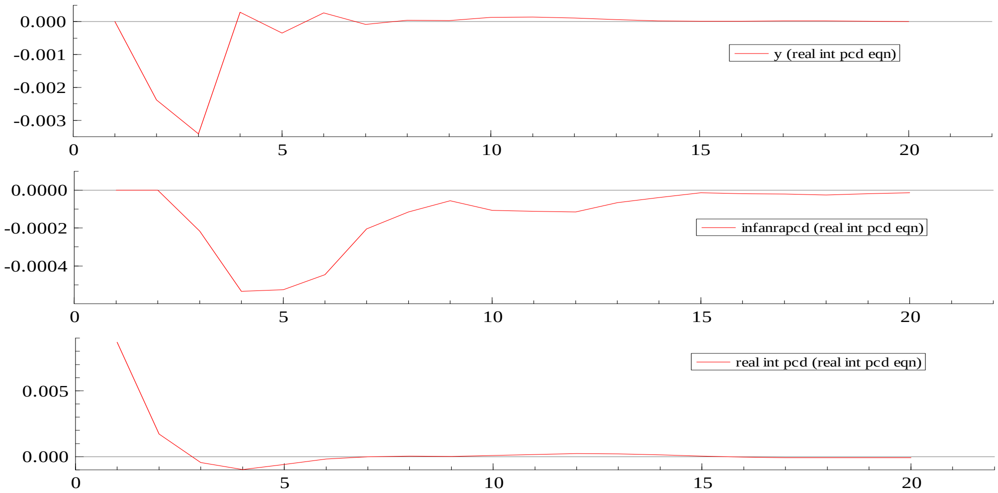
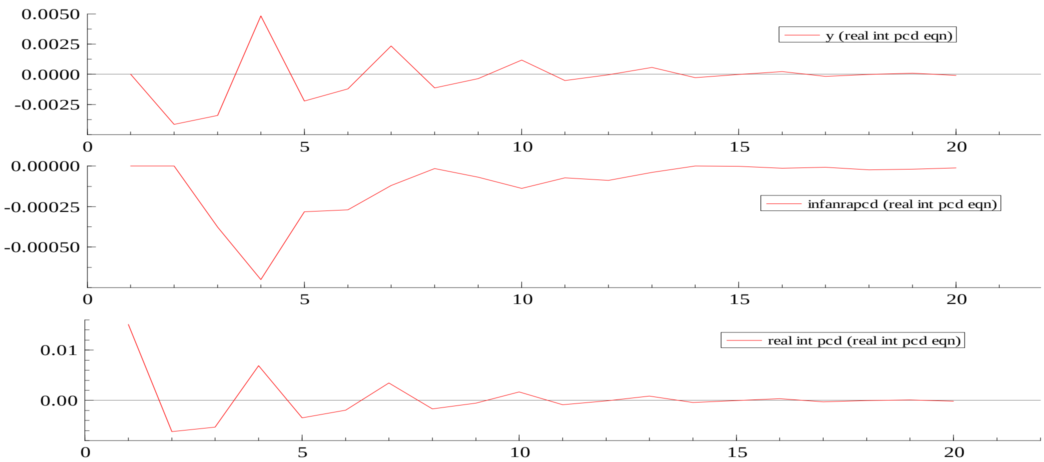
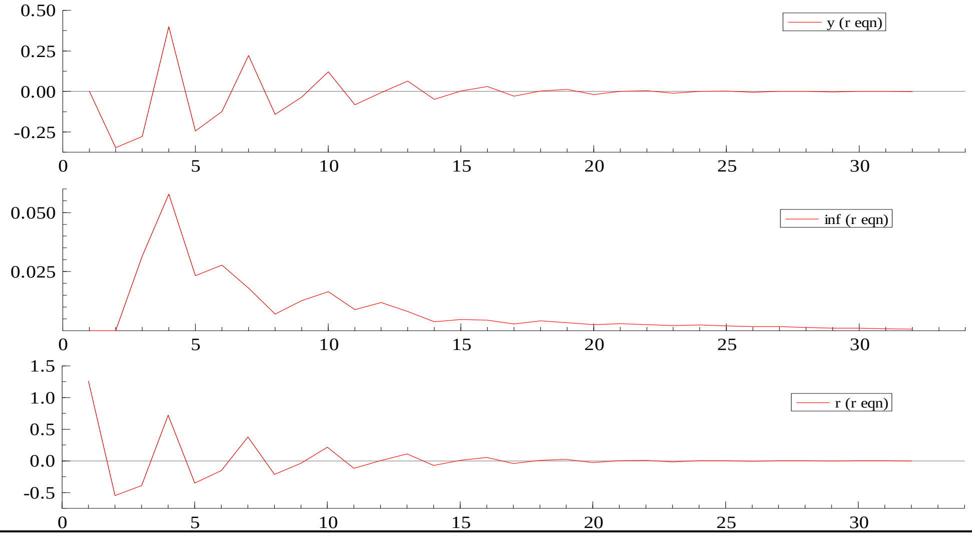


Fig. 12b. IMPULSE RESPONSE FUNCTION UNDER TAYLOR RULE



[34] Some authors use SUR estimation at this stage.

Fig. 12c. IMPULSE RESPONSE FUNCTION UNDER P&S1 RULE



Optimal responses are obtained using the Cholescky decomposition. The order of the responses in the two figures 12a and 12b is output, inflation and interest rate from the top to the bottom respectively.

Responses are similar for what concerns the signs (except for the P&S case for inflation), but the use of a Taylor rule or of the P&S1 implies more output gap and interest rate variability. In fact, although they come back to equilibrium as quickly as with the optimal rule, their fluctuations are larger, allowing for more macroeconomic instability.

CONCLUSIONS

I started my work with the question: "does it make sense to do monetary policy analysis with respect to a Central Bank taking into account a period in which that Central Bank does not exist?", and the answer I anticipated was “it probably does not”.

After my analysis, I can affirm that the answer is “it does not”, especially for the European central bank and the Euro Area. The main reason is because the features of the euro area have changed in the last years (in terms principally of less persistence in the inflation process) and the policy implications of those changes are quite important.

In particular, we found that the coefficient suggested as response of the interest rate to the current inflation is smaller then what indicated by Taylor (1993). This contrasts with results of Peersman and Smets (1998) who found an even smaller coefficient.

Our results seem to contrast also with the Taylor principle, i.e. that the response of interest rate to inflation has to be more that one to one. This is a principle which is valid for forward looking expectations models. Hence our optimal rule seems to capture the backward looking nature of our problem.

As a general conclusion then, contrary to what suggested by Taylor (1999b), who suggested that “[a] clear guideline, or policy rule [referring to his rule], for ECB decisions would go a long way toward reducing uncertainty and increasing economic stability throughout the globe”, and by Peersman and Smets (1998) who argued that “… *it may be worth considering a simple guideline like the one suggested by Taylor (1993) as a benchmark for analysing monetary policy in the euro area*”, ECB should not follow any Taylor rule, but rather the optimal rule we derived here, if the model we estimated is a reasonably good approximation of the way the euro area works. In fact, the adoption of theTaylor rule would imply an increase in the loss of the ECB of about 95 per cent. Other form of suggested rule, for instance those suggested by Peersman and Smets (1998), again imply a higher loss.

As for the interest rate smoothing, we found that it is optimal for the ECB to allow for a high degree of policy gradualism or interest rate smoothing, since the coefficient of the first lag of interests rate in the rule is 0.75. This result is not affected by the weight put on policy inertia in the loss function.

In the end, as concerns the response to the current value of output gap, we found that such a coefficient is higher than both the coefficient on inflation and the Taylor rule suggestion. From the literature, it is not clear, as for the inflation, which has to be the right magnitude of the coefficient on output gap, but our result is for instance in line Peersman and Smets (1998).

This may be explained by the relative low importance of it in the role to stabilize inflation implied by our model.

In any case, however, we found what other found in the recent literature on the estimation of policy rules for ECB regarding the period after 1999, i.e. that the ECB appears to react strongly to movements in real economic activity, or, as (Neumann, 2001, p. 14) puts it, "… *ECB's monetary policy is not just guided by the price stability objective but to a considerable degree also tries to stabilise the business cycle.*"

Another interesting result is that a trade off between the two objectives of output stabilization and inflation stabilization emerges. We can remember that for values of inflation variability lower than 0.13, the curve becomes very steep, i.e. the output variability increases a lot. In other words, only extremely uneven consideration for inflation would lead the ECB to choose a monetary policy rule which generates output gap variability outside the range 0.09 – 0.11.

In the end, we want to conclude with some suggestions for further research, that we could not pursue here since the addition of other regressors in equation(1) – (2) would have implied too few degrees of freedom. In particular we could specify a bigger and more complex model of the European economy, including for example the open economy aspects, and hence allowing for the effects of exchange rate; or more include the financial sector, considering financial variables such as stock prices. Those last variables may be also considered as good variables to be inserted in the objective function, because the necessity for a central bank to target the financial variables is still an open issue.

Nevertheless, the most important modification is to allow for asymmetries in the objective function. In fact, it is not clear why the central bank should experiment the same loss for the negative and the positive deviation of the variables from their targets.

APPENDIX A. Dynamic programming

Let $\beta \in (0, 1)$ be a discount factor. We want to choose an infinite sequence of "controls" $\{u_t\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t r(x_t, u_t) \qquad \text{(a)}$$

subject to $x_{t+1} = g(x_t, u_t)$, with x_0 given. We assume that $r_t(x_t, u_t)$ is a concave function and that the set $\{(x_{t+1}, x_t) : x_{t+1} \leq g_t(x_t, u_t), u_t \in R^k\}$ is convex and compact. Dynamic programming seeks a time-invariant *policy function h* mapping the *state* x_t into the control u_t, such that the sequence $\{u_s\}_{s=0}^{\infty}$ generated by iterating the two functions

$$\begin{aligned} u_t &= h(x_t) \\ x_{t+1} &= g(x_t, u_t) \end{aligned} \qquad \text{(b)}$$

starting from initial condition x_0 at $t = 0$ solve the original problem. A solution in the form of equations (b) is said to be *recursive*. To find the policy function *h* we need to know another function $V(x)$ that express the optimal value of the original problem, starting from an arbitrary initial condition $x \in X$. This is called the *value function*. In particular, define

$$V(x_0) = \max_{\{u_s\}_{s=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t r(x_t, u_t) \qquad \text{(c)}$$

where again the maximization is subject to $x_{t+1} = g(x_t, u_t)$, with x_0 given. Of course, we cannot possibly expect to know $V(x_0)$ until after we have solved the problem, but let's proceed on faith. If we knew $V(x_0)$, then the policy function *h* could be computed by solving for each $x \in X$ the problem

$$\max_{u} \{r(x, u) + \beta V(\tilde{x})\} \qquad \text{(d)}$$

where the maximization is subject to $\tilde{x} = g(x, u)$, with x given. Thus, we have exchanged either original problem of finding an infinite sequence of control that maximize expression (a) for the problem of finding the optimal value function V(x) and a function *h* that solves the continuum of maximization problems (d) – one maximum problem for each value of x. This exchange doesn't look like a progress, but we shall see that it often is.

Out task has become jointly to solve for V(x), h(x), which are linked by the *Bellman equation*

$$V(x) = \max_u \{r(x,u) + \beta V[g(x,u)]\} \qquad \text{(e)}$$

The maximize of the right hand side of equation (e) is a *policy function h(x)* that satisfies

$$V(x) = r[x, h(x)] + \beta V\{g[x, h(x)]\} \qquad \text{(f)}$$

Equation (e) or (f) is a *functional equation* to be solved for the pair of unknown function V(x), h(x).

Methods for solving the Bellman equation are based on mathematical structures that vary in their details depending on the precise nature of the functions *r* and *g*, but the description of those methods goes beyond the scope of this appendix, hence we will refer to Ljungqvist and Sargent (2004).

APPENDIX B. Derivation of the loss values

The covariance matrix Σ_{YY} for the goals variables is given by

$$\Sigma_{YY} \equiv E\left[Y_t Y_t^{'}\right] = C\Sigma_{XX} C' \quad \text{(a)}$$

where Σ_{XX} is the unconditional covariance matrix of the state variables. The latter fulfils the equation matrix equation

$$\Sigma_{XX} \equiv E\left[X_t X_t^{'}\right] = M\Sigma_{XX} M' + \Sigma_{\omega\omega} \quad \text{(b)}$$

where $\Sigma_{\omega\omega}$ is the variance covariance matrix of the disturbances whose estimation is obtained by the estimation of the model in equations (1)-(2).

We can use the relations vec(A + B) = vec(A) + vec(B) and vec(ABC) = ($C^{'} \otimes$ A) vec(B) on (b) (where vec (A) denotes the vector of stacked column vectors of the matrix A, and $\otimes$ denotes the Kronecker product) which results in

$$\begin{aligned} vec(\Sigma_{XX}) &= vec(M\Sigma_{XX} M') + vec(\Sigma_{\omega\omega}) \\ &= (M \otimes M) vec(\Sigma_{XX}) + vec(\Sigma_{\omega\omega}) \end{aligned} \quad \text{(c)}$$

Solving for vec(Σ_{XX}) we get

$$vec(\Sigma_{XX}) = [I - (M \otimes M)]^{-1} vec(\Sigma_{\omega\omega}) \quad \text{(d)}$$

REFERENCES

Adalid, A., Günter, C., McAdam, P., Siviero, S. (2005), *The Performance and Robustness of Interest-Rate Rules in Model of Euro Area*, ECB's Working Paper No. 479.

Angelini, P., Del Giovane, P., Siviero, S. and Terlizzese, D. (2002), *Monetary Policy Rules for the Euro Area: What Role for National Information?*, Temi di discussione, Banca d'Italia.

Angeloni, I., Aucremanne, L., Ehrmann, M., Gali, J., Levin, A., Smets, F. (2005), *Inflation Persistence in the Euro Area: Preliminary Summary of Findings*, drafted for the Inflation Persistence Network.

Ball, L. (1999), *Policy Rules for Open Economies*, in John B. Taylor, ed., Monetary Policy Rules. Chicago, Illinois: University of Chicago Press, 127-144.

Bank for International Settlements (1995), *Financial Structure and the Monetary Policy Transmission Mechanism*, Basle, Switzerland.

Bofim, A. N. and Rudebusch, G. D. (1997), *Opportunistic and Deliberate Disinflation Under Imperfect Credibility*, Federal Reserve Bank of San Francesco.

Breuss, F. (2002), *Was ECB's monetary policy optimal?*, Atlantic EconomicJournal, 30, 298-320.

Bryant, R. C., Hooper P. and Mann C., eds., (1993), *Evaluating Policy Regimes: New Research in Empirical Macroeconomics*, Brooking Institute, Washington, DC.

Casares, M. (2006), *ECB Interest-Rate Smoothing*, International Economic Trends, Federal Reserve Bank of ST. Louis.

Castelnuovo, E. and Surico, P. (2004), *Model Uncertainty, Optimal Monetary Policy and the Preferences of the Fed*, Scottish Journal of Political Economy, 51(1), 105-126.

Chow, G. C. (1975), *Analysis and Control of Dynamic Economic Systems*, John Wiley & Sons, New York.

Clarida, R., Gali, J., Gerlter, M. (1999), *The Science of Monetary Policy: A New Keynesian Perspective*, NBER Working Paper.

Clarida, R., Gali, J., Gerlter, M. (2000), *Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory*, The Quarterly Journal of Economic.

Clausen, V. and Hayo, B. (2002), *Monetary Policy in the Euro Area – Lesson from the First Five Years*, Center for European Integration Studies Working Paper.

Coenen, G. and Wieland, V. (2000), *A Small Estimated Euro Area Model with Rational Expectations and Nominal Rigidities*, ECB Working Paper No. 30.

Collard, F., Fève, P. and Langot, F. (2002), *Structural Inference and the Lucas Critique”*. Annales d’Economie et de Statistiques, 67/68, 183-2002.

Dieppe, A., Küster, K. and McAdam, P. (2004), *Optimal Monetary Policy Rules for the Euro Area: An Analysis Using the Area Wide Model*, ECB Working Paper No. 360.

Divino, J. A. C. A. and Pinto de Andrade, J. (2001), *Optimal Rules For Monetary Policy In Brazil*, Paper prepared for the REDIPEA Research Network coordinated by IPEA.

Engle, R. F., Hendry, D. F. and Richard, J. F. (1983), *Exogeneity*, Econometrica, Vol. 51, No. 2, 277-304.

Estrella, A. and Fuhrer, J. C. (1999), *Are ‘Deep’ Parameters Stable? The Lucas Critique as an Empirical Testable Hypothesis*, Federal Reserve Bank of Boston Working Papers, No. 99-4.

European Central Bank (2004), *The Monetary Policy of the ECB.*

European Central Bank (October 2001), *Monthly Bulletin*.

Fagan, G., Henry, G. and Mestre, R. (2001), *An area-wide model (awm) for the Euro area*, ECB’s Working Paper No. 42.

Fanelli, L. (2005), *Testing the New Keynesian Phillips Curve through Vector Autoregressive Models: Results from the Euro Area*. Paper prepared for the Conference: “The Keynesian Legacy in Macroeconomic Modelling”, Cassino - Italy - 16/17 September 2005.

Fendel, R. M. and Frenkel, M. R. (2006), *Five Years of Single European Monetary Policy in Practice: Is the Ecb Rule-Based?*, Contemporary Economic Policy, 24(1), 106-115.

Fourçans, A. and Vranceanu, R. (2002), *ECB Monetary Policy Rule: Some Theory and Empirical Evidence*, ESSEC Research Centre Working Paper No. 02008.

Gali, J. (2001), *European Inflation Dynamics*, European Economic Review, 45, 1237-1270.

Galí, J., Gerlach, S., Rotemberg, J., Uhlig, H. and Woodford, M. (2004), *The monetary policy strategy of the ECB reconsidered: Monitoring the European Central Bank 5*. CEPR.

Gali, J., Gerlter, M. (1999), *Inflation Dynamics: a Structural Econometrics Analysis*, Journal of Monetary Economics, 44(2), 195-222.

Gerdesmeier, D and Roffia, B. (2003), *Empirical Estimates of Reaction Functions for the Euro Area*, ECB Working Paper No. 206.

Gerlach, S. (2004), *Interest Rate Retting by the EBC: Words and Deeds*, CEPR Discussion Paper 4775.

Gerlach, S. and Schnabel, G. (1999), *The Taylor Rule And Interest Rates in the EMU Area: A Note,* Bank for International Settlements Working Paper No. 73.

Goodfriend, M. and King, R. (1997), *The New Neoclassical Synthesis and the role of Monetary Policy*, NBER Working Paper.

Green, W. H. (2003), *Econometric Analysis*, 5th edition, Prentice Hall.

Hansen, B. (1992), *Testing for Parameter Instability in Linear Models*, Journal of Policy Modeling, 14, 517-533.

Issing, O. (2003), Panel discussion at the conference "Inflation Targeting: Prospects and Problem", http://www.ecb.int/press/key/date/2003/html/sp031017_1.en.html.

Levin, A. T. and Williams J. C. (2003), *Robust Monetary Policy with Competing Reference Models*, Journal of Monetary Economics, 50, 945-975.

Lindé, J. (2000), *Monetary Policy Analysis in Backward-Looking Models*, Central Bank of Sweden Working Paper No. 114.

Lindé, J. (2001), *Testing for the Lucas Critique: a Quantitative Investigation*. American Economic Review, 91(4), 986-1005.

Ljungqvist, L. and Sargent, T. J. (2004), *Recursive Macroeconomic Theory*, 2nd edition, MIT Press.

Lubik, T. A. and Surico, P.(2006), *The Lucas Critique and the Stability of Empirical Models*, Richmond Federal Reserve Bank Working Paper.

Miguel, C. (2006), *ECB Interest-Rate Smoothing*, International Economic Trends, Federal Reserve Bank of St. Louis.

Mishkin, F. S. (2001), *Inflation Targeting*, Prepared for Brian Vane and Howard Vine, *An* Encyclopedia of Macroeconomics.

Monacelli, T. (2006), *Incomprensibile BCE*, on www.lavoce.info.

Monteforte, L. and Siviero, S. (2002), *The Economic Consequences of Euro Area Modelling Shortcuts*, Temi di discussione, Banca d'Italia.

Neumann, M. J.M. (2001), Background paper for EMU Monitor.

Peersman, G. and Smets, F. (1999), *Uncertainty and the Taylor rule in a simple model of the Euro-area economy*, Federal Reserve Bank of San Francesco.

Peersman, G. and Smets, F. (2001), *The monetary transmission mechanism in the Euro area: more evidence from VAR analysis*, ECB Working Paper No. 91.

Rudebusch, G. D. and Svensson, L. E. O. (1998), *Policy Rules for Inflation Targeting*, NBER Conference on Monetary Policy Rules.

Rudebusch, G. D. and Svensson, L. E. O. (2000), *Eurosystem Monetary Targeting: Lesson from U.S. Data*, NBER Working Paper.

Sack, B. (1998), *Does the Fed Act Gradually? A VAR Analysis*, Board of Governors of the Federal Reserve System.

Scheller, H. K. (2004), *The European Central Bank, History, Role and Functions*.

Sims, C. (2001), *A Review of Monetary Policy Rule*s, Journal of Economic Literature, XXXIX, 562-566.

Smets, F. and Wouters, (2002), *Monetary Policy In An Estimated Stochastic Dynamic General Equilibrium Model Of The Euro Area*, Federal Reserve Bank of San Francisco.

Smets, F. and Wouters, R (2003), *An Estimated Stochastic Dynamic General Equilibrium Model Of The Euro Area*, ECB Working Paper No. 171.

Stock, J. H. and Watson, M. W. (2001), *Vector Autoregression*, Journal of Economic perspectives, 15(4), 101-115.

Svensson, L. E. O. (1997), *Inflation targeting: Some Extensions*, NBER Woking Paper.

Svensson, L. E. O. (2000), *Open-Economy Inflation Targeting*, Journal of International Economics, 50 (1), 155-183.

Taylor, J. B. (), *The Role of the Exchange Rate in Monetary Policy Rules*, Carnegie-Rochester Conference Series on Public Policy, 39, 195-214.

Taylor, J. B. (1993), *Discretion versus Policy Rules in Practice*, Carnegie-Rochester Conference Series on Public Policy, 39, 195-214.

Taylor, J. B., ed., (1998), *The Robustness and Efficiency of Monetary Policy Rules as Guidelines for Interest Rate Setting by the European Central Bank*, Institute for International Economic Studies, Stockholm University, Seminar Paper No. 649.

Taylor, J. B., ed., (1999a), *Monetary Policy Rules*, NBER and University of Chicago Press, Chicago.

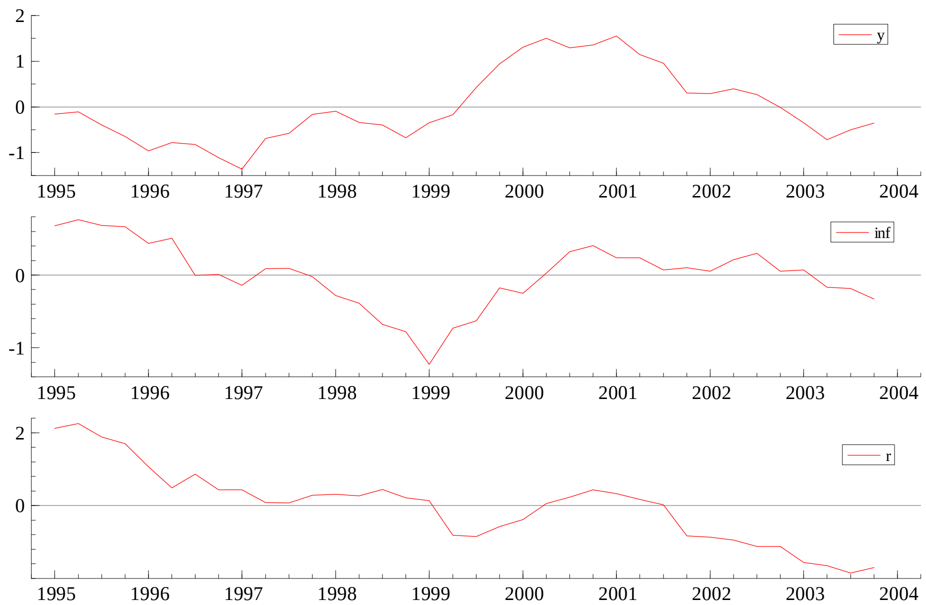
Taylor, J. B., ed., (1999b), *What the European Central Bank Needs to do*, Hoover Digest, No.1.

Ullrich, K. (2003), *A Comparison Between the Fed and the ECB: Taylor rules*, Discussion Paper No. 03-19, ZEW.

Zampolli, F. (2006), *Optimal Monetary Policy In A Regime-Switching Economy: The Response To Abrupt Shifts In Exchange Rate Dynamics*, Bank of England Working Paper No. 297.

FIGURES AND TABLES

Fig. 1. VARIABLES



y = output gap, inf = π = inflation, r= real interest rate. All the variables are in percentage points and demeaned.

Fig. 2. COMPARISON AMONG INLFATION RATES

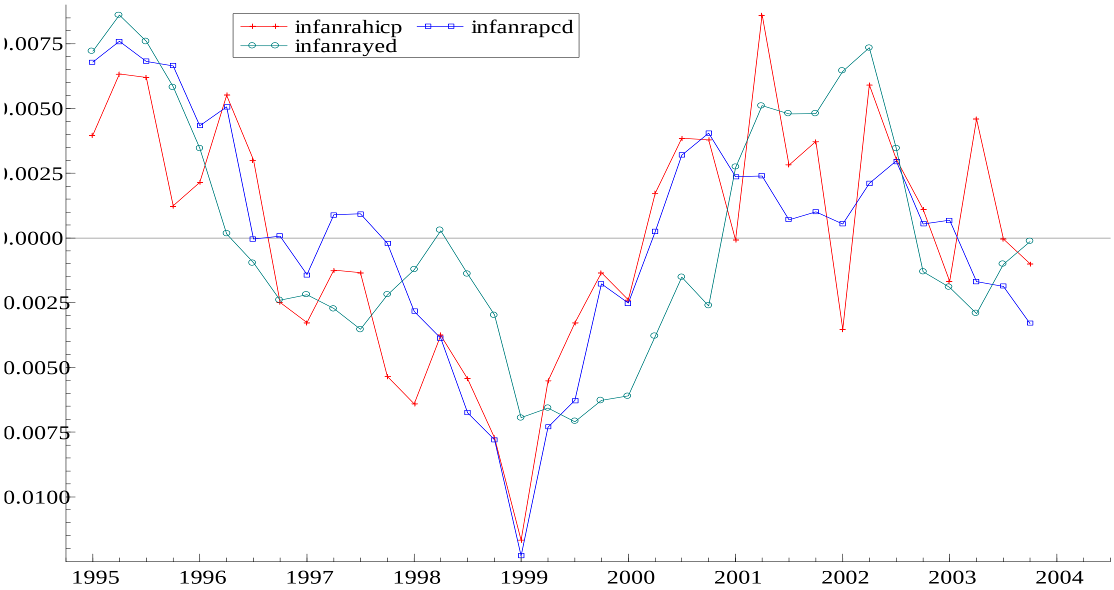
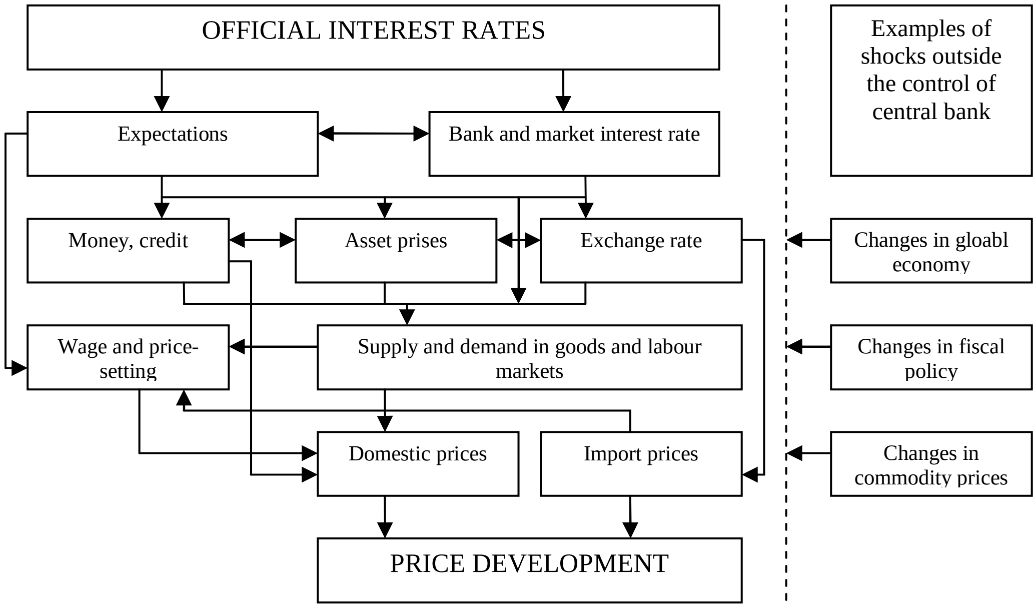


Table 1

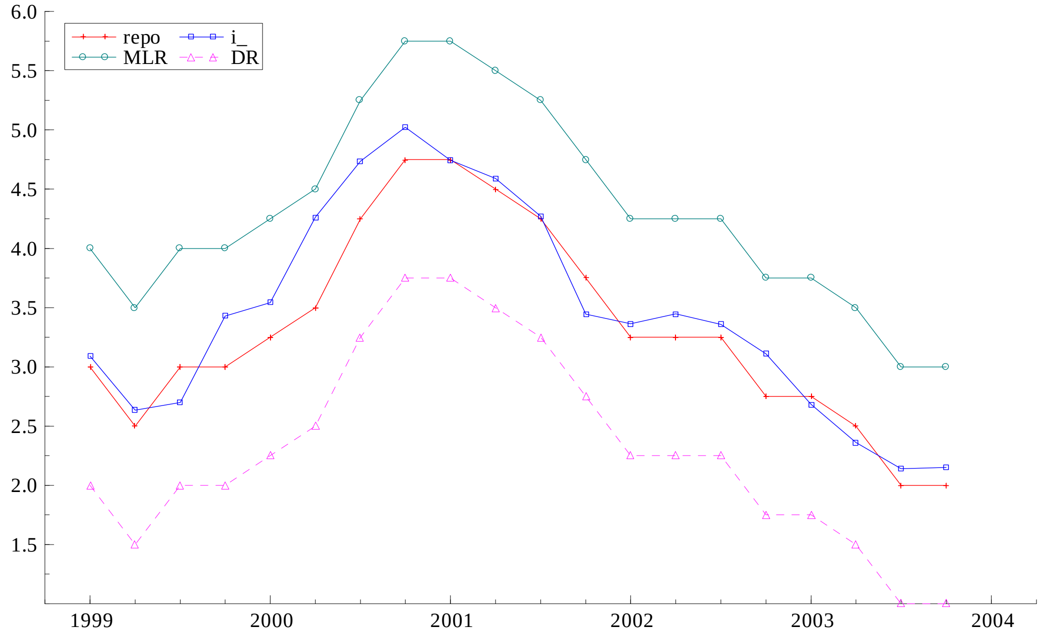
CORRELATION MATRIX			
	Infanrahicp	infanrapcd	infanrayed
infanrahicp	1.0000	0.8098	0.6197
infanrapcd		1.0000	0.7088
infanrayed			1.0000

Fig. 3. TRANSMISSION MECHANISM



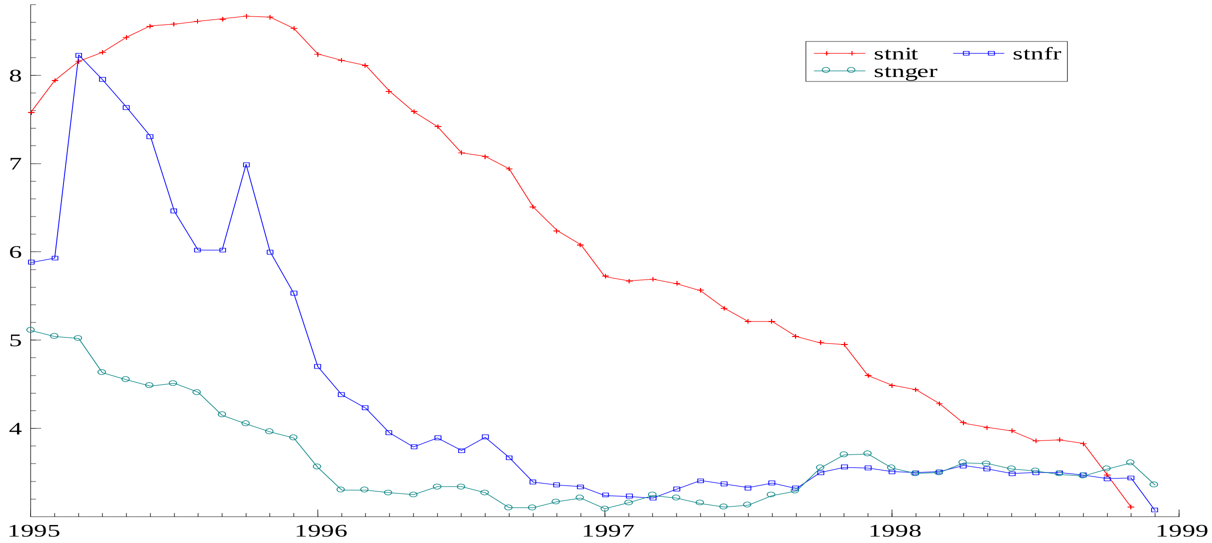
Source: ECB (2004)

Fig. 4. COMPARISON REPO-SHORT TERM INTEREST RATE



Repo in the the rate applied by ECB in the liquidity-providing reverse transactions based on a repurchase agreement, "i" is the short term nominal interest rate in the money market, MLR is the marginal lending rate plied by ECB to the bank system, and DR is the deposit rate again apply by ECB.

Fig. 5. SHORT TERM INTEREST RATES COMPARISON



stnit = short term nominal interest rate for Italy, stnfr = short term nominal interest rate for France, stnger = short term nominal interest rate for Germany.

The choice of the countries is quite popular in the literature when one wants to take a representative aggregate of Europe, since Germany, France and Italy produce about the 75 per cent of the total GDP. In our case, It was useful to take Italy because it was one of that countries got out from the European Monetary System with other four European countries, and hence with a potential necessity to use monetary policy in different way either to stabilize the exchange rate or to not loose the possibility to borrow from abroad which required higher interest rate to compensate exchange rate risks. This is reflected by the lower correlation of interest rate with France and Germany.

Table 2.

CORRELATION MATRIX			
	stnit	stnfr	stnger
stnit	1.0000	0.7407	0.4795
stnfr		1.0000	0.8676
stnger			1.0000

Table 4. COMPARISON WITH OTHER ESTIMATES

$$\pi_{t+1} = \alpha_1\pi_t + \alpha_3\pi_{t-2} + \alpha_4\pi_{t-3} + \gamma_1 y_t + \varepsilon_{t+1} \qquad y_{t+1} = \beta_1 y_t + \beta_3 y_{t-2} + \delta_1(i_t - \pi_t) + \delta_3(i_{t-2} - \pi_{t-2}) + \delta_4(i_{t-3} - \pi_{t-3}) + \eta_{t+1}$$

		Phillips Curve								IS Curve							
	Sample period	α_1	α_2	α_3	α_4	γ_1	γ_2	γ_3	$\gamma 4$	β_1	β_2	β_3	β_4	δ_1	δ_2	δ_3	δ_4
Mine	1996 2003	1.034*		-0.595*	0.391**	0.091***				1.234*		-0.38*		-0.274*		0.503*	-0.315**
Monteforte Siviero[†] (2002)	1978 1998	0.652*			0.348	0.088*				0.769*				-0.05*			
Coenen Weiland (2000)[††]	1974 1978	0.488*	0.099^{ns}			0.39*	0.22^{ns}			1.233*	-0.27*			-0.04***			
Peersman Smets[+] (1998)	1975 1997	0.45*	0.17^{ns}	0.06^{ns}	0.06^{ns}	0.33*				0.84*	0.1^{ns}			-0.1*			
Rudebusch Svensson EMU[++]	1971 1994	0.70	0.06	0.05	0.05	0.11				1.25	-0.42			-0.02			
Rudebusch Svensson (1998) US	1961 1996	0.70*	-0.10^{ns}	0.28*	0.12^{ns}	0.14*				1.16*	-0.25*			-0.10*			

The theoretical coefficient subscript numeration has to be read in the following way: coefficient number = corresponding lag; for example, δ_3 refers to the third real interest rate lag. This is to facilitate the comparison with the other models.

*, **, *** , ns mean that coefficient is significant at 1 per cent, 5 per cent, 10 per cent significance level and not significance respectively.

† They consider an aggregate of 3 countries: Germany, France and Italy.

†† They estimate a VAR for the euro area with endogenous variables inflation and output gap. Separately, they estimate the euro area IS Curve.

+ They consider Europe an aggregate of 5 countries: Germany, France, Austria, Belgium and Netherlands.

++ These estimate are reported in Taylor (1998) and they are based on a weighted GDP and inflation for an aggregate of Germany, France and Italy.

Table 3. VARIANCES OF INFLATION AND OUTPU GAP PERIOD 1995-1998

	MEAN	OUTPUT GAP STANDARD DEVIATION
Germany	0	2.1851e+009
France	0	2.2681e+009
Italy	0	1.7466e+009

	MEAN	OUTPUT GAP STANDARD DEVIATION
Germany	0.014970	0.0045152
France	0.014041	0.0060610
Italy	0.033139	0.014912

Fig. 6a. IMPULSE RESPONSE FUNCTIONS WITH VAR

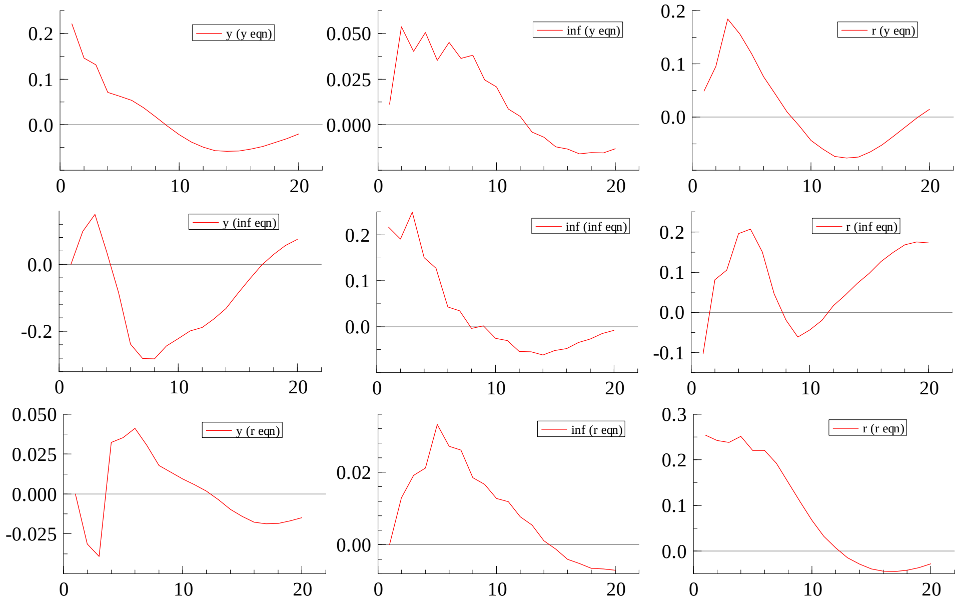
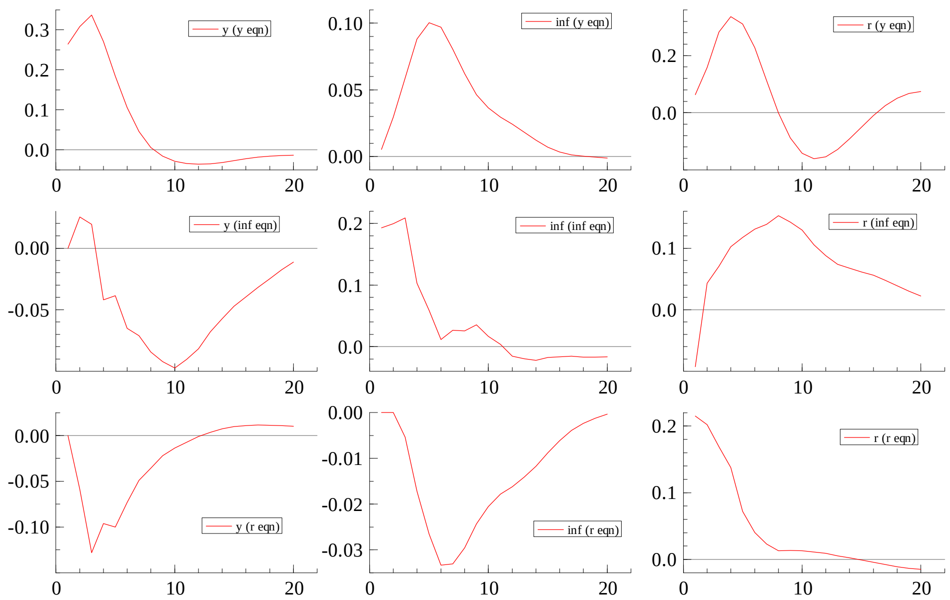


Fig. 6b. IMPULSE RESPONSE FUNCTIONS WITH STRUCTURAL EQUATIONS



Optimal responses are obtained using the Cholescky decomposition. The order of the responses in the two figures 6a and 6b is output, inflation and interest rate in the first, second and third column respectively. Shocks follow the same order on the rows.

Fig.9. M3 ANNUAL GROWTH RATE