ENDOUGENOUS AMENITIES AND SPATIAL STRUCTURE OF CITIES

Mihai TIVADAR*
PhD Student MEDEE-EQUIPPE, Lille 1 University, France
ATER Lille 2 University, France

Hubert JAYET+
MEDEE-EQUIPPE, Lille 1 University, France

We examine the dynamics of the urban social structures when the spatial repartition of amenities is endogenously modified by the spatial distribution of social groups. At each period, the equilibrium urban structure is determined by the repartition of amenities; but, between periods, this repartition changes, rich households generating local amenities in the locations they occupy, and therefore the social structure of the city changes. We show that when local amenities are endogenously generated, as the city develops, it passes from an American urban structure (rich located at periphery) to a European one (central localisation of rich).

1. INTRODUCTION

Our paper examines the endogenous dynamics of the social structure of cities when the spatial repartition of amenities is endogenously modified by the spatial distribution of the social groups. The idea of our model emerged from the observation of the contrasts existing between typical American and European cities structure. In most American cities, central locations are occupied by poor households, whereas in European cities, they are occupied by rich households.

For these contrasts, the literature proposed two explanations. In the standard urban models, the rich households are attracted by the city's central localisations when their transportation costs are much higher compared to the poor households.

Another explanation was proposed by Brueckner & alii (1999), based on the theory of local amenities. The European cities are characterized by a longer history. The amenities are mainly located in the centre (monuments, parks, boulevards, fine architecture, etc) which are the consequence of this history. If the rich demand for amenities is significant, such an advantage can be sufficient to attract the rich households to the central localisations, which corresponds to the typical European urban structure.

* mihai.tivadar@ed.univ-lille1.fr
+ hubert.jayer@univ-lille1.fr

According to Brueckner & alii (1999) the urban amenities are divided into three categories: natural amenities (which are generated by the topographic characteristics of the area), historical amenities (generated by the monuments, the buildings, the parks or other urban infrastructures which hold of the past) and modern amenities (which depend on the current conditions). In our paper, we suppose that the natural amenities do not cause differentiation of the urban space and we are interested only in the effects of the modern and historical amenities on the city's structure.

Our model is conceived in this theory framework. In this paper we are trying to overcome the limits of the original Brueckner's model: a static framework and exogenous amenities function. Thus, we explicitly consider the historical dimension of the process generating amenities: at each period, the equilibrium spatial structure of the city is determined by the spatial repartition of amenities; but, between periods, this repartition change, rich households generating amenities in the locations they occupy, and therefore the city's spatial structure changes. This dynamic framework leads to the idea that the actual social structure of the European cities is the consequence of a lock-in phenomenon: the rich households live in the centre because they remain in localisations which were peripheral at the beginning, but as the city develops, these localisations become central.

In a first part we present the theoretical model. Since the model is not solvable analytically, we are making a series of numerical simulations. The last part is devoted to the conclusions.

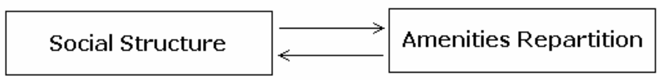
2 THE MODEL

We created a dynamic residential model, where the connection between periods is given by the transformation of the modern amenities into historical amenities. Our model belongs to the class of models without capital durability. This type of models was developed first by Alonso (1964), Mills (1967) and Muth (1969) within a static framework. We remain in the monocentric urban models tradition where the centre (CBD - Central Business District) is represented by a point in space and the only localisation variable is the distance to the centre (x).

There are two social classes, rich and poor households, differentiated by their income respectively y_1 and y_2 and by their preferences for the amenities. The utility of the households depends on the consumption of the composite good (z), whose price is standardized with the unit, on the living space (s) and the level of amenities ($a(x)$). We suppose that the rich households have stronger preferences for amenities than the poor. This assumption is explained by the fact that we regard the amenities as a superior good.

The model is in an open city framework (there are no costs of migration). Thus, the utility level of each category is exogenous, equal to the national level (u^t_i) and the ctiy's population at each time is endogenous.

The urban dynamic development is defined as a chain of static equilibriums, where the connection between periods is given by the transformation of the modern amenities into historical amenities. Thus, at each period we determine the equilibrium localisation of each category and the effects of this urban structure on the amenities level. These effects will be taken into account during the following period and will have an influence in the new decisions of localisation:



At each period, the households maximize their utility under budgetary constraint:

$$\max_{z,s,x} U_i(z,s,a^t(x)) \qquad \text{subject to} \qquad y_i^t - C_i^t(x) = z + R^t(x)s$$

where $C_i^t(x)$ is the commuting cost to (CBD) and $R^t(x)$ is la market land rent at period t.

At equilibrium, each household will reach a utility equal to the national level u^t_i. We define the bid-function as the maximum price per unit of land that the household can pay to residing at distance x while enjoying the utility level $u^t_{i:}$:

$$\psi_i^t(x,u_i^t) = \max\left\{\frac{\mathrm{y}_i^t - C_i^t(x) - z}{s} \,|\, U(z,s,a^t(x)) = u_i^t\right\}$$

When we solve this maximization we obtain the optimal lot size $s = S_{\mathrm{i}}^t(x,u_i^t)$, which is also called bid-max lot size. The city's structure will be the result of competition for the land between the various usages (residential, agricultural). Each location will be occupied by the highest bidder. Thus, the urban rent will be the upper envelope of the bid-functions and the agricultural rent (the opportunity cost of the land): $R^t(x) = \max\left\{\psi_i^t(x), RA^t\right\}$, where RA^t is the agricultural rent or the opportunity cost of the land at period t. The segregation points between social classes are given by the solution of equalization of the bid-functions $x_s^t \equiv \mathrm{sol}\left\{\psi_1^{\mathrm{t}}(x) = \psi_2^{\mathrm{t}}(x)\right\}$.

We define a binary spatial variable $K(x)$ that specifies the social category of the household living at distance x:

$$K(x)=\begin{cases}1, & \text{if } \psi_1^t(x)>\psi_2^t(x)\\ 2, & \text{if not}\end{cases}$$

The equilibrium residential boundary (or the city's border) is given by the point at which the bid-function of the category localised in the peripheral area of the city equals the agricultural rent: $x_f^t \equiv \text{sol}\left\{\psi_{K(x)}^{\text{t}}(x)=RA^t\right\}$.

We suppose that the land is allocated entirely to the residential use. Since the city is in a perfectly plane area, surface available for the residences to distance x is given by the perimeter of the circle $L(x)=2\pi x$. Hence the equilibrium household distribution is given as the ratio between the surface available for the residential use and the lot size of each house:

$$n^t(x)=\begin{cases}\dfrac{2\pi x}{S_{K(x)}^t(x)}, & x\in\left[0,x_f^t\right]\\ 0, & x>x_f^t\end{cases}$$

The household density is defined as the number of households per unit of land at distance x:

$$\rho^t(x)=\begin{cases}\dfrac{n^t(x)}{2\pi x}=\dfrac{1}{S_{K(x)}^t(x)}, & x\in\left[0,x_f^t\right]\\ 0, & x>x_f^t\end{cases}$$

The total number of households in the city is the sum of the number of households located at each distance from centre: $N^t=\int_0^{x_f^t} n^t(x)\mathrm{d}x$. We can distinguish the total number of households in the city ($N^t=N_1^t+N_2^t$), where $N_1^t=\int_0^{x_f^t}\left(2-K(x)\right)n^t(x)\,\mathrm{d}x$ are the rich households and $N_2^t=\int_0^{x_f^t}\left(K(x)-1\right)n^t(x)\mathrm{d}x$ the poor households.

The key of the model is amenities function $a^t(x)$. The basic assumption is that at each period, in the zones where the rich households are localised, as well as in their vicinity, the amenities level increases (modern amenities), this increase being added at the level of amenities inherited from the previous periods (i.e. the modern amenities become historical amenities). At the same time, the amenities decrease in the rich areas, near the poor areas, because their proximity constitutes a desamenity for the rich households.

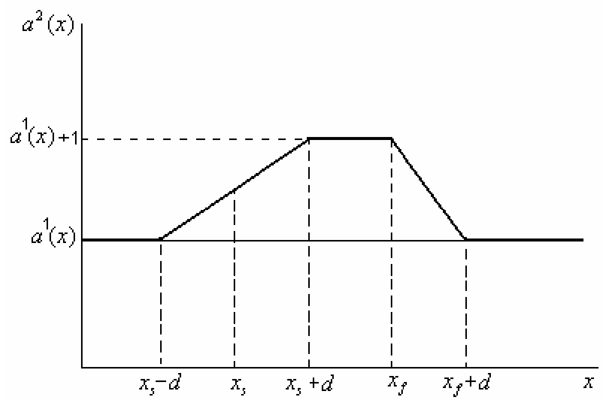
At the first period, there are no amenities or they are constant $a^0(x)=a$ (the city is located in a perfectly plane plain, without topographic specificities). This situation corresponds to the standard urban models in the tradition of Alonso (1964). Fujita (1989) shows that with identical transportation costs for the two social categories, the bid-functions are decreasing with the distance to the centre. Thereafter, there is only one point of segregation x_s and the city's border x_f is unique. This localisation schema corresponds is typical for the American cities: the rich households live the periphery and the poor in the downtown.

The amenities level at time *t* depends on their past level (historical amenities) and on the localization of the rich and poor households (modern amenities). In the rich areas and their proximity, the amenities are increasing. We can explain this assumption by a better quality of the buildings, but also of the environment. We suppose that, between periods, in the rich zones that are far away from the poor zones, the amenities increase with a unit compared to the previous period. In the proximity zones this amenities improvement is decreasing linearly with the distance to the segregation point. We define *d* as the distance from where one feels no longer positive externalities.

The amenities are influenced negatively by the proximity of poor areas. Thus, the level of amenities starts to decrease not at the segregation point but a certain distance before. To simplify the model, we can consider this distance equal to *d* which is called the proximity distance (the maximum distance where are social externalities between the two social classes).

For example, if there is only one segregation segregation x_s (the city divided in two completely segregated areas) and if the rich households live in the periphery, the amenities function at the second period is represented graphically:

Figure 1 : Amenity function at the second period



The dissymmetry of the amenity function in figure 1 is explained by the fact that in the proximity zone $x \in [x_s - d, x_s + d]$, there is a double effect. First, the amenities increase in the poor area, because of the proximity of rich households $x \in [x_s - d, x_s]$. But there is a negative effect in the rich area, because of the proximity of the poor households $x \in [x_s, x_s + d]$. Outside the city $x \in [x_f, x_f + d]$, since there is no proximity with the poor households, the only effect is the presence of rich households.

With this modelling, the amenities will be unlimited in time. To solve this problem, we supposed that they suffer a constant depreciation at a fixed rate $\delta, (0 < \delta < 1)$. Thus, the amenity function with constant depreciation, when there are *J* segregation points, is:

$$a^{t+1}(x) = \begin{cases} (1-\delta)\, a^t(x) + (1 - K(x)), & \text{zones without proximity} \\ \max \left\{ \begin{aligned} &(1-\delta)\, a^t(x) + \frac{x + d - x_s^t(j)}{2d} \\ &(1-\delta)\, a^t(x) + \frac{x_s^t(j) - x + d}{2d} \end{aligned} \right\}, & \text{proximity zones} \\ (2 - K(x_f^t))\left((1-\delta)\, a^t(x) + 1 + \frac{x_f^t - x}{d} \right), & \text{outside the city} \end{cases}$$

We note that if a rich zone is surrounded by the poor, the amenities are symmetrical: the two effects of proximity are identical from the both sides. This symmetry is lost when the rich households occupy the farther zone of the city centre.

With this formalisation, the amenities will be limited, and their maximum level in a stationary state will be reached in the areas which were occupied successively by the rich households. Thus, after an infinity of periods, this maximum level is $1/\delta$ [1]

3. NUMERICAL SIMULATIONS

In our simulations, we are analyzing two different scenarios: European and American urban structure. The reference scenario shows that a city can evolve from an American social structure to a European one. The second shows that sometime a city can be "blocked" in a social structure of American type.

In order to solve the numerical simulations, we use a Cobb Douglas utility function $U_i\left(z, s, a^t(x)\right) = z^{\alpha} s^{\beta} \left(a^t(x)\right)^{\gamma_i}$, with $\alpha + \beta = 1$. Because the rich households have stronger preferences for amenities than the poor and for computational reasons we pose $\gamma_1 = \gamma$ and $\gamma_2 = 0$. The transportation costs are linear with the distance and identical for the two social

[1] To determine that, we put the condition that the amenities where they increase more (in the rich areas) remain constant:: $a^{t+1}(x) = a^t(x) \Leftrightarrow (1-\delta)\, a^*(x) + 1 = a^*(x) \Leftrightarrow a^*(x) = 1/\delta$

categories: $C_i^t(x) = c^t x$. We choose identical costs in order to avoid the effect of the differentiated transportation costs on the city's structure and to highlight the role played by the amenities.

With this functional forms we are able do determine exactly the bid-rent functions $\psi_i^t(x, u_i^t) = A\left(y_i^t - c^t x\right)^{1/\beta} a^t(x)^{\gamma_i/\beta}(u_i^t)^{-1/\beta}$ and the bid-max lot size functions $S_i^t(x, u_i^t) = \alpha^{-\alpha/\beta}\left(y_i^t - c^t x\right)^{-\alpha/\beta} a^t(x)^{-\gamma_i/\beta}(u_i^t)^{1/\beta}$, where $A = \beta\alpha^{\alpha/\beta}$. The segregation point will be the solution of $x_s^t = \text{sol}\left\{\left(a^t(x)\right)^{\gamma}\left(y_1^t - c^t x\right)/\left(y_2^t - c^t x\right) = u_1^t/u_2^t\right\}$ and the city's border is $x_f^t = \text{sol}\left\{\left(y_{K(x_f)}^t - c^t x\right)\left(a^t(x_f)\right)^{\gamma} = \left(RA^t/A\right)^{\beta} u_{K(x_f)}^t\right\}$.

What differentiate the two series of simulations are the values of certain key parameters. According to Tivadar and Jayet (2006) the long term European equilibrium is more restrictive than the American equilibrium and the factors which support the existence of the European type exploit two dimensions. First, we mention the factors which increase the role played by the amenities in the space structuring: strong preference of the rich households for amenities and a weak depreciation of the amenities what leads to a high stationary level. The other factors have a direct impact on the biddings of the two categories, by increasing those of the rich households compared to the poor: strong utility and income differences.

In our simulations we will especially exploit the preferences of the rich households for amenities. Very strong preferences for amenities determine at long term a European urban structure whereas weak preferences determine "blocked" American structure.

3.1. The reference scenario: European urban structure

The European equilibrium represents an urban structure characterized by a central localisation of the rich households and a peripheral localisation of the poor households.

In our simulations, the exogenous variables are defined as $\mu_t = \mu_{t-1} + \tau_\mu t^{-\varepsilon}$, where μ is the variable in question (income, utility level and agricultural rent), τ_μ represents the increment rate of the variables μ, *t* is the period of simulation, and ε ($0 \le \varepsilon \le \infty$) is a speed parameter that influences the variation in time: a high value of ε means a slow evolution of the exogenous variables. For $\varepsilon = 0$, the variables will follow a linear evolution and for $\varepsilon = \infty$ they are constant in time. The transportation costs are constant and identical for the two categories, in order to avoid their influence on the households' localisation.

Table 1: Parameters value in the European scenario

Parameter	Rich Households	Poor Households
$\alpha / \beta / \gamma$	0,6 / 0,4 / 0,45	0,6 / 0,4 / 0
Income Income increment rate	100 5	90 4,95
Utility level Utility increment rate	10 0,5	7 0,3
Agricultural rent Increment rate of the agricultural rent	3 0,1	
Depreciation rate of amenities	0,10	
Proximity distance	5	
Amenity level at first period	1.5	
ε	1/8	

The income is expressed in K/period. The utility levels were determined to obtain a satisfactory solution. The proximity distance corresponds to 50 meters. Whatever their initial level, the amenities will be limited at $1/\delta$, which will be their stationary state. For a 10% depreciation rate, the amenities will bi limited to 10. We choose $\gamma_2 = 0$ (the poor do not have preferences for amenities) because we regard these amenities as a superior good. Moreover, if the rich and the poor have the same preferences for amenities ($\gamma_1 = \gamma_2$), the rich households will always occupy the peripheral area.

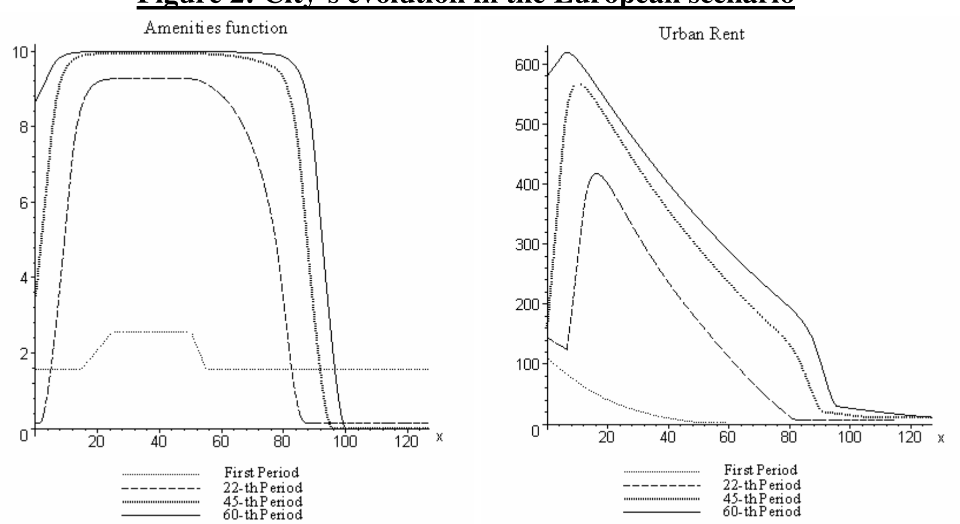
In this scenario the rich households' preferences for amenities are high (thus they will be attracted by the areas occupied before by the rich households). Whatever the initial value, the amenities will have the same stationary level. Thus, we can choose an initial level higher than the unit so that the two categories are present in the city. Also, for computational reasons, we set a rapid evolution of the exogenous variables (a low value of ε). With this set of parameters the amenities will play a significant role in the localisation decisions of households.

At the beginning, since the amenities are constant, the city is divided into two areas according to an American structure: the centre is occupied by the poor and the periphery by the rich (see Appendix 1.1). All functions have the usual forms of the standard urban models: decreasing convex bid-functions and urban rent with the distance. The houses size is an increasing function of distance and the household density is decreasing, with a discontinuity point at the segregation point. After this period the amenities increase in the area occupied by the rich households and in their proximity. The amenities are improved in the peripheral area which was inhabited successively by the rich households.

With time, the city size increases. The new rich households are attracted by the areas where, owing to the fact that their predecessors lived there or were localised in the proximity, the amenities are higher. The poor are remaining in the centre. But, wedged by the ring occupied by the rich households, they pile up until a part of them have interest to leave in periphery. The city is now composed by three areas: poor – rich – poor (Appendix 1.2). The functions have no longer the usual forms. The rich bin-function presents a strong increase in the rich area, according to their preferences for the amenities. They are ready to yield a part of their living space for amenities quality, which determines a concentration of the population.

More and more new poor households are delocalized towards the periphery, while the extension of the area occupied by the rich households gradually leads them to absorb the central zone. So, we find a European city with two areas, the rich living now in centre whereas the poor in suburbs. There is a strong comparative advantage in terms of amenities of the centre against the periphery (Appendix 1.3). This process of segregation is reinforced in time (Appendix 1.4) with a stronger differentiation between the two zones in terms of amenity quality, land prices and household density. . Practically, in terms of structure, the city does not change any more (Appendices 1.3 and 1.4). This evolution is explained by an amenity level closed to the stationary state and also by the concave evolution of the exogenous variables (and consequently the bid functions).

<u>Figure 2: City's evolution in the European scenario</u>



In figure 2, we represent the evolution of the amenity function and of the urban rent. This representation is very useful because it enables us to see the "lock-in" effect of the rich households. Their initial localization is peripheral, but with the city's development, their area does not represent any more the external one of the city, but the centre. The urban rent strongly increases in the rich area.

3.2 American scenario

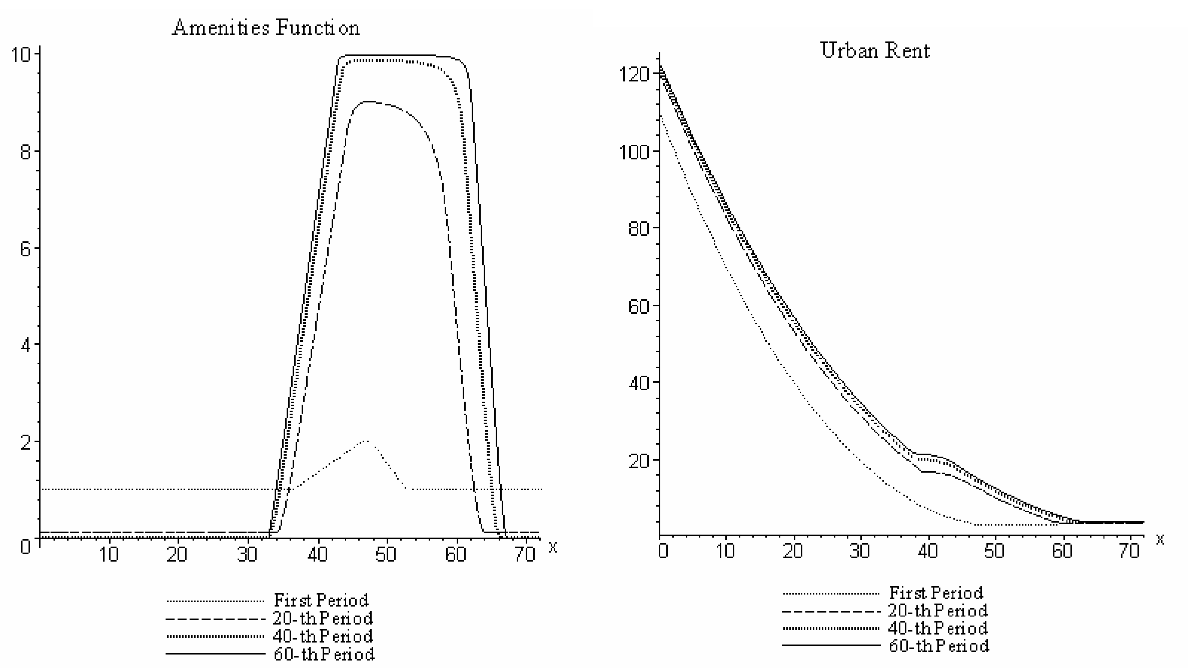
An American urban structure represents a spatial structure characterized by a complete segregation between the two social groups, with a central localisation of the poor households and a peripheral localisation of the rich households.

In order to reach this equilibrium, we start from the reference simulation and we decrease the role played by the amenities in the space structuring, decreasing the rich preference for amenities. Also we reduced the speed of variation of exogenous parameters:

Table 2: Parameters changes for the American scenario

Parameter	Rich Households	Poor Households
γ	0,15	0
ε	1	

Figure 3: City's evolution in the American scenario



For this set of parameters we find the same localisation tendencies as in the reference simulation, but the amenities cannot change the localisation of the two social classes (Appendices 2.1-2.4). Thus the city is completely segregated into two areas: a poor centre and a rich periphery. The amenities are concentrated in rich suburbs. The rent increases in time, more accentuated in the rich area. We note that at the last period of simulation (Appendix 2.4) the amenity function and bid-functions have form are very close to their steady state form (see Tivadar and Jayet, 2006).

4. CONCLUSIONS

The main result of paper is that the historical development of a city, concretized in the formation of local amenities, plays a determining role in the social structuring of the urban space.

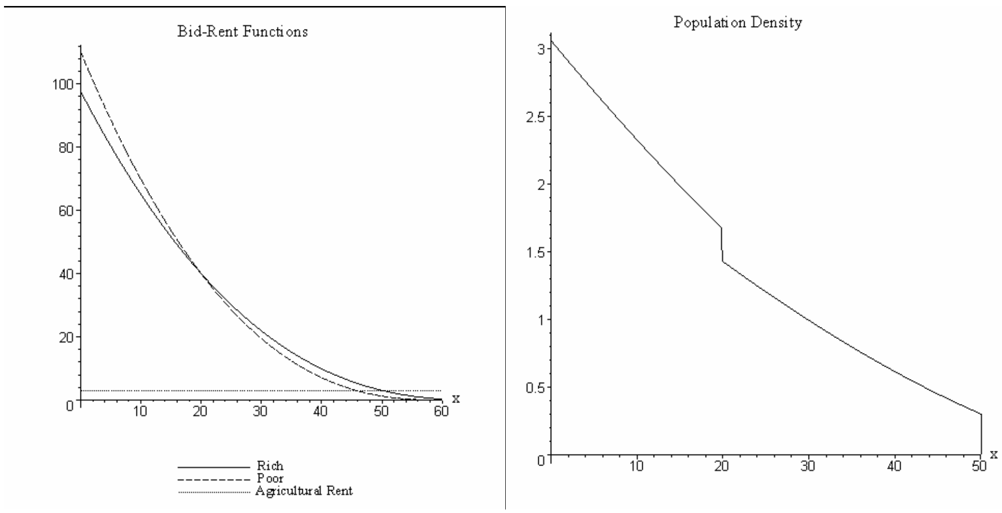
A possible explanation of contrasts between the American and European cities structure can be simply a difference in preferences, incomes and/or utility levels. In the case, with weak preferences of the rich households for amenities, in the long run, the rich are located in the periphery, while the poor households occupy the central area of the city. This type of social structure corresponds to the North-American cities. The rich households occupy the periphery and the amenities will be concentrated there.

In our dynamic simulations, we note that amenities can transform the structure of a city and that this process occurs in time. Thus another possible explanation for the difference between the spatial structure of the European and the North-American cities is the fact that the last ones are more recent and they did not arrive yet at their stationary state. This situation corresponds to the reference scenario. At the beginning the city has low amenities and the rich households locate towards the periphery, while the poor households occupy the centre. This type of social structure corresponds to the American cities, whose their history is recent, and thus, the amenities are very low. On the contrary, the European cities are "older". Therefore, initially the rich households were localised in the periphery, but because the city experienced a long development, the rich person remained there, which constitutes today the central area. Since the localization of the rich households determines an increase in the amenities level, the European centres have a strong comparative advantage in terms of amenities compared to the suburbs.

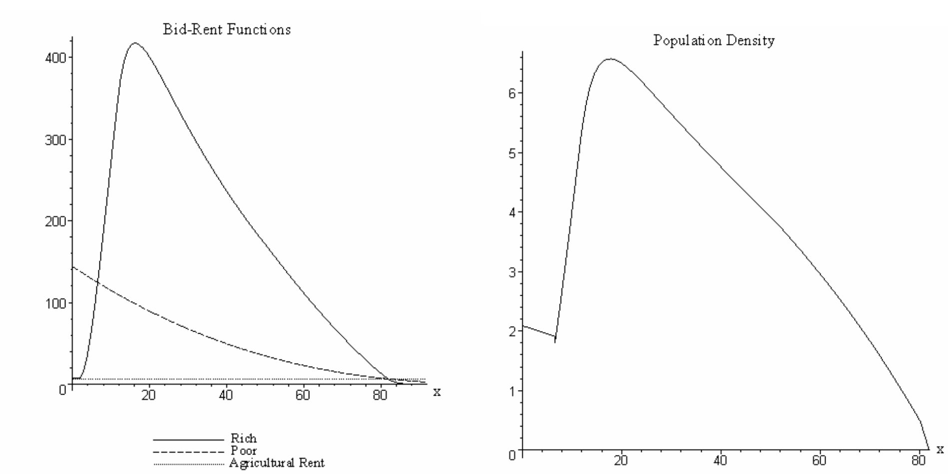
Appendix: Graphic results of the simulations

1. Reference scenario: European social structure

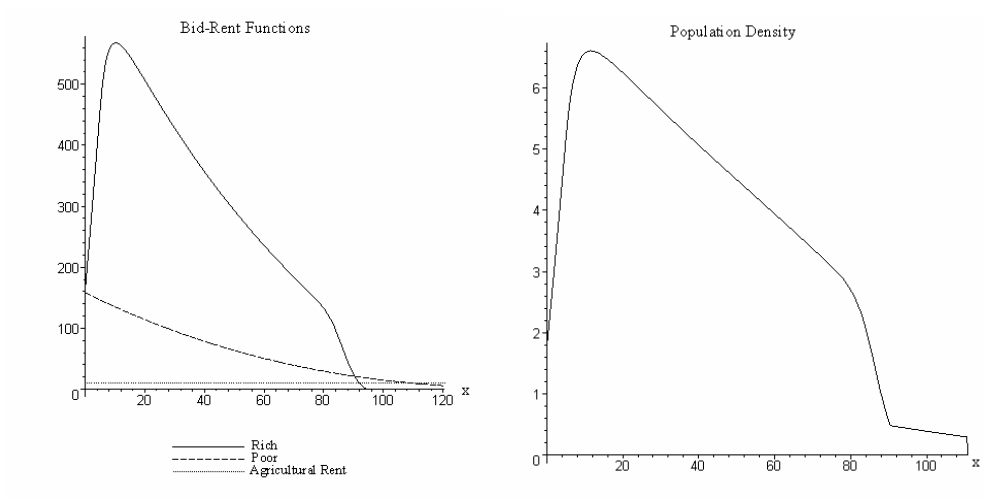
1.1. First period



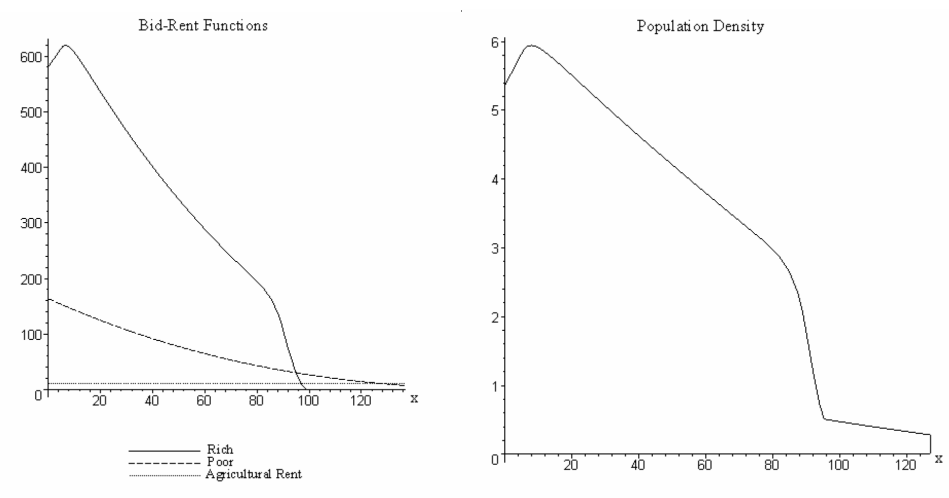
1.2. 19-th period



1.3. 65-th period

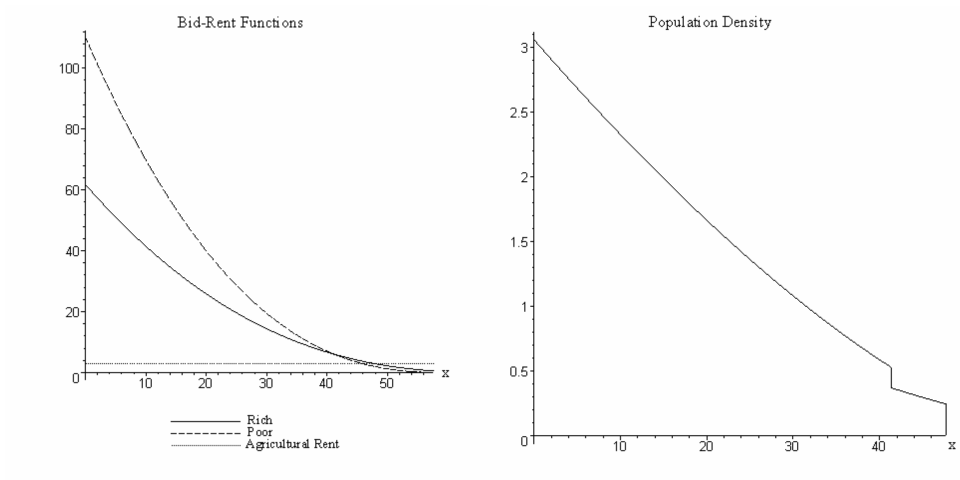


1.4. 72-th period

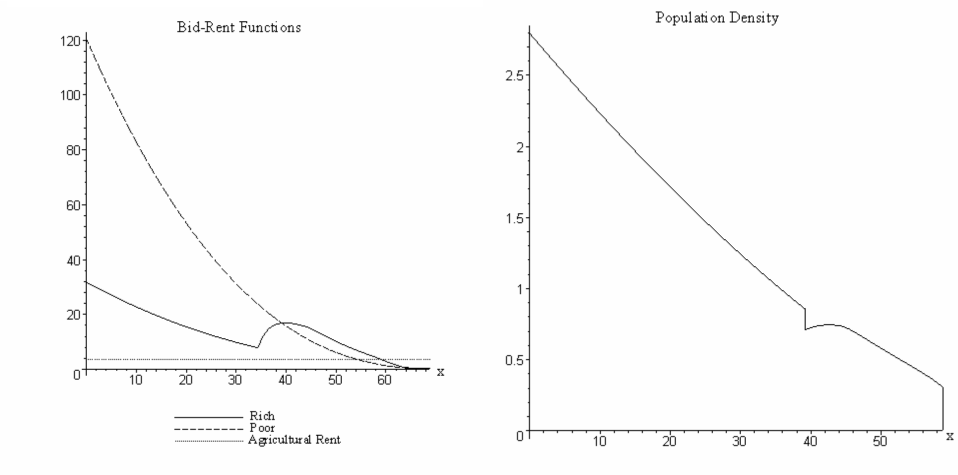


2. Second scenario: American social structure

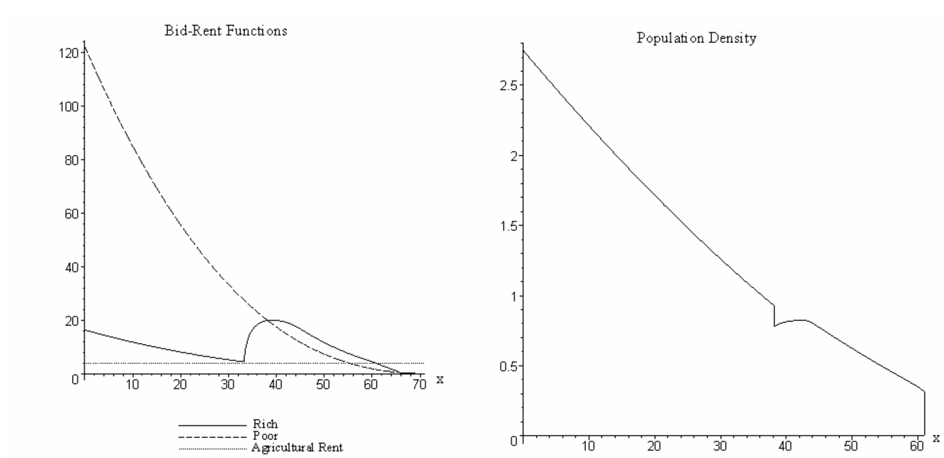
2.1. First period



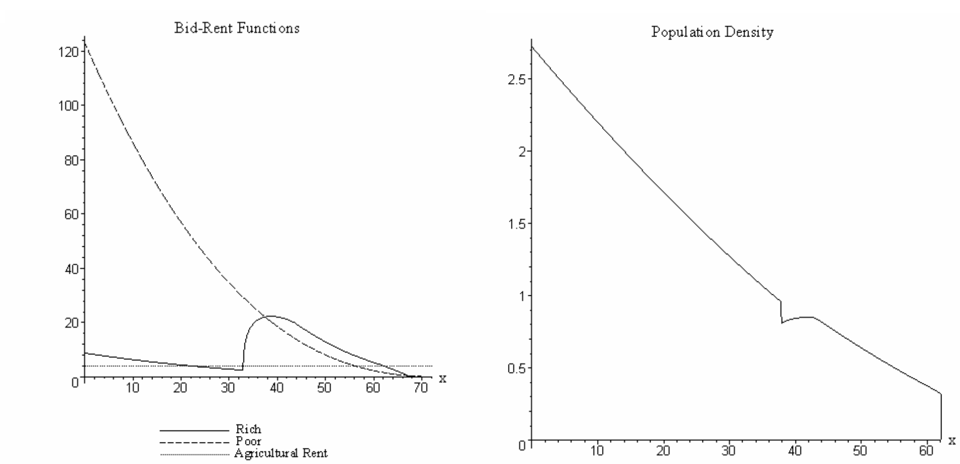
2.2. 20-th period



2.3. 40-th period



2.4. 60-th period



BIBLIOGRAPHY

ALONSO, William (1964): "Location and Land Use", *Cambridge, Harvard University Press*

BRUECKNER K. Jan, Jacques-François THISSE and Yves ZENOU (1999): "Why is central Paris rich and downtown Detroit poor?: An amenity-based theory"**,** *European Economic Review, Volume 43, Pages 91-107*

BRUECKNER K. Jan (2000): "Urban Growth Models with Durable Housing: An Overview", *Economics of Cities – Theoretical Perspectives, Chapitre 7, Pages 263-289*

FUJITA, Masahisa (1989): "Economic Theory: Land Use and City Size", *Cambridge University Press*

MILLS, E.S.(1967): "An Aggregative Model of Resource Allocation in a Metropolitan Area", *American Economic Review*, *Volume 57, Pages 197-210*

MIYAO, Takahiro (1987): "Dynamic Urban Models", *Handbook of Regional Science and Urban Economics , Volume 2, Chapitre 22, Pages 877-925*

MUTH, R.F. (1969): "Cities and Housing", *Chicago, University of Chicago Press*

TIVADAR, Mihai and JAYET, Hubert (2006): "Where it is better to live: in a European or American City", Work in progress