WHERE IT IS BETTER TO LIVE:
IN AN AMERICAN OR EUROPEAN CITY?

Mihai TIVADAR*
PhD Student MEDEE-EQUIPPE, Lille 1 University, France
ATER Lille 2 University, France

We examine the amenities consequences on the steady state social structure of a city and its efficiency when the spatial repartition of amenities is endogenously modified by the spatial distribution of social groups. Among the possible multiple long term equilibria we analyse two of them: a typical American structure (the poor located in the city's centre) and a European structure (the rich in the centre). The conditions of existence of a European equilibrium are more restrictive and included in the conditions necessary for an American equilibrium. An efficiency comparison between the two structure type shows that an American structure is more efficient.

1. INTRODUCTION

Our paper examines the role played by amenities in the social structuring of cities when the spatial repartition of amenities is endogenously modified by the spatial distribution of social classes. The idea of our model emerged from the observation of the contrasts existing between typical American and European cities structure. In most American cities, central locations are occupied by poor households, whereas in European cities, they are occupied by rich households.

For these contrasts, the literature proposed two explanations. In the standard urban models, the rich households are attracted by the city's central localisations when their transportation costs are much higher compared to the poor households.

Another explanation was proposed by Brueckner & alii (1999), based on the theory of local amenities. The European cities are characterized by a longer history. The amenities are mainly located in the centre (monuments, parks, boulevards, fine architecture, etc) which are the consequence of this history. If the rich demand for amenities is significant, such an advantage can be sufficient to attract the rich households to the central localisations, which corresponds to the typical European urban structure.

* mihai.tivadar@ed.univ-lille1.fr

According to Brueckner & alii (1999) the urban amenities are divided into three categories: natural amenities (which are generated by the topographic characteristics of the area), historical amenities (generated by the monuments, the buildings, the parks or other urban infrastructures which hold of the past) and modern amenities (which depend on the current conditions). In our paper, we suppose that the natural amenities do not cause differentiation of the urban space and we are interested only in the effects of the modern and historical amenities on the city's structure.

Our model is conceived in this theory framework. In this paper we are trying to overcome the limits of the original Brueckner's model: a static framework and exogenous amenities function. Thus, we explicitly consider the historical dimension of the process generating amenities: at each period, the equilibrium spatial structure of the city is determined by the spatial repartition of amenities; but, between periods, this repartition change, rich households generating amenities in the locations they occupy, and therefore the city's spatial structure changes.

We show that the endogenous generation of amenities has like consequence the existence of several long term equilibria. We are analysing only two types of equilibria: an American urban structure, with the poor located in the centre area and the rich at periphery, and a European equilibrium, characterized by a reversed location scheme: the rich in the centre and the poor in suburbs. The conditions of existence of a European equilibrium are more restrictive and included in the conditions necessary for an American equilibrium. We compare the two equilibria from an efficiency point of view. The efficiency comparison between these two opposite structures shows a superiority of American equilibrium.

The first part presents the theoretical model. Then we are analyzing the possibility of existence of multiple equilibria with an efficiency comparison. The last section is devoted to the conclusions.

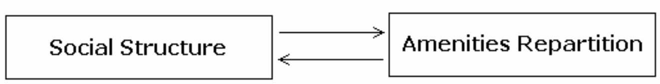
2 THE MODEL

We created a dynamic residential model, where the connection between periods is given by the transformation of the modern amenities into historical amenities. Our model belongs to the class of models without capital durability. This type of models was developed first by Alonso (1964), Mills (1967) and Muth (1969) within a static framework. We remain in the monocentric urban models tradition where the centre (CBD - Central Business District) is represented by a point in space and the only localisation variable is the distance to the centre (x).

There are two social classes, rich and poor households, differentiated by their income respectively y_1 and y_2 and by their preferences for the amenities. The utility of the households depends on the consumption of the composite good (z), whose price is standardized with the unit, on the living space (s) and the level of amenities ($a(x)$). We are using a Cobb-Douglas utility function $U_i\left(z,s,a^t(x)\right)=z^\alpha s^\beta \left(a^t(x)\right)^{\gamma_i}$, with $\alpha+\beta=1$. The rich households have stronger preferences for amenities than the poor $(\gamma_1>\gamma_2)$ and we pose $\gamma_1=\gamma$ and $\gamma_2=0$. This assumption is explained by the fact that we regard the amenities as a superior good. The transportation cost is linear with the distance and identical for the two social categories: $C_i^t(x)=c^t x$. We choose identical costs in order to avoid the effect of the differentiated transportation costs on the city's structure and to highlight the role played by the amenities.

The model is in an open city framework (there are no costs of migration). Thus, the utility level of each category is exogenous, equal to the national level (u^t_i) and the ctiy's population at each time is endogenous.

The urban dynamic development is defined as a chain of static equilibriums, where the connection between periods is given by the transformation of the modern amenities into historical amenities. Thus, at each period we determine the equilibrium localisation of each category and the effects of this urban structure on the amenities level. These effects will be taken into account during the following period and will have an influence in the new decisions of localisation:



At each period, the households maximize their utility under budgetary constraint:

$$\max_{z,s,x} U_i(z,s,a^t(x))=z^\alpha s^\beta a^t(x)^{\gamma_i} \qquad \text{b.c.} \qquad y_i^t-C_i^t(x)=z+R^t(x)s\,,$$

where $C_i^t(x)$ is the commuting cost to CBD and $R^t(x)$ is the market land rent at period t. At equilibrium, each household will reach a utility level equal to the national level u^t_i. We define the bid-function as the maximum price per space unit which the household can pay to reside at distance x by reaching the level of utility u^t_i:

$$\psi_i^t(x,u_i^t)=\max\left\{\frac{y_i^t-C_i^t(x)-z}{s}\,\middle|\,U(z,s,a^t(x))=u_i^t\right\}$$

This maximization gives the bid-function and the bid-surface functions:

$$\psi_i^t(x,u_i^t)=A\left(y_i^t-c^t x\right)^{1/\beta} a^t(x)^{\gamma_i/\beta}(u_i^t)^{-1/\beta}$$

$$S_i^t(x,u_i^t) = \alpha^{-\alpha/\beta}\left(y_i^t - c^t x\right)^{-\alpha/\beta} a^t(x)^{-\gamma_i/\beta}(u_i^t)^{1/\beta}$$

where $A = \beta\alpha^{\alpha/\beta}$.

The city's structure will be the result of competition for the land between the various usages (residential, agricultural). Each location will be occupied by the highest bidder. Thus, the urban rent will be the upper envelope of the bid-functions and the agricultural rent (the opportunity cost of the land):

$$R^t(x) = \max\left\{\psi_i^t(x), RA^t\right\}$$

where RA^t is the agricultural rent or the opportunity land cost at period *t*.

The segregation points between social classes are given by the solution of equalisation of the bid-functions:

$$x_s^t \equiv \mathrm{sol}\left\{\psi_1^t(x) = \psi_2^t(x)\right\} \Rightarrow x_s^t = \mathrm{sol}\left\{\frac{y_1^t - c^t x}{y_2^t - c^t x}\left(a^t(x)\right)^{\gamma} = \frac{u_1^t}{u_2^t}\right\}$$

We define a binary spatial variable $K(x)$ that specifies the social category of the household living at distance x:

$$K(x) = \begin{cases} 1, & \text{if } \psi_1^t(x) > \psi_2^t(x) \\ 2, & \text{if not} \end{cases}$$

The equilibrium residential boundary (or the city's border) is given by the point at which the bid-function of the category localised in the peripheral area of the city equals the agricultural rent:

$$x_f^t \equiv \mathrm{sol}\left\{\psi_{K(x)}^t(x) = RA^t\right\} \Rightarrow x_f^t = \mathrm{sol}\left\{\left(y_{K(x_f)}^t - c^t x\right)\left(a^t(x_f)\right)^{\gamma} = \left(\frac{RA}{A}\right)^{\beta} u_{K(x_f)}^t\right\}$$

We suppose that the land is allocated entirely to the residential use. Since our city is in a perfectly plane area, surface available for the residences to distance x is given by the perimeter of the circle $L(x) = 2\pi x$. Hence the equilibrium household distribution is given as the ratio between the surface available for the residential use and the lot size of each house:

$$n^t(x) = \begin{cases} \dfrac{2\pi x}{S_{K(x)}^t(x)}, & x \in \left[0, x_f^t\right] \\ 0, & x > x_f^t \end{cases}$$

The household density is defined as the number of households per unit of land at distance x:

$$\rho^t(x) = \begin{cases} \dfrac{n^t(x)}{2\pi x} = \dfrac{1}{S^t_{K(x)}(x)}, & x \in \left[0, x^t_f\right] \\ 0, & x > x^t_f \end{cases}$$

The total number of households in the city is the sum of the number of households located at each distance from centre: $N^t = \int_0^{x^t_f} n^t(x)\mathrm{d}x$. We can distinguish the population of each category ($N^t = N^t_1 + N^t_2$), where N^t_1 are the rich and N^t_2 the poor households:

$$N^t_1 = \int_0^{x^t_f} \left(2 - K(x)\right) n^t(x)\ \mathrm{d}x = 2\pi \int_0^{x^t_f} \left(2 - K(x)\right) \frac{x}{S^t_{K(x)}(x)}\ \mathrm{d}x \tag{1}$$

$$N^t_2 = \int_0^{x^t_f} \left(K(x) - 1\right) n^t(x)\mathrm{d}x = 2\pi \int_0^{x^t_f} \left(K(x) - 1\right) \frac{x}{S^t_{K(x)}(x)}\ \mathrm{d}x \tag{2}$$

The key of the model is amenities function $a^t(x)$. The basic assumption is that at each period, in the zones where the rich households are localised, as well as in their vicinity, the amenities level increases (modern amenities), this increase being added at the level of amenities inherited from the previous periods (i.e. the modern amenities become historical amenities). At the same time, the amenities decrease in the rich areas, near the poor areas, because their proximity constitutes a desamenity for the rich households.

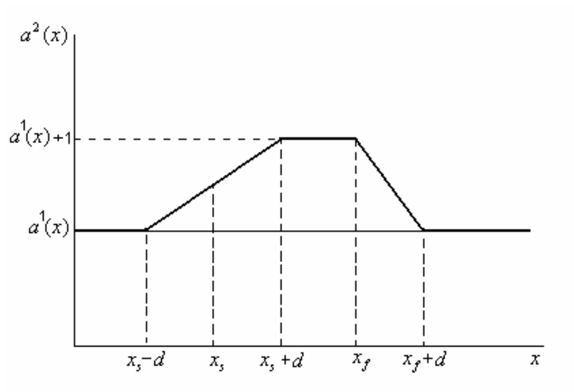
At the first period, there are no amenities or they are constant $a^0(x)=a$ (the city is located in a perfectly plane plain, without topographic specificities). This situation corresponds to the standard urban models in the tradition of Alonso (1964). Fujita (1989) shows that with identical transportation costs for the two social categories, the bid-functions are decreasing with the distance to the centre. Thereafter, there is only one point of segregation x_s and the city's border x_f is unique. This localisation schema corresponds is typical for the American cities: the rich households live the periphery and the poor in the downtown.

The amenities level at time t depends on their past level (historical amenities) and on the localisation of the rich and poor households (modern amenities). In the rich areas and their proximity, the amenities are increasing. We can explain this assumption by a better quality of the buildings, but also of the environment. We suppose that, between periods, in the rich zones that are far away from the poor zones, the amenities increase with a unit compared to the previous period. In the proximity zones this amenities improvement is decreasing linearly with the distance to the segregation point. We define d as the distance from where one feels no longer positive externalities.

The amenities are influenced negatively by the proximity of poor areas. Thus, the level of amenities starts to decrease not at the segregation point but a certain distance before. To simplify the model, we can consider this distance equal to d which is called the proximity distance (the maximum distance where are social externalities between the two social classes).

For example, if there is only one segregation segregation x_s (the city divided in two completely segregated areas) and if the rich households live in the periphery, the amenities function at the second period is represented graphically:

Figure 1 : Amenity function at the second period



The dissymmetry of the amenity function in figure 1 is explained by the fact that in the proximity zone $x \in [x_s - d, x_s + d]$, there is a double effect. First, the amenities increase in the poor area, because of the proximity of rich households $x \in [x_s - d, x_s]$. But there is a negative effect in the rich area, because of the proximity of the poor households $x \in [x_s, x_s + d]$. Outside the city $x \in [x_f, x_f + d]$, since there is no proximity with the poor households, the only effect is the presence of rich households.

With this modelling, the amenities will be unlimited in time. To solve this problem, we supposed that they suffer a constant depreciation at a fixed rate $\delta, (0 < \delta < 1)$. Thus, the amenity function with constant depreciation, when there are J segregation points, is:

$$a^{t+1}(x) = \begin{cases} (1-\delta)\,a^t(x) + (1 - K(x)), & \text{zones without proximity} \\ \max\left\{ \begin{aligned} &(1-\delta)\,a^t(x) + \frac{x + d - x_s^t(j)}{2d} \\ &(1-\delta)\,a^t(x) + \frac{x_s^t(j) - x + d}{2d} \end{aligned} \right\}, & \text{proximity zones} \\ (2 - K(x_f^t))\left((1-\delta)\,a^t(x) + 1 + \dfrac{x_f^t - x}{d} \right), & \text{outside the city} \end{cases}$$

We note that if a rich zone is surrounded by the poor, the amenities are symmetrical: the two effects of proximity are identical from the both sides. This symmetry is lost when the rich households occupy the farther zone of the city centre.

3 STATIONNARY EQLUIBRIA STUDY

In this part we will study the possibility of existence of multiple equilibria. Since there are an infinity possibilities of localization, we are interested only in two extreme cases: the rich located in the center versus the poor located in the center. Concretely, we want to determine under which conditions these two types of structure can be a long-run equilibrium.

3.1. Amenity function and bid-functions at stationary state

Before analysing the steady state urban forms, we need to determine the stationary state of the main functions in our model: the amenities and the bid functions. For a depreciation rate δ, the maximum level of amenities (in the zones which was inhabited successively by the rich households) is $1/\delta$[1]. For example, for a fixed depreciation of 10% the maximum level of the amenities is 10. In the zones inhabited successively by the poor households and which are not in the proximity of the rich zones, but also outside the city (except vicinity of a rich zone) the amenities suffered a continuous depreciation, thus they tend towards zero.

Now, we will determine the amenity function in the proximity areas. The condition that the amenities are in a stationary state is $a^{t+1}(x) = a^t(x)$. In the proximity zones where the rich are located farther than the poor, the steady state amenity function is increasing with the distance:

$$a^*(x) = \frac{x + d - x_s^*(j)}{2\delta d}$$

If in the proximity zones the rich live closer to centre than the poor, the amenity function is now decreasing with the distance to the centre:

$$a^*(x) = \frac{x_s^*(j) + d - x}{2\delta d}$$

[1] To determine this stationary level, we put the condition that where the amenities remain constant in the rich area: $a^{t+1}(x) = a^t(x) \Leftrightarrow (1-\delta)a^*(x) + 1 = a^*(x) \Leftrightarrow a^*(x) = 1/\delta$

If the most peripheral zone of the city is rich, a positive effect will exercise even out of the city. The steady amenity function is still decreasing with the distance but steeper than the previous one:

$$a^*(x) = \frac{x_f^* + d - x}{\delta d}$$

Thus, the global amenity function at stationary state is:

$$a^*(x) = \begin{cases} \dfrac{\left(2 - K(x)\right)}{\delta}, & \text{zones without proximity} \\ \max\left\{\begin{array}{c} \dfrac{x + d - x_s^*(j)}{2\delta d} \\ \dfrac{x_s^*(j) + d - x}{2\delta d} \end{array}\right\}, & \text{proximity zones} \\ \left(2 - K(x_f^t)\right)\left(\dfrac{x_f^* + d - x}{\delta d}\right), & \text{out of the city} \end{cases}$$

To simplify the notation, the steady state symbols of the variables are removed, but it is known that they are in their stationary state. For the poor households, the bid-function is easy to calculate, because they do not have preferences for the amenities so they have a usual form of the bid function (decreasing and convex function of the distance from the centre):

$$\psi_2(x, u_2) = A\left(y_2 - cx\right)^{1/\beta} (u_2)^{-1/\beta}$$

The rich bid function is sensitive to amenities, so its form will be strongly influenced by the amenities stationary level in every location. In the areas occupied successively by the poor households, and which are far away from the rich area the rich bid-function is zero because there are no amenities: $\psi_1^a(x) = 0$.

In the proximity areas where the amenities increase with the distance (the left side of a rich zone), to determine the stationary rich bid-function, it is necessary to replace the steady amenity function in the bid-function of the rich households:

$$\psi_1^b(x, u_1) = A\left(2d\delta\right)^{-\gamma/\beta}\left(y_1 - cx\right)^{1/\beta}(x + d - x_s)^{\gamma/\beta}\left(u_1\right)^{-1/\beta}$$

There is a double effect on the form of this function: a negative direct effect (the distance to the centre) and a positive one, played by the increase of the amenities level. Thus, this function is increasing until $\tilde{x} = \left[\gamma_1 y_1 + c\left(x_s - d\right)\right] / c\left(\gamma_1 + 1\right)$ and decreasing after this value. One can check easily that $\tilde{x} > x_s - d$, but $\tilde{x} < x_s + d$ only if $y_1 < c\left(x_s + d + 2d/\gamma_1\right)$. Therefore, the rich bid function in this interval is increasing and in certain situations (for example when the transportation costs are significant compared to incomes, the distance effect carries on the amenities effect) it can be decreasing starting from a certain distance $\tilde{x}$.

In the rich areas, where the amenities are constants at their higher level, the rich bid function take this form:

$$\psi_1^c(x,u_1) = A\delta^{-\gamma/\beta}\left(y_1 - cx\right)^{1/\beta}\left(u_1\right)^{-1/\beta}$$

We note that this function equal the rich bid-function without amenities multiplied by a constant. Thus, the bid-function is convex and decreasing with the distance to the centre. The rich bid-function in the proximity areas where the amenities are decreasing, the rich bid function is also decreasing and convex but steeper than $\psi_1^c(x,u_1)$ because the distance negative effect is reinforced by the decrease of amenity level:

$$\psi_1^d(x,u_1) = A\left(2\delta d\right)^{-\gamma/\beta}\left(y_1 - cx\right)^{1/\beta}(x_s + d - x)^{\gamma/\beta}\left(u_1\right)^{-1/\beta}$$

If the farthest area is occupied by the rich households, their bid function outside the city is even more stepper because the amenity function slope is also steeper

$$\psi_1^e(x,u_1) = A\left(d\delta\right)^{-\gamma/\beta}\left(y_1 - cx\right)^{1/\beta}(x_f + d - x)^{\gamma/\beta}\left(u_1\right)^{-1/\beta}$$

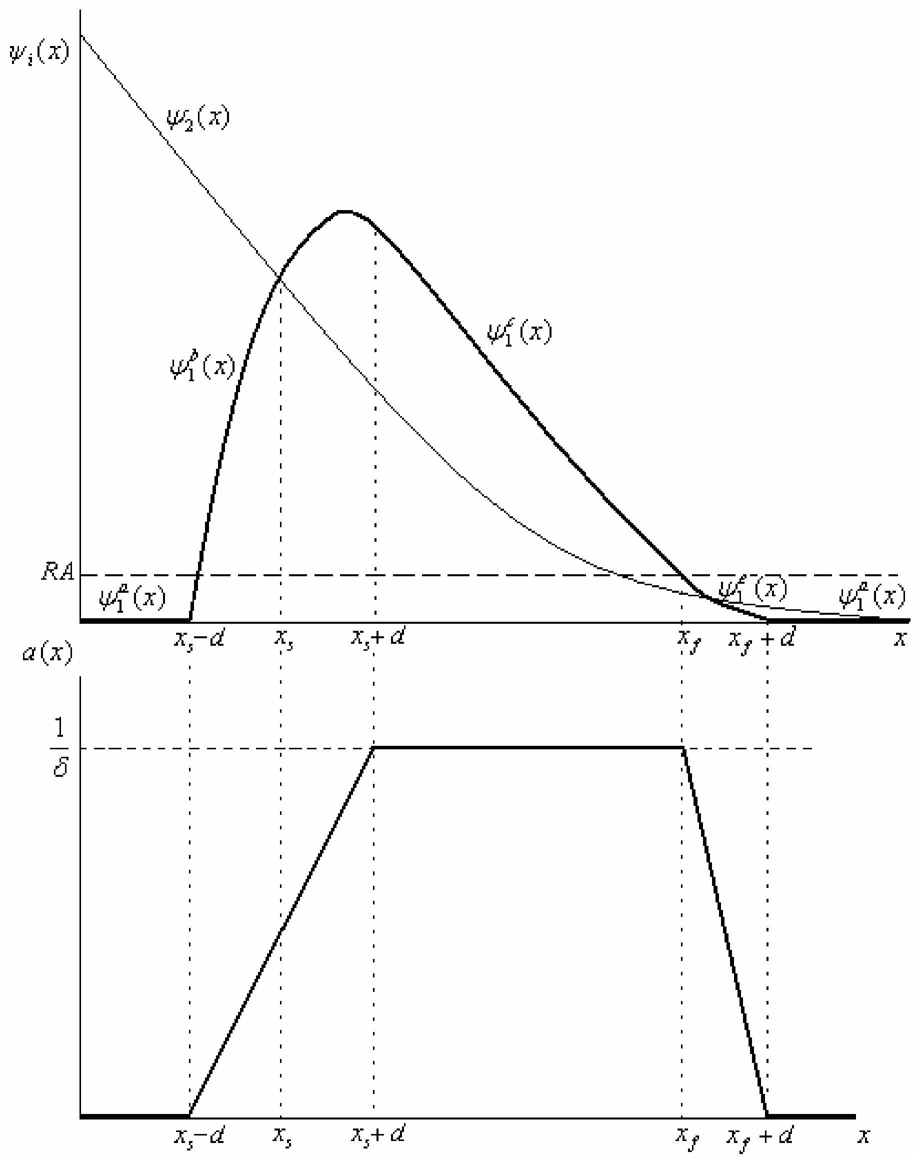
Now we can study of the steady state equilibria. We take the situation when all the variables are stationary and we want to show that the two spatial configurations (American and European type) can be a long-run equilibrium.

3.2. The existence of the American type equilibrium

We will analyze first the American type equilibrium, characterised by a central localisation of the poor households and a peripheral localisation of the rich. As depicted in Figure 2, the amenities are nulls in the centre occupied by the poor households, then increasing in the proximity zone, constant in the rich zone, and finally decreasing until zero outside the city. Since the poor households are insensitive to amenities, their bid-function $\psi_2(x)$ is continuous and decreasing. For the rich households, there are four situations. In the areas without amenities ($x \in \left(0, x_s - d\right)$ and $x \in \left(x_f + d, \infty\right)$), the rich bid function is zero: $\psi_1(x) = \psi_1^a(x) = 0$. In the proximity zone, where amenities are increasing with the distance ($x \in \left[x_s - d, x_s + d\right]$), the rich bid-function $\psi_1(x) = \psi_1^b(x)$ is increasing too, and in some situations, the distance effect can overcome the amenity effect and than the bid function becomes decreasing. In the rich zone where amenities are constant, their bid function

$\psi_1(x) = \psi_1^c(x)$ is decreasing because of the distance effect. Finally, at the city border ($x \in \left[x_f, x_f + d\right]$), the amenities reduction reinforces the distance effect, the bid-function $\psi_1(x) = \psi_1^e(x)$ becoming steeper:

Figure 2: The bid-functions and the amenities at American long-run equilibrium



We determine first the equilibrium level of the endogenous variables and then we will verify under which conditions this equilibrium exists. The segregation point is obtained by the equalisation of the bid functions in the proximity area: ($\psi_1^b(x) = \psi_2(x)$) :

$$x_s^a = \frac{1}{c}\left[\frac{(2\delta)^\gamma\, y_2 u_1 - y_1 u_2}{(2\delta)^\gamma\, u_1 - u_2}\right] \tag{3}$$

For a known agricultural rent, we can easily determine the city's border as the solution of $\psi_1^c(x) = RA$. The city's size will be function of the rich income and utility level:

$$x_f^a = \frac{1}{c}\left[y_1 - \left(\frac{RA}{A}\right)^{\beta} \delta^{\gamma} u_1 \right] \tag{4}$$

The first condition to verify is that the both social classes are present in the city. This means a positive segregation point (in order to have rich households in the city) and the presence of the poor household is conditioned by a segregation point inferior to city's border. Since the denominator of x_s^a is positive (see Appendix), these conditions can be rewritten:

$$x_s^a > 0 \Leftrightarrow u_2 < (2\delta)^{\gamma} \frac{y_2}{y_1} u_1 = u_2^M(u_1) \tag{5}$$

$$x_s^a < x_f^a \Leftrightarrow u_2 > (2\delta)^{\gamma} u_1 - 2^{\gamma} \left(\frac{A}{RA}\right)^{\beta} (y_1 - y_2) = u_2^m(u_1) \tag{6}$$

The condition (5) shows that the poor must have a lower utility than a maximum level $u_2^M(u_1)$ and higher than a minimum level $u_2^m(u_1)$. Now it is necessary to verify that this equilibrium is an American one: there is a single segregation point and the poor households live in the city's centre.

In the central area of the city $x < x_s^a - d$, the bid-function of the rich households are zero and those of the poor are positive. Thus, there isn't any segregation point and the zone will be occupied by the poor.

In the proximity area ($x \in \left[x_s^a - d, x_s^a + d\right]$), there is a point of segregation, but it should be verified that it is unique. This condition is surely respected if the rich bid-function is increasing in x_s, because the poor-bid function is decreasing. Thus, between the two bid-functions it will be only one intersection, and at the centre poor bid-function will be higher than the rich one.

If $cd > \gamma(y_1 - cx_s)$ the rich bid function $\psi_1^b(x)$ is decreasing in x_s^a. Then we will use Fujita (1989) technique, which consist in a comparison of the steepness of the two bid-functions. If the bid rent function of the poor households is steeper than that of rich, there is only one segregation point and the equilibrium location of poor households is closer to centre than that of rich households:

$$-\left.\frac{\partial \psi_1^b(x)}{\partial x}\right|_{x=x_s^a} < -\left.\frac{\partial \psi_2(x)}{\partial x}\right|_{x=x_s^a} \Rightarrow -\frac{\gamma}{d} < \frac{c(y_1 - y_2)}{(y_1 - cx_s^a)(y_2 - cx_s^a)}$$

This condition is always respected because the term on the left side is negative and the term on the right positive. In the peripheral area of the city ($x \in \left[x_s^a + d, x_f^a\right]$), we know that the two bid rent functions are continuous and decreasing with the distance. At the left limit of the zone, the rich bid function is higher than the poor bid function $\psi_1^c\left(x_s^a + d\right) > \psi_2\left(x_s^a + d\right)$. Thus, to verify that all locations of this area are occupied by the rich households and the pair of bid rents curves does not intersect, the rich bid function must be also higher than the poor function at the right limit of the zone: $\psi_1^c\left(x_f^a\right) > \psi_2\left(x_f^a\right) \Leftrightarrow u_2 > \delta^\gamma u_1 - \left(y_1 - y_2\right)\left(A / RA\right)^\beta$. This condition is true if the condition of presence of the rich households in the city (6) is respected[2]. Therefore, the only conditions necessary for an American equilibrium are those that ensure the presence of the two social groups in the city.

3.3. The existence of European type equilibrium

A European equilibrium is characterized by a central localisation of the rich households and a peripheral localisation of the poor households.

As we can see in Figure 3, the stationary amenities are constant at their higher level in the centre, decreasing in the proximity zone and zero in the poor zone and outside the city. This stationary form of the amenity functions corresponds well to the European cities situation, where the amenities are concentrated in the centre.

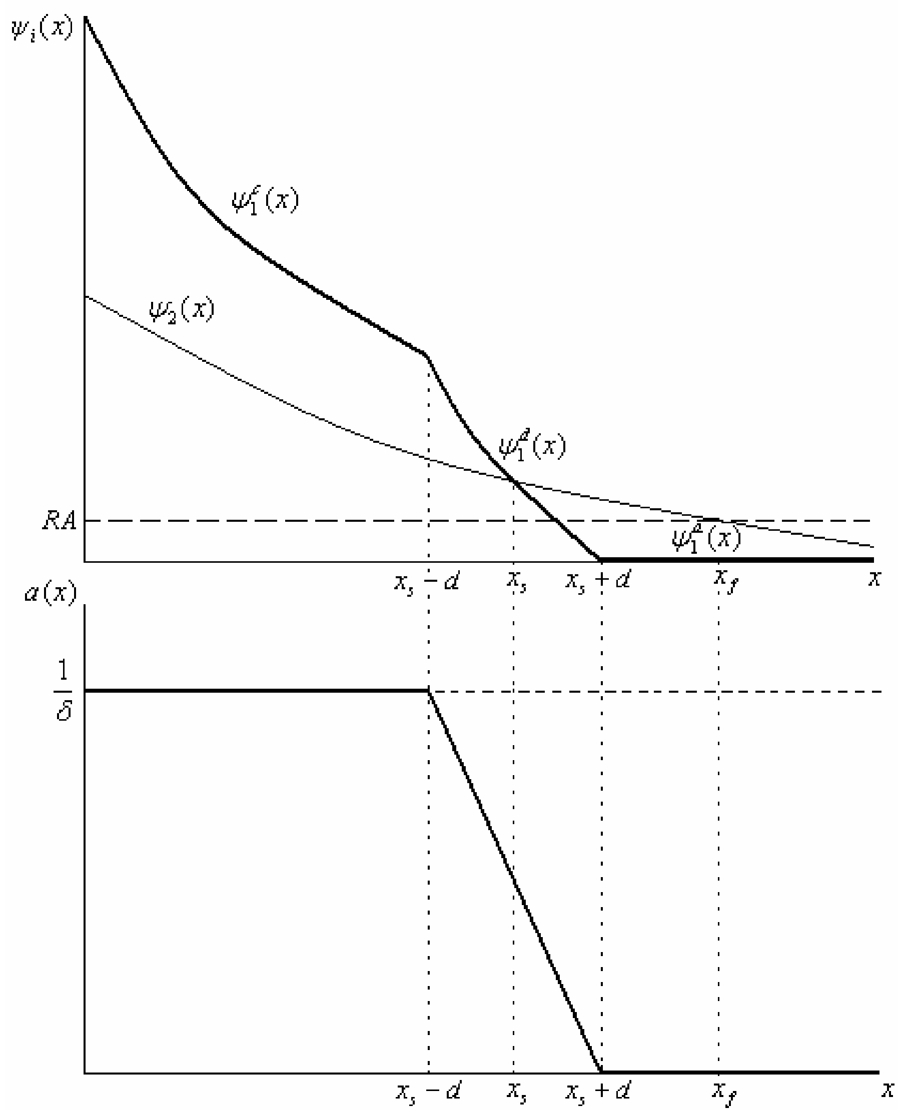
The rich bid function will be strongly influenced by the amenities function. In the central area $x \in \left(0, x_s - d\right)$, where the amenities are constant, the function ψ_1^c is decreasing and convex because of the distance effect. In the proximity zone $x \in \left[x_s - d, x_s + d\right]$, the rich function ψ_1^d becomes steeper, because the distance effect is reinforced by the amenities decrease. Finally, in the poor zone $x \in \left(x_s + d, x_f\right]$ and outside the city $x \in \left(x_f, \infty\right)$, where there are no amenities, the rich bid rent function is zero.

We will use the same approach as in the American case. We will first determine the equilibrium values of the structural endogenous variables (the segregation point and the city's boundary). With these values, we can then verify the conditions necessary for the existence of the European type equilibrium: presences of both categories in the city, uniqueness of the segregation point and central localisation of the rich households.

---

[2] $u_2 > u_2^m(u_1) > u_2^m(u_1)/2^\gamma = \delta^\gamma u_1 - \left(y_1 - y_2\right)\left(A / RA\right)^\beta$

Figure 3: The bid-functions and the amenities at European long-run equilibrium



The segregation point between the two social classes is obtained by the equalization of their biddings in the proximity area ($\psi_1^e(x) = \psi_2(x)$) :

$$x_s^e = \frac{1}{c}\left[\frac{(2\delta)^\gamma y_2 u_1 - y_1 u_2}{(2\delta)^\gamma u_1 - u_2}\right] = x_s^a \tag{7}$$

We find the same value of the segregation point as in the American case. That is explained by two facts. First, the poor bid function is identical in the two scenarios. Secondly, the rich bid functions in the two situations are symmetrical (reversed slope) and at the segregation point (in the middle of the proximity area) they are identical.

The city boundary depends this time on the utility level and the income of the poor households:

$$x_f^e = \frac{1}{c}\left[y_2 - \left(\frac{RA}{A}\right)^{\beta} u_2 \right] \tag{8}$$

As in the American case, we verify first the presence of the both categories in the city:

$$x_s^e > 0 \Leftrightarrow u_2 < (2\delta)^{\gamma} \frac{y_2}{y_1} u_1 = u_2^M (u_1) \tag{9}$$

$$x_s^e < x_f^e \Leftrightarrow u_2 > (2\delta)^{\gamma} u_1 - \left(\frac{A}{RA}\right)^{\beta} (y_1 - y_2) = u_2^{m\prime}(u_1) \tag{10}$$

Since the segregation point is identical for the two scenarios, the conditions (9) and (5) are identical. We note that the functions $u_2^m(u_1)$ from (6) and $u_2^{m\prime}(u_1)$ from (10) have the same slope $(2\delta)^{\gamma}$ and $u_2^{m\prime}(u_1) > u_2^m(u_1),\ \forall u_1$. This means that the condition (10) is more restrictive than the condition (6).

It should now be verified that the segregation point is unique and that the rich households live in the city centre and the poor households at periphery. For any localisation farther than $x > x_s^e + d$, the rich bid function is zero, while that of the poor households is positive. Thus, this area is occupied by the poor and there is not a segregation point.

In the proximity area $x \in \left[x_s^e - d, x_s^e + d \right]$, the two bid-functions are decreasing. So we will apply again the Fujita's steepness comparison technique. Therefore, the necessary and sufficient condition for a single segregation point and a central localisation of the rich is:

$$-\frac{\partial \psi_1^d(x)}{\partial x}\bigg|_{x=x_s^e} > -\frac{\partial \psi_2(x)}{\partial x}\bigg|_{x=x_s^e} \Rightarrow y_1 > y_2 + \frac{cd\left[(2\delta)^{\gamma} u_1 - u_2\right]^2}{\gamma (2\delta)^{\gamma} u_1 u_2} \tag{11}$$

Once again, European equilibrium is more restrictive than that American, because except the conditions of presence of the two social categories, European equilibrium requires an additional condition for the uniqueness of the segregation point and the "correct" localisation of the households.

According to the condition (11), the factors which support the European equilibrium existence are: a significant difference between incomes, strong preferences of the rich households for amenities, a short proximity distance and low transportation costs. Knowing that $(2\delta)^{\gamma} u_1 > u_2$ (see Appendix A), a low utility level of the rich households and strong level of the poor households also support the existence of this equilibrium.

It remains to be verified that the central area $x \in \left[0, x_s^e - d\right]$ is occupied exclusively by the rich households. The two bid-functions are continuous and decreasing in this area. Knowing that at the limit of this zone $\psi_1^c\left(x_s^e - d\right) > \psi_2\left(x_s^e - d\right)$ (because of the uniqueness condition of the segregation point in the proximity area), it is enough to verify that $\psi_1^c(0) > \psi_2(0)$:

$$y_1 > \delta^\gamma \frac{u_1}{u_2} y_2 \qquad (12)$$

We cannot determine between conditions (11) and (12) which one is more restrictive, but both presents the same elements supporting the European equilibrium. First, there are factors which increase the role played by the amenities in the space structuring: strong rich preference for amenities and a weak depreciation of the amenities what leads to high stationary level. The other factors have a direct impact on the bid capacity of the households, increasing the rich bid function compared to the poor households: strong difference between incomes, a small difference between utility levels.

3.4. The possibility of multiple equilibria

Because the presence of both social groups in the city needs some conditions on the utility levels of the households, we are seeking the couples $\left(u_1, u_2\right)$, for which the two localisation schemes can be equilibrium. For that, we will present another approach, more general, to find these presence conditions, which can be applied when the segregation point cannot be found analytically (for example when we don't have identical parameters of the utility functions). Since $x_s^a = x_s^e$, we will use only x_s to simplify the notation.

In the American scenario, the rich households occupy the peripheral area of the city and thus we can fix u_1 and then determine all the values of u_2 for which this spatial configuration is an equilibrium.

We know that there is a segregation point where the bid-functions are equal. Thus, we can express u_2 like a function of u_1 and x_s:

$$u_2 = f(x_s, u_1) = \left(2\delta\right)^\gamma \left(\frac{y_2 - cx_s}{y_1 - cx_s}\right) u_1 \qquad (13)$$

This function is decreasing on x_s because an enlargement of the poor area requires an increase in their bid function. Higher bid function needs a lower utility level, all other parameters constant.

We can determine the upper and lower value u_2 as a function of u_1. For that, we replace x_s with its minimal and maximal value in (13), which will give us the maximal and minimal value of u_2 :

$$u_2^M(u_1) = f(0, u_1) = (2\delta)^\gamma \frac{y_2}{y_1} u_1$$

$$u_2^m(u_1) = f(x_f^a, u_1) = (2\delta)^\gamma u_1 - 2^\gamma \left(\frac{A}{RA}\right)^\beta (y_1 - y_2)$$

It is not surprising that these expressions are identical to the conditions (5) and (6). The two functions are increasing and linear in u_1, $u_2^m(u_1)$ being steeper than $u_2^M(u_1)$[3]. The values of these functions in origins are $u_2^m(0) = 2^\gamma (A/RA)^\beta (y_2 - y_1) < 0$ and $u_2^M(0) = 0$. If $\delta < 1/2$, the functions have a slope lower than the unit. The intersection point of the two lines is: $u_1^{ii} = \delta^\gamma (A/RA)^\beta \, y_1$.

We must restrict the couples (u_1, u_2) only to the values with economic significance. The utility level cannot be negative, considering our specification of the utility function (Cobb-Douglas). Thus, for all the area $(0, u^i)$[4] $u_2^m(u_1) = 0$. With a depreciation rate of amenities $\delta < 1/2$, which seems plausible, the rich have a higher utility level than the poor households. Thus we can define the ensemble of couples (u_1, u_2) for which the American long terme equilibrium is possible: for $u_1 \in (0, u_1^{ii})$, $u_2 \in \left(\max\left(0, u_2^m(u_1)\right), u_2^M(u_1)\right)$.

We will use the same technique to find the necessary conditions for the European type equilibrium, knowing that the additional conditions (11) and (12) are necessary. This time we will fix u_2 because the poor households live in the periphery. Equalizing the two bid-functions at the segregation point we obtain u_1 as a function of x_s and u_2:

$$u_1 = g(x_s, u_2) = (2\delta)^{-\gamma} \left(\frac{y_1 - cx_s}{y_2 - cx_s}\right) u_2$$

---

[3] $\partial u_2^M / \partial u_1 = (2\delta)^{\gamma_1} (y_2/y_1) < \partial u_2^m / \partial u_1 = (2\delta)^{\gamma_1}$

[4] $u_1^{\,i} = \mathrm{sol}\left(u_2^m(u_1) = 0\right) = (y_1 - y_2)(A/RA)^\beta \, \delta^{-\gamma}$

This function is increasing on x_s. We calculate the extreme values of u_1 :

$$u_1^m(u_2) = g(0,u_2) = (2\delta)^{-\gamma} \frac{y_1}{y_2} u_2 \tag{14}$$

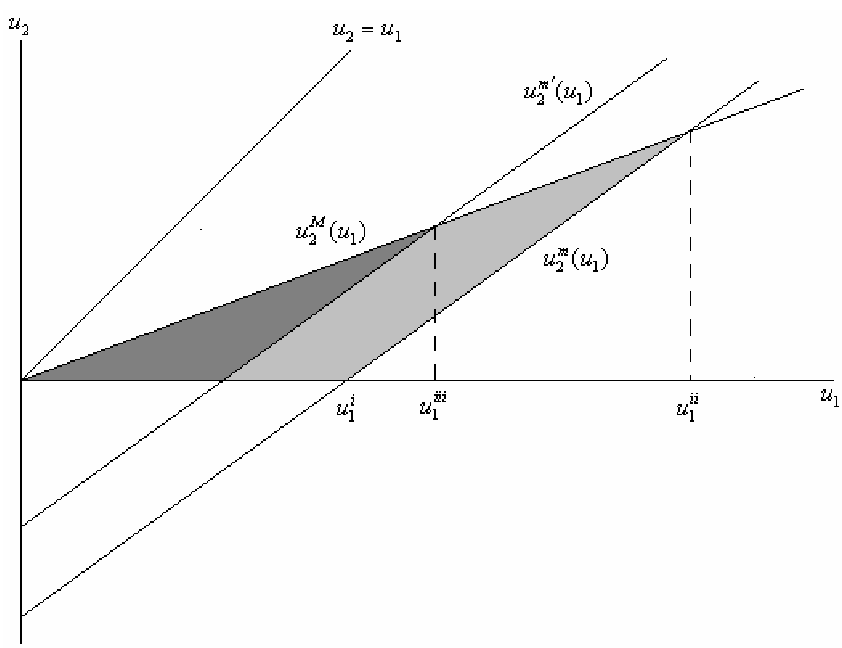
$$u_1^M(u_2) = g(x_f^e,u_2) = (2\delta)^{-\gamma} \left[u_2 + \left(\frac{A}{RA} \right)^{\beta} (y_1 - y_2) \right] \tag{15}$$

These two functions are increasing and linear with u_2, $u_1^m(0) = 0$ and $u_1^M(0) = (A/RA)^{\beta} (2\delta)^{-\gamma} (y_1 - y_2) > 0$. A comparison of the two scenarios conditions need to express the inversed functions of (14) and (15). Since the segregation point is identical in both scenarios, $u_1^m(u_2)$ is the inversed function of $u_2^M(u_1)$. We will find the same expression as the relation (10):

$$u_2^{m\prime}(u_1) = (2\delta)^{\gamma} u_1 - \left(\frac{A}{RA} \right)^{\beta} (y_1 - y_2)$$

With the assumption $\delta < 1/2$ we can define also the values of (u_1, u_2) that allow the European equilibrium (with the additional conditions (11) and (12)): for $u_1 \in (0, u_1^{iii})$[5], $u_2 \in \left(\max\left(0, u_2^{m\prime}(u_1)\right), u_2^M(u_1) \right)$.

Figure 4: The utility levels for which two equilibrium are possible



[5] $u_1^{iii} = (2\delta)^{-\gamma} (A/RA)^{\beta} y_1$ is the point of intersection between $u_2^M(u_1)$ et $u_2^{m\prime}(u_1)$

The couples (u_1,u_2) which simultaneously allow both equilibriums (when the conditions (11) and (12) are respected) are the same ones as those for European equilibrium because the European conditions are stricter and included in the American conditions. Graphically, theses couples of (u_1,u_2) are represented by the dark grey surface in the Figure 4. The clear grey surface represents the couples (u_1,u_2) which allow only the existence of the American equilibrium.

This analysis shows that under certain conditions, the two equilibriums are possible, but the conditions necessary for the European equilibrium are more restrictive. More, European equilibrium is "included" in the American one: if European equilibrium is possible, that American it is too, but not always the inverse.

4. OPTIMALITY COMPARISONS

We are interested to know which urban structure is more efficient: a European or American one. Thus, we will make comparisons in two cases: open-city and closed-city framework.

4.1. Surplus comparison in open-city framework

In this section we make an efficiency comparison between the two long term equilibrium urban structures within an open-city framework. In this case the utility levels of the two social groups are exogenous and identical for the two types of social structures. Thus, we will compare the surplus of the economy, which is represented only by the differential rent.

In the American case, the differential rent is:

$$RD^a = \int_0^{x_s} \left(\psi_2(x) - RA\right) \mathrm{d}x + \int_{x_s}^{x_s+d} \left(\psi_1^b(x) - RA\right) \mathrm{d}x + \int_{x_s+d}^{x_f^a} \left(\psi_1^c(x) - RA\right) \mathrm{d}x$$

and in a European spatial configuration, the differential rent becomes:

$$RD^e = \int_0^{x_s-d} \left(\psi_1^c(x) - RA\right) \mathrm{d}x + \int_{x_s-d}^{x_s} \left(\psi_1^e(x) - RA\right) \mathrm{d}x + \int_{x_s}^{x_f^e} \left(\psi_2(x) - RA\right) \mathrm{d}x$$

These two expressions cannot be calculated analytically, thus we will use numerical simulations to compare them. We choose first the level of the model parameters (preferences, incomes, transportation costs and the agricultural rent) and thereafter we determine all the couples (u_1,u_2) that respect the conditions of existence of the two types of equilibrium. For all these values of (u_1, u_2), we calculate the differential rent in the two scenarios and then we compare them.

Table 1 : The values of the stationary parameters

Parameter	**Rich Households**	**Poor Households**
$\alpha / \beta / \gamma$	0,6 / 0,4 / 0,25	0,6 / 0,4 / 0
Income	100	90
Transportation cost	1,5	
Agricultural rent	3	
Depreciation rate of amenities	0,10	
Proximity distance	5	

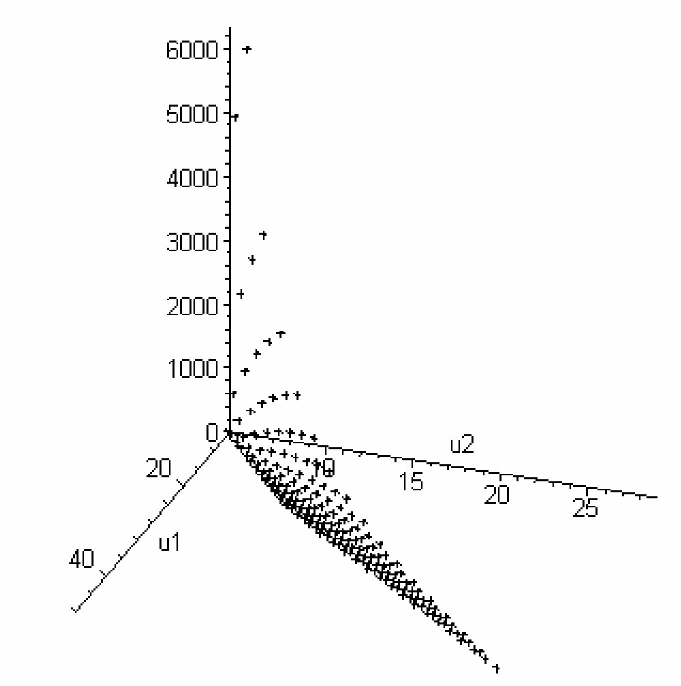
We choose $\gamma_2 = 0$ (the poor do not have preferences for the amenities) because we regard these amenities as a higher good. The incomes are expressed in K/period and the proximity distance corresponds to 50 meters.

According the conditions of presences of both categories in the city, the maximum value of u_1 for the American scenario is $u_1^{ii} = 58,46$ and for the European one is $u_1^{iii} = 49,16$. Since we are seeking the couples (u_1, u_2) which allow the existence of both equilibriums, the maximum value selected is u_1^{iii}.

In our simulations, we make evolve the two utility levels from 0 to u_1^{iii}, with a 0.1 step. After this series of simulations, we note that except some extreme values, for all (u_1, u_2), the American differential rent is higher than the European one. These exceptions appear when the utility levels are very high and there is a very important difference between these levels (the rich utility is almost the double of the poor). In these cases, the differential rents in the two situations are almost identical.

In Figure 5 we represented in 3D, the couples (u_1, u_2) which allow the existence of the two equilibriums and the difference between the American and European differential rent. In this graph, we can see that around the couple $(u_1 = 20,\ u_2 = 10)$ the difference between the differential rents is very small (see negative). Also, on this graph the constraints on the two levels of utility are visible. With low utility levels, the difference between the two differential rents is important.

Figure 5: Difference of the differential rents between the two urban structures



4.1. Welfare comparison in closed-city framework

In a closed-city framework, there is not migration between cities. Thus, the population of the city is exogenous and the utility levels become endogenous. The question that we try to find an answer is that for two cities with the same population, which structure is better for the citizens: the European or American one? For a given population ($\overline{n}_1$ and $\overline{n}_2$), we "let" the households locate following an American and European scheme, and then we compare their utility levels.

The equilibrium system is given by the equations determining the segregation point (expression (3) for the American scenario and (7) for the European scenario), by the equations determining the city's boundary (expression (4) and (8)) and the population constraints $\{N_1 = \overline{n}_1, N_2 = \overline{n}_2\}$, where N_1 and N_2 are the rich and the poor population determined in the model from equations (1) and (2).

The American population constraints are:

$$\left\{\begin{array}{l} \dfrac{2\pi}{B}\left(u_1^a\right)^{-1/\beta}(2\delta d)^{-\gamma/\beta}\displaystyle\int_{x_s^a}^{x_s^a+d} x\left(y_1-cx\right)^{\alpha/\beta}\left(x+d-x_s^a\right)^{\gamma/\beta}dx+\dfrac{2\pi}{B}\left(u_1^a\right)^{-1/\beta}(\delta)^{-\gamma/\beta}\displaystyle\int_{x_s^a+d}^{x_f^a} x\left(y_1-cx\right)^{\alpha/\beta\,\gamma/\beta}dx=\overline{n}_1 \\ \dfrac{2\pi}{B}\left(u_2^a\right)^{-1/\beta}\displaystyle\int_0^{x_S^a} x\left(y_2-cx\right)^{\alpha/\beta\,\gamma/\beta}dx=\overline{n}_2 \end{array}\right\}$$

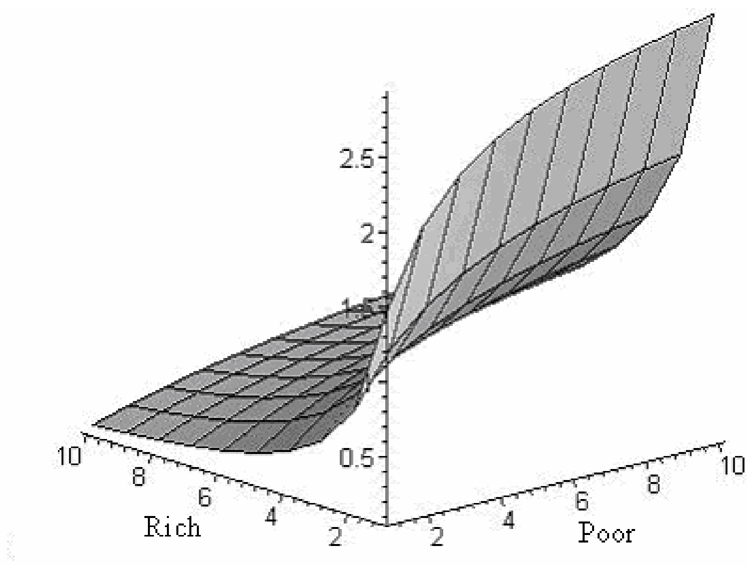
For the European case, these constraints are:

$$\left\{\begin{array}{l}\frac{2\pi}{B}\left(u_1^e\right)^{-1/\beta}\left(\delta\right)^{-\gamma/\beta}\int_0^{x_s^e-d}x\left(y_1-cx\right)^{\alpha/\beta\,\gamma/\beta}dx+\frac{2\pi}{B}\left(u_1^e\right)^{-1/\beta}\left(2\delta d\right)^{-\gamma/\beta}\int_{x_s^e-d}^{x_s^e}x\left(y_1-cx\right)^{\alpha/\beta}\left(x_s^e+d-x\right)^{\gamma/\beta}dx=\overline{n}_1\\ \frac{2\pi}{B}\left(u_2^e\right)^{-1/\beta}\int_{x_s^e}^{x_f^e}x\left(y_2-cx\right)^{\alpha/\beta\,\gamma/\beta}dx=\overline{n}_2\end{array}\right\}$$

Those systems cannot be solved analytically, thus we will use numerical simulations to find the long term equilibrium values of the two utility levels. We use the same parameters as Table 1, except the preference parameters. For calculability reasons, we choose $\alpha = 0.5$ and $\beta = 0.5$.

In this simulation the households number of each category varies. For every couple of population rich-poor we obtain a matrix with equilibrium values of the endogenous variables that satisfy the equilibrium conditions for both urban structures. The maximum number of household of each type simulated is 100.000. We present the results graphically, in three dimensions, where the horizontal axes are the number of households of each type and the vertical axe is the difference of utility levels between the two scenarios.

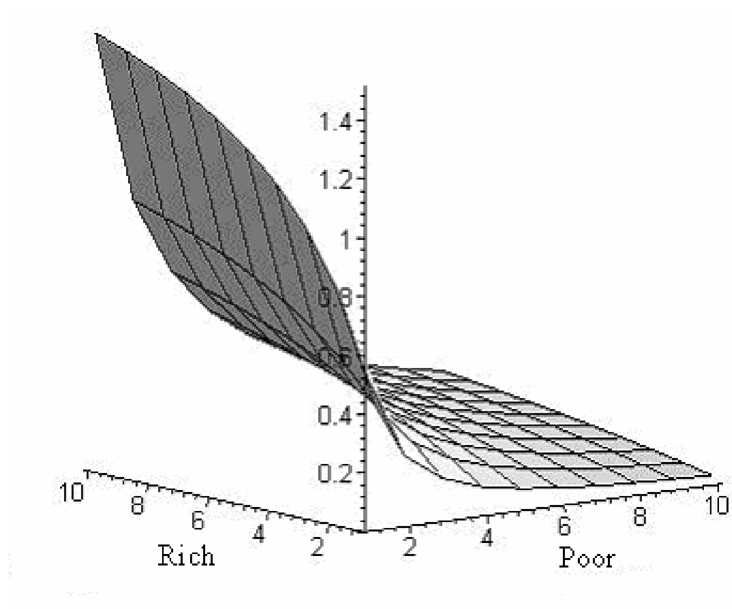
Figure 6: Utility difference for the rich households



As depicted in Figure 6, the difference of the rich utility level between American and European scenario is always positive. We remark that a linear evolution of the two populations has a differentiated impact on the rich utility levels. In a small city the difference of rich utility between scenarios is bigger than in a big city. More poor households and less rich households live in the city, higher is this difference. Contrary, an important rich population relative to the poor determine a convergence of the rich utility levels in the American and European structure.

In Figure 7 we present the same graphic but the vertical axe is represented now the difference of utility level between the American and the European scenario for the poor households. We find the same superiority for the American structure, but the differences in the utility levels are smaller for the poor than the rich households. As for the rich, for big cities, there is a convergence of the poor welfare in the two scenarios. But the predominant factor for this convergence is the number of poor households in the city, while an important number of rich determines a divergence of the utility levels of the poor between the American and the European structure.

<u>Figure 7: Utility difference for the poor households</u>



To explain this superiority of the American structure, the most evident factor is the city's physical size. Because the city boundary is determined by bid function of the social category which lives in the periphery, the American structure city is bigger than the European one. This phenomenon is not followed by an proportional increase of the households' density. Thus, there is less competition for land and then larger houses which improve the welfare of the households, even that commuting costs are higher.

We were interested to know if there are others factors to explain the American structure superiority. We made also an analytical welfare comparison between the two urban structures for cities of same physical size. We fixed the total amount of land of the city (the surface) and we determined the intervals which the utility levels can vary. We found that always, in the American structure, the utility levers are higher for both categories that in the European structure. Thus, we think that the European structure isn't the natural tendency of localisation, and the population switch has negative effects on the households' welfare. The city will be too crowed: in the centre, the rich has low house surface, because of the lower commuting costs and of the higher level of amenities, and the poor have also less space, because the commuting costs are now to high for them.

5. CONCLUSIONS

The study of stationary equilibriums shows that the conditions of existence of a European equilibrium are more restrictive and included in those for the American type. This is the result of two factors. First there are the utility level conditions of presence of both categories in the city that are more restrictive for the European scenario: all the couples of utility levels that allow the American equilibrium also allow the European one. Secondly, the European equilibrium requires supplementary conditions necessary to attract the rich in the city centre.

If the parameters satisfy the conditions (9) (10) (11) and (12) for the existence of the European equilibrium, then the two equilibriums are possible and we cannot predict which will arrive. According to these conditions the factors which support the European equilibrium play on two levels: the role played by the amenities in the localisation decisions of the rich households (preference of the rich households, their depreciation) and on the bid functions of the two social groups (the ratio between the incomes and the utility levels of the two households types).

A surplus comparison between the two urban structures in an open-city framework shows a superiority of the American structure, except some extreme utility values. This American structure superiority is confirmed by a welfare comparison in a closed-city framework: the households' utility are higher in the American structure city than in the European structure city.

Appendix: Relations between the utility levels of the two social categories

To check that the denominator of x_s (in both scenarios) is positive ($(2\delta)^{\gamma} u_1 - u_2 > 0$), we will use the indirect utility functions:

$$V_i(x) = \alpha^{\alpha}\beta^{\beta}(y_i - cx)a(x)^{\gamma_i}\psi_i(x)^{-\beta}$$

At equilibrium, the indirect utility function must be equal in all the localizations to the exogenous utility level of each category:

$$u_1 = \alpha^{\alpha}\beta^{\beta}(y_1 - cx)a(x)^{\gamma}\psi_1(x)^{-\beta}$$

$$u_2 = \alpha^{\alpha}\beta^{\beta}(y_2 - cx)\psi_2(x)^{-\beta}$$

The condition to check $(2\delta)^{\gamma} u_1 > u_2$ becomes :

$$(2\delta)^{\gamma}(y_1 - cx)a(x)^{\gamma}\psi_1(x)^{-\beta} > (y_2 - cx)\psi_2(x)^{-\beta}$$

The point where the two bid-functions are equal (with the urban revenue) is the point of segregation. By replacing *x* with x_s and after simplification, we obtain $y_1 > y_2$, which is always true.

It is also noted that if $\delta < 1/2 \Rightarrow (2\delta)^{\gamma} < 1$ and thus since $(2\delta)^{\gamma} u_1 > u_2$, u_1 must be higher than u_2. This assumption appears probable because a depreciation rate of $1/2$ is very high.

BIBLIOGRAPHY

ALONSO, William (1964): "Location and Land Use", *Cambridge, Harvard University Press*

BRUECKNER K. Jan, THISSE Jacques-François and ZENOU Yves (1999) : "Why is central Paris rich and downtown Detroit poor?: An amenity-based theory"**,** *European Economic Review, Volume 43, Pages 91-107*

FUJITA, Masahisa (1989): "Economic Theory: Land Use and City Size", *Cambridge University Press*

MILLS, E.S.(1967): "An Aggregative Model of Resource Allocation in a Metropolitan Area", *American Economic Review*, *Volume 57, Pages 197-210*

MUTH, R.F. (1969): "Cities and Housing", *Chicago, University of Chicago Press*