OWNERSHIP STRUCTURES, TRANSPARENCY AND ECONOMIC POWER[1]

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Abstract: A simple algebraic model of an ownership structure leading to the Leontief`s input-output scheme is developed and used to eliminate indirect ownership relations and evaluate the final distribution of property among individual owners. A concept of transparency of an ownership structure is defined. Implications of non-transparency for general equilibrium theory, profit distribution and decision making are discussed.

Keywords: Ownership structure, primary owners, privatization illusion, secondary owners, transparency

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1. INTRODUCTION

One of the basic paradigms of neo-classical economics reflected in general equilibrium theory and welfare economics is an assumption about economic organization of the society based on private ownership of production factors and services and their use to maximize "selfish" benefits of owners. Individuals as consumers are maximizing utility subject to budget constraint having on the right hand side incomes from selling production factors and services owned by them and the revenues from profits of firms they are co-owning, firms are maximizing profits and invisible hand of competition leads to Pareto optimal equilibrium states (Arrow 1951, Debreu 1959, Feldman 1986). One can call such an ideal picture a "family capitalism"; everything is owned by households and there are no indirect ownership relations.

Facing reality one can observe a significantly different picture: a universe of corporations and non-transparent networks of ownership relations. Citizens are owners of a fraction shares, but the ownership is dominated by big anonymous companies, banks and funds, who are co-owning a significant part of national property on institutional basis. A citizen A has a share in corporation B, corporation B has a share in corporation C, corporation C has a share in corporation D, and corporation D has a share in corporation B. Is there some relation between citizen A and corporation D? One can call such a structure a "capitalism of agents".

The legitimate question is: can anonymous institution as institution own anything? Because of transaction costs modern economy cannot be governed by individual owners directly, so the system of agents had been developed consisting of intermediary institutions and their professional management, mostly distinct from owners. But in principle intermediary institutions are only authorized some of the property rights as agents and on behalf and for benefit of individual owners. The final owners of national property can be only individuals or their non-profit associations.[3]

Accepting this point of view one can ask a rather technical question: In non-transparent network of ownership relations is there a possibility to disclose a final assignment of the whole national property to individual owners only (Turnovec, 1999)? Can we decompose the ownership structure of "capitalism of agents" to a "family capitalism" structure? In the paper we are trying to answer this question.

A simple algebraic model of ownership structures is formulated reflecting direct and indirect ownership relations.[4] An iterative process of eliminating indirect relations is proposed. It is shown that this process converges to the ownership structure in which all intermediary indirect relations are eliminated and the property is fully attributed to individual owners. Concept of

[3] "Property rights are of course human rights, i.e., rights which are possessed by human beings. The introduction of the wholly false distinction between property rights and human rights in many policy discussion is surely one the all time great semantic flimflams" (Jensen and Meckling, 1976).

[4] Speaking about direct relation we have in mind relation between individual A and company B providing that individual A owns a share in company B, while indirect relation means that individual A, having a share in company B and not having a share in company C, has through company B a relation to company C that is co-owned by company B.

transparency of a given observable ownership structure is introduced: an ownership structure is called to be transparent if the iterative process leads to elimination of indirect relations in finite number of iterations, otherwise the structure is considered not to be transparent. And finally, it is shown that using Leontief type model it is possible to evaluate the final distribution of property exactly (not as an approximation) even in the case of non-transparent ownership structure.

The idea of transparency based on convergence properties of an iterative process of indirect relations elimination, was proposed in Turnovec, 1999. New contribution presented in this paper is an extension of Leontief's input-output methodology on structural analysis of ownership relations.

2. MODEL OF OWNERSHIP STRUCTURES

Let us consider two types of economic agents: the primary owners, who can own, but cannot be owned (citizens, citizens' non-profit associations, state, municipalities, etc.), and the secondary owners, who can be owned and at the same time can own (companies, corporations).

Let

m be the number of primary owners, $i = 1, 2, ..., m$,

n be the number of secondary owners (companies), $j = 1, 2, ..., n$,

s^0_{ji} be the direct share of the primary owner i in the secondary owner j (as a proportion of total number of shares),

t^0_{jk} be the direct share of the secondary owner (company) k in the secondary owner (company) j.

Then the n x m matrix

$$\mathbf{S}_0 = (s^0_{ji})$$

where the row j expresses shares of the primary owners $i = 1, 2, ..., m$ in the secondary owner j, and the column i expresses the shares of the primary owner i in the secondary owners $j = 1, 2, ..., n$, will be called a matrix of primary property distribution, and the n x n matrix

$$\mathbf{T}_0 = (t^0_{jk})$$

where the row j expresses shares of the secondary owners $k = 1, 2, ..., n$ in the secondary owner j, and the column k expresses shares of secondary owner k in the secondary owners $j = 1, 2, ..., n$, will be called a matrix of secondary property distribution. The couple

$$(\mathbf{S}_0, \mathbf{T}_0)$$

characterizes an initial property distribution in an economy.

Clearly

$$\sum_{i=1}^{m} s^0_{ji} + \sum_{k=1}^{n} t^0_{jk} = 1$$

for any j = 1, 2, ..., n.

Example 1

Let us consider a hypothetical initial ownership structure with the three primary owners P_1, P_2, P_3, and the three companies C_1, C_2, C_3 (secondary owners), described in Table 1.

Table 1

	Matrix S_0			*Matrix T_0*			
	P_1	*P_2*	*P_3*	*C_1*	*C_2*	*C_3*	*total*
C_1	*0.4*	*0.2*	*0.1*	*0*	*0.3*	*0*	*1*
C_2	*0.55*	*0.25*	*0*	*0.2*	*0*	*0*	*1*
C_3	*0.3*	*0.3*	*0.2*	*0.1*	*0.1*	*0*	*1*

In this case

$$\mathbf{S}_0 = \begin{pmatrix} 0.4 & 0.2 & 0.1 \\ 0.55 & 0.25 & 0 \\ 0.3 & 0.3 & 0.2 \end{pmatrix}$$

and

$$\mathbf{T}_0 = \begin{pmatrix} 0 & 0.3 & 0 \\ 0.2 & 0 & 0 \\ 0.1 & 0.1 & 0 \end{pmatrix}.$$

Matrices $\mathbf{S}_0$ and $\mathbf{T}_0$ provide an observable property distribution.

If $\mathbf{T}_0 = \mathbf{0}_{nn}$, where $\mathbf{0}_{nn}$ is the nxn zero matrix, we have a very simple and transparent structure, when only primary owners own companies and there exists no indirect ownership.

However in real economies we do not have such transparent structures, and that can lead to situations when it is not so easy to see who owns what. If a primary owner A has a share in a secondary owner B, secondary owner B has a share in secondary owner C, and secondary owner C has a share in secondary owner D, then there exist direct ownership relations between A and B and B and C, and indirect ownership relations between A and C, A and D and B and D. If moreover D has a share in B, then the situation is completely unclear. The problem is how to evaluate direct and indirect ownership relations, and to identify the part of company C which is owned by primary owner A etc.

Assuming $\mathbf{T}_0 \neq \mathbf{0}_{nn}$ let us consider a primary owner i. A share of a primary owner P_i in a company (secondary owner) C_j is composed from his direct share in C_j, but also from his indirect share following from his shares in other secondary owners that are co-owning secondary owner C_j. We shall say that this relation generates reallocation of primary indirect ownership of the first degree:

$$s_{ji}^{1} = s_{ji}^{0} + \sum_{r=1}^{n} t_{jr}^{0} s_{ri}^{0}$$

Then, consider a secondary owner C_j. His residual share in the company k (what remains after reallocation of indirect ownership of the first degree) is given by appropriate fractions of the shares that follows from his direct shares in other companies that co-own company j. We shall speak about reallocation of secondary indirect ownership of the first degree:

$$t_{jk}^{1} = \sum_{r=1}^{n} t_{jr}^{0} t_{rk}^{0}$$

In matrix form we have

$$\mathbf{S}_1 = \mathbf{S}_0 + \mathbf{T}_0 \mathbf{S}_0$$

$$\mathbf{T}_1 = \mathbf{T}_0 \mathbf{T}_0 = \mathbf{T}_0^{\,2}$$

So, considering indirect relations, we can obtain a decomposition of property on direct (following from registered shares of primary owners) component and indirect component (following from indirect relations). We shall call initial distribution ($\mathbf{S}_0$, $\mathbf{T}_0$) a distribution of zero degree, and the distribution ($\mathbf{S}_1$, $\mathbf{T}_1$) a distribution of the first degree, where $\mathbf{S}_0$ provides the original direct shares of primary owners, $\mathbf{T}_0$ provides original direct shares of secondary owners, $\mathbf{S}_1$ provides the direct and indirect shares of primary owners following from direct shares of secondary owners owned by primary owners, and $\mathbf{T}_1$ provides indirect shares of secondary owners following from his direct shares in other secondary owners.

Example 2

In ownership structure from Table 1 the matrix of secondary owners shares is non-zero, so there exist indirect ownership relations. Taking into account indirect relations, we obtain a more precise distribution:

$$S_1 = \begin{pmatrix} 0.4 & 0.2 & 0.1 \\ 0.55 & 0.25 & 0 \\ 0.3 & 0.3 & 0.2 \end{pmatrix} + \begin{pmatrix} 0 & 0.3 & 0 \\ 0.2 & 0 & 0 \\ 0.1 & 0.1 & 0 \end{pmatrix} \begin{pmatrix} 0.4 & 0.3 & 0.1 \\ 0.55 & 0.25 & 0 \\ 0.3 & 0.3 & 0.2 \end{pmatrix} =$$

$$\begin{pmatrix} 0.4 & 0.2 & 0.1 \\ 0.55 & 0.25 & 0 \\ 0.3 & 0.3 & 0.2 \end{pmatrix} + \begin{pmatrix} 0.165 & 0.075 & 0 \\ 0.08 & 0.04 & 0.02 \\ 0.095 & 0.045 & 0.01 \end{pmatrix} = \begin{pmatrix} 0.565 & 0.275 & 0.1 \\ 0.63 & 0.29 & 0.02 \\ 0.395 & 0.345 & 0.21 \end{pmatrix}$$

and

$$T_1 = \begin{pmatrix} 0 & 0.3 & 0 \\ 0.2 & 0 & 0 \\ 0.1 & 0.1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0.3 & 0 \\ 0.2 & 0 & 0 \\ 0.1 & 0.1 & 0 \end{pmatrix} = \begin{pmatrix} 0.06 & 0 & 0 \\ 0 & 0.06 & 0 \\ 0.02 & 0.03 & 0 \end{pmatrix}.$$

The recalculated distribution is set out in Table 2.

Table 2

	Matrix S_1			*Matrix T_1*			
	P_1	*P_2*	*P_3*	*C_1*	*C_2*	*C_3*	*total*
C_1	*0.565*	*0.275*	*0.1*	*0.06*	*0*	*0*	*1*
C_2	*0.63*	*0.29*	*0.02*	*0*	*0.06*	*0*	*1*
C_3	*0.395*	*0.345*	*0.21*	*0.02*	*0.03*	*0*	*1*

Now we have a new distribution ($\mathbf{S}_1$, $\mathbf{T}_1$) taking into account indirect relations of the first degree. Matrix $\mathbf{T}_1$ is non-zero, so we have not disclosed final distribution of property among primary owners.

We can repeat all our considerations to produce a distribution of the second degree as

$$\mathbf{S}_2 = \mathbf{S}_0 + \mathbf{T}_0\mathbf{S}_1 = \mathbf{S}_0 + \mathbf{T}_0(\mathbf{S}_0 + \mathbf{T}_0\mathbf{S}_0) =$$

$$= (\mathbf{I} + \mathbf{T}_0^1 + \mathbf{T}_0^2)\mathbf{S}_0$$

$$\mathbf{T}_2 = \mathbf{T}_0\mathbf{T}_1 = \mathbf{T}_0^{\,3}$$

etc..

In the general case

$$\mathbf{S}_r = \mathbf{S}_0 + \mathbf{T}_0\mathbf{S}_{r-1}$$

$$\mathbf{T}_r = \mathbf{T}_0\mathbf{T}_{r-1}$$

(r = 1, 2, ..., k, ...), or

$$\mathbf{S}_r = \left(\mathbf{I} + \sum_{j=1}^{r} \mathbf{T}_0^j \right) \mathbf{S}_0$$

$$\mathbf{T}_r = \mathbf{T}_0^{r+1}$$

To eliminate indirect relations there should exist a positive integer r such that

$$\mathbf{T}_r = \mathbf{T}_0^r = \mathbf{0}_{nxn}$$

Example 3

In Table 3 and 4 we have next two iterations of our eliminating process. We can still observe some residual indirect property relations.

Table 3

	Matrix S_2			*Matrix T_2*			
	P_1	P_2	P_3	C_1	C_2	C_3	*total*
C_1	*0.589*	*0.287*	*0.106*	*0*	*0.018*	*0*	*1*
C_2	*0.663*	*0.305*	*0.02*	*0.012*	*0*	*0*	*1*
C_3	*0.4195*	*0.3565*	*0.212*	*0.006*	*0.006*	*0*	*1*

Table 4

	Matrix S_2			*Matrix T_2*			
	P_1	P_2	P_3	C_1	C_2	C_3	*total*
C_1	*0.5989*	*0.2915*	*0.1060*	*0.0036*	*0*	*0*	*1*
C_2	*0.6678*	*0.3074*	*0.0212*	*0*	*0.0036*	*0*	*1*
C_3	*0.4252*	*0.3592*	*0.2126*	*0.0012*	*0.0018*	*0*	*1*

In fact we state that in this particular case we shall never be able to find final assignment of property to the primer owners.

Example 4

Let us consider now the simple structure in Table 5.

Table 5

	Matrix S_0			*Matrix T_0*			
	P_1	P_2	P_3	C_1	C_2	C_3	*total*
C_1	*0.6*	*0.2*	*0.1*	*0*	*0.1*	*0*	*1*
C_2	*0.4*	*0.1*	*0.2*	*0*	*0*	*0.3*	*1*
C_3	*0.3*	*0.3*	*0.4*	*0*	*0*	*0*	*1*

In Table 6 we have the result of the second iteration. In this particular case we succeeded to eliminate indirect relations and identify final distribution of property among the primary owners.

Table 6

	Matrix S_2			*Matrix T_2*			
	P_1	*P_2*	*P_3*	*C_1*	*C_2*	*C_3*	*total*
C_1	*0.649*	*0.219*	*0.132*	*0*	*0*	*0*	*1*
C_2	*0.490*	*0.190*	*0.320*	*0*	*0*	*0*	*1*
C_3	*0.300*	*0.300*	*0.400*	*0*	*0*	*0*	*1*

3. TRANSPARENCY

Intuitively: a concept of transparency of a property structure should be related to possibility to eliminate indirect relations and to find a final assignment of the total property to primary owners only.

Within the framework of the model described above the sequence of matrices $\mathbf{T}_0$, $\mathbf{T}_1$, $\mathbf{T}_2$, ... can be used for quantification of the concept of transparency of a property distribution.

If we accept as an axiom that finally any distribution of property is distribution among the primary owners only, then transparency of a particular initial distribution can be measured by a distance of primary distribution from the final distribution taking into account all degrees of indirect links.

The maximum of transparency is achieved when $\mathbf{T}_0 = \mathbf{0}_{nn}$. In this case primary distribution is transparent in the sense that any property is related to primary owners only and no indirect relations appear.

We shall say that a particular property structure ($\mathbf{S}_0$, $\mathbf{T}_0$) such that $\mathbf{T}_0 \neq \mathbf{0}_{nn}$ is k-transparent, if in property distribution ($\mathbf{S}_k$, $\mathbf{T}_k$) of degree k it holds that $\mathbf{T}_k = \mathbf{0}_{nn}$, while in property distribution of degree k-1 ($\mathbf{S}_{k-1}$, $\mathbf{T}_{k-1}$) it holds that $\mathbf{T}_{k-1} \neq \mathbf{0}_{nn}$.

A property structure is non-transparent, if for any positive integer k it holds that $\mathbf{T}_k \neq \mathbf{0}_{nn}$.

Lemma 1

Let **A** be a square n x n matrix such that the sequence

$$\mathbf{A}^1, \mathbf{A}^2, \ldots, \mathbf{A}^k, \ldots$$

of powers of the matrix **A** converges to zero matrix, i.e.

$$\lim_{k \to \infty} \mathbf{A}^k = \mathbf{0}_{nxn}$$

Then

1. either there exist a positive integer $s \le n$ such that

$$\mathbf{A}^{s-1} \ne \mathbf{0}_{nxn} \text{ and } \mathbf{A}^s = \mathbf{0}_{nxn}$$

or

$$\mathbf{A}^k \ne \mathbf{0}_{nxn}$$

for any positive integer k.

2. matrix **I** - **A** is non-singular (**I** being n x n identity matrix) and

$$(\mathbf{I} - \mathbf{A})^{-1} = \sum_{k=0}^{\infty} \mathbf{A}^k$$

PROOF of the first part is based on properties of so called nilpotent matrices (e.g. Archibald, 1968), second part on Leontief, 1956.

The matrix $\mathbf{T}_0$ of order n is assumed to be such that

$$\sum_{k=1}^{n} t_{jk}^{0} < 1$$

Then clearly the sequence of its powers converges to zero matrix and conditions of the theorem are satisfied. From Lemma 1 it follows that either there exists k such that $k \le n$, $\mathbf{T}_k \ne \mathbf{0}_{nxn}$ and $\mathbf{T}_{k+1} = \mathbf{0}_{nxn}$, or $\mathbf{T}_r \ne \mathbf{0}_{nxn}$ for any integer r.

Example 5

Let us consider the matrix $\boldsymbol{T}_0$ *from the first property structure:*

$$T_0 = \begin{pmatrix} 0 & 0.3 & 0 \\ 0.2 & 0 & 0 \\ 0.1 & 0.1 & 0 \end{pmatrix}$$

Then

$$T_1 = \begin{pmatrix} 0 & 0.3 & 0 \\ 0.2 & 0 & 0 \\ 0.1 & 0.1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0.3 & 0 \\ 0.2 & 0 & 0 \\ 0.1 & 0.1 & 0 \end{pmatrix} = \begin{pmatrix} 0.06 & 0 & 0 \\ 0 & 0.06 & 0 \\ 0.02 & 0.03 & 0 \end{pmatrix}$$

and

$$T_2 = T_0^3 = \begin{pmatrix} 0 & 0.018 & 0 \\ 0.012 & 0 & 0 \\ 0.006 & 0.006 & 0 \end{pmatrix}$$

In our case n = 3, and we have $\boldsymbol{T}_1 \neq \boldsymbol{0}_{nxn}$*, and* $\boldsymbol{T}_2 \neq \boldsymbol{0}_{nxn}$*, where* $\mathbf{T}_2 = \mathbf{T}_0^3$*, hence the alternative b) of the lemma statement appears. The property structure is not transparent.*

4. ONE INTERESTING IDENTITY

Let us assume that there exist a final distribution of property among primary owners without any indirect links. Let x_{ji} be a full (direct and indirect) share of primary owner i in corporation j. Let us call the n x m matrix $\mathbf{X} = (x_{ji})$ a matrix of final distribution. In case of a transparent ownership structure we know that

$$\mathbf{X} = \mathbf{S}_t = \left(\mathbf{I} + \sum_{j=1}^{t} \mathbf{T}_0^j \right) \mathbf{S}_0$$

where $t \leq n$, n is the number of secondary owners.

Question: is it possible to evaluate exactly the matrix $\mathbf{X}$ also in case when initial ownership structure is not transparent?

Lemma 2

Let $(\mathbf{S}_0, \mathbf{T}_0)$ be an initial ownership structure such that

$$s_{ji}^0 \geq 0,\ t_{jk}^0 \geq 0$$

and

$$\sum_{i=1}^{m} s_{ji}^0 + \sum_{k=1}^{n} t_{jk}^0 = 1$$

and there exist a non-negative integer r such that

$$\sum_{k=1}^{n} t^{r}_{jk} < 1$$

for all j = 1, 2, ..., n. Then the sequence $\mathbf{S}_r$ converges and

$$\lim_{r\to\infty} \mathbf{S}_r = (\sum_{r=0}^{\infty} \mathbf{T}_r)\mathbf{S}_0 = (\sum_{r=0}^{\infty} \mathbf{T}_0^r)\mathbf{S}_0 = (\mathbf{I} - \mathbf{T}_0)^{-1}\mathbf{S}_0 = \mathbf{X}$$

We obtained an identity that is well-known from Leontief's input-output models:

$$\mathbf{X} = (\mathbf{I} - \mathbf{T}_0)^{-1}\mathbf{S}_0$$

Example 6

In property structure from Example 1, which is not transparent, we have

$$(\mathbf{I} - \mathbf{T}_0) = \begin{pmatrix} 1 & -0.3 & 0 \\ -0.2 & 1 & 0 \\ -0.1 & -0.1 & 1 \end{pmatrix}$$

and

$$(\mathbf{I} - \mathbf{T}_0)^{-1} = \begin{pmatrix} 1.06383 & 0.319149 & 0 \\ 0.212766 & 1.06383 & 0 \\ 0.12766 & 0.138298 & 1 \end{pmatrix}$$

Then

$$\mathbf{X} = (\mathbf{I} - \mathbf{T}_0)^{-1}\mathbf{S}_0 =$$

$$\begin{pmatrix} 1.06383 & 0.319149 & 0 \\ 0.212766 & 1.06383 & 0 \\ 0.12766 & 0.138298 & 1 \end{pmatrix} \begin{pmatrix} 0.4 & 0.2 & 0.1 \\ 0.55 & 0.25 & 0 \\ 0.3 & 0.3 & 0.2 \end{pmatrix} =$$

$$\begin{pmatrix} 0.601064 & 0.292553 & 0.106383 \\ 0.670213 & 0.308511 & 0.021277 \\ 0.427128 & 0.360106 & 0.212766 \end{pmatrix}$$

which gives the final distribution of full shares (direct and indirect) of primary owners in corporations (secondary owners) after elimination of indirect links ("family capitalism" type of corporate governance in "capitalism of agents" ownership structure from Table 1).

5. WHO GETS THE PROFITS?

If $\mathbf{T}_0 = \mathbf{0}_{nxn}$, where $\mathbf{0}_{nxn}$ is nxn zero matrix, we can speak about the "family capitalism" structure, if $\mathbf{T}_0$ is non-zero matrix, we can speak about "capitalism of agents" structure. We are living in the world of corporate stakeholders (the capitalism of agents). In the latter case the corporate governance (decision making rights and profit shares of stakeholders) can be based on matrices $\mathbf{S}_0$ and $\mathbf{T}_0$, but it is theoretically possible (while, perhaps, not very practical) to simulate the "family capitalism" governance based on the matrix $\mathbf{X}$.

Example 7

Let us consider a hypothetical initial ownership structure from Example 1, with the three primary owners P_1, P_2, P_3, and the three companies C_1, C_2, C_3 (secondary owners), described in Table 1.

In Table 7 we have the final distribution of shares

Table 7

	Matrix **X**			*lim* $\mathbf{T}_r$			
	P_1	P_2	P_3	C_1	C_2	C_3	*total*
C_1	*0.601064*	*0.2292553*	*0.106383*	*0*	*0*	*0*	*1*
C_2	*0.670213*	*0.308511*	*0.021277*	*0*	*0*	*0*	*1*
C_3	*0.427128*	*0.360106*	*0.212766*	*0*	*0*	*0*	*1*

To illustrate the difference in the distribution of profits in "capitalism of agents" and "family capitalism" type of corporate governance, let us assume that profits of corporations C_1, C_2 and C_3 are 100 in all three cases:

$$\pi = \begin{pmatrix} 100 & 100 & 100 \end{pmatrix}$$

Then in "capitalism of agents" the profits will be distributed among all six actors:

$$\pi(\mathbf{S}_0, \ \mathbf{T}_0) = (100 \quad 100 \quad 100)\begin{pmatrix} 0.4 & 0.2 & 0.1 & 0 & 0.3 & 0 \\ 0.55 & 0.25 & 0 & 0.2 & 0 & 0 \\ 0.3 & 0.3 & 0.2 & 0.1 & 0.1 & 0 \end{pmatrix} =$$

$$\begin{pmatrix} 125 & 75 & 30 & 30 & 40 & 0 \end{pmatrix}$$

i.e. 125 for P_1, 75 for P_2, 30 for P_3, 30 for C_1, 40 for C_2 and 0 for C_3. In "family capitalism" type of governance the profits will be distributed only among the primary owners:

$$\pi\mathbf{X} = (100 \quad 100 \quad 100)\begin{pmatrix} 0.601064 & 0.292553 & 0.106383 \\ 0.670213 & 0.308511 & 0.021277 \\ 0.427128 & 0.360106 & 0.212766 \end{pmatrix} =$$

$$(169.8404 \quad 96.11702 \quad 34.04255)$$

i.e. 169.8404 for P_1, 96.11702 for P_2 and 34.04255 for P_3, nothing for corporate shareholders.

6. WHO HOLDS THE POWER?

To simplify our consideration, let us assume that decision making rights are given by shares. In voting the share-holders weights are counting and according to required quota (say, majority of more than 50%, or qualified two-third majority) the group of share-holders with required majority of shares wins). The decision making power of owners clearly depends on what distribution of property is considered in voting: $\mathbf{S}_0$ and $\mathbf{T}_0$, or $\mathbf{X}$.

It is known that a distribution of votes among the members of a committee is not a sufficient characteristic of their voting power or an influence distribution. So called power indices are used to estimate an influence of the members of a committee as a function of a voting rule and of a structure of representation in a committee.[5]

[5] In 1954 Lloyd Shapley and Martin Shubik published a short paper in the American Political Science Review, proposing that the Shapley value for cooperative characteristic function form games could serve as a measure of voting power in committees. In 1965 John Banzhaf proposed a new index of voting power. Since that more than twenty new definitions (with more or less satisfactory theoretical justification) of so called power indices have been published.

The majority of proposed power indices are based on the game theoretical model of simple games in characteristic function form and on different concepts of "decisiveness" of members of a committee with respect to winning coalitions. They usually express probability of members of the body to be “decisive” in a given sense.

For illustration let us consider one of the most frequently used power indices proposed by John Banzhaf, the so called Banzhaf power index. All possible winning coalitions are considered. Each of the winning coalition is analyzed and the so called "swing" voters are identified: i.e. those who by changing their vote from "yes" to "no" could change the coalition from winning to losing. The “voting power” of individual members is then measured as a probability to have a swing.

Example 8

We shall apply Banzhaf power index to measure voting power of owners in property structure from Example 1.

In Table 8 we provide voting weights in decision making about particular corporations according to the observable property structure $\mathbf{S}_0$, $\mathbf{T}_0$ and according to the final distribution $\mathbf{X}$ with primary owners only.

Table 8
Voting weights of primary and secondary owners

	Observable distribution S0, T0			Final distribution X		
	voting weights in decision making about			voting weights in decision making about		
	C1	C2	C3	C1	C2	C3
P1	0.4	0.55	0.3	0.601064	0.670213	0.427128
P2	0.2	0.25	0.3	0.292553	0.308511	0.360106
P3	0.1	0	0.2	0.106383	0.021277	0.212766
C1	0	0.2	0.1	0	0	0
C2	0.3	0	0.1	0	0	0
C3	0	0	0	0	0	0

In Table 9 we have the results of voting power evaluation (Banzhaf index) for the voting weights generated by observable property distribution $\mathbf{S}_0$, $\mathbf{T}_0$.

Table 9
Distribution of voting power in corporations when observable property distribution is used for voting weights

	Decisional power (Banzhaf index) in %					
	simple majority			qualified majority		
	voting power in decision making about			voting power in decision making about		
	C1	C2	C3	C1	C2	C3
P1	41.67	100	30.77	50	60	30.43
P2	25	0	30.77	10	20	30.43
P3	8.33	0	23.08	10	0	13.04
C1	0	0	7.69	0	20	13.04
C2	25	0	7.69	30	0	13.04
C3	0	0	0	0	0	0

In Table 10 we provide the results of voting power evaluation (Banzhaf index) for the voting weights generated by the final property distribution **X**.

Table 10
Distribution of voting power in corporations when final property distribution is used for voting weights

	Decisional power (Banzhaf index) in %					
	simple majority			qualified majority		
	voting power in decision making about			voting power in decision making about		
	C1	C2	C3	C1	C2	C3
P1	100	100	33.33	60	100	50
P2	0	0	33.33	20	0	50
P3	0	0	33.33	20	0	0
C1	0	0	0	0	0	0
C2	0	0	0	0	0	0
C3	0	0	0	0	0	0

7. A PRIVITAZION ILLUSION

Using the structural approach described above we can try to answer the question: How much privatized is an "almost fully" privatized economy?

Let w_j be the weight of a company j (e.g. the market value, value of assets etc.). Considering a distribution ($\mathbf{S}_r$, $\mathbf{T}_r$) of degree r, we can evaluate the corresponding distribution of the total property in an economy as

$$p_i^r = \frac{\sum_{j=1}^{n} s_{ij}^r w_j}{\sum_{j=1}^{n} w_j}, \quad d_k^r = \frac{\sum_{j=1}^{n} t_{kj}^r w_j}{\sum_{j=1}^{n} w_j}$$

where p_i^r is the share of the i-th primary owner and d_k^r is the share of the k-th company (secondary owner) in the total property according to distribution of the degree r. Let us illustrate by a simple example that a primary distribution of national property can significantly differ from the final distribution reflecting indirect links.

Example 9

Let us assume that an economy consists of the following 5 actors: the state S, group of individual investors M, two banks B1 and B2, investment fund F and a group of industrial enterprises I. In Table 11 we provide a hypothetical primary property distribution in such an economy.

Table 11

	S	*M*	*B1*	*B2*	*F*	*I*	*total*	*weights*
B1	*0.6*	*0.3*	*0*	*0*	*0.1*	*0*	*1*	*10*
B2	*0.7*	*0.2*	*0.1*	*0*	*0*	*0*	*1*	*5*
F	*0*	*0*	*0*	*1*	*0*	*0*	*1*	*35*
I	*0*	*0.3*	*0*	*0*	*0.7*	*0*	*1*	*50*
total share	*0.095*	*0.19*	*0.005*	*0.35*	*0.36*	*0*		

We can see that, with respect to initial property distribution, the total share of state of national property is 9.5%.

Let us use Lemma 2. In our particular case

$$,(I - T_0) = \begin{pmatrix} 1 & 0 & -0.1 & 0 \\ -0.1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -0.7 & 1 \end{pmatrix}$$

and

$$(I - T_0)^{-1} = \begin{pmatrix} 1.010101 & 0.10101 & 0.10101 & 0 \\ 0.10101 & 1.010101 & 0.010101 & 0 \\ 0.10101 & 1.010101 & 1.010101 & 0 \\ 0.070707 & 0.707071 & 0.707071 & 1 \end{pmatrix}$$

Then

$$X = (I - T_0)^{-1} S_0 =$$

$$\begin{pmatrix} 1.010101 & 0.10101 & 0.10101 & 0 \\ 0.10101 & 1.010101 & 0.010101 & 0 \\ 0.10101 & 1.010101 & 1.010101 & 0 \\ 0.070707 & 0.707071 & 0.707071 & 1 \end{pmatrix} \begin{pmatrix} 0.6 & 0.3 \\ 0.7 & 0.2 \\ 0 & 0 \\ 0 & 0.3 \end{pmatrix} =$$

$$\begin{pmatrix} 0.676768 & 0.323232 \\ 0.767677 & 0.232323 \\ 0.767677 & 0.232323 \\ 0.537374 & 0.462626 \end{pmatrix}$$

and the final distribution of shares, after elimination of indirect links, will look like follows:

Table 12

	S	*M*	*B1*	*B2*	*F*	*I*	*total*	*weights*
B1	*0.676768*	*0.323232*	*0*	*0*	*0*	*0*	*1*	*10*
B2	*0.767677*	*0.232323*	*0*	*0*	*0*	*0*	*1*	*5*
F	*0.767677*	*0.232323*	*0*	*0*	*0*	*0*	*1*	*35*
I	*0.537374*	*0.4626265*	*0*	*0*	*0*	*0*	*1*	*50*
total share	*0.6434343*	*0.3565657*	*0*	*0*	*0*	*0*	*1*	

8. CASE STUDY: CZECH BANKING SECTOR IN 1997

In this part we demonstrate possibility of practical implementation of our model on an analysis of property structure of the core banking sector in the Czech Republic at the end of 1997. There were five major banks, representing almost 90% of the total assests of the Czech banking sector (R. Matoušek, 1998):

CS	Česká spořitelna (Czech Saving Bank),
CP	Česká pojišťovna (Czech Insurance),
KB	Komerční banka (Commercial Bank),
IPB	Investiční a poštovní banka (Investment and Post bank),
CSOB	Československá obchodní banka (Czecho-Slovak Trade Bank).

As primary owners we have:

FNM	Fond národního majetku (Fund of National Property), state agency,
CNB	Česká národní banka (Czech National Bank), central bank,
MF	Ministerstvo financí (Ministry of Finance), state agency,
Mun.	Sdružení měst (Association of Municipalities),
BH	Bank Holding, non-state,
JRING	J. Ring stock comp., non-state,
PPF I	First Privatization Holding, non-state,
BNY	The Bank of New York,
Nomura	Nomura Group,
MB	The Midland Bank,
BTI	The Bankers Trust Investment,
SR	Slovak Republic,
others	minority investors (mostly from voucher privatization).

The secondary owners are:

SPIF-ČS	Spořitelní privatizační investiční fond - Český (investment fund),
SPIF-VS	Spořitelní privatizační investiční fond - výnosový (investment fund),
PPF	První privatizační fond (investment fund),
PIF	První investiční fond (investment fund),
RIF	Restituční investiční fond (investment fund),
IPF-K	Investiční privatizační fond Komerční banky (investment fund),
VS	Vojenské stavby (stock company).

The structure is incomplete, because some of our primary owners are in fact secondary owners as well (owned mostly by foreign capital), but to have a closed system for illustrative purposes, we shall not go deeper.

Table 13 gives the initial ownership distribution (end of 1997). In Table 18 we obtained final ownership distribution after elimination of indirect relations. We can see for example that difference between final and initial distribution can mean difference between majority control and minority (Komerční banka). While the matrix $\mathbf{S}_0$ of initial distribution is pretty sparse, the matrix $\mathbf{X}$ of final distribution allocates additional fractions of property to all primary owners.

Table 13
Initial property distribution in the banking sector of the Czech Republic, end of 1997, in relative shares

	Primary owners (matrix S_0)													Secondary owners (matrix T_0)											
	FNM	CNB	MF	Mun.	BH	JRING	PP F I	BNY	No- mura	MB	BTI	SR	others	CS	CP	KB	IPB	CSOB	SPIF-C	SPIF-V	PPF	PIF	RIF	IPF-K	VS
CS	0.528			0.1475									0.1195		0.101	0.028			0.051	0.025					
CP	0.3025												0.1771				0.1718	0.14			0.2086				
KB	0.4874							0.1292					0.2983	0.0153								0.0121	0.0356	0.0221	
IPB	0.3149				0.1497				0.502				0.4066												0.0786
CSOB	0.1959	0.2651	0.1959									0.2578	0.0853												
SPIF-C										0.3			0.4495	0.2505											
SPIF-V										0.3	0.1		0.3501	0.2499											
PPF							1																		
PIF													0.862		0.138										
RIF	0.2037												0.6953		0.101										
IPF-KB													0.7099			0.2901									
VS					0.411	0.427							0.162												

Table 14
Matrix $\mathbf{T}_0$

0	0.101	0.028	0	0	0.051	0.025	0	0	0	0	0
0	0	0	0.1718	0.14	0	0	0.2086	0	0	0	0
0.0153	0	0	0	0	0	0	0	0.0121	0.0356	0.0221	0
0	0	0	0	0	0	0	0	0	0	0	0.0786
0	0	0	0	0	0	0	0	0	0	0	0
0.2505	0	0	0	0	0	0	0	0	0	0	0
0.2499	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0.138	0	0	0	0	0	0	0	0	0	0
0	0.101	0	0	0	0	0	0	0	0	0	0
0	0	0.2901	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

Table 15
Matrix ($\mathbf{I}$ - $\mathbf{T}_0$)

1	-0.101	-0.028	0	0	-0.051	-0.025	0	0	0	0	0
0	1	0	-0.1718	-0.14	0	0	-0.2086	0	0	0	0
-0.0153	0	1	0	0	0	0	0	-0.0121	-0.0356	-0.0221	0
0	0	0	1	0	0	0	0	0	0	0	-0.0786
0	0	0	0	1	0	0	0	0	0	0	0
-0.2505	0	0	0	0	1	0	0	0	0	0	0
-0.2499	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0
0	-0.138	0	0	0	0	0	0	1	0	0	0
0	-0.101	0	0	0	0	0	0	0	1	0	0
0	0	-0.2901	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	1

Table 16
Matrix $(\mathbf{I} - \mathbf{T}_0)^{-1}$

1.01984	0.103155	0.02874	0.017722	0.014442	0.052012	0.025496	0.021518	0.000348	0.001023	0.000635	0.001393
0	1	0	0.1718	0.14	0	0	0.2086	0	0	0	0.013503
0.015704	0.006888	1.006895	0.001183	0.000964	0.000801	0.000393	0.001437	0.012183	0.035845	0.022252	0.000093
0	0	0	1	0	0	0	0	0	0	0	0.0786
0	0	0	0	1	0	0	0	0	0	0	0
0.25547	0.02584	0.007199	0.004439	0.003618	1.013029	0.006387	0.00539	0.000087	0.000256	0.000159	0.000349
0.254858	0.025778	0.007182	0.004429	0.003609	0.012998	1.006371	0.005377	0.000087	0.000256	0.000159	0.000348
0	0	0	0	0	0	0	1	0	0	0	0
0	0.138	0	0.023708	0.01932	0	0	0.028787	1	0	0	0.001863
0	0.101	0	0.017352	0.01414	0	0	0.021069	0	1	0	0.001364
0.004556	0.001998	0.2921	0.000343	0.00028	0.000232	0.000114	0.000417	0.003534	0.010399	1.006455	0.000027
0	0	0	0	0	0	0	0	0	0	0	1

Table 17
Matrix $\mathbf{S}_0$

	FNM	CNB	MF	Mun.	BH	JRING	PPF I	BNY	Nomura	MB	BTI	SR	others
CS	0.528	0	0	0.148	0	0	0	0	0	0	0	0	0.12
CP	0.303	0	0	0	0	0	0	0	0	0	0	0	0.177
KB	0.487	0	0	0	0	0	0	0.129	0	0	0	0	0.298
IPB	0.315	0	0	0	0.15	0	0	0	0.05	0	0	0	0.407
CSOB	0.196	0.265	0.196	0	0	0	0	0	0	0	0	0.258	0.085
SPIF-C	0	0	0	0	0	0	0	0	0	0.3	0	0	0.45
SPIF-V	0	0	0	0	0	0	0	0	0	0.3	0.1	0	0.35
PPF	0	0	0	0	0	0	1	0	0	0	0	0	0
PIF	0	0	0	0	0	0	0	0	0	0	0	0	0.862
RIF	0.204	0	0	0	0	0	0	0	0	0	0	0	0.695
IPF-KB	0	0	0	0	0	0	0	0	0	0	0	0	0.71
VS	0	0	0	0	0.411	0.427	0	0	0	0	0	0	0.162

Table 18
Matrix $\mathbf{X} = (\mathbf{I} - \mathbf{T}_0)^{-1}\mathbf{S}_0$
Final property distribution after elimination of indirect relations

	FNM	CNB	MF	Mun.	BH	JRING	PPF I	BNY	Nomura	MB	BTI	SR	others	Σ
CS	0.592306	0.003829	0.002829	0.150426	0.003225	0.000595	0.021518	0.003713	0.00089	0.023252	0.00255	0.003723	0.191144	1
CP	0.384026	0.037114	0.027426	0	0.031268	0.005766	0.2086	0	0.008624	0	0	0.036092	0.261083	1
KB	0.508999	0.000256	0.000189	0.002316	0.000215	0.00004	0.001437	0.130091	0.000059	0.000358	0.000039	0.000249	0.355752	1
IPB	0.3149	0	0	0	0.182005	0.033562	0	0	0.0502	0	0	0	0.419333	1
CSOB	0.1959	0.2651	0.1959	0	0	0	0	0	0	0	0	0.2578	0.0853	1
SPIF-C	0.148373	0.000959	0.000709	0.037682	0.000808	0.000149	0.00539	0.00093	0.000223	0.305825	0.000639	0.000933	0.497381	1
SPIF-V	0.148017	0.000957	0.000707	0.037592	0.000806	0.000149	0.005377	0.000928	0.000222	0.305811	0.100637	0.00093	0.397867	1
PPF	0	0	0	0	0	0	1	0	0	0	0	0	0	1
PIF	0.052996	0.005122	0.003785	0	0.004315	0.000796	0.028787	0	0.00119	0	0	0.004981	0.89803	1
RIF	0.242487	0.003749	0.00277	0	0.003158	0.000582	0.021069	0	0.000871	0	0	0.003645	0.721669	1
IPF-KB	0.147661	0.000074	0.000055	0.000672	0.000062	0.000012	0.000417	0.037739	0.000017	0.000104	0.000011	0.000072	0.813104	1
VS	0	0	0	0	0.411	0.427	0	0	0	0	0	0	0.162	1

....

9. SOME IMPLICATIONS

There can be a significant difference between a primary "face" image of the ownership structure and a "true" position of the subjects of property rights. This difference as a difference between "family capitalism" and "capitalism of agents" types of corporate governance has serious theoretical implications.

Just few questions:

a) How the profits are and should be distributed? We established that the final allocation of property to the individual property owners, after elimination indirect relations, is

$$\mathbf{X} = (\mathbf{I} - \mathbf{S}_0)^{-1}\mathbf{S}_0$$

while only listed direct initial distribution $\mathbf{S}_0$ is taken into account.

b) What are the implications for voting power in the corporate governance (Maeland, 1991, Gambarelli, 1994)? How the decision making power is and should be distributed: according to $\mathbf{X}$ or according to $\mathbf{S}_0$?[6]

c) Another issue for theoretical research is an implication of non-transparency of ownership structures on general equilibrium and welfare theory. Indirect ownership relations clearly generate externalities in profit maximization doctrine of general equilibrium theory: total profit of one company might depend on profits of other companies.

Many problems associated with the inadequacy of the current general equilibrium theory and welfare economics can be related to the theory of agency relationships (principal-agent problem). An agency relationship is a contract under which one or more persons (the principal(s)) engage another person (the agent) to perform some service on their behalf which involves delegating some decision making authority to the agent and providing some incentive scheme for the agent to maximize the welfare of the principal. Agency relations have been intensively investigated on the level of the firm (see e.g. Jensen and Meckling 1976, Varian 1992). But here we face the whole economy level of the principal-agent problem. Indirect ownership relations, generally viewed as full ownership relations, are frequently just agency relations. We are living in economy of agents behaving as owners. There is a hierarchical structure of agents in economy. Primary owners are principals and secondary institutional owners are in many cases just labels for agents. But in the network of indirect ownership relations an agent A becomes a principal with respect to some other agent B, the agent B becomes a principal with respect to some other agent C, and C can become a principal with respect to A, principal of his principals. So finally it is not clear who is an agent and who is his principal. Such situation can be considered a market imperfection and can lead to market

[6] An agenda for future research is to apply the methodology developed here to the control structures that are given not only by direct shares, but by hierarchical relations in networks of principals and agents. Extension of voting models and power indices methodology for such structures could bring new ideas also into studies of political behaviour.

failures.[7]

It is interesting that one of the major differences between the USA on the one hand and Germany and Japan on the other is in the role of corporations as each other's shareholders. In the USA it is rare that one corporation owns large block of shares in other companies; in some situations this is even forbidden by law. It is not so in Germany and Japan where high proportions of company shares are held by other corporations (Marer, 2000).

A hierarchical principal-agent problem and corporate governance design within the framework of general equilibrium theory and welfare economics is a challenge for economic theory

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[7] Not understanding clearly distinction between principals and agents and absence of agency relation regulation was one of the reasons of problematic results of Czech privatization (see Bohatá, 1998, Schwartz, 1997).