Reexamining Real Interest Rate Parity

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Abstract

Much research has investigated the real interest rate parity (RIRP) hypothesis that argues that real interest rates should equalize across countries. However, the empirical findings have been mixed. Early results, assuming stationarity of real rates, rejected the hypothesis. However, more recent tests that assume real rates to be nonstationary have found more support for RIRP. The objective of this paper is to consider whether the inconclusive results of previous RIRP tests may stem from the different approaches used in constructing real interest rates. The results for six different methods for calculating the real interest rate indicate that unit root results are very sensitive to how the real interest rate is computed. Testing 36 different bivariate combinations of the real interest rates in four OECD countries we find very limited support for the RIRP hypothesis. Furthermore, we also find very limited support for the existence of a world interest rate when we test the real rates jointly as a group.

Keywords: Real interest rate calculations; Real Interest Rate Parity; unit roots.

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1 Introduction

Since the end of the Bretton Woods era, international capital markets have become increasingly integrated, especially among the industrialized countries. In integrated financial markets, domestic investors can buy foreign assets and foreign investors can buy domestic assets. Therefore, assets with identical risk and liquidity should command the same expected return, regardless of locations. This is a basic postulate of real interest parity (RIRP) such that

$$r_t^e = r_t^{e*} \tag{1}$$

where r_t^e and r_t^{e*} are the *ex ante* real rate in home country and foreign countries, respectively. Arbitrage should encourage a tendency toward parity for international real interest rates.

Although much research has investigated such an international connection between the real interest rates, the empirical findings have been mixed. Early tests reject the real interest parity, whereas more recent tests using cointegration methods mostly support the validity of RIRP. The early RIRP literature assumed that real rates were stationary, and thus used standard regression techniques to test whether the computed real interest rate in one country was closely linked with another country's real interest rate. Mostly these tests provided very limited evidence for real interest rate parity. More recent tests have allowed for the possibility of nonstationary time series process of the real interest rates. Therefore, they have examined a potential common long-run relationship between two random walk real interest rate series. Such studies generally found evidence of mean-reverting real interest rate differentials and thus supported the validity of long-run RIRP hypothesis.

Why do some empirical studies fail to replicate the theoretical RIRP relationship, while others seem to support the theory? It is possible that the real interest rate series that researchers used in RIRP tests contribute to this conflicting evidence. The central problem in testing and estimating the linkage between domestic and foreign real interest rates relies on the fact that expected inflation, and hence the *ex ante* real interest rate, is unobservable. The *ex ante* real rates, which is defined as the difference between the nominal interest rate and the expected rate of future inflation, depend substantially on inflationary expectations of economic agents. Different assumptions about the inflationary expectations may lead to different conclusions about the validity of the RIRP hypothesis.

The objective of this paper is to consider whether the mixed results in previous research of the RIRP hypothesis arise as a result of different real interest rate calculations used in the past. We test the RIRP hypothesis for all combinations of four OECD countries that do not have any explicit exchange rate ties, namely: Japan, Switzerland, the United Kingdom, and the United States. To test the robustness of the results, six different real interest rates methodologies are calculated for each country. Hence, 36 different bivariate relationships are tested using both stationary and nonstationary regression methods. Furthermore, the possibility that all four countries share a common world interest rate is also examined.

The next section presents the background that provides the basis for testing the RIRP hypothesis. Section three discusses the data sources, the methodology and summarizes results of the constructed real interest rates. The fourth section presents the results of both the stationary and nonstationary RIRP tests, followed by concluding remarks in the final section.

2 Real Interest Rate Parity

When agents form their expectations rationally and there is no barrier to trade or capital flow, real interest rates should be equalized across countries. The RIRP can be viewed as a more general indicator of whether countries are integrated or autonomous, which has important policy implications such that it constrains the ability of domestic monetary authorities to intervene in foreign exchange markets. The RIRP relies upon four parity conditions, the Fisher relation in each country, *ex ante* purchasing power parity (EPPP) and the uncovered interest parity (UIP) - equations (2)-(5)

$$r_t = i_t - \pi^e_{t+1} \tag{2}$$

$$r^*_t = i^*_t - \pi^{e*}_{t+1} \tag{3}$$

$$s^e_{t+1} - s_t = \pi^e_{t+1} - \pi^{e*}_{t+1} \tag{4}$$

$$s^e_{t+1} - s_t = i_t - i^*_t \tag{5}$$

where $s^e_{t+1} = E_t(s_{t+1}|\phi_t)$, s_t is the logarithm of the spot exchange rate (domestic price of a foreign currency), s^e_{t+1} is the logarithm of the spot exchange rate expected to prevail at time $t+1$, and an asterisk denotes a foreign variable. Equations (2) and (3) follow the definition of the Fisher conditions for the domestic and foreign countries. Inflation rates are related across countries via the EPPP in equation (4) such that the expected exchange rate depreciation should be equal to expected inflation differential over the same period. The UIP relates the nominal interest rate differential to the expected exchange rate depreciation as in equation (5). Combining equations (2)-(5) yields the real interest parity relationship in equation (1).

The early studies, such as Mishkin (1984), Mark (1985) and Cumby and Mishkin (1986), utilized standard regression methods to test the existence of RIRP by implicitly assuming that real interest rates were stationary. In general, such tests found a lack of interest rate equalization across countries. In contrast, recent studies, assuming that real interest rates are nonstationary, have found strong comovement using cointegration techniques. For example, Goodwin and Grennes (1994) and Modjtahedi (1987) and Kugler and Neusser (1993) found the U.S. real interest rate to have a predictive content for other OECD countries, and Ferreira (2003) also found support for RIRP between developing and emerging countries.

The idea of cointegration can be related to the concept of long-run equilibrium between time series when one allows for the possibility of nonstationarity in the underlying series. If a linear combination of nonstationary ($I(1)$) variables is stationary ($I(0)$), then the variables are said to be cointegrated. The existence of a cointegrating vector implies that the two variables cannot move too far apart. If the real interest rates between two countries are cointegrated, for the RIRP to hold, the cointegrating vector must be [1,-1]. If the cointegrating vector differs from the unit vector then the real rates do not follow each other sufficiently to equalize, but are merely comoving. Briefly, the idea of cointegration is based on a vector autoregressive (VAR) model

$$Y_t = \Phi_1 Y_{t-1} + ... + \Phi_k Y_{t-k} + U_t, \qquad t = 1,2,...,T \tag{6}$$

where Y_t is the $n \times 1$ vector of $I(1)$ variables and U_t is a vector of white noise errors.[1] We rewrite this

[1]We only briefly discuss cointegration, because it is frequently used in the literature. For a more general discussion, see

equation as

$$\Delta Y_t = \Pi_1 Y_{t-1} + \Pi_2 \Delta Y_{t-1} ... + \Pi_k \Delta Y_{t-k+1} + U_t \tag{7}$$

where $\Pi_1 = -I + \sum_{i=1}^{k} \Phi_i$ and $\Pi_j = -\sum_{i=j}^{k} \Phi_i$ for $j = 2, ..., k$. The vector of interest is Π_1 which indicates the long-run relationship between the variables in Y_t. The rank of the Π matrix (r) conveys important information about the cointegrating behavior of the variables. If the matrix Π_1 has zero rank, then there is no cointegration among the $I(1)$ variable. The reduced rank ($r < n$) of the matrix Π_1 implies that there are r cointegrating vector among nonstationary variables. Lastly, the full rank ($r = n$) of Π_1 implies that all variables are stationary to begin with. Note that in the bivariate case we test, the Π_1 has to be of a rank = 1 to support the RIRP. Specifically, if we find a unit rank, then the estimated cointegrating vector must be [1, -1] to satisfy the RIRP condition. To establish the rank of the Π_1 matrix, we use the trace test and maximum eigenvalue test of Johansen (1991) and only estimate the long run cointegrating vector in the cases where a single cointegrating vector exits.

3 Constructing Real Interest Rates

This study analyzes the three-month eurocurrency deposit rates and monthly consumer price index for the 1978:09 to 2004:07 period for the United States, the United Kingdom , Japan, and Switzerland.[2] Constructing real interest rates is a difficult task. Conceptually, one must be careful in defining how agents develop their methods of inflation forecasting. As no single method can be found to have a clear superior forecasting accuracy, we present six methods used in the prior literature. Each country's *ex ante* real interest rates (r^e) are constructed using: (*i*) the *ex post* real interest rate, (*ii*) the AR(4) inflation forecast, (*iii*) Mishkin's linear projection, (*iv*) the recursive least squares regression, (*v*) the rolling regression method, and (*vi*) the regime-switching model.[3]

Johansen (1988).

[2]Specifically, the data come from the *International Financial Statistics* CD-ROM by the IMF. The interest rates and CPI are series *60ea* and *64*, respectively. The Paris interbank offer rate is used for pound sterling. For other currencies, the London interbank offer rates are used. The eurocurrencies are used to avoid governmental restrictions.

[3]The agent is assumed to use a five-year forecast window in the rolling regression framework, thus eliminating the first 5 years of data. Therefore the sample period is adjusted to 1983:08 to 2004:06 for comparability from different approaches of measuring the real interest rate.

3.1 Methods of Constructing Real Interest Rates

To mimic how agents form their expectations about future inflation and hence construct the real interest rate, we consider the six selected methods. They vary in terms of the underlying assumptions of the agents' available information set. The *ex post* real rate assumes that agents have rational expectations. Thus, researchers assume that the expected inflation can be proxied by the actual inflation (π_{t+1}), with a random inflation forecast error. Kugler and Neusser (1993) , Gagnon and Unferth (1995), and Goodwin and Grennes (1994) are examples of papers that use the *ex post* rates for the empirical methodology.

Alternatively, time-series forecasting methods can be useful to extrapolate a forecast of future inflation based on the past history of the inflation rate. Each technique assumes different available information set that agents utilize in their forecasting models. For instance, a method that is straightforward to implement is the AR representation, which expresses the current observable data as a function of the past observations. An example of RIRP tests using the AR specification is Baharumshah et al. (2005), who use an AR(1) specification for estimating expected inflation. In contrast, Mishkin (1984), Cumby and Mishkin (1986), Huizinga and Mishkin (1984) expand the autoregressive approach by adding some macroeconomic variables to an AR model of the expected inflation. This approach, hereafter referred as the "Mishkin approach," implies that the *ex ante* real rate equals the expected real return on a one-period bond, conditional on available information at time t:

$$r_t^e = X_t \beta + u_t \tag{8}$$

where $u_t = r_t^e - P(r_t^e|X_t)$ are the projection errors, $P(r_t^e|X_t)$ is a linear projection of r^e into X_t, and u_t is orthogonal to X_t. Mishkin's choice of X_t includes four lags of the inflation rate, one lag of money growth (M1), the nominal eurodollar interest rate, and a fourth-order time polynomial.

The above forecasting methods assume that agents have the full data sample to estimate the coefficients even in the beginning of the forecasting period. To relax this assumption, one can use an out-of-sample forecasting method. We select two related out-of-sample methods, the rolling regression and the recursive least squares methods, which differ in how they constrain the data available to the agent but allows a similar updating scheme of the inflation forecasting coefficient.[4] The rolling regression technique requires a fixed sample size, which is a 5-year moving-interval of data on inflation in our estimation. The approach is accomplished by adjusting the starting and end points of the data, when agents move across time. The recursive least squares approach, on the other hand, estimates the coefficients every period once new information about the rate of inflation is revealed and updates the forecasts accordingly. In this technique, the sample size grows each period. Thus, the precision of the estimates would increase as new observations are added, if the true coefficients are fixed across time. For the rolling regression and the recursive least squares approaches, the 3-month forecasts of future rates of inflation are obtained after a sequence of autoregressive moving-average ARMA(p,q) estimations of the inflation rate.[5] The choice of p and q are selected according to the Box-Jenkins model selection criterion.[6]

Prior research has found some evidence of a shift in inflation regimes. For example, Huizinga and Mishkin (1984) find that a significant shift in the stochastic process of real rates occurs around October 1979 when the Fed changed its policy procedure. Garcia and Perron (1996) considered such shifts in the behavior of the U.S. real interest rate, by allowing agents to incorporate possible regime switching in both mean and variance. The three-state Markov-switching mean-variance model accounts for regime shifts in an autoregressive model of the *ex post* real rate in the following way:

$$(y_t - \mu_{s_t}) = \phi_1(y_{t-1} - \mu_{s_{t-1}}) + \phi_2(y_{t-2} - \mu_{s_{t-2}}) + e_t, \tag{9}$$

where e_t is normally distributed with mean 0 and variance $\sigma^2_{s_t}$, and where y_t is an AR(2) process of the *ex post* real rate and μ_{s_t} and $\sigma^2_{s_t}$ is the mean and variance switching parameters when state S_{jt} is realized for $j = 1,2,3$, respectively. The state variable, S_t, is unknown *a priori*, but is governed by the Markov-switching transition probabilities.[7]

[4] Junttila (2001), for example, uses a rolling regression technique to allow agents only the information that was available at the time of the forecast.

[5] Our data on the nominal interest rate depend on a 3-month holding period. Therefore, the annualization of the inflation rate is calculated as $\pi_t = \ln(\frac{P_t}{P_{t-3}})^4$. Moreover, a one-period ahead forecast of inflation is computed by projecting 3-month ahead to coincide with the time to maturity of the bond.

[6] The fitted ARMA(p,q) models for each country's inflation rate are the following: Japan - ARMA$(2,3)$, Switzerland - ARMA$(2,3)$, UK - ARMA$(4,3)$, and US - ARMA$(1,4)$.

[7] To determine the log likelihood function, we use the Gibbs-sampling procedure that generates a sample from the marginal density without requiring the marginal density distribution itself. See Casella and George (1992) for examples

3.2 Time Series Distribution of Real Interest Rates

Much of the differences between the early and recent tests of RIRP rest on the assumptions about how the real interest rate is distributed, in particular the stationarity property of real rate series. Thus, it is important to examine how each country's real interest rates behave across time. For each country, the means of the real interest rates from different approaches appear to be similar, as shown in Table 1. Same pattern of standard deviations is also observed, except for the real interest rate from the Mishkin approach that appears to have a slightly lower variability.

In spite of the resemblance in the means and the standard deviations, the real rates from various approaches are significantly different in terms of the interpretation of the times series distribution. Since testing for the time series distribution is difficult due to the low power of unit root tests, we present results using four different tests, namely: the augmented Dickey-Fuller test (ADF), the Phillips-Perron unit root test (PP), the Dickey-Fuller test with GLS detrending (DF-GLS) introduced by Elliot, Rothenberg and Stock (1996), and the Ng-Perron test.

The most commonly used method to test for unit roots is the ADF test. For a time series process y_t, the ADF test is carried out by estimating

$$\Delta y_t = a + \alpha y_{t-1} + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + \varepsilon_t \tag{10}$$

The augmented terms, Δy_{t-j} for $j = 1, ..., k$, are included to correct the serial correlation of the disturbances ε_t and the number of k lags are selected by information criteria. The null hypothesis of a unit root ($\alpha = 0$) is tested against the alternative hypothesis of stationarity ($\alpha < 0$).[8] The PP test is an alternative non-parametric approach to deal with autocorrelation in the error term and allows for heterogeneity of the variance. Phillips and Perron (1988) proposed the nonparametric test statistic as follows:

$$Z_t = \frac{s_e}{s} t_{\hat{\alpha}} - \frac{1}{2} \frac{(s^2 - s_e^2)}{s(T^{-2} \sum_1^T y_{t-1}^2)^{1/2}} \tag{11}$$

where $\hat{\alpha}$ is the OLS estimate of α without the augmented differencing terms, $t_{\hat{\alpha}}$ is the t-ratio of α, s_e

and applications of the Gibbs-sampling algorithm.

[8]The test statistic for ADF test is evaluated using the t-ratio for α and MacKinnon's updated version of critical values.

is the coefficient standard error, and s^2 is a consistent estimate of the error variance. [9]

The DF-GLS and Ng-Perron unit root tests purport to solve the problem of low power and serious size distortions in the ADF and PP tests when the moving-average component in the series y_t is significant and negative. Both of the DF-GLS and Ng-Perron tests are based on simple modifications of the ADF and the PP tests, respectively. By GLS-detrending the data series prior to running the test regression, the tests yield substantial power gains. Therefore, we include these tests for comparison purposes. [10]

Table 2 summarizes the unit root tests of the real interest rates for the different countries. The results of the unit root tests show substantial differences across the different methodologies. Focusing on the summary column that is reported at the end of Table 2, one can observe that the conclusion differs between methodologies of constructing real interest rates. Across the four countries, we find the real rate to be $I(0)$ 27 times versus 59 times that we find the real rate to be $I(1)$. This implies that the real rate would be found to be an $I(1)$ process in about two thirds of the time. For the individual countries, Japan and the U.S. are the most likely countries to have nonstationary real rates (with 75 percent chance), followed by Switzerland and the U.K. that have about a two thirds chance of being $I(1)$ processes. Examining across the methodologies, the Mishkin approach is most likely to yield an $I(1)$ series, with the *ex post* approach being the least likely to produce an $I(1)$ series. The latter finding is consistent with the conjecture by Sun and Phillips (2004), who argue that an *ex post* calculation is less likely to lead to a unit root result due to the added noise component.

4 Real Interest Rates Linkages

Since we are unable to reach a clear conclusion about the distribution of the real interest rate, we test the RIRP concept using both stationary and nonstationary methods. Table 3 examines the relationship using stationary methods, which test all bivariate combinations of the four countries' real interest rates using the methodology proposed by Mishkin (1984). If RIRP holds, then the intercept

[9] We use a kernel sum-of-covariances estimator with Bartlett weights in combination with the Newey-West bandwidth selection method to obtain estimators of the residual spectrum at frequency zero. The asymptotic distribution of the PP t-ratio is the same as that of the ADF statistics, so we report MacKinnon's critical values for this test.

[10] See Elliott et al. (1996) for details on the DF-GLS test, and Perron and Ng (1996) for Monte Carlo simulations of the size and power of the Ng-Perron test.

coefficient, α, should equal zero and the slope coefficient, β, should be unity, as reported in the fourth and eighth columns. The results show that the RIRP hypothesis can be rejected in all combinations. Therefore, we do not find any evidence that real interest rates equalize across countries, using standard regression methods. However, we do find some connection. In all cases the bivariate combinations show some connection. This connection differs substantially between different methods of calculating the interest rate. In some cases, the differences in conclusions by researchers are so large that a researcher studying the Japan-Switzerland connection of real interest rates would either conclude that there is a very small connection, a β of 0.280, or a very high connection with a β of 0.828 depending on the method of calculating the real interest rate.

If the underlying real rates are found to be unit roots, then the appropriate test of the RIRP hypothesis would be to test for mean reversion of the real interest rate differential. If unit root tests indicate nonstationary real rate differential, then the persistent deviations from parity imply the rejection of RIRP. According to mixed evidence we found in Table 2 regarding whether the real rates are nonstationary or not, we proceed in this section by testing all real rate combinations with a cointegration methodology, assuming that all real rates are nonstationary. Table 4 shows the results of Johansen's maximum likelihood tests for each country-pair. We report the trace and maximum eigenvalue statistics in columns three to six, with the optimal lag length in the second column. Column seven provides a summary of the number of cointegrating vectors, with the last two columns showing the test of RIRP for the cases where a single cointegrating vector exists.[11]

The results in Table 4 indicates that the conclusions of the bivariate Johansen cointegration tests are sensitive to the methodologies used to calculate the real interest rate. The results provide very limited support for RIRP even if all real interest rates are assumed to be unit roots. Most of the combinations have either zero or two cointegrating vectors. If zero cointegrating vector exists, then the real interest rate differential is not mean reverting. On the contrary, if the bivariate combination has two cointegrating vectors, the real interest rates are stationary to begin with. Note that the same

[11] Since the Johansen cointegration test relies on the assumption of Gaussian error term in a VAR system, the lag order must be selected a priori in order to correct for serial autocorrelation. The lag order of an unrestricted VAR is searched within the maximum lag of 12. The selection of lag length is based on the Hannan-Quinn information criterion (HQC) and then residuals of the VAR model for a chosen lag are tested for no serial correlation with the LM test, $HQC = \ln(\hat{\sigma}^2) + 2T^{-1}k\ln[\ln(T)]$

country pair can have either zero, one or two cointegrating vectors depending on the method of calculation of the real interest rate used. For example, the Japan-UK combination has one case of zero cointegrating vector, three cases of one cointegrating vector, and two cases of two cointegrating vectors. Consequently, the types of method used to calculate the real interest rate yield very different conclusions. If the researcher used, for instance, the rolling regression approach to construct the real rates and assumed that all rates were $I(1)$, they would conclude that at least one cointegrating vector existed in all cases.[12] However, if one uses the Mishkin approach, most countries appear to have zero cointegrating vectors. Our findings indicate that only six of the 36 combinations appear to have a single cointegrating vector. We fail to reject a $[1,-1]'$ vector and hence support the validity of RIRP among these real interest rates in all these six cases. Since these combinations are constructed by various methods, there does not exists any clear pattern to explain why one can find support for the RIRP in certain cases.

Testing the bivariate combinations is not efficient if the true underlying real interest rate is a world interest rate that is shared by all the countries in our sample. To examine this possibility, we test for the mean reversion using a multivariate Johansen test. The findings are reported in Table 5. The Table shows very different conclusions depending on the type of methodology used. One cannot reject the existence of four cointegrating vectors for the *ex post*, recursive and rolling regression real rates using the Trace test. Such a finding implies that the real rates are not cointegrated, but are individually stationary. Nonetheless, the autoregressive methods results in a single cointegrating vector, implying some sort of connection between the rates. Only in the case of the regime-switching model do we find support for the hypothesis of a world interest rate, as three cointegrating vectors are significant. Finally, Mishkin's method for calculating the real rate results in a finding of zero cointegrating relationships.

[12]The researcher ought to continue to check for a second cointegrating vector. However, many times researchers stop their search for the number of cointegrating vector once they find one cointegrating vector.

5 Conclusions

The results show that tests of the times series process of Japanese, Swiss, U.K., and U.S. *ex ante* real interest rates lead to mixed results under different approaches of constructing the real interest rates. Due to the inconclusive results of stationarity tests, we evaluate the RIRP condition using both standard regression tests and cointegration tests. If all real interest rate series are treated as stationary then the RIRP is soundly rejected. Some comovements exist between countries, but not sufficient comovements to equalize real rates across countries. The estimated comovements are quite different between different methodologies of constructing the real rate, indicating substantial sensitivity to the underlying methods of constructing the real rates.

Furthermore the mean reversion of real interest differentials is tested, assuming that the real rates are nonstationary. Although the results differ markedly depending on the method of real interest rate measurement, the bivariate combinations present very limited evidence of real interest equalization. Similarly, in the multivariate tests we find little evidence of any cointegration between the four interest rates, and in no case do we find any evidence of a single underlying world real interest rate.

The results indicate that the methodology for constructing the real rates matters significantly to the outcome of the RIRP tests. Thus, this paper has two conclusions. First, it is important for researchers to allow for many different types of real interest rate calculations to make sure the results are robust to the method used. Secondly, our results for a number of different real interest rate calculations show that, in the case of these four countries, the RIRP hypothesis is unlikely to hold in the form that has been proposed by researchers so far.

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Table 1: Statistics for Real Interest Rates

Statistics	Ex Post	AR(4)	Mishkin	Rolling Reg.	Recursive	Switching
Japan						
Mean	2.286	2.181	2.328	2.237	2.306	2.328
Standard deviation	2.537	2.367	1.894	2.315	2.375	2.002
Switzerland						
Mean	1.967	1.909	1.951	1.961	2.002	1.910
Standard deviation	2.148	2.102	1.324	1.959	2.109	1.772
The UK						
Mean	4.566	4.523	4.702	4.614	4.694	4.492
Standard deviation	2.730	2.506	2.213	2.440	2.356	2.295
The US						
Mean	2.873	2.836	2.960	2.888	2.967	3.505
Standard deviation	2.534	2.526	2.398	2.497	2.486	2.128

The sample period is 1983M08 - 2004M04.

Table 2: Results of Unit Root Tests.

Statistics	Ex Post	AR(4)	Mishkin	Rolling Reg.	Recursive	Switching	$I(0)/I(1)$
Japan							
ADF test:	-1.564	-1.745	-1.693	-1.757	-1.622	-1.615	0/6
	(9)	(7)	(5)	(4)	(4)	(9)	
PP test:	-5.237*	-3.944*	-1.642	-4.025*	-3.815*	-3.767*	5/1
	(43)	(18)	(9)	(15)	(12)	(35)	
DF-GLS test:	0.030	-0.293	0.372	-1.445	-1.417	0.328	0/6
	(9)	(15)	(8)	(6)	(6)	(10)	
Ng-Perron test:	-0.981	-0.973	0.287	-0.659	-0.768	-0.500	0/6
	(9)	(15)	(8)	(6)	(6)	(10)	
Switzerland							
ADF test:	-1.376	-1.389	-0.695	-2.400*	-2.305*	-1.260	2/4
	(9)	(9)	(3)	(4)	(4)	(9)	
PP test:	-5.759*	-6.999*	-1.263	-5.456*	-5.432*	-4.713*	5/1
	(35)	(42)	(6)	(29)	(22)	(10)	
DF-GLS test:	-0.513	-1.408	-1.586	-1.253	-1.175	-0.616	0/6
	(9)	(10)	(14)	(10)	(10)	(9)	
Ng-Perron test:	-1.460	-1.067	-2.823*	0.329	0.923	-0.283	1/5
	(9)	(10)	(14)	(10)	(10)	(9)	
The UK							
ADF test:	-1.317	-1.313	-1.069	-1.555	-1.181	-1.073	0/6
	(10)	(7)	(5)	(2)	(7)	(9)	
PP test:	-2.063*	-2.914*	-1.188	-2.058*	-4.006*	-2.462*	5/1
	(46)	(64)	(97)	(34)	(24)	(77)	
DF-GLS test:	-2.206*	-1.983*	-0.733	-1.444	-1.114	-1.651	2/4
	(10)	(9)	(10)	(9)	(9)	(9)	
Ng-Perron test:	-2.905*	-1.662	-0.207	-1.855	-1.817	-1.228	1/5
	(10)	(9)	(10)	(9)	(9)	(9)	
The US							
ADF test:	-1.947*	-1.793	-1.419	-1.707	-2.070*	-1.660	2/4
	(3)	(3)	(2)	(3)	(2)	(2)	
PP test:	-2.655*	-3.752*	-1.529	-2.557*	-2.572*	-1.892	4/2
	(73)	(89)	(8)	(36)	(24)	(13)	
DF-GLS test:	0.674	-0.104	0.293	-1.521	-1.282	1.656	0/6
	(12)	(15)	(10)	(5)	(10)	(11)	
Ng-Perron test:	-0.018	1.061	0.062	-0.719	0.407	-1.234	0/6
	(12)	(15)	(10)	(5)	(10)	(11)	
$I(0)/I(1)$	7/9	5/11	1/15	5/11	6/10	3/13	

The sample period is 1983M08 - 2004M04. The numbers in parentheses are the number of lagged terms (or bandwidth for the Phillips-Perron test). The number of augmented terms in the ADF unit root test is based on SIC. Bandwidths in the Phillips-Perron unit root tests are determined by Newey-West using the Bartlett kernel. Lag length of the DF-GLS and Ng-Perron tests are based on the modified AIC. Critical value for the ADF, PP and DF-GLS tests is -1.942. The asymptotic critical value of the Ng-Perron test is -1.980. * indicates rejection of the unit root null hypothesis at a 5% significance level. The last column and row represent the summary of unit root results of which the first and second numbers represent the number of stationary and nonstationary conclusions, respectively.

Table 3: Results of Standard Regression Method

Country	α	β	$H_0 : \alpha = 0, \beta = 1$	Country	α	β	$H_0 : \alpha = 0, \beta = 1$
Japan-Switzerland				**Switzerland-UK**			
Ex post	1.730	0.282	73.33*	Ex post	0.329	0.358	319.78*
	(0.294)	(0.089)			(0.358)	(0.065)	
AR(4)	1.466	0.374	76.30*	AR(4)	-0.057	0.435	382.30*
	(0.277)	(0.074)			(0.333)	(0.060)	
Mishkin	0.712	0.828	7.49*	Mishkin	0.257	0.360	448.90*
	(0.268)	(0.080)			(0.297)	(0.061)	
Recursive	1.745	0.280	93.25*	Recursive	0.015	0.424	362.05*
	(0.296)	(0.075)			(0.354)	(0.064)	
Rolling Reg.	1.717	0.265	86.36*	Rolling Reg.	0.335	0.352	379.67*
	(0.311)	(0.079)			(0.347)	(0.064)	
Switching	1.744	0.306	90.39*	Switching	0.333	0.351	564.71*
	(0.252)	(0.077)			(0.321)	(0.058)	
Japan-UK				**Switzerland-US**			
Ex post	0.435	0.405	159.11*	Ex post	1.344	0.217	95.61*
	(0.388)	(0.069)			(0.367)	(0.094)	
AR(4)	-0.073	0.498	204.98*	AR(4)	1.349	0.197	137.22*
	(0.342)	(0.058)			(0.363)	(0.086)	
Mishkin	-0.854	0.677	277.37*	Mishkin	1.460	0.166	235.99*
	(0.273)	(0.049)			(0.329)	(0.076)	
Recursive	-0.104	0.514	203.16*	Recursive	1.414	0.198	132.80*
	(0.342)	(0.058)			(0.347)	(0.084)	
Rolling Reg.	0.149	0.452	237.96*	Rolling Reg.	1.507	0.157	137.01*
	(0.357)	(0.054)			(0.328)	(0.084)	
Switching	0.290	0.454	294.74*	Switching	1.725	0.053	262.02*
	(0.315)	(0.051)			(0.291)	(0.069)	
Japan-US				**US-UK**			
Ex post	1.353	0.325	87.75*	Ex post	1.066	0.396	56.25*
	(0.256)	(0.074)			(0.587)	(0.118)	
AR(4)	1.224	0.337	121.73*	AR(4)	0.775	0.456	57.90*
	(0.255)	(0.061)			(0.570)	(0.116)	
Mishkin	0.924	0.474	87.48*	Mishkin	-1.046	0.852	78.31*
	(0.238)	(0.056)			(0.430)	(0.089)	
Recursive	1.399	0.306	130.19*	Recursive	0.726	0.478	55.80*
	(0.257)	(0.061)			(0.601)	(0.122)	
Rolling Reg.	1.253	0.340	129.95*	Rolling Reg.	0.794	0.454	50.73*
	(0.249)	(0.058)			(0.581)	(0.124)	
Switching	0.821	0.430	189.52*	Switching	0.289	0.454	295.51*
	(0.326)	(0.057)			(0.315)	(0.051)	

* denotes the rejection of the null hypothesis at 5% significance level. The second and third columns represent the estimated intercept and slope coefficients α and β of the standard regression model with the corresponding standard errors in the parentheses. The fourth column represents the test statistics of joint significance whether coefficients $\alpha = 0$ and $\beta = 1$. The critical values for χ^2_2 distribution are 5.991 and 9.210 at 5% and 1%, respectively.

Table 4: Johansen Cointegration Results for Bivariate System

Country	Lag	Trace		Max. Eigen		# Coint. Vectors	Coint. Estimates	RIRP $[1 \ -1]$
		$r=0$	$r \leq 1$	$r=0$	$r=1$			
Japan-Switzerland								
Ex post	10	9.956	2.276	7.680	2.276	0		
AR(4)	8	13.758	2.796	10.962	2.796	0		
Mishkin	4	7.788	1.192	6.597	1.192	0		
Recursive	3	51.093*	9.150*	41.943*	9.150*	2		
Rolling Reg.	3	44.585*	9.353*	35.232*	9.353*	2		
Switching	10	6.839	1.632	5.207	1.632	0		
Japan-UK								
Ex post	11	20.127*	2.552	17.574*	2.552	1	$[1 \ -1.104]$	0.604
AR(4)	8	23.702*	2.403	21.299*	2.403	1	$[1 \ -1.152]$	0.385
Mishkin	6	8.455	0.010	8.445	0.010	0		
Recursive	8	23.384*	4.246*	19.137*	4.246*	2		
Rolling Reg.	3	39.221*	7.631*	31.591*	7.631*	2		
Switching	11	18.042*	1.765	16.722*	1.765	1	$[1 \ -1.138]$	0.431
Japan-US								
Ex post	10	8.774	2.450	6.325	2.450	0		
AR(4)	8	7.903	1.151	6.752	1.151	0		
Mishkin	4	9.847	0.519	9.327	0.519	0		
Recursive	3	24.187*	6.163*	17.574*	6.613*	2		
Rolling Reg.	4	16.607*	4.590*	12.016	4.590*	2/0		
Switching	7	17.396*	1.024	16.372*	1.024	1	$[1 \ -1.139]$	0.576
Switzerland - UK								
Ex post	11	18.203*	4.299*	13.904	4.299*	0/2		
AR(4)	8	21.599*	3.618	17.981*	3.618	1	$[1 \ -0.731]$	0.197
Mishkin	4	14.489	1.481	13.008	1.481	0		
Recursive	8	29.269*	6.330*	22.940*	6.330*	2		
Rolling Reg.	3	48.479*	10.977*	37.502*	10.977*	2		
Switching	4	32.658*	7.975*	24.683*	7.975*	2		
Switzerland - US								
Ex post	10	8.112	2.654	5.453	2.659	0		
AR(4)	10	5.723	0.556	5.167	0.556	0		
Mishkin	4	14.689	0.638	14.051	0.638	0		
Recursive	3	36.502*	8.730*	27.772*	8.730*	2		
Rolling Reg.	4	26.940*	7.228*	19.712*	7.228*	2		
Switching	4	25.631*	8.419*	17.211*	8.419*	2		
US - UK								
Ex post	11	17.424*	4.269*	12.974*	4.269*	2		
AR(4)	10	13.076	2.925	10.151	2.925	0		
Mishkin	4	16.158*	0.793	15.365*	0.793	1	$[1 \ -0.768]$	0.178
Recursive	8	17.311*	4.676*	12.635	4.767*	2/0		
Rolling Reg.	3	22.252*	5.929*	16.323*	5.929*	2		
Switching	12	12.271	2.228	10.043	2.228	0		

The first column represents the lag order of the cointegration test as chosen based on HQC. The maximum lag length is set at 12. r denotes a hypothesized number of cointegrating vectors under the null hypothesis. * denotes significance at 5% level. The trace test critical values at the 0.05 level are 15.41 (for $r = 0$) and 3.76 (for $r \leq 1$). The maximum eigenvalue test critical values at the 0.05 level are 14.07 (for $r = 0$) and 3.76 (for $r = 1$). The entries in the 'Coint. Estimate' column are the estimated cointegrating vectors, normalized on the first country real interest rate in the country pair. The last column is the probability of the LR test (which is distributed as a χ^2_1) for examining whether the null hypothesis of the cointegrating vector is equal to $[1 \ -1]'$.

Table 5: Johansen Cointegration Results for Multivariate System: Japan, Switzerland, U.K., and U.S.

	Ex Post	AR(4)	Mishkin	Recursive	Rolling Reg.	Switching
Trace test						
$r = 0$	80.843*	57.161*	36.578	100.68*	92.183*	81.484*
$r \leq 1$	44.471*	29.475	16.433	46.800*	48.869*	42.888*
$r \leq 2$	15.581*	11.429	5.862	18.888*	17.523*	17.474*
$r \leq 3$	4.917*	2.480	0.061	6.222*	5.528*	2.429
Max. eigenvalue test						
$r = 0$	36.372*	27.685*	20.157	53.883*	43.324*	38.596*
$r = 1$	28.890*	18.045	10.559	27.912*	31.346*	25.414*
$r = 2$	10.665	8.949	5.802	12.667	11.995	15.044*
$r = 3$	4.917*	2.480	0.061	6.222*	5.528*	2.429
# Coint. vectors	4/2	1/1	0/0	4/2	4/2	3

* denotes significance at 5% level. The trace test critical values at the 0.05 level are 47.21 (for $r = 0$), 29.68 (for $r \leq 1$), 15.41 (for $r \leq 2$), and 3.76 (for $r \leq 3$). The maximum eigenvalue test critical values at the 0.05 level are 27.07 (for $r = 0$), 20.97 (for $r = 1$), 14.07 (for $r = 2$), and 3.76 (for $r = 3$). The lag length was chosen by using the BIC information criteria and was 4 in all cases, except in the case of the rolling regression and the recursive least squares specification where the optimal lag length is 3.