Indeterminacy of free entry equilibria: general approach and macroeconomic applications

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Abstract

Free entry equilibria are usually determined by resorting to the zero profit condition. We plead instead for a strict application of the Nash equilibrium concept to a symmetric one-stage game played by actual and potential producers, who have a decreasing average cost function without sunk costs. Equilibrium then appears as typically indeterminate, with a number of active firms varying between an upper bound imposed by profitability and a lower bound required by sustainability. This indeterminacy may have significant macroeconomic implications, since it opens the way to coordination failures and to the emergence of endogenous fluctuations generated by the coordination process. The paper presents a general framework for the analysis of free entry equilibria, applies this framework to the standard regimes of price and quantity competition used in macroeconomic modelling, and illustrates dynamic aggregate implications in a simple macroeconomic model.

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1 Introduction

Free entry is commonly associated with zero profits. Under free entry and exit, positive profits are supposed to stimulate creation of new firms and negative profits to induce destruction of existent firms. A free entry equilibrium may thus be seen as a stationary state, characterized by the zero profit condition, of a dynamic process of net business formation. This view is implicit in the concept of long run perfectly competitive equilibrium, and is naturally extensive to monopolistic competition, where the relevant scales of individual firms also appear as negligible with respect to market size. As long as profits remain positive, any entrant is then able to reproduce in an unreactive environment the operating conditions and the proceeds of a high number of successful incumbents.

This line of argument ceases to hold, however, when a potential entrant has to compete with a few incumbents only, all producing under internal economies of scale. In this context, a simple replication of the incumbents' performance cannot guarantee identical success to the entrant, whose environment may be seriously perturbed by that replication. Entry must now be examined as a *strategic* decision, generally in a complex context, where timing and information considerations are at stake. Faced with this difficulty, a common attitude in macroeconomic modelling is to take the zero profit condition as an acceptable approximation and to leave more sophisticated approaches to industrial organization theory. This was already the position adopted in one of the first macroeconomic papers emphasizing the role of increasing returns and imperfect competition, and treating the number of producers as endogenous: "The story [behind the solution concept based on the zero profit condition] can only be defended as an approximation. Entry and exit are complicated phenomena, involving difficult game theoretic issues that defy neat analytic formulation" (Weitzman, 1982, p. 797). So, we seem to be trapped in a dilemma: either to force a solution concept devised for non-strategic forms of competition into the domain of oligopoly, or to resort to industrial organization tools that may prove too complex and also too specific for an accurate macroeconomic use. The point we want to make in this paper is that we are by no means doomed to that dilemma. A straightforward application of the concept of Nash equilibrium to static symmetric games reproducing standard regimes of oligopolistic competition offers in fact a simple way out.

To be explicit, we owe to Shubik (1959, 1984) the idea that entry can be modelled as a one-stage game between actual and potential entrants, depicted as "firms-in-being". At an equilibrium of such a game, along with *active* profit maximizing firms, there may well be *inactive* firms that optimally decide not to produce, on the basis of correct conjectures about the actions of the former. This asymmetry might be the consequence of some advantage of incumbents over potential entrants, creating a barrier to entry. But it can also prevail in a completely symmetric game where all players are *a priori* indistinguishable. *Ex ante* symmetry is in fact required for an accurate representation of a *perfectly contestable market*, characterized by costless entry and exit and no disadvantage for potential entrants relative to incumbents (Baumol, Panzar and Willig, 1982).

For a market to be perfectly contestable any observable profile of incumbents' strategies must be *sustainable*, that is, no potential entrant may be able to make a profit by becoming active. This requirement might suggest that, as in the dynamic story of business formation, equilibrium profits are necessarily close to zero as soon as entry is free, or the market perfectly contestable. And this is indeed true if incumbents' capacity to earn positive profits extends to any potential entrant, always in a position to attract enough customers either by slightly undercutting incumbents' prices (as in the Bertrand oligopoly[1]), or by simply imitating incumbents' pricing behavior within the Chamberlinian "large group" where an individual price decision has no sensible repercussions on the industry price level (as in Dixit and Stiglitz, 1977). Sustainability is however not independent of the regime of competition, and may well refer instead, with different implications, to Cournot oligopoly (Novshek, 1980; Brock and Scheinkman, 1983) or to price competition within a "small group" producing differentiated goods, in a modified Dixit-Stiglitz setting (d'Aspremont, Dos Santos Ferreira and Gérard-Varet, 1996). It then appears that the zero profit condition is by no means necessary for sustainability, and that multiple free entry equilibria may quite generally exist along with the one at break-even prices (d'Aspremont, Dos Santos Ferreira and Gérard-Varet, 2000). In those equilibria, the strategies of active firms entail positive profits and are nevertheless sustainable because potential entrants, taking them into account, realize that, whatever they do, demand will be insufficient for attaining the scale at which production becomes profitable. Under these circumstances, there is of course no sensible reason for the incumbents to accommodate entry.

Multiplicity of equilibria raises coordination issues, which may have important macroeconomic implications. The first issue concerns *coordination failures*, whose scope – as we will show – is *broadened* through the asymmetry allowed between active and inactive firms, so that their existence ceases to be confined to regimes of competition displaying strategic complementarity (as in Cooper and John, 1988). But since symmetry is preserved among actual producers the simplicity of symmetric Nash equilibria remains intact. The second issue raised by equilibrium multiplicity concerns the emergence of *sunspot fluctuations* independently of dynamic indeterminacy, as firms in each industry need to coordinate on some extrinsic, potentially varying, public signal (Dos Santos Ferreira and Lloyd-Braga, 2003, and Dos Santos Ferreira and Dufourt, 2005).[2]

The examples put forward in the last two quoted papers might suggest that the indeterminacy we are considering is quite specific, even if it appears in different modelling contexts. An objective of the present paper is to show that it is on the contrary a very robust property of oligopolistic competition in contestable markets. For that purpose we provide a *unified conceptual and analyt-*

[1] A recent game-theoretic formulation of the Bertrand oligopoly under free entry, leading precisely to the long run competitive outcome, has been proposed by Yano (2005).

[2] A related idea has been explored by Chatterjee, Cooper and Ravikumar (1993) in a model where equilibrium multiplicity results from the heterogeneity of producers' participation costs and from the existence of demand externalities. Indeterminacy is however a pervasive property of free entry equilibria which does not depend upon heterogeneity.

ical framework for the study of free entry equilibria, covering different regimes of oligopolistic competition and different specifications of internal increasing returns.

We carry out this task in section 2, by (*i*) defining a general concept of free entry equilibrium, (*ii*) introducing a canonical model where strategies are represented by prices, and (*iii*) establishing equilibrium conditions on incumbents' prices under general specifications of cost and demand functions.

These conditions are applied in section 3 to the three standard regimes that have been used in macroeconomic modelling: quantity competition in a homogeneous oligopoly (Cournot) and price competition in a differentiated oligopoly, when the products are the elements of a composite good (Dixit-Stiglitz, adapted to the "small group" case) and when they are spatially dispersed along the circle (Salop, 1979). In each one of these cases, we determine an interval of admissible numbers of active firms in free entry equilibria. We observe that, under positive but *arbitrarily small* economies of scale, this interval contains typically (although not always) more than one integer.[3] Our indeterminacy results are summarized in self-contained, ready-to-use, claims, providing a toolbox for various types of macroeconomic applications.

We illustrate in section 4, by using a very simple and standard overlapping generations model with money as sole asset and labor as sole input, the potentially important implications for the economy as a whole of this fundamental indeterminacy. In particular, we argue that it is unlikely that coordination among firms will systematically select in all sectors and in all periods, as implicit in the zero profit condition, the *least profitable* equilibrium for active firms, a possibility which would imply that incumbents are always ready to accommodate entry. As soon as this possibility is discarded, some coordination process must be assumed, which may well lead to Pareto dominated outcomes or generate endogenous aggregate fluctuations. We conclude in section 5.

2 Oligopolistic equilibrium with free entry

In this section, we first introduce a game theoretic framework applying to perfectly contestable oligopolistic markets and exploiting symmetry of strategy profiles (in the spirit of Cooper and John, 1988). In this context, we define a comprehensive concept of free entry equilibrium, characterized by two conditions, *profitability* and *sustainability*. Second we formulate, under general assumptions on demand and cost functions, a *canonical model* where incumbents' strategies are *represented* by a price, even when involving a quantity or a location in the characteristics space, as in Cournotian or spatial competition,

[3] Free entry equilibrium is unique in the oligopolistic variant of the Dixit-Stiglitz model, with linear (affine) cost (Claim 4). Sustainability and profitability, the two free entry equilibrium conditions, are then equivalent to the zero profit condition (if we disregard the so-called integer problem). But this equivalence is not robust: indeterminacy appears as soon as we modify the specification of the cost function by assuming strict concavity (Claim 3), or as we switch to a different form of product differentiation while keeping the assumption of cost linearity (Claim 5).

respectively. Third we show that a free entry equilibrium requires the common price set by the incumbents to be a *critical* point of their profit function (for optimality), above the *break-even* price (for profitability) and below the *limit* price (for sustainability). These bounds on incumbents' prices then translate into a non-degenerate admissible interval to which the number of active firms should belong.

2.1 The concept of free entry equilibrium

Free entry means absence of any entry barrier accounting for some advantage of incumbents over entrants. Under free entry all firms, whether established or not, are supposed to benefit from full equality of opportunities. But this does not imply that they are assured of equality of results. In game theoretic terms, firms are assumed to play a symmetric game, the equilibria of which need however not be symmetric. These equilibria may in any case display a primary kind of asymmetry, the one which concerns us here, involving the distinction between active and inactive firms.

To be explicit, consider a symmetric game played by N competing oligopolistic firms, each one with the strategy space $\mathbb{S}$ and the payoff function $\mathbf{\Pi} : \mathbb{S}^N \to \mathbb{R}$. A firm is *inactive* if it chooses an element of the subset $\mathbb{S}_0$ of strategies leading to zero output, and it is *active* if it chooses a strategy in the complementary subset. The nature of the subset $\mathbb{S}_0$ results from the particular specification of the model, $\mathbb{S}_0$ being for instance equal to $\{0\}$, in quantity competition games, or to the set of prices higher than any buyer's reservation price, in price competition games. We admit that the payoff function is constant with respect to any of its arguments over $\mathbb{S}_0$, if this set has more than one element. Now consider strategy profiles $\mathbf{s} \in \mathbb{S}^N$ that are symmetric within the class of n active firms $(0 < n < N)$,[4] all choosing $s_n \in \mathbb{S} \diagdown \mathbb{S}_0$ while $N - n$ inactive firms indifferently choose some element of $\mathbb{S}_0$. It is clear that the relevant information in $\mathbf{s}$ is completely contained in the pair (s_n, n). Similarly, as the vector $\mathbf{s}_{-i} \in \mathbb{S}^{N-1}$ of strategies of the $N - 1$ competitors of any firm i has $n - \delta$ elements equal to s_n (with $\delta = 1$ if firm i is active and $\delta = 0$ if it is inactive) and $N - 1 - (n - \delta)$ elements belonging to $\mathbb{S}_0$, it can be fully characterized by the triplet (s_n, n, δ). The profit $\mathbf{\Pi}(s_i, \mathbf{s}_{-i})$ of any firm i, choosing strategy $s_i = s$ and facing a profile $\mathbf{s}_{-i}$ of its competitors' strategies with such characterization, can then be denoted accordingly by $\Pi(s, s_n, n, \delta)$.

If we apply the Nash equilibrium concept to this framework, for a pair (s_n, n) to characterize an equilibrium, the profit $\Pi(s, s_n, n, 1)$ of an active firm must reach a maximum at $s = s_n$, and the profit $\Pi(s, s_n, n, 0)$ of an inactive firm must reach a maximum at any $s_0 \in \mathbb{S}_0$. Also, if we take free entry as comprehending free exit, so that sunk costs are excluded, inactivity always results in zero profits, so that any equilibrium (s_n, n) must also verify $\Pi(s_n, s_n, n, 1) \geq 0$ and $\Pi(s_0, s_n, n, 0) = 0$. This standard application of the Nash equilibrium concept differs from the usual understanding of a free entry (and exit) equilibrium,

[4] We admit that $n > 0$ in order to eliminate trivial equilibria, and that $n < N$ to put aside the case where entry is impossible for lack of further participants.

which is to require, first that $\Pi(s, s_n, n, 1)$ reach a maximum non-negative value at $s = s_n$, and second that there be no equilibrium with $n+1$ active firms (symmetric with respect to these firms).[5] The second condition means that, for any strategy $s_{n+1} \in \mathbb{S} \diagdown \mathbb{S}_0$, if $\Pi(s, s_{n+1}, n+1, 1)$ is maximized on $\mathbb{S} \diagdown \mathbb{S}_0$ at $s = s_{n+1}$ then $\Pi(s_{n+1}, s_{n+1}, n+1, 1) < 0$. Neglecting the so-called "integer problem" (as n belongs to $\mathbb{N}^*$, not to $\mathbb{R}_+$, $\Pi(s_n, s_n, n, 1) > 0$ cannot be excluded), this condition may be identified to the *zero profit condition*, commonly seen as implied by free entry.

We stick instead to the standard Nash equilibrium concept, leading to the following definition.

Definition 1 *A non-trivial symmetric* ***free entry equilibrium*** *is a pair* (s_n, n) *in* $(\mathbb{S} \diagdown \mathbb{S}_0) \times \{1, ..., N-1\}$ *satisfying two conditions:*

$$\begin{aligned} \max_{s \in \mathbb{S}} \Pi(s, s_n, n, 1) &= \Pi(s_n, s_n, n, 1) \geq 0 \text{ (\textbf{profitability}) and} \\ \max_{s \in \mathbb{S}} \Pi(s, s_n, n, 0) &= \max_{s \in \mathbb{S}_0} \Pi(s, s_n, n, 0) = 0 \text{ (\textbf{sustainability}).} \end{aligned}$$

A strategy profile characterized by the pair $(s_n, n) \in (\mathbb{S} \diagdown \mathbb{S}_0) \times \{1, ..., N-1\}$ can be an equilibrium only if it is *profitable* for any active firm to choose the strategy s_n, meaning that no higher profit is attainable either while staying active ($\Pi(\cdot, s_n, n, 1)$ is maximized at s_n) or through becoming inactive ($\Pi(s_n, s_n, n, 1)$ is non-negative). It must also be *sustainable* with respect to inactive firms, which should not be able to obtain a positive profit by becoming active.

2.2 A canonical pricing model

We consider an industry with decreasing demand $D(P)$ for either a homogeneous or a composite good sold at price P. The good is potentially produced under internal increasing returns by N firms with the same increasing cost function $C(y)$, with $C(0) = 0$ (no sunk costs) and such that average cost $C(y)/y$ is decreasing on $(0, \infty)$. As in the preceding subsection, we restrict our analysis to equilibria which are symmetric with respect to n active firms ($1 \leq n < N$), all choosing the same strategy $s_n \in \mathbb{S} \diagdown \mathbb{S}_0 \subset \mathbb{R}_+$. In order to obtain a simple unified framework applying to different regimes of competition, we shall always *represent* this strategy by the price p_n at which any active firm intends to sell

[5] The underlying rationale for the second condition is that entry/exit are first stage strategies of a sequential game, in which price and/or quantity decisions are taken at the second stage. For a strategy profile with n active firms to be a sub-game perfect equilibrium, the equilibrium profit expected for n active firms at the second stage must be non-negative in order to induce entry of these n firms at the first stage, and the corresponding profit for $n+1$ active firms must be negative, otherwise a further competitor would always want to enter. This argument implicitly assumes that all entrants necessarily benefit from *equal treatment* at the second stage equilibrium, which is by no means implied by free entry. Hence, sequentiality (with equilibrium sub-game perfection) is not enough to validate the usual interpretation of free entry equilibrium.

its output. Thus, any firm deciding to supply quantity y at price p, and facing demand $d(p, p_n, n, \delta)$, has to solve a problem that can be stated as follows:

$$\max_{(p,y)\in\mathbb{R}_+^2} \{py - C(y) : y \leq d(p, p_n, n, \delta)\}. \tag{1}$$

Clearly, a pair (p, y) such that $0 < y < d(p, p_n, n, \delta)$ cannot be a solution to this problem, since the profit is increasing in y if $C(y)/y \leq p$. Thus, the firm will always decide either to produce $y = d(p, p_n, n, \delta)$ or to stay inactive (*i.e.* to choose $y = 0$), so that we may stick to the *canonical program* in the single decision variable p

$$\max_{p\in\mathbb{R}_+} \{pd(p, p_n, n, \delta) - C(d(p, p_n, n, \delta))\}, \tag{CP}$$

and then check that the maximum profit is non-negative, taking otherwise $y = 0$ as the optimal decision.

One sees immediately that the canonical program (CP) covers the case where firms produce differentiated goods and compete in prices. It is less evident yet true that it also covers the case of a homogeneous oligopoly with Cournotian firms. Indeed, given symmetry with respect to n active firms, each one of these firms chooses $p_n = P$ and $y_n = D(P)/n$, while $N - n$ inactive firms all choose $y = 0$. The residual demand at price p for any firm, whether active ($\delta = 1$) or inactive ($\delta = 0$) is $D(p) - (n - \delta) y_n = D(p) - (1 - \delta/n) D(p_n) \equiv d(p, p_n, n, \delta)$. As $y = d(p, p_n, n, \delta)$ if and only if $D(p) = (n - \delta) y_n + y$, program (CP) is indeed equivalent to the standard program of the Cournotian firm, namely $\max_{y\in\mathbb{R}_+} \left\{D^{-1}((n - \delta) y_n + y) y - C(y)\right\}$.

We now introduce the two following general assumptions on the (average) cost and demand functions:

A1 The function $C(y)/y$ is twice differentiable and has a negative, non-decreasing elasticity ($\epsilon_y C(y) - 1 < 0$, $\epsilon_{yy}^2 C(y) \geq 0$)[6] in the interval $(0, \infty)$.

A2 For any triplet (p_n, n, δ), the function $d(\cdot, p_n, n, \delta)$ is twice differentiable in the interval $(0, \widetilde{p}(p_n, n, \delta))$ in which it is positive (where $\widetilde{p}(p_n, n, \delta) \in (0, \infty]$ is the supremum of buyers' reservation prices), and has in this interval a negative, decreasing elasticity ($\epsilon_p d(\cdot, p_n, n, \delta) < 0$, $\epsilon_{pp}^2 d(\cdot, p_n, n, \delta) > 0$), such that

$$\frac{1}{\lim_{p\to 0} \epsilon_p d(p, p_n, n, \delta)} < \lim_{y\to\infty} \epsilon_y C(y) - 1 \text{ and} \tag{2}$$

$$\frac{1}{\lim_{p\to\widetilde{p}(p_n,n,\delta)} \epsilon_p d(p, p_n, n, \delta)} > \lim_{y\to 0} \epsilon_y C(y) - 1. \tag{3}$$

Furthermore, the function $d(p, \cdot)$ is increasing in p_n and δ, and non-increasing in n, as long as its value remains positive.

[6] We denote $\epsilon_x f(x, y) \equiv (\partial f(x, y)/\partial x) x / f(x, y)$ the partial elasticity of f at (x, y) with respect to x. All related elasticity notations are self-explanatory.

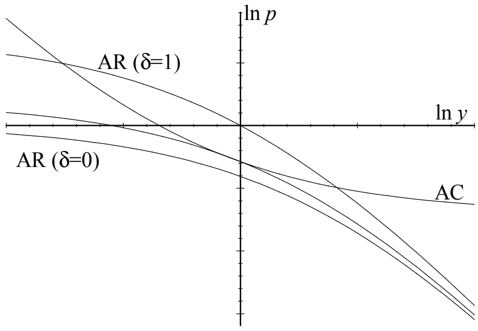

Figure 1: Average cost and revenue curves

Assumption (A1) states that the average cost curve AC is decreasing and convex when represented in the space $(\ln y, \ln p)$, and assumption (A2) that the average revenue curve AR (given by the inverse of function $d(\cdot, p_n, n, \delta)$) is decreasing and strictly concave in the same space (see Figure 1). Also, by inequalities (2) and (3) on the limit values of the slopes of these curves, average cost is higher than average revenue both for y close to zero and for y close to infinity.

Profitability requires the average revenue curve of an active firm AR ($\delta = 1$) to be higher than the average cost curve AC for intermediate values of y (as represented in Figure 1). Sustainability requires by contrast that the average revenue curve of an inactive firm AR ($\delta = 0$) be lower than the average cost curve AC for all values of y. The two conditions are compatible because average revenue is increasing in δ by assumption (A2).

2.3 Equilibrium conditions

Under the assumptions of the preceding subsection, we can reformulate these two conditions in terms of the price p_n set by any active firm. Indeed, for this price to characterize a symmetric equilibrium, it must clearly be a *critical* point of the profit function in the canonical program (CP), at least equal to the *break-even* price (so that profitability may be satisfied). It must also be at most equal to the *limit* price deterring entry (so that sustainability may be ensured). We are now going to examine sufficiency of these conditions.

Definition 2 *A **critical price** p_n^* is a positive price that, when simultaneously set by n active firms, satisfies the first order condition necessary for an interior solution of (CP), that is, solves the equation of marginal revenue with marginal cost:*

$$p_n \left(1 + 1/\epsilon_p d\left(p_n, p_n, n, 1\right)\right) = C'\left(d\left(p_n, p_n, n, 1\right)\right). \tag{FOC}$$

If the critical price entails non-negative profits, this first order condition is in fact sufficient for an interior solution of program (CP), so that profitability is then satisfied, as stated in the following lemma:

Lemma 1 (Profitability) *Under assumptions (A1) and (A2), the symmetric strategy profile represented by the pair* $(p_n^*, n) \in \mathbb{R}_{++} \times \{1, ..., N-1\}$ *satisfies the profitability condition if and only if* p_n^* *is a critical price entailing non-negative profits, that is, leading to a revenue-cost ratio at least equal to one:*

$$g\left(p_n^*, n\right) \equiv \frac{p_n^* d\left(p_n^*, p_n^*, n, 1\right)}{C\left(d\left(p_n^*, p_n^*, n, 1\right)\right)} \geq 1. \tag{PNNC}$$

Proof. See Appendix. ■

Now, if the function $g(\cdot, n)$ is non-decreasing, the profit non-negativity condition can equivalently be expressed by requiring that the critical price be at least equal to the break-even price:

Definition 3 *The* ***break-even price*** $\underline{p}(n)$ *is the lowest price* p_n *which, when set by all the active firms, allows them to get non-negative profits:* $\underline{p}(n) \equiv \inf \mathcal{P}(n)$*, with*

$$\mathcal{P}(n) \equiv \{p_n \in (0, \infty) : d(p_n, p_n, n, 1) > 0 \text{ and } g(p_n, n) \geq 1\} \tag{4}$$

(by convention, $\underline{p}(n) = \infty$ *if* $\mathcal{P}(n) = \varnothing$*).*

From this definition, it is clear that the inequality $p_n \geq \underline{p}(n)$ is a necessary and sufficient condition for profit non-negativity if $g(\cdot, n)$ is increasing. In order to discuss this property, we may consider the demand to the industry $D(P) = \alpha(n) nd(p_n, p_n, n, 1)$ at $P = p_n/\alpha(n)$, where $\alpha(n)$ is an aggregating factor to be used when the product is a composite good ($\alpha(n) \equiv 1$, otherwise).[7] Indeed, the elasticity of $g(\cdot, n)$ is then seen to be

$$\epsilon_{p_n} g(p_n, n) = 1 + (1 - \epsilon_y C(d(p_n, p_n, n, 1)))\, \epsilon_P D(p_n/\alpha(n)), \tag{5}$$

positive, by assumption (A1), if $\epsilon_P D(P) \geq -1$ for any P. Otherwise, when the elasticity of demand to the industry takes values smaller than -1, so that the monotonicity of $g(\cdot, n)$ is not guaranteed, the condition $p_n \geq \underline{p}(n)$ remains necessary, but sufficiency is lost. Profits may then become negative at high prices, inducing too low expenditure levels (in particular to cover fixed costs).

Finally, recall that the *limit price* has been defined as "the highest common price which the established seller(s) believe they can charge without inducing at least one increment to entry" (Bain, 1949, p. 454). This is the price leading to an average revenue curve of the potential entrant which is just below the average cost curve (Modigliani, 1958), as represented by the dotted curve tangent to curve AC in Figure 1.

[7] The definition of the price index ensures that the aggregate expenditure $PD(P)$ in the industry is indeed equal to the sum $np_n d(p_n, p_n, n, 1)$ of firms revenues.

Definition 4 *The* ***limit price*** $\overline{p}(n)$ *is the highest price* p_n *which, when set by all the active firms, prevents an inactive firm from getting positive profits:* $\overline{p}(n) \equiv \sup \widetilde{\mathcal{P}}(n)$*, with*

$$\widetilde{\mathcal{P}}(n) \equiv \left\{ p_n \in (0,\infty) : \max_{p \in (0,\widetilde{p}(p_n,n,0))} G(p,p_n,n) \leq 1 \right\}, \tag{6}$$

with $G(p,p_n,n) \equiv \frac{pd(p,p_n,n,0)}{C(d(p,p_n,n,0))}$ *and* $\widetilde{p}(p_n,n,0)$ *as defined in assumption (A2).*

Observe that the elasticity with respect to p of the revenue-cost ratio G is

$$\epsilon_p G(p,p_n,n) = 1 + (1 - \epsilon_y C(d(p,p_n,n,0)))\, \epsilon_p(d(p,p_n,n,0)) \tag{7}$$

which, by assumptions (2) and (3), is positive for p close to zero and negative for p close to $\widetilde{p}(p_n,n,0)$, implying that $G(\cdot,p_n,n)$ has indeed an interior maximum. Thus, we can determine the limit price $\overline{p}(n)$ as the solution in p_n to equations:

$$pd(p,p_n,n,0) = C(d(p,p_n,n,0)) \tag{8}$$

$$-\epsilon_p(d(p,p_n,n,0)) = \frac{1}{1-\epsilon_y C(d(p,p_n,n,0))}, \tag{9}$$

namely the zero profit condition and the first order condition to maximization of $G(\cdot,p_n,n)$, respectively. Notice that, as $d(p,p_n,n,0)$ is non-increasing in n and increasing in p_n by assumption (A2), $\overline{p}(\cdot)$ is a non-decreasing function.

Given Definition 4, we can now reformulate the sustainability condition by reference to the limit price $\overline{p}(n)$.

Lemma 2 (Sustainability) *Under assumptions (A1) and (A2), the condition* $p_n \leq \overline{p}(n)$ *is necessary and sufficient for* $(p_n,n) \in \mathbb{R}_{++} \times \{1,...,N-1\}$ *to satisfy the sustainability condition.*

Proof. See Appendix. ■

We summarize in the following proposition the results stated in the two lemmata and in the discussion of the break-even price as a greatest lower bound to profitable prices.

Proposition 1 *Under assumptions (A1) and (A2), and the additional restriction on the demand to the industry that* $\epsilon_P D(P) \in [-1,0)$ *for any* P*, a symmetric profile characterized by* $(p_n^*,n) \in \mathbb{R}_{++} \times \{1,...,N-1\}$ *is a free entry equilibrium if and only if* p_n^* *is a critical price between the break-even price and the limit price:* $\underline{p}(n) \leq p_n^* \leq \overline{p}(n)$*.*

With elastic demand to the industry, profit non negativity may impose an upper bound, as already noted, on the price p_n. By (5), if the elasticity of demand to the industry is non-increasing, the set $\mathcal{P}(n)$ as defined in (4) is an interval $\left[\underline{p}(n), \underline{\underline{p}}(n)\right]$, and we obtain the following proposition.

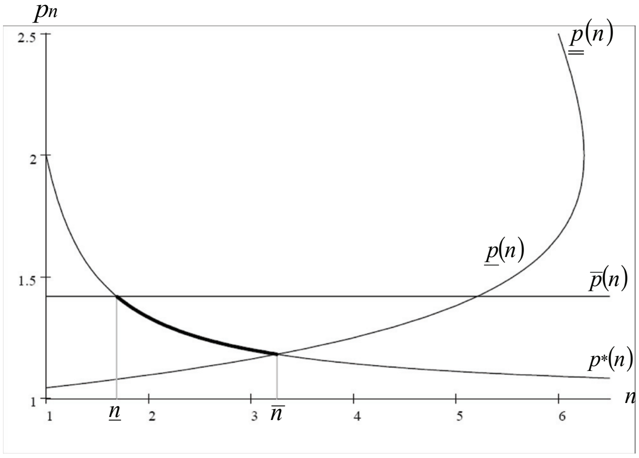

Figure 2: Critical, limit and break-even prices

Proposition 2 *Under assumptions (A1) and (A2), and the additional restriction on the demand to the industry that $\epsilon_P D(\cdot)$ be non-increasing, a symmetric profile characterized by $(p_n^*, n) \in \mathbb{R}_{++} \times \{1, ..., N-1\}$ is a free entry equilibrium if and only if p_n^* is a critical price such that $\underline{p}(n) \leq p_n^* \leq \min\left\{\underline{\underline{p}}(n), \overline{p}(n)\right\}$, with $\underline{\underline{p}}(n) \equiv \sup \mathcal{P}(n)$.*

By referring to these two propositions, we may define an admissible set of values of n that allow a pair (p_n^*, n) to characterize a free entry equilibrium. Recall that the function $\overline{p}(\cdot)$ is non-decreasing. So is the function $\underline{p}(\cdot)$, since g is non-increasing in n and increasing in p_n in a neighborhood of $\underline{p}(n)$. We thus obtain in Figure 2 a typical illustration of equilibrium conditions in the space (n, p_n).[8] We see that the condition on the critical price $p^*(n)$ (that it lie between the break-even price $\underline{p}(n)$ and the limit price $\overline{p}(n)$, as in the thick segment of the critical price curve), translates into a condition on the number n of active firms, which should belong to the interval $[\underline{n}, \overline{n}]$.

Multiplicity of free entry equilibria results from existence of more than one integer in this interval, as illustrated in Figure 2. It is worthwhile to emphasize that this source of equilibrium multiplicity differs from the one usually considered in the coordination failures literature and popularized by the seminal paper of Cooper and John (1988). In this literature, multiplicity of symmetric equilibria associated with the same n (thus resulting, in our framework, in a multi-valued function $p^*(\cdot)$) relies on strategic complementarity, which amounts

[8] The case represented in Figure 2 corresponds to Cournot competition, with a constant marginal cost (normalized to one) and a share of individual fixed cost in total expenditure (at unit price) equal to 0.04, when demand to the industry has constant elasticity -2 (so that Proposition 2 applies).

to require that the best response of p be an increasing function of p_n.[9] No such condition is necessary in our approach, because symmetry is now imposed only within each class of active and inactive firms. Hence, this approach allows an enlargement of the scope for coordination failures in macroeconomic models, while preserving the relevant symmetry leading to simplicity in aggregation.

3 Application to standard regimes in macroeconomic modelling

To illustrate the robustness of multiple free entry equilibria, we now apply the previous framework to the standard regimes of oligopolistic competition used in macroeconomic modelling. These are the Cournot homogeneous oligopoly first used by Hart (1982) and two versions of the differentiated oligopoly in prices: the one with the CES aggregator introduced by Dixit and Stiglitz (1977) and adopted by Blanchard and Kiyotaki (1987), and the spatial version of Salop (1979) adapted by Weitzman (1982). Before applying our framework to these regimes, it is however useful to introduce simplifying specifications of the cost and demand functions.

3.1 Common specifications and features

We shall use in the following the cost function $C(y) = c(\phi + y^\gamma)$ if $y > 0$, with $\phi \geq 0$, $0 < \gamma \leq 1$ and $\phi + 1 - \gamma > 0$, and such that $C(0) = 0$. We thus cover the two sources of decreasing average cost namely, if $\phi > 0$, the existence of a fixed (non sunk) cost and, if $\gamma < 1$, the existence of internal economies of scale accounting for decreasing marginal cost. This cost function can be derived from the technological constraint $y \leq (F(k,l) - \phi)^{1/\gamma}$, where F is a neoclassical production function, homogeneous of degree one. Of course, the two sources of internal increasing returns may appear either combined or alone, but they cannot be simultaneously excluded. Finally, notice that, as $\epsilon(C(y)/y) = -(\phi + (1-\gamma)y^\gamma)/(\phi + y^\gamma)$ assumption (A1) is clearly satisfied.

Since this greatly simplifies calculations, unit-elasticity of demand to the industry will also be assumed in the following: $D(P) = b/P$, with $b > 0$, which of course excludes equilibrium with a single active firm ($n \geq 2$). Under this assumption, the price p_n and the quantity y_n chosen by each one of the n active firms in a symmetric profile must verify

$$b = np_n y_n = nP\alpha(n)\, y_n, \tag{10}$$

where $\alpha(n)$ is the aggregating factor, used in the case where the industry produces a composite good (otherwise, $\alpha(n) \equiv 1$). The critical price p_n^* may be

[9] In fact, as shown in Cooper and John (1988), the necessary condition for existence of multiple *symmetric* equilibria is even stronger: the slope of the best response curve must take values larger than one (not only positive) to allow existence of more than one intersection with the principal diagonal of the (p_n, p) space.

expressed as the product of marginal cost $c\gamma y_n^{\gamma-1}$ (with $y_n = b/np_n^*$) and the markup factor $1/\left(1+1/\epsilon_p d\left(p_n^*, p_n^*, n, 1\right)\right)$:

$$p_n^* = c\gamma \left(\frac{b}{np_n^*}\right)^{\gamma-1} \frac{\epsilon_p d\left(p_n^*, p_n^*, n, 1\right)}{1+\epsilon_p d\left(p_n^*, p_n^*, n, 1\right)}. \tag{11}$$

Under conditions that will be satisfied in each one of the regimes that we propose to examine and on which the markup factor depends, this equation implicitly defines the equilibrium price as a function of the number of active firms: $p_n^* = p^*(n)$.

By Proposition 1, a strategy profile represented by the pair (p_n, n) is a free entry equilibrium if and only if p_n is the critical price $p^*(n)$ and if it lies between the break-even price $\underline{p}(n)$ and the limit price $\overline{p}(n)$. One major advantage of assuming unit-elasticity of demand is that the break-even price is then *regime-independent*, given by

$$\underline{p}(n) = \frac{b}{n^{1-1/\gamma}\left(b/c - n\phi\right)^{1/\gamma}}, \tag{12}$$

while the limit-price will have to be determined in each one of the regimes we are going to consider. The condition $\underline{p}(n) \le p^*(n) \le \overline{p}(n)$ will then be used to compute the admissible (non-degenerate) interval $[\underline{n}, \overline{n}]$ specific to each regime.

3.2 Quantity competition in the homogeneous oligopoly: the Cournot model

As already shown in subsection 2.2, the residual demand for a Cournotian firm can be expressed in a symmetric configuration as

$$d(p, p_n, n, \delta) = b\left(\frac{1}{p} - \frac{1-\delta/n}{p_n}\right), \tag{13}$$

where p is the market price aimed at by the firm and the price $p_n = b/ny_n$ *represents* the strategy y_n expected from each one of its $n-\delta$ active competitors (with $\delta = 1$ if the firm is itself active in the reference situation, $\delta = 0$ otherwise). The first and second partial elasticities of $d(\cdot, p_n, n, \delta)$, for p in the interval $(0, p_n/(1-\delta/n))$ in which individual demand is positive and finite, are

$$\begin{aligned} \epsilon_p d(p, p_n, n, \delta) &= -\frac{1}{1-(1-\delta/n)(p/p_n)} < 0 \text{ and} & (14) \\ \epsilon_{pp}^2 d(p, p_n, n, \delta) &= \frac{(1-\delta/n)(p/p_n)}{1-(1-\delta/n)(p/p_n)} > 0, & (15) \end{aligned}$$

satisfying assumption (A2). It is easy to check that all remaining conditions of this assumption are also verified.

As $\epsilon_p d(p_n, p_n, n, 1) = -n$, we obtain from equation (11) the following expression for the critical price:

$$p^*(n) = b\left(c/b\right)^{1/\gamma}\left(\gamma n^{1-\gamma}\frac{n}{n-1}\right)^{1/\gamma} \equiv b\left(c/b\right)^{1/\gamma}\left(\Psi(n)\right)^{1/\gamma}, \tag{16}$$

where $n/(n-1) \equiv \mu(n)$ is the markup factor on marginal cost. Profit non-negativity ($p^*(n) \geq \underline{p}(n)$) requires, by (12),

$$(c/b)\,\phi\gamma n^2 + (1-\gamma)\,n - 1 \leq 0,$$

imposing the following upper bound on the number of active firms:

$$\overline{n} = \begin{cases} \sqrt{\left(\frac{1-\gamma}{2\gamma}\frac{b}{c\phi}\right)^2 + \frac{1}{\gamma}\frac{b}{c\phi}} - \frac{1-\gamma}{2\gamma}\frac{b}{c\phi} & \text{if} \quad \phi > 0 \\ 1/(1-\gamma) & \text{if} \quad \phi = 0 \end{cases}. \tag{17}$$

Notice that $\overline{n} \leq 1/(1-\gamma)$, so that $\epsilon_n \Psi(n) = 1-\gamma-1/(n-1) \leq -(1-\gamma)^2/\gamma$ for $n \leq \overline{n}$, implying that Ψ is decreasing, a property that will be used in the next section.

Next, by using equations (8) and (9), and as shown in the Appendix, we can determine the limit price $\overline{p}(n)$:

$$\overline{p}(n) = b\,(c/b)^{1/\gamma}\left(\frac{1}{\gamma(1-\phi\overline{n}c/b)} - 1\right)\frac{1}{(1-\gamma+\phi(\gamma\overline{n}-1)\,c/b)^{1/\gamma}}. \tag{18}$$

By (16), the inequality $p^*(n) \leq \overline{p}(n)$ expressing the sustainability condition, imposes the following lower bound on the number of active firms:

$$\underline{n} = \Psi^{-1}\left(\left(\frac{1}{\gamma(1-\phi\overline{n}c/b)} - 1\right)^{\gamma}\frac{1}{1-\gamma+\phi(\gamma\overline{n}-1)\,c/b}\right). \tag{19}$$

From the double inequality $\underline{n} \leq n \leq \overline{n}$, and by making the necessary computations for the two benchmark cases of zero fixed cost with decreasing marginal cost ($\phi = 0$, with $\gamma < 1$) and of constant marginal cost ($\gamma = 1$, with $\phi > 0$), we can thus claim:

Claim 1 *In the symmetric homogeneous oligopoly under quantity competition, with demand b/P ($b > 0$) and cost cy^γ ($c > 0$ and $1/2 \leq \gamma < 1$), there exists a free entry equilibrium with a number n of active firms, for any n in the interval $[\underline{n}, \overline{n}]$, with $\overline{n} = 1/(1-\gamma) \geq 2$ and $\underline{n} = \Psi^{-1}\left(1/\gamma^\gamma(1-\gamma)^{1-\gamma}\right) < \overline{n}$, where $\Psi(n) \equiv \gamma n^{1-\gamma}\mu(n)$, $\mu(n) \equiv n/(n-1)$.*

Claim 2 *In the symmetric homogeneous oligopoly under quantity competition, with demand b/P ($b > 0$) and cost $c(\phi + y)$ ($c > 0$ and $0 < \phi \leq b/4c$), there exists a free entry equilibrium with a number n of active firms, for any n in the interval $[\underline{n}, \overline{n}]$, with $\overline{n} = \sqrt{b/c\phi} \geq 2$ and $\underline{n} = \overline{n}/(2-1/\overline{n}) \in (\overline{n}/2, 2\overline{n}/3]$.*

Notice that indeterminacy can be directly established in the case of constant marginal cost (Claim 2). Indeed, the admissible interval $[\underline{n}, \overline{n}]$ contains more than one integer for $\overline{n} \geq 3$, that is, for a *small enough* degree of economies of scale, as determined by the share $c\phi/b$ of individual fixed cost in aggregate expenditure. However, the impact of an increment in n on the equilibrium price, through the markup factor, is larger when n is small, which results from a high degree of economies of scale.

3.3 Price competition in a differentiated oligopoly: the Dixit-Stiglitz model

Now take a composite good with quantity index $Y = \left(\sum_{j=1}^{N} y_j^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}$, the aggregator for constant elasticity of substitution $\sigma \in (1,\infty)$. In a symmetric configuration (y_n, n) with n active firms, we have $Y = \alpha(n)\, n y_n$ with $\alpha(n) = n^{1/(\sigma-1)}$, the *aggregating factor* expressing preference for variety or economies of scope, according to the specific use of the composite good, in consumption or in production. The composite good is priced $P(p, p_n, n, \delta) = \left(p^{1-\sigma} + (n-\delta)\, p_n^{1-\sigma}\right)^{1/(1-\sigma)}$, so that the price index P is manipulable by the firm setting p, given the price p_n set by each one of the n active firms and according to its status, whether active ($\delta = 1$) or inactive ($\delta = 0$). In the monopolistic competition regime applying to the Chamberlinian "large group", P is taken as given by all firms, because each one of them produces at a scale that is insignificant with respect to market size. By contrast, we assume a "small group", where each firm takes into account the non-negligible impact of its price decision on the industry price index P. The regime we are assuming is thus one of *oligopolistic* price competition with product differentiation (see d'Aspremont, Dos Santos Ferreira and Gérard-Varet, 1996).

The demand for each product is

$$d(p, p_n, n, \delta) = \frac{b p^{-\sigma}}{p^{1-\sigma} + (n-\delta)\, p_n^{1-\sigma}}, \tag{20}$$

with first and second partial elasticities with respect to p:

$$\epsilon_p d(p, p_n, n, \delta) = -\frac{p^{1-\sigma} + \sigma(n-\delta)\, p_n^{1-\sigma}}{p^{1-\sigma} + (n-\delta)\, p_n^{1-\sigma}} < 0 \text{ and} \tag{21}$$

$$\epsilon_{pp}^2 d(p, p_n, n, \delta) = \frac{(\sigma-1)^2 (n-\delta)\, p^{1-\sigma} p_n^{1-\sigma}}{\left(p^{1-\sigma} + \sigma(n-\delta)\, p_n^{1-\sigma}\right)\left(p^{1-\sigma} + (n-\delta)\, p_n^{1-\sigma}\right)} > 0 \tag{22}$$

These properties of demand elasticity satisfy assumption (A2), the other conditions of which are all verified, as it can be readily checked.

As $\epsilon_p d(p_n, p_n, n, 1) = -(1/n + (1-1/n)\,\sigma)$, we obtain from equation (11) the following expression for the critical price:

$$p^*(n) = b\,(c/b)^{1/\gamma}\left(\gamma n^{1-\gamma}\left(1 + \frac{1}{(1-1/n)(\sigma-1)}\right)\right)^{1/\gamma}, \tag{23}$$

where $1 + 1/(1-1/n)(\sigma-1) \equiv \mu(n)$ is the markup factor on marginal cost. By (12) and (23), profit non-negativity ($p^*(n) \geq \underline{p}(n)$) is equivalent to:

$$(c/b)\,\phi\gamma\sigma n^2 + ((1-\gamma)(\sigma-1) - \gamma(1 + (c/b)\,\phi(\sigma-1)))\, n - (\sigma-1)(1-\gamma) \leq 0, \tag{24}$$

imposing an upper bound $\overline{n}$ on the number of active firms, corresponding to the unique positive solution of (24), taken as an equality. As earlier, we give the expressions for $\overline{n}$ in the two benchmark cases of constant marginal cost

$$\overline{n} = 1 + \frac{1}{\sigma}\left(\frac{b}{c\phi} - 1\right), \text{ with } \sigma \leq \frac{b}{c\phi} - 1, \text{ if } \gamma = 1, \tag{25}$$

and zero fixed cost

$$\overline{n} = \frac{\sigma - 1}{\sigma - 1/(1-\gamma)}, \text{ with } \frac{1}{1-\gamma} < \sigma \leq \frac{1+\gamma}{1-\gamma}, \text{ if } \phi = 0. \tag{26}$$

Notice that the upper bound on σ ensures in each case that $\overline{n} \geq 2$.

By using equations (8) and (9), we also obtain, as shown in the Appendix, the following expression for the limit price $\overline{p}(n)$ in the two benchmark cases ($\gamma = 1$ and $\phi = 0$):

$$\overline{p}(n) = \frac{b/\overline{n}}{(b/c\overline{n} - \phi)^{1/\gamma}}\left(\frac{n}{\overline{n} - 1}\right)^{1/(\sigma-1)}. \tag{27}$$

Hence, by (23) and recalling that $\alpha(n) = n^{1/(\sigma-1)}$, the condition $p^*(n) \leq \overline{p}(n)$ for sustainability takes the form:

$$\Psi(n) \equiv \gamma n^{1-\gamma}\mu(n)\,\alpha(n)^{-\gamma} \leq \frac{1/\overline{n}^{\gamma}}{1/\overline{n} - c\phi/b}\left(\frac{1}{\overline{n}-1}\right)^{\gamma/(\sigma-1)}. \tag{28}$$

The function Ψ is decreasing for $n \leq \overline{n}$ in the two benchmark cases, because the two negative effects of a change in n, through the aggregating factor α and through the markup factor μ, together dominate the positive effect through the marginal cost:

$$\begin{aligned} \epsilon_n \Psi(n) &= 1 - \gamma - \frac{1}{1/n + (1 - 1/n)\sigma}\frac{1}{n-1} - \frac{\gamma}{\sigma - 1} < 0 \text{ if } \gamma = 1 \\ &\leq -\frac{(\sigma(1-\gamma) - 1)^2}{(\sigma - 1)\gamma} < 0 \text{ if } n \leq \frac{\sigma - 1}{\sigma - 1/(1-\gamma)} \;(= \overline{n} \text{ if } \phi = 0) \end{aligned} \tag{29}$$

Hence, the sustainability condition can be reformulated as follows in the two benchmark cases:

$$n \geq \Psi^{-1}\left(\frac{1/\overline{n}^{\gamma}}{1/\overline{n} - c\phi/b}\left(\frac{1}{\overline{n}-1}\right)^{\gamma/(\sigma-1)}\right) \equiv \underline{n}. \tag{30}$$

From (28) and (25)-(26), it is easy to check that $\underline{n} < \overline{n}$, so that the admissible interval $[\underline{n}, \overline{n}]$ is non-degenerate. However, it is interesting to observe that, in the case of a constant marginal cost ($\gamma = 1$), there is at most one free entry equilibrium (the one selected by the zero profit condition). Indeed, observe that $n = \overline{n} - 1$ violates condition (28), implying $\overline{n} - 1 < \underline{n}$, or $\overline{n} - \underline{n} < 1$. Hence, indeterminacy is excluded in this case, but existence itself is not ensured under free entry.[10] Still, indeterminacy appears easily for $\gamma < 1$.

To summarize,

[10] Take for instance $\gamma = b = c = 1$, $\phi = .05$ and $\sigma = 5$. The admissible interval $[\underline{n}, \overline{n}] = [4.005, 4.8]$ does not contain any integer. Profitability requires the number of active firms to be no larger than 4. With $n = 4$, the critical price is 1.3333, larger than the break-even price 1.25, but also larger than the limit price 1.3328, so that sustainability is not ensured.

Claim 3 *Consider a symmetric oligopoly with differentiated goods, linked by the constant elasticity of substitution* σ*, aggregated into a composite good whose demand is* b/P*, and produced according to the cost function* cy^{γ}*. Then, under the parameter restrictions* $b > 0$*,* $c > 0$ *and* $1 < 1/(1-\gamma) < \sigma \leq (1+\gamma)/(1-\gamma)$*, there exists a free entry equilibrium with a number* n *of active firms, for any* n *in the interval* $[\underline{n}, \overline{n}]$*, the limits of which are* $\overline{n} = (\sigma - 1)/(\sigma - 1/(1-\gamma))$ *and* $\underline{n} = \Psi^{-1}\left(\overline{n}^{1-\gamma}(\overline{n}-1)^{-\gamma/(\sigma-1)}\right)$*, with* $\Psi(n) \equiv \gamma n^{1-\gamma}\mu(n)\alpha(n)^{-\gamma}$*,* $\mu(n) = 1 + n/(n-1)(\sigma-1)$ *and* $\alpha(n) = n^{1/(\sigma-1)}$*. The equilibrium price of the composite good is* $P = b^{1-1/\gamma}c^{1/\gamma}(\Psi(n))^{1/\gamma}$*, decreasing in* n*.*

Claim 4 *Consider a symmetric oligopoly with differentiated goods, linked by the constant elasticity of substitution* σ*, aggregated into a composite good whose demand is* b/P*, and produced according to the cost function* $c(\phi + y)$*. Then, under the parameter restrictions* $b > 0$*,* $c > 0$*,* $\phi > 0$ *and* $1 < \sigma \leq b/c\phi - 1$*, there exists at most one free entry equilibrium with a number* $\lfloor \overline{n} \rfloor = \lfloor 1 + (b/c\phi - 1)/\sigma \rfloor$ *of active firms (if* $\lfloor \overline{n} \rfloor \geq \underline{n} \equiv \Psi^{-1}\left((\overline{n}-1)^{-1/(\sigma-1)}/(1-\overline{n}c\phi/b)\right)$*, with* $\Psi(n) \equiv \mu(n)/\alpha(n)$*,* μ *and* α *as defined in Claim 3).*

3.4 Price competition in a differentiated oligopoly: the Salop spatial model

In the industrial organization literature, spatial competition is a popular alternative to non-address models relying on CES or quadratic consumers' utility functions. Although much less frequent in macroeconomic modelling, it has for instance been used by Weitzman (1982), who introduced a macroeconomic version of Salop's (1979) model of the circular city, and by Pagano (1990). The space of characteristics of the industry good is represented by a circle with perimeter equal to 1, on which consumers' locations are uniformly distributed with unit density. A consumer devoting a positive budget b to the purchase of that good and located at point x between two firms j and $j+1$, which are themselves located at a_j and a_{j+1} respectively, will buy from firm j if $p_j + \tau(x - a_j) < p_{j+1} + \tau(a_{j+1} - x)$, where p_j and p_{j+1} are the prices set by the two firms and τ is the (subjective) transportation rate in money equivalent units. The marginal consumer who is indifferent between the two suppliers is the one located at point $x_{(j,j+1)} = (a_j + a_{j+1})/2 + (p_{j+1} - p_j)/2\tau$, so that the market area of firm j is

$$x_{(j,j+1)} - x_{(j-1,j)} = \frac{a_{j+1} - a_{j-1}}{2} + \frac{(p_{j-1} + p_{j+1})/2 - p_j}{\tau}, \tag{31}$$

which is independent upon its location a_j. However, although indifferent about its precise location within its market area, firm j is assumed to set its price p_j on the basis of its conjectures not only of the prices p_{j-1} and p_{j+1} but also of the *locations* a_{j-1} and a_{j+1} chosen by its neighbors. This implies in particular that, when inactive at the strategy profile taken as reference, a deviating firm *does not* conjecture that the locations of its competitors are going to be benevolently accommodated in response to its decision to deviate into activity. As a consequence, any firm, through its pricing decision, is able to manipulate its market area within the segment separating its two neighbors, but the length of this segment is $2/n$ if the firm is active and $1/n$ if it is inactive (assuming that locations are symmetric with respect to the n active firms).

On the basis of symmetry with respect to both locations and prices, we obtain the following expression for demand to the representative firm (with $\delta = 1$ if it is active, $\delta = 0$ otherwise):

$$d(p, p_n, n, \delta) = (b/\tau p)\left((1+\delta)\tau/2n + p_n - p\right), \text{ with } p \in (0, (1+\delta)\tau/2n + p_n]. \tag{32}$$

The first and second partial elasticities of $d(\cdot, p_n, n, \delta)$ with respect to p are:

$$\epsilon_p d(p, p_n, n, \delta) = -\frac{(1+\delta)\tau/2n + p_n}{(1+\delta)\tau/2n + p_n - p} < 0 \text{ and} \tag{33}$$

$$\epsilon_{pp}^2 d(p, p_n, n, \delta) = \frac{p}{(1+\delta)\tau/2n + p_n - p} > 0. \tag{34}$$

All the conditions of assumption (A2) are again satisfied.

For simplicity, we shall only consider in this subsection the case of constant marginal cost ($\gamma = 1$). Since $\epsilon_p d(p_n, p_n, n, 1) = -(1 + np_n/\tau)$, we obtain from equation (11) the following expression for the critical price:

$$p^*(n) = c\left(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{\tau/c}{n}}\right) \equiv c\Psi(n), \tag{35}$$

where Ψ, now identical to the markup factor μ, is a decreasing function. Profit non-negativity ($p^*(n) \geq \underline{p}(n)$) imposes, by (12) and this equation, an upper bound on the number n of active firms:

$$n \leq \frac{\tau/c}{\sqrt{\tau/c}\sqrt{c\phi/b}\left(1 + \sqrt{\tau/c}\sqrt{c\phi/b}\right)} \equiv \frac{\tau/c}{\beta(1+\beta)} \equiv \overline{n}, \tag{36}$$

which is increasing in $\tau/c \in (0, \infty)$ (the ratio of the transportation rate to the unit production cost, representing the degree of product differentiation and determining the markup factor) and decreasing in $c\phi/b \in (0, 1/2)$ (the share of individual fixed cost in aggregate expenditure). It is easy to check that the parameter β is equal to the rate of markup on marginal cost $\mu(\overline{n}) - 1$ at the break-even price, and represents the corresponding degree of economies of scale.[11]

Finally, equations (8) and (9) lead, as shown in the Appendix, to the limit price

$$\overline{p}(n) = c\left((1+\beta)^2 - \frac{\tau/c}{2n}\right), \tag{37}$$

[11] The degree of economies of scale is given by the inverse of the elasticity of the cost function minus one: $(\phi + y)/y - 1 = \phi/y$, that is, the ratio of fixed cost to variable cost. At the break-even price, $\beta = \phi/y_{\overline{n}} = \mu(\overline{n}) - 1$.

and, after a straightforward computation, to the lower bound imposed by sustainability ($p^*(n) \leq \overline{p}(n)$):

$$n \geq \frac{\tau/c}{\left(\sqrt{2(1+\beta)^2+1/4}-1\right)^2-1/4} \equiv \underline{n}. \tag{38}$$

The admissible interval $[\underline{n}, \overline{n}]$ is again non-degenerate, but more can easily be told about its amplitude. We see that $\overline{n} = (\tau/c)\overline{\mathcal{N}}(\beta)$ and $\underline{n} = (\tau/c)\underline{\mathcal{N}}(\beta)$, where these two functions of β are given by (36) and (38), respectively. They both increase with β from zero to infinity, and their ratio has the limit values:

$$\lim_{\beta\to 0}\frac{\overline{\mathcal{N}}(\beta)}{\underline{\mathcal{N}}(\beta)} = \lim_{\beta\to 0}\frac{\overline{\mathcal{N}}'(\beta)}{\underline{\mathcal{N}}'(\beta)} = \lim_{\beta\to 0}\frac{4(1+\beta)\left(\sqrt{2(1+\beta)^2+1/4}-1\right)}{(1+2\beta)\sqrt{2(1+\beta)^2+1/4}} = \frac{4}{3} \tag{39}$$

(using L'Hospital's rule), and

$$\lim_{\beta\to\infty}\frac{\overline{\mathcal{N}}(\beta)}{\underline{\mathcal{N}}(\beta)} = \lim_{\beta\to\infty}\frac{\left(\sqrt{2(1+\beta)^2+1/4}-1\right)^2-1/4}{\beta(1+\beta)} = 2. \tag{40}$$

To summarize,

Claim 5 *In a symmetric oligopoly with spatially differentiated goods, located on a circle where the transportation rate is τ, the production cost is $c(\phi+y)$ and the aggregate demand is b/P, there exists, under the parameter restrictions $\tau > 0$, $b > 0$, $c > 0$ and $0 < \phi < b/2c$, a free entry equilibrium with a number n of active firms, for any n in the interval $[\underline{n}, \overline{n}]$, such that $\overline{n} = (\tau/c)/\beta(1+\beta)$, with $\beta \equiv \sqrt{\tau/c}\sqrt{c\phi/b}$, and $\underline{n} = (\tau/c)/\left(\left(\sqrt{2(1+\beta)^2+1/4}-1\right)^2-1/4\right) \in (\overline{n}/2, 3\overline{n}/4)$.*

Thus, $\overline{n} \geq 4$ is a sufficient condition for the existence of more than one integer in the admissible interval $[\underline{n}, \overline{n}]$, and hence for indeterminacy. It results from a high enough degree of product differentiation (measured by τ/c), associated with sufficient market power, and/or from a low enough share $c\phi/b$ of individual fixed cost in aggregate expenditure. The potential amplitude of indeterminacy and of its effects through the markup factor on the equilibrium price increases with the degree β of economies of scale at the break-even price, an average of τ/c and $c\phi/b$.

3.5 Comparative numerical examples

In order to compare the amplitude of the effects of equilibrium indeterminacy on the market price in the two benchmark specifications of the cost function (zero fixed cost and constant marginal cost) and in the different regimes of competition examined above, we consider three different values of the degree of economies of scale

$$\beta \equiv \frac{1}{\epsilon_y C\left(y_{\overline{n}}\right)} - 1 = \frac{\phi}{\gamma y_{\overline{n}}^{\gamma}} + \frac{1}{\gamma} - 1, \tag{41}$$

namely $\beta = .25$, $.11$ and $.02$. The following table indicates, for the Cournot and Dixit-Stiglitz models with decreasing marginal cost and zero fixed cost, the corresponding extreme values of the (integer) number of active firms ($\lceil \underline{n} \rceil$ and $\lfloor \overline{n} \rfloor$), of the markup factor μ ($\underline{\mu} = \mu\left(\lfloor \overline{n} \rfloor\right)$ and $\overline{\mu} = \mu\left(\lceil \underline{n} \rceil\right)$) and of the function Ψ expressing the combined effects of the variations in the marginal cost and in the markup and aggregating factors ($\underline{\Psi} = \Psi\left(\lfloor \overline{n} \rfloor\right)$ and $\overline{\Psi} = \Psi\left(\lceil \underline{n} \rceil\right)$). In order to facilitate the comparison between the two competition regimes, we have chosen for the elasticity of substitution the value $\sigma = 2 + 1/\beta$ which leads to the same $\overline{n} = 1 + 1/\beta$ (and $\mu\left(\overline{n}\right) = 1 + \beta$).

			Cournot				Dixit-Stiglitz			
β	$\lfloor \overline{n} \rfloor$	$\underline{\mu}$	$\lceil \underline{n} \rceil$	$\overline{\mu}$	$\overline{\Psi}$	$\underline{\Psi}$	$\lceil \underline{n} \rceil$	$\overline{\mu}$	$\overline{\Psi}$	$\underline{\Psi}$
.25	5	1.25	3	1.5	1.49	1.38	3	1.3	1.09	1.07
.11	10	1.11	4	1.33	1.38	1.26	5	1.124	1.029	1.023
.02	50	1.02	17	1.06	1.10	1.08	17	1.021	1.0019	1.0015

Table 1: $\phi = 0$, $\gamma < 1$

We see that the amplitude of the potential variations of n is preserved when switching from Cournot oligopoly to price competition in a Dixit-Stiglitz setting, but that the variability of $\mu\left(n\right)$ and $\Psi\left(n\right)$ is significantly diminished.

We find in Table 2 the equivalent numerical comparisons for the Cournot and Salop models with constant marginal cost and positive fixed cost. We have chosen a value for the ratio of the transportation rate to the unit production cost (measuring the degree of product differentiation) $\tau/c = \left(1+\beta\right)^2$, with $\beta = \sqrt{\tau/c}\sqrt{c\phi/b}$, that leads to the same value of $\overline{n} = 1+1/\beta$ (and $\mu\left(\overline{n}\right) = 1+\beta$) in both regimes of competition.

			Cournot		Salop	
β	$\lfloor \overline{n} \rfloor$	$\underline{\mu} = \underline{\Psi}$	$\lceil \underline{n} \rceil$	$\overline{\mu} = \overline{\Psi}$	$\lceil \underline{n} \rceil$	$\overline{\mu} = \overline{\Psi}$
.25	5	1.25	3	1.5	4	1.300
.11	10	1.11	6	1.2	8	1.136
.02	50	1.02	26	1.04	38	1.027

Table 2: $\phi > 0$, $\gamma = 1$

Inspection of this table shows that switching from Cournotian competition to price competition in a spatial setting tends again to decrease the variability of $\mu\left(n\right)$ (or $\Psi\left(n\right)$), although in a smaller extent than in the former case.

4 Dynamic aggregate implications

Equilibrium indeterminacy calls for some selection procedure, allowing coordination of firms conjectures and resulting decisions. In this perspective, the usual zero profit condition appears as a particular selection rule, picking up the equilibrium associated with the greatest integer in the admissible interval $[\underline{n}, \overline{n}]$. But, as long as other profitable strategy profiles are sustainable, there is no reason to suppose that firms always coordinate on the least profitable of them. This section explores the potential aggregate implications of this coordination problem, using a very simple macroeconomic model with overlapping generations of identical consumers and a large number of differentiated industries.

4.1 Consumers

The economy is composed by overlapping generations of "young" and "old" consumers. A generation of identical consumers of unit mass is born at each date t and lives for two periods. Consumers work only when young, receiving in this period wage earnings and dividends from firms (which are equally held by young consumers), and can only save in the form of money, which brings no interest. They consume only when old, using past money savings. In addition, we assume that the preferences of old consumers are defined over varieties $i = 1, ..., m$ of goods produced by the m industries, with a constant elasticity of substitution that we take equal to unity. This implies that consumption may be represented by the aggregate $\overline{Y} = m\left(\prod_{i=1}^{m} Y_i^{1/m}\right)$, which can be bought at the corresponding price index $\overline{P} = \prod_{i=1}^{m} P_i^{1/m}$.

Assuming identical homothetic preferences, we can simply refer to the choices of an aggregate representative young consumer, born at t and wishing to maximize $E_t U(\overline{Y}_{t+1}) - V(L_t)$ subject to the constraints $\overline{P}_{t+1}\overline{Y}_{t+1} \leq M_t$ and $M_t \leq w_t L_t + \mathcal{D}_t$, where M_t is money demand, L_t is labor, supplied at the nominal wage w_t, and $\mathcal{D}_t$ is the total amount of dividends received from firms. The first-order conditions for this program may be written as

$$E_t\left(U'(\overline{Y}_{t+1})\frac{w_t}{\overline{P}_{t+1}}\right) = V'\left(L_t\right), \tag{42}$$

with the two former constraints binding at the optimum. For simplicity, we shall from now on restrict our attention to the case of a utility function which is isoelastic in consumption and linear in labor, i.e. $U\left(Y\right) = Y^{\rho}/\rho$ and $V(L) = vL$, with $\rho \in (0, 1)$ and $v \in (0, \infty)$. With these assumptions, it is easy to see from the optimality conditions that the young consumer will save in the form of money all its income available at t, supplying any positive amount of labor provided that the nominal wage is at least equal to the nominal reservation wage $\underline{w}_t = v/E_t\left(\overline{Y}_{t+1}^{\rho-1}\overline{P}_{t+1}^{-1}\right)$.

4.2 The production sector

We adopt the usual macroeconomic modelling of monopolistic competition (Blanchard and Kiyotaki, 1987) by assuming that the economy is composed of a large number m of industries producing differentiated goods. However, we depart from this modelling by relaxing the assumption that all sectors are represented by a single firm acting in a *monopolistic* position. Instead, as seen in the preceding section, we consider that in each industry a number N of potential producers compete in an *oligopolistic* setting, deciding in particular to be active or not according to their (correct) conjectures of other producers' behavior. Each firm wishes to maximize profits $py - wl$ using the same production function[12] $y = l^{1/\gamma}$, with $0 < \gamma < 1$, and decides to set its price and to choose its activity level according to these conjectures and to the sectoral demand $D(P_i) = \left(\overline{PY}/m\right)/P_i$ (see program (CP) above). As shown in section 3, at a symmetric free-entry equilibrium with n_i active firms in industry i, each one of these firms chooses the same critical price, equal (or proportional, in the case of an industry producing a composite good) to $P_i = \left(\overline{PY}/m\right)^{1-1/\gamma} w^{1/\gamma} \left[\Psi_i(n_i)\right]^{1/\gamma}$, where Ψ_i is a decreasing function with a specification depending on the regime of competition prevailing in the industry. For simplicity, we assume symmetry across sectors as concerns the competition regime, so that $\Psi_i = \Psi$ for all i. Also, under appropriate conditions on γ, stated for instance in Claims 1 and 3, there will typically exist a non-degenerate interval $[\underline{n}, \overline{n}]$ of admissible numbers of active firms at equilibrium, producing a total sectoral amount of good $D(P_i) = \left(\overline{PY}/m\right)^{1/\gamma} w^{-1/\gamma} \left[\Psi(n_i)\right]^{-1/\gamma}$.

4.3 Equilibrium

It is easy to derive from the previous subsections the general equilibrium of this economy. Assuming a constant stock of money, $M_t = M$ for $t = 0, ..., \infty$, the total aggregate consumption demand is equal to the real purchasing power of money holdings of old consumers, $M/\overline{P}_t$. Total production, as represented by the index $\overline{Y}_t$, satisfies:

$$\overline{Y}_t = m \left(\frac{\overline{P}_t \overline{Y}_t}{m w_t} \right)^{1/\gamma} \left(\prod_{i=1}^{m} \Psi(n_{i,t})^{1/m} \right)^{-1/\gamma}. \tag{43}$$

Equilibrium in the output market then requires $\overline{Y}_t = M/\overline{P}_t$. Labor market equilibrium implies that the nominal wage equalizes the nominal reservation wage:

$$w_t = v/E_t \left(\overline{Y}_{t+1}^{\rho-1} \overline{P}_{t+1}^{-1} \right). \tag{44}$$

Finally, the money market clears by Walras law. Combining these two equations and using $\overline{Y_t} = M/\overline{P}_t$ (and $\overline{Y}_{t+1} = M/\overline{P}_{t+1}$), it is straightforward to show that

[12] We might have chosen the specification $y = l - \phi$ instead, by assuming either Cournot or spatial competition (the CES product differentiation case being excluded by claim 4).

the general equilibrium of this economy may be represented by the following (non-autonomous) one-dimensional dynamic system:

$$\overline{Y}_t = m^{1-1/\gamma} v^{-1/\gamma} \left(\prod_{i=1}^{m} \Psi(n_{i,t})^{1/m} \right)^{-1/\gamma} \left(E_t \left(\overline{Y}_{t+1}^{\rho} \right) \right)^{1/\gamma}. \qquad \text{(DS)}$$

Clearly, if the coordination process selects a time-invariant number of active firms n_i in each sector, the system (DS) has a deterministic stationary equilibrium $\overline{Y}^* = \left(m^{1-\gamma} v \overline{\Psi} \right)^{1/(\rho-\gamma)}$, with $\overline{\Psi} = \left(\prod_{i=1}^{m} \Psi(n_i)^{1/m} \right)$, for each admissible m-uple (n_i). Obviously, any such equilibrium is indeterminate in the dynamic sense as long as $\rho \geq \gamma$. This indeterminacy condition, imposing a degree of increasing returns to scale high enough compared to the degree of concavity of the utility function, is typical in this kind of model. Although in this very simple setup without capital accumulation and with labor as the unique input in production, indeterminacy may well occur for empirically reasonable degrees of increasing returns and risk aversion, richer dynamic models in line with the Real Business Cycle literature are often found to be indeterminate only for relatively high increasing returns, an assumption which is widely discussed and criticized in the empirical literature (see the survey by Benhabib and Farmer, 1999, for further discussion on this issue).

Since we want to focus on an alternative source of indeterminacy, we will assume in the following that the non-trivial steady state $\overline{Y}^*$ is determinate in the dynamic sense, that is, that $\rho < \gamma$. Given this condition, it is clear that, if the coordination problem within each industry were implicitly solved by referring to the zero profit condition (*i.e.*, by making n_i equal to the greatest integer value $\lfloor \overline{n} \rfloor$ in the interval $[\underline{n}, \overline{n}]$, for each $i = 1, ..., m$), the unique non-explosive trajectory would require that output jump instantaneously and permanently to its long-run stationary value $\overline{Y}^* = \left(m^{1-\gamma} v \Psi \left(\lfloor \overline{n} \rfloor \right) \right)^{1/(\rho-\gamma)}$. No deterministic or stochastic fluctuations remaining in a compact neighborhood of the steady-state would then be possible. However, as long as the coordination problem is not given such a simple solution (implying that incumbents, despite their interests, are always ready to accommodate entry), neither endogenous fluctuations nor multiplicity of Pareto-ranked deterministic steady states can be excluded.

4.4 Coordination failures

Multiplicity of sectoral free entry equilibria naturally translates into multiplicity of steady states, which may be Pareto-ranked. For instance, if we assume complete symmetry across sectors ($n_i = n \in [\underline{n}, \overline{n}]$ for any industry i, so that $\overline{\Psi} = \Psi(n)$), the representative consumer's utility at a steady state with a number n of active firms per sector is

$$\begin{aligned}\frac{\overline{Y}^{*\rho}}{\rho}-vL^{*} &= \frac{\overline{Y}^{*\rho}}{\rho}-vmn\left(\frac{\overline{Y}^{*}}{mn\alpha\left(n\right)}\right)^{\gamma} \\ &= \frac{\left(m^{1-\gamma}v\Psi\left(n\right)\right)^{\rho/(\rho-\gamma)}}{\rho}\left(1-\frac{\rho}{\gamma\mu\left(n\right)}\right)\equiv\mathcal{U}\left(n\right), \qquad (45)\end{aligned}$$

using $\overline{Y}^{*}=\left(m^{1-\gamma}v\Psi\left(n\right)\right)^{1/(\rho-\gamma)}$ and $\Psi(n)=\gamma n^{1-\gamma}\mu\left(n\right)\alpha\left(n\right)^{-\gamma}$ (by (16) and (28)).

The elasticity of $\mathcal{U}$ is

$$\epsilon_n\mathcal{U}\left(n\right)=\frac{1}{\gamma/\rho-1}\left(-\frac{\mu\left(n\right)-1}{\mu\left(n\right)-\rho/\gamma}\epsilon_n\mu\left(n\right)-\left(1-\gamma\left(1+\epsilon_n\alpha\left(n\right)\right)\right)\right), \qquad (46)$$

giving, in Cournot competition, by Claim 1 and for $n\leq\overline{n}$:

$$\begin{aligned}\epsilon_n\mathcal{U}\left(n\right) &= \frac{1}{\gamma/\rho-1}\left(\frac{1}{\rho/\gamma+\left(1-\rho/\gamma\right)n}\frac{1}{n-1}-\left(1-\gamma\right)\right) \\ &\geq \frac{1}{\gamma/\rho-1}\frac{1-\gamma}{1-\rho}\left(1/\gamma-1-\left(1-\rho\right)\right)=\epsilon_n\mathcal{U}\left(\overline{n}\right), \qquad (47)\end{aligned}$$

an expression which is non-negative if $1/\gamma-1\geq1-\rho$. Hence, all multiple steady states associated with the different integers in the admissible interval $[\underline{n},\overline{n}]$ are in this case Pareto-ranked: the utility of the representative consumer in any generation born at $t=0,...,\infty$ is increasing in n and so is, at $t=0$, the utility of the old consumer of the generation born at $t=-1$, who spends M for consumption at a price $\overline{P}^{*}$ that is decreasing in n (since Ψ is a decreasing function). In this case, the zero profit condition selects a *Pareto-efficient* steady state.

However, in the alternative configuration $1/\gamma-1<1-\rho$, consumer's utility is maximized at $\widehat{n}\in[\underline{n},\overline{n})$ (with n taken as continuous). Pareto-ranking of steady states (increasing with n) will then concern those, if multiple, that are associated with integers in the interval $[\underline{n},\widehat{n}]$. As for the steady states associated with integers in the interval $[\widehat{n},\overline{n}]$, all but the old consumers born at $t=-1$ will prefer those with a *higher degree of concentration*, entailing less consumption but more leisure. There is a trade-off between inefficiency due to market power (decreasing with n) and technological inefficiency (increasing with n), the consequences of the latter receiving now a higher weight in the representative consumer's preferences. Thus, the zero profit condition ceases to be a criterion of Pareto optimality in this case.

To conclude on this point, notice that we obtain in the price competition regime, by Claim 3 and for $n \leq \overline{n}$,

$$\begin{aligned}
&\epsilon_n \mathcal{U}(n) = \frac{1}{\gamma/\rho - 1} \times \\
&\left(\frac{1}{1 + (1 - 1/n)(\sigma - 1)(1 - \rho/\gamma)} \frac{1}{(n-1)\sigma + 1} \frac{n}{n-1} - \frac{\sigma(1-\gamma) - 1}{\sigma - 1} \right) \\
\geq \;& \frac{\sigma(1-\gamma) - 1}{(\gamma/\rho - 1)(1-\rho)(\sigma - 1)} \left((\sigma - 1)\gamma (1/\gamma - 1)^2 - (1-\rho) \right) = \epsilon_n \mathcal{U}(\overline{n}), \qquad (48)
\end{aligned}$$

leading essentially to the same result as before.

4.5 Fluctuations driven by sunspots

Let us now illustrate how the fundamental static indeterminacy inherent in free entry equilibria may be the source of important sunspot-driven fluctuations, even when the steady-state is determinate in the dynamic sense (when $\rho < \gamma$). Multiplicity of equilibria requires that firms within each industry find a coordination scheme allowing them to rationalize one equilibrium among the potential ones – a point that has been left implicit when discussing the possibility of coordination failures. We shall therefore consider that firms tackle this coordination problem by referring to some idiosyncratic signal indicating the situation that is about to be realized at some date in this particular sector.

To be explicit, assume that the admissible interval $[\underline{n}, \overline{n}]$ contains $K \in \{2, ..., N-2\}$ integers $n_k \equiv \lceil \underline{n} \rceil + k - 1$ (where $\lceil \underline{n} \rceil$ is the smallest integer larger than or equal to $\underline{n}$), with $k = 1, ..., K$. A *signal* for industry i at date t may then be seen as a realization $k_{i,t}$ of a random variable with arbitrary distribution over the discrete support $\{1, ..., K\}$, ensuring coordination within this sector on the number $n_{k_{i,t}}$ of active firms. Also, we can define a *state* of the economy as a whole at time t as a vector $F_t = (f_{t1}, ..., f_{tK})$ of proportions of industries that have received the signal k ($k = 1, ..., K$) at this period. Looking for stationary stochastic equilibria of our economy, we assume that there is an arbitrary number $R \in \mathbb{N} \setminus \{0, 1\}$ of such states, indexed by $r = 1, ..., R$ and such that with each r we associate the vector of proportions F_r. Besides, we assume that the transition between states across periods is described by a $(R \times R)$ row-stochastic transition matrix $\mathbf{T}$ with elements T_{ij} satisfying $T_{ij} \equiv \Pr(r' = j \mid r = i)$, where a prime stands for next period.

It is now straightforward to derive from equation (DS) the level of output $\overline{Y}_r$ associated with any state $r \in \{1, ..., R\}$ of the economy in a stationary stochastic equilibrium (with $\overline{Y}_1 \leq ... \leq \overline{Y}_R$, by convention):

$$\overline{Y}_r = m \left(\frac{1}{mv} \frac{\sum_{r'=1}^{R} T_{rr'} \overline{Y}_{r'}^{\rho}}{\prod_{k=1}^{K} \Psi(n_k)^{f_{rk}}} \right)^{1/\gamma} \equiv \eta_r \left(\overline{Y}_1, ..., \overline{Y}_R \right). \qquad \text{(SSE)}$$

Since $\overline{Y}_1^{\rho} \leq \sum_{r'=1}^{R} T_{rr'} \overline{Y}_{r'}^{\rho} \leq \overline{Y}_R^{\rho}$ and $\Psi(n_K) \leq \prod_{k=1}^{K} \Psi(n_k)^{f_{rk}} \leq \Psi(n_1)$, we have:

$$m\left(\frac{1}{mv}\frac{\overline{Y}_1^{\rho}}{\Psi(n_1)}\right)^{1/\gamma} \leq \overline{Y}_1 \leq \overline{Y}_R \leq m\left(\frac{1}{mv}\frac{\overline{Y}_R^{\rho}}{\Psi(n_K)}\right)^{1/\gamma}, \tag{49}$$

and hence, for $\rho < \gamma$,

$$\underline{Y} \equiv \left(m^{1-\gamma}v\Psi(n_1)\right)^{-1/(\gamma-\rho)} \leq \overline{Y}_1 \leq \overline{Y}_R \leq \left(m^{1-\gamma}v\Psi(n_K)\right)^{-1/(\gamma-\rho)} \equiv \overline{\overline{Y}}. \tag{50}$$

Thus, η as defined in (SSE) is a continuous mapping of the set $\left[\underline{Y},\overline{\overline{Y}}\right]^R$ into itself. By Brouwer's fixpoint theorem, there exists a solution $\left(\overline{Y}_1^*,...,\overline{Y}_R^*\right) = \eta\left(\overline{Y}_1^*,...,\overline{Y}_R^*\right)$ to system (SSE).

Obviously, for a non-degenerate transition matrix $\mathbf{T}$, aggregate real output $\overline{Y}$ will fluctuate stochastically among its R potential (generically different) values. It can also be checked that, with degenerate transition matrices (in particular, when all the rows of $\mathbf{T}$ belong to the canonical basis of $\mathbb{R}^R$), real output will necessarily tend to a deterministic cycle of order $q \leq R$ (possibly after a transition period of finite time). Without intrinsic uncertainty, in an economy where the zero profit condition would imply an instantaneous and permanent coordination on a single deterministic steady-state, we see that simple coordination procedures called for by the multiplicity of free entry equilibria may well be the source of periodic or aperiodic, deterministic or stochastic cycles (Dos Santos Ferreira and Dufourt, 2005, exploit this possibility within a fully specified and calibrated model of the business cycle).

5 Conclusion

We have argued in this paper that, in spite of an almost universal convention, zero profits should not be imposed, beyond the realm of non-strategic forms of competition, as an equilibrium condition under free entry. A straightforward application of the Nash equilibrium concept to standard one-shot games portraying different regimes of oligopolistic competition typically entails indeterminacy of free entry equilibria. These equilibria are characterized by two conditions: *profitability* (the price should be no smaller than the break-even price) and *sustainability* (the price should be no larger than the limit price). These conditions define a non-degenerate interval of admissible numbers of active firms, that typically contains more than one integer. The zero profit condition then appears as no more than a particular selection criterion, picking up the least profitable (and not necessarily the Pareto dominant) equilibrium, associated with the highest integer in this interval. Taking into account this type of indeterminacy in macroeconomic modelling enlarges the scope for coordination failures, which cease in particular to depend upon strategic complementarity, and opens the way to the existence of sunspot equilibria in contexts where, because of only moderately increasing returns to scale, sunspots are ruled out by dynamic determinacy.

Appendix

Proof of Lemma 1 (Profitability):

Take a pair $(p_n^*, n) \in \mathbb{R}_{++} \times \{1, ..., N-1\}$. Equation (FOC) in Definition 2, namely the equality of marginal revenue and marginal cost, is the necessary first order condition for an interior solution of (CP). Hence, the condition that p_n^* be a critical price entailing non-negative profits is clearly necessary for profitability. Let us examine if it is sufficient. A sufficient second order condition for a *local* maximum is that the marginal revenue decrease faster with y (increase faster with p) than the marginal cost, that is, that the elasticity with respect to p of the left-hand side of (FOC) be larger than the corresponding elasticity of the right-hand side:

$$1 - \frac{\epsilon_{pp}^2 d\left(p_n^*, p_n^*, n, 1\right)}{1 + \epsilon_p d\left(p_n^*, p_n^*, n, 1\right)} > \epsilon_y C'\left(d\left(p_n^*, p_n^*, n, 1\right)\right) \epsilon_p d\left(p_n^*, p_n^*, n, 1\right). \qquad \text{(SOC)}$$

By assumption (A2) and given (FOC) (implying $1 + \epsilon_p d\left(p_n^*, p_n^*, n, 1\right) < 0$), the left-hand side of inequality (SOC) is larger than one, so that the inequality is satisfied if the right-hand side is not larger than one. This is always the case if marginal cost is non-decreasing. Otherwise, if $\epsilon_y C'\left(y^*\right) < 0$ (with $y^* = d\left(p_n^*, p_n^*, n, 1\right)$), the profit non-negativity condition (PNNC) which, by (FOC), can be expressed for a critical price as

$$\epsilon_p d\left(p_n^*, p_n^*, n, 1\right) \geq \frac{1}{\epsilon_y C\left(d\left(p_n^*, p_n^*, n, 1\right)\right) - 1} \qquad \text{(PNNC*)}$$

implies that the right-hand side of (SOC) is indeed at most equal to

$$\frac{\epsilon_y C'\left(y^*\right)}{\epsilon_y C\left(y^*\right) - 1} = 1 + \frac{\epsilon_{yy}^2 C\left(y^*\right)}{\epsilon_y C\left(y^*\right) - 1} \leq 1,$$

by assumption (A1), thus verifying condition (SOC). Hence, the profit function has a *local* interior non-negative maximum at any critical price p_n^* satisfying (PNNC). This maximum is in fact a *global* maximum. Indeed, if there were two maxima, they would be separated by a minimum, satisfying (FOC) and violating (SOC), hence (PNNC) (and (PNNC*)). But, if profit has a negative minimum at some price, then it cannot have a positive maximum at a higher price, since $\epsilon_p d\left(\cdot, p_n^*, n, 1\right)$ is decreasing and $\epsilon_y C\left(d\left(\cdot, p_n^*, n, 1\right)\right) - 1$ is non-increasing, so that (PNNC*) cannot be satisfied at this higher price.■

Proof of Lemma 2 (Sustainability):

As $d(p, \cdot, n, 0)$ is increasing (by assumption (A2)), and $C(y)/y$ decreasing (by assumption (A1)), the revenue-cost ratio $G(p, p_n, n)$ is increasing in p_n. By definition of the limit price $\overline{p}(n)$, $G\left(\widehat{p}, \overline{p}(n), n\right) = 1$ at $\widehat{p}$ maximizing $G\left(\cdot, \overline{p}(n), n\right)$. Hence, $p_n \leq \overline{p}(n)$ is clearly a necessary condition for sustainability ($G\left(\widehat{p}, p_n, n\right) > 1$ if $p_n > \overline{p}(n)$). For sufficiency, consider $p_n < \overline{p}(n)$ and $G(p, p_n, n) > 1$ for some p. Then, $G\left(p, \overline{p}(n), n\right) > 1$, contradicting the definition of $\overline{p}(n)$.■

Computation of the limit price

Cournot:

From (8) and (13) with $\delta = 0$, we get

$$py = b\left(1 - p/p_n\right) = c\left(\phi + y^\gamma\right), \tag{51}$$

so that

$$p_n = \frac{b}{b - c\left(\phi + y^\gamma\right)} \frac{c\left(\phi + y^\gamma\right)}{y}. \tag{52}$$

From (9), we get

$$\frac{p}{p_n} = \frac{\gamma y^\gamma}{\phi + y^\gamma}, \tag{53}$$

an equality which, together with (51) and (17), gives

$$\phi + y^\gamma = \left(1-\gamma\right) b/2c + \sqrt{\left(\left(1-\gamma\right) b/2c\right)^2 + \phi\gamma b/c} = \left(1-\gamma\right) b/c + \phi\gamma\overline{n}. \tag{54}$$

By inserting this result in (52), we obtain the following expression for the limit price:

$$\overline{p}\left(n\right) = b^{1-1/\gamma} c^{1/\gamma} \left(\frac{1}{\gamma\left(1 - \phi\overline{n}c/b\right)} - 1\right) \frac{1}{\left(1 - \gamma + \phi\left(\gamma\overline{n} - 1\right) c/b\right)^{1/\gamma}}. \tag{55}$$

Dixit-Stiglitz:

From (8) and (20) with $\delta = 0$, we obtain

$$py = \frac{bp^{1-\sigma}}{p^{1-\sigma} + np_n^{1-\sigma}} = c\left(\phi + y^\gamma\right), \tag{56}$$

and then

$$np_n^{1-\sigma} = \left(\frac{b}{c\left(\phi + y^\gamma\right)} - 1\right) p^{1-\sigma}. \tag{57}$$

From (9), we get

$$\frac{p^{1-\sigma} + \sigma np_n^{1-\sigma}}{p^{1-\sigma} + np_n^{1-\sigma}} = \frac{\phi + y^\gamma}{\phi\gamma + \left(1-\gamma\right)\left(\phi + y^\gamma\right)}, \tag{58}$$

which, together with the former equation, gives

$$\left(1-\gamma\right)\left(\phi + y^\gamma\right)^2 - \left(\frac{b\left(\sigma\left(1-\gamma\right) - 1\right)}{c\left(\sigma - 1\right)} - \phi\gamma\right)\left(\phi + y^\gamma\right) - \frac{b}{c}\phi\gamma\frac{\sigma}{\sigma - 1} = 0. \tag{59}$$

Limiting our computation to the two benchmark cases of constant marginal cost and zero fixed cost, and using (25) and (26), we obtain

$$\phi + y^\gamma = \frac{b\phi\sigma}{b + c\phi\left(\sigma - 1\right)} = \frac{b}{c\overline{n}} \text{ if } \gamma = 1 \tag{60}$$

$$= \frac{b\left(\sigma - 1/\left(1-\gamma\right)\right)}{c\left(\sigma - 1\right)} = \frac{b}{c\overline{n}} \text{ if } \phi = 0. \tag{61}$$

By inserting these results into (57), we finally get the following expression of the limit price (valid when $\gamma = 1$ or $\phi = 0$):

$$\overline{p}(n) = \frac{b/\overline{n}}{(b/c\overline{n} - \phi)^{1/\gamma}} \left(\frac{n}{\overline{n} - 1} \right)^{1/(\sigma - 1)}. \tag{62}$$

Salop:
From (8) and (32) with $\delta = 0$, we obtain

$$py = (b/\tau)(p_n - p + \tau/2n) = c(\phi + y), \tag{63}$$

and from (9), we get

$$\frac{p_n + \tau/2n}{p_n - p + \tau/2n} = 1 + \frac{y}{\phi}, \tag{64}$$

together leading to $y = \sqrt{b\phi/\tau}$ and $p = c\left(1 + \sqrt{\phi\tau/b}\right)$. By inserting these expressions in (63), we obtain the following expression for the limit price:

$$\overline{p}(n) = c\left(1 + \sqrt{\tau/c}\sqrt{c\phi/b}\right)^2 - \tau/2n. \tag{65}$$

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