SM². A different approach to the definition of potential output.

A. Dramais(*)

A. Methodology

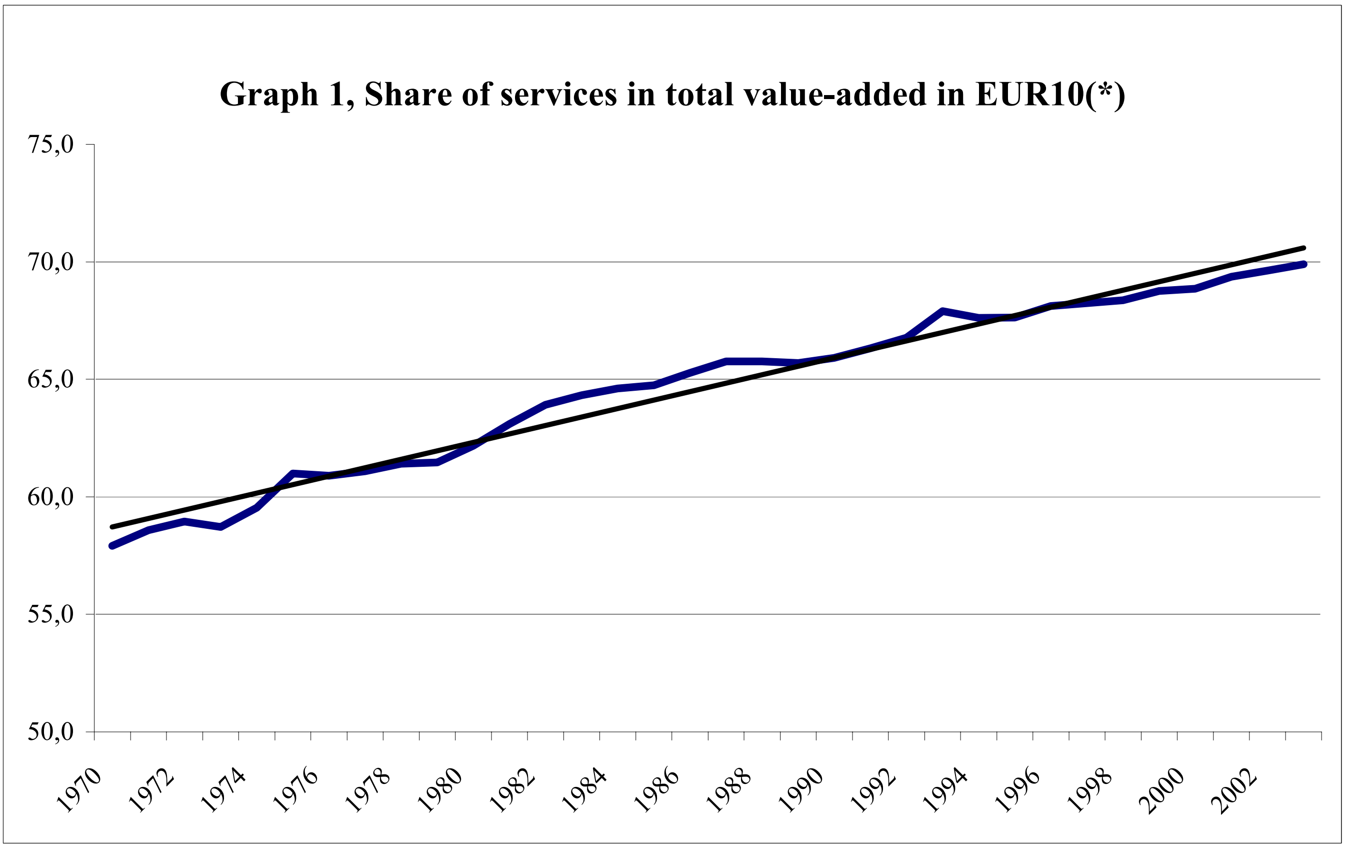
1. The problem.

According to the macroeconomic database AMECO of DG ECFIN the sectoral breakdown of GDP in 2003 (last year available) for the EUR25 aggregate was (in bio €):

Total value-added:	8999	100.0 %
Agriculture, fisheries & forestry:	187	2.1 %
Industry, excluding construction	1913	21.2 %
Construction	511	5.7 %
Services	6388	71.0 %

As shown by graph 1, this results from a long-term increasing trend.

In other words a large majority of GDP is located in a field where measurement issues are considerable and the notion of "potential" output a lot more vague than in the other three broad sectors.(see e.g. Zvi Griliches (1992 and 1994))



(*) Eurozone, minus Ireland and Luxemburg (see section A.3 below)

(*) *Directorate general for Economic and Financial Affairs, European Commission. The views expressed in this paper are those of the author and should in no way be attributed to the European Commission.*

As a consequence, potential output as measured with the usual Cobb-Douglas

$$GDP^* = A.\ K^{*\alpha}\ .\ L^{*(1-\alpha)}\ .\ e^{\gamma t} \qquad (1)$$

With K^* = potential capital stock

L^* = potential employment

γ = independent technical progress term

is becoming more and more uncertain, irrespective of the difficulties in defining exactly what should be potential capital, potential employment and independent technical progress!

One way to deal with these uncertainties was to use another starting point and use Bayesian techniques as done by Planas, Rossi and Fiorentini (2005)

Another approach could be to fuzzycise the production function (1). Since the latter is linear in logs, a fuzzy estimation can be envisaged along the lines presented by Buckley, Eslami and Feuring (2002) and by Xu and Li (2001).

A third (which is retained here) is to build a Small Macro Sectoral Module hereafter called SM^2 which explicitly link potential output to its sectoral component which may or may not be fuzzy sets themselves.

2. The sectoral relations.

2.1. Main assumptions.

The main hypothesis underlying the structure of SM^2 is the existence of a potential production frontier linking *total output* (not value-added) to factors of production labour (L) and capital K), energy inputs (E), material inputs (M) and services (S), For sector s thus we have

$$X_s = F(L^*_s, K^*_s, E_s{}^*_{1,\ldots n}, M_s{}^*_{1,\ldots m}, S_s{}^*_{1\ldots p}) \qquad (2)$$

Such a general production frontier would however be impossible to estimate given the very large number of explanatory variables against a rather small dataset.

It is thus necessary to make use of the weak separability assumption between inputs (see, e.g., Berndt and Christensen (1973)) so that (2) could be rewritten as a set of four relations (dropping the index s for clarity)

$$X^* = F_1(L^*, K^*, E^*, M^*, S^*) \qquad (3a)$$

$$E^* = F_2(E^*_1,\ldots,E^*_n) \qquad (3b)$$

$$M^* = F_3(M^*_1,\ldots,M^*_m) \qquad (3c)$$

$$S^* = F_4(S^*_1,\ldots,S^*_p) \qquad (3d)$$

The basic implication is that the choice between labour, capital, energy, material and services aggregated inputs is influenced by all variables pertaining to those categories whereas inside an aggregate group, the choice between the individual inputs is only influenced by variables pertaining to that group.

2.2 Derivations of parameters.

The derivation of parameters for functions (2) or (3a to 3d) may be approached from two directions:

i. Postulate an analytical form for F(.) and use Lagrangian techniques in order to obtain the derived optimal demand relations under the usual cost minimisation assumption. This works well with a Cobb-Douglas formulation but become quickly awkward when one use a more general functional form able to provide a second order approximation to any arbitrary production frontier with no particular restriction on the elasticity of substitution between inputs.

ii. Make use of the Shephard-Samuelson duality theorem stating that given some regularity conditions a technology may be equivalently represented either by a production frontier F(.) or its dual cost frontier

$$C = c(X, p_{1,\ldots j,\ldots nt}) \qquad (4)$$

With C the total cost of output for sector s

p_j = unit cost of input j for sector s

If the duality theorem holds then input demand relations are obtained for input j by simple partial derivatives of relation (4) with respect to the prices, i.e.

$$\text{Demand for input j} = \left(\frac{\delta C}{\delta pj}\right)$$

Since partial derivatives are as a rule easier to compute than Lagrangian, the duality approach will be retained here.

2.3. The duality approach.

Duality theory is not new since mathematically it rests on a theorem by Minkowski (1911): every closed convex set in R^N can be characterised as the intersection of its supporting half spaces.

Assume that we are given a M factor production frontier F with $X = F(q_1, q_2, \ldots, q_M)$ the maximal amount of output X that can be produced with the inputs q_i during a given time period. From now on the vector of output quantities will be represented by q so

$$X = F(q) \qquad (5)$$

If F satisfies some regularity conditions (defined below) then the total cost for the producer may be computed as

$$C(X : p) = \min_q \{p'q : F(q) \geq X\} \qquad (6)$$

where p is the vector of unit costs ($p_1, p_2, \ldots, p_M$) associated with the M inputs q_i

Shephard (1953, 1967) and, before him, Samuelson (1947) demonstrated that the reverse is also true: if the cost frontier satisfies the same regularity conditions than F, the cost frontier (6) determines uniquely the production frontier (5). Starting from the cost frontier, the production frontier is the solution of

$$F(q) = \max_X \{X : C(X : p) \leq p'q \text{ for every } p \geq 0\} \qquad (7)$$

The regularity conditions on F(.) are

i. $F(q) > 0$ for all $q > 0$ (F is positive)

ii. $F(\lambda q) = \lambda F(q)$ for every $\lambda>0$, $q>0$ (F is homogenous of degree one in q)

iii. $F(\lambda q' + (1-\lambda)q'') \geq \lambda F(q') + (1-\lambda)F(q'')$ for $q', q'' > 0$ and $0 \leq \lambda \leq 1$ (F is concave)

Since F is concave over $\{q: q>0\}$ F is a continuous function over the positive orthant. Furthermore F can be extended from the positive orthant $\{ q : q>0\}$ to the non-negative orthant $\{q : q\geq 0\}$.

Finally, if F satisfies (i), (ii) and (iii) for $X>0$ and $p>0$ the cost function $C(X; p)$ factors into

$$C(X; p) = c(p).X \qquad (8)$$

where c(p) also satisfies the regularity conditions.

2.4. Choice of a functional form for the cost frontier.

Several functional forms may be chosen for C, with the condition that they should be capable of providing a second-order numerical approximation to an arbitrary function (Lawrence Lau, 1974)

The most commodious one is the Generalised Leontief cost function proposed by Diewert (1971, 1974)

$$C(p) = \sum_i \sum_j b_{i,j}\, p_i^{1/2} p_j^{1/2} \qquad (9)$$

Applying Shephard's lemma the cost-minimising inputs are given by

$$q_i^* = X^*. \sum_j b_{i,j}.(p_j^*/p_i^*)^{1/2} \quad \text{or}$$

$$q_i^*/X^* = \sum_j b_{i,j}.(p_j^*/p_i^*)^{1/2} \qquad (10)$$

Thus the optimal technical coefficients are *linear function* of optimal *relative* unit costs. This is clearly preferable to the more popular Translog production and cost frontier proposed notably by Jorgenson (1973) and still used in productivity studies. With the Translog frontier one has that the *value shares* are linear functions of the logarithm of the price *levels*

$$\left(\frac{p*_i\, q*_i}{\sum_k p*_k .q*_k}\right) = \alpha_i + \sum_j \gamma_{i,j} .\ln p*_j \qquad (11)$$

Since trends in price levels are more evident than in relative magnitudes, the estimation of (11) is more likely to be plagued by co linearity problems than (10)

The Diewert functions satisfies the Shephard-Samuelson conditions provided that all $b_{i,j} \geq 0$ and at least one $b_{i,j} > 0$.

It may also be noted that if all $b_{i,j} = 0$ for $i \neq j$ the function collapse to the usual Leontief fixed coefficient function $q_i/X^*=b_{ii}$ [1]

Finally, the partial elasticity of substitution between any pair of inputs is given from Uzawa (1962) as

$$\sigma_{ij} = \left[\sum_k p*_k\, q*_k\right]^{1/2} b_{ij}.\left[p*_i{}^{-1/2}\, p*_j{}^{-1/2}\right].\left[X*/(q*_i\, q*_j\right] \qquad (12)$$

for $i \neq j$

If we define $P*.X* = \sum_k p*_k\, q*_k$ and $Z*_i = (p*_i.q*_i)/(P*X*)$ (13) may be written

$$\sigma_{ij} = 1/2.\left(\frac{b_{ij}}{Z_i * Z_j *}\right)\left(\frac{p_i *}{P*}\right)^{1/2}\left(\frac{p_j *}{P*}\right)^{1/2} \qquad (13)$$

The cross-price elasticities are given by

$$\varepsilon(q_i */p_j *) = 1/2\frac{bij}{Zj.}\left(\frac{p_i *}{P*}\right)^{1/2}\left(\frac{p_j *}{P*}\right)^{1/2} = \sigma_{ij} Z_i \qquad (14)$$

and the own-price elasticity by

$$\varepsilon(qi */pi *) = -1/2\frac{1}{Z_i *}.\left(\frac{p_i *}{P*}\right)^{1/2}\sum_{i\neq j} bij\left(\frac{p*_j}{P*}\right)^{1/2} \qquad (15)$$

If all b_{ij} are > 0 the σ_{ij} are all positive and all inputs are substitutable. In the same way all own-price elasticities are negative and all cross-price elasticities positive.

Finally independent technical progress terms may be introduced by allowing the diagonal element of the technical coefficient matrix to contain a time trend e.g..$b_{ii} = \delta_{ii} + c_i.t$

[1] In fact the major inconvenience of the Diewert approach is that as in all linear functions the parameter are sensitive to the unit used and are less easy to interpret than elasicities.

2.5. Constraints.

Relation (11) may be written in terms of value shares as

$$\frac{p_i * .q_i *}{\sum_k p_k * q_k *} = \sum_j b_{ij} p_i^{\ 1/2} .p_j^{\ 1/2} \quad \text{for I = 1,…n} \qquad (16)$$

Since value share must sum identically to one the n relations (11) are not independent and that must be taken into account in the estimation (see e.g. Barten (1969))

Second, since relations (10) or (16) are derived from an optimisation process the Slutzky conditions should be respected. Hence $b_{ij} = b_{ji}$ for all pair i j which introduces $\frac{n}{2}$ (n-1) linear restrictions on the parameters.

3. Empirical application

3.1 The data.

The raw materials have basically to come from Input/Output (I/O) tables and sectoral data from Eurostat both in current and constant prices. The problem is that the work on I/O tables was interrupted during many years and has only recently been revised under the new ESA95 standard but the situation is far from clear-cut.

For the Eurozone countries all Members, [except Ireland (who got derogation and produced a table for 1998) and Luxemburg (who does not produce any table…)] have tables for 1995, 1997 and 1999. They cover thus about 99% of the EU12 aggregate which is good enough.

For 1996 are missing Belgium, Germany, France and Austria (+ Ireland and Luxemburg). That year is thus unusable as such unless we interpolate the missing countries for which we have 1995 and 1997 plus the marginal totals for 1996.

For 1998, we miss Belgium, France and Austria (+Luxemburg) so hardly better than 1996, France being too large for being ignored unless one interpolate.

For 2000, we miss Greece and Portugal (+Ireland and Luxemburg). The only solution would be to extrapolate the 1999 matrices of Greece and Portugal on the basis of the marginal totals which are available in sectoral statistics.

For 2001, Denmark, Spain and Finland are added to those missing in 2000.

Finally for 2002 only Finland sent a table (according to the regulations countries have 36 months after the end of a year to send the table related to that year!).

As far as the new Member countries are concerned, six out of ten have produced some tables: Estonia has a I/O table for the year 1997, Hungary has three continuous years 1998 to 2000, Malta has 2000 and 2001, Poland has a continuous series of yearly tables from 1995 to 1999, the Slovak Republic goes one better with 1995 to 2000 and finally Slovenia has tables for1996, 2000 and 2001. Thus only Poland and the Slovak Republic would be usable from 1995 to 2000 with an extrapolation for Poland. However their structure is still so different from the one of the 15 former Member states that they are likely to be “disturbing”.

So with some creativity and the use of RAS techniques (Friedlander (1961) Bacharach (1965,1970), Snower (1990) and Toh (1998), it would be possible to have a EU10 aggregate for six consecutive years i.e. 60 individual observations for each cell of the I/O table. If one wants to remain statistically "pure", then the number of observations is reduced by one-half to 30.

The number of sectors is 59 which excludes out of hand equation (2).

Hence we would have to use the weak separability assumption with, say, 5 sectors in the energy subgroup, 29 (which could be redefined into about 10 groups to reduce the number of coefficient to be estimated) in the material input part and 25 (which can also be reduced to 10 broader sectors) in the service part.

3.2. Uncertainty.

The uncertainty can be introduced in SM² basically in two ways:

- Estimate everything by Bayesian analysis but there might be a problem of size with the joint probability distribution)

- Fuzzycise part of the analysis mostly the one dealing with services. Since the input demand function in the Diewert approach are linear function of relative unit costs, standard fuzzy methods could probably be used.

After discussion with experts of both approaches , it does not seem that the fuzzy approach could be appropriate and after a first approximation with standard methods, a Bayesian angle can be added to the analysis if it seems worth it.

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B. Practical approach

1. The available data

A problem with the approach described at the end of section A.3.1 above is the need for a large number of sectoral deflators in agreement with the input/output classification in order to express all magnitudes in constant prices. Now, although Eurostat claims to have current and constant prices I/O matrices, the constant price country coverage is presently limited to four countries, Germany, Greece Sweden and Hungary, which of course made it useless for the proposed analysis. Furthermore, there seems to be no easy way to find this price information in the industrial statistics contained in New Chronos, since the delivery of such indices is not compulsory. Thus, the time and sectoral coverage vary from country to country, most particularly in the Service part..

In order to have at least a first shot, I have aggregated the 59x59 Input/Output matrix into a 2x2 framework: sector m being all material I/O [sectors 1 (agriculture) to 34 (construction)] and sector s all services [from sector 35 (automobile trade and repair) to 59 (domestic services to households)]. In this highly aggregated case, deflators can be found or constructed without too many problems..

As an example, I have given on the following page the resulting table for the Euro10 group (Euro zone minus Ireland and Luxemburg) in 2000.

Some remarks on that table:

1. In an ideal I/O table, sectors would be so specialised that none would consume its own product. Hence the diagonal elements would be zero and can be used to store sectoral imports. Here of course the degree of aggregation makes that impossible.
2. Although total inputs are not too far away between the two sectors it is fairly evident that they mostly consume their own intermediate products. The "material" sectors consume only ± 20 % of services in their intermediate inputs whereas the service sectors consume about 30 % of material inputs.
3. The following columns give final demand elements from which one can compute GDP (after subtracting imports). We can see that private consumption is well distributed (about 40% for services and 60 % for the rest). On the other hand, public consumption is of course mostly services (public administration, education and health) while conversely, gross investment, changes in inventories and exports are massively composed of material products.
4. Below the I/O table strictly speaking (the 2x2 left-hand corner) are given the elements of value-added, the sum of which is net national income. We see that the contribution of the service sector is far larger than that of the material sector confirming its labour-intensive nature. Conversely, the external supply (imports) is mostly going into the material sector. As a result, sectoral total output at basic prices are quite similar
5. Finally, after some valuation adjustments to go from basic prices to purchasers' prices, total supply is indeed equal to total demand, as it should be. It is also clear that total output or demand in the Euro zone is more than twice larger than GDP. For a small open economy like Belgium it is even three times as large.

I/O compressed table for Euro10 (Eurozone w/o IRL and LUX) - 2000

	Material demand	Service demand	Intermediate demand
Material outputs	2738	807	3545
Service outputs	690	1826	2516
Total	3428	2633	6061
Wage bill	1033	2183	
Net taxes on production	21	81	
Depreciation	180	393	
Net Surplus	575	1295	
Value added	1809	3952	
Imports	1881	373	
Total supply, basic prices	7117	6958	
Trade and transport margins	1002	-1002	
Indirect taxes minus subsidies	500	187	
Total supply, purchasers' prices	8619	6143	14762

Private Consumption	Public Consumption	Gross fixed Capital Formation	Changes in Inventories	Exports	Final Demand	Total Demand
1912	64	1203	26	1869	5073	8619
1795	1223	181	-1	429	3627	6143
3707	1288	1384	25	2298	8701	14762

2. Typical equations related to the Diewert production function and dual cost function.

In the two by two framework, we have eight sectoral relations, four for each aggregated sector: labour, capital, material inputs and service inputs. All are linear functions of the square root of relative prices.

The material sector will be represented by M, the service sector by S

Let XT_i be the output of sector i, i = M, S

$X_{i,j}$ is the input of product i by sector j, i,j = M, S.

L_i is employment in sector i

K_i is the capital stock of sector i

W_i is the average wage cost in sector i

Uck_i the unit cost of capital goods in sector i

PM the aggregate deflator of material goods[2]

PS the aggregate deflator of services[2]

We have thus for sector M

$$\frac{L_M}{XT_M} = a_{1,1} + a_{1,2}.(\frac{Uck_M}{W_M})^{1/2} + a_{1,3}.(\frac{PM}{W_M})^{1/2} + a1,4(\frac{PS}{W_M})^{1/2} \quad (17)$$

$$\frac{K_M}{XT_M} = a_{2,1}(.\frac{W_M}{Uck_M})^{1/2} + a_{2,2} + a_{2,3}(\frac{PM}{Uck_M})^{1/2} + a_{2,4}.(\frac{PS}{Uck_M})^{1/2} \quad (18)$$

$$\frac{X_{M,M}}{XT_M} = a_{3,1}(.\frac{W_M}{PM})^{1/2} + a_{3,2}.(\frac{Uck_M}{PM})^{1/2} + a_{3,3} + a_{3,4}.(\frac{PS}{PM})^{1/2} \quad (19)$$

$$\frac{X_{S,M}}{XT_M} = a_{4,1}.(\frac{W_M}{PS})^{1/2} + a_{4,2}.(\frac{Uck_M}{PS})^{1/2} + a_{4,3.}(\frac{PM}{PS})^{1/2} + a_{4,4} \quad (20)$$

and for sector S

$$\frac{L_S}{XT_S} = b_{1,1} + b_{1,2}.(\frac{Uck_S}{W_S})^{1/2} + b_{1,3}.(\frac{PM}{W_S})^{1/2} + b1,4(\frac{PS}{W_S})^{1/2} \quad (21)$$

$$\frac{K_S}{XT_S} = b_{2,1}(.\frac{W_S}{Uck_S})^{1/2} + b_{2,2} + b_{2,3}(\frac{PM}{Uck_S})^{1/2} + b_{2,4}.(\frac{PS}{Uck_S})^{1/2} \quad (22)$$

$$\frac{X_{M,S}}{XT_S} = b_{3,1}(.\frac{W_S}{PM})^{1/2} + b_{3,2}.(\frac{Uck_S}{PM})^{1/2} + b_{3,3} + b_{3,4}.(\frac{PS}{PM})^{1/2} \quad (23)$$

$$\frac{X_{S,S}}{XT_S} = b_{4,1}.(\frac{W_S}{PS})^{1/2} + b_{4,2}.(\frac{Uck_S}{PS})^{1/2} + b_{4,3.}(\frac{PM}{PS})^{1/2} + b_{4,4} \quad (24)$$

[2] It was also necessary to assume that the Law of One Price applies and that the price of material goods and of services is the same in both sectors.

The 2x2 matrix of technical coefficients TC obtains directly from relations (19), (20), (23) and (24). Once the vector 2x1 of final demand FD is given, total outputs could be computed from the basic I/O accounting identity

$$XT = (I - TC)^{-1} \,.\, FD \qquad (25)$$

In a full macro-sectoral model, sectoral final demand categories should be estimated by the use of the same methodology as the technical coefficients, maximising in this case a utility function of a generalised Leontief or Translog form. Here, once a forecast is gotten for the macroeconomic components they can be sectorialised by the use of ratios summing over unity taken from the last available observations[3]

$$DF_M = R_M \,.\, Dftot \qquad (26)$$

$$DF_S = R_S \,.\, Dftot \qquad (27)$$

With $R_M + R_S = 1$

As far as prices and unit costs are concerned, at such level of aggregations they are available without much problems. Sectoral wage bill divided by sectoral employment give the average wage cost and the unit cost of capital is computed via the Jorgenson approach, i.e. $Uck_i = PV.(r_l + \delta_i)$ with PV the price of capital goods, r_l the long-run interest rate and δ_i the depreciation rates. All input prices are then recomputed as index 1995=1.0.

As stated above, for the estimation of an Euro10 model a cross section of countries have to be used with a full true sample only for 1995, 1997 and 1999 thus 30 observations. With the RAS techniques, it can be extended to 1996, 1998 and 2000 i.e. 60 observations. Given the constraints on the parameters, the number of degrees of freedom should be enough..

[3] Given the high degree of aggregation such ratios are very stable from one year to the other.

C. Estimation results.

1. Introduction

The equations described in the former section were estimated first without constraints, using the procedures described in Balestra-Nerlove (1966) for a sample composed of a chronological series (1995-2000) of cross-section (10 I/O tables from €-area countries).

A problem when using panel data regression techniques is to detect and introduce specific country and/or time effects Given the rather short chronological time period covered, only country effects were analysed.

There are two ways of dealing with the issue when one believes that there are strong country effects : the simplest is to suppress the constant term and add 10 country binaries, the estimation being done by OLS. The other is to work at the residual level and distinguish a country effect in the definition of the residual variance-covariance matrix. In this case, a two step approach has to be followed.

2. The equations.

2.1. Country effects through dummies

$$Y = X\beta + BIN\gamma + \varepsilon \quad (28)$$

Where Y is a 60x1 vector of ratios of a given input (labour, capital, material inputs, service inputs) to total output of an aggregated sector (material or services outputs)

X is a 60x3 matrix of the square roots of the relative prices of inputs

BIN is a 60x10 matrix of dummies : one column per country with ones at the level of that country in the Y vector (position 1 to 6 for Belgium, 7 to 12 for Germany until 55 to 60 for Finland) and zeros elsewhere.

ε is a vector 60x1 of random residuals $N(0, \sigma)$

The estimation was done with the panel regression instruction of WinRats 6.0 and are presented in table one .

Table 1. Regression with fixed country effects represented by dummies.
(the coefficient of the dummies are not presented in this table)
(standard errors of coefficients are in brackets below the estimated values)
(coefficient not statistically different from zero are in red)

Dependant	X1	X2	X3	Adjusted R^2
Lm/XTm	0. 154	2.315	-1.571	0.9975
	(0.033)	(0.273)	(0.533)	
Km/XTm	-0.835	1.729	-0.932	0.9811
	(0.475)	(0.360)	‘(0.568)	
Xmm/XTm	0.202	0.0133	-0.383	0.9801
	(0.062)	(0.0065)	(0.095)	
Xsm/XTm	0.040	0.0036	0.046	0.9923
	(0.022)	(0.0027)	(0.033)	
Ls/XTs	0.412	1.856	-1.717	0.9981
	(0.103)	(0.589)	0.513)	
Ks/XTs	-0.373	1.833	-1.549	0.9975
	(0.525)	(0.666)	(0.561)	
Xms/XTs	0.031	0.045	0.448	0.9828
	(0.065)	(0.012)	(0.063)	
Xss/XTs	-0.083	-0.001	-0.198	0.9924
	(0.069)	(0.013)	(0.077)	

Results are somewhat mixed: in both sector the labour demand function is the best one, with all coefficients significant but with the relative price of services having the “wrong” sign. This is the case in practically all equations, which seems to indicate that, on the whole, services are complements (rather than substitutes) for all other inputs. The relative price of labour is also badly signed in 3 out of the 6 equations where it appears but is never significantly different from zero when it is negatively signed.

2.2. Country effects within the residuals.

The model becomes

$$Y = X\beta + \varepsilon \qquad (29)$$

Where X is now 60x4 via the addition of a column of 1 for the constant term

.
We also assume that the residual term can be partitioned in two effects

$\varepsilon_{i,t} = \mu_i + \nu_{i,}t$ with μ_i representing the country effects and $\nu_{i,}t$ the random residual.(I = 1,…,N, t =1,…,T)

The two residual items are assumed to be independent so that the variance covariance of the residuals $\varepsilon_{i,t}$ is block-diagonal of the form

$$Eu.u' = \Omega = \sigma^2 \begin{Bmatrix} A & 0 & \dots & 0 \\ 0 & A & \dots & 0 \\ .. & .. & \dots & \\ 0 & 0 & \dots & A \end{Bmatrix} \qquad (30)$$

with A a TxT matrix of the form

1 ρ ρ … ρ
ρ 1 ρ … ρ
. . .. ….. ..
ρ ρ ρ … 1

with ρ the ratio of the variance of μ_i (σ^2_μ) to σ^2

With such a structure of residuals an OLS estimation of model (2) would give unbiased and efficient coefficients $\hat{\beta}$ but they would not be minimum variance nor, in general asymptotically efficient.

A two-stage approach has thus to be adopted as proposed initially by Zellner (1962) and Telser(1964).

Model (2) can be rewritten as

$$y_{i,t} = (\beta_1 + \mu_i) + \sum_{k=2}^{K} \beta^k . x_{1,t}^k + v_{i,t} \qquad (31)$$

If enough observations are available OLS estimates of parameters β may be obtained country by country and are designed by $\hat{\beta}_i^k$. Then the following approach will lead to efficient estimates of all parameters and of the variance-covariance matrices.

$$\hat{\beta}^k = \frac{1}{N}\sum_{i=1}^{N} \hat{\beta}_i^k \qquad k = 2,\ldots, K \qquad (32)$$

$$\bar{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it} \qquad (33)$$

$$\bar{x}_i^k = \frac{1}{T}\sum_{t=1}^{T} x_{i,t}^k \qquad (34)$$

$$\hat{\beta}_1 = \frac{1}{N}\sum_{i=1}^{N} (\bar{y}_i - \sum_{k=2}^{K} \hat{\beta}^k . \bar{x}_i^k) \qquad (35)$$

$$\hat{\sigma}_\mu^2 = \frac{1}{N}\sum_{i=1}^{N} [\ \bar{y}_i - \hat{\beta}_1 - \sum_{k=2}^{K} \bar{x}_i^k \hat{\beta}^k\ ]^2 \qquad (36)$$

$$\hat{\sigma}^2 = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} [y_{i,t} - \hat{\beta}_1 - \sum_{k=2}^{K} \hat{\beta}^k . x_{i,t}^k\ ]^2 \qquad (37)$$

$$\hat{\rho} = \hat{\sigma}_\mu^2 / \hat{\sigma}^2 \qquad (38)$$

With these estimates one can compute the variance covariance matrices and the minimum-variance, linear, unbiased estimators of β are then given by (in matrix notation)

$$\hat{\beta} = [X'\Omega^{-1} X\ ]^{-1} X'\Omega^{-1} y \qquad (39)$$

with a variance-covariance matrix V($\hat{\beta}$) that can be estimated by

$$\hat{V}(\hat{\beta}) = \frac{\hat{u}'\Omega^{*-1}\hat{u}}{NT - K} . [X'\Omega^{*-1} X\ ]^{-1} \qquad (40)$$

with $\Omega^* = \dfrac{\Omega}{\hat{\sigma}^2}$ and $\hat{u} = y - X\hat{\beta}$

Provided that the û are distributed according to a multivariate normal distribution with zero means and variance-covariance $\sigma^2\Omega^*$, $\hat{\beta}$ given by (4.8) is the maximum likelihood estimate of β.

Estimation results are given in table 2

Table 2. Input demand functions estimated by GLS
(standard errors are within brackets and non significant coefficients are in red)

Dependant variable	Constant	X1	X2	X3	Corrected R^2
Lm/XTm	-0.2390	0.1638	2.2886	-1.4357	0.9956
	(0.373)	(0.036)	(0.299)	(0.517)	
Km/XTm	0.8422	-0.6216	1.7685	-0.9859	0.9665
	(0.085)	(0.400)	(0.395)	(0.625)	
Xmm/XTm	0.4780	0.1819	0.0141	-0.3542	0.9655
	(0.062)	(0.069)	(0.007)	(0.106)	
Xsm/XTm	-0.058	0.054	0.005	0.063	0.9874
	(0.061)	(0.025)	(0.003)	(0.026)	
Ls/XTs	1.4485	0.4171	1.8762	-1.6894	0.9967
	(0.627)	(0.113)	(0.645)	(0.562)	
Ks/XTs	2.6592	-0.5718	1.9097	-1.4417	0.9955
	(0.300)	(0.579)	(0.733)	(0.621)	
Xms/XTs	-0.373	0.042	0.043	0.446	0.9694
	(0.079)	(0.071)	(0.014)	(0.069)	
Xss/XTs	0.5393	-0.063	-0.0016	-0.2011	0.9866
	(0.070)	(0.075)	(0.015)	(0.084)	

The results are not much different than those of table 1. Constant shift variables are thus a good approximation of the final results with a more sophisticated econometric approach. The major advantage of the latter is of course that it reduces seriously the number of coefficient to be estimated per equation from 13 with binaries to 4 with GLS, The number of "wrong signs" is also lower except for the relative prices of services which is practically always negative and significant. It is also clear that the symmetry constraints are not coming in free estimation.

As a final trial before going into system estimations, the labour and capital demand were estimated jointly with SURE 'Seemingly Unrelated Regressions", introducing the constraint that the coefficient of X1 (cost of capital over wage in Lm and wage over cost of capital in Km) should be equal. The estimation was made with dummies, as in table one.

Table 3. SURE estimation of labour and capital demand equations. Material and services sectors.

Dependant variable	X1	X2	X3	Corrected R^2
Free estimation, material sector				
Lm/XTm	0.1545	2.3164	-1.5472	0.9966
	(0.033)	(0.273)	(0.471)	
Km/XTm	-0.7491	1.7934	-1.0697	0.9743
	(0.366)	(0.361)	(0.573)	
Constrained est.				
Lm/XTm	0.1350	2.4157	-1.8539	0.9965
	(0.032)	(0.271)	(0.453)	
Km/XTm	0.1350	2.1422	-2.2315	0.9716
	(0.032)	(0.347)	(0.329)	
Free estimation, services sector				
Ls/XTs	0.413	1.868	-1.699	0.9974
	(0.103)	(0.589)	(0.513)	
Ks/XTs	-0.563	1.888	-1.427	0.9965
	(0.529)	(0.670)	(0.564)	
Constrained est.				
Ls/XTs	0.360	1.824	-1.890	0.9974
	(0.099)	(0.590)	(0.502)	
Ks/XTs	0.360	1.402	-1.763	0.9963
	(0.099)	(0.626)	(0.544)	

The experiment is highly satisfactory since the sum of squared residuals hardly changes as shown by the corrected R^2 and the coefficients in the constrained equations are all significant.. The SURE part in WinRats 6.0 however does not allow for symmetry constraints which should be done in the non-linear system estimation part. But there, the introduction of dummy variables lead to too large a sample and the estimation breaks down.

A simple test could however be done in a recursive way : given the high quality of these equations, use their coefficients in the remaining two equations and estimate only the remaining parameters, under constraint. In that way a symmetric βij can easily be produced.

Table 4. Material and service inputs equations, under constraint of labour and capital demand coefficients

Dependant variable	X1	X2	X3	Corrected R^2
Xmm/XTm	2.4157	2.143	3.341	0.589
			(2.284)	
Xsm/XTm	-1.8539	-2.2315	3.431	0.570
			(2.284)	
Xsm/XTs	1.824	1.402	4.745	0.755
			(0.878)	
Xss/XTs	-1.890	-1.763	4.745	0.742
			(0.878)	

The matrix of coefficients would thus be

Table 5 Matrix of coefficient $\beta_{ij,}$ material sector, recursive estimation

Relative prices	Labour	Capital	Material inputs	Service inputs
Labour demand	1	0.135	2.416	-1.854
Capital demand	0.135	1	2.143	-2.231
Material demand	2.416	2.143	1	3.341
Service demand	-1.854	-2.231	3.341	1

Table 6 Matrix of coefficient $\beta_{ij,}$ service sector, recursive estimation.

Relative prices	Labour	Capital	Material inputs	Service inputs
Labour demand	1	0.360	1.824	-1.890
Capital demand	0.360	1	1.402	-1.763
Material demand	1.824	1.402	1	4.745
Service demand	-1.890	-1.763	4.745	1

In these matrices the 1 on the diagonal represent the constant term as given by the 10 dummy variables.

Although the symmetry constraint is respected, the sign constraint is not: for the dual approach to be valid, the β_{ij} coefficients should all be ≥ 0 with at least one of them > 0 It is also clear that negative coefficients are concentrated in the last line and column i.e. the one dealing with services. However, if some β_{ij} are negative, the dual cost function can still provide a valid representation of technology for a range of input prices. Diewert has indeed shown that if the parameters β_{ij} are such that $\nabla c(p^*) \geq 0$ for some $p^* \geq 0$ and that $\nabla^2 c(p^*)$ is a negative semi-definite matrix of rank N-1, there is a neighbourhood of prices around p* where c(p) satisfies the conditions of a valid dual cost function.

A full simultaneous FIML approach with symmetry constraints etc. has also be used with a generalised Balestra Nerlove formulation: the size of the problem being four times the former example.

Y pools all four explained variables together and become 240x1. The explanatory variables matrix become 240 x 16 in four diagonal boxes 60x4 with zeros elsewhere, the coefficients form a 16x1 vector. The procedure is then be the same as before with one more dimension r representing the equations and the Ω matrix is 240x240 with 40 diagonal blocks 6x6.
Formally

$$Y = \begin{matrix} Yl \\ Yk \\ Ym \\ Ys \end{matrix} = \begin{matrix} X^l & 0 & 0 & 0 \\ 0 & X^k & 0 & 0 \\ 0 & 0 & X^m & 0 \\ 0 & 0 & 0 & X^s \end{matrix} \begin{matrix} \beta^l \\ \beta^k \\ \beta^m \\ \beta^s \end{matrix} + \mu + \varepsilon \qquad (41)$$

The computation would be the same as before with 40 OLS estimates to start with and four different μ_i (one series per equation) and hence four different ρ from which the Ω^* and Ω matrices could be computed..

Finally a matrix of symmetry constraints is added to the problem, but looking at the results in table 7 and 8, these massive computation do not solve the problem of negativity in the service line and row and given confidence intervals, are usually not statistically different from those obtained in table 5 and 6.

Table 7. Matrix of coefficient $\beta_{ij,}$ material sector, joint estimation.

Relative prices	Labour	Capital	Material inputs	Service inputs
Labour demand	0.723	0. 204	2.400	-1.422
Capital demand	0.204	0.935	1.987	-2.333
Material demand	2.400	1.987	0.575	0.503
Service demand	-1.422	-2.333	0.503	0.611

Table 8. Matrix of coefficient $\beta_{ij,}$ service sector, joint estimation.

Relative prices	Labour	Capital	Material inputs	Service inputs
Labour demand	1.299	0.388	1.638	-1.723
Capital demand	0.388	0.935	1.203	-1.888
Material demand	1.638	1.203	0.385	2.910
Service demand	-1.723	-1.888	2.910	0.569

As a result, substitution elasticities and cross-prices elasticities for service demand with respect to all other inputs are nearly always negative and own-price elasticities nearly always positive. Services are thus different from the other inputs in some quality at least, a point that should more thoroughly investigated.

D. Some facts about the service sector in the Euro area.

If one looks at the composition of XT_S (total output of services), one observes that about 62 % of it has clearly a complementary nature with respect to the production of material goods: once a sellable product has been produced, it must be transported to the distribution point, it must be insured, it must be financed (e.g. in order to cover the time delay between production and storage on the one hand and final consumption on the other hand.) Finally in order to produce enterprises need informatics support, some R&D and other services.[4]

In the I/O tables all that covers a substantial part of trade, transport, post and communication, financial intermediation, insurance, informatics, R&D, other services to enterprises i.e. more than 60% of the total, the rest being public goods or semi public goods activities, like public administration, health, education, sewage collection and handling, plus all activities peculiar to households which inherently come from increases in the standard of living like hotels and restaurants, associative, cultural, sport and recreational activities and finally personal and domestic services.

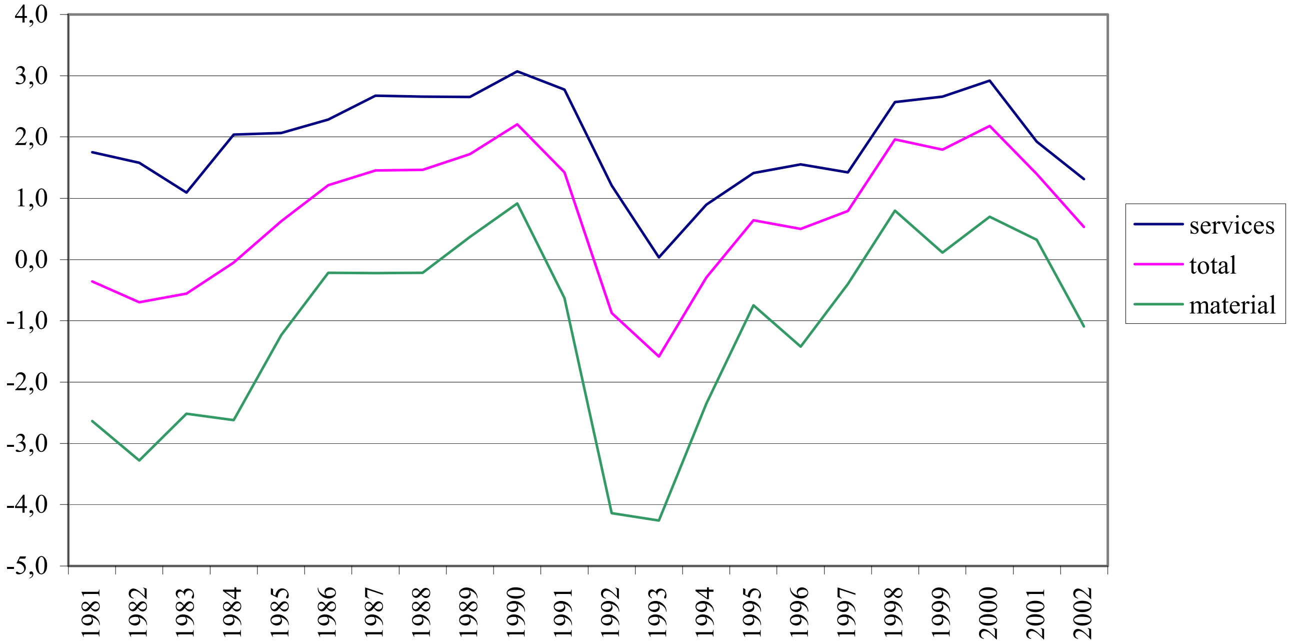
[4] This of course, does not preclude substitution between providers of these services within the broad I/O categories.

So, even when taking into account that part of transport, insurance, etc. is not directly linked to material production, the complementary nature of all these inputs is undisputable. Beside, the remaining part is growing basically with increases in the standard of living whatever its source but since material production also involves payment of wage and non-wage income, growth should also be closely linked at least in long run trends.

Furthermore, about 90% of the part of services going directly into final demand goes to total consumption and very little in capital formation or exports.

It is also clear that the high labour demand characteristic of 1995 to 2000 years (1.4% growth of total employment per year) is **totally** coming from the service sector: employment in that sector grew by 2.2% per year, whereas the employment in the material sector *decreased* by 0.3% per year during the same period.

Graph 2. Total, services & material employment yearly rates of growth.
(in percent, EUR10(*))



(*) Euro area without Ireland and Luxemburg

Graph 2 also shows that between 1981 and 2002 the growth of employment in services has **always** been positive whereas the growth of employment in the material sector has been negative in most years with small positive excursions only in 1990 and in 1998-2001. On the other hand, cyclical fluctuations in both series are quite similar which is also in favour of the complementary assumption.

The same was true for growth:: total value-added (i.e. GDP) grew in real terms by 2.4% p.y. distributed into a growth of 1.1% p.y. for the material value-added against 3.0% p.y. for the value-added of services. The apparent productivity of labour was thus growing at 1.4 % for the aggregate of agriculture, construction and manufacturing industries and at 0.8% for the service sector!

E. Final equations for total input demands of services and supply of material products.

This being so, I have tested econometrically a demand equation for the total input of services in real terms (XT_S) with the real total output of material products (XT_M) as a proxy for complementarity effects, real net disposable income of households (YD) as a proxy for purchasing-power-connected effects and the relative price of services with respect to the GDP deflator (Pr).as a cost element.

Using once again the panel regression of rats 6.0, the best results were obtained in a log-linear model with generalised LS

$$\text{Ln } XT_S = \underset{(0.523)}{1.292} + \underset{(0.082)}{0.716} \ln XT_M + \underset{(0.103)}{0.324} \ln YD \underset{(0.126)}{-0.480} \ln PR \qquad R^2 = 0.9995$$

Thus the sum of the quantitative elasticities is slightly above 1 but not statistically different from one and the relative price elasticity has in this case the right sign..

In order to have total output we need an estimation for XT_M. I used for that one a directly estimated Diewert production function[5] with Km and Lm as argument,. i.e.

$$XT_M = \alpha_1 \sqrt{Lm} + \alpha_2 \sqrt{Km} + \tfrac{1}{2} [\sqrt{Lm} \;\; \sqrt{Km}] \boxed{\begin{matrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{matrix}} \begin{vmatrix} \sqrt{Lm} \\ \sqrt{Km} \end{vmatrix} + \text{dummies}$$

under the usual symmetry constraint $b_{12}=b_{21}$.

The estimation proved successful with parameters given in the following table

Table7. Parameters of a Diewert production function applied to material output.

.Parameters	Estimation	Standard error
α_1	42.335	20.065
α_2	37.938	15.663
b_{11}	1.049	0.455
b_{22}	11.592	2.512
$b_{21}=b_{12}$	3.725	0.805
R^2	0.9968	

Of course total output $XT = XT_S + XT_M$

[5] This function has a variable elasticity of substitution and is a second order numerical approximation to any arbitrary production function.

This close the SM² exercise for the present since a finer decomposition of services would require inexistent deflators and the I/O avenue is therefore unusable until more years and more deflators are available for more countries..

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