Scale and structure effects of final demand shocks: measuring interindustry linkages in national and regional economies

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ABSTRACT. In traditional linkage analysis, either at national or regional level, output multipliers are calculated from the Leontief inverse, imposing unitary final demand shocks with a fixed (predetermined) structure. In this paper, output multipliers result from solving an optimization problem, with two important advantages. First, the final demand structure is not fixed in advance. Second, the maximum output impact can be decomposed in two significant effects: a (homothetic-) scale one, depending on the magnitude of the positive shock applied to a pre-existing final demand structure and a structure effect, resulting from the sectoral final demand output maximizing changes. This method can be very helpful in measuring interindustry linkages and choosing (a certain kind of) key sectors in a national or regional economy. An empirical application is made in the paper, using Portuguese national and regional (Azores) input-output data.

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1. INTRODUCTION

In studying the structure of a national or regional economy working according to the Leontief model hypothesis, a central role is devoted to final demand multipliers, i.e., the elements of the Leontief inverse used to measure the impacts of change(s) in one (several) component(s) of final demand on output, value added or employment.

However, the common use of this kind of multipliers, dating back to Rasmussen(1956), suffers from an important drawback, namely that it limits itself to particular changes in final demand, such as a unitary shock in each sector and zero elsewhere in the backward multipliers case, and a unitary shock in all sectors at once in the forward multipliers case. This limitation, pointed out by Skolka(1986), reduces the usefulness of the Rasmussen multipliers.

It can even be argued that the use of traditional multipliers leads to an inadequate invasion of macroeconomic concepts over a genuine multisectoral analysis. Let us consider, for instance, a unit increase in total final demand. From a macroeconomic point of view it is by definition indifferent to know in advance how this monetary unit is distributed among sectors, because these sectors are not individually considered. But from a multisectoral point of view it is crucial to know if this unit is, for example, entirely directed to one particular sector or otherwise evenly distributed among all the sectors.

In the first case, the new situation (after the final demand increase) is more different from the initial one comparing to the second case. This difference does not exist in an aggregate macroeconomic analysis. In a disaggregated intersectoral analysis it should not be ignored.

For this kind of comparisons between different situations the traditional Leontief/Rasmussen multipliers are inappropriate, because they are unable to compare output (value added, employment) impacts of changes in final demand originating new vectors equidistant to the initial vector. New methods are needed.

One interesting approach to this problem is the pioneering work of Ciaschini (1989; 1993; 2002) based on the so-called singular value decomposition method.

In this paper a different and easier approach is followed. Solving an appropriately designed optimization problem, two important advantages are obtained. First, the final demand structure subsequent to a final demand shock is not fixed in advance, so overcoming an important limitation of traditional linkage measures. Second, the maximum output impact can be decomposed in two significant effects: a homothetical scale one, depending on the magnitude of the positive shock applied to a pre-existing final demand structure, and a structure effect, resulting from the sectoral final demand output maximizing changes.

This method, explained and formalized in section 2, originates a new kind of (what can be termed) *distance multipliers* and may prove itself to be helpful in measuring interindustry linkages and choosing (a certain kind of) key sectors in a national or regional economy.

An empirical application of the method is made in for the Portuguese national and regional (Azores) input-output data (section 3). The paper concludes with a summary of the main results (section 4).

2. EUCLIDEAN DISTANCE INTERSECTORAL MULTIPLIERS

Context of analysis

When a standard Leontief model[1] $\boldsymbol{x} = \mathbf{L}\boldsymbol{f}$ is used for studying the potentialities of growth of an economy in response to final demand shocks at least three problems can be considered:

a) find, for a new situation the largest increase in production resulting from an unitary increase in final demand supposing that no sector decreases its final demand in this new situation relatively to the initial one. This problem is easily solved using the Rasmussen multipliers. The unitary increase is final demand should be affected to the sector i such that the Rasmussen multiplier $\sum_j l_{ji}$ is maximum.
b) find the largest increase in production following a unitary increase in final demand assuming that the final demand for each sector can vary (supposing that this variation will not lead for that sector to a negative final demand in the new situation; negative final demand for a given sector has no meaning with the possible exception of the existence of large stocks for that sector in the initial situation – a case that we rule out). This problem again is easily solved. All the final demand (total value of final demand in the initial situation plus one additional monetary unit) should be affected to the sector i of the largest $\Sigma_j\ b_{ji}$ and for the other sectors final demand should be zero.

These two problems are easily solved but both are of a limited interest because of its lack of realism more pronounced of course for the second problem. For the first problem the macroeconomic bias is clear. It is supposed that it’s possible to increase in one monetary unit the final demand of any sector and at the same time keep constant final demand for the other sectors, assumption which a genuine multisectoral analysis can not accept. That is why it’s worthwhile to consider a third problem

c) find the variations of the vector of final demand inside a given neighborhood of a initial vector that maximize (or minimize) the distance of the resulting vector of production in the new situation relatively to the initial production vector

One important characteristic of this third problem is the use of the Euclidean distance between vectors to measure the variations in relation to the initial situation. A vector resulting from concentrating all the final demand increase in one sector is at a greater distance from the original final demand vector, than a vector resulting from evenly distributing a final demand increase of the same magnitude, which means that the Euclidean distance effectively distinguishes two situations that must be treated as different. So, a genuinely multisectoral analysis should focus on the comparison between final demand variations originating new vectors located at the *same distance* from the original vector. In the same way, the output impact of these final demand variations should be measured by the Euclidean distances between the new and the original output vectors.

Methodology

In studying the structure of a national (or regional) economy, suppose that we have to find the vector that maximizes the total output attainable in the next period. Formally, let's call $\mathbf{y}^s$ the initial final demand vector and $\mathbf{x}^s$ the corresponding output vector, given by the familiar IO relation $\mathbf{x}^s=\mathbf{L}\,\mathbf{y}^s$, with $\mathbf{L}$ being the Leontief inverse. Given a vicinity β of $\mathbf{y}^s$, $V(\mathbf{y}^s,\beta)$, the objective is to find the vector $\mathbf{y}^* \in V$ such that the distance between $\mathbf{x}^*(\mathbf{y}^*)$ and $\mathbf{x}^s$ is maximum.

Note that this is not the case of calculating the output growth resulting from a unitary increase in final demand. This problem is easily dealt with traditional multipliers. In this case, what we want is to find from among all the vectors at a certain distance of $\mathbf{y}^s$, the vector that maximizes the variation of the resulting output vector relatively to the initial vector, $\mathbf{x}^s$.

Consider, for simplicity, that $\beta = 1$. In this case, a vector at a unitary distance of $\mathbf{y}^s$ is not necessarily a final demand vector for which the summation of all its elements exceeds in exactly one monetary unity the summation of all the elements of the initial vector. This only holds when all the (unitary) final demand increase concentrates in one sector. In general, and excluding this particular case, it is a vector representing a monetary expenditure greater by more than one unity than the total expenditure of vector $\mathbf{y}^s$ in more than one unity.

Particularly in studies of economic growth it is much more interesting to consider the output impacts of final demand vectors at a given distance from an initial vector than merely attending to the output growth of unitary final demand increases.

Suppose we want to study the impact upon the distance from the initial output vector $\mathbf{x}^s$ to the vector $\mathbf{x^*}$ of a final demand change from $\mathbf{y}^s$ to $\mathbf{y^*}$, in which:

$$\Sigma\,(y_j - y^*_j)^2 = \beta^2$$

It is a case of maximizing (with β equal to 1, by hypothesis):

$(\mathbf{x} - \mathbf{x}^s)'\,(\mathbf{x} - \mathbf{x}^s)$, (the signal ' means transpose)

subject to:

$(\mathbf{y} - \mathbf{y}^s)'\,(\mathbf{y} - \mathbf{y}^s) = 1.$

As $\mathbf{x}^s = \mathbf{L}\,\mathbf{y}^s$, the corresponding *Lagrangean* is:

$(\mathbf{y} - \mathbf{y}^s)'\,\mathbf{L'L}\,(\mathbf{y} - \mathbf{y}^s) - \lambda[(\mathbf{y} - \mathbf{y}^s)'\,(\mathbf{y} - \mathbf{y}^s)].$

After differentiating and equalizing to zero:

(2.1) $\mathbf{L'L\,(y - y^s) = \lambda (y - y^s)}$.

Since **L'L** is symmetric, all its eigen values are real. Since it a case of maximizing a definitive positive quadratic form, all the eigen values are positive.

Besides, multiplying both members of **(2.1)** by $\mathbf{(y - y^s)'}$ and considering only vectors **y** such as $\mathbf{(y - y^s)'\,(y - y^s)} = 1$, we have:

$\mathbf{(y - y^s)'\,L'L\,(y - y^s)} = \lambda$,

and so, the maximum distance between **x** and $\mathbf{x^s}$ is obtained for the greatest value of λ, that is, for the greatest eigen value and the minimum distance for the smallest one.

An economy is the more variable relatively to final demand structures, the greater the amplitude of variation of the distance between **x** and $\mathbf{x^s}$, in response to a unitary final demand shock.

A demand management economic policy may focus on maximizing output and employment, and in this case it will try to attain the vector **y*** that maximizes the distance between **x** and $\mathbf{x^s}$. An economic policy focused on an inflation target will generally try to attain a vector **y** that minimizes this distance.

The amplitude of variation attainable for the distance between **x** and $\mathbf{x^s}$ can be measured by the difference $s(\mathbf{L'L}) = (\lambda_{max} - \lambda_{min})$, that is, the *spread* of **L'L** and it is certainly an important property of each technological structure **A,** and its corresponding Leontief inverse, **L**.

An important property of technological structures

Some linear algebra results can be used to further advance the research of this property of technological structures.

It is known (Marcus *et al,* 1967) that:

$2 \max c_{ij} \leq s(\mathbf{L'L}) < [2||\mathbf{L'L}||^2 - 2/n\ (tr\ \mathbf{L'L})^2]^{1/2}$

in which by c_{ij} , $i \neq j$ we mean the off-main diagonal elements of **L'L** and in which the norm is Euclidean, that is, with any **N,** $||\mathbf{N}|| = (\sum n_{ij}^2)^{1/2}$.

It is easy to see that $tr\ \mathbf{L'L} = ||\mathbf{L}||^2$.

Besides, by the general norm and Euclidean norm properties:

$||\mathbf{L'L}|| \leq ||\mathbf{L}||.||\mathbf{L'}|| = ||\mathbf{L}||^2$,

so that,

$2 \max c_{ij} \leq s(\mathbf{L'L}) < (2-2/n)^{1/2}\ ||\mathbf{L}||^2 \approx \sqrt{2}\ ||\mathbf{L}||^2$.

This shows the importance for this analysis of the maximum value of off-main diagonal values of **L'L** and of the summation of the square elements of **L**.

An increase in the value of **L** elements (that is, of the elements of **A**), leads necessarily to an increase in the elements of **L'L,** since **L** is a matrix of positive elements. This implies, if the increase is sufficiently intense, an increase in the amplitude of the possible output variations in response to a unitary final demand shock.

With a more “full” technological structure, the final demand management is more important than with a less “full” one. This property is a potentially useful measure of the economic complexity of an economy, alternative to that proposed in Amaral *et al* (2006).

As an example, consider the case of an economy with just two sectors and where, for simplifying purposes, there are only identical inputs:

$$A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

Table 2.1 summarizes the possible values for *a* and *b,* and the corresponding values for the *spread,* where it is clear that this increases when the values of a and b grow.

Homothetic-scale and structure effects

As we saw previously, there are be two vectors of final demand variations that result in maximum output movement, the vector with all the final demand components increasing and the other symmetric to this. If we are interested in the increasing output vector, we will consider the vector **Δy*** with all the components positive. The corresponding output vector, **Δx*,** is **LΔy*,** and this variation can be decomposed in two components: a scale effect and a structure effect.

Without structural changes we would have a proportional increase in all sectors,

$$\Delta\mathbf{x}^* = \delta_0 \mathbf{x}$$

However, in general, we don't have this proportional change. On the contrary, $\Delta\mathbf{x}^*$ is a result of the combination of economic expansion according to the existing structure and economic development given by structural changes in the economy. That is,

$$\Delta\mathbf{x}^* = SC + ST ,$$

where *SC* and *ST* are *scale vector* and *structural change vector*. For the scale effect, we have:

$$\delta = \min\left\{\frac{\Delta x_1^*}{x_1}, \frac{\Delta x_2^*}{x_2}, \cdots, \frac{\Delta x_n^*}{x_n}\right\}$$

$$SC = \delta\mathbf{x} ,$$

so that *ST* is obtained[1] by

$$ST = \Delta\mathbf{x}^* - SC$$

We have now the result,

$$\left\|\Delta\mathbf{x}^*\right\|_2^2 = \left\|SC\right\|_2^2 + \left\|ST\right\|_2^2 + 2\left|SC\right|\left|ST\right| \cos(SC, ST)$$

In the empirical application, we present the values for the length of $\Delta\mathbf{x}^*$, *SC* and *ST*, in order to compare the measure of the effects in scale and in structural change with the overall effect.

3. AN APPLICATION TO A REGIONAL AND NATIONAL IO TABLE

[1] An identical decomposition can be made for the "optimal" impulse vector of final demand, $\Delta\mathbf{y}^*$

In this section we make an application of the results presented in the previous one to Portugal (national level) and one of its regions, Azores. This is a small regional economy, with about 1-2% of GDP and 3% of the population of Portugal.

The Azorean economy is little diversified and is characterized by a predominance of services and also a significant weight of the agriculture sector (table 3.1). The only IO table available for Azores is for the year 1998 and it mirrors this reality of the regional economy. It is built for 15 sectors. Industry is aggregated in a single sector. In order to allow comparability of results, the national IO table was aggregated in the same 15 sectors (see Appendix).

Table 3.2 summarizes some results for the national and regional economy. The maximum effect is stronger for the national matrix, while the minimum distance is similar. In other words, the national economy has a larger capacity of reaction concerning a shock of unitary distance to the final demand. As a consequence, the spread for the national economy is substantially higher to that of Azores. In both cases, the effect of structural change is much more important than the scale effect, particularly in the case of the regional economy, where almost all the global effect is originated by this component. Of course, this is in accordance with the characteristics already pointed out for the Azorean economy and the importance that, in this context, structural changes represents for the insular economies, sometimes characterised by important restrictions at the level of productive structures.

According to these "optimal" structural change effects, how do the observed production structure diverge from this "optimum"? The following figures show the different impact on different sectors, where the mining and quarrying for Portugal and electricity, gas and water for Azores stand out.

A way of obtaining a measure for the degree of similarity, or of divergence, between the new vectors and the observed structures is through the computation of the cosine of the angle formed by the two vectors. A value close to unity means overlapping or, in other words, expansion according to the same pattern (just growth in scale), while a value close to zero means that the two vectors are orthogonal or, in other words, expansion in agreement with a quite different structure.

For the case of Portugal, the value obtained was 0.72, while for Azores it was only 0.34, meaning that, in the case of Azores, the maximum impact is obtained with a production vector very different from the existing structure, as we had already seen previously.

4. CONCLUSIONS

In this paper we present a new kind of intersectoral output multipliers that can be used to overcome a serious limitation of traditional Leontief/Rasmussen multipliers, namely the obligation to consider a fixed (predetermined) structure of final demand.

Solving a properly designed optimization problem, one can calculate the impact on sectoral outputs of a shock in final demand along all vectors at a certain distance from the initial final demand vector can be calculated.

Along the spectrum of all possible new final demand vectors, a particular one plays an important role for economic policy: the vector maximizing output growth if the objective is to promote employment; the vector minimizing output growth if the objective is to control inflation or, for example, minimize CO2 emissions.

An important property of productive structures is the so-called *spread* of technological matrix, the difference between the maximizing and the minimizing impacts.

In the maximizing case, an interesting exercise consists in decomposing the total impact in two effects: a homothetical scale effect, when the economy grows along the initial structure; a structure effect, given by the change in structure caused by the maximizing purpose at hand.

An empirical exercise is made in the paper, using Portuguese national and regional (Azores) IO tables.

FOOTNOTES

1. For a good exposition of this model, see, for instance, Miller and Blair(1985). The meaning of x, y and L is the conventional one (output and final demand vectors and the Leontief inverse, respectively).

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APPENDIX: Sectors used in subsection 3

1	Agriculture, hunting and forestry
2	Fishing and fish products
3	Mining and quarrying
4	Manufacturing
5	Electricity, gas and water
6	Construction
7	Trade and repairing automobile
8	Hotels, restaurants
9	Transports and communications
10	Financial services
11	Real estate services, renting
12	Public administration, defence and social security
13	Education services
14	Health and Social Services
15	Other services

TABLES

Table 2.1: *Spread* of matrix A for different values of *a* and *b*

		b									
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
a	0	0	0.41	0.87	1.45	2.27	3.56	5.86	10.77	24.69	99.72
	0.1	0	0.56	1.21	2.08	3.41	5.74	10.67	24.61	99.65	
	0.2	0	0.81	1.78	3.17	5.56	10.52	24.49	99.56		
	0.3	0	1.22	2.77	5.25	10.28	24.31	99.41			
	0.4	0	1.96	4.69	9.88	24.00	99.17				
	0.5	0	3.47	9.07	23.44	98.77					
	0.6	0	7.11	22.22	97.96						
	0.7	0	18.75	96.00							
	0.8	0	88.89								
	0.9	0									

Table 3.1 Production structure: Portugal and Azores

	Portugal 1999	Azores 1998
Primary	3.4	10.1
Secondary	29.1	14.9
Construction	10.8	10.2
Services	56.7	64.8

Table 3.2: *Distance* multipliers: Portugal and Azores

	Portugal 1999	Azores 1998
λmax	3.67	2.60
λmin	0.80	0.88
Spread (**L'L**)	2.87	1.72
Total effect	1.84	1.61
Scale effect	0.47	0.09
Structural change effect	1.40	1.58

FIGURES

Figure 3.1 - Azores

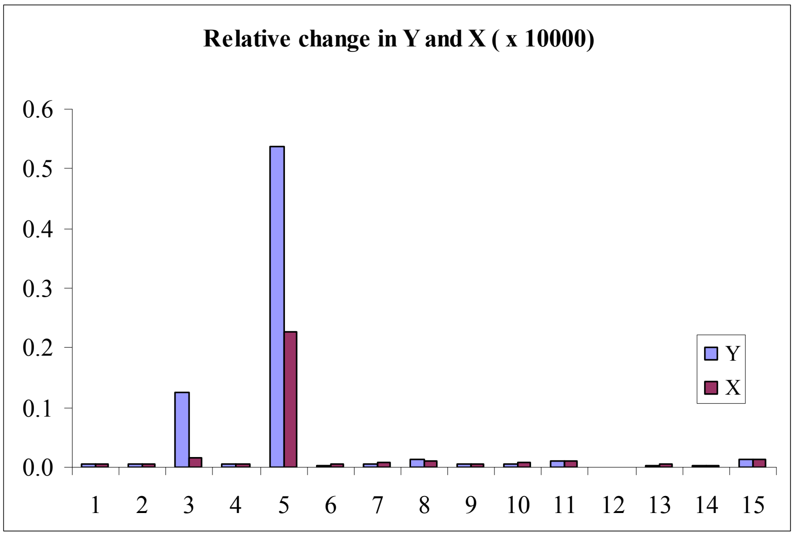

Figure 3.2 - Portugal