Estimating prices and excess demand and trade costs in a spatial price equilibrium model[1]

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Abstract

This article treats the estimation of regional prices, excess demand and trade costs, for homogeneous products in a spatial price equilibrium model. The estimation is formulated as a bilevel program, with the upper level objective to minimize the weighted sum of squared deviations of estimated from observed values of prices and excess demand. The estimation is restricted to optimal solutions of the transport cost minimization problem, parametrized by a trade cost function, the parameters of which are also to be determined. The estimation is applied to data for Benin, and the results compared to those of empirical studies.

Keywords: spatial price equilibrium, bilevel programming, trade costs
JEL classification: C13, F11

1 Introduction

Spatial price equilibrium (SPE) models with homogeneous goods have been used in agricultural sector analysis at least since the publications of Judge and Wallace (1958) and Takayama and Judge (1964). This article is concerned with an SPE model for homogeneous primary agricultural products in Benin. The model has twelve regions (administrative departments) and seven goods. For each region there is data on annual supply, demand and price, and there is also a table of distances between each pair of regions. To indicate the generality of the employed method, the trade cost minimization component of the model for each product k is put as a linear program in standard form,

$$\begin{aligned} \min \quad & c_k x_k \\ \text{subject to} \quad & A x_k \geq q_k \\ & x_k \geq 0 \end{aligned} \tag{1}$$

where c_k is a $1 \times n(n\text{-}1)$ vector of trade costs, x_k an $n(n\text{-}1) \times 1$ vector of trade flows, q_k is an $n \times 1$ vector of excess demand and A is an $n \times n(n\text{-}1)$ matrix of "0", "1" and "-1" arranged in such a way that for the i^{th} row, there is a "1" in all col-

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umns corresponding to flows into region *i*, a "-1" in all columns corresponding to flows out of region *i* and "0" elsewhere.

If a model is to be used in a positive way, it is desirable that it is capable of reproducing real world behaviour ex-post, and consequently we would like to interpret real world observations as model solutions. This requires that the ex-post data satisfies the Kuhn-Tucker (KT) conditions for an optimal solution to the transportation problem. For several reasons, the KT conditions are likely to be violated by ex-post data: The good may not be quite homogeneous; errors may arise when observations are aggregated over time and space, there can be measurement errors involved etc. Given such errors, some calibration procedure is required in order to fit the model to the ex-post data.

Traditionally, the calibration of SPE models has been handled by solving the transportation model with observed or engineered trade costs, subject to market clearing constraints for given regional excess demand quantities, and using the Lagrange multiplicators associated with the market clearing constraints to determine the regional prices (e.g. Judge and Wallace (1958), Litzenberg, McCarl and Polito (1982), Peeters (1990), Kawaguchi, Suzuki and Kaiser (1997) and Guajardo and Elizondo (2003)). This implies that any disturbances of observed trade costs and excess demand are accepted, and that all corrections needed to satisfy the KT conditions are undertaken on the price positions, for which only a single observation is used (the numerator price).

Whereas this certainly may be a plausible way of proceeding in some instances, it is equally easy to imagine situations where there are observations of regional prices available and the observations of trade costs and excess demand are associated with errors. Then the traditional procedure described above is inefficient, because the price observations are ignored. It is also unable of identifying autarky regions; an observed nonzero regional excess demand, however tiny, enforces a fixed price difference (equal trade cost) to some other region.

A general approach to this type of estimation problem is to recognize that *given* a set of trade costs and regional excess demand, trade flows and regional price differences result from solving of the transportation model. Thus, the estimation problem at hand is to select the parameters so that they, together with the solution variables of the transportation problem, minimize some estimation criterion. Viewed that way, the problem falls within the class of *bilevel programs*, prominently exemplified by the Stackelberg game. In terms of a principal-agent problem, the leader is the person conducting the estimation, the leader's cost function is the estimation criterion, his decision variables the parameters of the transportation problem. The follower's problem is the transportation model with parameters given by the leader.

Furthermore, the situation at hand is sometimes (e.g. in Dempe, 1997) described as the optimistic or weak approach. In economic terms that would mean that if the follower is indifferent regarding two solutions, he chooses the one preferred by the leader. In mathematical terms it means that if the solution of the inner problem is not a singleton, the leader is allowed to choose that value from the *set* of solutions of the inner problem that minimizes the estimation criterion. This property simplifies the solution of the bilevel estimation problem compared to the general bilevel program, where the weak approach cannot be assumed *a priori*. In the case at hand, it has special implications for the prices, which are not fully identified in a solution to the transportation problem; only price differences are. The weak approach justifies that we, among all sets of prices satisfying those price differences, choose those that are closest to the observed prices.

2 A bilevel estimation program

The mathematical representation of the bilevel programming problem in this application departs from a representation of the transportation problem by its first order conditions, with a weighted least squares objective function penalizing deviations from observations of prices and excess demand. The first order conditions here are cast as a *linear complementarity problem* (LCP), thus formulating the estimation problem explicitly as a *mathematical program with equilibrium constraints*—the branch of literature from which the solution method is borrowed.

$$\min_{q,p,c,x,\beta} \sum_k \left(w_k^p \left(p_k - p_k^o\right)' \left(p_k - p_k^o\right) + w_k^q \left(q_k - q_k^o\right)' \left(q_k - q_k^o\right) \right) \tag{2}$$

$$\begin{bmatrix} u_k \\ v_k \end{bmatrix} - \begin{bmatrix} 0 & -A' \\ A & 0 \end{bmatrix} \begin{bmatrix} x_k \\ p_k \end{bmatrix} = \begin{bmatrix} c_k' \\ -q_k \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} u_k \\ v_k \end{bmatrix} \geq 0, \begin{bmatrix} x_k \\ p_k \end{bmatrix} \geq 0 \tag{4}$$

$$\begin{bmatrix} u_k \\ v_k \end{bmatrix}' \begin{bmatrix} x_k \\ p_k \end{bmatrix} = 0 \tag{5}$$

$$c_k' = \beta_1 \delta^{\beta_2} \tag{6}$$

The objective function (2) minimizes the weighted sum of squared deviations of estimated prices and excess demand from observations. If observations of trade flows and costs were available, those could be similarly included into the objective. Equations 3-6 form an LCP that is equivalent to the Kuhn-Tucker conditions for the LP (1), with p_k the dual vector of the constraints in the LP, and u_k and v_k slack vectors. Equation 6 is a function relating the trade cost between any two regions to the distance δ between them, parametrized by β_1 and β_2.

Trade costs are expressed per weight unit, and in order to economize on degrees of freedom, the trade costs per weight unit were assumed to be equal for all products. This would be reasonable if all products were equally perishable and with similar prices, which is not perfectly true for the set of products at hand: cassava, yams, maize, rice, sorghum, beans and peanuts. On the other hand, IFPRI (2004) does not find that traders in Benin discriminate between different agricultural products when setting transportation rates, supporting the use of a single trade cost function for all products.

3 Solution method

An optimization problem constrained by an LCP falls in the class of *mathematical programs with equilibrium constraints* (MPEC), that started to attract attention in the literature in the 1990's, evidenced by the publication of two books on the subject (Luo, Pang and Ralph (1996), Outrata, Kocvara and Zowe (1998)). The solver NLPEC for GAMS (see NLPEC solver manual) solves MPECs via smooth reformulation of the complementarity constraints. The method used in this paper is one of the reformulations implemented in NLPEC:

The complementary slackness constraint (5) is the equation causing trouble when attempting to solve the problem (2-6), because it makes the feasible space non-convex and has "corner". The key idea of the smooth reformulations is to replace (5) by a sequence of increasingly accurate approximations. Several such reformulations are available, and after extensive testing with synthetic data, a method where a penalty function minimizes the *complementarity gap* was chosen.

Before proceeding, we note that data is unlikely to support solutions with zero price for any product. Thus, the slack vectors v_k can be fixed to zero, reducing the problem somewhat. Then the remaining complementary slackness condition $u_k'x_k = 0$ is removed, and instead a penalty term $\mu(u_k'x_k)$ is added to the objective function with μ a nonnegative real number. The resulting system is solved repeatedly, with μ initially set to a small number and then stepwise increased, each time using the previous solution as starting point, until the complementarity gap $u_k'x_k$ is zero. The estimation problem then is:

$$\min_{q,p,c,x,\beta} \sum_k \left(w_k^p \left(p_k - p_k^o\right)' \left(p_k - p_k^o\right) + w_k^q \left(q_k - q_k^o\right)' \left(q_k - q_k^o\right) + \mu\left(u_k' x_k\right) \right) \quad (7)$$

$$\begin{bmatrix} u_k \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & -A' \\ A & 0 \end{bmatrix} \begin{bmatrix} x_k \\ p_k \end{bmatrix} = \begin{bmatrix} c_k' \\ -q_k \end{bmatrix} \quad (8)$$

$$u_k \geq 0, x_k \geq 0 \quad (9)$$

$$c_k' = \beta_1 \delta^{\beta_2} \quad (10)$$

with w_k^p and w_k^q weights to be defined below. Note that when $\mu \to \infty$, the original problem is recovered. Testing with different sequences of μ and different synthetic data constellations revealed that this method is *not guaranteed* to find the (existing, see e.g. Dempe, 1997) global minimum for the problem size at hand. However, of the methods tested it performed on average and in median best, measured by the sum of squared deviations obtained, on similarly structured problems.

In an attempt to verify that the iterative approximation method finds the unique global minimum, or at least a point close to it, for the incumbent data, the problem was also reformulated as a mixed integer programming problem, with binary variables in a so-called "big M" construct switching the complementary slackness conditions. To reduce the size of the problem, only one product (maize) was included, and the problem initialised with the solution obtained by the iterative approximation described above. The so obtained problem in 132 binary variables was solved with a branch-and-bound algorithm (the solver SBB in GAMS on the NEOS server). The solver terminated after 38 minutes and 1.6 million iterations without any significant improvement of the objective, though still with a possible gap (between best found and best possible) of round 10% of the objective function value. For the entire problem (around 900 binary variables) the solver terminated due to limited system resources (memory). As a comparison, the iterative smooth approximation solution of the entire problem solves in about 20 seconds on a standard workstation. So even if a better solution may exist, it is difficult to find.

4 Assigning weights

The objective function of the problem (2-6) literally compares apples to pears. It actually does more than that, because it also weighs an error in the *price* of one commodity to the error in *quantity* of another. In order to make the estimator more efficient, the error terms need to be weighed by the inverse of their variances, which in this case are unknown.

In other circumstances, one approach would be to estimate the variances simultaneous with the parameters, either using maximum likelihood or by iteratively computing the sample variances from the residuals of previous estimation steps. To this end, one could assume that prices and quantities of each commodity constitute two homoscedastic groups with variances σ_p^2 and σ_q^2 (or some more complex matrix function of those variances). Endogenously determined σ_p^2 and σ_q^2, would likely result in one group having variance close to zero and the other a very high variance. The reason for this is that either observed prices or quantities always can be matched perfectly by the estimates in this model. If the ratio of the variances σ_p^2/σ_q^2 is shifted towards zero, prices will be matched perfectly and the objective value be depending only on the inverse of σ_q^2, and vice versa.

Thus, some external source or assumption must be used to assign weights. In this analysis, we *assumed* that variances are proportional to the absolute size of the variables at hand. More specifically we assumed that the variances of prices are proportional to the observed price and to the inverse market share of the current region. The variances of excess demand were assumed to be proportional to the sum of regional supply and demand, the sum being motivated by the fact that the variance of a difference is the sum of the variances. The weights, shown in tables 4 for prices and 5 for excess demand, were computed as

$$w_{ik}^{p} = \frac{1}{p_{ik}} \frac{nd_{ik}}{\sum_{j} d_{jk}}, \quad w_{ik}^{q} = \frac{1}{d_{ik} + s_{ik}}$$

5 Data

The data used in the estimation is the data that is available in the BenImpact model data base for 2001. Regional demand stems from the Benin statistic agency ONASA (several publications), as do regional prices. Regional supply is based on ONASA-data on yields and acreage on the level of the administrative units sub-prefectures, of which there are 77 in Benin. Both yields and levels, however, fluctuate strongly between adjacent time periods as well as regions, so the data was scaled by fitting it to yield and acreage trends estimated for “agri-ecological zones” (AEZ) based on survey data from van den Akker. AEZ are eight agronomically homogeneous but spatially discontinuous geographical units. Data on prices and excess demand is shown in tables 1 and 2.

The model and the estimations run at the level of departments, which are administrative units that are aggregates of sub-prefectures and of which there are twelve in the model. Distances between departments have been computed using a table of line-of-sight-distances between the principal market places in each department. In praxis, sometimes different market places are important for different products, so that the selection of principal market places had to be a compromise if not one unique distance matrix was to be used for each product. The distance matrix used is shown in table 3.

6 Results of other studies

There are some other sources of trade cost estimates available for Benin. One such source is IFPRI 2004, performing a survey of traders in Benin. They find that on distances of 160 km, large trucks are used, and that motorized transport on average costs 0.28 USD/ton/km. Converted to FCFA using an exchange rate of 700FCFA/USD this corresponds to 31.36 FCFA per kg for 160 km. It is not clear to the author of this article if those rates also contain other mark-ups than transportation costs.

Van den Akker kindly supplied her survey results to the BenImpact team. She comes up with transport cost estimates for maize that, when fitted to the trade cost function used in this article, correspond to a distance elasticity of transport costs of 0.37, 0.71 and 0.41 for southern, central and northern Benin respectively (own computations). For a typical truck operated distance of 160 km, this amounts to transportation costs of 9.60, 17.92 and 10.51 FCFA per kg. These numbers are supposed to contain only transportation costs and not other costs connected to trade, whence we expect our estimated trade costs to be somewhat higher. For maize, van den Akker finds that marketing costs and profit each amount to approximately as much as the transportation costs.

Finally, there are estimates of distance elasticities of trade costs from other studies, prominently in the gravity literature. Hummels (1999) estimates a trade cost function similar to ours but with ad-valorem trade costs and finds a distance elasticity of 0.27 (all products), and commodity specific elasticities "tightly clustered in the 0.2 to 0.3 range" (ibid p.11).

The main reason for expecting a concave trade cost function with a distance elasticity of less than unity is that trade takes place with a multitude of means, ranging from transportation by foot over bicycles, motorcycles, modified ordinary automobiles, small trucks up to large trucks (IFPRI 2004), all with different fix charges and costs per km. If always the cheapest available means of transportation were used for a given haul, this would result in concave trade costs as illustrated in figure 1, where the heavy grey line shows the graph of a trade cost function as of equation 6. However, most distances in the model are relatively long and therefore could be operated by a more homogeneous class of transportation means, allowing the function to be closer to linear. Having all this in mind, we would expect our estimated trade cost function to be such that the elasticity is between 0.2 and 0.9, and the function value for 160 km to be around 30 FCFA per kg.

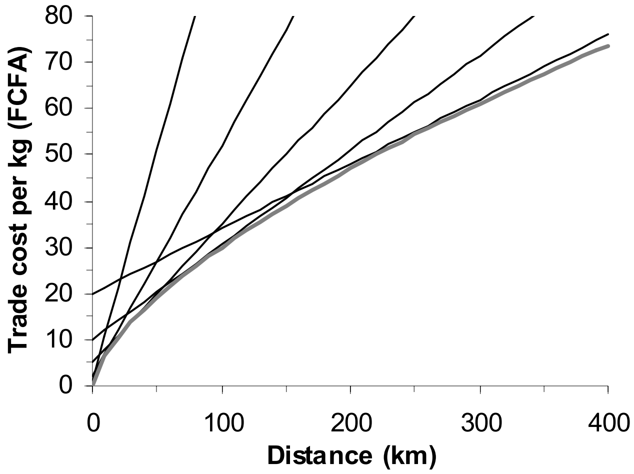

Figure 1: Degression of trade costs resulting from a heterogeneous class of means of transportation.

7 Results of estimation

The quality of the estimates were evaluated using the R-squared measure, computed for each product separately according to the formula

$$R^2 = \frac{SSR}{SST} \text{ with } SST = \sum_r (\hat{p}_r - \bar{p}_r)^2 \text{ , } SSE = \sum_r (\hat{p}_r - p_r)^2$$

and $SSR = SST - SSE$, and the same for excess demand. A bar denotes the sample mean and a hat denotes the estimated value. Tables 6 and 7 show the estimated prices and excess demand, table 8 the computed R-squared values. The R-squares for the three products cassava, maize and rice the R-squares for prices turn out negative, indicating that the data only partially support the assumed model. Indeed, a look at the data reveals that for those products, excess demand is sometimes positive where the price is low and vice versa (see discussion below).

The estimated parameters of the trade cost function turn out $(\beta_1,\beta_2) = (0.147,1.000)$ corresponding to a trade cost at 160 km of 23.5 FCFA/kg. The cost for 160 km is of plausible magnitude, judged by the results of van den Akker and IFPRI. However, the distance elasticity β_2 of unity, implying a linear trade cost function, appears high compared to the considerations expressed in the previous section "Results of other studies".

In an attempt to obtain better estimates, an additional estimation was performed, using only products for which R-squared was positive, thus not contradicting the assumed model. That meant omitting cassava, maize and rice. That results in parameter estimates of $(\beta_1,\beta_2) = (1.558,0.643)$, corresponding to a trade cost for 160 km of 40.7 FCFA per kg, the estimated prices and excess demand shown in tables 9 and 10, and the measures of determination shown in table 11. Those estimates, as far as the trade cost function is concerned, is more in line with other results, albeit a result of manual selection of data. –Probably, similar estimates could be produced by randomly generated price and excess demand data if the 40% share of observations least supporting the model were discarded.

8 Discussion

The estimation results were not stable when the composition of the sample was modified, as demonstrated by the rather different results obtained when using a subset of the products. This signals that there are problems with the model specification, with the data or both. The list of potential specification errors is long:

(i) Lack of temporal disaggregation. In the tropical country of Benin, there are two production seasons, with somewhat different time windows in the south and in the north. Thus, production, demand and trade takes place within shorter time frames and may even reverse within a year. Van den Akker uses four time periods. This setup was tried but discarded, as it in addition to temporary disaggregated data requires the estimation of a storage function and urges considerations of uncertainty.

(ii) Products may not be homogeneous. For some products there are local as well as commercial varieties, which may sell at different prices.

(iii) Great circle distances neither reflect the state of maintenance and other qualities of the road network nor actual distances of road to travel.

(iv) Congestion effects are not considered. According to IFPRI, congestion in the transportation system sometimes occur during cotton harvest.

(vi) External trade, occurring via the harbour in Cotonou or across the borders to neighbour countries, is not considered due to lack of data on prices and quantities. The only net trade allowed is by a constant, derived from the sample, and attributed entirely to Cotonou.

The data problems for a country suffering under deficits in all kinds of infrastructure and low literacy (33.6% according to CIA 2006) are obvious. Official statistics frequently appear to be more "guesstimates" than the results of actual measurements, and utterly sparse. The data used in the estimations has gone through a gap-filling process already before entering the estimation, and is still not complete (one price is missing for yams). For some products and regions there are obvious problems. As an example, the cassava price in the department Couffo is the lowest of all regions even though Couffo has the second to largest excess demand, clearly contradicting the assumption of spatial arbitraging. Indeed, the coefficient of correlation between prices and excess demand is negative (-0.138) for cassava, though positive for all other products. However, trade does occur and is not likely to take place at a loss, so this is more likely to be a data than a specification problem.

To conclude, the estimation is connected with some severe difficulties. Not only is the available data barely supporting the assumed model, but furthermore, the bilevel program is difficult to solve due to non-convexities. However, the method is workable, delivering reasonable estimates compared to expert knowledge and other trade cost studies. Furthermore, the given model, it is difficult to see how the available data could be used more efficiently. The estimation uses all available information, and, by the proper use of weights, attaches more confidence to items that for some reason are believed to be more certain (have less variance).

The bilevel programming approach to the estimation of constrained programming models can, as noted above, be extended to include observations also of trade flows and trade costs in a similar manner, as well as a time series of observations. One could also attempt similar (bilevel) techniques to estimate parameters of general LP models, or, with additional second order conditions to NLP models, and could thus be of interest to a wider range of modellers as an alternative to separate estimations or calibration methods.

9 Tables

Abbreviations for regions (Littoral = Cotonou)

ALI	Alibori	COL	Collines	MON	Mono
ATA	Atacora	COU	Couffo	OUE	Oueme
ATL	Atlantique	DON	Donga	PLA	Plateau
BOR	Borgou	LIT	Littoral	ZOU	Zou

Abbreviations for products:

CASS	Cassava	PULS	Beans	RICE	Rice
SORM	Sorghum and millet	MAIZ	Maize	YAMS	Yams
		PEAN	Groundnut		

Other abbreviations

n.a. = not available, P = price, Q = excess demand

Table 1: Price observations for regions and products

	CASS	SORM	PULS	MAIZ	PEAN	RICE	YAMS
ALI	172.42	107.06	219.74	88.52	250.45	266.31	73.93
ATA	171.70	113.21	182.53	96.45	225.54	266.36	72.83
ATL	156.99	186.65	269.81	104.10	286.02	277.70	136.28
BOR	182.98	112.39	231.28	91.68	236.80	291.55	59.56
COL	153.40	111.35	219.01	86.83	233.85	246.89	79.29
COU	117.67	163.74	224.43	85.91	197.73	273.59	119.74
DON	166.81	106.76	201.69	87.70	175.50	235.24	n.a.
LIT	159.55	205.29	288.96	127.37	364.50	288.62	138.55
MON	165.71	204.19	253.03	110.65	294.84	264.46	127.13
OUE	142.66	191.01	285.67	92.06	324.33	248.11	126.91
PLA	156.02	168.16	292.00	94.35	264.97	248.43	96.18
ZOU	149.85	132.42	219.66	101.41	220.39	237.59	118.71

Source: BenImpact database for 2001, based on data from ONASA.

Table 2: Regional excess demand observations

	CASS	SORM	PULS	MAIZ	PEAN	RICE	YAMS
ALI	60.76	-20.45	4.94	-17.54	-12.88	5.18	91.95
ATA	-20.77	-4.45	0.85	-7.10	1.65	5.04	-13.77
ATL	-102.96	0.08	2.10	23.07	1.97	17.04	7.43
BOR	-19.50	11.51	6.68	-45.59	0.13	9.99	-242.02
COL	-134.79	-2.04	-4.84	11.41	-11.84	2.25	-129.94
COU	82.68	6.86	-3.87	-6.08	-7.95	10.31	108.25
DON	-7.14	3.93	3.10	-1.85	4.95	6.29	57.79
LIT	103.32	0.06	2.03	29.46	5.22	37.31	6.51
MON	31.75	7.62	1.58	-12.44	1.68	6.36	68.67
OUE	42.39	21.64	4.81	48.30	10.63	14.26	4.56
PLA	-229.93	15.21	0.08	-37.34	1.55	7.88	-8.64
ZOU	14.57	3.20	-0.71	29.08	-14.27	11.27	-21.41

Source: BenImpact database for 2001, based on data from ONASA and van den Akker.

Table 3: Distance matrix

	ALI	ATA	ATL	BOR	COL	COU	DON	LIT	MON	OUE	PLA	ZOU
ALI		300	574	213	425	592	347	628	631	658	570	502
ATA			485	212	336	503	78	539	542	569	481	413
ATL				361	149	102	407	54	63	84	140	72
BOR					212	379	134	415	418	445	357	289
COL						167	258	203	206	233	145	77
COU							425	106	39	136	158	90
DON								461	464	491	403	335
LIT									67	30	138	126
MON										97	197	129
OUE											108	156
PLA												68

Source: Own computations using a map of Benin.

Table 4: Weights for price disturbances

	CASS	SORM	PULS	MAIZ	PEAN	RICE	YAMS
ALI	0.446	1.155	0.769	0.554	0.29	0.258	2.834
ATA	0.136	1.966	0.628	0.171	0.744	0.267	3.634
ATL	1.293	0.003	0.231	2.244	0.268	0.474	0.079
BOR	0.373	2.644	1.031	0.522	0.431	0.328	3.832
COL	0.589	0.215	0.56	1.497	0.383	0.335	0.065
COU	1.449	0.325	0.211	0.611	0.271	0.299	1.335
DON	0.059	0.813	0.359	0.118	0.818	0.227	15.644
LIT	0.924	0.002	0.129	0.683	0.223	0.998	0.067
MON	0.631	0.284	0.129	0.451	0.116	0.186	0.815
OUE	0.966	0.986	0.377	2.527	0.588	0.517	0.078
PLA	0.437	0.681	0.207	1.511	0.344	0.245	0.056
ZOU	0.666	0.232	0.614	1.463	0.441	0.373	0.048

Source: Own computations.

Table 5: Weights for excess demand

	CASS	SORM	PULS	MAIZ	PEAN	RICE	YAMS
ALI	1.161	1.877	7.438	1.97	4.476	7.898	0.5
ATA	1.533	1.574	8.602	5.481	4.959	7.452	0.261
ATL	0.204	1230.172	21.374	0.74	12.518	5.863	13.186
BOR	0.668	1.484	5.19	1.282	7.614	6.748	0.179
COL	0.325	11.913	5.505	1.305	4.259	5.211	0.729
COU	0.411	13.756	11.086	2.404	6.704	9.198	0.874
DON	3.837	5.235	20.935	11.328	7.289	13.243	0.491
LIT	0.56	1399.573	49.265	3.394	18.692	2.68	15.357
MON	0.595	12.846	50.817	2.164	36.331	15.597	1.32
OUE	0.452	3.528	14.49	0.916	7.048	5.275	10.906
PLA	0.278	6.575	15.385	0.747	9.709	12.695	6.174
ZOU	0.567	20.176	6.503	1.401	3.718	8.555	3.402

Source: Own computations.

Table 6: Price estimates with all products included

	CASS	SORM	PULS	MAIZ	PEAN	RICE	YAMS
ALI	189.21	88.06	219.74	87.42	250.45	266.31	87.24
ATA	145.18	113.21	192.91	87.27	218.92	262.73	72.83
ATL	148.70	172.30	256.64	109.14	267.20	263.46	108.96
BOR	157.95	119.32	224.03	56.16	227.14	293.84	55.99
COL	126.84	150.43	234.77	87.27	245.33	262.73	87.10
COU	151.34	174.94	241.67	94.17	259.57	271.09	111.61
DON	138.28	112.57	204.36	75.83	207.47	274.17	75.65
LIT	156.63	170.83	248.71	109.73	275.13	255.54	116.89
MON	157.07	180.66	247.39	99.89	265.29	265.37	117.33
OUE	152.22	175.23	253.12	105.32	279.53	259.94	112.49
PLA	136.37	171.71	256.05	89.47	266.61	275.79	96.64
ZOU	138.14	161.73	246.07	98.57	256.63	274.03	98.40

Source: Own estimation.

Table 7: Excess demand estimates with all products included (empty = autarky)

	CASS	SORM	PULS	MAIZ	PEAN	RICE	YAMS
ALI	60.22	-25.34					89.93
ATA	-21.17		-2.62			-2.21	
ATL	-99.99	0.07	4.79	18.69	1.50	21.49	7.35
BOR	-20.42	5.33	0.94	-48.13		4.66	-247.69
COL	-136.70	-2.81	-2.36	8.92	-13.21	-4.66	-131.33
COU	81.18	6.19	-1.99	-7.43	-8.81	13.15	107.09
DON	-7.30		1.68			2.21	55.73
LIT	104.40	0.09	3.20	28.51	4.91	47.04	6.45
MON	30.71	10.59	1.99	-13.94	1.52	8.03	67.90
OUE	43.73	32.48	8.77	43.38	9.80	19.21	4.46
PLA	-227.74	13.81	0.96	-43.38	0.95	9.93	-8.80
ZOU	13.48	2.74	1.39	26.76	-15.83	14.32	-21.71

Source: Own estimation.

Table 8: Measures of determination with all products included

		mean	sse	sst	ssr	R2
CASS	P	157.98	5031	3113	-1918	-0.62
CASS	Q	-14.97	26	104029	104003	1.00
SORM	P	150.18	5163	17822	12658	0.71
SORM	Q	3.60	227	1250	1023	0.82
PULS	P	240.65	5588	14594	9006	0.62
PULS	Q	1.40	111	129	18	0.14
MAIZ	P	97.25	2217	1623	-594	-0.37
MAIZ	Q	1.12	464	8721	8256	0.95
PEAN	P	256.24	17654	31893	14238	0.45
PEAN	Q	-1.60	200	719	519	0.72
RICE	P	262.07	5306	3924	-1382	-0.35
RICE	Q	11.10	336	940	605	0.64
YAMS	P	104.46	2249	8280	6030	0.73
YAMS	Q	-5.89	234	104109	103875	1.00

Source: Own computations.

Table 9: Price estimates with product subset

	CASS	SORM	PULS	MAIZ	PEAN	RICE	YAMS
ALI	n.a.	90.20	219.74	n.a.	250.45	n.a.	95.93
ATA	n.a.	113.21	182.56	n.a.	225.54	n.a.	72.83
ATL	n.a.	182.73	265.60	n.a.	270.65	n.a.	112.77
BOR	n.a.	114.13	231.34	n.a.	236.80	n.a.	47.01
COL	n.a.	144.11	226.72	n.a.	233.77	n.a.	74.69
COU	n.a.	184.59	235.12	n.a.	249.96	n.a.	116.53
DON	n.a.	108.61	208.21	n.a.	201.90	n.a.	83.32
LIT	n.a.	165.28	245.35	n.a.	281.20	n.a.	122.12
MON	n.a.	188.54	251.55	n.a.	266.39	n.a.	122.47
OUE	n.a.	179.16	259.23	n.a.	285.59	n.a.	116.73
PLA	n.a.	182.32	264.92	n.a.	269.77	n.a.	85.11
ZOU	n.a.	158.84	241.44	n.a.	246.29	n.a.	88.41

Source: Own estimation.

Table 10: Excess demand estimates with product subset (empty = autarky)

	CASS	SORM	PULS	MAIZ	PEAN	RICE	YAMS
ALI	n.a.	-20.24		n.a.		n.a.	89.93
ATA	n.a.		-2.62	n.a.		n.a.	
ATL	n.a.	0.08	4.79	n.a.	1.50	n.a.	7.35
BOR	n.a.		0.94	n.a.		n.a.	-247.69
COL	n.a.	-2.00	-1.33	n.a.	-13.21	n.a.	-131.33
COU	n.a.	6.89	-1.99	n.a.	-8.81	n.a.	107.09
DON	n.a.		1.68	n.a.		n.a.	55.73
LIT	n.a.	0.09	3.20	n.a.	4.91	n.a.	6.45
MON	n.a.	10.59	1.99	n.a.	1.52	n.a.	67.90
OUE	n.a.	32.48	8.77	n.a.	9.80	n.a.	4.46
PLA	n.a.	15.27	1.33	n.a.	0.95	n.a.	-8.80
ZOU	n.a.			n.a.	-15.83	n.a.	-21.71

Source: Own estimation.

Table 11: Measure of determination with product subset

		mean	sse	sst	ssr	R2
SORM	P	150.18	4698	17822	13123	0.74
SORM	Q	3.60	304	1250	945	0.76
PULS	P	240.65	4045	14594	10549	0.72
PULS	Q	1.40	114	129	15	0.12
PEAN	P	256.24	13605	31893	18288	0.57
PEAN	Q	-1.60	200	719	519	0.72
YAMS	P	104.46	2661	8280	5618	0.68
YAMS	Q	-5.89	234	104109	103875	1.00

Source: Own computations.

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