

SAINT 1.01

A Standard CGE Model for Advanced Analysis of Indirect Taxation

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MANUAL

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1 Introduction

This manual give a documentation of the CGE model SAINT together with some advices over how to use the program. The SAINT model is a development of the IFPRI Standard model¹ with the purpose of making it more suitable for advanced analysis of taxes on commodities and similar policy instruments, but it would probably be a useful model in a lot of applications where a little bit more sophisticated structure of intermediate inputs is needed. The model is written with the Swedish tax system and the structure of Swedish National Accounts in mind but would probably be suitable for most European countries.

In many countries commodity tax rates are distinguished between different users, a feature that is not possible to model in the original version of the standard model. This model allows for differences in tax rates and trade margins between different actors. The law of one price is imposed in basic prices while in the standard model there is one single purchaser price. The production and consumer behaviour functions are much more flexible than in the standard model giving a possibility to distinguish between complements and substitutes both in intermediate goods and in final consumption.

In order to make the model recursive dynamic, a couple of equations determining investments capital stocks and foreign assets are included. Consumption of the households is determined from both income and wealth and not just from income as in the standard model. Moreover some of the features of the standard model that is not very important in the studies of developed countries are removed to simplify the model and a different set of macro closures are included in the model. These changes have made it necessary to change almost every equation in the model. What's left from the standard model is thus mostly the structure of the program while the mathematical formulations of the models differ quite a lot.

There are also some tools for simplifying data manipulations included in the model. These tools make it easy to run sensitivity analysis on different kinds of manipulations and simplify documentation of the manipulations made. Moreover just like the standard model there are a lot of diagnostic test producing error messages if indata is inconsistent with the model and advices the users what corrections of data that is needed.

This project is a work in progress with the purpose of developing a standardized model suitable for sensitivity analysis between different model specifications. In this paper the version SAINT.1.01 is described. Comments and hints over how to introduce different kind of model specifications into the model are welcome and also of course reports over bugs in the code.

In section two the structure of indata for the model is described, section three highlights some of the most important developments of the model according to the standard model, section four describe the tools for data manipulations, section five give a description of the technical structure of the program and the relation between the different files in the package. Finally section six gives a complete mathematical formulation of the model.

¹ Lofgren et al 2002.

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2 Indata

The model is developed with the purpose of being as consistent as possible with the system of national accounts. In national accounts the payments in the economy is reported both in purchaser prices, i.e. the amount of money paid by the final user, and divided into trade margins, that goes to the retail sector, taxes, that goes to the governmental sector and payments at basic prices i.e. the part of the price that end up at the producer. This structure is also the main structure of SAINT.

The word activity will in the following be used for a specific producing technology. Most users will probably use the activities as equivalent to industries but it is also possible to define different activities within a specific industry describing different technologies used in the industry.

A social accounting matrix (SAM) is a comprehensive, economy wide data framework, typically representing the economy of a nation. It is a square matrix in which each account is represented by a column and a row. Each cell shows the payment from the account of its column to the account of its row. Thus the incomes of an account appear along its row and the expenditure along its column. If the matrix is balanced so that column sums equal row sums all expenditure equal revenue for all accounts.

The basic structure of the SAM is shown in table 1 below. The government are specified with a main government account and specific accounts for all kinds of tax income. The tax account is needed to distinguish different kind of taxes. In a specific application, the SAM may exclude any (or all) of the individual tax accounts.

The use table of the SAM should be in basic prices. The rows for trade margins, unit taxes and ad valorem taxes are to be specified in sub matrices with commodities in rows and users in columns. The investments(S-I) column is to be specified in the four sub matrices, investments at basic prices, ad valorem taxes, unit taxes and trade margins, all with activities in columns and commodities in rows.

There are thus seven sub matrices that has to be included in the indata file, although some of them may in specific applications be zero matrices if the taxes or trade margins are not present. The seven submatrices are the following:

1. tqmat(C,P) Unit taxes on commodities with user in columns and commodities in rows.
2. tvmat(C,P) Ad valorem taxes on commodities with user in columns and commodities in rows.
3. tmmat(C,P) Trade margins on commodities with user in columns and commodities in rows.

In these three matrices as well as in the SAM investments should be aggregated into one column called S-I. Data over investments should than be disaggregated over industries in the following four matrices:

4. binvmat(C,A) Investments at basic prices with activity in columns and commodities in rows.

5. tqinv(C,A) Unit taxes on commodities used for investments with activity in columns and commodities in rows.
6. tvinv(C,A) Ad valorem taxes on commodities used for investments with activity in columns and commodities in rows.
7. tminv(C,A) Trade margins on commodities used for investments with activity in columns and commodities in rows.

The difference between this SAM and the SAM of the standard model is that taxes on commodities and trade margins³ are removed from commodity column to the columns of different purchasers. Trade margins are then distributed to the retail commodity in the column of trade margins while the taxes of course are distributed to the government as usual. Another difference not shown in table 1 is that an account for financial income is included that transforms capital income from national industries and from abroad to domestic and foreign capital owners.

In the indata file you may define physical quantities of factors of production and depreciation rates for different kind of capital. If you do not have this information or not want to use it, these could also be calculated in the model. You may also define realistic commodity prices. If you don't do that prices will be set to 1. You should also define foreign net asset position.

To get a better picture over how the data should be organized see the indata.inc that is included in the zipped model library available at <http://www.natskolan.se/research/saint.htm>

³ In the standard model trade margins are named "transactions cost".

Table 1 the basic structure of the SAM

| | | | | | | | | | | | | | | | | |
|------------------|-------------|--------------|------------------|-------|-------|-----------|----------|---------|---------|---------|------------|-------------|-------|----|--------|-------|
| Activities A* | Activi ties | Commo dities | Trade marg | Q tax | V tax | Act taxes | Fact tax | Ins tax | Imp tax | Exp tax | Facio rs F | house holds | gov | si | row | Total |
| Commo- dities C* | Use bas p | Make Bas pr | Distr to retails | | | | | | | | | C | G | I | Export | |
| Trade margin | T M | | | | | | | | | | | TM | TM | TM | TM | |
| Quantity Taxes | QT | | | | | | | | | | | QT | QT | QT | | |
| Value taxes | VT | | | | | | | | | | | VT | VT | VT | | |
| Activity taxes | AT | | | | | | | | | | | | | | | |
| Factor taxes | | | | | | | | | | | F tax | | | | | |
| Taxes on ins | | | | | | | | | | | | TAX | | | | |
| Imp tax | | Imp tax | | | | | | | | | | | | | | |
| Exp tax | | Exp tax | | | | | | | | | | | | | | |
| Factors F | VA | | | | | | | | | | | | | | F inc | |
| househo lds | | | | | | | | | | | F inc | | Trans | | Trans | |
| gov | | | | TAX | TAX | TAX | TAX | TAX | TAX | TAX | | | | | Trans | |
| si | | | | | | | | | | | | H sav | G sav | | F sav | |
| row | | imports | | | | | | | | | F inc | | Trans | | | |
| total | | | | | | | | | | | | | | | | |

3 Description of some specific features of the model

The model explains all payments that are recorded in the SAM. As a consequence the model follows the SAM disaggregation of factors, activities, commodities and institutions. It is written as a set of simultaneous equations that define the behaviour of the different actors. A complete description of the model is found in section 6. In this section some specific features that differ from the standard model are described in some more detail.

3.1 Modelling trade margins and commodity taxes.

The most important difference between this model and the standard model is the treatment of commodity taxes and trade margins. In this section the motivation for the chosen way of modelling is explained.

The law of one price is normally imposed in a CGE model. In reality a lot of divergence from this law is observed. In a CGE model where the law of one price is imposed in purchaser prices and quantities are normalized with an assumed price of one, the implication is that the different purchasers have different quantity units. In the following this kind of quantity unit will be called value based. By instead impose the law of one price in basic prices and allow for differences in trade margins we have an opportunity to impose physical quantities and get consistency with data over costs by distribute the price between an equal basic price and a trade margin that differs between purchasers. As a lot of tax rates are based on physical units, in the case of Sweden for example energy taxes, this solution will improve the ability to model those taxes in a realistic way. Properly modelling of trade margins is important for evaluating unit taxes since the size of the trade margins will determine the percentage increase in the cost from a specific unit tax rate.

Another advantage of this kind of modelling may be that the equilibrium conditions will be more realistic. An example where this way of modelling may improve the equilibrium condition is the electricity market in Scandinavia. The price less tax really paid of the consumer per MWh differs a lot between small households and large firms, both due to differences in unit price and difference in the share of the cost that is fixed cost. Meaning that a value based quantity differs a lot between sectors in terms of MWh. Anyhow the equilibrium at Nordpool is probably best described as equilibrium in MWh rather than equilibrium in values. If the demand is increased from a sector that buy to low prices (probably large firms) the increase in physical units would be greater for a specific increase in the value based unit than if the increase in demand is from a sector that buy to high prices (probably households without electrical heating). Thus the equilibrium conditions may be more realistic if the quantity units are a physical quantity as we normally think of the equality between supply and demand as equality in physical quantities.

It may be the case for some commodities that it is cheaper to produce to large purchasers and that the restriction in supply thus rather is a restriction in value units than in physical unit. In that case traditional models are to be preferred but if we believe that the difference between prices for large and small purchasers is more driven from differences in transactions costs than from differences in production costs the way of modelling used in this model may be

more realistic than a value based physical unit. However the main motivation for the way of modelling trade margins in this model is to simplify a realistic modelling of unit taxes.

Demand equations in the model have the following structure. As an example the demand of household consumption is shown below:

$$(1 + tv_{c,h}) (PQ_c + tq_{c,h} + TM_{c,h}) = \begin{matrix} h \in H \\ cgh \in CGH \\ c \in C \end{matrix} \\ QCGH_{h,cgh} \cdot PCGH_{h,cgh} \cdot \left(\sum_{c' \in C'} \delta_{h,c',cgh}^{cgh} \cdot QH_{h,c',cgh}^{-\rho_{h,cgh}^{cgh}} \right)^{-1} \cdot \sum_{c \in C} \delta_{h,c,cgh}^{cgh} \cdot QH_{h,c,cgh}^{-\rho_{h,cgh}^{cgh} - 1}$$

where

- PQ_c = basic price for the commodity
- $tq_{c,h}$ = the unit taxrate for this commodity and this household
- $TM_{c,h}$ = the Trade Margin for this commodity and this household
- $tv_{c,h}$ = the ad valorem taxrate for this commodity and this household
- $QCGH_{h,cgh}$ = quantity of aggregated commodity cgh in household h
- $PCGH_{h,cgh}$ = price of aggregated commodity cgh in household h
- $QH_{h,c,cgh}$ = quantity consumed of com c by household h in group cgh
- $\delta_{h,c,hacom}^{hacom}$ = share parameter in nested ces utility
- $\rho_{h,hacom}^{hacom}$ = CES expenditure system exponent

Note that the left hand side of this equation is the purchaser price that will differ across actors since the tax rates, tq and tv , and trade margins TM differs between different purchasers, while PQ is basic price that would be the same in the whole economy. CGH is a commodity group. The purpose of these groups will be explained in the next section.

It turns out that the definition of trade margins in national accounts and in the context of this model not necessarily are the same. In national accounts trade margins are defined as the part of the price that goes to a middleman in the retail sector and thus as the difference between the amount paid of the user less taxes and the amount received by the producer. In this model trade margins should be the difference between price really paid and a price that would fulfil the law of one price. Anyhow, if we believe that normally the law of one price holds from producer to retail sector, the trade margins from National Accounts may be a good starting point to calculate trade margins for the model. For commodities that are not subject to unit taxation, trade margins from National Accounts and an assumed price of one will probably be good enough to get data that is realistic enough.

It is important to consider though that the sizes of the trade margins in national accounts are a function of the specification of the industries. It is only when the retailer is not classified under the same industry code as the producer that we will have any trade margins reported in national accounts. In the Swedish national accounts the electricity trading companies are classified under the same code as the producers and not in sector 50-52 meaning that trade margins according to national account is equal to zero. In that case trade margins have to be recalculated to be used in this model.

For calculation of trade margins on commodities that are subject to unit taxation the following method would be a practical solution. Start with a matrix of tax rates and tax paid of different purchasers. Derive the use in physical units consistent with these data. Assume a basic price PQ_c and calculate the trade margins as the difference between the amount of money really

paid and the amount imposed by the assumed price. Choosing a price that makes the sum of calculated trade margins equal to the sum of trade margins from National Account will leave row and column totals unchanged and the SAM will still be balanced.

3.2 Fix trade margins

Most often differences in the average price paid by different actors are larger than the differences in marginal prices. In case of electricity for example due to high fix cost the average price for household are much higher than their marginal price. If we assume household to optimize their use according to marginal price this have to be taken care of. In SAINT this is done by the FTM-variable that converts a part of the trade margins to a pure transaction not dependent upon the amount consumed of the commodity and not showing up in the demand equation for that commodity. It only shows up in the LES equation to reduce the income available for spending in marginal prices and in the activity price function to determine total cost of inputs.

The calculation of the Fix trade margins is done in the include-file FIXTMINC.GMS. This will be done automatically if you turn on the procedure by setting the FIXTMPAR = 1 in the options section of SAINTBASE.GMS. Define the parameter margprice(C,PNI) to a number that for each commodity and purchaser defines the maximum value of marginal trade margins. (0.3 means that the TM is set to maximum 30% of the basic price in your base solution.) The rest of the trade margins become fix trade margins. In the program tmmat is divided into two matrices, margtm and fixtm. Note though that the model do not support fix trade margins on exports.

3.3 The consumer behaviour system

The consumer behaviour system is modelled as a LES-system over commodity groups and CES-equations within the commodity groups. For a specific commodity there is a CES-equation between imports and domestic production, the so called Armington equation. This system could be used to sort out substitutes in the same commodity group with a rather high elasticity of substitution and impose a high negative number on the Frisch parameter in order to get a low elasticity of substitution between the different commodity groups. One may of course also define commodity groups with complements using low elasticity of substitution between the commodities in that kind of groups.

CGH in the equations below refers to the set of commodity groups. This set is declared in the include file ELASx.INC where you also should define a parameter called usehodef(C,CGH) that will determine what commodities that would be included in each commodity group. In these include file you should also define your parameters for the LES system as well as the elasticities in the CES functions. You can define several such files for different utility functions and then easily make sensitivity analysis by switching between different include files. Thus you can experiment with different structure of commodity groups as well as different elasticities. This system is extremely flexible and you can easily change the properties of the consumer behaviour system. In many studies distinguishing between

substitutes and complements is important and than a model with these kind of properties is needed.

The household consumption system above the Armington equation consists of the following equations.

The Les equations (demand for CGH in household h):

$$QCGH_{h,cgh} \cdot PCGH_{h,cgh} = PCGH_{h,cgh} \cdot \gamma_{cgh,h} \\ + \beta_{cgh,h} \cdot \left(EH_h - \sum_{c' \in C'} FTM_{c',h} (1 + tv_{c',h}) - \sum_{cgh' \in CGH'} PCGH_{h,cgh'} \cdot \gamma_{cgh',h} \right) \quad \begin{matrix} h \in H \\ cgh \in CGH \end{matrix}$$

The CES equations (aggregation of single commodities to CGH)

$$QCGH_{h,cgh} = \alpha_{h,cgh}^{cgh} \cdot \left(\sum_{c \in C} \delta_{h,c,cgh}^{cgh} \cdot QH_{h,c,cgh}^{-\rho_{h,cgh}^{cgh}} \right)^{\frac{-1}{\rho_{h,cgh}^{cgh}}} \quad \begin{matrix} h \in H \\ cgh \in CGH \end{matrix}$$

Households commodity demand (FOC of the CES equation)

For every commodity group this equation would only be defined for the commodities in that group

$$(1 + tv_{c,h}) (PQ_c + tq_{c,h} + TM_{c,h}) = \quad h \in H \\ QCGH_{h,cgh} \cdot PCGH_{h,cgh} \cdot \left(\sum_{c' \in C'} \delta_{h,c',cgh}^{cgh} \cdot QH_{h,c',cgh}^{-\rho_{h,cgh}^{cgh}} \right)^{-1} \cdot \sum_{c \in C} \delta_{h,c,cgh}^{cgh} \cdot QH_{h,c,cgh}^{-\rho_{h,cgh}^{cgh} - 1} \quad \begin{matrix} cgh \in CGH \\ c \in C \end{matrix}$$

Where:

- EH_h = household consumption expenditure (fix trade margins included)
- PQ_c = basic price for the commodity
- $tq_{c,h}$ = the unit taxrate for this commodity and this household
- $TM_{c,h}$ = the Marginal Trade Margin for this commodity and this household
- $FTM_{c,h}$ = the FIX Trade Margin for this commodity and this household
- $tv_{c,h}$ = the ad valorem taxrate for this commodity and this household
- $QH_{h,c,cgh}$ = the quantity demanded for this commodity and this household in the cgh nest
- $QCGH_{h,cgh}$ = the quantity of aggregate CGH consumed by household H
- $PCGH_{h,cgh}$ = the price of aggregate CGH consumed by household H
- $\delta_{h,c,cgh}^{cgh}$ = share parameter in the ces utility function for the CGH nest and houshold H
- $-\rho_{h,cgh}^{cgh}$ = exponent in the ces utility function for the CGH nest

Aggregate household consumption is a function of income, marginal propensity to save and the wealth of the household as in standard macroeconomic textbooks. Including wealth in aggregate consumption functions is not very common in standard CGE models though. This feature has no impact on static simulations. However in dynamic simulations this will be an important feature. The aggregate consumption function will be discussed in some more detail in section 3.5 "Dynamics of the Model".

3.4 Production functions

The intermediate goods are first divided into two main subsets CLEO and CSUBI where CLEO is intermediate goods where the use is strictly proportional to output and CSUBI is intermediate inputs with substitution possibilities. The total quantity of the aggregated non substitutable intermediate goods is denoted QLEO and the total quantity of substitutable intermediates is denoted QSUBI. The aggregate of substitutable intermediate goods and labour is denoted QSUB and the aggregate of QSUB and capital is denoted QSC. Total output of an activity is denoted QA and can be a mix of different outputs produced with fixed output coefficients.

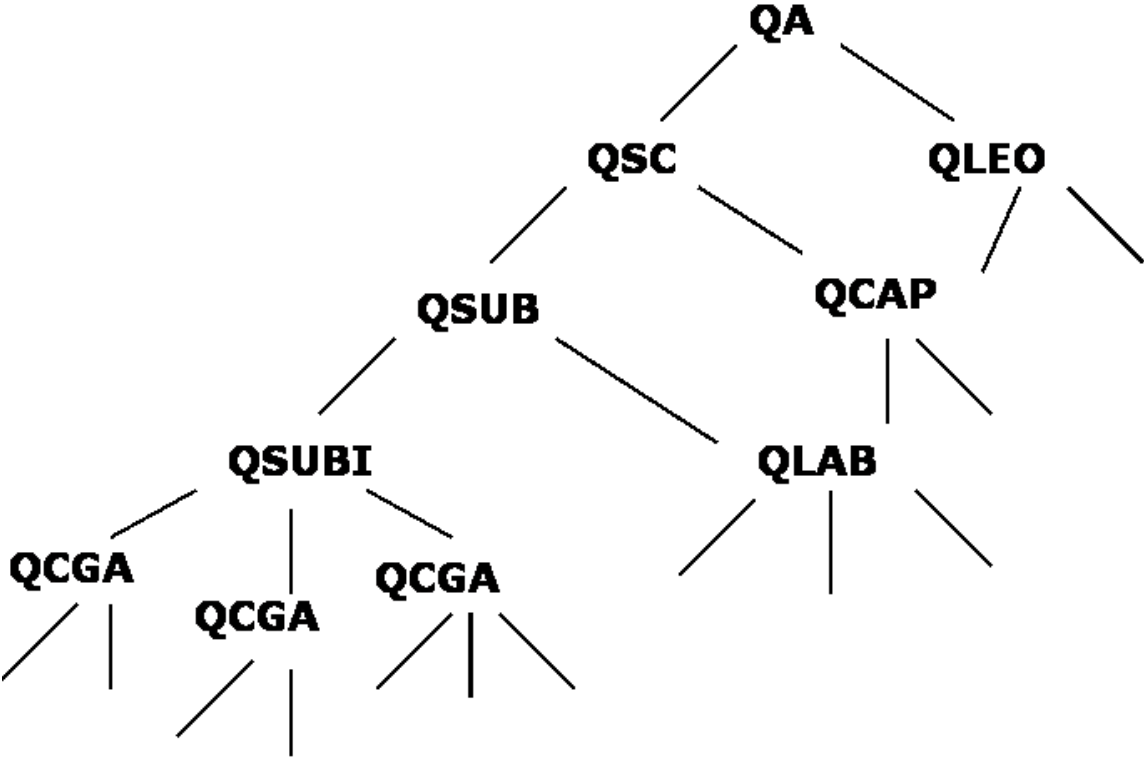
There are possibilities to define several capital goods in each activity. The set of different capital goods is FCAP. The total number of different capital good in the model will thus be equal to the number of activities times the number of different capital goods. There are possibilities to define several kinds of labour. The set of different kind of labour is denoted FLAB and the quantity of each kind of labour is denoted QFL_{flab} .

The different kind of substitutable intermediates could be aggregated into groups. The set of different intermediate groups is called CGA and the total quantity in such a group is denoted $QCGA_{cga}$. It is possible to define different elasticities within these groups and between them giving a possibility to distinguish between substitutable and complementary commodities and the production function regarding intermediate inputs is thus very flexible. The basic idea is the same as for the different commodity groups in household consumption. For sensitivity analysis over model structure just change what ELASx.INC file that is included in the options section of SAINTBASE.GMS. In these files different structure of commodity groups could be defined as well as different values of the elasticities.

The basic structure of the production function is shown in figure 1. At top nest there is leontief technology and all other nest is CES. The elasticity of substitution has to be symmetric i.e. to be the same between every pair of QCGA, as well as between every pair of commodities included in the same QCGA group. In the same way the elasticity of substitution has to be the same between every two kinds of capital or labour.

If no commodities are defined as substitutable the QSUBI nest will not be included in the model and all intermediates will be included in QLEO. If one or more commodities are defined as substitutable at least one QCGA group has to be defined for the program to work, although if just one group is defined it will have no impact on the model structure.

Figure 1 The nest structure of production functions in the model



3.5 macro closures in static simulations

3.5.1 Closures for the government budget

There are five different closures for the government available. Closure 1 has a flexible saving rate meaning that when you run a simulation the budget deficit will change. The other four closures have a fixed government saving and changes instead different tax rates. Closure number 2 and 3 change the income tax, closure 4 changes the labour tax and closure 5 change taxes on capital. The difference between 2 and 3 is whether the change should be equal in absolute terms or if the relative change should be the same for all selected institutions. The user chooses what closure to use by defining a value to the parameter govclos.

3.5.2 Closures for the rest of the world and savings investments

The parameter rowclos defines the closure for the balance of payments. If you have a closed economy model, so that you do not have any rest of the world account, the value of rowclos should be 4. If you have a fixed exchange rate the value should be 2. Closure 1 and 3 have flexible exchange rates and the difference between them is that in closure 1 foreign savings, i.e. the current account surplus, is fixed while it is flexible in closure 3.

The parameter siclos defines the closure for savings and investments. If you have chosen rowclos 3 you have to choose siclos 6 that fixes both marginal propensity to consume and investments. In that case foreign savings will adjust to get equilibrium between savings and investments.

Siclos 3 have flexible investment meaning that total savings determines the amount of investments. In siclos 1, 2 investments and government consumption is fixed and determines the amount of household saving. In siclos 4 and 5 investment and government consumption is a fixed proportion of domestic absorption and determines the amount of saving.

The difference between 1 and 2 as well as between 4 and 5 is whether the change in mpc should be the same in percentage units or percentage rate for different households.

3.6 Dynamics of the model

There is a recursive dynamic simulation file included in SAINT. The most important dynamic equations will be presented in this section. In this section the subscript t denote time but will only be present in equations where the variables refer to different time periods.

3.6.1 Capital stock and investments

Capital stock for each kind of capital in each activity in a specific time period is calculated from investments and depreciations:

$$QF_{fcap,a,t+1} = QF_{fcap,a,t} \cdot (1 - depreate_{a,fcap}) + QAGGINV_{a,fcap,t}$$

Where:

$QF_{fcap,a,t}$ = Capital stock for this capital good and activity at year t

$depreate_{a,fcap}$ = Depreciation rate for this capital good and activity

$QAGGINV_{a,fcap,t}$ = Investments in this capital good and activity at year t

Investments in each capital good are a function of Tobin Q. Each capital good consists of a fixed coefficient vector of commodities. Investment demand in each commodity and industry is calculated from:

$$QINV_{c,a,fcap} = IADJ_{a,fcap} \cdot \overline{qinv}_{c,a,fcap}$$

Where:

$QINV_{c,a,fcap}$ = Investment demand for this specific commodity c in activity a to be used in the formation of capital good $fcap$.

$IADJ_{a,fcap}$ = Investment scaling factor in activity a for capital good $fcap$ (the relation between this years investments and investments in the base year)

$\overline{qinv}_{c,a,fcap}$ = Investment demand in base year for this commodity c in the formation of capital good $fcap$ in activity a

The variable IADJ for a specific year is calculated from the Tobin Q of the previous year under an assumption that expectations are formed from a random walk model i.e. this years relative prices and demanded quantities are the best prognosis for the future prices and quantities. The motivation for using previous years Tobin Q is partly computational but also justified from an assumption that investments are planned one year before they are realised. Investment in a specific capital good and activity would thus be equal to:

$$IADJ_{a,fcap,t+1} \cdot QAGGINV_{a,fcap,t} = QF_{fcap,a,t} \cdot depreate_{a,fcap} \cdot \left(\frac{WF_{fcap,t} \cdot wfdist_{fcap,a,t}}{(depreate_{a,fcap} + RETURN_t) \cdot PI_{a,fcap,t}} \right)^{invpar_{a,fcap}}$$

where:

- $QAGGINV0_{a,fcap}$ = Investments in this capital good $fcap$ and activity a at base year
 $WF_{fcap,t}$ = economy-wide rent for capital good $fcap$ at year t
 $WFDIST_{fcap,a,t}$ = rent distortion variable for capital good $fcap$ in activity a at year t
 $RETURN_t$ = investors required rate of return for investments in this country at year t
 $PI_{a,fcap,t}$ = the price of this capital good for this activity at year t

Note that since the expression in the parenthesis is equal to the Tobin Q investments would be equal to depreciation i.e. net investments would be equal to 0 if Tobin Q is equal to one.

3.6.2 Determination of foreign net asset position

Foreign liabilities are the value of domestic asset owned by foreigners. In the model there is no distinguishing between shares and bonds. Or more correctly all domestic assets are treated as shares and thus valued from future expectations about the profitability of domestic capital. Expectations are assumed to be determined from a random walk model with current return as the best prognosis of future returns giving the following expression for foreign liabilities.

$$FLP = \sum_{a \in A} finout_a \cdot \sum_{fcap \in FCAP} \frac{GRPROF_{a,fcap}}{deprate_{a,fcap} + RETURN}$$

where

- FLP = The value of foreign liabilities
 $finout_a$ = The share of foreign owners in activity a
 $GRPROF_{a,fcap}$ = Return to capital good $fcap$ in industry a before depreciations
 $deprate_{a,fcap}$ = Depreciation rate for this capital good and activity
 $RETURN$ = investors required rate of return for investments in this country

Foreign asset owned by domestic residents is calculated as accumulated investments abroad. Since foreign assets are not explicitly modelled they cannot be valued from their return and are just supposed to give the world rate of capital return. Moreover the investments abroad are calculated from financial account under the assumption that domestic investments always attract a constant share of the invested capital from abroad.⁴ Since the financial account is the net of foreigner's investments in the domestic country and domestic citizen's investments abroad the following expression gives the development of foreign assets over time where we correct the financial account with foreigners investments in domestic capital assets:

$$FAP_{t+1} = FAP_t - FSAV_t + \sum_{a \in A} finout_a \cdot \sum_{fcap \in FCAP} PI_{a,fcap,t} \cdot QAGGINV_{a,fcap,t}$$

⁴A drawback of this kind of model may be that the assumption of constant foreign ownership over time may not be appropriate. However it is not quite clear how to model the development of this share since differences if domestic and world return would theoretically have the same impact on domestic and foreign investors.

Where:

FAP_t = Foreign assets owned of domestic citizens at time t

$FSAV_t$ = The financial account in domestic currency at time t , note that positive investments abroad is equal to negative financial account.

$finout_a$ = The share of foreign ownership in activity a

$QAGGINV_{a,fcap,t}$ = Investments in this capital good and activity at year t

The foreign net asset position is of course the net between assets and liabilities.

$$FNAP = FAP - FLP$$

Where

$FNAP$ = foreign net asset position

Financial income from abroad in the balance of payments is determined from foreign asset position time the rate of return in the rest of the world.

$$finin = factinret * FAP$$

$finin$ = Capital income from abroad.

$factinret$ = Capital return in the rest of the world.

3.6.3 The intertemporal budget constraint of the nation

The aggregate consumption function is a Keynesian function but where the intercept is a function of household wealth.

$$EH_h = MPC_h \cdot \left((1 - TINS_h) \cdot YH_h - trnsfr_{row,h} \right) + cint_h \cdot WEALTH_h$$

Where:

EH_h = total consumption expenditure in household h

MPC_h = marginal propensity to consume in household h

$TINS_h$ = income tax on total income in household h

YH_h = Income in household h

$trnsf_{row,h}$ = net transfer payments to other country from household h

$cint_h$ = the impact from wealth on consumption in household h

$WEALTH_h$ = the wealth of household h

The implication from this equation is that private consumption is not only determined from household income but also from households assets which is important for the dynamics of the model. The wealth of the households are determined from their share of domestic capital and foreign assets.

$$WEALTH_h = \sum_{a \in A} shifin_{h,a} \cdot (1 - finout_a) \cdot \sum_{fcap \in FCAP} \frac{GRPROF_{a,fcap}}{deprate_{a,fcap} + RETURN} + shifinin_h \cdot FAP$$

Where:

$shifin_{h,a}$ = the share of the capital stock in activity a owned by household h

$shifinin_h$ = the share of foreign assets owned by household h

The required rate of return is a function of the foreign net asset position.

$$RETURN = \frac{balreturn}{e^{FNAP/GNPN}}$$

Where:

balreturn = return when FNAP equals zero.

GNPN = nominal gross national product

This formulation makes the model not deviate very far from the intertemporal budget constraint of the nation since accumulated deficits in the current account will increase required return to invest in the country. An increase in required return will decrease investments as well as consumption (since Tobin Q as well as household wealth will decrease from an increase in RETURN) and thus increase current account.

3.6.4 Calibration of investments and physical capital stocks

Dynamic simulations may be easier to evaluate if your base scenario starts from a base solution in long run equilibrium. Since there is no productivity growth in the model such a scenario would be a scenario where nothing happens over time. That will make it easy to evaluate the ceteris paribus effects from your experiment. A long run equilibrium requires that Tobin Q is equal to one, that the size of the investments in base year are optimal, and that capital inflow is equal to foreigners share of reinvestments, meaning that a current account with capital return net for depreciations would be balanced.

In the options section of SAINTBASE you should give the parameter `tobinqpar` the value of 1 or 2 if you want the model to calculate depreciation rates and capital stocks that make Tobin Q be equal to 1 and base year investments being optimal. In that case the depreciation rates that you eventually have defined in your `indata` will be replaced with the depreciation rates calculated by the model. If you give the value of 2 to `tobinqpar` the model will adjust the current account surplus by adjusting government savings and government transfers from abroad, if `tobinqpar` is equal to 1 the current account is left unchanged. **If you want a base scenario in long run equilibrium `tobinqpar` should be equal to 2.**

3.6.5 Macro closures in dynamic simulations

In dynamic simulations the closures for government budget fixes government expenditure as share of absorption instead of its level. Moreover a sixth closure is included that fixes both government saving and the tax rate and makes the share of government expenditure flexible. For the balance of payments and savings investments the number of options is reduced since more of these variables are determined in the model. The user chooses between three different values of `siclos` that determine the closure for both balance of payments and savings investments. In the first two closures we have a fixed exchange rate and savings adjusts in order to finance the level of investments determined in the model. `Siclos 3` have a flexible exchange rate and marginal propensity to consume is fixed. The difference between `siclos 1` and `2` is whether the change in `mpc` should be the same in percentage units or percentage rate for different households.

4 Data manipulations

Before you start working with your indata make sure you have a balanced SAM! If the Sam is balanced from the beginning the manipulations described in this section will leave it so. There is no balancing tool included in this model.

4.1 Tools for manual data manipulations

There are often necessary to adjust real life data to make it consistent with the model. One example is that the model estimates capital stocks from operating surplus under an assumption of capital stocks to be in long run equilibrium. In some industries this may be a highly inappropriate assumption and at least one have to make adjustments to avoid negative capital stocks. For this kind of manual manipulations you should use the matrix SAMMAN. The model will then add that matrix of manipulations to the original SAM and thus you will always keep your original indata intact. Make sure that always make balanced manipulations i.e. that SAMMAN have the same row and column totals and that TRCMAN, TQMAN and TVMAN adds up to the totals in the manipulated SAM. It is easy to store displays from the manipulation matrix together with the resulting database from each run of the model and thus to keep track of which set of manipulations that generated the different results.

4.2 Tools for automatic data manipulations

As the model do not work if exports exceeds domestic production an optional procedure is included that subtract an amount equal to the difference between exports and domestic production from both import and export of the commodity if exports is less than domestic production. This procedure is in the include file: NETTRADE.INC. The procedure also produces a vector, DOMEXP(C), for all commodities with the quota of domestic production over exports and a vector; EXPKORR(C), of all the manipulations it makes.

Another automatic data manipulating tool deals with fix trade margins. The user may define the largest tolerated deviation in pre tax price for different kind of actors. The program will then distribute some of the trade margins to a fix trade margins unrelated to the purchased quantity. See section 3.2 for more details.

There is a program dealing with secondary products BWSEC.GMS included in the model directory. This program calculates industry specific technological coefficient using the Bohlin Widell method of minimizing variances.⁵ These coefficients is then used to remove vectors of inputs, trade margins, ad valorem- and unit taxes together with the secondary products to the industry where this product is the main product. As indata this program needs a mapping of all commodities to an industry where it is the main product (the matrix primact), together with information over what secondary product that it not should remove in the different industries (the matrix secprod) and whether it should use ITA or CTA or a mix between them in the calculation of technological coefficients. The program will produce new indata matrices, a matrix of industry specific technological coefficients and five matrices of all the

⁵ For details see Bohlin, Widell 2006

manipulations that is done; makecorr(A,C), usecorr(C,A), SYMTM(C,P), SYMTQ(C,P) and SYMTV(C,P).

4.3 Recommended working procedure for data manipulations

Start with running the SAINTBASE with the original data with the nettrade procedure activated. You will probably end up with a lot of error messages about inconsistencies. Start with adjusting the trade vectors. One option is to just subtract the adjustments from the EXPKORR(C) into the SAMMAN(AC,AC). However this will impose an assumption that all domestic production is exported of these commodities. If one rejects that assumption it would be preferable to subtract a higher amount from the export and import vectors in SAMMAN(AC,AC). Furthermore one may investigate the DOMEXP(C), for commodities where the quota is near one and decrease trade even in these commodities under an assumption that there exist reexport and that in reality a larger share of domestic production is sold domestically.

The next step would be to adjust the SAM, TRCMAT, TQMAT, TVMAT, BINVMAT, TRCINV, TQINV and TVINV, in order to avoid all error messages. Just click on the error message and you will get to a display of a matrix in the LST file that tells you what manipulation you have to do. After that you can inactivate nettrade.gms and start running simulations.

After that you may use the BWSEC program for sensitivity analysis over different treatment of secondary products. To do that you should run SAINT_SEC.GMS. Search for *Matrices to be copied into the new indata file*: Copy the matrices and paste them into a new indata file. You may now run the same simulations using this indata file.

4.4 Creating indata from output

In some cases you may want to produce an indata file from the result of a simulation. Starting DATACREATOR.GMS from the saved workfile will do the job. After that you just search for *new data* in datacreator.lst to find the matrices to be copied and paste into the new indatafile.

5 Technical structure of the program

The model consists of a set of files. For running the model you first have to run SAINTBASE.GMS to get a base solution. This solution has to be saved in a workfile. Print s=save/name1 in the GAMS parameter entry field. Note that you first have to create a map called save in your working directory.

You can then run the SAINTSIM.GMS or SAINTDYNSIM.GMS by restarting from the saved workfile. Print r=save/name1 s=save/name2 in the GAMS parameter entry field. After that you can run short display programs by typing r=save/name2 in the GAMS parameter entry field in order to display the result. You can also print r=save/name2.gdx=name2.gdx in the GAMS parameter entry field to get all your sets, parameters and variables in a.gdx file.

If you make sensitivity analysis over the different options in the SAINTSIM.GMS or SAINTDYNSIM.GMS files you can restart them from the same base solution. However running sensitivity analysis over different options in SAINTBASE.GMS require that you save the different executions of SAINTBASE.GMS under a number of different names that you can restart the SAINTSIM.GMS and SAINTDYNSIM.GMS from.

5.1 Include files

Under execution SAINTBASE will call the following include files.

INDATA.INC

In this file the indata should be specified. The model is distributed with an indata file of up made data. You should specify your own indata by creating a similar datafile. Choices over different in data files are to be made in the OPTIONS section of SAINTBASE.GMS

ELAS1.INC

The model is distributed with an example of an elasticity file called ELAS1.INC. The elasticity file defines the structure of model equations. You could define a number of different elasticity files with different names. The elasticity include files has to be consistent with the indatafile so that the same items are included in the set of commodities. Thus you probably have to define a specific set of elasticity files for every specific indata file. It is easy to run sensitivity analysis by calling for different elasticity files in the OPTIONS section of SAINTBASE.GMS But you have to be sure that you in each run chose an elasticity file consistent with the indata file chosen.

NETTRADE.INC

This include file will only be included if the procedure is activated in the Options section of SAINTBASE. The files subtract an equal amount from imports and exports if exports exceed domestic production.

FIXTM.INC

This include file will only be included if the procedure is activated in the Options section of SAINTBASE. It is calculating fix trade margins by dividing the matrix of trade margins in two different matrices, margtm and fixtm. The procedure could be used to decrease changes in unit price for different actors.

DIAGNOSTICS.INC

This file will create error messages if your data is not consistent with the model. Do not turn it off!

VARINIT.INC

This file is used to initialize variables. It is not only included in SAINTBASE but also in the simulations files.

REPBASE.INC

This file produces reports over your base solution.

Under the execution of the simulation files, SAINTSIM.GMS and SAINTDYNSIM.GMS the following include files will be included.

REPPAR.INC, DYNREPPAR.INC

In these files sets and parameters for reporting the results are defined. All variables of the model have two corresponding report parameters. One for the level (X added to variable name) and one for the percentage change (XP added to variable name). There are also some report parameters for creation of tables and for consistency and solution checks.

SIMREP.INC, DYNSIMREP.INC

These files are included after each solution of the model and assign values to the report parameters for levels.

SIMSAM.INC

This file calculates the SAM in each solution of the model and is included in SIMREP.INC and DYNSIMREP.INC.

REPPERC.INC, DYNREPPERC.INC

These files are included after the solver loop and assign values to the report parameters for percentage change.

Under the execution of SAINT_SEC.GMS the following include file are included.

INSEC.INC

This file is used to tell SAINT_SEC.GMS what secondary products to remove and which industry that is the main producer of each commodity. You do this by assigning the two matrices secprod and primact.

5.2 Reports of the results

In the code there just a few display statements included. These are mostly used to display potentially inconsistencies in the indata. In order to reduce the size of the LST file and make it easier to get the results the user are interested in, it is assumed that the user should run separate short display programs in order to display the result. In that way the user would get a separate lst file with the results and can freely choose what variables and parameters to include in the display. In the zipped model directory an example of a display program “display.gms” is included programmed to be restarted from the saved base solution. In the end of SAINTBASE some excel reports are produced through the xldump facility. To choose what reports you want to produce search for 14. SOLUTION REPORTS.

For creating of excel reports of simulations three examples of report programs are included in the model directory. The two programs simxldump.gms and dynxldump.gms should be used for static and dynamic simulations respectively. The program carbon.gms is included as an example over how the user may write programs for creating specific tables in excel. That program is for static simulations. All the report programs should be restarted from the saved simulations workfile or be included with an include statement at the end of the simulation file.

6 A complete mathematical formulation of the model

6.1 SETS

Sets defining different kinds of accounts in the SAM

| | |
|------------|--|
| AC | global set for model accounts - aggregated microsam accounts |
| ACNT(AC) | all elements in AC except TOTAL |
| P(AC) | all purchasers |
| PNI(P) | all purchasers except investments |
| A(PNI) | activities |
| PNE(PNI) | all purchasers except exports and investments |
| F(AC) | factors |
| FLAB(F) | labour |
| FCAP(F) | capital |
| INS(P) | institutions |
| INSD(PNI) | domestic institutions |
| INSNG(INS) | non-government institutions |
| H(INSD) | households |

Sets used to define the nest structures

| | |
|-----|---|
| CGH | set to define commodity groups in household consumption |
| CGA | set to define commodity groups in intermediate use |

Sets used to define different kinds of commodities

| | |
|----------|---|
| C(AC) | commodities |
| CD(C) | commodities with domestic sales of output |
| CDN(C) | commodities without domestic sales of output |
| CE(C) | exported commodities |
| CEN(C) | non-export commodities |
| CM(C) | imported commodities |
| CMN(C) | non-imported commodities |
| CX(C) | commodities with output |
| CTR(C) | commodities used for trade margins |
| CLEO(C) | commodities with leontief technology in the production function |
| CSUBI(C) | commodities with ces technology in the production function |
| CINV(C) | fixed investment goods |
| CGOV(C) | commodities consumed by government |

6.2 VARIABLES

Variables where the first letter is a P is prices, Q is quantities and Y is income.

| | |
|---------------------------|---|
| CPI | consumer price index (based on purchaser prices) |
| DMPC | change in marginal propensity to consume for selected inst |
| DTINS | change in domestic institution tax share |
| EG | total current government expenditure |
| EH _h | household consumption expenditure |
| EXR | exchange rate |
| FSAV | The financial account in domestic currency, note that positive investments abroad is equal to negative financial account. |
| FTM _{c,pni} | fix part of trade margins on commodity <i>c</i> purchased of <i>pni</i> |
| FTMINV _{c,a} | fix part of trade margins on investments on <i>c</i> in activity <i>a</i> |
| GADJ | government demand scaling factor |
| GOVSHR | govt consumption share of absorption |
| GRPROF _{a,fcap} | gross return from capital <i>fcap</i> in activity <i>a</i> |
| GSAV | government savings |
| IADJ _{a,fcap} | investment scaling factor in activity <i>a</i> for capital <i>fcap</i> |
| INVSHR | investment share of absorption |
| PI _{a,fcap} | price per unit of investments of <i>fcap</i> in activity <i>a</i> |
| MPC _{insd} | marginal propensity to consume for dom non-gov inst <i>insd</i> |
| MPCADJ | savings rate scaling factor |
| PA _a | output price of activity <i>a</i> |
| PCAP _a | price of aggregate capital in activity <i>a</i> |
| PCGA _{a,cga} | price of intermediate aggregate <i>cga</i> in activity <i>a</i> |
| PCGH _{h,cgh} | price of aggregated commodity <i>cgh</i> in household <i>h</i> |
| PDS _c | supply price for com'y <i>c</i> produced & sold domestically |
| PE _c | price of exports of commodity <i>c</i> in national currency |
| PLAB _a | price of labour aggregate in activity <i>a</i> |
| PLEO _a | part of price for aggregate leontief intermediates in activity <i>a</i> |
| PM _c | price of imports of commodity <i>c</i> in national currency |
| PQ _c | price of composite good <i>c</i> (basic price i.e. without taxes and trade margins) |
| PSC _a | part of price covering aggregate <i>qsc</i> (production - leontief inputs) |
| PSUB _a | part of price covering aggregate substitutable inputs less operating surplus in activity <i>a</i> |
| PSUBI _a | part of price for aggregate substitutable intermediates in activity <i>a</i> |
| PWE _c | world price of exports in foreign currency |
| PWM _c | world price of imports in foreign currency |
| PX _c | average output price of commodity <i>c</i> |
| QA _a | level of domestic activity in activity <i>a</i> |
| QAGGINV _{a,fcap} | quantity of aggregate investments <i>fcap</i> in activity <i>a</i> |
| QCAP _a | quantity price of capital aggregate in activity <i>a</i> |
| QCGA _{a,cgh} | quantity of aggregated commodity <i>cga</i> in activity <i>a</i> |
| QCGH _{h,cgh} | quantity of aggregated commodity <i>cgh</i> in household <i>h</i> |
| QD _c | quantity of domestic sales of commodity <i>c</i> |
| QE _c | quantity of exports of commodity <i>c</i> |
| QF _{f,a} | quantity demanded of factor <i>f</i> from activity <i>a</i> |

| | |
|------------------------------|---|
| QFS _f | quantity of factor supply |
| QG _c | quantity of government consumption |
| QH _{h,c,cgh} | quantity consumed of com <i>c</i> by household <i>h</i> in group <i>cgh</i> |
| QINT _{c,a} | quantity of intermediate demand for <i>c</i> from activity <i>a</i> |
| QINTA _{csubi,a,cga} | quantity of intermediate dem. for <i>c</i> from activity <i>a</i> in group <i>cga</i> |
| QINV _{c,a,fcap} | quantity of fixed <i>fcap</i> investment demand of <i>c</i> in activity <i>a</i> |
| QLAB _a | quantity of labour aggregate in activity <i>a</i> |
| QLEO _a | quantity of aggregate leontief intermediate input in activity <i>a</i> |
| QM _c | quantity of imports of commodity <i>c</i> |
| QQ _c | quantity of composite goods supply of commodity <i>c</i> |
| QSC _a | quantity of production less leontief inputs in activity <i>a</i> |
| QSUB _a | quantity of aggregate substitutables part of production less capital in activity <i>a</i> |
| QSUBI _a | quantity of aggregate substitutable intermediates in activity <i>a</i> |
| QT _c | quantity of trade and transport demand for commodity <i>c</i> |
| QX _c | quantity of aggregate marketed commodity <i>c</i> output |
| TABS | total absorption |
| TFIN | rate of direct tax on financial return |
| TFINADJ | scaling factor for tax on returns from financial assets |
| TFLAB _{flab} | rate of direct tax on labour (soc sec and income tax) |
| TINS _{insd} | rate of direct tax on domestic institutions <i>insd</i> |
| TINSADJ | direct tax scaling factor |
| TLABADJ | labour tax scaling factor |
| TM _{c,pni} | trade margins on commodity <i>c</i> when purchased of <i>pni</i> , (always domestic currency even for exports) |
| TMI _{c,a} | trade margins on com <i>C</i> when purchased of <i>A</i> for investments |
| WALRAS | savings-investment imbalance (should be zero) |
| WALRASSQR | Walras squared |
| WEALTH _h | the wealth of household <i>h</i> |
| WF _f | economy-wide wage (rent) for factor <i>f</i> |
| WFDIST _{f,a} | factor wage distortion variable for factor <i>f</i> in activity <i>a</i> |
| YFIN _a | total financial income of domestic financial asset <i>a</i> |
| YFLAB _{flab} | total labour income from domestic and foreign activities |
| YG | current government income |
| YH _h | Household income |
| YIFL _{insd,flab} | income of institution ins from labour <i>flab</i> |
| YIFIN _{insd} | financial income of institution <i>insd</i> |

6.3 PARAMETERS

Parameters other than tax rates

| | |
|------------------------------|--|
| α_a^{cga} | shift parameter for ces activity production function cga |
| α_a^{cap} | shift parameter for CES activity production function capital |
| α_h^{cgh} | shift parameter in nested ces utility function |
| α_a^{lab} | shift parameter for CES activity production function labour |
| α_c^q | shift parameter for Armington function |
| α_a^{sc} | shift parameter for CES activity production function qsc nest |
| α_a^{sub} | shift parameter for CES activity production function sub nest |
| α_a^{subi} | shift parameter for CES activity production function subi |
| α_c^t | shift parameter for CET function |
| $\beta_{cgh,h}$ | marg share of hhd cons on commodity group cga |
| $cint_h$ | the marginal increase in consumption from an increase in wealth |
| $cwts_{c,h}$ | consumer price index weights |
| δ_c^{arm} | share parameter for CET function |
| $\delta_{fcap,a}^{cap}$ | share parameter for CES activity production function capital |
| δ_c^{cet} | share parameter for Armington function |
| $\delta_{csubi,a,cga}^{cga}$ | share parameter for ces activity production function cga |
| $\delta_{h,c,cgh}^{cgh}$ | share parameter in nested ces utility |
| $\delta_{flab,a}^{lab}$ | share parameter for CES activity production function labour |
| δ_c^q | share parameter for import demand equation |
| δ_a^{sc} | share parameter for CES activity production function qsc nest |
| δ_a^{sub} | share parameter for CES activity production function sub nest |
| $\delta_{a,cga}^{subi}$ | share parameter for CES act. production function intermediates |
| δ_c^t | share parameter for export supply equation |
| <i>factinret</i> | return on foreign assets to be used in saintdynam.gms |
| <i>FAP</i> | foreign asset position this year |
| <i>finin</i> | financial income from abroad in foreign currency |
| <i>finout_a</i> | share of foreign financial income in total financial income from industry A |
| <i>FLP</i> | foreign liabilities this year |
| <i>FNAP</i> | foreign net asset position this year |
| <i>ftmq_{c,pmi}</i> | fix part of trade margins in quantities of the trade commodity |
| <i>ftmqinv_{c,a}</i> | fix part of trade margins in quantities of the trade commodity for investments |
| $\gamma_{cgh,h}$ | per-cap subsist cons of marketed com c for hhd h |
| $ica_{c,a}$ | leontief intermediate input c per unit of aggregate intermediate |
| $inta_a$ | aggregate intermediate input coefficient |
| isc_a | aggregate substitutables coefficient |
| $iwts_{c,a,fcap}$ | quantity commodity c in one unit of investment in activity a |

| | |
|-------------------------|---|
| $labin_{flab}$ | labour income from abroad in foreign currency |
| $labout_{flab}$ | share of foreign labour income in total labour income from domestic activities |
| $mpc01_{insd}$ | 0-1 par for potential flexing of savings rates |
| $mpcbar_{insd}$ | marg prop to consume for dom non-gov inst ins (exog part) |
| $qbar_g$ | exogenous (unscaled) government demand |
| $qbar_{inv_{c,a,fcap}}$ | exogenous (unscaled) investment demand by activity |
| $qdst_c$ | inventory investment by sector of origin |
| $return$ | required rate of returns on investments |
| ρ_c^{arm} | Armington function exponent |
| ρ_a^{cap} | CES activity production function exponent capital |
| ρ_c^{cet} | CET function exponent |
| ρ_a^{ega} | ces production function exponent cga |
| $\rho_{h,cgh}^{egh}$ | CES expenditure system exponent |
| ρ_a^{lab} | CES activity production function exponent labour |
| ρ_c^q | Armington function exponent |
| ρ_a^{sc} | CES activity production function exponent qsc nest |
| ρ_a^{sub} | CES activity production function exponent qsub nest |
| ρ_a^{subi} | CES activity production function exponent subst. intermediates |
| ρ_c^t | Export demand function exponent |
| $shifl_{insd,flab}$ | share of dom. inst'on i in income of labour <i>flab</i> |
| $shifin_{insd,a}$ | share of dom. inst'on i in income from financial asset <i>a</i> |
| $shifinin_{insd}$ | share of dom. inst'on i in financial income from abroad |
| $shtr_c$ | share of commodity <i>c</i> in transactions |
| $supernum_h$ | LES supernumerary income |
| $\theta_{a,c}$ | yield of commodity <i>c</i> per unit of activity <i>a</i> |
| $tins01_{insd}$ | 0-1 par for potential flexing of dir tax rates |
| $transfr_{p,p}$ | transfers fr. inst. or factor <i>ac</i> to institution ins (All transfers from rest of the world are in foreign currency all transfers to rest of the world are in domestic currency) |
| $trq_{c,pni}$ | quantity of trade margins |
| $trqi_{c,a}$ | quantity of trade margins in investments |

Tax rates

| | |
|------------------|--|
| ta_a | rate of tax on producer gross output value |
| te_c | rate of tax on exports |
| $tflbar_{flab}$ | rate of direct tax on labour in base (soc sec tax) |
| $tfinbar$ | rate of direct tax on financial return in base |
| $tinsbar_{insd}$ | rate of (exog part of) direct tax on dom inst ins |
| tar_c | rate of import tariff |
| $tq_{c,pni}$ | unit tax on commodity <i>c</i> when purchased of purchaser purch |
| $tqi_{c,a}$ | unit tax on commodity <i>c</i> when used as investment in activity <i>a</i> |
| $tv_{c,pni}$ | value tax on commodity <i>c</i> when purchased of purchaser purch |
| $tvi_{c,a}$ | value tax on commodity <i>c</i> when used as investment in activity <i>a</i> |

6.4 EQUATIONS

First column gives the name of the equation, second column its specification and last column its domain.

6.4.1 Price Block

Import price $PM_c = pwm_c \cdot (1 + tar_c) \cdot EXR$ $c \in CM$

The price in domestic currency for an imported good is equal to the world market price of that good, multiplied with 1 plus the ad valorem import tariff and the exchange rate defined as domestic currency over foreign currency.

Export price $PE_c = pwe_c \cdot (1 - te_c) \cdot EXR - TM_{c, 'row'}$ $c \in CE$

The price in domestic currency for an imported good is equal to the world market price of that good, multiplied with 1 minus the ad valorem export tax and the exchange rate defined as domestic currency over foreign currency.

Absorption $PQ_c \cdot QQ_c = PDS_c \cdot QD_c + PM_c \cdot QM_c$ $c \in CD$

The money paid for domestic absorption is equal to the domestic price multiplied with total quantity but must also be equal to the money paid for imports plus the money paid for goods that are produced and sold domestically.

Value of domestic production $PX_c \cdot QX_c = PDS_c \cdot QD_c + PE_c \cdot QE_c$ $c \in CX$

Total sales of domestic firms are equal to their sales at the domestic market plus their sales in the international market.

Activity price from

demand side $PA_a = \sum_{c \in C} PX_c \cdot \theta_{a,c}$ $a \in A$

The price of total output in an activity is equal to the sum of the price of each produced commodity times the output share of that commodity in one unit of production.

Input price of aggregate

Leontief intermediates $PLEO_a = \sum_{cleo \in C} (PQ_{cleo} + tq_{cleo,a} + TM_{cleo,a}) \cdot (1 + tv_{cleo,a}) \cdot ica_{cleo,a}$ $a \in A$

The price of the aggregate of those intermediate goods that are used in fixed proportion to output is equal to the sum of the share of that commodity in one unit of output times basic price of each commodity together with taxes and trade margins.

Activity price from

$$\begin{aligned} \text{supply side} \quad PA_a \cdot (1 - ta_a) \cdot QA_a &= \sum_{c \in C} FTM_{c,a} \cdot (1 + tv_{c,a}) \\ &+ PSC_a \cdot QSC_a + PLEO_a \cdot QLEO_a \end{aligned} \quad a \in A$$

Total income in an activity is equal to the price of its output times the quantity of output less activity taxes and it is also equal to total cost in the industry. Total cost is equal to fixed trade margins and the cost of the two main aggregates in the production function.

Price of substitutable costs

$$\text{plus operating surplus} \quad PSC_a \cdot QSC_a = PCAP_a \cdot QCAP_a + PSUB_a \cdot QSUB_a \quad a \in A$$

Price of substitutable costs less Capital

$$PSUB_a \cdot QSUB_a = PLAB_a \cdot QLAB_a + PSUBI_a \cdot QSUBI_a \quad a \in A$$

The sum of labour cost and cost for substitutable intermediates.

Consumer price

$$\text{Index} \quad CPI = \sum_{c \in C} \sum_{h \in H} (1 + tv_{c,h}) \cdot (PQ_c + tq_{c,h} + TM_{c,h}) \cdot cwts_{c,h}$$

6.4.2 Production Block

For simplicity the production function is divided into several equations aggregating together the different aggregates.

Demand for aggregate

$$\text{Leontief intermediates} \quad QLEO_a = inta_a \cdot QA_a \quad a \in A$$

Total quantity of intermediate inputs that are used in fixed proportions to output is always equal to that fixed proportion.

Demand for substitutable intermediates plus the capital stock

$$QSC_a = isc_a \cdot QA_a \quad a \in A$$

The quantity of all inputs with substitution possibilities has to be a constant share of output since they cannot be substituted against anything else.

QSC part of

$$\text{production function} \quad QSC_a = \alpha_a^{sc} \cdot \left(\delta_a^{sc} \cdot QCAP_a^{-\rho_a^{sc}} + (1 - \delta_a^{sc}) \cdot QSUB_a^{-\rho_a^{sc}} \right)^{\frac{-1}{\rho_a^{sc}}} \quad a \in A$$

The quantity of total substitutable inputs is a CES function of capital and other substitutable inputs.

QSUB part of

production function
$$QSUB_a = \alpha_a^{sub} \cdot \left(\delta_a^{sub} \cdot QLAB_a^{-\rho_a^{sub}} + (1 - \delta_a^{sub}) \cdot QSUBI_a^{-\rho_a^{sub}} \right)^{\frac{-1}{\rho_a^{sub}}} \quad a \in A$$

QSUB part of production function for activities not using substitutable intermediates

$$QSUB_a = QLAB_a \quad a \in A$$

The quantity of other substitutable inputs than capital is a CES function of total labour and total substitutable intermediate inputs if substitutable intermediate inputs are used in the activity. For other activities it is simply equal to aggregated labour.

Aggregation of different capital good

$$QCAP_a = \alpha_a^{cap} \cdot \left(\sum_{fcap \in FCAP} \delta_{fcap,a}^{cap} \cdot QF_{fcap,a}^{-\rho_a^{cap}} \right)^{\frac{-1}{\rho_a^{cap}}} \quad a \in A$$

The total quantity of capital is equal to a CES function of all different capital goods.

Aggregation of different intermediate aggregates

$$QSUBI_a = \alpha_a^{subi} \cdot \left(\sum_{cga \in CGA} \delta_{a,cga}^{subi} \cdot QCGA_{a,cga}^{-\rho_a^{subi}} \right)^{\frac{-1}{\rho_a^{subi}}} \quad a \in A$$

The total quantity of substitutable intermediate goods is a CES function of the different commodity groups defined by the user.

Aggregation of different labour aggregates

$$QLAB_a = \alpha_a^{lab} \cdot \left(\sum_{flab \in FLAB} \delta_{flab,a}^{lab} \cdot QF0_{flab,a}^{-\rho_a^{lab}} \right)^{\frac{-1}{\rho_a^{lab}}} \quad a \in A$$

The total quantity of labour is a CES function of the different labour categories defined by the user.

Aggregation of different substitutable intermediates

$$QCGA_{a,cga} = \alpha_a^{cga} \cdot \left(\sum_{csubi \in CSUBI} \delta_{csubi,a,cga}^{cga} \cdot QINTA_{csubi,a,cga}^{-\rho_{a,cga}^{cga}} \right)^{\frac{-1}{\rho_{a,cga}^{cga}}} \quad a \in A$$

$$cga \in CGA$$

The total quantity in each commodity group is a CES function of the specific substitutable intermediate goods.

**First order condition
QSC part of PF**

$$\frac{QCAP_a}{QSUB_a} = \left(\frac{PSUB_a \cdot \delta_a^{sc}}{PCAP_a \cdot (1 - \delta_a^{sc})} \right)^{\frac{1}{1 + \rho_a^{sc}}} \quad a \in A$$

**First order condition
QSUB part of PF**

$$\frac{QLAB_a}{QSUBI_a} = \left(\frac{PSUBI_a \cdot \delta_a^{sub}}{PLAB_a \cdot (1 - \delta_a^{sub})} \right)^{\frac{1}{1 + \rho_a^{sub}}} \quad a \in A$$

**First order condition
capital**

$$\begin{aligned} & WF_{fcap} \cdot \overline{WFDIST}_{fcap,a} \\ &= PCAP_a \cdot QCAP_a \cdot \left(\sum_{fcap' \in FCAP'} \delta_{fcap',a}^{cap} \cdot QF_{fcap',a}^{-\rho_a^{cap}} \right)^{-1} \cdot \delta_{fcap,a}^{cap} \cdot QF_{fcap,a}^{-\rho_a^{cap}-1} \quad \begin{array}{l} a \in A \\ f \in FCAP \end{array} \end{aligned}$$

This is the demand equation for a specific capital good

**First order condition
labour**

$$\begin{aligned} & WF_{flab} \cdot \overline{WFDIST}_{flab,a} = \\ & PLAB_a \cdot QLAB_a \cdot \left(\sum_{flab' \in FLAB'} \delta_{flab',a}^{lab} \cdot QF_{flab',a}^{-\rho_a^{lab}} \right)^{-1} \delta_{flab,a}^{lab} \cdot QF_{flab,a}^{-\rho_a^{lab}-1} \quad \begin{array}{l} a \in A \\ f \in FLAB \end{array} \end{aligned}$$

This is the demand equation for a specific category of labour

First order condition intermediate aggregates

$$\begin{array}{l} a \in A \\ f \in FLAB \end{array}$$

$$PCGA_a = PSUBI_a \cdot QSUBI_a \cdot \left(\sum_{cga' \in CGA'} \delta_{a,cga'}^{subi} \cdot QCGA_{a,cga'}^{-\rho_a^{subi}} \right)^{-1} \delta_{a,cga}^{subi} \cdot QCGA_{a,cga}^{-\rho_a^{subi}-1}$$

First order condition substitutable intermediates

$$a \in A$$

$$c \in CSUBI$$

$$cga \in CGA$$

$$(1 + tv_{csubi,a}) (PQ_{csubi} + tq_{csubi,a} + TM_{csubi,a}) =$$

$$PCGA_{a,cga} \cdot QCGA_{a,cga} \cdot \left(\sum_{csubi' \in CSUBI'} \delta_{csubi',a,cga}^{cga} \cdot QINTA_{csubi',a,cga}^{-\rho_a^{cga}} \right)^{-1} \delta_{csubi,a,cga}^{cga} \cdot QINTA_{csubi,a,cga}^{-\rho_a^{cga}-1}$$

This is the demand equation for a specific substitutable intermediate good. For every commodity group this equation would only be defined for the commodities in that group.

Aggregation of intermediate demand of substitutables

$$QINT_{csubi,a} = \sum_{cga \in CGA} QINTA_{csubi,a,cga}$$

$$a \in A$$

$$c \in CSUBI$$

In practise this is not a sum since for every commodity QINTA is only defined for one CGA. This equation is needed only because we need the variable to be defined with and without the cga index.

Disaggregated demand of leontief intermediates

$$QINT_{cleo,a} = ica_{cleo,a} \cdot QLEO_a$$

$$a \in A$$

$$c \in CLEO$$

This is the demand equation for a specific leontief intermediate good The parameter ica defines the share of every specific commodity in one unit of aggregate leontief intermediates.

Aggregation of output from

different industries (the make matrix) $QX_c = \sum_{a \in A} \theta_{a,c} \cdot QA_a$ $c \in C$

Total output of a specific commodity is the sum of the produced quantity of that commodity in all activities. The model assumes no substitution between outputs i.e secondary products must be produced in specific proportions to the main product. If this assumption is regarded unrealistic the user may prefer to redistribute secondary products usin BWSEC.GMS

6.4.3 Investment block

Gross return to a specific capital $GRPROF_{a,fcap} = WF_{fcap} \cdot \overline{WFDIST}_{fcap,a} \cdot QF_{fcap,a} \quad a \in A$

This equation calculates gross return of a specific capital good in a specific activity. It is used in the calculation of foreign liabilities in the dynamic simulation file.

Investment demand $QINV_{c,a,fcap} = IADJ_{a,fcap} \cdot \overline{qinv}_{c,a,fcap} \quad a \in A$
 $c \in C$
 $fcap \in FCAP$

In static simulations the variable IADJ changes total investment if the siclos 3 is chosen. In other cases it is fixed and thus investment is equal to the value in base. In dynamic simulation investments is endogenous. Between every solution of the model IADJ is recalculated according to Tobin q in for that capital good in that activity.

Unit price of aggregate investment

$$PI_{a,fcap} = \sum_{c \in C} (1 + tvi_{c,a}) (PQ_c + tqi_{c,a} + TMI_{c,a}) \cdot iwts_{c,a,fcap} \quad a \in A$$

$$fcap \in FCAP$$

The price of a capital good is equal to the sum of the prices on the goods it is made of times the share of these goods in one unit of the capital good.

Aggregate investment demand

$$QAGGINV_{a,fcap} = \sum_{c \in C} \frac{(1 + tvi_{c,a}) (PQ_c + tqi_{c,a} + TMI_{c,a}) \cdot QINV_{c,a,fcap}}{PI_{a,fcap}} \quad a \in A$$

$$fcap \in FCAP$$

The quantity if investment is equal to the money spent on investments divided by the price per unit of that capital good.

6.4.4 Trade Block

CET function $QX_c = \alpha_c^t \cdot \left(\delta_c^t \cdot QE_c^{\rho_c^{cet}} + (1 - \delta_c^t) \cdot QD_c^{\rho_c^{cet}} \right)^{\frac{1}{\rho_c^{cet}}} \quad c \in (CE \cap CD)$

The CET equation addresses the allocation of marketed domestic output to two alternative markets, the domestic market and exports. It reflects the assumption of imperfect transformability between the two destinations.

Export - domestic supply ratio $\frac{QE_c}{QD_c} = \left(\frac{PE_c}{PDS_c} \cdot \frac{1 - \delta_c^t}{\delta_c^t} \right)^{\frac{1}{1 + \rho_c^t}} \quad c \in (CE \cap CD)$

Producers are assumed to allocate their selling efforts between the domestic and foreign market according to the relative prices on the different markets.

Note that this is not the f.o.c of the CET equation if $\rho_c^t \neq \rho_c^{cet}$ giving a possibility to relax the assumption of optimal behaviour in international trade.

**Output transformation
when not both
export and domestic
consumption exist**

$$QX_c = QD_c + QE_c \quad c \in (CD \cap CEN) \cup (CE \cap CDN)$$

Armington
$$QQ_c = \alpha_c^q \cdot \left(\delta_c^q \cdot QM_c^{-\rho_c^{arm}} + (1 - \delta_c^q) \cdot QD_c^{-\rho_c^{arm}} \right)^{\frac{1}{\rho_c^{arm}}} \quad c \in (CM \cap CD)$$

The Armington equation assumes imperfect substitutability between domestic production and imports.

Import - domestic supply ratio
$$\frac{QM_c}{QD_c} = \left(\frac{PDS_c}{PM_c} \cdot \frac{\delta_c^q}{1 - \delta_c^q} \right)^{\frac{1}{1 + \rho_c^q}}$$

$$c \in (CM \cap CD)$$

The choice between domestic production and imports will depend on their relative price. Note that this is not the f.o.c of the Armington equation if $\rho_c^q \neq \rho_c^{arm}$ giving a possibility to relax the assumption of optimal behaviour in international trade.

**Armington when not both import
and domestic production**

$$QQ_c = QD_c + QM_c \quad c \in (CD \cap CMN) \cup (CM \cap CDN)$$

6.4.5 Trade Margins Block

Demand for trade margins (retail service)

$$c \in CTR$$

$$QT_{ctr} = shtr_{ctr} \cdot \left(\begin{aligned} & \sum_{a \in A} \sum_{c \in C} trq_{c,a} \cdot QINT_{c,a} + \sum_{h \in H} \sum_{c \in C} trq_{c,h} \cdot \sum_{cgh \in CGH} QH_{h,c,cgh} + \sum_{a \in A} \sum_{c \in C} trqi_{c,a} \cdot \sum_{fcap \in FCAP} QINV_{c,a,fcap} \\ & + \sum_{c \in C} trq_{c,gov} \cdot QG_c + \sum_{c \in C} trq_{c,row} \cdot QE_c + \sum_{pni \in PNI} \sum_{c \in C} fmq_{c,pni} + \sum_{a \in A} \sum_{c \in C} fmq_{inv_{c,a}} \end{aligned} \right)$$

The sum of all trade margins for all purchasers and commodities defines total demand of retail services.

Determination of trade margins
$$TM_{c,pni} = \sum_{ctr \in CTR} trq_{c,pni} \cdot shtr_{ctr} \cdot PQ_{ctr} \quad c \in C$$

$$pni \in PNI$$

Determination of trade margins

on investments

$$TMI_{c,a} = \sum_{ctr \in CTR} trqi_{c,a} \cdot shtr_{ctr} \cdot PQ_{ctr} \quad \begin{array}{l} c \in C \\ a \in A \end{array}$$

The size of the trade margin will depend on the amount of retail service that is required when this commodity is purchased by this actor and the unit price of retail services. If retail services consist of more than one commodity i.e if the set CTR consist of more than one element this equation will become a sum over the different commodities in retail services.

Fix trade margins

$$FTM_{c,pni} = \sum_{ctr \in CTR} ftmq_{c,pni} \cdot shtr_{ctr} \cdot PQ_{ctr} \quad \begin{array}{l} c \in C \\ pni \in PNI \end{array}$$

Fix TM on investments

$$FTMINV_{c,a} = \sum_{ctr \in CTR} ftmq_{inv_{c,a}} \cdot shtr_{ctr} \cdot PQ_{ctr} \quad \begin{array}{l} c \in C \\ a \in A \end{array}$$

Fix trade margins is independent of the quantity purchased. It is used to get more realistic price of a marginal unit of the good.

6.4.6 Institution block

Labour income

$$YFLAB_{flab} = \sum_{a \in A} WF_{flab} \cdot WFDIST_{flab,a} \cdot QF_{flab,a} + labin_{flab} \cdot EXR \quad flab \in FLAB$$

Total labour income is equal to the wage rate times the amount of labour in each activity plus labour income from abroad.

Income of financial assets

$$YFIN_a = \sum_{fcap \in FCAP} WF_{fcap} \cdot WFDIST_{fcap,a} \cdot QF_{fcap,a} \quad fin \in FIN$$

Income of the shares in an activity is equal to operating surplus. All profits are assumed to be paid out to the households.

Institutional labour income

$flab \in FLAB$

$i \in INSD$

$$YIFL_{i,flab} = shifl_{i,flab} \cdot \left[(1 - tflab_{flab}) \cdot YFLAB_{flab} - labour_{flab} \cdot \sum_{a \in A} WF_{flab} \cdot wfdist_{flab,a} \cdot QF_{flab,a} \right]$$

Labour income to a specific institution (most often household) are equal to this institutions share of labour income less labour taxes and labour income to other countries.

Institutional financial income

$$YIFIN_i = \sum_{a \in A} \left[shifin_{i,a} \cdot (1 - tfin) \cdot (1 - finout_a) \cdot YFIN_a \right] + shifinin_i \cdot (1 - tfin) \cdot finin \cdot EXR \quad i \in INSD$$

Financial income to a specific institution is equal to this institutions share of the return from domestic financial assets less capital taxes and financial income to other countries plus this institutions share of the return from foreign assets less taxes.

Income of domestic households

$h \in H$

$$YH_h = \sum_{flab \in FLAB} YIFL_{h,flab} + YIFIN_h + trnsfr_{h,gov} \cdot CPI + trnsfr_{h,row} \cdot EXR$$

Total income of a domestic household is equal to the sum of labour income financial income, transfers from the government and transfers from the rest of the world.

Wealth of the household

$h \in H$

$$WEALTH_h = \sum_{a \in A} shifin_{h,a} \cdot (1 - finout_a) \cdot \sum_{fcap \in FCAP} \frac{GRPROF_{a,fcap}}{deprate_{a,fcap} + RETURN} + shifinin_h \cdot FAP$$

Total wealth of a household is equal to the value of their share of domestic financial assets plus the value of their share of foreign assets.

Households' consumption expenditure

$h \in H$

$$EH_h = MPC_h \cdot \left((1 - TINS_h) \cdot YH_h - trnsfr_{row,h} \right) + cint_h \cdot WEALTH_h$$

Total consumption in a household is equal to MPC times their income less taxes and transfers to other countries plus an intercept that is dependent on the wealth of the household. In dynamic simulations the WEALTH will change over time and thus influence household saving/consumption decision.

The EH Les equations

$h \in H$

$cgh \in CGH$

$$QCGH_{h,cgh} \cdot PCGH_{h,cgh} = PCGH_{h,cgh} \cdot \gamma_{cgh,h} + \beta_{cgh,h} \cdot \left(EH_h - \sum_{c' \in C'} FTM_{c',h} (1 + tv_{c',h}) - \sum_{cgh' \in CGH'} PCGH_{h,cgh'} \cdot \gamma_{cgh',h} \right)$$

This is the LES equation defining fixed shares of spending on the different commodity groups for that part of household expenditure that is not used up for subsidiary consumption.

The EH CES equations

$$h \in H$$

$$cgh \in CGH$$

$$QCGH_{h,cgh} = \alpha_{h,cgh}^{cgh} \cdot \left(\sum_{c \in C} \delta_{h,c,cgh}^{cgh} \cdot QH_{h,c,cgh}^{-\rho_{h,cgh}^{cgh}} \right)^{\frac{-1}{\rho_{h,cgh}^{cgh}}}$$

The total quantity of a commodity group is a CES equation of the specific commodities.

Households commodity demand

$$h \in H$$

$$cgh \in CGH$$

$$c \in C$$

$$(1 + tv_{c,h}) (PQ_c + tq_{c,h} + TM_{c,h}) =$$

$$QCGH_{h,cgh} \cdot PCGH_{h,cgh} \cdot \left(\sum_{c' \in C'} \delta_{h,c',cgh}^{cgh} \cdot QH_{h,c',cgh}^{-\rho_{h,cgh}^{cgh}} \right)^{-1} \cdot \sum_{c \in C} \delta_{h,c,cgh}^{cgh} \cdot QH_{h,c,cgh}^{-\rho_{h,cgh}^{cgh} - 1}$$

The first order condition of the CES equation determines the demand for a specific commodity.

Government consumption demand

$$QG_c = GADJ \cdot qbarg_c \quad c \in CGOV$$

GADJ is a fixed adjustment factor for the size of government consumption. It becomes flexible in static simulations when siclos 4 or 5 is used where it is instade government consumption as share of domestic absorption that is constant.

Government revenue

$$YG = \sum_{h \in H} TINS_h \cdot YH_h + \sum_{flab \in FLAB} \left(tflab_{flab} \cdot YFLAB_{flab} - labout_{flab} \cdot \sum_{a \in A} WF_{flab} \cdot WFDIST_{flab,a} \cdot QF_{flab,a} \right)$$

$$+ tfin \cdot \left(\sum_{a \in A} (1 - finout_a) \cdot YFIN + finin \cdot EXR \right) + \sum_{c \in CM} tar_c \cdot pwm_c \cdot QM_c \cdot EXR + \sum_{c \in CE} te_c \cdot pwe_c \cdot QE_c \cdot EXR$$

$$+ \sum_{flab \in FLAB} YIFL_{gov,flab} + YIFIN_{gov} + transfr_{gov,row} \cdot EXR + \sum_{a \in A} \sum_{c \in C} [tq_{c,a} \cdot QINT_{c,a} + tv_{c,a} \cdot (PQ_c + tq_{c,a} + TM_{c,a}) \cdot QINT_{c,a}]$$

$$+ \sum_{a \in A} ta_a \cdot PA_a \cdot QA_a + \sum_{h \in H} \sum_{c \in C} \sum_{cgh \in CGH} [tq_{c,h} \cdot QH_{h,c,cgh} + tv_{c,h} \cdot (PQ_c + tq_{c,h} + TM_{c,h}) \cdot QH_{h,c,cgh}]$$

$$+ \sum_{a \in A} \sum_{c \in C} \left[tqi_{c,a} \cdot \sum_{fcap \in FCAP} QINV_{c,a,fcap} + tv_{c,a} \cdot (PQ_c + tqi_{c,a} + TMI_{c,a}) \cdot \sum_{fcap \in FCAP} QINV_{c,a,fcap} \right]$$

$$+ \sum_{c \in C} [tq_{c,gov} \cdot QG_c + tv_{c,gov} \cdot (PQ_c + tq_{c,gov} + TM_{c,gov}) \cdot QG_c]$$

$$+ \sum_{pmi \in PNI} \sum_{c \in C} (tv_{c,pmi} \cdot FTM_{c,pmi}) + \sum_{a \in A} \sum_{c \in C} (tvi_{c,a} \cdot FTMINV_{c,a})$$

Government revenue is the sum of all kinds of taxes plus factor income and transfers from abroad.

Government expenditure

$$EG = \sum_{c \in C} (PQ_c + tq_{c,gov} + TM_{c,gov}) \cdot (1 + tv_{c,gov}) \cdot QG_c \\ + \sum_{h \in H} trnsfr_{h,gov} \cdot \bar{CPI} + trnsfr_{row,gov} + \sum_{c \in C} (FTM_{c,gov}) \cdot (1 + tv_{c,gov})$$

Government expenditure is equal to government consumption plus transfers to households and other countries.

6.4.7 System constraints block

Factor market $\sum_{a \in A} QF_{f,a} = QFS_f \quad f \in F$

Total factor supply is equal to factor supply in all activities.

Market for retail services $c \in CTR$

$$QQ_{ctr} = \sum_{a \in A} QINT_{ctr,a} + \sum_{h \in H} \sum_{cgh \in CGH} QH_{h,ctr,cgh} + QG_{ctr} + \sum_{a \in A} \sum_{fcap \in FCAP} QINV_{ctr,a,fcap} + qdst_{ctr} + QT_{ctr}$$

Commodity market for other commodities than retail services $c \in CNTR$

$$QQ_c = \sum_{a \in A} QINT_{c,a} + \sum_{h \in H} \sum_{cgh \in CGH} QH_{h,c,cgh} + QG_c + \sum_{a \in A} \sum_{fcap \in FCAP} QINV_{c,a,fcap} + qdst_c$$

Supply equal demand in all commodity markets. This equation is divided into two since QT is only defined for retail services.

Current account balance

$$\sum_{c \in CM} pwm_c \cdot QM_c \cdot EXR + \sum_{flab \in FLAB} labour_{flab} \cdot \sum_{a \in A} WF_{flab} \cdot WFDIST_{flab,a} \cdot QF_{flab,a} \\ + \sum_{a \in A} \sum_{fcap \in FCAP} finout_a \cdot GRPROF_{a,fcap} + \sum_{i \in I} trnsfr_{row,i} \\ = \sum_{c \in CE} pwe_c \cdot QE_c \cdot EXR + \sum_{flab \in FLAB} labin_{flab} \cdot EXR + \\ finin \cdot EXR + \sum_{i \in I} trnsfr_{i,row} \cdot EXR + FSAV$$

If we have a surplus in the current account it has to be equal to foreign saving (financial account). Note that factor payments to rest of the world are dependent on domestic wage levels and profit rates. In dynamic simulations financial return from other countries (finin) will be dependent on foreign asset position. Transfers from abroad are fix in foreign currency while transfers to other countries are fixed in domestic currency.

Government balance $YG = EG + GSAV$

Government saving is equal to income less expenditure.

Direct institutional tax rates

$$TINS_h = tinsbar_h \cdot (1 + TINSADJ \cdot tins01_h) + DTINS \cdot tins01_h \quad h \in H$$

The direct income tax can be adjusted in specific closures. In govclos 2 DTINS is adjusted while in govclos 3 TINSADJ is adjusted.

Labour tax rates

$$TFLAB_{flab} = tflbar_{flab} \cdot (1 + TLABADJ) \quad f \in FLAB$$

In govclos 4 the labour tax rate is adjusted through the adjustment factor TLABADJ for the government saving to remain constant.

Capital tax rates

$$TFIN = tfinbar \cdot (1 + TFINADJ)$$

In govclos 5 the capital tax rate is adjusted through the adjustment factor TFINADJ for the government saving to remain constant.

Households marginal propensity to consume

$$MPC_h = mpcbar_h \cdot (1 + MPCADJ \cdot mpc01_h) + DMPC \cdot mpc01_h \quad h \in H$$

In specific closures the marginal propensity to consume is adjusted. In siclos 1 and 4 DMPC is adjusted while in siclos 2 and 5 MPCADJ is adjusted.

Saving investment balance

$$\begin{aligned} & \sum_{h \in H} \left[(1 - MPC_h) \cdot (1 - TINS_h) \cdot YH_h - trnsfr_{row',h} \right] - cint_h \cdot WEALTH_h \\ & + GSAV + FSAV = \\ & \sum_{c \in C} \sum_{a \in A} (PQ_c + tqi_{c,a} + TMI_{c,a}) \cdot (1 + tvi_{c,a}) \cdot \sum_{fcap \in FCAP} QINV_{c,afcap} \\ & + \sum_{c \in C} PQ_c \cdot qdst_c + \sum_{a \in A} \sum_{c \in C} \left[(1 + tvi_{c,a}) \cdot FTMINV_{c,a} \right] + walras \end{aligned}$$

Total saving is always equal to total investments if walras law holds. If that not hold the variable walras will not be equal to zero and an error message will be received.

6.4.8 Closure block

Total absorption

$$\begin{aligned}
 TABS &= \sum_{h \in H} \sum_{c \in C} (1 + tv_{c,h}) \cdot (PQ_c + tq_{c,h} + TM_{c,h}) \cdot \sum_{cgh \in CGH} QH_{h,c,cgh} \\
 &+ \sum_{c \in C} (1 + tv_{c,gov}) \cdot (PQ_c + tq_{c,gov} + TM_{c,gov}) \cdot QG_c \\
 &+ \sum_{c \in C} \sum_{a \in A} \sum_{fcap \in FCAP} (1 + tvi_{c,a}) \cdot (PQ_c + tqi_{c,a} + TMI_{c,ga}) \cdot QINV_{c,a,fcap} + \sum_{c \in C} PQ_c \cdot qdst_c
 \end{aligned}$$

Ratio of investments to absorption

$$\begin{aligned}
 INVSHR \cdot TABS &= \sum_{c \in C} \sum_{a \in A} \sum_{fcap \in FCAP} (1 + tvi_{c,a}) \cdot (PQ_c + tqi_{c,a} + TMI_{c,ga}) \cdot QINV_{c,a,fcap} \\
 &+ \sum_{c \in C} \sum_{a \in A} FTMINV_{c,a} \cdot (1 + tvi_{c,a}) + \sum_{c \in C} PQ_c \cdot qdst_c
 \end{aligned}$$

Ratio of government consumption to absorption

$$GOVSHR \cdot TABS = \sum_{c \in C} (PQ_c + tq_{c,gov} + TM_{c,gov}) \cdot (1 + tv_{c,gov}) \cdot QG_c + \sum_{c \in C} (1 + tv_{c,gov}) \cdot FTM_{c,gov}$$

These three equations are needed in SICLOS 4 and 5 in SAINTSIM where investments and government consumption is not fixed in levels but as shares of domestic absorption. The last equation is needed in SAINTDYNAMSIM as well.

References

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