

**ASSESSING ECONOMIC AND FISCAL REFORMS IN LEBANON.  
A DYNAMIC CGE ANALYSIS WITH DEBT CONSTRAINTS**

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**Abstract**

*Since the early Nineties, Lebanon has been undertaking a number of economic reforms, covering in particular international trade and internal fiscal policy issues. Simultaneously, debt has been skyrocketing, partially justified by reconstruction needs after the end of the civil war. Fostering economic growth seems to be the only way out of the debt trap, but reforms intended to stimulate growth may well have adverse short run effects on public and external deficits. We construct a dynamic open economy CGE model with debt constraints in the sense that external debt requires physical capital as collateral. This model allows us to study the effects of a number of important economic policy issues (fiscal policy reform, WTO membership, FDI) in a multisectoral growth setting under the realistic assumption that debt constraints relax when the economy starts growing.*

**Key words:** dynamic CGE, Lebanon, trade liberalisation, FDI, political stability

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## 1 Introduction

Lebanon has taken major steps towards integration in the world trading system. On the one hand, an Association Agreement<sup>1</sup> with the European Union was signed June 17, 2002, in Luxembourg. On the other hand, an “observer” status at the WTO was granted since 1999. Both decisions constitute important achievements towards trade liberalisation and further opening up to foreign capital flows. Besides, Lebanon is a member of the GAFTA<sup>2</sup> since its creation in 1997 and joined the Agadir Agreement in 2004. A number of bilateral treaties with OECD as well as other world countries are intended to further promote the free exchange of goods, services and capital.

The preferential as well as the non-discriminatory trade liberalisation process takes place in the immediate aftermath of the introduction of a number of internal fiscal policy reform measures. These have been undertaken with the primary goal of addressing fiscal imbalances, which have been steadily growing since the mid Nineties. Important modernization steps have involved the introduction of the VAT in 2002 and of a General Income Tax in 2003. This was preceded (in 2002) by the introduction of a new system for the deduction at source of the income tax on salaries and wages. Such measures, broadening and consolidating the tax base, are expected to counteract the likely negative fiscal impacts of trade liberalisation due to the loss in public revenues. Even after the tariff system reform of 1993 which brought about a strong simplification of the tariff structure, the protection level in Lebanon is still somewhat high if compared to other developing countries (see Haddad, 2004). The high reliance of government revenues on tariff proceeds has been pointed out by Nashashibi (2002) and reinforced by Dessus and Ghaleb (2004); the latter authors report that even after the 2001 unilateral tariff cut tariff proceeds amount to about one quarter of government revenues. According to an older paper by Ghesquiere (1998), the preferential liberalisation with the EU will account for a revenue loss equal to 4.2% of GDP over the agreement implementation period.

As to the expected benefits of trade liberalisation, previous studies suggest only limited gains in terms of trade creation, lower import prices, higher consumer welfare or increased competition in the domestic economy. See, e. g., Martin (1996, 2000) who studies the impact of the implementation of the Agreement by means of a simple static CGE model. In his study, losses from trade diversion are found to exceed the gains from trade creation, resulting in a net welfare loss of 0.3% of GDP. This is due to the high share of Lebanon’s imports from the EU (around of 50%) and to the broadly poor export performance. Similar results are obtained by Dessus and Ghaleb (2004) under the assumption, however, of a non-discriminatory

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<sup>1</sup> Henceforth “the Agreement”.

<sup>2</sup> Greater Arab Free Trade Agreement

trade liberalisation. They quantify the loss of public revenues at about 17% with only a negligible increase in GDP (0.1%). Somewhat larger benefits (GDP growth of 0.4%) are achieved by introducing further reforms, in particular by removing regulations with anticompetitive effect.

The static structure of these models does not allow for the quantification of long-run welfare gains. It is one aim of this paper to close this gap and allow for productive investment and sectoral reallocation of capital, which may possibly induce additional GDP growth and positive welfare effects.

In order to account for investment, we also need to consider FDI, i. e. the import of resources from the world capital market. We model Lebanon's integration in the global economy with limited capability of international borrowing. Specifically, we follow Barro, Mankiw and Sala-i-Martin (1995) in assuming that external debt requires collateral and that only physical capital can serve this purpose. Due to lack of data, however, we do not consider human capital as the limiting factor in capital demand, but rather adopt the approach of Cohen and Sachs (1986), who postulate that only a fraction  $\nu$  of the actual capital stock may be used as collateral. Using this setup, which, to our knowledge, is novel in the CGE literature, we calibrate a dynamic multi-sectoral CGE model on the most recent National Accounts' data (MOET<sup>3</sup>, 2003).

Our simulations confirm that positive effects of trade liberalization on aggregate economic activity in Lebanon arise only in a long term perspective. This is consistent with the economic effects of increasing Lebanon's access to the world capital market. Positive developments in political stability, for instance due to Syrian withdrawal from Lebanon or peaceful settlement of ethnic or religious disputes raise the reliability of Lebanon as a debtor on the world capital market and thus lower the collateral requirements. We show that progress along these lines is economically much more important than all efforts of trade liberalization. However, it should be borne in mind that trade liberalization and political stabilization are by no means rivalrous. Quite to the contrary, they may positively influence each other and the EU's Association Agreement is actually intended to make progress along both fronts.

The sequel of the paper is organised as follows. Section 2 provides an overview on main agreements on trade and investments involving Lebanon. Section 3 describes the structure of the CGE model that we use to quantify the effects. Section 4 briefly presents main calibration issues and the solution technique. Simulations of gradual non-discriminatory tariff reductions and economic benefits of political stability are carried out in Section 5. The main conclusions are summarised in the final section of this paper.

## 2 *Main agreements on trade and investments*

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<sup>3</sup> Ministry of Economy and Trade of the Republic of Lebanon

Lebanon's integration process in the global economy is based on several trade agreements and various treaties. The former include multilateral agreements (as in the case of WTO) and agreements at regional and bilateral level. Main regional agreements are the Euro-Mediterranean Agreement, enhancing North-South integration as well as the GAFTA and the Agadir Agreement promoting South-South integration. Bilateral agreements have been signed with, Syria (1993), Kuwait (1996), Egypt (1998) and United Arab Emirates (2000). The Agreement for an Economic Free Trade Zone with Jordan (signed in 1992) is expected to be ratified before long. The latter involves a number of treaties on investments promotion and protection as well as on double taxation.

The Euro-Mediterranean Agreement is part a wider programme, known as the Euro-Mediterranean Barcelona Process, involving a number of Mediterranean Countries. This follows the 1977 Cooperation Agreement (signed on 3<sup>rd</sup> May 1977) which grants Lebanese industrial exports duty-free access to EU markets (after satisfying strict rules of origins). The Agreement covers several areas of cooperation, including the progressive liberalization of trade in goods through a gradual phasing out of tariff and non-tariff barriers, with the ultimate goal of establishing a Mediterranean Free Trade Area. In addition, the Agreement seeks to liberalise trade in services and the rights of establishment, while widening the FDI potential in Lebanon.

The ratification by all parties being a compulsory condition for the Association Agreement to enter into force<sup>4</sup>, Lebanon and the European Community signed<sup>5</sup> a bilateral Interim Agreement. This reproduces the provisions contained in the Agreement on trade and trade-related matters. From the fifth year onward, customs duties on industrial imports from the EU will be gradually reduced by 12% per year until they reach zero in year 12. The phasing of tariff elimination will take into account the extent to which such imports compete with domestic production. Reductions in tariffs on agricultural products, fisheries and processed agricultural products are scheduled in a single shot in the fifth year. Customs duties on processed agricultural products are to fall by a maximum of 30% of the original duty, provided that imports currently subject to a 5% tariff will be fully liberalized. Lebanon's exports of industrial products to the EU will continue to be allowed free of customs duties (with the exception of certain listed sensitive agricultural and agro-industrial imports) as already granted in the Cooperation Agreement.

The GAFTA deals essentially with trade in goods. Tariffs reduction commitment foresees a 10% per year reduction over a period of 10 years, beginning in 1998. As foreseen by the agreement, Lebanon (as well as each other partner) was allowed to draw up a list of products to be excluded from the tariff reduction scheme for the first three years to allow industry restructuring. The trade liberalization process has been accelerated in 2002 and is expected to achieve full reduction by 2005.

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<sup>4</sup> Lebanon ratified the Agreement in December 2002

<sup>5</sup> March 2003

The Agadir Agreement was signed in May 2001 by two Maghreb Countries (Morocco and Tunisia) and two Mashreq Countries (Egypt and Jordan) in order to establish a free trade area open to the other Maghreb<sup>6</sup> and Mashreq<sup>7</sup> Countries. According to Hamoudeh (2002) the process for achieving the free trade area should start in 2003 with a 65% reduction in tariffs, followed by a further 15% in 2004, and a 10% in 2005 and 2006. Agadir Member Countries are allowed to cumulate the Agadir rules of origin with those of the Euro-Mediterranean Agreement, while granting their exports an easier access to the EU markets.

In February 1999 Lebanon submitted its application for accession to the WTO and two months later (April 1999) was granted the observer status. A National Committee on Accession to the WTO was established in May 1999 with the task of investigating the existing legal framework in order to ensure conformity of Lebanon's foreign trade regime to the WTO requirements. Following the submission of its Memorandum on the Foreign Trade Regime in May 2001, the first round of negotiations was held in October 2002 and a second round in December 2003. The WTO membership will eventually constitute a proof for a stable and conducive investment environment. The investors' perceived commercial risk in investing in Lebanon will descend, while attracting new FDI.

Further investment promotion is sought through a number of treaties with Armenia, Australia, Austria, Azerbaijan, Belarus, Benelux, Bulgaria, Canada, Chile, China, Croatia, Cuba, Cyprus, Czech Republic, Egypt, Finland, France, Gabon, Germany, Greece, Hungary, Iran, Italy, Kuwait, Malaysia, Morocco, the OPEC Fund, Pakistan, Romania, Russia, Slovakia, Spain, Sweden, Switzerland, Syria, Tunisia, Ukraine, Uruguay, the United Arab Emirates, the United Kingdom and Yemen. Treaties for the avoidance of double taxation have already been ratified with 20 countries and signed with eight additional countries.

### 3 *The Model*

#### 3.1 *A dynamic CGE Model*

The Lebanese economy is modelled as a dynamic small open economy (cf. Devarajan and Go (1998)), i. e. the domestic country is a price taker on international markets. International borrowing is constrained by the requirement of collateral, adopting ideas of Cohen and Sachs (1986) and Barro, Mankiw and Sala-i-Martin (1995). Specifically, external debt requires collateral and only a fraction  $\nu$  of physical capital can serve this purpose.

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<sup>6</sup> Algeria, Libya, Mauritania

<sup>7</sup> Israel, Lebanon, Syria and the Gaza Strip

The model is formulated in discrete time. In each period  $t$  a population of  $\Omega_t$  identical individuals grows at a constant exogenous rate of  $\gamma_\Omega$ . The population cannot migrate. Labor, however, can move freely between domestic production activities.

The economy consists of  $N$  mono-product industries (activities), each one producing a specific commodity indexed  $m=1,2,\dots,M$ <sup>8</sup>. Since the number of primary factors is lower than  $M$  (number of tradable commodities), an “overspecialisation” problem arises from the assumption of constant returns to scale technologies (see Samuelson 1953). This is solved through the adoption of the familiar Armington approach. Under this assumption, domestic actors consider commodities with identical statistical classification but different country of origin as imperfect substitutes. Imported and domestically produced commodities are used to “produce” an aggregate commodity, the Armington good, which is defined by a conventional CES function, unambiguously identified by the scale parameter,  $\vartheta_m$ , the elasticity,  $\mu$  and the share parameter,  $\varphi_m$ . Armington goods can be used either for consumption, which may be public or private, or for investment or as intermediate for production. The specifications of the CES function for each of the four types of Armington aggregates (private and public consumption, intermediates and investment) are to be found in the appendix (equations A.6, A.16, A.25, A.34). On the export side, an analogous approach is based on so-called CET functions, as shown below.

Aggregate quantities (e. g. for consumption or investment) are also obtained from CES aggregators defined by the scale parameter,  $\zeta$ , the elasticity,  $\kappa$  and the share parameter,  $\psi_m$ . The three CES aggregators (for private and public consumption as well as for investment) are reported in the appendix (equations A.21, A.30, A.39).

Perfect competition is assumed on all markets. Production factors are fully homogeneous and mobile across sectors.

### 3.2 Production

Sectoral production  $Q_t^n$  employs value added  $V_t^n$  and  $M$  intermediate inputs according to the following Leontief fixed coefficients’ production function:

$$Q_t^n = \min \left\{ V_t^n, \frac{x_t^{1,n}}{a_{1,n}}, \frac{x_t^{2,n}}{a_{2,n}}, \dots, \frac{x_t^{m,n}}{a_{m,n}}, \dots, \frac{x_t^{M,n}}{a_{M,n}} \right\}, \quad n = 1, 2, \dots, N, \quad (1)$$

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<sup>8</sup> Under this assumption, the total number of produced commodities  $M$  equals the total number of industries  $N$ .

where  $x_t^{m,n}$  indicates the intermediate input  $m$  used for production of commodity  $n$  and  $a_{m,n}$  is the corresponding fixed input requirement. Each intermediate commodity  $m$  is an Armington aggregate of domestic origin  $D_t^{m,n,F}$  and imported origin  $M_t^{m,n,F}$  (see equation A.6). Value added,  $V_t^n$  is produced with three primary inputs: physical capital  $K_t^n$ , labour services  $u_t^n \Omega_t^n$  (being  $u_t^n$  the number of per-capita worked hours in sector  $n$  during period  $t$ ) and land  $La_t^n$ . Value added of firm  $n$  is generated under a Cobb-Douglas technology specified as follows:

$$V_t^n = as_n \cdot A_t^n \left[ (K_t^n)^{bk_n} \cdot (u_t^n \Omega_t^n)^{bh_n} \cdot (La_t^n)^{bla_n} \right], \quad (2)$$

where  $as_n$  is a fixed (time-invariant) technology parameter and  $A_t^n$  is the total factor productivity, both sector specific. The assumption of constant returns to scale implies:

$$bk_n + bh_n + bla_n = 1. \quad (3)$$

Since firms operate in a fully competitive environment  $bk_n$ , the production elasticity of capital services represents the capital income share.

Profit maximization requires cost minimization. Since the cost of intermediates is constant for a given amount of output, the cost minimization problem involves the minimization of the total cost of value added:

$$VC_t^n \equiv p_t^n r_t \cdot K_t^n + q_t \cdot u_t^n \Omega_t^n + pla_t^n \cdot La_t^n, \quad (4)$$

under the value added production function constraint. The minimal value added cost is:

$$VC_t^n = \frac{(p_t^n r_t)^{bk_n} (q_t)^{bh_n} (pla_t^n)^{bla_n}}{(bk_n)^{bk_n} (bh_n)^{bh_n} (bla_n)^{bla_n} \cdot as_n \cdot A_t^n} \cdot y_t^n, \quad (5)$$

where

$$P_t^{V,n} = \frac{(p_t^n r_t)^{bk_n} (q_t)^{bh_n} (pla_t^n)^{bla_n}}{(bk_n)^{bk_n} (bh_n)^{bh_n} (bla_n)^{bla_n} \cdot as_n \cdot A_t^n}, \quad (6)$$

is the price of value added. Primary factors' demand functions are:

$$K_t^n = \frac{bk_n}{P_t^{V,n} p_t^n r_t} \cdot y_t^n, \quad (7)$$

$$u_t^n \Omega_t^n = \frac{bh_n}{q_t} P_t^{V,n} \cdot y_t^n, \quad (8)$$

$$La_t^n = \frac{bla_n}{pla_t^n} P_t^{V,n} \cdot y_t^n. \quad (9)$$

The maximization of value added profit:

$$\Pi_t^{V,n} = P_t^{V,n} \cdot V_t^n - VC_t^n, \quad (10)$$

gives the optimal demand of primary factors’.

Domestic production  $Q_t^n$  ( $n=1,2,\dots,N$ ) satisfies domestic demand for domestic goods,  $D_t^m$  ( $m=1,2,\dots,M$ ) and foreign demand  $E_t^m$ . We denote with  $P_t^{m,D,P}$  the producer price (indicated by the superscript  $\bullet,\bullet,P$ ) of the domestic sales<sup>9</sup> of sector  $m$  and by  $P_t^{m,E,P}$  the producer price of exports<sup>10</sup> of the same sector  $m$ . For a given production  $Q_t^m$ , firms maximise the value of total sales (where  $P_t^{m,P}$  is the producer price of commodity  $m$ ):

$$P_t^{m,P} \cdot Q_t^m = P_t^{m,D,P} \cdot D_t^m + P_t^{m,E,P} \cdot E_t^m, \quad (11)$$

under the CET restriction:

$$Q_t^m = \chi_m \left[ \phi_m (D_t^m)^{\sigma_\varepsilon} + (1-\phi_m) (E_t^m)^{\sigma_\varepsilon} \right]^{\frac{1}{\sigma_\varepsilon}}, \quad (12)$$

as constraint. In this function  $\chi_m > 0$  is a scale parameter,  $\phi_m$  is the domestic sales’ share parameter, and  $\sigma_\varepsilon < 1$  is related to the elasticity of transformation between domestic and export commodities:  $\varepsilon = \frac{1}{1-\sigma_\varepsilon}$ .

Solving this problem determines the optimal amount of domestically sold and exported goods (see equation A.10 and A.11 in the appendix) as well as the producer price of the composite good:

$$P_t^{m,P} = \frac{1}{\chi_m} \cdot \left[ (\phi_m)^\varepsilon (P_t^{m,D,P})^{1-\varepsilon} + (1-\phi_m)^\varepsilon (P_t^{m,E,P})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \quad (13)$$

Private capital dynamics follow the standard neoclassical capital accumulation equation:

$$K_{t+1} = I_t + (1-\delta) K_t, \quad (14)$$

<sup>9</sup> The producer price of the domestic sales is assumed to be net of any general sales’ tax (GST).

<sup>10</sup> The producer price of exports may entail export subsidies.



where  $I_t$  are aggregate investments (see Section 3.3) and  $\delta$  is the capital depreciation rate. The investment good  $I_t$  has a structure similar to the consumption composite: It is a CES composite (see equation A.30 in the appendix) of  $m$  Armington aggregates (see equation A.25 in the appendix). This CES composite has nominal investment price  $P_t^I$ . Investments are financed through private savings,  $S_t$ , government savings,  $S_t^G$  and net foreign debt,  $D_{t+1} - (1 + \bar{r})D_t$ , which is subject to collateral requirements (see Section 3.5):

$$I_t = S_t + S_t^G + D_{t+1} - (1 + \bar{r})D_t. \quad (15)$$

### 3.3 Consumption and leisure

Differently from Devarajan and Go (1998), infinitely-lived households can choose between consumption and leisure. In each period  $t$  they are endowed with one unit of time, part of which is supplied on the labour market. The remaining time is devoted to leisure activities. The representative agent is represented by the following instantaneous utility function:

$$U_t(c_t, u_t) = \left[ (1 - \eta) \frac{(c_t)^{1 - \alpha} - 1}{1 - \alpha} + \eta \frac{(1 - u_t)^{1 - \beta} - 1}{1 - \beta} \right] \cdot \Omega_t, \quad (16)$$

where  $c_t$  is the per-capita composite consumption at time  $t$  and  $u_t$  is the fraction of time spent working. Parameter  $\alpha$  ( $\beta$ ) is the inverse of the elasticity of substitution between consumption (leisure) at any two points of time and  $\eta$  is the leisure share parameter.

At the beginning of her life  $t = 0$ , the representative agent maximizes her discounted utility:

$$U_0 = \sum_{t=0}^{+\infty} \left( \frac{1}{1 + \rho} \right)^t \cdot U_t, \quad (17)$$

with being  $\rho$  the consumer's positive and constant rate of time preference.

Households' assets are ownership claims on two types of durables, productive capital and land. We denote the aggregate household's productive capital assets by  $K_t$  and land assets by  $La_t$ . Households behave competitively, taking as given the domestic interest rate  $r_t$ , the price of investment goods  $P_t^I$  and the wage rate  $w_t$ , paid per unit of labour services. The total income of the aggregate of households encompasses labour income,  $w_t \cdot u_t \Omega_t$ , and asset income. This is the sum of capital rents,  $P_t^I r_t \cdot K_t$ , and land rents,  $P_t^{La} \cdot La_t$ . In addition, households receive a net

lump sum transfer  $T_t^G$  from the government and foreign remittances  $T_t^W$ . These two latter variables grow at the exogenous rate  $\gamma_\Omega$  in steady-state. Total disposable income is given by:

$$Y_t \equiv (1-\tau) \left[ (1-\tau_L) w_t \cdot u_t \Omega_t + (1-\tau_K) P_t^I r_t \cdot K_t + (1-\tau_{La}) P_t^{La} \cdot La_t \right] + T_t^G + T_t^W, \quad (18)$$

where  $\tau$  is a general income tax rate and  $\tau_L$ ,  $\tau_K$ ,  $\tau_{La}$  are factor income tax rates on labour, capital, land income respectively.

Net income is allocated to consumption and to savings. In each period  $t$ , households choose among a variety of domestic and imported goods. As illustrated in Section 3.1, each consumption good  $c_t^m$  is an Armington aggregate of domestic goods  $D_t^{m,C}$  and imports  $M_t^{m,C}$  (see equation A.16). The final composite good is the result of a CES-aggregation (see equation A.21) of each of the  $m$  commodities. Its market price is  $P_t^C$ . Savings  $S_t$  are used to accumulate productive capital or may be rented out on the international capital market. The budget constraint of the household is given by:

$$\Omega_t \cdot P_t^C c_t + S_t + (1+\bar{r}) D_t = Y_t + D_{t+1}. \quad (19)$$

The household optimisation problem is to maximize the overall utility  $U_0$  in equation (17), subject to the budget constraint in equation (19), the stock of initial assets (capital,  $K_0$ , land,  $La_0$ , and debts,  $D_0$ ) and the borrowing constraint in equation (24). The solution of the optimisation problem gives the optimal demand for leisure:

$$(1-u_t)^\beta \equiv (l_t)^\beta = \frac{1-\eta}{\eta} \frac{P_t^C (c_t)^\alpha}{(1-\tau)(1-\tau_L) w_t}, \quad (20)$$

being the Euler condition given by:

$$\left( \frac{c_{t+1}}{c_t} \right)^\alpha \frac{P_{t+1}^C}{P_t^C} \cdot \frac{\nu \cdot P_{t+1}^I - P_t^I}{\nu \cdot P_{t+1}^I} = \frac{1+\bar{r}}{1+\rho} - \frac{1+(1-\tau)(1-\tau_K) r_{t+1} - \delta}{(1+\rho)\nu}. \quad (21)$$

### 3.4 Government behaviour

Public revenues  $R_t$  include general income taxes and single factor taxes on labour, capital and land income. Additionally the government raises import taxes on imported quantities and collects indirect taxes.

$$\begin{aligned}
R_t = & \tau \left[ (1 - \tau_L) w_t \cdot u_t \Omega_t + (1 - \tau_K) P_t^I r_t \cdot K_t + (1 - \tau_{La}) P_t^{La} \cdot La_t \right] + \\
& + \tau_L (w_t \cdot u_t \Omega_t) + \tau_K (P_t^I r_t \cdot K_t) + \tau_{La} (P_t^{La} \cdot La_t) + \\
& + \sum_{m=1}^M \omega_m \cdot \left[ P_t^{m,D,P} D_t^m + (1 + \mu_m) P_t^{m,M,W} \right] + \sum_{m=1}^M \mu_m \cdot P_t^{m,M,W} M_t^m
\end{aligned} \tag{22}$$

with  $M_t^m \equiv M_t^{m,C} + M_t^{m,I} + M_t^{m,G} + M_t^{m,F}$ .

Government outlays  $O_t$  consist of purchases of consumption goods and services in the aggregate quantity  $G_t$ , payments abroad,  $P_t^G$ , and net savings  $S_t^G$ . Direct lump transfers  $T_t^G$ , in the amount of revenues exceeding expenditures and savings, are paid to consumers.

$$O_t = P_t^G G_t + B_t^G + S_t^G + T_t^G. \tag{23}$$

Government consumption of commodity  $m$ ,  $G_t^m$  is an Armington aggregate of domestic good  $D_t^{m,G}$  and imported commodity  $M_t^{m,G}$ . The aggregate government consumption  $G_t$  is a CES-composite of all commodities  $G_t^m$ . The variables payments abroad,  $B_t^G$  and net savings  $S_t^G$  grow in the steady-state at the exogenous rate  $\gamma_\Omega$ .

### 3.5 Foreign trade and international borrowing

International linkages of the domestic economy encompass trade as well as financial flows. Due to data limitations, the model currently allows for just a single trading partner which is also the only foreign direct investor. Trade relations are modelled taking into account import tariffs and indirect taxes raised on imported goods. These accrue to the government. Export activity is neither subsidised nor subject to tariffs or taxes. Quotas are excluded. Non-tariff barriers are not considered due to lack of reliable data.

As mentioned in Section 3.3, households are allowed to borrow from abroad while financing a share of private capital accumulation. The so called collateral rule (as introduced in Barro, Mankiw, Sala-i-Martin (1995) and further developed in Penalver (2000)) requires that households' external borrowing is constrained to a fraction of the existing physical capital which is used as collateral for foreign debt:

$$D_t \leq \nu \cdot K_t \quad \text{with} \quad 0 < \nu < 1. \tag{24}$$

Hence foreign debt evolves according to

$$D_{t+1} = (1 + \bar{r}) D_t + (I_t - S_t). \tag{25}$$

### 3.6 Market clearing

Factor market clearing requires:

$$K_t = \sum_{n=1}^N K_t^n, \quad (26)$$

$$L_t = \sum_{n=1}^N L_t^n, \quad (27)$$

$$La_t = \sum_{n=1}^N La_t^n. \quad (28)$$

On the domestic goods markets, equilibrium is given by:

$$D_t^m = D_t^{m,C} + D_t^{m,I} + D_t^{m,G} + D_t^{m,F}. \quad (29)$$

The equilibrium of the balance of payments requires that financial inflows, due to exports and transfer payments from the rest of the world as well as from new foreign debt, equal financial outflows due to imports, government payments abroad, foreign debt reimbursement and interest payments on debt:

$$\sum_{n=1}^N P_t^{m,E,W} \cdot E_t^m + T_t^W + D_{t+1} = \sum_{n=1}^N P_t^{m,M,W} \cdot M_t^m + B_t^G + (1 + \bar{r}) D_t. \quad (30)$$

## 4 Model calibration and solution technique

The model is calibrated so that its steady-state solution reproduces Lebanon's economic performance in 1997, i. e. basically prior to its recent efforts of integrating into the world economy, cf. Section 2. Unless otherwise specified, data are taken from National Accounts' (MOET, 2003). The social accounting matrix (SAM) is based on these data and is the result of the authors' own calculations<sup>11</sup>. The Sam distinguishes 8 production sectors and 3 production factors.

Calibration of model parameters is based on the SAM and some additional assumptions. In particular, the population growth rate is assumed to equal to 1.4% per annum according to World Bank (2004) and the world interest rate  $\bar{r}$  is exogenously set at 4%. Substitution elasticities are obtained from the existing literature wherever possible.

On the demand side, the elasticity of substitution between different commodities is set at 0.9, while the Armington elasticity of substitution between domestic and imported good is set at 0.5 (see Devarajan, Go, Li,

<sup>11</sup> We thank Salam Said for very helpful research assistance.

1999). On the supply side the elasticity of substitution between domestic and exports is -2.3. In the utility function, the inverse of the consumer's elasticity of intertemporal substitution and the elasticity of leisure are set equal to 1.

Once elasticity values have been fixed, model share and scale parameters are calibrated from the SAM. To save space, only few aspects of the calibration shall be discussed. In particular, the parameter  $\nu$ , denoting the fraction of foreign debt to domestic capital, is not a free parameter but it is the result of a calibration on the SAM. Subtracting consumption and investment expenditures from households' net factor income (including transfers from the government as well as from abroad) and relating this figure to gross capital income we obtain  $\nu = 0,393$ .

The model is programmed in Gauss and solved with the method of backward integration, cf. Brunner and Strulik (2002). In this method, the algorithm sets off in an arbitrarily small neighborhood of the post-shock steady state and iterates backwards on the saddle path. Since time is reversed in this method, all instable trajectories become stable in the sense that they converge to the true saddle path. Hence, choosing a starting value arbitrarily close to the post-shock steady state gives excellent approximations to the saddle path.

## 5 *Simulation scenarios and results*

As illustrated in Section 2, Lebanon implements trade liberalisation mainly through bilateral and regional agreements. However, since previous studies demonstrated that the preferential (see Martin, (1996, 2000)) as well as the non-discriminatory (see Dessus und Ghaleb (2004)) trade liberalisation have only minor static effects, we want to demonstrate that the dynamic effects (in an MFN sense ) are more important. In two further scenarios, we demonstrate, however, that the economic impacts of advances in political stability are slightly larger.

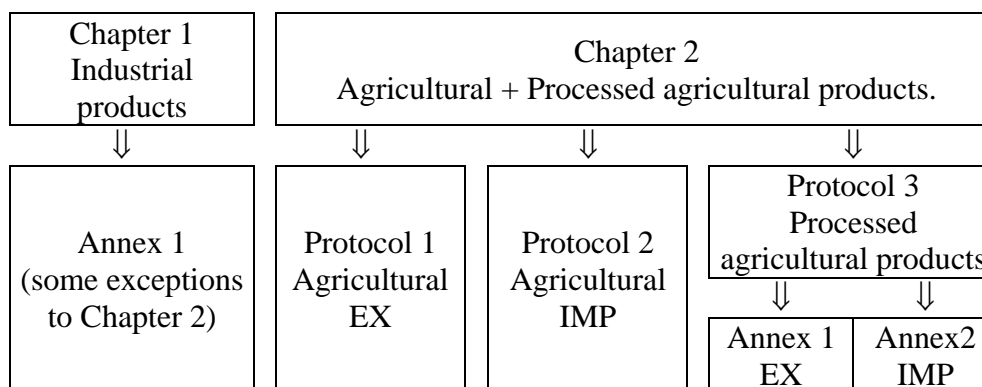
Our analysis begins at the time when the Interim Agreement was signed, so that the time period  $t = 0$  corresponds approximately to 2003 in real time, and our model is therefore currently situated almost in the middle of the 5-year transition period granted to Lebanon by the Interim Agreement for further structural reforms. We simulate a gradual reduction of tariff rates as specified in the Agreement, but counterfactually assume that this reduction applies to all trading partners rather than to just the EU countries.

### 5.1 *Gradual non-discriminatory tariff reductions*

Title II of the Interim Agreement (see Table 1) is devoted to the free movement of goods and states that the free trade area will be established "over a transitional period not exceeding [i.e. of] 12 years [...]". Goods are divided into two categories, industrial products (chapter 1), and agricultural,

fisheries and processed agricultural products (chapter 2). The second category is further divided into two classes, the former including agricultural products and the latter the processed agricultural products<sup>12</sup>. Exceptions to the provisions of chapter 1 are allowed for products listed in Annex 1 of the Agreement. These are agricultural and processed agricultural products as oil derivatives or silk, cotton and wool.

**Table 1** - Title II of the Interim Agreement (schematic structure)



Customs' duties and charges with equivalent effect on imports of industrial products into Lebanon are to be abolished according to the schedule represented in table 2. A five-year transition period is allowed for appropriate structural reforms aimed at improving the competitiveness of the economy. By the fifth year the progressive reduction of tariffs' will start according to the schedule indicated in the right column of the table (Interim Agreement, Art. 5).

**Table 2** - Tariffs abolishment schedule according to the Interim Agreement

Time Frame	Rate (with reference to the basic rate)
5 <sup>th</sup> year	88%
6 <sup>th</sup> year	76%
7 <sup>th</sup> year	64%
8 <sup>th</sup> year	52%
9 <sup>th</sup> year	40%
10 <sup>th</sup> year	28%
11 <sup>th</sup> year	16%
12 <sup>th</sup> year	0%

Consistently with the multi-sectoral structure of the model, this tariff abolishment regime will affect sector 2 (Energy & Water) and sector 3 (Manufacturing), which are the non-agricultural sectors of the economy.

Agricultural and fisheries' imports (category 2, class 1) are addressed in Protocol 2. Tariff reduction on these products will occur in a single shot in the fifth Agreement year. Since the reduction will not affect all goods categories equally, we will adopt an aggregate tariff rate, as calculated on the base of Protocol 2. The depicted tariffs' abatement framework will be

<sup>12</sup> The classification at hand is based on the *Combined Nomenclature 2002 (CN 2002)* and of the *Lebanese Custom Code (LCC)*.

applied to sector 1 (Agriculture). Processed agricultural products (category 2, class 2), including those listed in Annex 1, will be granted a less binding treatment according to the general statement of Art. 9, to be interpreted alongside Protocol 3. These product categories are however aggregated to the manufacturing sector (sector 3) in our model and will therefore be treated according to the provisions of chapter 1.

The provisions of the Interim Agreement are implemented in the model through an unexpected exogenous variation of the import tariff rate  $\theta_m$ . Table 3 illustrates the (calibrated) benchmark value of  $\theta_m$  and its subsequent variations for each import sector.

**Table 3** - Import tariff rate subsequent variations for each import sector

Time Frame	1. Agriculture	2. Energy & Water	3. Manufacturing
Benchmark	0,076	0,046	0,036
5 <sup>th</sup> year	0,053	0,041	0,031
6 <sup>th</sup> year	0,053	0,035	0,027
7 <sup>th</sup> year	0,053	0,030	0,023
8 <sup>th</sup> year	0,053	0,024	0,018
9 <sup>th</sup> year	0,053	0,019	0,014
10 <sup>th</sup> year	0,053	0,013	0,010
11 <sup>th</sup> year	0,053	0,007	0,006
12 <sup>th</sup> year	0,053	0,002	0,001
13 <sup>th</sup> year	0,053	0,000	0,000

## 5.2 *Economic benefits of political stability*

In order to quantify the economic benefits of political stability, we take Moody's rating system as indicator. We first simulate an improvement in political stability, assuming an upgrade in Moody's rating by 3 notches over a 20-year-period. Secondly, we calculate the economic effects of the deterioration in political environment due to the assassination of the former Prime Minister, Rafiq al Hariri. The related fall in creditworthiness is reflected by a minor but instantaneous downgrade (by 1 notch)<sup>13</sup>.

The variations in Moody's rating are depicted through an exogenous increase (decrease) in the fraction  $\nu$  of foreign debt to productive capital. The increase (decrease) is calculated to be equal to 36% (12%). Being the initial fraction of debt slightly less than 40%, this means a final value of around 53% (35%) in the first (second) case.

<sup>13</sup> See Moody's Rating Action of March 24, 2005.

### 5.3 Simulation results

The first scenario implies a reduction of import tariff rate by the fifth year of the agreement. This induces an immediate decrease in the domestic price of each imported commodity  $m$  :

$$P_t^{m,M,D} = (1 + \omega_m) \left[ (1 + \theta_m) P_t^{m,M,W} \right], \quad (31)$$

which instantaneously affects the demand prices of the four Armington aggregate  $m$  :  $P_t^{m,C}$  (price of consumption),  $P_t^{m,I}$  (price of investment),  $P_t^{m,G}$  (price of government consumption) and  $P_t^{m,F}$  (price of intermediate good). These price variations affect the overall price levels  $P_t^C$ ,  $P_t^I$  and  $P_t^G$  of the 3 composite commodities.

The immediate effect of the lowering of the domestic price of imports is an increase in the demand for foreign goods by 1.6%. The generalised price reduction following from the abolishment of tariffs stimulates demand for private consumption (which increases by 1.2%) and investment (which increases by 3%). Government consumption remains constant, being exogenously given. Figure 1 shows the dynamics of aggregate private consumption and aggregate investment.

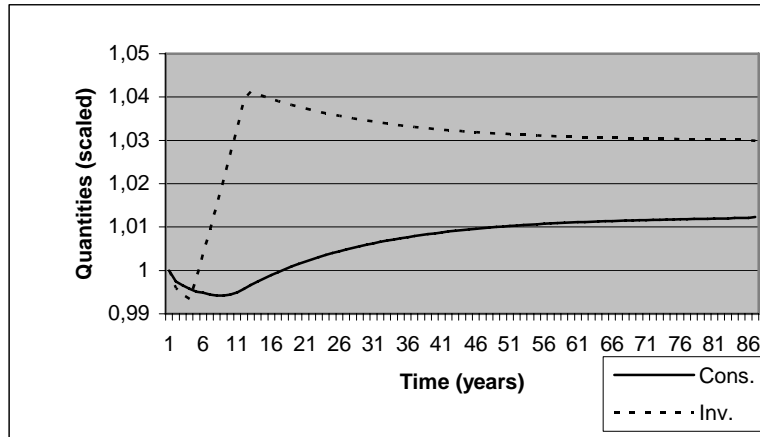


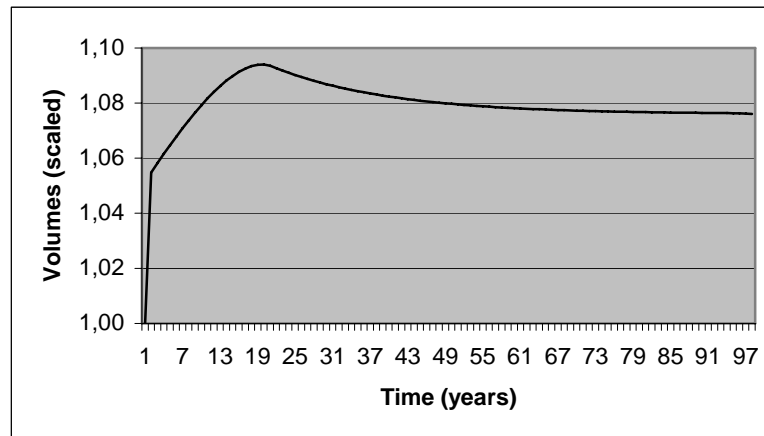
Figure 1 - Aggregate private demand (scenario 1 - trade liberalisation)

The overall benefits of trade liberalisation include a rise in GDP by 2.7%, coupled however with very poor rise in consumer welfare (0.07%). A possible reason for this may lie in the larger weight given by households to present and near future consumption rather than to distant future consumption. The initial fall in consumption may therefore have greater influence on overall welfare than postponed yet increased consumption. These results are in line with the findings of other studies (see for instance Martin (1996, 2000)). As demonstrated by Dessus und Ghaleb (2004), public revenues suffer a relevant fall, which in our case is around 9.3%.

A progressive increase in external borrowing possibilities, as following from major political stability, enlarges households' savings. The first

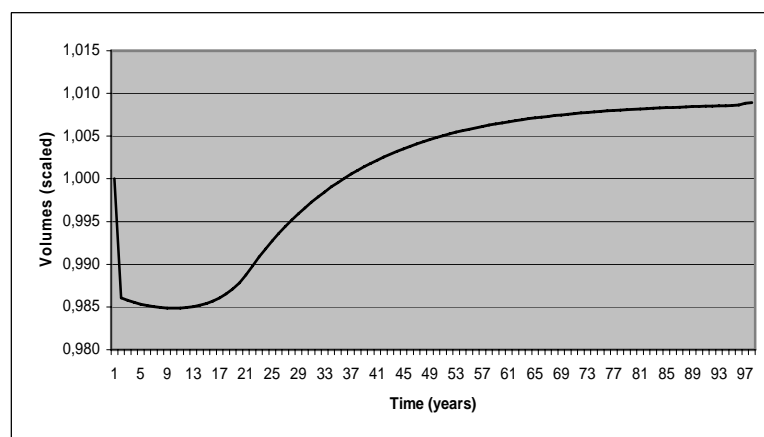


consequence is an increase in the demand for investments, which grows in the long run by 7.6%, according to the non-monotonic transition path showed in Figure 2. After a strong increase during the first periods, the demand for investment reaches its highest value at around the last period of exogenous increase in  $\nu$  and then descends to its steady-state value. The



**Figure 2** - Gross Investment demand (scenario 2 - political stabilisation)

initial short term boost in investment demand influences private consumption demand, which shows an opposite non monotonic transition path (see Figure 3) which ends up with an increase of 0.9%. A further

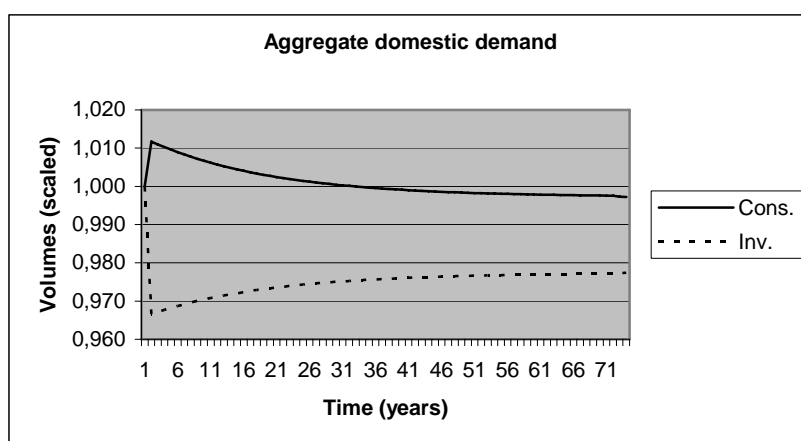


**Figure 3** - Consumption demand (scenario 2 - political stabilisation)

increase on the demand side is represented by exports, which grow steadily from the first transition period as far as achieving a 20.9% increase in the new steady-state. On the supply side, consequences of the improvement in the political environment include a growth of the productive capital by 7.6%, which is supported by a FDI boost (in value) of 41.4% relatively to the old steady-state. The overall demand increase brings about a long run GDP growth of 3.4%.

By contrast, a rapid (though limited) deterioration in political environment due to a destabilising incident as the assassination of the

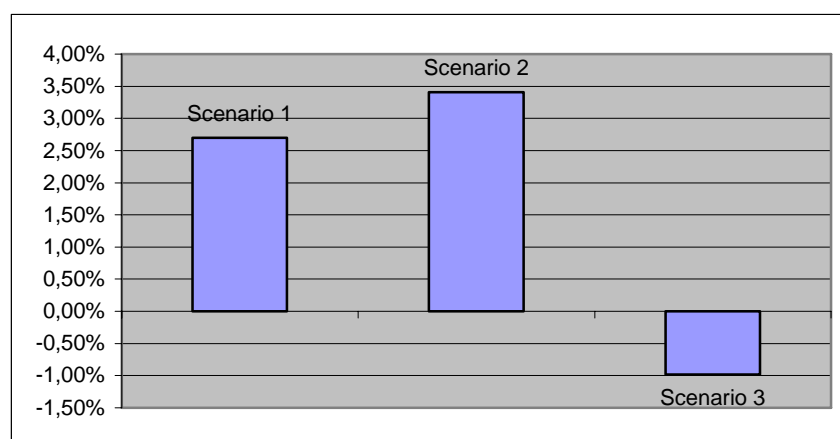
former Prime Minister shows to have generalised negative effects. The underlying economic reasons are conceptually the same as those in the opposite situation. Due to the fall in creditworthiness on global markets, households' foreign borrowing constraint becomes stricter, thus limiting productive investments' possibilities. Figure 4 shows the transition path of domestic consumption and investment. The second falls rapidly since the



**Figure 4** - Aggregate domestic demand (scenario 3 - political destabilisation)

negative shock, leaving productive capital exceeding demand requirements. On the other hand, due to stronger constraint on international borrowing, households modify their consumption and savings' path and more income is allocated to consumption. These two effects together bring about a short term rise in consumption. Fall in creditworthiness imply also deterioration in exports: In the new steady-state exports shrink by 6.1% relatively to the benchmark. The overall negative effect is quantified by a GDP fall by 1% in the long term.

A comparison of the three scenarios shows the quantitative relevance of political stability in comparison with trade liberalisation. Figure 5 depicts the long term GDP effects in the three scenarios. Leaving the positive effect of an increase in creditworthiness aside, evidence suggests how single destabilising incidents as the one quantified here may undermine the positive effects of trade liberalisation.



**Figure 5** - Effects on GDP in the three simulation scenarios

## 6 Conclusions

The main aim of this paper is to study the effects of Lebanon's integration into the world economy with a dynamic CGE model. Unlike previous studies, the intertemporal structure of the model allows a global quantification of the effects, accounting also for long term effects. Additionally, the analysis takes into consideration the role of FDI and their immediate linkage to Lebanon's creditworthiness on the world capital market.

We show that a non-discriminatory trade liberalisation brings about significant effects in the long term, in any case heavily affecting public revenues. Slightly major affects are to be expected from a deeper integration in the world economy, through increasing FDI due to positive developments in political stability.

In spite of the model capability in capturing not only the long-run effects of trade liberalisation but also the positive impacts of improvements in the political environment, the model does not assume imperfect competition. In consideration of the current situation of the Lebanese economy (as documented by MOET (2003a)), this proves to be a drawback, to be removed by further research.

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## *Appendixes*

### *Appendix A. Glossary*

$a_{m,n}$	fixed input (sector $m$ ) requirement for production (sector $n$ )
$A_t^n$	total factor productivity (sector $n$ )
$as_n$	fixed technology parameter (sector $n$ )
$B_t^G$	government payments abroad
$bh_n$	production elasticity of labour (sector $n$ )
$bk_n$	production elasticity of capital (sector $n$ )
$bla_n$	production elasticity of land (sector $n$ )
$c_t$	per-capita composite consumption demand
$c_t^m$	per-capita consumption demand of Armington aggregate $m$
$D_t$	foreign debt
$D_t^m$	domestic demand of domestic goods (sector $m$ )
$D_t^{m,C}$	consumption demand of domestic commodity $m$
$D_t^{m,F}$	intermediates' demand of domestic commodity $m$
$D_t^{m,n,F}$	intermediates' demand of domestic commodity $m$ (sector $n$ )
$D_t^{m,G}$	government consumption demand of domestic good $m$
$D_t^{m,I}$	investment demand of domestic commodity $m$
$E_t^m$	export demand (sector $m$ )
$G_t$	government composite consumption demand
$G_t^m$	government consumption demand of Armington aggregate $m$

$I_t$	composite investment demand
$I_t^m$	investment demand of Armington aggregate $m$
$K_t^n$	physical capital (sector $n$ )
$La_t^n$	land assets (sector $n$ )
$m$	generic commodity
$M$	total number of commodities
$M_t^m$	domestic demand of imports (sector $m$ )
$M_t^{m,C}$	consumption demand of imported commodity $m$
$M_t^{m,F}$	intermediates' demand of imported commodity $m$
$M_t^{m,n,F}$	intermediates' demand of imported commodity $m$ (sector $n$ )
$M_t^{m,G}$	government consumption demand of imported commodity $m$
$M_t^{m,I}$	investment demand of imported commodity $m$
$n$	generic mono-product industry (activity)
$N$	total number of mono-product industries (activities)
$O_t$	government outlays
$P_t^C$	optimal price of the aggregate consumption commodity
$P_t^G$	optimal price of the aggregate government consumption commodity
$P_t^I$	optimal price of the aggregate investment commodity
$P_t^{m,C}$	demand price of Armington consumption aggregate $m$
$P_t^{m,I}$	demand price of Armington investment aggregate $m$
$P_t^{m,F}$	demand price of Armington intermediate aggregate $m$
$P_t^{m,G}$	demand price of Armington government consumption aggregate $m$
$P_t^{m,D,D}$	demand price of domestic good $i$
$P_t^{m,D,P}$	producer price of the domestic sales (sector $m$ )
$P_t^{m,E,P}$	producer price of exports (sector $m$ )
$P_t^{m,E,W}$	world price of exports (sector $m$ )
$P_t^{m,M,D}$	market price of import good (sector $m$ )
$P_t^{m,M,W}$	world price of the import good (sector $m$ )
$P_t^{m,P}$	optimal producer price (sector $m$ )
$Q_t^n$	production (sector $n$ )
$r_t$	domestic interest rate
$\bar{r}$	world interest rate
$R_t$	government revenues
$S_t$	private savings
$S_t^G$	government net savings
$T_t^G$	net lump transfer from the government
$T_t^W$	foreign remittances
$u_t^n$	number of per-capita worked hours (sector $n$ )
$U_0$	overall utility

$U_t$	instantaneous utility
$V_t^n$	value added (sector $n$ )
$w_t$	wage rate
$x_t^{m,n}$	intermediate input (sector $m$ ) for production (sector $n$ )
$\alpha$	inverse of the constant elasticity of substitution in the instantaneous utility function (consumption)
$\beta$	inverse of the constant elasticity of substitution in the instantaneous utility function (leisure)
$\chi_m$	scale parameter in the CET function (sector $m$ )
$\delta$	capital depreciation rate
$\varepsilon$	elasticity of transformation in the CET function
$\phi_m$	domestic sales' share parameter in the CET function (sector $m$ )
$\gamma_\Omega$	population growth rate
$\eta$	leisure share parameter
$\varphi_m$	share parameter in the CES function (Armington composite $m$ )
$\vartheta_m$	scale parameter in the CES function (Armington composite $m$ )
$\kappa$	elasticity of substitution in the CES function (final commodity aggregate)
$\mu$	elasticity of substitution in the CES function (Armington composite)
$\theta_m$	import tariff rate on commodity $m$
$\rho$	consumer's rate of time preference.
$\Pi_t^{V,n}$	value added profit (sector $m$ )
$\sigma_\varepsilon$	substitution parameter in the CET function
$\sigma_\kappa$	substitution parameter in the CES function (final commodity aggregate)
$\sigma_\mu$	substitution parameter in the CES function (Armington composite)
$\tau$	general income tax rate
$\tau_L$	labour income tax rate
$\tau_K$	capital income tax rate
$\tau_{La}$	land income tax rate
$\upsilon$	fraction of foreign debt to domestic capital
$\omega_m$	GST on commodity $m$
$\Omega_t$	population at time $t$
$\psi_m$	commodity $m$ 's share parameter in the CES function (final commodity aggregate)
$\zeta$	scale parameter in the CES function (final commodity aggregate)

**Appendix B. List of equations**

The model consists of equations (1) - (31) and of the equations listed in this appendix.

### B.1 Prices and parameter relations

$$P_t^{m,M,D} = (1 + \omega_m) \left[ (1 + \theta_m) P_t^{m,M,W} \right] \quad (\text{A.1})$$

$$P_t^{m,D,D} = (1 + \omega_m) \cdot P_t^{m,D,P} \quad (\text{A.2})$$

$$\mu = \frac{1}{1 - \sigma_\mu} \quad (\text{A.3})$$

$$\kappa = \frac{1}{1 - \sigma_\kappa} \quad (\text{A.4})$$

### B.2 Production

#### B.2.1 Armington composite

$$P_t^{m,F} \cdot x_t^{m,n} = P_t^{m,D,D} \cdot D_t^{m,n,F} + P_t^{m,M,D} \cdot M_t^{m,n,F} \quad (\text{A.5})$$

$$x_t^{m,n} = \mathcal{G}_i \left[ \varphi_i (D_t^{m,n,F})^{\sigma_\mu} + (1 - \varphi_i) (M_t^{m,n,F})^{\sigma_\mu} \right]^{\frac{1}{\sigma_\mu}} \quad (\text{A.6})$$

$$D_t^{m,n,F} = \frac{(\varphi_m)^\mu}{\mathcal{G}_m} \cdot \frac{\left[ (\varphi_m)^\mu (P_t^{m,D,D})^{1-\mu} + (1 - \varphi_m)^\mu (P_t^{m,M,D})^{1-\mu} \right]^{\frac{\mu}{1-\mu}}}{(P_t^{m,D,D})^\mu} x_t^{m,n} \quad (\text{A.7})$$

$$M_t^{m,n,F} = \frac{(1 - \varphi_m)^\mu}{\mathcal{G}_m} \cdot \frac{\left[ (\varphi_m)^\mu (P_t^{m,D,D})^{1-\mu} + (1 - \varphi_m)^\mu (P_t^{m,M,D})^{1-\mu} \right]^{\frac{\mu}{1-\mu}}}{(P_t^{m,M,D})^\mu} x_t^{m,n} \quad (\text{A.8})$$

$$P_t^{m,F} = \frac{1}{\mathcal{G}_m} \cdot \left[ (\varphi_m)^\mu (P_t^{m,D,D})^{1-\mu} + (1 - \varphi_m)^\mu (P_t^{m,M,D})^{1-\mu} \right]^{\frac{1}{1-\mu}} \quad (\text{A.9})$$

#### B.2.2 CET composite

$$P_t^{m,P} \cdot Q_t^m = P_t^{m,D,P} \cdot D_t^m + P_t^{m,E,P} \cdot E_t^m \quad (\text{A.10})$$



$$Q_t^m = \chi_m \left[ \phi_m (D_t^m)^{\sigma_\varepsilon} + (1-\phi_m) (E_t^m)^{\sigma_\varepsilon} \right]^{\frac{1}{\sigma_\varepsilon}} \quad (\text{A.11})$$

$$D_t^m = \frac{(\phi_m)^\varepsilon \cdot \left[ (\phi_m)^\varepsilon (P_t^{m,D,P})^{1-\varepsilon} + (1-\phi_m)^\varepsilon (P_t^{m,E,P})^{1-\varepsilon} \right]^{\frac{\varepsilon}{1-\varepsilon}}}{\chi_m (P_t^{m,D,P})^\varepsilon} Q_t^m \quad (\text{A.12})$$

$$E_t^m = \frac{(1-\phi_m)^\varepsilon \cdot \left[ (\phi_m)^\varepsilon (P_t^{m,D,P})^{1-\varepsilon} + (1-\phi_m)^\varepsilon (P_t^{m,E,P})^{1-\varepsilon} \right]^{\frac{\varepsilon}{1-\varepsilon}}}{\chi_m (P_t^{m,E,P})^\varepsilon} Q_t^m \quad (\text{A.13})$$

$$P_t^{m,P} = \frac{1}{\chi_m} \cdot \left[ (\phi_m)^\varepsilon (P_t^{m,D,P})^{1-\varepsilon} + (1-\phi_m)^\varepsilon (P_t^{m,E,P})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad (\text{A.14})$$

### B.3 Consumption and leisure

#### B.3.1 Armington (consumption) composite

$$P_t^{m,C} \cdot c_t^m = P_t^{m,D,D} \cdot D_t^{m,C} + P_t^{m,M,D} \cdot M_t^{m,C} \quad (\text{A.15})$$

$$c_t^m = \mathcal{G}_m \left[ \phi_m (D_t^{m,C})^{\sigma_\mu} + (1-\phi_m) (M_t^{m,C})^{\sigma_\mu} \right]^{\frac{1}{\sigma_\mu}} \quad (\text{A.16})$$

$$D_t^{m,C} = \frac{(\phi_m)^\mu \cdot \left[ (\phi_m)^\mu (P_t^{m,D,D})^{1-\mu} + (1-\phi_m)^\mu (P_t^{m,M,D})^{1-\mu} \right]^{\frac{\mu}{1-\mu}}}{\mathcal{G}_m (P_t^{m,D,D})^\mu} c_t^m \quad (\text{A.17})$$

$$M_t^{m,C} = \frac{(1-\phi_m)^\mu \cdot \left[ (\phi_m)^\mu (P_t^{m,D,D})^{1-\mu} + (1-\phi_m)^\mu (P_t^{m,M,D})^{1-\mu} \right]^{\frac{\mu}{1-\mu}}}{\mathcal{G}_m (P_t^{m,M,D})^\mu} c_t^m \quad (\text{A.18})$$

$$P_t^{m,C} = \frac{1}{\mathcal{G}_m} \cdot \left[ (\phi_m)^\mu (P_t^{m,D,D})^{1-\mu} + (1-\phi_m)^\mu (P_t^{m,M,D})^{1-\mu} \right]^{\frac{1}{1-\mu}} \quad (\text{A.19})$$

#### B.3.2 Final aggregate consumption good

$$P_t^C \cdot c_t = \sum_{m=1}^M P_t^{m,C} c_t^m \quad (\text{A.20})$$

$$c_t = \zeta \cdot \left[ \sum_{m=1}^M \psi_m \cdot (c_t^m)^{\sigma_\kappa} \right]^{\frac{1}{\sigma_\kappa}} \quad (\text{A.21})$$

$$c_t^m = \zeta^{\kappa-1} \left( \frac{\psi_m}{P_t^{m,C}} P_t^C \right)^\kappa \cdot c_t \quad (\text{A.22})$$

$$P_t^C = \frac{1}{\zeta} \cdot \left[ \sum_{m=1}^M (\psi_m)^\kappa (P_t^{m,C})^{1-\kappa} \right]^{\frac{1}{1-\kappa}} \quad (\text{A.23})$$

### B.3.3 Armington (investment) composite

$$P_t^{m,I} \cdot I_t^m = P_t^{m,D,D} \cdot D_t^{m,I} + P_t^{m,M,D} \cdot M_t^{m,I} \quad (\text{A.24})$$

$$I_t^m = \mathcal{G}_m \left[ \varphi_m (D_t^{m,I})^{\sigma_\mu} + (1-\varphi_m) (M_t^{m,I})^{\sigma_\mu} \right]^{\frac{1}{\sigma_\mu}} \quad (\text{A.25})$$

$$D_t^{m,I} = \frac{(\varphi_m)^\mu \cdot \left[ (\varphi_m)^\mu (P_t^{m,D,D})^{1-\mu} + (1-\varphi_m)^\mu (P_t^{m,M,D})^{1-\mu} \right]^{\frac{\mu}{1-\mu}}}{\mathcal{G}_m (P_t^{m,D,D})^\mu} I_t^m \quad (\text{A.26})$$

$$M_t^{m,I} = \frac{(1-\varphi_m)^\mu \cdot \left[ (\varphi_m)^\mu (P_t^{m,D,D})^{1-\mu} + (1-\varphi_m)^\mu (P_t^{m,M,D})^{1-\mu} \right]^{\frac{\mu}{1-\mu}}}{\mathcal{G}_m (P_t^{m,M,D})^\mu} I_t^m \quad (\text{A.27})$$

$$P_t^{m,I} = \frac{1}{\mathcal{G}_m} \cdot \left[ (\varphi_m)^\mu (P_t^{m,D,D})^{1-\mu} + (1-\varphi_m)^\mu (P_t^{m,M,D})^{1-\mu} \right]^{\frac{1}{1-\mu}} \quad (\text{A.28})$$

### B.3.4 Final aggregate investment good

$$P_t^I \cdot I_t = \sum_{m=1}^M P_t^{m,I} I_t^m \quad (\text{A.29})$$

$$I_t = \zeta \cdot \left[ \sum_{m=1}^M \psi_m \cdot (I_t^m)^{\sigma_\kappa} \right]^{\frac{1}{\sigma_\kappa}} \quad (\text{A.30})$$

$$I_t^m = \zeta^{\kappa-1} \left( \frac{\psi_m}{P_t^{m,l}} P_t^l \right)^\kappa \cdot I_t \quad (\text{A.31})$$

$$P_t^l = \frac{1}{\zeta} \cdot \left[ \sum_{m=1}^M (\psi_m)^\kappa (P_t^{m,l})^{1-\kappa} \right]^{\frac{1}{1-\kappa}} \quad (\text{A.32})$$

## B.4 Government behaviour

### B.4.1 Armington composite

$$P_t^{m,G} \cdot G_t^m = P_t^{m,D,D} \cdot D_t^{m,G} + P_t^{m,M,D} \cdot M_t^{m,G} \quad (\text{A.33})$$

$$G_t^m = \mathcal{G}_m \left[ \varphi_m (D_t^{m,G})^{\sigma_\mu} + (1-\varphi_m) (M_t^{m,G})^{\sigma_\mu} \right]^{\frac{1}{\sigma_\mu}} \quad (\text{A.34})$$

$$D_t^{m,G} = \frac{(\varphi_m)^\mu}{\mathcal{G}_m} \cdot \frac{\left[ (\varphi_m)^\mu (P_t^{m,D,D})^{1-\mu} + (1-\varphi_m)^\mu (P_t^{m,M,D})^{1-\mu} \right]^{\frac{\mu}{1-\mu}}}{(P_t^{m,D,D})^\mu} G_t^m \quad (\text{A.35})$$

$$M_t^{m,G} = \frac{(1-\varphi_m)^\mu}{\mathcal{G}_m} \cdot \frac{\left[ (\varphi_m)^\mu (P_t^{m,D,D})^{1-\mu} + (1-\varphi_m)^\mu (P_t^{m,M,D})^{1-\mu} \right]^{\frac{\mu}{1-\mu}}}{(P_t^{m,M,D})^\mu} G_t^m \quad (\text{A.36})$$

$$P_t^{m,G} = \frac{1}{\mathcal{G}_m} \cdot \left[ (\varphi_m)^\mu (P_t^{m,D,D})^{1-\mu} + (1-\varphi_m)^\mu (P_t^{m,M,D})^{1-\mu} \right]^{\frac{1}{1-\mu}} \quad (\text{A.37})$$

### B.4.2 Final aggregate government consumption good

$$P_t^G \cdot G_t = \sum_{m=1}^M P_t^{m,G} G_t^m \quad (\text{A.38})$$

$$G_t = \zeta \cdot \left[ \sum_{m=1}^M \psi_m \cdot (G_t^m)^{\sigma_\kappa} \right]^{\frac{1}{\sigma_\kappa}} \quad (\text{A.39})$$

$$G_t^m = \zeta^{\kappa-1} \left( \frac{\psi_m}{P_t^{m,G}} P_t^G \right)^\kappa \cdot G_t \quad (\text{A.40})$$

$$P_t^G = \frac{1}{\zeta} \cdot \left[ \sum_{m=1}^M (\psi_m)^\kappa (P_t^{m,G})^{1-\kappa} \right]^{\frac{1}{1-\kappa}} \quad (\text{A.41})$$