Various methods of balancing of the macro SAM of Tunisia during the year 2000

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Abstract

This paper has the aim to present the various methods of balancing applied to the macroeconomic Social Accounting Matrix (macro SAM) of Tunisia during the year 2000. These methods (method of entropy, method of least squares....) were used by the modellers of Computable General Equilibrium (CGE) whose want to balance the totals in columns and the totals in lines of the SAM of the developing countries in question. To be able to be regarded as the base of data of a CGE model, the matrix must check the principle balance of the totals in columns and in lines of each one of these accounts.

Introduction

        The Social Accounting Matrix (SAM) is regarded as the instrument necessary to examine and establish the relations between the economic development and the social development for the Developing Countries. Known under the name of the database of the Computable General Equilibrium (CGE) models, the SAM knew these last years an improvement of the level of its structure and especially of the level of its form. With the aim of overcoming the problem of imbalance of the SAM resulting from several statistical sources in the case of the Developing Countries, the modellers used several methods of balancing. In this document, we present four of the most used methods who has the object to balance the macro SAM of Tunisia during the year 2000.

        This study consists of three sections. The first section will present the initial macro SAM of Tunisia during the year 2000. Then the second will stress the various methods of balancing. Lastly, the third section will be devoted to the adjusted macro SAM of Tunisia based on the four methods.

1. The initial macro MCS: unbalanced

    The macro SAM is the aggregate form of the SAM. In our case, we devote itself to the macro SAM of Tunisia during the year 2000. This matrix is square since it has 9 accounts in lines (resources) and 9 accounts in columns (uses):

- activities
- products
- labour factor
- capital factor
- households
- companies (firms)
- State
- Rest of the world (ROW)
- Saving - Investment

        This decomposition can be carried out in an arbitrary way, by gathering for example the two factors of production in only one account. Our choice is inspired by diagrammatical form of the SAM adapted by the IEQ (Institute of Quantitative Economy, Tunis) with some modifications made by the author [1].

        Generally, we can detect macroeconomic aggregates based on any SAM. As it is the case in our macro SAM of Tunisia.

        The sum of the value-added (Dinar (D) 23,099.8 million) is equal to the sum of the value-added versed to the account of the Labour factor (D 8,415.9 million) and to the value-added

versed to the account of the Capital factor (D 5,081.9 million). However this value corresponds to the GDP of 2000 at the factor costs (f.c.). We notice that 41,56% of the GDP at the factor costs are versed with the account of the Labour factor, whereas at the same time, the remainder is versed with the account of the Capital factor. Since 1996, this percentage remains on the average of 41% to 42 %. This explains the capitalisation of the Tunisian GDP.

The indirect taxes (D 3,585.5 million) are subdivided in taxes indirect by related to the production (D 2,066.8 million) and in indirect taxes on the imports (D 1,518.7 million). The capital factor income (D 13,497.8 million) is shared between the households (D 8,415.9 million) and the companies (D 5,081.9 million i.e. 37,65%) while the factor income work is entirely versed with households (D 5,081.9 million).

The gross income of the households consists of the payments of the factors of production (Labour and Capital) and the received transfers of other institutions (Firms, State and ROW). This income is broken down for final consumption (D 16,181.4 million), for transfers addressed to the companies and the State (including the direct taxes). The remainder, as for him, is turned towards the saving. The final household consumption accounts for 79,57% of total expenditure of the households, whereas the saving is 9,68%.

The companies pour dividends with the households and the RDM, taxes and dividends in the State, and the remainder is preserved like a saving. The latter consists of 57,76% of the total expenditure of the companies.

In our macro SAM, the State account gathers the Central administration (headquarters), the local communities and the organizations of social security. The public revenue consists of indirect taxes (D 3,585.5 million), of direct taxes (income tax and the benefit) and other received transfers of other institutions. The great saving is 17,73% of the total public expenditure in 2000 and it was 15,62% during the year 1996.

The balance of the currant account of the rest of the world corresponds to the saving of this account. In 2000, this balance was estimated at D 1,133.6 million (D 499 million in 1996), that is to say 4,9% of the GDP with the f.c. (3% in 1996).

Table 1 :   **The macro Social Accounting Matrix of Tunisia during the year 2000 (Million Dinar)**

	activities	products	Labour	capital	household	firms	State	ROW	saving - Investment	Total
activities		35096,5						11868,4		46964,9
products	23865,1				16181,4		4165		7309,9	51521,4
labour	9602									9602
capital	13497,8									13497,8
household			9602	8415,9		358,56	1513,67	1066,51		20956,64
firms				5081,9	286,13		335,18	12,4		5715,61
State		3585,5			1849,6	556,357		106,94		6098,397
ROW		12839,3			50,12	1209,81	56,903			14156,133
saving - Investment					1968,8	2906,3	1309,1	1133,6		7317,8
Total	46964,9	51521,3	9602	13497,8	20336,05	5031,027	7379,853	14187,85	7309,9	

Source : accounts by the author

GDP (f.c.) = D 23,099.8 million
PIB (m. p.) = D 26,685.3 million

According to the preceding macro SAM of Tunisia, we notice that there is an inequality on the level of the totals in lines (resources) and columns (uses) of the following accounts: The four accounts of institutions (households, companies, State and RDM) and accounts of Saving-Investment. This inequality is caused by the data of statistics gathering from several sources (National Institute of the Statistics of Tunis, the Central Bank of Tunisia....). To overcome this problem of imbalance, we use various methods of balancing.

2. Various methods of balancing

These last years, several alternatives follow one after the other whose object is to balance the totals in lines and in columns of the accounts on the level of the SAM. These experiments led to the finding of the solution of balancing using a series of methods which were used following the availability of the statistical data of the country in question, the software available and the suitable discipline of the modeller.

Among these series, we chose four of the most common methods:
- method of Entropy
- method of least squares
- method of the linear program by minimizing the norm L1 of the adjustments
- method of the linear program by minimizing the norm L infinite of the adjustments

2.1 Method of Entropy

We suppose that our initial macro and square SAM (M) is made of several elements:

$a_{i,j}$ with i =1…9 (in lines) and j =1….9 (in columns). Each element $a_{i,j}$ consists with a transfer of an account j of column (uses) on an account i of line (resource). The final SAM ($\hat{M}$) is regarded as a matrix estimated by the method of entropy. It is also made of several elements: $\hat{a}_{i,j}$ avec i =1…9 (en lines) and j =1….9 (in columns).

We point out that the objective of the use of this method is to estimate a new matrix whose principle of balance (equality) between the totals in lines and columns is checked. This principle can be written in the following mathematical form:

$$\sum_{i=1}^{9} \hat{a}_{i,j} = \sum_{j=1}^{9} \hat{a}_{i,j} \qquad \text{(eq.1)}$$

The program of the Entropy method applied to our matrix consists in minimizing the objective function of the entropy between M and $\hat{M}$ subject to the constraint of the equation of the equality principle. This program is written in this form:

$$Min \ z_1 = \sum_{i=1}^{9}\sum_{j=1}^{9} \hat{a}_{i,j} \left[\log \left(\frac{\hat{a}_{i,j}}{a_{i,j}} \right) - 1 \right] \qquad (prog.1)$$

$$s.c. \qquad \sum_{i=1}^{9} \hat{a}_{i,j} = \sum_{j=1}^{9} \hat{a}_{i,j}$$

2.2 Method of least squares

We point out that the principle of least square is to minimize the sum of the square errors. This principle can be applied in our case.

We suppose that the elements $a_{i,j}$ and $\hat{a}_{i,j}$ are the values of the initial SAM and the values of the estimated SAM (or finale), respectively.

The program of this method minimizes the sum of the square errors subject to the equation of the principle of preceding equality (eq.1).

This program of minimization can be illustrated in this form:

$$Min \ z_2 = \sum_{i=1}^{9}\sum_{j=1}^{9} a_{i,j}^2 \left[\left(\frac{\hat{a}_{i,j}}{a_{i,j}} - 1 \right) \right]^2 \qquad (prog.2)$$

$$s.t. \qquad \sum_{i=1}^{9} \hat{a}_{i,j} = \sum_{j=1}^{9} \hat{a}_{i,j}$$

2.3 Method of the linear program by minimizing the norm L1 of the adjustments

In this method, we add two types of elements:

$d_{i,j}^{+}$: element which defines the positive difference between $a_{i,j}$ and $\hat{a}_{i,j}$

$$with : \quad d_{i,j}^{+} = \max \left[\left(a_{i,j} - \hat{a}_{I,j} \right), 0 \right]$$

$d_{i,j}^{-}$: element which defines the negative difference between $a_{i,j}$ and $\hat{a}_{i,j}$

$$with : \quad d_{i,j}^{-} = \max \left[-\left(a_{i,j} - \hat{a}_{I,j} \right), 0 \right]$$

This method consists in minimizing the sum of these two preceding elements under two constraints:
- the equation of the principle of equality (eq.1)
- the equation of equality enters the difference between $d^{+}_{i,j}$ and $d^{-}_{i,j}$ and the difference between $a_{i,j}$ and $\hat{a}_{i,j}$

Thus, our program of minimization is written in its mathematical form:

$$Min \; z_3 = \sum_{i=1}^{9} \sum_{j=1}^{9} \left[\left(d^{+}_{i,j} + d^{-}_{i,j} \right) \Big/ a_{i,j} \right] \qquad \text{(prog.3)}$$

$$\text{s.t.} \quad \sum_{i=1}^{9} \hat{a}_{i,j} = \sum_{j=1}^{9} \hat{a}_{i,j}$$

$$d^{+}_{i,j} - d^{-}_{i,j} = a_{i,j} - \hat{a}_{i,j}$$

2.4 Method of the linear program by minimizing the norm L infinite of the adjustments

This method is differed with the third method only on the level of the objective function. Thus, the program of minimization is illustrated as follows:

$$Min \; z_{i,j} = d^{+}_{i,j} + d^{-}_{i,j} \qquad \text{(prog.4)}$$

$$\text{s.t.} \quad \sum_{i=1}^{9} \hat{a}_{i,j} = \sum_{j=1}^{9} \hat{a}_{i,j}$$

$$d^{+}_{i,j} - d^{-}_{i,j} = a_{i,j} - \hat{a}_{i,j}$$

3. The adjusted macro MCS of Tunisia during year 2000

After having presenting the algebraic program of minimization of the four methods, we focus on the results. In our study, we used the software GAMS to be able to solve these programs numerically. For each program, we obtained an adjusted and square macro SAM. These matrixes differ from a program to another what obliges us to choose a matrix which reflects in the best way the initial macro SAM. This choice remains arbitrary since in all the matrixes, the principle of equality between the totals in lines and the totals in columns of each account is checked. The adjusted macro SAM of Tunisia during the year 2000 of the four methods are presented as follows:

Table 2 :   **The adjusted macro SAM of Tunisia during the year 2000 (in Million Dinars)**
by the Entropy *method*

	activities	products	labour	capital	household	firms	State	ROW	saving - Investment	Total
activities		34938,06						11867,205		46805,265
products	23973,329				16446,808		3823,076		7383,647	51626,86
labour	9545,781									9545,781
capital	13286,155									13286,155
household			9545,781	8450,137		378,323	1369,694	1053,952		20797,887
firms				4836,018	271,183		286,886	11,614		5405,701
State		3906,176			2048,073	650,014		117,021		6721,284
ROW		12782,624			50,717	1291,703	52,001			14177,045
Saving - Investment					1981,106	3085,659	1189,628	1127,254		7383,647
Total	46805,265	51626,86	9545,781	13286,155	20797,887	5405,699	6721,285	14177,046	7383,647	

Source : accounted by the author

GDP (c. f.) = D 22,831.936 million
GDP (p.m.) = D 26,738.112 million

Table 3 : The adjusted macro SAM of Tunisia during the year 2000 (in Millions Dinars) by the Least square method

	activities	products	labour	capital	household	firms	State	ROW	saving - Investment	Total
activities		35065,068						11860,222		46925,29
products	23896,532				16266,654		4066,475		7335,478	51565,139
labourl	9575,089									9575,089
capital	13453,668									13453,668
household			9575,089	8406,21		383,311	1332,89	1004,51		20702,01
firms				5047,459	261,379		126,649	0		5435,487
State		3684,025			2033,38	764,888		228,719		6711,012
ROW		12816,046			112,12	1296,561	0			14224,727
saving - Investment					2028,477	2990,728	1184,997	1131,276		7335,478
Total	46925,289	51565,139	9575,089	13453,669	20702,01	5435,488	6711,011	14224,727	7335,478	

Source : accounted by the author

GDP (c.f.) = D 23,028.757 million
GDP(p.m.) = D 26,712.782 million

Table 4 :   **The adjusted macro SAM of Tunisia during the year (in Millions Dinars)**
by the linear program method by minimizing the norm L1 of the adjustments

	activities	products	labour	capital	household	firms	State	ROW	Saving - Investment	Total
activities		34443,634						11836,683		46280,317
products	23865,1				16804,99		2880,544		7317,8	50868,434
labour	9602									9602
capital	12813,217									12813,217
household			9602	8415,9		358,56	1516,67	1066,51		20959,64
firms				4397,317	286,13		335,18	12,4		5031,027
State		3585,5			1849,6	556,357		106,94		6098,397
ROW		12839,3			50,12	1209,81	56,903			14156,133
Saving - Investment					1968,8	2906,3	1309,1	1133,6		7317,8
Total	46280,317	50868,434	9602	12813,217	20959,64	5031,027	6098,397	14156,133	7317,8	

Source : accounted by the author

GDP (c.f.) = D 22,415.217 million
GDP(p.m.) = D 26,000.717 million

Table 5 : **The adjusted macro SAM of Tunisia during the year (in Millions Dinars)**
by the linear program by minimizing the norm L infinite of the adjustments

	activités	produits	travail	capital	ménages	entreprises	Etat	RDM	Epargne - Investissement	Total
activités		34943,056						11714,956		46658,012
produits	23865,1				16334,844		4011,556		7309,8	51521,3
travail	9448,556									9448,556
capital	13344,356									13344,356
ménages			9448,556	8262,456		502,191	1363,226	1066,51		20642,939
entreprises				5081,9	286,13		181,736	0		5549,766
Etat		3738,944			2003,044	709,801		260,384		6712,173
RDM		12839,3			50,12	1286,03				14175,45
Epargne - Investissement					1968,8	3051,744	1155,656	1133,6		7309,8
Total	46658,012	51521,3	9448,556	13344,356	20642,938	5549,766	6712,174	14175,45	7309,8	

Source : accounted by the author

GDP (c.f.) =   D 22,792.912 million
GDP(p.m.) =   D 26,531.856 million

Within the framework of the choice of the best method, we carry out a comparative approach between the four methods. This approach consists in calculating the value D for each method.

$$D_r = \sum_{i=1}^{9} \sum_{j=1}^{9} \left| a_{i,j} - \hat{a}_{i,j,r} \right|$$

with : $i = 1... 9$ (in lines)

$\quad j = 1... 9$ (in columns)

$\quad r = 1... 4$ (four programs: *prog1 prog2 prog3 prog4*)

We obtain the following inequality between the four values of D :

$$D_{prog2} < D_{prog4} < D_{prog1} < D_{prog3}$$

From the SAM adjusted through *the prog2* and *prog4,* we notice the values of the elements are null: [$a_{6,8} = 0$ (in *the prog2* and *the prog4*) and $a_{8,7} = 0$ (in *the prog2*)].

Whereas these values ($a_{6,8}$ and $a_{8,7}$) are not null in the initial macro SAM. From where these two programs do not reflect the latter. But in the two other methods, they are different from zero. Moreover, the value of D is very low in the method of Entropy. Therefore the latter can be regarded as the best method since it has the criteria that to reflect the better way the initial macro SAM.

Conclusion

We succeeded in our study to apply the four methods to balance the macro SAM of Tunisia during the year 2000 and choosing one among the four. But this choice remains adequate and valid only in the case of our macro SAM of Tunisia. The results of these methods differ from an exercise to another since they are based on the intensity of imbalance and on the initial statistics on the level of the matrix. But the application of these four methods remains a good strategy to be able to compare them and choose one of them. This application can also be related to the micro SAM.

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