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High-Order Consumption Moments and Asset Pricing

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Abstract

This paper investigates the role of the hypothesis of incomplete consumption insurance in explaining the equity premium and the return on the risk-free asset. Using a Taylor series expansion of the individual's marginal utility of consumption around the conditional expectation of consumption, we derive an approximate equilibrium model for expected returns. In this model, the priced risk factors are the cross-moments of return with the moments of the cross-sectional distribution of individual consumption and the signs of the risk factor coefficients are driven by preference assumptions. We demonstrate that if consumers exhibit decreasing and convex absolute prudence, then the cross-sectional mean and skewness of individual consumption yield a larger equity premium if their cross-moments with the excess market portfolio return are positive, while the cross-sectional variance and kurtosis always reduce the equity premium explained by the model. Using the data from the U.S. Consumer Expenditure Survey (CEX), we find that, in contrast to the complete consumption insurance model, the model with heterogeneous consumers explains the observed equity premium and risk-free rate with economically realistic values of the relative risk aversion (RRA) coefficient (less than 1.8) and the time discount factor when the cross-sectional skewness of individual consumption, combined with the cross-sectional mean and variance, is taken into account.

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1 Introduction

Numerous studies have focused on a representative-agent consumption capital asset pricing model in which asset prices are determined by the consumption and asset allocation decisions of a single representative investor with conventional power utility. Empirical evidence is that this model is inconsistent with the data on consumption and asset returns in many respects. In particular, a reasonably parameterized representative-agent model generates a mean equity premium which is substantially lower than that observed in data. This is the equity premium puzzle. Another anomaly with the same model is that it yields the risk-free rate which is too high compared to the observed mean return on the risk-free asset. This is the risk-free rate puzzle.

One plausible response to the equity premium and risk-free rate puzzles is to argue that the poor empirical performance of the representative-agent model is due to the fact that this model abstracts from the lack of certain types of insurance such as insurance against the idiosyncratic shocks to the households' income or divorce, for example.[1] The potential for the incomplete consumption insurance model to explain the equilibrium behavior of stock and bond returns, both in terms of the level of equilibrium rates and the discrepancy between equity and bond returns, was first suggested by Mehra and Prescott (1985). Weil (1992) studies a two-period model in which consumers face, in addition to aggregate dividend risk, idiosyncratic and undiversifiable labor income risk and shows that decreasing absolute risk aversion and decreasing absolute prudence are sufficient to guarantee that the model predicts a smaller bond return and a larger equity premium than a representative-agent model calibrated on the basis of aggregate data solely. In the infinite horizon setting, individuals are able to make risk-free loans to one another and borrow to buffer any short-lived jump in their consumption. This reasoning suggests that in the infinite horizon economy, the additional demand for savings induced by incomplete consumption insurance will generally be smaller than that in a two-period model. Hence, incomplete consumption insurance may have little impact on interest rates. Aiyagari and Gertler (1991), Bewley (1982), Heaton and D. Lucas (1992, 1995, 1996), Huggett (1993), Lucas (1994), Mankiw (1986), and Telmer (1993) confirm this intuition.[2]

[1] Complete consumption insurance implies that consumers can use financial markets to diversify away any idiosyncratic differences in their consumption streams. It follows that under the assumption of complete consumption insurance, aggregate consumption per capita can be used in place of individual consumption and, hence, the pricing implications of a complete consumption insurance model are similar to those of the representative-consumer economy. With incomplete consumption insurance, individuals are not able to self-insure against uninsurable risks and, therefore, are heterogeneous.

[2] In the Bewley (1982), Lucas (1994), Mankiw (1986), and Telmer (1993) models, consumers face uninsurable income risk and borrowing or short-selling constraints, whereas Aiyagari and Gertler (1991) and Heaton and D. Lucas (1992, 1995, 1996) calibrate an economy in which consumers face uninsurable income risk and transaction or borrowing costs. Aiyagari and Gertler (1991) and Heaton and D. Lucas (1992, 1995,

Unlike earlier work which assumes that the idiosyncratic income shocks are transitory and homoskedastic, Constantinides and Duffie (1996) model the time-series process of each consumer's ratio of labor income to aggregate income as nonstationary and heteroskedastic. Given the joint process of arbitrage-free asset prices, dividends, and aggregate income satisfying a certain joint restriction, Constantinides and Duffie (1996) show that in the equilibrium of an economy with heterogeneity in the form of uninsurable, persistent, and heteroskedastic labor income shocks, the pricing kernel is a function not only of per capita consumption growth, but also of the cross-sectional variance of the logarithmic individual consumption growth rate. One of the key features of the Constantinides and Duffie (1996) model is that idiosyncratic shocks to labor income must be persistent. However, using the data from the Panel Study of Income Dynamics (PSID) Heaton and D. Lucas (1996) and Storesletten, Telmer, and Yaron (1997) show that the conclusion whether labor income shocks are persistent or not depends on auxiliary modelling assumptions. Brav, Constantinides, and Géczy (2002) test empirically the Constantinides and Duffie (1996) pricing kernel using the CEX database and find that this stochastic discount factor (SDF) fails to explain the equity premium.

Balduzzi and Yao (2000) derive a SDF which differs from the Constantinides and Duffie (1996) pricing kernel in that the second pricing factor is the difference of the cross-sectional variance of log consumption and not the cross-sectional variance of the log consumption growth rate. Although this SDF specification allows to explain the equity premium with a value of the RRA coefficient which is substantially lower than that obtained using the conventional representative-agent model, the value of risk aversion needed to explain the equity premium remains rather high (larger than 9).

Cogley (2002) uses a Taylor series expansion of the individual's intertemporal marginal rate of substitution (IMRS) and develops an equilibrium factor model in which the pricing factors for the equity premium are the cross-moments of the excess market portfolio return with the first three moments of the cross-sectional distribution of log consumption growth. He finds that this model is not able to explain the observed mean equity premium with economically realistic value of risk aversion even when the model includes the first three cross-sectional moments of log consumption growth.[3]

In contrast to the previous studies, Brav, Constantinides, and Géczy (2002) find empirical evidence for the importance of the incomplete consumption insurance hypothesis for explaining the equity premium. They find that the skewness of the cross-sectional distri-

1996) show that the pricing implications of an incomplete market model do not differ substantially from those of a representative-consumer model, unless the ratio of the net supply of bonds to aggregate income is restricted to an unrealistically low level.

[3]With low (below 5) values of the RRA coefficient, the model can explain only about one-fourth of the observed mean equity premium.

bution of the individual consumption growth rate, combined with the mean and variance, plays an important role in explaining the excess market portfolio return. Their calibration result is that the SDF, given by a third-order Taylor series expansion of the equal-weighted average of the household's IMRS, reproduces the observed mean premium of the market portfolio return over the risk-free rate with low and economically plausible (between two and four) value of the RRA coefficient.

To investigate the role of the hypothesis of incomplete consumption insurance in explaining the equity premium and the risk-free rate of return, we use a Taylor series expansion of the individual's marginal utility of consumption around the conditional expectation of consumption and derive an approximate equilibrium model for expected returns.[4] In this model, the priced risk factors are the cross-moments of return with the moments of the cross-sectional distribution of individual consumption and the signs of the risk factor coefficients are driven by preference assumptions. The attractiveness of this approach comes from the possibility of avoiding an ad hoc specification of preferences and considering a general class of utility functions when addressing the question of the sign of the effect of a particular moment of the cross-sectional distribution of individual consumption on the expected excess market portfolio return and risk-free interest rate, while an ad hoc specification of the utility function is necessary when taking a Taylor series expansion of the agent's IMRS or the mean of the individual's IMRS.[5] We show that if the agent's preferences exhibit decreasing and convex absolute prudence, then the cross-sectional mean and skewness of individual consumption help to explain the equity premium if their cross-moments with the excess market portfolio return are positive, while the cross-sectional variance and kurtosis always reduce the equity premium explained by the model.

Using the data from the CEX, in our empirical investigation we apply several approaches to assess the plausibility of the approximate equilibrium model for reproducing different features of expected returns. First, we perform a Hansen-Jagannathan (1991) volatility bound analysis. In this part, we assess the plausibility of the SDF for the market premium by studying the mean and standard deviation of the pricing kernel for different values of the risk aversion coefficient. The purpose of the calibration exercise is to test whether the observed mean equity premium and risk-free rate can be explained with economically plausible values of the preference parameters. Finally, we investigate the conditional version of the Euler equations for the equity premium and the risk-free rate using a non-linear

[4]Mankiw (1986) and Dittmar (2002) take a Taylor series expansion of the individual's marginal utility of consumption around the unconditional expectation of consumption.

[5]All we need to know to answer the question whether considering a particular moment of the cross-sectional distribution of individual consumption generates a smaller or, on the contrary, larger predicted asset return, is the sign of its cross-moment with return and the sign of the corresponding derivative of the utility function. Considering some special form of preferences is, however, necessary when assessing the size of that effect.

generalized method of moments (GMM) estimation approach. Here, we exploit information about time-series properties of consumption and asset returns. In each of the three parts, the empirical analysis is performed under the assumption of limited asset market participation. Like Brav, Constantinides, and Géczy (2002), we find an important role played by the hypothesis of incomplete consumption insurance in explaining asset returns. Our result is that, in contrast to the complete consumption insurance model, both the equity premium and the risk-free rate may be explained with economically realistic values of the RRA coefficient (less than 1.8) and the time discount factor when the cross-sectional skewness of individual consumption, combined with the cross-sectional mean and variance, is taken into account. This result is robust to the threshold value in the definition of assetholders and the estimation procedure.

The rest of the paper proceeds as follows. In Section 2, we derive an approximate equilibrium model for the expected equity premium and risk-free rate using a general class of utility functions. Section 3 describes the data and presents the empirical results under the CRRA preferences. Section 4 concludes.

2 An Approximate Equilibrium Asset Pricing Model

Consider an economy in which an agent maximizes expected lifetime discounted utility:

$$E_t\left[\sum_{j=0}^{\infty}\delta^j u\left(C_{k,t+j}\right)\right]. \tag{1}$$

In (1), δ is the time discount factor, $C_{k,t}$ is the individual k's consumption in period t, $u(\cdot)$ is a single-period von Neumann-Morgenstern utility function, and $E_t[\cdot]$ denotes an expectation which is conditional on the period-t information set, Ω_t, that is common to all agents.[6]

Let us consider a set of agents, $k = 1, ..., K$, that participate in asset markets. In equilibrium, the investor k's optimal consumption profile must satisfy the following first-order condition:

$$\delta E_t\left[u'\left(C_{k,t+1}\right)R_{i,t+1}\right] = u'\left(C_{k,t}\right),\ k = 1, ..., K,\ i = 1, ..., I. \tag{2}$$

The right-hand side of (2) is the marginal utility cost of decreasing consumption by $dC_{k,t}$ in period t. The left-hand side is the increase in expected utility in period $t+1$ which results from investing $dC_{k,t}$ in asset i in period t and consuming the proceeds in period $t+1$. $R_{i,t+1}$ is the simple gross return on asset i and I is the number of traded securities.

[6] We assume $u(\cdot)$ to be increasing, strictly concave, and differentiable.

For the excess return on asset i over some reference asset j, equation (2) can be rewritten as

$$E_t\left[u'\left(C_{k,t+1}\right)\left(R_{i,t+1}-R_{j,t+1}\right)\right]=0,\ k=1,...,K,\ i=1,...,I. \tag{3}$$

Assume that $u(\cdot)$ is $N+1$ times differentiable and take an N-order Taylor series expansion of the individual k's marginal utility around the conditional expectation of consumption, $h_{t+1}=E_t\left[C_{k,t+1}\right]$, which is assumed to be the same for all investors:

$$u'\left(C_{k,t}\right)=\sum_{n=0}^{N}\frac{1}{n!}u^{(n+1)}\left(h_t\right)\left(C_{k,t}-h_t\right)^n,\ k=1,...,K.^7 \tag{4}$$

Substituting (4) into (2) yields

$$\delta\sum_{n=0}^{N}\frac{1}{n!}u^{(n+1)}\left(h_{t+1}\right)\cdot E_t\left[\left(C_{k,t+1}-h_{t+1}\right)^n R_{i,t+1}\right]=\sum_{n=0}^{N}\frac{1}{n!}u^{(n+1)}\left(h_t\right)\left(C_{k,t}-h_t\right)^n, \tag{5}$$

$k=1,...,K,\ i=1,...,I.$

We can rearrange (5) to explicitly determine expected asset returns:

$$\begin{aligned}E_t\left[R_{i,t+1}\right]&=\delta^{-1}\sum_{n=0}^{N}\frac{1}{n!}\frac{u^{(n+1)}\left(h_t\right)}{u'\left(h_{t+1}\right)}\left(C_{k,t}-h_t\right)^n\\&-\sum_{n=1}^{N}\frac{1}{n!}\frac{u^{(n+1)}\left(h_{t+1}\right)}{u'\left(h_{t+1}\right)}\cdot E_t\left[\left(C_{k,t+1}-h_{t+1}\right)^n R_{i,t+1}\right],\end{aligned} \tag{6}$$

$k=1,...,K,\ i=1,...,I.$

These equations can now be summed over investors and then divided by the number of investors in the economy to yield the following approximate relationship between expected asset returns and priced risk factors:

$$E_t\left[R_{i,t+1}\right]=\delta^{-1}\sum_{n=0}^{N}\frac{1}{n!}\frac{u^{(n+1)}\left(h_t\right)}{u'\left(h_{t+1}\right)}Z_{n,t}-\sum_{n=1}^{N}\frac{1}{n!}\frac{u^{(n+1)}\left(h_{t+1}\right)}{u'\left(h_{t+1}\right)}\cdot E_t\left[Z_{n,t+1}R_{i,t+1}\right], \tag{7}$$

$i=1,...,I$, where $Z_{n,t}=\frac{1}{K}\sum_{k=1}^{K}\left(C_{k,t}-h_t\right)^n$.[8] This is an approximate equilibrium asset pricing model in which the priced risk factors are the cross-moments of return with the moments of the cross-sectional distribution of individual consumption.

[7] Here, and throughout the paper, $u^{(n)}(\cdot)$ denotes the nth derivative of $u(\cdot)$. Mankiw (1986) limits his analysis to a second-order Taylor approximation of the agent's marginal utility of consumption ($N=2$). We can interpret h_{t+1} as a reference consumption level. Since h_{t+1} is supposed to be the same for all investors, we may assume that it is equal to the conditional expectation of aggregate consumption per capita, $h_{t+1}=E_t\left[C_{t+1}\right]$. Under the assumption $h_{t+1}=E\left[C_{t+1}\middle|C_t,C_{t-1},...\right]$, h_{t+1} can be defined as an external habit.

[8] A major problem with testing equation (6) directly is the observation error in reported individual consumption. Averaging over investors seems to mitigate the measurement error effect. However, it is quite plausible that the observation error in individual consumption makes it difficult to precisely estimate the cross-moments of return with the high-order moments of the cross-sectional distribution of individual consumption. An issue of the measurement error effect will be addressed in Section 2.3.

The multifactor pricing model (7) can be seen as an attractive alternative to the multifactor models based on the Arbitrage Pricing Theory (APT). The first attractive feature of model (7) is that, in contrast to the APT which does not provide the identification of the risk factors, the set of factors and the form of the pricing kernel obtain endogenously from the first-order condition of a single investor's intertemporal consumption and portfolio choice problem. That allows to avoid some serious problems arising from an ad hoc specification of a factor structure.[9] The APT pricing models are agnostic about the preferences of the investors. Attractive feature of model (7) is that the signs of the risk factor coefficients are driven by preference assumptions, while they are unrestricted in the multifactor models based on the APT. The problem with both the multifactor pricing model (7) and the multifactor models based on the APT approach is the unknown number of risk factors. In the case of model (7), this problem translates into deciding at which point to truncate the Taylor series expansion. This issue is explored in Section 3.3.

As previously mentioned, the conclusion about the role of consumer heterogeneity in explaining asset returns depends on the assumed degree of shock persistence. A virtue of our approach is that it dos not need to make any assumption about shock persistence. That differs our approach from those by Aiyagari and Gertler (1991), Bewley (1982), Constantinides and Duffie (1996), Heaton and D. Lucas (1992, 1995, 1996), Huggett (1993), Lucas (1994), Mankiw (1986), and Telmer (1993).

2.1 Uninsurable Background Risk and the Equity Premium

The intuition that relaxing the assumption of complete consumption insurance has the potential for explaining the equity premium puzzle is due to recognizing the fact that in the real world, consumers face, in addition to the risk associated with the portfolio choice, multiple uninsurable and idiosyncratic risks such as loss of employment or divorce, for example.[10]

If risks are substitutes, then the presence of an exogenous risk should reduce the demand for any other independent risk.[11] Nevertheless, the presence of one undesirable risk can make another undesirable risk desirable. This is the case of complementarity in independent

[9] See Campbell, Lo, and MacKinlay (1997). First, choosing factors without regard to economic theory may lead to overfitting the data. The second potential danger is the lack of power of tests which ignore the theoretical restrictions implied by a structural equilibrium model.

[10] Mankiw (1986), for instance, argues that if aggregate shocks to consumption are not dispersed equally across all consumers, then the level of the equity premium is in part attributable to the distribution of aggregate shocks among the population. Specifically, he takes a second-order Taylor series expansion of the agent's marginal utility around the unconditional expectation of consumption which is assumed to be the same for all individuals and shows that the expected excess return on the market portfolio over the return on the risk-free asset depends on the cross-moment of the equity premium with the cross-sectional variance of individual consumption.

[11] See Gollier and Pratt (1996), Kimball (1993), Pratt and Zeckhauser (1987), and Samuelson (1963).

risks.[12] Whether risks are substitutes or complements may depend on their nature. It is possible that the effect of adding one risk to another one is mixed and it is rather a question of which effect, substitutability or complementarity in independent risks, is dominating.

Weil (1992) demonstrates that if consumers exhibit decreasing absolute risk aversion and decreasing absolute prudence (i.e., the absolute level of precautionary savings declines as wealth rises), then neglecting the existence of the undiversifiable labor income risk leads to an underprediction of the magnitude of the equity premium.[13] Gollier (2001) shows that if absolute risk aversion is decreasing and convex and/or absolute risk aversion and absolute prudence are decreasing, the presence of the uninsurable background risk in wealth raises the aversion of a decision maker to any other independent risk. If at least one of these sufficient conditions is satisfied, such preferences preserve substitutability in the uninsurable background risk in wealth and the portfolio risk and, therefore, can help in solving the equity premium puzzle. Since decreasing and convex absolute risk aversion and decreasing absolute prudence are widely recognized as realistic assumptions, these results are overwhelmingly in favor of substitutability in the uninsurable background risk in wealth and the portfolio risk. It follows that the presence of the uninsurable background risk in wealth should reduce the agent's optimal demand for any risky asset and, therefore, should increase the expected equity premium.

For the excess return on the market portfolio over the risk-free rate, $RP_{t+1} = R_{M,t+1} - R_{F,t+1}$, equation (7) reduces to

$$E_t\left[RP_{t+1}\right] = -\sum_{n=1}^{N} \frac{1}{n!} \frac{u^{(n+1)}\left(h_{t+1}\right)}{u'\left(h_{t+1}\right)} \cdot E_t\left[Z_{n,t+1} RP_{t+1}\right].^{14} \tag{8}$$

[12] See Ross (1999).

[13] If consumers' tastes exhibit decreasing absolute risk aversion and decreasing absolute prudence, then the nonavailability of insurance against an additional idiosyncratic and undiversifiable labor income risk makes consumers more unwilling to bear aggregate dividend risk and the equilibrium return premium on equity rises relative to the full-insurance case.

[14] Another way to represent this equation is to rewrite it in terms of the deviations of individual consumption from per capita consumption and the cross-moments of excess return with per capita consumption. In particular, when a second-order Taylor approximation of marginal utility ($N = 2$) is taken, (8) can be rewritten as

$$E_t\left[RP_{t+1}\right] = -\frac{u''\left(h_{t+1}\right)}{u'\left(h_{t+1}\right)} \cdot E_t\left[\left(C_{t+1} - h_{t+1}\right) RP_{t+1}\right] - \frac{1}{2} \frac{u'''\left(h_{t+1}\right)}{u'\left(h_{t+1}\right)} \times$$
$$E_t\left[\left(C_{t+1} - h_{t+1}\right)^2 RP_{t+1}\right] - \frac{1}{2} \frac{u'''\left(h_{t+1}\right)}{u'\left(h_{t+1}\right)} \cdot E_t\left[\frac{\sum_{k=1}^{K}\left(C_{k,t+1} - C_{t+1}\right)^2}{K} RP_{t+1}\right],$$

where C_{t+1} denotes aggregate consumption per capita. The first two terms on the right-hand side of this equation show how relative asset yields depend on the second and third cross-moments of excess return with per capita consumption, while the third term reflects influence of consumer heterogeneity on the expected excess return. For a higher-order Taylor approximation, a similar equation in terms of higher-order cross-moments can be derived. See Mankiw (1986).

Within the complete consumption insurance framework, $C_{k,t+1} = C_{t+1}$, $k = 1, ..., K$, and, consequently, equation (8) reduces to that in the representative-agent setting:

$$E_t\left[RP_{t+1}\right] = -\sum_{n=1}^{N} \frac{1}{n!} \frac{u^{(n+1)}\left(h_{t+1}\right)}{u'\left(h_{t+1}\right)} \cdot E_t\left[Z^a_{n,t+1} RP_{t+1}\right], \tag{9}$$

where $Z^a_{n,t+1} = \left(C_{t+1} - h_{t+1}\right)^n$ and $Z^a_{1,t+1} = Z_{1,t+1}$ at all t.

The cross-moments of the excess market portfolio return with the moments of the cross-sectional distribution of individual consumption can be calculated from data on individual consumption expenditures and the excess return on the market portfolio. It follows that to determine which effect (substitutability or complementarity in the portfolio risk and the background risk in wealth) is generated by each of the moments of the cross-sectional distribution of individual consumption, it suffices to sign the first five derivatives of $u(\cdot)$. As is conventional in the literature, we assume that the marginal utility of consumption is positive ($u'(\cdot) > 0$) and decreasing ($u''(\cdot) < 0$), and an agent is prudent ($u'''(\cdot) > 0$).[15] We now turn to the signs of the fourth and fifth derivatives of $u(\cdot)$. Assume that absolute prudence, $AP(\cdot)$, is decreasing.[16]

Proposition 1 *Absolute prudence is decreasing (DAP) if and only if* $u''''(\cdot) < -AP(\cdot)\, u'''(\cdot)$. *The condition* $u''''(\cdot) < 0$ *is necessary for DAP.*

Proof. DAP implies that

$$AP'(\cdot) = -\frac{u''''(\cdot)\, u''(\cdot) - \left(u'''(\cdot)\right)^2}{\left(u''(\cdot)\right)^2} < 0. \tag{10}$$

In order to prove that the condition $u''''(\cdot) < 0$ is necessary for DAP suppose, in contrast, that $u''''(\cdot) \geqslant 0$. When $u''''(\cdot) \geqslant 0$, $u''''(\cdot)\, u''(\cdot) \leqslant 0$ and, therefore, $AP'(\cdot) > 0$, what contradicts the assumption that absolute prudence is decreasing.

Inequality (10) means that $u''''(\cdot)\, u''(\cdot) - \left(u'''(\cdot)\right)^2 > 0$ is the necessary and sufficient condition for DAP. We can rewrite this condition as

$$u''''(\cdot) < \frac{\left(u'''(\cdot)\right)^2}{u''(\cdot)} = -AP(\cdot)\, u'''(\cdot). \tag{11}$$

Since an agent is assumed to be prudent, the term on the right-hand side of (11) is negative. ■

[15] Kimball (1990) defines "prudence" as a measure of the sensitivity of the optimal choice of a decision variable to risk (of the intensity of the precautionary saving motive in the context of the consumption-saving decision under uncertainty). A precautionary saving motive is positive when $-u'(\cdot)$ is concave ($u'''(\cdot) > 0$) just as an individual is risk averse when $u(\cdot)$ is concave.

[16] $AP(\cdot) = -\frac{u'''(\cdot)}{u''(\cdot)} > 0$. Intuitively, the willingness to save is an increasing function of the expected marginal utility of future wealth. Since marginal utility is decreasing in wealth, the absolute level of precautionary savings must also be expected to decline as wealth rises.

A natural assumption is that, likewise absolute risk aversion, absolute prudence is convex (the absolute level of precautionary savings is decreasing in wealth at a decreasing rate).

Proposition 2 *Absolute prudence is convex (CAP) if and only if* $u''''' (\cdot) > -2AP' (\cdot) \cdot u''' (\cdot) - AP (\cdot) u'''' (\cdot)$. *If preferences exhibit prudence and decreasing absolute prudence, then* $u''''' (\cdot) > 0$ *is the necessary condition for CAP.*

Proof. Absolute prudence is convex if the following condition is satisfied:

$$AP'' (\cdot) = -\frac{A - B}{C} > 0, \tag{12}$$

where $A = (u'' (\cdot))^2 (u''''' (\cdot) u'' (\cdot) - u''' (\cdot) u'''' (\cdot))$, $B = 2u'' (\cdot) u''' (\cdot) (u'''' (\cdot) u'' (\cdot) - (u''' (\cdot))^2)$, and $C = (u'' (\cdot))^4$.

To prove that $u''''' (\cdot) > 0$ is necessary for CAP under prudence and DAP, assume that $u''''' (\cdot) \leqslant 0$. An agent is prudent $(AP (\cdot) > 0)$ if and only if $u''' (\cdot) > 0$. By Proposition 1, we know that the necessary condition for DAP is that $u'''' (\cdot) < 0$. Then, under prudence and DAP, $A > 0$. Since $u'''' (\cdot) u'' (\cdot) - (u''' (\cdot))^2 > 0$ is the necessary and sufficient condition for DAP, prudence and DAP also imply that $B < 0$. In consequence, $AP'' (\cdot) < 0$, what contradicts the initial assumption that absolute prudence is convex.

It follows from (12) that the necessary and sufficient condition for CAP is $A - B < 0$. This condition can be written as follows:

$$u''''' > \frac{2u''' (\cdot) \left(u'''' (\cdot) u'' (\cdot) - (u''' (\cdot))^2\right)}{(u'' (\cdot))^2} + \frac{u''' (\cdot) u'''' (\cdot)}{u'' (\cdot)} \tag{13}$$

or, equivalently,

$$u''''' > -2AP' (\cdot) u''' (\cdot) - AP (\cdot) u'''' (\cdot) . \tag{14}$$

Under prudence and DAP, the term $-2AP' (\cdot) u''' (\cdot) - AP (\cdot) u'''' (\cdot)$ is positive.[17] ■

So, we obtain that under DAP and CAP, $u'''' (\cdot) < 0$ (the necessary condition for DAP) and $u''''' (\cdot) > 0$ (this condition is necessary for CAP). Combined with the conditions $u' (\cdot) > 0$, $u'' (\cdot) < 0$, and $u''' (\cdot) > 0$, it follows that the cross-sectional mean and skewness of individual consumption help to explain the equity premium if their cross-moments with the excess market portfolio return are positive. Since the cross-moments of the equity premium with the cross-sectional variance and kurtosis of individual consumption are always positive, taking them into account reduces the equity premium explained by model (8).

[17] If an agent exhibits prudence, then $AP(\cdot) > 0$ and $u''' (\cdot) > 0$. The condition $u'''' (\cdot) < 0$ is necessary for DAP.

2.2 Uninsurable Background Risk and the Risk-Free Rate

According to (7), the equilibrium rate of return on the risk-free asset is

$$E_t\left[R_{F,t+1}\right] = \delta^{-1} \sum_{n=0}^{N} \frac{1}{n!} \frac{u^{(n+1)}\left(h_t\right)}{u'\left(h_{t+1}\right)} Z_{n,t} - \sum_{n=1}^{N} \frac{1}{n!} \frac{u^{(n+1)}\left(h_{t+1}\right)}{u'\left(h_{t+1}\right)} \cdot E_t\left[Z_{n,t+1} R_{F,t+1}\right] \tag{15}$$

or, equivalently,

$$\begin{aligned} E_t\left[R_{F,t+1}\right] &= \left(\delta \frac{u'\left(h_{t+1}\right)}{u'\left(h_t\right)}\right)^{-1} + \left(\delta^{-1} \frac{u''\left(h_t\right)}{u'\left(h_{t+1}\right)} Z_{1,t} - \frac{u''\left(h_{t+1}\right)}{u'\left(h_{t+1}\right)} \cdot E_t\left[Z_{1,t+1} R_{F,t+1}\right]\right) \\ &+ \sum_{n=2}^{N} \frac{1}{n!} \left(\delta^{-1} \frac{u^{(n+1)}\left(h_t\right)}{u'\left(h_{t+1}\right)} Z_{n,t} - \frac{u^{(n+1)}\left(h_{t+1}\right)}{u'\left(h_{t+1}\right)} \cdot E_t\left[Z_{n,t+1} R_{F,t+1}\right]\right).^{18} \end{aligned} \tag{16}$$

When consumption insurance is complete, the expected risk-free rate is

$$\begin{aligned} E_t\left[R_{F,t+1}\right] &= \left(\delta \frac{u'\left(h_{t+1}\right)}{u'\left(h_t\right)}\right)^{-1} + \left(\delta^{-1} \frac{u''\left(h_t\right)}{u'\left(h_{t+1}\right)} Z^a_{1,t} - \frac{u''\left(h_{t+1}\right)}{u'\left(h_{t+1}\right)} \cdot E_t\left[Z^a_{1,t+1} R_{F,t+1}\right]\right) \\ &+ \sum_{n=2}^{N} \frac{1}{n!} \left(\delta^{-1} \frac{u^{(n+1)}\left(h_t\right)}{u'\left(h_{t+1}\right)} Z^a_{n,t} - \frac{u^{(n+1)}\left(h_{t+1}\right)}{u'\left(h_{t+1}\right)} \cdot E_t\left[Z^a_{n,t+1} R_{F,t+1}\right]\right) \end{aligned} \tag{17}$$

with $Z^a_{1,t} = Z_{1,t}$ at all t.

The expected return on the risk-free asset in equations (16) and (17) is expressed as a sum of three terms. The first term, $\left(\delta \frac{u'(h_{t+1})}{u'(h_t)}\right)^{-1}$, characterizes the effect of preference for the present. Since the agent's utility function is concave, the investor has preferences for smoothing his consumption over time. In order to make the agent not to smooth his consumption, the risk-free rate must be larger than $\left(\delta \frac{u'(h_{t+1})}{u'(h_t)}\right)^{-1}$ (the consumption smoothing effect). This effect is reflected by the second term on the right-hand side of equations (16) and (17). The size of the consumption smoothing effect depends on the degree of concavity of the agent's utility function.[19] When the agent is prudent and, hence, wants to save more in order to self-insure against uninsurable risks, the risk-free rate must be lower than $\left(\delta \frac{u'(h_{t+1})}{u'(h_t)}\right)^{-1}$ to sustain the equilibrium (the precautionary saving effect). The precautionary saving effect is represented by the third term on the right-hand side of equations (16) and (17). Since $Z^a_{1,t} = Z_{1,t}$ at all t, the first two terms on the right-hand side of equations (16) and (17) are the same and, therefore, taking into account consumer heterogeneity has the potential for explaining the risk-free rate puzzle if the third term on the right-hand side of (16) is less than that in (17).

[18] Likewise (8), this equation can also be rewritten in terms of the cross-moments of return with per capita consumption and the cross-moments of return with the moments of the cross-sectional distribution of individual consumption.

[19] The more concave the agent's utility function, the higher the risk-free rate needed to compensate the agent for not smoothing his consumption over time.

2.3 Measurement Error Issue

A well documented potential problem with using household level data is the large measurement error in reported individual consumption.[20] The widely used solution to mitigate the impact of measurement error consists in averaging over the level of consumption or consumption growth. Since measurement error is not observable, the choice of the optimal method remains somewhat arbitrary and depends on what type of measurement error is assumed.[21]

We assume that the observation error in the consumption level is additive. Since individual consumption is assumed to be misreported by some stochastic dollar amount $\epsilon_{k,t}$, the observed consumption level is $C_{k,t} = C^*_{k,t} + \epsilon_{k,t}$, where $C^*_{k,t}$ is the true level of the agent k's consumption in period t. We further assume that for all k at all t, $\epsilon_{k,t} \sim D\left(0, \sigma^2_{\epsilon,t}\right)$ and $\epsilon_{k,t}$ is independent of the true consumption level.

By the law of large numbers, when $K \longrightarrow \infty$, $\frac{1}{K}\sum_{k=1}^{K} C_{k,t} \xrightarrow{P} E\left[C_{k,t}\right] = E\left[C^*_{k,t}\right]$ and, hence, averaging over the level of consumption should mitigate the additive idiosyncratic measurement error effect. It follows that when $K \longrightarrow \infty$, $Z^a_{n,t} \longrightarrow Z^{a*}_{n,t}$ for all n at all t and $Z_{1,t} \longrightarrow Z^*_{1,t}$ at all t.[22]

It may be shown that

$$Z_{2,t} \xrightarrow{P} E\left[C_{k,t} - h_t\right]^2 = E\left[C^*_{k,t} - h_t\right]^2 + \sigma^2_{\epsilon,t}, \tag{18}$$

$$Z_{3,t} \xrightarrow{P} E\left[C_{k,t} - h_t\right]^3 = E\left[C^*_{k,t} - h_t\right]^3 + E\left[\epsilon_{k,t}\right]^3 \tag{19}$$

and

$$Z_{4,t} \xrightarrow{P} E\left[C_{k,t} - h_t\right]^4 = E\left[C^*_{k,t} - h_t\right]^4 + 6\sigma^2_{\epsilon,t} E\left[C^*_{k,t} - h_t\right]^2 + E\left[\epsilon_{k,t}\right]^4. \tag{20}$$

Therefore, when the number of households in a sample is large, equations (9) and (17) yield asymptotically unbiased estimates of the coefficient of risk aversion and δ. The same is also true for equations (8) and (16) with $N = 1$. Under the assumption of consumer heterogeneity, a Taylor series expansion of order higher than 1 can lead to biased estimates of both the risk aversion parameter and the time discount factor δ. Observe that with measurement error of the type assumed here, little may be said about the signs and magnitudes of the biases in the estimates of the coefficient of risk aversion and δ.[23] However,

[20] See Runkle (1991) and Zeldes (1989).

[21] Additive measurement error suggests averaging over the level of consumption, while in the case of multiplicative measurement error, averaging over consumption growth may be preferable.

[22] The sign "*" means that a value is calculated using true levels of consumption.

[23] In Section 3.5 below, we perform an empirical analysis of the effect of additive measurement error in the consumption level on the estimates of the preference parameters in the context of the hypothesis of incomplete consumption insurance.

as we can see from (18)-(20), it seems to be plausible that the magnitude of the bias in the estimates of the moments of the cross-sectional distribution of individual consumption and, therefore, the cross-moments of return with the moments of the cross-sectional distribution of individual consumption increases as the order of a Taylor series expansion rises.

3 Empirical Results

In Section 2, we have shown that the hypothesis of incomplete consumption insurance has the potential for explaining both the excess market portfolio return and the rate of return on the risk-free asset. In this section, we assess the quantitative importance of the hypotheses of incomplete consumption insurance and limited asset market participation for explaining the equity premium and the risk-free rate.

A class of utility functions widely used in the literature is the set of utility functions exhibiting an harmonic absolute risk aversion (HARA). HARA utility functions take the following form:

$$u\left(C_t\right) = a\left(b + \frac{cC_t}{\gamma}\right)^{1-\gamma}, \tag{21}$$

where a, b, and c are constants, $b + \frac{cC_t}{\gamma} > 0$, and $a\frac{c(1-\gamma)}{\gamma} > 0$.[24]

There are three special cases of HARA utility functions.[25] Constant relative risk aversion (CRRA) utility functions can be obtained by selecting $b = 0$:

$$u\left(C_t\right) = a\left(\frac{cC_t}{\gamma}\right)^{1-\gamma}. \tag{22}$$

In the special case when $a = \frac{(c/\gamma)^{\gamma-1}}{(1-\gamma)}$, we get $u\left(C_t\right) = \frac{C_t^{1-\gamma}}{1-\gamma}$, where γ is the RRA coefficient, $\gamma \neq 1$.[26]

If $\gamma \longrightarrow \infty$, we obtain constant absolute risk aversion (CARA) utility functions:

$$u\left(C_t\right) = -exp\left(-\frac{c}{b}C_t\right). \tag{23}$$

With $\gamma = -1$, we get quadratic utility functions:

$$u\left(C_t\right) = a\left(b + \frac{cC_t}{\gamma}\right)^{2}. \tag{24}$$

It is easy to check that when the first five derivatives of the HARA utility function exist, $u'(\cdot) > 0$ and $u''(\cdot) < 0$ imply $u'''(\cdot) > 0$, $u''''(\cdot) < 0$, and $u'''''(\cdot) > 0$. Given the

[24] The last inequality is necessary to insure that $u'(\cdot) > 0$.

[25] See Golier (2001).

[26] A logarithmic utility specification, $u\left(C_t\right) = log\left(C_t\right)$, corresponds to the case when $\gamma = 1$.

results obtained in Section 2, it follows that any HARA class utility function may be used when addressing the question of the effect of a particular cross-moment of return with the moment of the cross-sectional distribution of individual consumption on the expected equity premium and risk-free rate. For the equations derived in Section 2 to be scale-invariant, we need $b = 0$ when using a HARA class preference specification. That corresponds to the CRRA preferences.

Assuming the CRRA homogeneous preferences, $u(C_{k,t}) = \frac{C_{k,t}^{1-\gamma}-1}{1-\gamma}$, $k = 1, ..., K$, we can rewrite (8) and (16) as

$$E_t[RP_{t+1}] = -E_t\left[\left(\sum_{n=1}^{N}\frac{1}{n!}(-1)^n\left(\prod_{l=0}^{n-1}(\gamma+l)\right)\frac{Z_{n,t+1}}{h_{t+1}^n}\right)RP_{t+1}\right] \tag{25}$$

and

$$\begin{aligned}E_t[R_{F,t+1}] &= \delta^{-1}\left(\frac{h_{t+1}}{h_t}\right)^{\gamma}\left(1+\sum_{n=1}^{N}\frac{1}{n!}(-1)^n\left(\prod_{l=0}^{n-1}(\gamma+l)\right)\frac{Z_{n,t}}{h_t^n}\right)\\ &-E_t\left[\sum_{n=1}^{N}\frac{1}{n!}(-1)^n\left(\prod_{l=0}^{n-1}(\gamma+l)\right)\frac{Z_{n,t+1}}{h_{t+1}^n}R_{F,t+1}\right].^{27}\end{aligned} \tag{26}$$

If consumption insurance is complete, then $Z_{n,t+1} = Z_{n,t+1}^a$ and, hence, equations (25) and (26) can be rewritten as

$$E_t[RP_{t+1}] = -E_t\left[\left(\sum_{n=1}^{N}\frac{1}{n!}(-1)^n\left(\prod_{l=0}^{n-1}(\gamma+l)\right)\frac{Z_{n,t+1}^a}{h_{t+1}^n}\right)RP_{t+1}\right] \tag{27}$$

and

$$\begin{aligned}E_t[R_{F,t+1}] &= \delta^{-1}\left(\frac{h_{t+1}}{h_t}\right)^{\gamma}\left(1+\sum_{n=1}^{N}\frac{1}{n!}(-1)^n\left(\prod_{l=0}^{n-1}(\gamma+l)\right)\frac{Z_{n,t}^a}{h_t^n}\right)\\ &-E_t\left[\sum_{n=1}^{N}\frac{1}{n!}(-1)^n\left(\prod_{l=0}^{n-1}(\gamma+l)\right)\frac{Z_{n,t+1}^a}{h_{t+1}^n}R_{F,t+1}\right],\end{aligned} \tag{28}$$

respectively.

We first focus on the mean and standard deviation of the SDF derived in Section 2. In this part, we perform the Hansen-Jagannathan volatility bound analysis to explore the potential for our model to explain the market premium. The second part is a model calibration. Here, we look for the values of the risk aversion coefficient γ and the time discount factor δ which allow to fit the observed mean equity premium and risk-free rate. In the

[27] Under the CRRA preferences, $\frac{Z_{n,t+1}}{h_{t+1}^n} = \frac{1}{K}\sum_{k=1}^{K}\left(\frac{C_{k,t+1}-h_{t+1}}{h_{t+1}}\right)^n$. If h_{t+1} is the reference level of consumption, we can define $\frac{C_{k,t+1}-h_{t+1}}{h_{t+1}}$ as a surplus consumption ratio, as in Campbell and Cochrane (1999).

third part, we exploit the time series properties of consumption and asset returns and use a non-linear GMM estimation approach to test the conditional Euler equations for the excess market portfolio return and the risk-free rate.

3.1 Description of the Data

The Consumption Data. The consumption data used in our analysis are taken from the CEX.[28] Our measure of consumption is consumption of nondurables and services (NDS). For each household, we calculate monthly consumption expenditures for all the disaggregate consumption categories offered by the CEX. Then, we deflate obtained values in 1982-84 dollars with the CPI's (not seasonally adjusted, urban consumers) for appropriate consumption categories.[29] Aggregating the household's monthly consumption across these categories is made according to the National Income and Product Account definitions of consumption aggregates. In order to transform our consumption data to a per capita basis, we normalize the consumption of each household by dividing it by the number of family members in the household.

The Returns Data. The measure of the nominal market return is the value-weighted return (capital gain plus dividends) on all stocks listed on the NYSE and AMEX obtained from the Center for Research in Security Prices (CRSP) of the University of Chicago. The real monthly market return is calculated as the nominal market return divided by the 1-month inflation rate based on the deflator defined for NDS consumption. The nominal monthly risk-free rate of interest is the 1-month Treasury bill return from CRSP. The real monthly risk-free interest rate is calculated as the nominal risk-free rate divided by the 1-month inflation rate. Market premium is calculated as the difference between the real market return and the real risk-free rate of interest.

[28]The CEX data available cover the period from 1979:10 to 1996:2. It is a collection of data on approximately 5000 households per quarter in the U.S. Each household in the sample is interviewed every three months over five consecutive quarters (the first interview is practice and is not included in the published data set). As households complete their participation, they are dropped and new households move into the sample. Thus, each quarter about 20% of the consumer units are new. The second through fifth interviews use uniform questionnaires to collect demographic and family characteristics as well as data on monthly consumption expenditures for the previous three months made by households in the survey (demographic variables are based upon heads of households). Various income information is collected in the second and fifth interviews as well as information on the employment of each household member. As opposed to the PSID, which offers only food consumption data on an annual basis, the CEX contains highly detailed data on monthly consumption expenditures (food consumption is likely to be one of the most stable consumption components; furthermore, as is pointed out by Carroll (1994), 95% of the measured food consumption in the PSID is noise due to the absence of interview training). The CEX attempts to account for an estimated 70% of total household consumption expenditures. Since the CEX is designed with the purpose of collecting consumption data, measurement error in consumption is likely to be smaller for the CEX consumption data compared to the PSID consumption data.

[29]The CPI series are obtained from the Bureau of Labor Statistics through CITIBASE.

Asset Holders. For the consumer units completing their participation in the first through third quarters of 1986, the Bureau of Labor Statistics has changed, beginning the first quarter of 1986, the consumer unit identification numbers so that the identification numbers for the same household in 1985 (when this household has been interviewed for the first time) and in 1986 (when it has completed its participation) are not the same. To match consumer units between the 1985 and 1986 data tapes, we use household characteristics which allow us to identify consumer units uniquely. As a result, we manage to match 47.0% of households between the 1985 and 1986 data tapes. The detailed description of the procedure used to match consumer units is given in Appendix A.

In the fifth (final) interview, the household is asked to report end-of-period estimated market value of all stocks, bonds, mutual funds, and other such securities (market value of all securities) held by the consumer unit on the last day of the previous month as well as the difference in this estimated market value compared with the value of all securities held a year ago last month. Using these two values, we calculate asset holdings at the beginning of a 12-month recall period. The consumer unit is considered as an assetholder if the household's asset holdings at the beginning of a 12-month recall period exceed a certain threshold.

To assess the quantitative importance of limited participation of households in asset markets, we consider four sets of households. The first set (SET1) consists of all consumer units irrespectively of the reported market value of all securities. To take into consideration that only a fraction of households participates in asset markets, we use three sets of households defined as assetholders: the first one (SET2) consists of consumer units whose asset holdings are equal to or exceed $2 in 1999 dollars, the two others consist of all households with reported total assets equal to or exceeding $10000 (SET3) and $20000 (SET4).[30] Per capita consumption of a set of households is calculated as the equal-weighted average of normalized consumption expenditures of the households in the set. Obtained per capita consumption is seasonally adjusted by using the X-11 seasonal adjustment program.[31] We seasonally adjust the normalized consumption of each household by using the additive adjustments obtained from per capita consumption.

[30] Over the period 1991-1996 about 18% of households, for whom the market value of all securities held a year ago last month is not missing, reported asset holdings of $1 at the beginning of a 12-month recall period. That occurs when the household reported owning securities without precising their value (see Vissing-Jorgensen (1998)). Following Vissing-Jorgensen (1998), we classify these households as nonassetholders.

[31] Ferson and Harvey (1992) point out that since the X-11 program uses past and future information in the time-averaging it performs, this type of seasonal adjustment may induce spurious correlation between the error terms of a model and lagged values of the variables and, hence, may cause improper rejections of the model based on tests of overidentifying restrictions. As alternatives to using the X-11 program, Brav and Géczy (1995) propose to use a simpler linear filter (Davidson and MacKinnon (1993)) or the Ferson-Harvey (1992) method of incorporating forms of seasonal habit persistence directly in the Euler equation.

Data Selection Criteria. We drop from the sample nonurban households, households residing in student housing, households with incomplete income responses, and households who do not have a fifth interview. Following Brav, Constantinides, and Géczy (2002), in any given month we drop from the sample households that report in that month as zero either their food consumption or their NDS consumption, or their total consumption, as well as households with missing information on the above items. Additionally, we keep in the sample only households whose head is between 19 and 75 years of age.

3.2 Estimation of the Conditional Expectation of Consumption

Since the conditional expectation of consumption, h_{t+1}, is supposed to be the same for all investors, we may assume it to be equal to the conditional expectation of aggregate consumption per capita, $h_{t+1} = E_t \left[C_{t+1} \right]$. Assuming, as in Campbell and Cochrane (1999), a random walk model of consumption,

$$\Delta c_{t+1} = g + \eta_{t+1}, \tag{29}$$

where $\Delta c_{t+1} = log \frac{C_{t+1}}{C_t}$ and $\eta_{t+1} \sim N\left(0, \sigma_\eta^2\right)$, we get

$$h_{t+1} = exp\left(g + \frac{\sigma_\eta^2}{2}\right) C_t. \tag{30}$$

Table I presents the usual ML parameter estimates and tests of model (29).

3.3 Required Order of a Taylor Series Expansion

One approach to determine the order at which the expansion should be truncated is to allow data to motivate the point of truncation.[32] This approach consists in repeating the estimation of the model for increasing values of N and truncating the expansion at the point when further increasing in N does not significantly affect the estimation results. As Dittmar (2002) points out, there are at least two difficulties with allowing data to determine the required order of a Taylor series expansion. The first one is the possibility of overfitting the data. Another problem is that when a high-order expansion is taken, preference theory no longer guides in determining the signs of the priced risk factors. To avoid the last problem, Dittmar (2002) proposes to let preference arguments determine the point of truncation. He shows that increasing marginal utility, risk aversion, decreasing absolute risk aversion, and decreasing absolute prudence imply the fourth derivative of utility functions to be negative. Since preference assumptions do not guide in determining the signs of the higher-order derivatives, Dittmar (2002) assumes that the Taylor series expansion terms of order higher

[32] See Bansal, Hsieh, and Viswanathan (1993).

than three do not matter for asset pricing and truncates a Taylor series expansion after the cubic term.[33] His point of view is that the advantage coming from signing the Taylor series expansion terms outweighs a loss of power due to omitting the terms of order four and higher.

In this paper, we let both preference theory and the data guide the truncation. The restriction of decreasing absolute prudence allows us to sign the fifth derivative of utility functions and, therefore, pursue the expansion further than it is usually done. Following Dittmar (2002), we should truncate a Taylor series expansion after the cross-moment of return with the cross-sectional kurtosis of individual consumption. The question here is whether the cross-moments of return with the first four moments of the cross-sectional distribution of individual consumption can be estimated precisely given the previously mentioned potentially severe effect of measurement error on the estimates of the high-order moments of the cross-sectional distribution of individual consumption. To answer this question, we estimate the cross-moments $E\left[Z_{n,t+1}RP_{t+1}\right]$ and $E_t\left[Z_{n,t+1}RP_{t+1}\right]$ for n ranging from 1 to 4 using an iterated GMM approach.[34] We find that the precision of estimation decreases as n rises so that for all the sets of households classified as assetholders, the null hypotheses $E\left[Z_{n,t+1}RP_{t+1}\right] = 0$ and $E_t\left[Z_{n,t+1}RP_{t+1}\right] = 0$ are not rejected statistically at the 5% level for $n = 4$. This result confirms the conjecture that the observation error in reported individual consumption can make it difficult to precisely estimate the high-order cross-sectional moments of individual consumption and, therefore, their cross-moments with the excess market portfolio return.

We find that the cross-moments of the equity premium with the cross-sectional variance and skewness of individual consumption are both positive. It follows that the variance of the cross-sectional distribution of individual consumption represents the effect of complementarity in the portfolio risk and the background risk in wealth, while the cross-sectional skewness of individual consumption represents the effect of substitutability. Since both the unconditional and conditional cross-moments of the equity premium with the cross-sectional skewness of individual consumption are still estimated precisely, we limit our empirical investigation to the first three moments of the cross-sectional distribution of individual consumption.[35]

[33]Brav, Constantinides, and Géczy (2002) also limit their analysis to a third-order approximation when using a Taylor series expansion of the equal-weighted average of the household's IMRS.

[34]When the conditional cross-moments are estimated, the set of instruments consists of a constant and the term inside brackets lagged one period.

[35]Given the same problem with estimating high-order moments due to measurement error, Cogley (2002) also stops at a third-order polynomial when taking a Taylor series expansion of the individual's IMRS.

3.4 The Hansen-Jagannathan Volatility Bound Analysis

In this section, we assess the plausibility of the pricing kernel derived in Section 2 by studying its mean and standard deviation for different values of the risk aversion coefficient.

Assuming that a candidate SDF $M_t^*(m)$ may be formed as a linear combination of asset returns, $M_t^*(m) = m + (R_t - E[R_t])' \lambda_m$, Hansen and Jagannathan (1991) show that a lower bound on the volatility of any SDF M_t, that has unconditional mean m and satisfies the first-order condition $E[M_t R_t] = \iota$, is given by

$$\sigma(M_t^*(m)) = \left((\iota - mE[R_t])' \Sigma^{-1} (\iota - mE[R_t])\right)^{1/2}, \tag{31}$$

where ι is the vector of ones, R_t is the vector of time-t asset gross returns, and Σ is the unconditional variance-covariance matrix of asset returns.

When an unconditionally risk-free asset (or, more generally, an unconditional zero-beta asset[36]) exists, the first-order condition for the risk-free interest rate, $E[M_t R_{F,t}] = 1$, implies that the unconditional expectation of the SDF is the reciprocal of the expected gross return on this asset, $m = \frac{1}{E[R_{F,t}]}$. However, the restriction of nonsingularity of the second-moment matrix of asset returns, Σ, implies that there is no unconditionally risk-free asset or combination of assets[37] and, hence, m must be treated as an unknown parameter.

If excess returns are used, condition (31) becomes

$$\sigma(M_t^*(m)) = \left(m^2 E[R_t^e]' \widetilde{\Sigma}^{-1} E[R_t^e]\right)^{1/2}. \tag{32}$$

Here, R_t^e is the vector of excess returns and $\widetilde{\Sigma}$ denotes the variance-covariance matrix of excess returns.[38] Working with excess returns, we are allowed to assume that there is an unconditionally risk-free asset.[39] When such an asset exists, m is no longer an unknown parameter and may be calculated from data on the risk-free interest rate.

If there is a single excess return, the lower bound on the volatility of the SDF is given by

$$\sigma(M_t^*(m)) = m \frac{E\left[R_{i,t}^e\right]}{\sigma\left(R_{i,t}^e\right)}. \tag{33}$$

It means that the Sharpe ratio of any excess return $R_{i,t}^e$ must obey

$$\frac{E\left[R_{i,t}^e\right]}{\sigma\left(R_{i,t}^e\right)} \leqslant \frac{\sigma(M_t^*(m))}{m} = \frac{\sigma(M_t^*(m))}{E[M_t^*(m)]}. \tag{34}$$

[36] An asset which unconditional covariance with the SDF is zero.

[37] At least, its identity is not known beforehand.

[38] $R_{i,t}^e = R_{i,t} - R_{j,t}$ is the excess return on asset i over some reference asset j.

[39] This asset is usually used as a reference one.

The Hansen-Jagannathan or Sharpe ratio inequality (34) implies that for the SDF to be consistent with a given set of asset return data, it must lie above a ray from the origin with slope equal to the Sharpe ratio of the single risky excess return $R^e_{i,t}$, $\frac{E\left[R^e_{i,t}\right]}{\sigma\left(R^e_{i,t}\right)}$. The only excess return used in this empirical research is the excess value-weighted return on the market portfolio over the risk-free rate. Over the period from 1979:11 to 1996:2, the mean market premium is 0.73% per month with a standard deviation of 4.24%. The dotted line in Figure 1 indicates the lower volatility bound for the pricing kernels implied by the Sharpe ratio inequality (34) and the average monthly excess return on the U.S. stock market.

Our benchmark case is the conventional representative-agent model. For this model, the SDF in the Euler equation for the market premium is $\widetilde{M}_{t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$. The black "boxes" in Figure 1 represent mean-standard deviation pairs implied by the SDF $\widetilde{M}_{t+1}$ for γ ranging from 1 to 13 with increments of 1 for the set of all households in the sample (SET1). The white "boxes" denote mean-standard deviation points for the set of households whose asset holdings are equal to or exceed \$2 (SET2). The "triangles" and the "crosses" are used for the groups of households who reported total assets equal to or exceeding \$10000 (SET3) and \$20000 (SET4), respectively.

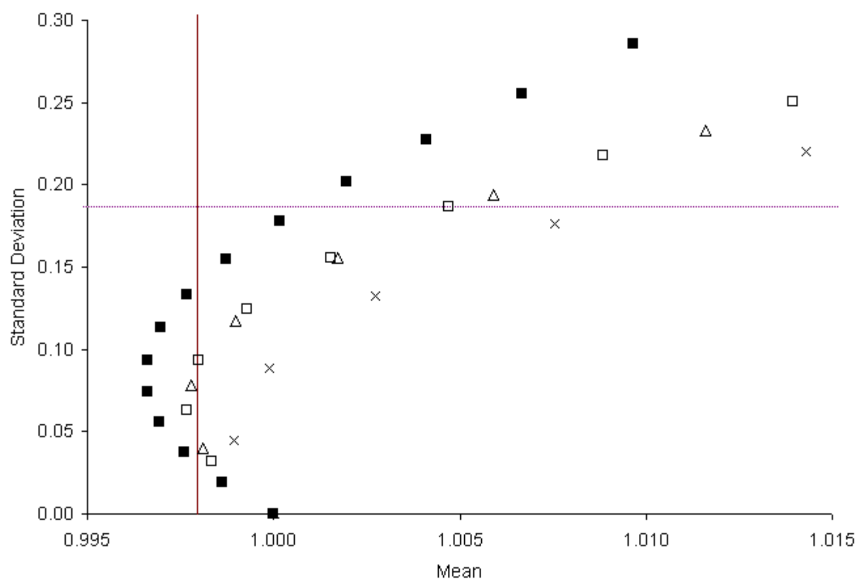

Figure 1: Standard deviation-mean diagram. $\widetilde{M}_{t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$. The dotted line indicates the lower volatility bound for SDFs implied by the Sharpe ratio inequality (34) and the average monthly excess return on the U.S. stock market. The solid vertical line at $\frac{1}{E\left[R_{F,t+1}\right]} = 0.9979$ indicates $E\left[\widetilde{M}_{t+1}\right]$ for δ equal to 1. The CEX data, 1979:10 to 1996:2.

To explain a Sharpe ratio of 0.17, the conventional representative-agent model needs a risk aversion coefficient $\gamma \geqslant 10$ for SET1, $\gamma \geqslant 6$ for SET2, and $\gamma \geqslant 5$ for SET3 and SET4. Although, as expected, the coefficient of risk aversion, γ, at which the mean-standard deviation points implied by $\widetilde{M}_{t+1}$ enter the feasible region, decreases in asset holdings, it is only slightly different across the sets of consumer units defined as assetholders.

From

$$m = E\left[M_{t+1}\right] = E\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right] = E\left[\delta\widetilde{M}_{t+1}\right] = \frac{1}{E\left[R_{F,t+1}\right]}, \tag{35}$$

it follows that $E\left[\widetilde{M}_{t+1}\right] = \frac{1}{\delta E\left[R_{F,t+1}\right]}$ and, hence, $\delta = \frac{1}{E\left[R_{F,t+1}\right]\cdot E\left[\widetilde{M}_{t+1}\right]}$. Given that over the period from 1979:11 to 1996:2 the sample mean real risk-free rate is 0.21% per month, the unconditional mean of the SDF is $m = \frac{1}{E\left[R_{F,t+1}\right]} = 0.9979$. The solid vertical line at $\frac{1}{E\left[R_{F,t+1}\right]} = 0.9979$ in Figure 1 indicates the mean of $\widetilde{M}_{t+1}$ for δ set to 1. By looking at Figure 1, we can see that when the mean-standard deviation points are in the admissible region for SDFs, the mean of $\widetilde{M}_{t+1}$ is larger than the reciprocal of the mean real risk-free interest rate, what implies δ less than 1. However, for economically plausible (less than 3) values of the risk aversion coefficient, $\widetilde{M}_{t+1}$ has, in most cases, a mean which is less than $\frac{1}{E\left[R_{F,t+1}\right]}$ and, then, the Euler equation for the risk-free rate can be satisfied only with δ larger than 1.[40] This is the risk-free rate puzzle. At the same time, for the values of the RRA coefficient in the conventional range, the SDF has a standard deviation which is much less than that required by the Sharpe ratio. This is the equity premium puzzle.

Now, let us take a Taylor series expansion of the representative-agent's marginal utility of consumption around the conditional expectation of aggregate consumption per capita. That implies the following pricing kernel:

$$\widetilde{M}_{t+1} = 1 + \sum_{n=1}^{N} \frac{1}{n!}\left(-1\right)^n \left(\prod_{l=0}^{n-1}\left(\gamma + l\right)\right) \frac{Z^a_{n,t+1}}{h^n_{t+1}}. \tag{36}$$

The Hansen-Jagannathan volatility bound analysis results for $\widetilde{M}_{t+1}$ given by (36) for a third-order ($N = 3$) Taylor series expansion of the representative-agent's marginal utility of consumption are presented in Figure 2. These results show that even when a third-order Taylor series expansion is taken and only a fraction of consumers is assumed to participate in asset markets, relatively high risk aversion is needed to make the mean-standard deviation points enter the feasible region.

If consumption insurance is incomplete, then individuals are heterogeneous and, hence,

[40] For SET1, for example, the value of δ larger than 1 is required for fitting the return on the risk-free asset for any value of γ between 2 and 7.

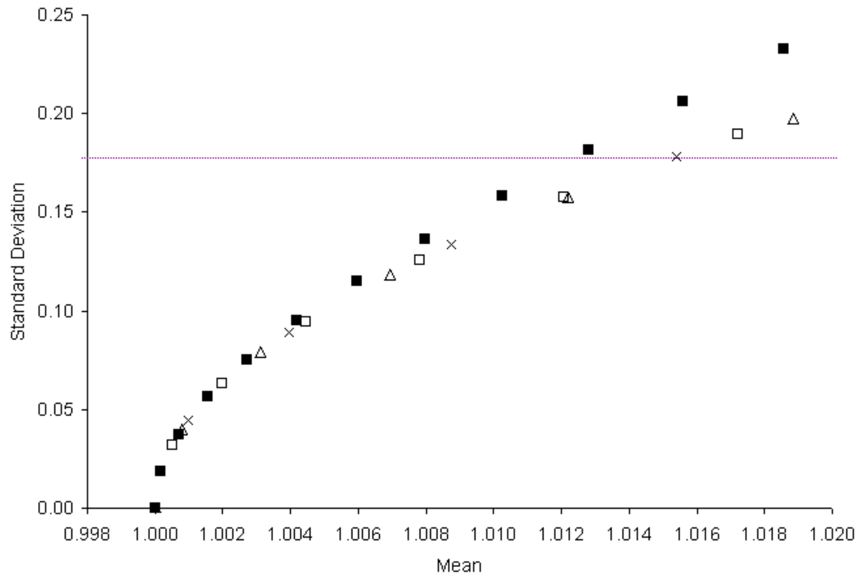

Figure 2: Standard deviation-mean diagram. $\widetilde{M}_{t+1} = 1+\sum_{n=1}^{3} \frac{1}{n!} (-1)^n \left(\prod_{l=0}^{n-1} (\gamma + l)\right) \frac{Z^a_{n,t+1}}{h^n_{t+1}}$. The dotted line indicates the lower volatility bound for SDFs implied by the Sharpe ratio inequality (34) and the average monthly excess return on the U.S. stock market. The CEX data, 1979:10 to 1996:2.

the SDF in the Euler equation for the equity premium is

$$\widetilde{M}_{t+1} = 1 + \sum_{n=1}^{N} \frac{1}{n!} (-1)^n \left(\prod_{l=0}^{n-1} (\gamma + l)\right) \frac{Z_{n,t+1}}{h^n_{t+1}}. \tag{37}$$

The results for the SDF specification (37) corresponding to a third-order ($N = 3$) Taylor series expansion of the agent's marginal utility of consumption are shown in Figure 3. The first mean-standard variation point above the horizontal axis corresponds to the RRA coefficient of 0.1, successive points have relative risk aversion of 0.2, 0.3, and so on. Given the results in Figure 3, we may conclude that taking into account consumer heterogeneity substantially affects the mean and standard deviation of the SDF. The Hansen-Jagannathan analysis shows that the mean-standard deviation points enter the feasible region at plausible values of γ (less than 1), unlike the case when a representative agent within each group of households is assumed.[41] For a given value of γ, volatility of the SDF is only slightly affected by the size of asset holdings, so that the value of risk aversion allowing to explain the market premium does not significantly differ across sets of households.

[41] Although both the volatility bound and the mean-standard deviation points implied by the SDF are estimated with error, it seems, however, to be improbable that this error is so large to substantially affect the results and to make the points enter the feasible region at implausibly high values of γ.

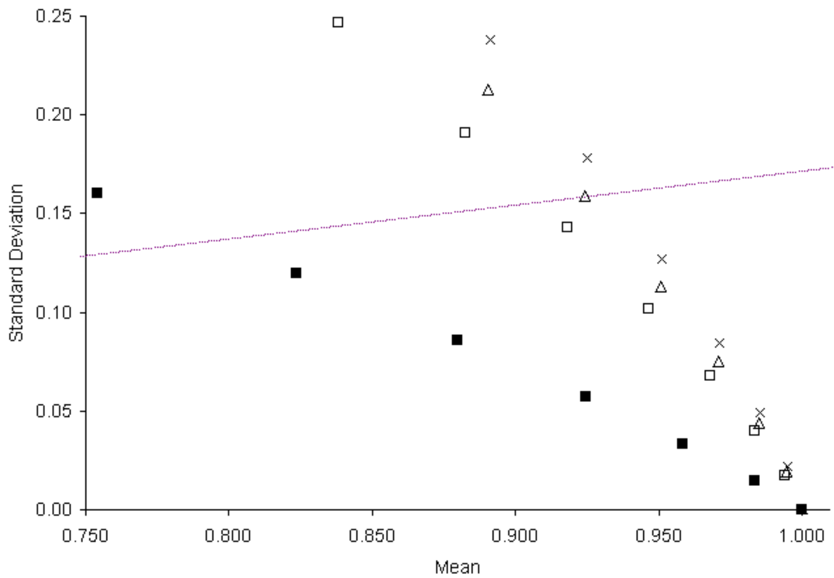

Figure 3: Standard deviation-mean diagram. $\widetilde{M}_{t+1} = 1+\sum_{n=1}^{3} \frac{1}{n!} (-1)^n \left(\prod_{l=0}^{n-1} (\gamma + l)\right) \frac{Z_{n,t+1}}{h_{t+1}^n}$. The dotted line indicates the lower volatility bound for SDFs implied by the Sharpe ratio inequality (34) and the average monthly excess return on the U.S. stock market. The CEX data, 1979:10 to 1996:2.

3.5 Model Calibration

In this section, we address the question of whether the size of the adverse effect of the independent background risk in wealth on the attitude towards the portfolio risk is such that it makes it possible to explain the observed mean excess return on the market portfolio and risk-free rate with economically realistic values of the preference parameters. The results are presented in Table II.

As in the preceding section, our benchmark case is the conventional representative-agent model. For this model, we calculate the unexplained mean equity premium as

$$v_1 = \frac{1}{T} \sum_{t=0}^{T-1} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} RP_{t+1} \tag{38}$$

for the values of the risk aversion coefficient γ increasing from 0 with increments of 0.1.[42] When the RRA coefficient is set to zero, the unexplained mean premium is equal to the sample mean of the excess market portfolio return.[43] In Table II, we report the values of γ for which the unexplained mean premium of the value-weighted market portfolio becomes

[42] See Brav, Constantinides, and Géczy (2002).

[43] Over the period from 1979:11 to 1996:2, the mean premium of the value-weighted market portfolio is 0.73% per month.

negative. As we can see in Table II, even when limited asset market participation is taken into consideration, the conventional representative-agent model is able to fit the observed mean equity premium only if an individual is assumed to be implausibly risk averse.

When a Taylor series expansion of the representative-agent's marginal utility of consumption is taken, we calculate the statistic v_2 as

$$v_2 = \frac{1}{T} \sum_{t=0}^{T-1} \left(1 + \sum_{n=1}^{N} \frac{1}{n!} (-1)^n \left(\prod_{l=0}^{n-1} (\gamma + l) \right) \frac{Z^a_{n,t+1}}{h^n_{t+1}} \right) RP_{t+1}. \tag{39}$$

The results in Table II show that a Taylor series expansion of any order fails to explain the mean equity premium with an economically plausible value of the RRA coefficient. When a first-order Taylor series expansion is taken, the mean premium of the value-weighted market portfolio can be explained with the RRA coefficient ranging from 22.4 to 113 for different sets of households. In the case of a third-order Taylor series expansion, the mean equity premium can be explained with the values of the RRA coefficient which are slightly lower than those in the first-order Taylor series expansion case (between 23.2 and 63), but, nevertheless, remain too large to be recognized as economically plausible. As the representative-agent's marginal utility of consumption is expanded as a Taylor series up to terms capturing the third cross-moment of the excess market portfolio return with aggregate consumption, the statistic v_2 increases as the RRA coefficient rises, so that there is no positive value of γ allowing to fit the observed mean premium of the value-weighted market portfolio. Under the assumption of limited asset market participation, we find some evidence that the risk aversion coefficient decreases as the threshold value in the definition of assetholders is raised. When the threshold value is quite large ($10000 in 1999 dollars or larger), one can explain the mean equity premium with γ which is lower than that in the conventional representative-agent model. However, even after limited participation is taken into consideration, the model fails to explain the mean excess market portfolio return with an economically realistic value of risk aversion.

Assuming incomplete consumption insurance, we calculate the unexplained mean equity premium as

$$v_3 = \frac{1}{T} \sum_{t=0}^{T-1} \left(1 + \sum_{n=1}^{N} \frac{1}{n!} (-1)^n \left(\prod_{l=0}^{n-1} (\gamma + l) \right) \frac{Z_{n,t+1}}{h^n_{t+1}} \right) RP_{t+1}. \tag{40}$$

In contrast to the complete consumption insurance case, taking into account consumer heterogeneity allows to fit the mean excess market portfolio return with an economically plausible (between 1 and 1.8) value of the RRA coefficient when the agent's marginal utility of consumption is expanded as a Taylor series up to cubic terms. Under the hypothesis of limited asset market participation, empirical evidence that the RRA coefficient decreases as the threshold value rises is weak. There is no positive value of the RRA coefficient allowing to explain the mean equity premium when the agent's marginal utility of consumption is expanded as a Taylor series up to terms capturing the cross-sectional variance of individual consumption.[44] Given the values of the risk aversion parameter γ which allow to explain the observed mean premium of the value-weighted market portfolio, we estimate the time discount factor δ needed to fit the observed mean risk-free rate as

$$\delta = \frac{\frac{1}{T}\sum_{t=0}^{T-1}\left[\left(\frac{h_{t+1}}{h_t}\right)^{\gamma}\left(1+\sum_{n=1}^{N}\frac{1}{n!}\left(-1\right)^n\left(\prod_{l=0}^{n-1}\left(\gamma+l\right)\right)\frac{Z_{n,t}}{h_t^n}\right)\right]}{\frac{1}{T}\sum_{t=0}^{T-1}\left[\left(1+\sum_{n=1}^{N}\frac{1}{n!}\left(-1\right)^n\left(\prod_{l=0}^{n-1}\left(\gamma+l\right)\right)\frac{Z_{n,t+1}}{h_{t+1}^n}\right)R_{F,t+1}\right]}. \tag{41}$$

To test whether the obtained results are susceptible to additive measurement error in the consumption level, we assume that observation error is normally distributed with zero mean, $\epsilon_{k,t} \sim N\left(0, \sigma_{\epsilon,t}^2\right)$, and independent of true consumption. We further assume that the cross-sectional variance of measurement error is 20% of the cross-sectional variance of the household consumption observed in the data, $\sigma_{\epsilon,t}^2 = 0.2\frac{1}{K}\sum_{k=1}^{K}\left(C_{k,t}-C_t\right)^2$. The row "allowing for observation error" in Table II presents the results obtained when $\widetilde{C}_{k,t} = C_{k,t}+\epsilon_{k,t}$ is used in the calibration. These results illustrate that in a small sample framework with the measurement error of the type analyzed here, the estimate of γ will be biased upward. Empirical evidence is that, in contrast to the estimate of γ, the estimate of δ is quite sensitive to idiosyncratic observation error in the consumption level.

3.6 GMM Results

The Hansen-Jagannathan volatility bound analysis and calibration results show that the complete consumption insurance model does not perform well. However, taking into account asymmetry of the cross-sectional distribution of individual consumption allows to explain both the equity premium and the return on the risk-free asset with economically realistic values of the preference parameters. Given this result, in this section we limit our analysis to the incomplete consumption insurance case.

An iterated GMM approach is used to test Euler equations and estimate model parameters. We estimate the Euler equation

$$E_t\left[\left(1+\sum_{n=1}^{N}\frac{1}{n!}\left(-1\right)^n\left(\prod_{l=0}^{n-1}\left(\gamma+l\right)\right)\frac{Z_{n,t+1}}{h_{t+1}^n}\right)RP_{t+1}\right]=0 \tag{42}$$

[44] Although Brav, Constantinides, and Géczy (2002) take a Taylor series expansion of the equal-weighted sum of the household's IMRS and not that of the agent's marginal utility of consumption, as in our work, their results are similar to ours. Specifically, they find that when the SDF is expressed in terms of the cross-sectional mean and variance of the household consumption growth rate, the average unexplained premium increases as the RRA coefficient rises. When a Taylor series expansion captures the cross-sectional skewness, in addition to the mean and variance, the average unexplained excess return on the market portfolio is less than that for the SDF given by the equal-weighted sum of the household's IMRS.

for the excess value-weighted return and the Euler equation

$$\delta E_t \left[\left(1 + \sum_{n=1}^{N} \frac{1}{n!} (-1)^n \left(\prod_{l=0}^{n-1} (\gamma + l) \right) \frac{Z_{n,t+1}}{h_{t+1}^n} \right) R_{F,t+1} \right] =$$
$$\left(\frac{h_{t+1}}{h_t} \right)^{\gamma} \left(1 + \sum_{n=1}^{N} \frac{1}{n!} (-1)^n \left(\prod_{l=0}^{n-1} (\gamma + l) \right) \frac{Z_{n,t}}{h_t^n} \right) \tag{43}$$

for the gross return on the real risk-free interest rate jointly exploiting three sets of instruments. The first instrument set (INSTR1) consists of a constant, the real value-weighted market return, the real risk-free rate, and the real consumption growth rate lagged one period. The second set of instruments (INSTR2) is the first set extended with the same variables lagged an additional period. The third set (INSTR3) has a constant, the real value-weighted market return, the real risk-free rate, and the real consumption growth rate lagged one, two, and three periods.

To study the role of the first three moments of the cross-sectional distribution of individual consumption in explaining the equity premium and risk-free rate puzzles, we truncate the series expansion after the cubic term and estimate the Euler equations with $N = 3$. The model is able to fit the excess return on the market portfolio and the risk-free rate with an economically plausible (less than 1.5) and statistically significant value of risk aversion for any set of households whatever the set of instruments (see Table III). The estimate of the RRA coefficient is decreasing in asset holdings, as anticipated. According to Hansen's test of the overidentifying restrictions, the model is not rejected statistically at the 5% level.

4 Conclusions

Empirical evidence suggests that the complete consumption insurance model fails to fit the observed equity premium with an economically plausible value of risk aversion. This result is robust to the threshold value in the definition of assetholders and the used analysis method.

The impact of incomplete consumption insurance on the expected equity premium is mixed. We find that the cross-moments of the excess market portfolio return with the cross-sectional variance and skewness of individual consumption are both positive. It follows that the cross-sectional variance of individual consumption represents the effect of complementarity in the portfolio risk and the background risk in wealth, while the cross-sectional skewness represents the effect of substitutability. The empirical results show that the effect of substitutability dominates. Since the used in our empirical investigation CRRA preferences exhibit decreasing and convex absolute risk aversion and decreasing absolute prudence, this result is in line with the results of Weil (1992) and Gollier (2001).

Another important result is that both the equity premium and the risk-free rate may be explained with economically realistic values of the RRA coefficient (less than 1.8) and the time discount factor when the cross-sectional skewness of individual consumption, combined with the cross-sectional mean and variance, is taken into account. This result is robust to the threshold value in the definition of assetholders and the estimation procedure.

APPENDIX A: Matching Consumer Units between the 1985 and 1986 Data Tapes

In the CEX, each household is interviewed every three months over five consecutive quarters. The initial interview collects demographic and family characteristics and is not placed on the tape. Each quarter, consumer units that have completed their final interview in the previous quarter (about one-fifth of the sample) are replaced by new households introduced for the first time. The households remained on the tape complete their participation. For the consumer units completing their participation in the first through third quarters of 1986, the Bureau of Labor Statistics has changed beginning the first quarter of 1986 the consumer unit identification numbers (NEWID). As a result, the NEWIDs for the same household in 1985 (when this household has been interviewed for the first time) and in 1986 (when it has completed its participation) are no longer the same.

To match consumer units between the 1985 and 1986 data tapes, we use the following household characteristics:

AGE_REF - age of reference person,
COMP1 - number of males age 16 and over in family,
COMP2 - number of females age 16 and over in family,
COMP3 - number of males age 2 through 15 in family,
COMP4 - number of females age 2 through 15 in family,
COMP5 - number of members under age 2 in family,
BLS_URBN - area of residence (urban/rural),
BUILDING - description of building,
EDUC_REF - education of reference person,
MARITAL1 - marital status of reference person,
ORIGIN1 - origin or ancestry of reference person,
POPSIZE - population size of the primary sampling unit,
REF_RACE - race of reference person,
REGION - region (for urban areas only),
SEX_REF - sex of reference person.

The values of the variables SEX_REF, ORIGIN1, and REF_RACE must be the same for the same household on both the 1985 and 1986 data tapes. Moreover, the CEX is constructed so that the values of the variables BLS_URBN, REGION, and BUILDING are also the same for the same consumer unit over all interviews.

As a rule, the variable POPSIZE also has the same value for the same consumer unit over all interviews. However, on the 1985 and 1986 data tapes this variable is coded differently:

	1985	1986
1	More than 4 million	More than 4 million
2	1.25 million - 4 million	1.20 million - 4 million
3	0.385 - 1.249 million	0.33 - 1.19 million
4	75 - 384.9 thousand	75 - 329.9 thousand
5	Less than 75 thousand	Less than 75 thousand

It follows that in the case of these two years, the same consumer unit has the same code for POPSIZE only if in 1985 this code is 1, 2, or 5. If a consumer unit has in 1985 the code 3, then it can have in 1986 the code 2 or 3. Households having in 1985 the code 4 can have the code 3 or 4 in 1986. It is valid for all households living in the Northeast, the Midwest, and the South region. For consumer units residing in the West, the variable POPSIZE is suppressed by the Bureau of Labor Statistics on the 1986 data tape.

It is more difficult to deal with the variables MARITAL1, EDUC_REF, COMP1, COMP2, COMP3, COMP4, and COMP5 which can take different values over interviews. For these variables, we determine the set of all possible values that they can take in 1986 given their values in 1985.

The variable MARITAL1 is coded as follows:

1 Married
2 Widowed
3 Divorced
4 Separated
5 Never married

If a reference person is married today, then tomorrow he may be either married or widowed, or divorced, or separated. If he is widowed or divorced, he may either keep this status or be married. In the case, when a reference person is separated, he remains to be separated or else becomes to be widowed or divorced. A never married person can either keep this marital status or be married.

Let $COMP1_{h,t}$ denote the number of males age 16 and over in family h in period t, $COMP2_{h,t}$ - the number of females age 16 and over, $COMP3_{h,t}$ - the number of males age 2 through 15, $COMP4_{h,t}$ - the number of females age 2 through 15, $COMP5_{h,t}$ - the number of members under age 2 in family, $SUM\_T_{h,s} = COMP3_{h,s} + COMP4_{h,s} + COMP5_{h,s}$ - the total number of children age 15 and under in family in any period s ($s > t$, $t = 2, 3, 4$), $SUM\_M_{h,s} = COMP1_{h,s} + COMP3_{h,s}$ and $SUM\_F_{h,s} = COMP2_{h,s} + COMP4_{h,s}$ - the total number of males age 2 and over and the total number of females age 2 and over, respectively.

We apply the following restrictions:

$$COMP5_{h,t} \leqslant SUM\_T_{h,s} \tag{A.1}$$

and

$$COMP3_{h,t} \leqslant SUM\_M_{h,s} \leqslant COMP1_{h,t} + COMP3_{h,t} + COMP5_{h,t}. \tag{A.2}$$

It follows that

$$0 \leqslant SUM\_M_{h,s} - COMP3_{h,t} \leqslant COMP1_{h,t} + COMP5_{h,t}. \tag{A.3}$$

Similarly, for $SUM\_F_{h,s}$ we get

$$COMP4_{h,t} \leqslant SUM\_F_{h,s} \leqslant COMP2_{h,t} + COMP4_{h,t} + COMP5_{h,t} \tag{A.4}$$

and

$$0 \leqslant SUM\_F_{h,s} - COMP4_{h,t} \leqslant COMP2_{h,t} + COMP5_{h,t}. \tag{A.5}$$

Finally,

$$\begin{aligned} SUM\_M_{h,s} + SUM\_F_{h,s} \quad \leqslant \quad & COMP1_{h,t} + COMP2_{h,t} + COMP3_{h,t} \\ & + COMP4_{h,t} + COMP5_{h,t}. \end{aligned} \tag{A.6}$$

In the CEX, the variable EDUC_REF is coded as follows:

1 Elementary (1-8 years)
2 High school (1-4 years), less than High school graduate
3 High school graduate (4 years)
4 College (1-4 years), less than College graduate
5 College graduate (4 years)
6 More than 4 years of college
7 Never attended school

Changing the code from 7 to 0 enables us to introduce the restriction:

$$EDUC\_REF_{h,s} - EDUC\_REF_{h,t} \leqslant 1. \tag{A.7}$$

We apply the following restriction to the value of the variable AGE_REF:

$$AGE\_REF_{h,t} \leqslant AGE\_REF_{h,s}. \tag{A.8}$$

To match consumer units between the 1985 and 1986 data tapes, we define three groups of households. The first group consists of consumer units that should have the following stream of interviews: the second interview in the second quarter of 1985, the third - in the third quarter of 1985, the fourth - in the fourth quarter of 1985, and the fifth interview in the first quarter of 1986. For this group, we construct two data sets. In the 1985 data set, we include all households who completed at least one of the interviews in 1985. The

1986 data set consists of households who completed their fifth interview in the first quarter of 1986. As a second group are considered consumer units whose second interview should happened in the third quarter of 1985, the third - in the fourth quarter of 1985, the fourth - in the first quarter of 1986, and the fifth interview should happened in the second quarter of 1986. For this group, the 1985 data set includes all consumer units who completed at least one interview in 1985 and the 1986 data set consists of households who completed at least one of the interviews in 1986. In the third group of households, we include consumer units that should have the second interview in the fourth quarter of 1985, the third - in the first quarter of 1986, the fourth - in the second quarter of 1986, and the fifth interview in the third quarter of 1986. For this group of consumer units, the 1985 data set consists of all households who completed their second interview in the fourth quarter of 1985 and the 1986 data set consists of consumer units who completed in 1986 at least one of the interviews.

So, for each group of consumer units there are two data sets (one for 1985 and another one for 1986). To find the same household in both data sets, each consumer unit included in the 1986 data set is compared with each household included in the 1985 data set for the same group of households. As a result, for the first group of consumer units there are 521 households (in 1019 included in the 1986 data set, 51.1%), for which we find households with appropriate characteristics in the corresponding 1985 data set. For the second group, this number is 756 (in 1612 consumer units, 49.6%). For the third group, we match 807 households (in 1800, 44.8%). For all the three groups, the number of matched households is 2084 (in 4431 included in the 1986 data sets, 47.0%).[45]

[45] In order to see how well this procedure works, we test it using the 1986 and 1987 data tapes (in both these years, the same household has the same NEWID) for the first group of consumer units constructed in the same way as in the case of using the 1985 and 1986 data tapes. There are 1465 households who completed their fifth interview in the first quarter of 1987 and 1787 consumer units included in the 1986 data set. Only 1344 in 1465 households have at least one interview happened in 1986. As a result, we match 1281 households (95.3% of the real number). In 96% of the cases, there is only one household in the 1986 data set corresponding to the household included in the 1987 data set. In only 4% of the cases, there are more than 1 (between 2 and 4) corresponding households.

APPENDIX B: Tables

Table I

Parameter Estimates and Tests of Model $\Delta c_{t+1} = g + \eta_{t+1}$

The sampling period is from 1979:10 to 1996:2. Four sets of households are considered. The first set (SET1) consists of all consumer units irrespectively of the reported market value of all securities. We use three sets of households classified as assetholders: SET2 consists of households who reported asset holdings equal to or exceeding \$2 in 1999 dollars, SET3 and SET4 consist of households who reported total assets equal to or exceeding \$10000 and \$20000, respectively. The model is estimated by ML. Standard errors in parentheses.

Parameters	SET1	SET2	SET3	SET4
g	0.0016	0.0022	0.0027	0.0021
	(0.0014)	(0.0023)	(0.0029)	(0.0032)
σ_η^2	0.0004	0.0010	0.0016	0.0020

Table II

Calibration Exercise

Values of the Preference Parameters Needed to Explain the Observed Mean Equity Premium and Risk-Free Rate

The sampling period is from 1979:10 to 1996:2. Four sets of households are considered. The first set (SET1) consists of all consumer units irrespectively of the reported market value of all securities. We use three sets of households classified as assetholders: SET2 consists of households who reported asset holdings equal to or exceeding \$2 in 1999 dollars, SET3 and SET4 consist of households who reported total assets equal to or exceeding \$10000 and \$20000, respectively. Panel A provides the results under the assumption of complete consumption insurance. In panel B, we report the results under incomplete consumption insurance. Under the assumption of incomplete consumption insurance, we present the results for both the raw consumption data (the row "raw consumption data") and under the assumption that 20% of the observed cross-sectional variance of individual consumption is noise (the row "data allowing for observation error"). The sign "-" means that there is no positive value of the RRA coefficient allowing to explain the observed mean equity premium.

Model	Parameters	SET1	SET2	SET3	SET4
Panel A: Complete consumption insurance					
1. Standard representative-agent model	γ	36.00	51.00	30.00	36.00
2. First-order Taylor series expansion	γ	113.00	76.00	27.00	22.40
3. Second-order Taylor series expansion		-	-	-	-
4. Third-order Taylor series expansion	γ	36.00	63.00	23.40	23.20
Panel B: Incomplete consumption insurance					
1. First-order Taylor series expansion	γ	113.00	76.00	27.00	22.40
2. Second-order Taylor series expansion		-	-	-	-
3. Third-order Taylor series expansion:					
- raw consumption data	γ	1.06	1.55	1.38	1.41
	δ	0.9840	1.3098	1.0073	0.9169
- data allowing for observation error	γ	1.13	1.75	1.45	1.51
	δ	0.9847	0.8895	1.0095	0.9141

Table III

GMM Results under Incomplete Consumption Insurance

The sampling period is from 1979:10 to 1996:2. Four sets of households are considered. The first set (SET1) consists of all consumer units irrespectively of the reported market value of all securities. We use three sets of households classified as assetholders: SET2 consists of households who reported asset holdings equal to or exceeding \$2 in 1999 dollars, SET3 and SET4 consist of households who reported total assets equal to or exceeding \$10000 and \$20000, respectively. The agent's marginal utility of consumption is expanded as a Taylor series up to cubic terms $(N = 3)$. The Euler equation for the excess value-weighted market return is estimated jointly with the Euler equation for the real risk-free interest rate using an iterated GMM approach (standard errors in parentheses). Three sets of instruments are exploited. The first instrument set (INSTR1) consists of a constant, the real value-weighted market return, the real risk-free rate, and the real consumption growth rate lagged one period. The second set of instruments (INSTR2) is the first set extended with the same variables lagged an additional period. The third set (INSTR3) has a constant, the real value-weighted market return, the real risk-free rate, and the real consumption growth rate lagged one, two, and three periods. The J statistic is Hansen's test of the overidentifying restrictions. The P value is the marginal significance level associated with the J statistic.

Parameters	SET1	SET2	SET3	SET4
		INSTR1		
γ	1.2126	1.1282	0.0715	0.0848
	(0.0505)	(0.0780)	(0.0189)	(0.0223)
δ	0.9901	1.0113	0.9978	0.9977
	(0.2349)	(0.0223)	(0.0005)	(0.0005)
J statistic	4.9871	7.1099	7.6519	7.7094
P value	0.7590	0.5248	0.4682	0.4624
		INSTR2		
γ	1.1495	1.4911	1.3999	1.4609
	(0.0296)	(0.0574)	(0.0554)	(0.0584)
δ	0.9878	0.9854	1.0138	0.9733
	(0.0517)	(0.0971)	(0.0595)	(0.1049)
J statistic	6.5296	8.9943	8.2458	7.6449
P value	0.9813	0.9137	0.9412	0.9587
		INSTR3		
γ	1.0611	0.0128	0.0077	0.0091
	(0.0206)	(0.0016)	(0.0014)	(0.0011)
δ	0.9977	0.9988	0.9982	0.9979
	(0.0231)	(0.0001)	(0.0001)	(0.0001)
J statistic	8.9334	9.4825	9.5840	9.6172
P value	0.9977	0.9964	0.9961	0.9960

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