Using Flexible McFadden Export Supply and Import Demand Functions for Bilateral Trade Policy Analysis

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Abstract

The aim of the paper is twofold. The first objective is to show the limitations in employing the CES and CET functions for modelling bilateral trade flows as is commonly done in trade models. We suggest to use a flexible form such as the Symmetric Generalized McFadden Function (SGMF) which is flexible from second order. The other aim refers to the critique on the handling of model parameters in calibrated policy models, the so called idiot's law of elasticities. In general, employed elasticities violate the theoretical conditions and must be adjusted to comply with the symmetry, homogeneity, budget and curvature condition. Parameters of flexible functional forms of the second-order type can be calibrated with relatively little effort. We describe the calibration procedure developed to obtain model parameters consistent with economic theory. The method used is a two-stage minimisation program.

1 Introduction

Advances over the last decades with regard to trade theory and methodology for analysing international trade are substantial. Rigorous application of duality theory allows for more structural complexity with simultaneously adhering to the theoretical conditions. This holds for the analyses of supply as well as demand in both general and partial equilibrium models. However, without the concurrent development of new functional forms it would not have been possible to make full use of the potential of this theory in empirical investigations. For decades, the Cobb-Douglas, the Constant Elasticity of Substitution (CES) and, with some limitation, also the Leontief functional forms dominate quantitative assessments of policies and technical change on supply, demand and trade as well as providing examples for text books on applications in microeconomic theory. As of today, especially the CES function and, its mirror part, the Constant Elasticity of Transformation (CET) function enjoy still widespread use in computable general equilibrium (CGE) models with a multi-country focus. If imported goods are differentiated by country of origin; i.e. if they are not homogeneous then the CES form also is often applied.[1] Though used in many empirical studies involving CGE models these functional forms may cause difficulties with regard to being theoretical consistent. In addition, they include a number of maintained hypotheses which limit their applicability. To those who use the model results it is usually rather difficult to distinguish the models based on their algebraic functional form, and the maintained hypotheses hidden in these functions cannot easily be detected, especially by those not familiar with modelling (Frohberg 2001).

A rather severe limitation is the restricted substitutability among the arguments of the functions mentioned. The Leontief form does not allow for substitution, the Cobb Douglas function maintains the substitution elasticity to be unity and the CES one allows a sufficient range for substitution only if there are not more than two

[1] There are only a few studies applying other functional forms for trade analysis (for example Hanson et al., 1993).

arguments. Due to this single constant elasticity of substitution or transformation they do not have the flexibility needed to reflect adequately second-order effects if three or more independent variables are included in the analysis.

Especially the approach used for implementing the assumption that imported goods are differentiated by country of origin was recently criticised also on theoretical grounds (Gros 1987, Panagariya 2000, Panagariya and Duttagupta 2001). If products are not homogeneous they are characterised by imperfect substitutability. Such a condition was originally assumed by Armington (1969) with regard to domestically produced and imported goods, both being aggregated to a composite consumption item. Panagariya (2000) argues that the assumption of imperfect substitutability by country of origin implies some monopoly power of the corresponding country related with relatively high optimal tariffs. He, therefore, claims that there is an inherent contradiction between this postulate and the small-country assumption both of which are very often made simultaneously in models designed for empirical trade analysis. In this respect, CGE modellers are criticised to generate benefits for small countries from preferential liberalisation (Robinson and Thierfelder 1999) by recourse to an inadequate model specification and inappropriate parameter values. Results of these models, it is claimed, do not match with those derived by applying the theory without compromise (Bhagwati, Greenaway, Panagariya 1998). The debate on the gains from preferential trade liberalisation and “new regionalism” as well as the discussion on the tools employed for trade policy analysis is still going on. Schiff and Winters (2003) criticise CGE models because of their typically ad hoc estimates of behavioural parameters and moreover they argue that CGE models overestimate the terms of trade benefits to the members of regional trade agreements (RTAs) even because they question the monopoly power of small countries implicitly supposed by the Armington approach. In a recent paper Burfisher, Robinson and Thierfelder (2003) survey this dispute in particular with respect to the different tools applied to analyse RTAs and they contradict the arguments of Panagariya and Schiff and Winters. They consider CGE models to become the work-horse of policy analysis as they provide the most appropriate tool for examining the impact of trade liberalisation on world prices, trade and welfare within a framework that has a consistent foundation in microeconomics and trade theory.

Regarding the degree of substitutability, Hillberry et al. (2001) as well as McDaniel and Balistreri (2002) postulate that too small numerical values for the substitution parameters are presumed in multi-country CGE models for quantifying the degree to which agents respond to differences in bilateral trade costs. Instead most of the buying pattern is explained by the so called taste (Armington efficiency) parameter. The authors find that Armington taste parameters in CGE models are like error terms in econometric models, in that they contain the unexplained variance in the dependent variable. A low substitutability limits the model’s response to changes in trade policies caused by changes in relative bilateral trade costs. This explains the widely-held belief in policymaking circles that empirical analyses based among others on the CES function understate the effects of policy changes on the economy. Moreover, these authors demonstrate that not only the quantitative effects of liberalisation but also the qualitative ones i.e. losses or benefits, are sensitive to the choice of the CES substitution parameter. Similar problems arise in models depicting export possibilities of a country in a mirror like fashion to those of imports and using for this purpose the CET function. Besides limiting substitution possibilities in their conventional form, the CES and CET functions also imply a homothetic structure. Using them for differentiating products in trade analysis implies that changes in the total quantities of imports or exports leaves their respective country shares unaltered.

Another empirical feature which is rather useful in modelling but quite often rejected by empirical tests is weak separability (Saito 2002). If the elasticity of substitution is econometrically estimated then usually it is done for the relation between domestically produced goods sold at internal market and the aggregate of the imported good from all sources. The substitution elasticities for the various imports, each of which differentiated by country of origin, are even less often estimated. The reason for this one-sided approach is simply data availability. If separability and homotheticity are given the substitution elasticities at the second stage can be estimated independently of those at the first level in a theoretically consistent way. These two properties are, therefore, very desirable because they lead to a less complex model structure. Unfortunately, they are difficult to test since this requires that they are simultaneously econometrically estimated. However, in the relative few cases where their statistical significance was checked they had to be quite often rejected.
To summarise the debate on the influence of CGE models on policy and further the effect of functional forms on simulation results, it is important to distinguish small stylised models and complex applied models. Stylised models developed for example by Panagariya and Duttagupta (2001) indicate how sensibly models react to the selection of CES parameters and how switching of substitution elasticities may turn a loss of welfare into a gain and vice versa, but they may be misused when pushed beyond their domain of applicability (Devarajan, Robinson 2002). Large applied models reviewed by Robinson and Thierfelder on the other hand, have been criticised for their ad hoc parameterisation and their black box character (Dawkins et al. 2001). However, modellers should keep in mind the limitations of the functional forms usually employed in both kinds of models, when interpreting simulation result. We think empirical models may gain from the experimentation with more advanced functional forms, they exhibit advantages some of which are demonstrated in the following.

2 Model Specification

To illustrate the advantages of the flexible SGMF for modelling bilateral trade flows, we used the standard CGE model developed at IFPRI (Löfgren, Harris, Robinson 2001) with a complete and consistent data set for the country Mozambique.[2] The IFPRI model is based on a Social Accounting Matrix (SAM), implemented in GAMS and solved as a mixed complementarity program (MCP). The GAMS code is written in a manner that provides the necessary flexibility to change single parts of the model and gives the analyst the possibility to supplement additional economic accounts being of special interest. The main focus of such tools is a numerical implementation of theoretical structures to provide insights about the effects of policy or other changes given theory is maintained, rather than to test it. As a consequence, model results depend on the functional form employed and strongly rely on the set of parameters either specified entirely by the modeller or partly endogenously determined. Though, regardless of being used directly or represented implicitly by a set of parameters, substitution elasticities are key parameters in policy-oriented models and thus crucial in determining the quantitative and sometimes also the qualitative results (McDaniel and Balistreri 2002). However, the standard IFPRI

[2] The flexible SGMF and NQQES supply and demand functions are usually implemented in the partial agricultural equilibrium models developed at IAMO, in particular, flexible bilateral trade systems were introduced in a recent model of Croatia's agricultural and food sector to analyse the effects of the country's various trade agreements on agriculture and food processing.

model distinguishes domestic and foreign goods assuming the well known Armington aggregation, but it does not cover bilateral trade flows by countries of origin and destination. Thus, we suppose 5 illustrative trading partners with different trade shares for single traded goods and identical initial import and export world market prices for all imports from and exports to Mozambique. We specified second stage trade functions applying the CES/CET and the SGMF alternatively. In addition, we replace consumer demand for marketed commodities originally represented by a least expenditure system (LES) by a normalized quadratic-quadratic expenditure system (NQEES). Representatively, for the demand functions derived from the NQQES we describe the procedure to calibrate a set of parameters consistent with economic theory. Section 2.1 briefly describes the functional forms additionally included into the standard model, before the calibration procedure is explained in section 2.2. Finally section 2.3 illustrates some empirical details concerning the advantages of the SGMF for modelling bilateral trade flows.

Figure 1: Major SAM Accounts of the Standard IFPRI CGE Model

Receipts	Expenditures					
	Activities	Commodities	Factors	Domestic Institutions	Rest of World	Totals
Activities		Market sales		Home consumption		Activity income
Commodities	Intermediate Inputs	Transactions costs		Final market demands	Exports	Commodity demand
Factors					Transfers	Factor income
Domestic Institutions	Taxes	Tariffs, Taxes	Income, Taxes	Transfers, Taxes, Savings	Transfers, Savings	Institution income
Rest of World		Imports				Foreign exchange outflow
Totals	Activity spending	Commodity supply	Factor spending	Institution spending	Foreign exchange inflow	

Source: taken from Löfgren, Robinson and Thierfelder 2002

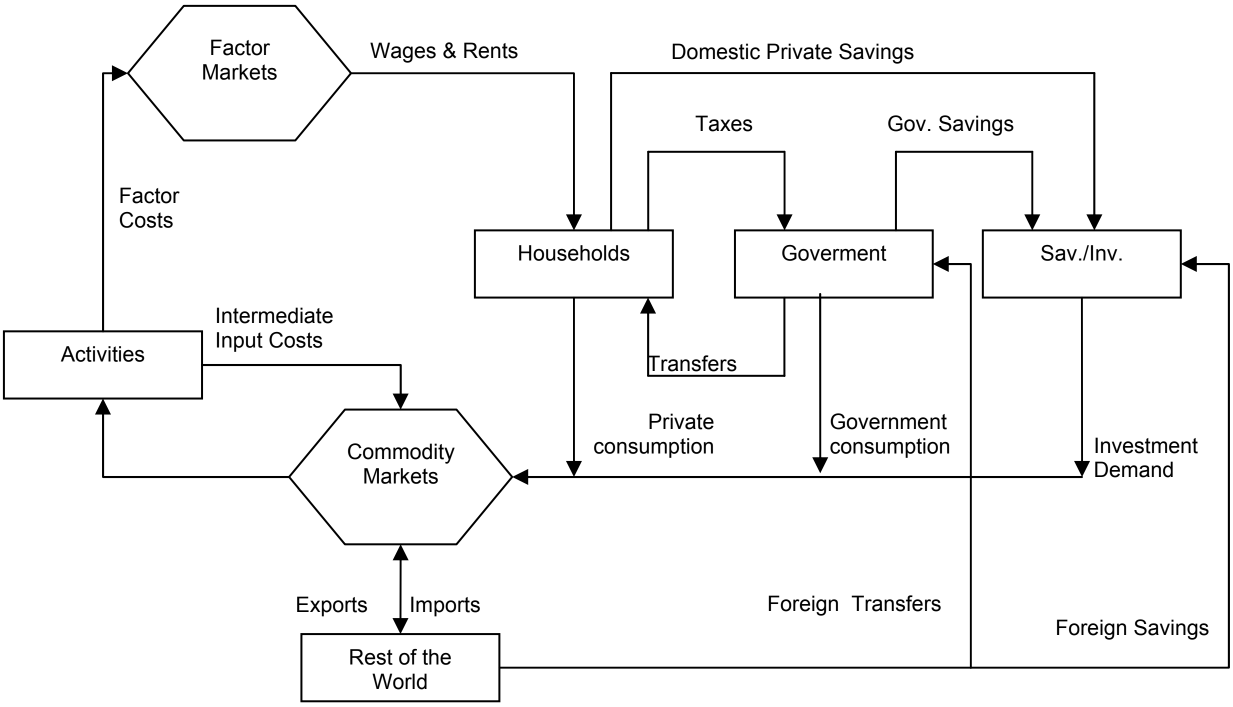
2.1 Functional Forms

In this section we describe those functions which we linked to the standard IFPRI CGE. A detailed presentation of the complete model is included in the documentation published by the Trade and Macroeconomic Division of the International Food Policy Research Institute (Löfgren et al. 2001). Figures 1 and 2 give a general idea of the aggregated SAM and the model structure. The SAM is the data framework that records the flows of payments.[3] The model explains these flows and is calibrated to the SAM. Like the data framework it can be flexibly disaggregated. The major

[3] A detailed documentation to build a conventional SAM is provided in the UN Publication “System of National Accounts” (1993).

accounts depict commodity markets, activities, factors, domestic institutions and the rest of the world. To model bilateral trade flows the account “rest of the world” must be disaggregated.

Figure 2: Stylised Model Structure of the Standard IFPRI CGE Model



Source: taken from Löfgren, Robinson and Thierfelder 2002

2.1.1 Bilateral trade

At the second stage of trading the substitutability among foreign goods is represented. In other words, exporters and importers differentiate between countries of destination and origin, respectively. Equation (1) shows the CET aggregation of bilateral export quantities and prices of a certain country. The Price PE_i is the aggregate export price of a composite commodity and is equal to the marginal revenue of the aggregate export commodity QE_i.

(1) $$QE_i = \Omega_i \cdot \left(\sum_r \upsilon_{i,r} \cdot QER_{i,r}{}^{\tau_i} \right)^{\frac{1}{\tau_i}} \quad and \quad PE_i = \frac{1}{\Omega_i} \cdot \left(\sum_r \upsilon_{i,r}{}^{\frac{1}{1-\tau_i}} \cdot PER_{i,r}{}^{\frac{-\tau_i}{1-\tau_i}} \right)^{\frac{\tau_i - 1}{\tau_i}}$$

where meaning of the variables and parameters is as follows:

i	commodity index
r,s,t	country indices
QE_i	aggregate export quantity of Mozambique of commodity i
$QER_{i,r}$	bilateral export to country r
PE_i	price of the aggregate export of Mozambique of commodity i
$PER_{i,r}$	price of exports to country r
Ω_i	efficiency parameter of the CET
τ_i	exponent of the CET
ν_i	share parameter of the CET

Equation (2) defines the CES aggregation of bilateral import quantities and of the aggregate prices for imports of Mozambique. The aggregate import price of commodity i equals the marginal cost of aggregate import. Bilateral import quantities are given in equation (3)

$$(2) \qquad QM_i = \Psi_i \cdot \left(\sum_r \varphi_{i,r} \cdot QMR_{i,r}^{-\rho_i} \right)^{\frac{-1}{\rho_i}} \; and \; PM_i = \frac{1}{\Psi_i} \cdot \left[\sum_r \varphi_{i,r}^{\frac{1}{\rho_i+1}} \cdot PMR_{i,r}^{\frac{\rho_i}{\rho_i+1}} \right]^{\frac{\rho_i+1}{\rho_i}}$$

$$(3) \qquad QMR_{i,r} = \frac{QM_i}{\Psi_c} \cdot PMR_{i,r}^{\frac{-1}{\rho_i+1}} \cdot \varphi_{i,r}^{\frac{1}{\rho_i+1}} \cdot \left(\sum_s \varphi_s^{\frac{1}{\rho_i+1}} \cdot PMR_{i,s}^{\frac{\rho_i}{\rho_i+1}} \right)^{\frac{1}{\rho_i}}$$

where the meaning of the variables and parameters is as follows:

QM_i	aggregate import quantity of Mozambique of commodity i
$QMR_{i,r}$	bilateral import of Mozambique from country r
PM_i	price of the aggregate import of Mozambique
$PMR_{i,r}$	price of imports of Mozambique from country r
Ψ_i	bilateral "taste" parameter of CES
ρ_i	exponents of the CES
φ_i	share parameter of CES

In all cases in our example, the bilateral exports and imports involve more than two countries. Hence, flexibility of the CES and CET functions representing the second stage of decision making regarding trade is not given. However, at the first stage consistency in theory and flexibility are guaranteed.
Alternatively, for the second stage of decision making on trade, i.e. for bilateral exports and imports we specified flexible bilateral import demand and export supply functions based on employing the Symmetric Generalized McFadden Function (SGMF) to represent the costs of importing the total quantity QM (4). Using Shephards lemma, the first derivatives of the SGMF cost function with respect to the bilateral import prices result in the bilateral import demand equations.[4] Likewise, the derivatives of the SGMF's revenue version with regard to the bilateral export prices yield bilateral export supply equations.[5] According to economic theory, the matrix of second derivatives of the import cost version of the SGMF must be negative semi-definite and those of the export revenue version positive semi-definite. Equation (5) depicts the price of aggregate import, PM, and in (6) the equations for bilateral import quantities, QMR, are shown. The σ_{rs} is a symmetric negative semi-definite matrix of substitution parameters and α_r β_r are predetermined parameters of the McFadden cost function.

$$(4) \qquad C = \sum_r \beta_r PMR_r QM + 0.5 \frac{\sum_r \sum_s \sigma_{rs} PMR_r PMR_s}{\sum_r \alpha_r PMR_r} QM$$

where the meaning of the variables and parameters is as follows:

C	total costs of importing the aggregate quantity QM of a certain commodity
QM	aggregate import of commodity i by Mozambique

[4] For ease of presentation, this equation and the following ones do not include the commodity index. The calibration has to be carried out separately for each traded product considered in the analysis.

[5] Bilateral export supply is obtained as first derivatives of the revenue version of the SGMF; these equations are similar to those for bilateral imports and are not listed. Besides the variables included in these equations the only other difference regards the signs of parameters which must be consistent with economic theory.

PMR_r	price of bilateral imports from country r
α_r	parameter for normalising the cost function
β_r	parameter of the linear response of costs to changes in imports from country r
σ_{rs}	symmetric negative semi-definite matrix of substitution parameters

(5) $$\frac{\partial C}{\partial QM} = PM = \sum_r \beta_r PMR_r + 0.5 \frac{\sum_r \sum_s \sigma_{rs} PMR_r PMR_s}{\sum_r \alpha_r PMR_r}$$

(6)
$$\frac{\partial C}{\partial PMR_r} = QMR_r = \beta_r QM + \frac{\sum_s \sigma_{rs} PMR_s}{\sum_s \alpha_s PMR_s} QM - 0.5 \frac{\alpha_r \sum_s \sum_t \sigma_{st} PMR_s PMR_t}{\left(\sum_s \alpha_s PMR_s\right)^2} QM$$

where the meaning of the variables and parameters is as follows:

PM	price of aggregate import of a certain commodity by Mozambique
QMR_r	bilateral imports from country r of a certain commodity

and all other variables and all parameters as described for equation (4).

While the focus of the analysis relates to the impact of using different functional forms for representing the bilateral trade relations a single country has with its trading partners little attention is placed on the effects trade has on the latter countries. Hence, their production and demand conditions are not explicitly modelled. Rather, they are implicitly represented in their export supply and import demand functions. By using linear equations for both of them they are kept quite simple. For any specific commodity, equation (7) shows the export function and equation (8) the one for imports.

(7) $$QE_{i,r} = ce_{i,r} + se_{i,r}\, pwe_{i,r}\left(1 + te_{i,r}\right)$$

(8) $$QM_{i,r} = cm_{i,r} + sm_{i,r}\, pwm_{i,r}\left(1 + tm_{i,r}\right)$$

where the meaning of the variables is as follows:

QE_{ir}	quantity exported by a trading partner r to the destination Mozambique
pwe_{ir}	fob world market price for country r relevant for exporting to Mozambique
te_{ir}	subsidy of country r paid for exporting to Mozambique
QM_{ir}	quantity imported from Mozambique by country r
pwm_{ir}	cif world market price for country r relevant for importing from Mozambique
tm_{ir}	tariff of country r levied for imports originating from Mozambique

ce_{ir} , se_{ir} , cm_{ir} , sm_{ir} : parameters of linear export supply and import demand functions.

2.1.2 Household consumption of marketed goods

On the demand side we replace the LES explaining household consumption in the standard IFPRI model by a flexible functional form for modelling household consumption for marketed commodities. Demand equations for home-produced goods were not modified. The flexible demand functions of the consumption block are obtained by applying Roy's identity to the Normalised Quadratic-Quadratic Expenditure System (NQQES) developed by Ryan and Wales (1996, 1999). The NQQES has some attractive features for depicting the preferences of consumers in a mathematical model for demand analysis. The first one relates also to second-order flexibility. Again this implies that an arbitrary but theoretically consistent set of values describing the level of utility, the quantities consumed as well as the first derivatives of the latter with regard to prices and income can be depicted by this function at a reference point. In addition, this demand system exhibits a third order flexibility since it has sufficient free parameters to permit the second derivatives of these goods with respect to income being arbitrary and nonzero at that point (Lau 1986). This property has relevance for analysing consumption under uncertainty. The third derivative of the utility function plays a critical role in comparative static's and a functional form with even third order flexibility should be chosen, because there is the need to know not only the level of the elasticity but also its rate of change. The second feature of the NQQES concerns derived Engel curves which are quadratic in income with linearity as a special case.[6]

Consumption functions derived from the NQQES are consistent with the four fundamental properties of demand theory. These functions add up and are homogenous of degree zero in prices and total expenditure when a linear budget constraint is specified. Their compensated price responses are symmetric and form a negative semi-definite matrix which is the consequence of consistent preferences and the concavity of the expenditure function (Deaton and Muellbauer 1992). The system of consumption functions derived from the NQQES is given in equation (9) below using some auxiliary functions f, g, h and their first partial derivatives with respect to prices described by f_i, g_i, and h_i for making the structure more obvious as shown in (10).

$$(9) \qquad qd_i(pd, y) = \frac{h_i}{g}(y-f)^2 + \frac{g_i}{g}(y-f) + f_i$$

$$g = \sum_k pd_k b_k + \tfrac{1}{2}\left(\frac{\sum_{k,l} B_{kl}\, pd_k\, pd_l}{\sum_k \alpha_k\, pd_k}\right) \quad g_i = \frac{\partial g}{\partial pd_i} = b_i + \frac{\sum_k B_{ik}\, pd_k}{\sum_k \alpha_k\, pd_k} - \tfrac{1}{2}\frac{\alpha_i \sum_{k,l} B_{kl}\, pd_k\, pd_l}{\left(\sum_k \alpha_k\, pd_k\right)^2}$$

$$(10) \qquad f = \sum_k pd_k d_k \quad f_i = \frac{\partial f}{\partial pd_i} = d_i$$

$$h = \sum_k a_k \log(pd_k), \ \sum_k a_k = 0 \quad h_i = \frac{\partial h}{\partial pd_i} = \frac{a_i}{pd_i}$$

where the meaning of the variables and parameters is as follows:

qd_i	consumption of good i
pd_i	consumer price of good i
y	total consumption expenditure
$a_i, b_i, d_i, \alpha_i, B_i$	parameters of the NQQES
i,j,k,l	indexes over the set of commodities for consumption

[6] Engel curves are derived by assigning the utility-maximizing commodity bundle to each point on the income expansion path, holding prices constant. While linear Engel curves, very often employed, assume a proportional increase in demand for each consumption item, non-linear Engel curves are more in line with empirical evidence, which suggests that growing income modifies consumption patterns and expenditure shares disproportionally and thus, functional forms involving non-linear Engel curves are more suitable for empirical analysis.

2.2 Calibration of model parameters

Flexible functional forms of the second-order type can be calibrated with relatively little effort. This is briefly explained for the consumer demand qd derived from the NQQES and defined in equation (9) using the auxiliary functions f, h and g and their derivatives h_i, f_i and g_i with respect to price p and consumption expenditure y which are shown in equation (10). A set of constraints for normalisation is listed in equation (11). Again, B, a, b and d are parameters of the NQQES.

(11) $\sum a_i = 0,\ \ \sum b_i = 1,\ \sum d_i pd_i = 0,\ \sum B_{ij} pd_i = 0,\ \ with\ B_{ij} = B_{ji}$

In an initial step, a set of realistic demand elasticities with regard to prices, ε_{ij}^0, and income, ε_i^{y0}, for the country of interest is to be determined. In general, the initial price and income elasticities violate theoretical conditions and must be adjusted in order to comply with them; i.e. symmetry, homogeneity, the budget and the curvature conditions. For obtaining in a first step theoretically consistent elasticities, ε_{ij} and ε_i^y, function Z^0 shown in (12) is solved. Based on weights, w_{ij}^0 and w_i^{y0}, this minimises the squared deviation between the elasticities initially set, ε_{ij}^0 and ε_i^{y0}, and those to be found for meeting the requirements of demand theory listed in equations (13) and (14).[7] Adherence to symmetry, homogeneity, adding up and the budget constraint by the NQQES is assured if it meets the conditions in (11). However, concavity of the substitution matrix is not inherent and must be imposed. A procedure for achieving this, shown in equation (13), was suggested by Diewert and Wales (1987) and Ryan and Wales (1999). We impose the concavity by using the Cholesky decomposition for the Slutsky matrix, where L is a lower triangular matrix and L^T its transpose.

(12) $Z^0(\varepsilon,\varepsilon^y) = \min_{\varepsilon,\varepsilon^y}\left[\sum_{i,j} w_{ij}^0\ (\varepsilon_{ij} - \varepsilon_{ij}^0)^2 + \sum_i w_i^{y0}\ (\varepsilon_i^y - \varepsilon_i^{y0})^2\right]$

(13) $Symmetry: S_{ij} = \varepsilon_{ij}\frac{qd_i}{pd_j} + \varepsilon_i^y\frac{qd_i qd_j}{y} = S_{ji},\ \ Curvature: S = -LL^T$

(14) $Adding\,up: \sum_i \varepsilon_i^y \frac{qd_i pd_i}{y} = 1,\ \ Homogeneity: \sum_j \varepsilon_{ij} = -\varepsilon_i^y,\ Budget: \sum_i qd_i pd_i = y$

In a second step, values for the parameters of the NQQES (a, b, d, B) are determined by solving again an objective function, Z^F in equation (15), which minimises the squared deviation of the elasticities to be finally used (ε_{ij}^F and ε_i^{yF}) and those which were arrived at in the first step of calibration given the observed consumer demand defined in equation (9),subject to the conditions in (11) and again subject to the constraints given in equations (13) and (14).

(15) $Z^F(a,b,d,B) = \min_{a,b,d,B}\left[\sum_{i,j} w_{ij}(\varepsilon_{ij} - \varepsilon_{ij}^F)^2 + \sum_i w_i^y(\varepsilon_i^y - \varepsilon_i^{yF})^2\right]$

Because the NQQES in its restricted form is flexible the value of Z^F must be zero since it entails minimizing the deviation of second order effects of a theoretical consistent set of values and those of the flexible function. In other words, the elasticities ε_{ij} and ε_i^y obtained in the first step of the procedure are perfectly matched by the elasticities ε_{ij}^F and ε_i^{yF}, respectively. Hence, the important step in the

[7] During step 1 additional constraints like bounds may be placed on single elasticities to not allow implausible values. S_{ij} is the Slutsky substitution matrix and L,L^T is a lower triangular matrix and L^T its transpose.

calibration procedure is step one; i.e. to find a set of elasticities which fulfils all theoretical conditions.

The same calibration procedure is applied to fit the parameters of the bilateral import and export functions, all derived from the SGMF revenue and cost functions respectively. Despite the deficiencies of calibrated models mentioned by their critics, calibration has become a widely used technique of empirical economic investigation and we developed a procedure which leads towards a better practice in the calibration process requiring the modeller to verify at least the consistency between model parameters specified and economic theory.

Besides these theoretical and mathematical-statistical considerations there is another worry creeping into demand analysis. This concerns the range of variables considered in investigating demand especially for policy recommendations. It is more and more recognised that there is a whole set of variables affecting welfare which is kept out from being included in the analysis. This concerns by and large income disparity and environmental aspects. Leaving aside these issues and returning to the mathematical properties it is worth mentioning that given the aim of demand analysis is to find the correct set of demand functions, complete with well-defined and precisely estimated elasticities, one is some way from perfection as Deaton and Muellbauer (1992) point out.

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2.3 Empirical details

To compare both functional forms specified for bilateral trade flows and to show the properties of the SGMF, a set of own- and cross-price elasticities for the bilateral CES and CET functions was derived in a first step. This has been done by computing the derivatives of bilateral trade quantities with respect to their respective bilateral trade prices, multiplied by the ratio of initial prices and quantities, given for bilateral imports in (16), where the indizes r and s refer to countries and i to the imported commodity.

(16) $$\varepsilon_{i,rs}^{import} = \frac{\partial QMR_{i,r}}{\partial PMR_{i,s}} \cdot \frac{PMR_{i,s}}{QMR_{i,r}}$$

These (point) elasticities are taken as initial values for calibrating the parameters of the MacFadden functions by repeating the procedure described in section 2.2.

The CES trade functions fulfil all required theoretical conditions and, therefore, yield a set of consistent elasticities. Being flexible of 2^{nd} degree the SGMF function can implicitly represent precisely the same elasticities as the CES specification. To keep the exposition relatively simple, values for only one imported good are reported. The commodity industrial goods (CIND) was chosen for this purpose. In the database for Mozambique aggregate imports of industrial goods account for about 36% of GDP and cover 74% of the total import value. We assume country R1 delivers 50 % of total imports of industrial goods, country R2 delivers 25 %, R3 12.5 %, R4 10 % whereas the rest of 2.5 % originates from country R5. Table 1 shows the computed price elasticities for both functional forms. On the left half, it depicts the CES price elasticities, given the substitution parameter for imports (ρ^{import}) is –0.5, the calibrated SGMF price elasticities are depicted in the right half of that table.

Table 1: CES price elasticities of bilateral imports[1,2] given the substitution parameter ρ is –0.5 (left half) and computed elasticities of the same type using the SGMF (right half)

	R1	R2	R3	R4	R5	R1	R2	R3	R4	R5
R1	**-1,0000**	0,5000	0,2500	0,2000	0,0500	**-1,0000**	0,5000	0,2500	0,2000	0,0500
R2	1,0000	**-1,5000**	0,2500	0,2000	0,0500	1,0000	**-1,5000**	0,2500	0,2000	0,0500
R3	1,0000	0,5000	**-1,7500**	0,2000	0,0500	1,0000	0,5000	**-1,7500**	0,2000	0,0500
R4	1,0000	0,5000	0,2500	**-1,8000**	0,0500	1,0000	0,5000	0,2500	**-1,8000**	0,0500
R5	1,0000	0,5000	0,2500	0,2000	**-1,9500**	1,0000	0,5000	0,2500	0,2000	**-1,9500**

1: The numbers refer to bilateral imports of industrial goods by Mozambique
2: The column and row indizes R1 to R5 refer tothe trading partners of the country analysed
Source: own calculations

The numbers are arranged in such a way that columns represent the country specific price changes and rows the countries' quantity responses for the commodity industrial goods. The CES specification implies the same cross price elasticities for all countries; i.e. a price change in one country triggers percentage wise the same response in all other countries. This is a very strong (maintained) hypothesis of this functional form a modeller should be aware of when implementing a second stage Armington approach for modelling bilateral trade flows. Own price elasticities are to be interpreted as the response of imports by Mozambique from the corresponding country due to a change in the price of the country concerned. For example, the value of the own price elasticity for the country R1 is –1,00. Hence, a one percentage decline in the countries price of industrial goods to be shipped to Mozambique will lead to an increase in Mozambique's imports from this country by 1,00 %. In addition, imports from other countries change too when the price of R1 varies. Since all these cross price elasticities have a positive sign imports from these countries respond in the opposite way to an price change of R1. This can be seen from the column headed "R1" in Table 1. The right half of this table shows the exact and implicit representation of these elasticities after parameterisation of the SGMF. However, just the structure of trade flows supposed here, exhibits one shortcoming of the CES; irrespectively of how important a trading partner is, if in one country the price of a certain product to be exported to Mozambique changes the relative quantity effect will be alike in all other countries also shipping to that destination. As mentioned above, for the aggregate commodity "industrial goods" both the bilateral trade system derived from the SGMF and the CES adheres to the homogeneity, symmetry and curvature conditions. The latter condition implies non-positive values for own price elasticities and the first leads to having at least one cross price elasticity to be non-negative. Furthermore, since all cross price elasticities of the CES have the same value, if a single one needs to be positive all others must be too. In other words, the relative import decline from a country after this increased its export price is offset by expanded buying from all other countries. Complementarity between two or more countries of origin cannot be depicted using the CES but can be modelled applying the SGMF.

To demonstrate the difference in import quantity responses to price changes between the CES and the SGMF functions is difficult because the latter always can approximate the price elasticities of the former. Therefore, what we want to point out in the following two important characteristics of the SGMF. These are its ability to depict complementarity in imports and secondly to account for differences in the relative responses to a price change of a single trading partner. In the right half of Table 2 we show the SGMF elasticities calibrated, if only one initial elasticity, here the own-price elasticity of R1 is modified. We suppose a value of –5.00 which is fixed during the calibration procedure without imposing any further constraint on other values but maintaining again all theoretical conditions. As is to be expected the SGMF yields a set of new elasticities which show significant deviations from the initial ones obtained from the CES given the value for the substitution parameter is -0.5. After changing a single elasticity theoretical conditions are not anymore met if the remaining ones of the initial set are not adjusted either. Hence, the remaining ones have to be altered keeping the value of the changed one fixed.[8] Likewise the substitution parameter of the CES had to be changed if this function is to depict an own price elasticity of imports of –5.00 for R1. In our example a value of –0.9 for the substitution parameter corresponds to this assumed elasticity. Furthermore, all the other price elasticities derived from the CES alter as well given the new value of the substitution parameter. They are depicted in the left half of Table 2.

Table 2: CES price elasticities given the substitution parameter of bilateral imports[1,2] ρ is –0.9 (left half) and computed SGMF elasticities of the same type (right half) assuming a fixed own-price elasticity of –5,00 for R1

	R1	R2	R3	R4	R5	R1	R2	R3	R4	R5
	-5,0000	2,5000	1,2500	1,0000	0,2500	**-5,0000**	3,1418	1,0310	0,7250	0,1021
R2	5,0000	**-7,5000**	1,2500	1,0000	0,2500	6,2837	**-6,2842**	0,0003	0,0002	0,0001
R3	5, 0000	2,5000	**-8,7500**	1,0000	0,2500	4,1242	0,0005	**-4,1249**	0,0002	0,0001
R4	5,0000	2,5000	1,2500	**-9,0000**	0,2500	3,6249	0,0005	0,0003	**-3,6257**	0,0001
R5	5,0000	2,5000	1,2500	1,0000	**-9,7500**	2,0427	0,0005	0,0003	0,0002	**-2,0436**

1: The numbers refer to bilateral imports of industrial goods by Mozambique
2: The column and row indizes R1 to R5 refer tothe trading partners of the country analysed
Source: own calculations

Supposing a complementary relationship between imports from country R1 and R2, the result from calibration depends on the constraints set for single initial elasticities. First we fixed again the cross price elasticity of R1, the outcome is depicted in the left half of Table 3. In a second run all initial values are free to vary only the constraint on the sign must be met. Results of this calibration process are given in the right half of that table.

Table 3: SGMF price elasticities of bilateral imports[1,2] assuming complementarity of imports from R1 and R2 if the own-price elasticity of R1 is fixed (left half) and not fixed (right half)

	R1	R2	R3	R4	R5	R1	R2	R3	R4	R5
R1	**-5,0000**	-0,0050	2,8966	1,9321	0,1762	**-0,4707**	-0,3278	0,4212	0,3148	0,0625
R2	-0,0100	**-0,0015**	0,0003	0,0002	0,0111	-0,6555	**-0,4567**	0,5907	0,4410	0,0806
R3	11,5865	0,0005	**-11,5873**	0,0002	0,0001	1,6846	1,1814	**-2,8663**	0,0002	0,0001
R4	9,6606	0,0005	0,0003	**-9,6614**	0,0001	1,5742	1,1024	0,0003	**-2,6769**	0,0001
R5	3,5249	0,1105	0,0003	0,0002	**-3,6359**	1,2493	0,8055	0,0003	0,0002	**-2,0552**

1: The numbers refer to bilateral imports of industrial goods by Mozambique
2: The column and row indizes R1 to R5 refer tothe trading partners of the country analysed
Source: own calculations

However, the outcome of calibration is obtained by minimizing the squared deviations of initial values, thus the modeller is not released from setting plausible default values which means that he should have an idea of probable substitution possibilities between imports stemming from different countries of origin and exports delivered to various countries of destinations.

[8] Putting particular weighting factors on single elasticities will change the whole set again, thus the modeller has the possibility to influence the outcome of the calibration procedure.

Conclusion

In this paper the theoretical framework of computable trade models for policy analysis is discussed. Advances in both theory and methodology suggest using improved functional forms in models for policy analysis. It is shown that frequently applied functional forms such as the Cobb-Douglas and CES/CET maintain strong hypothesises. The question is by how much dominate these functional forms the results. Recent literature on policy modelling pays much attention to two main aspects; the first concerns the dependence of model results on the model structure and functional forms, the second underlines the dependence of simulation results on model parameters. Mainly the handling of the latter in calibrated models is criticised. It is suggested that further investigation regarding structural and parametric sensitivity analysis could shed more light on these issues. Using the functional forms employed in the IAMO partial equilibrium models for the agricultural sector the calibration procedure applied to obtain model parameters consistent with economic theory is explained. It could be shown for the numerical example carried out with the standard IFPRI model, namely for modelling imports of industrial goods that the CES also keeps these theoretical properties but at the same time it is not flexible to distinguish elasticities of substitution across different pairs of goods, furthermore the CES cannot consider complementary relations in bilateral trade flows. In this respect the implementation of flexible import demand and export supply functions derived from the Symmetric Generalized McFadden cost and revenue functions is promising and will be further investigated.

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